How to find magnetic nulls and reconstruct field topology with MMS data?

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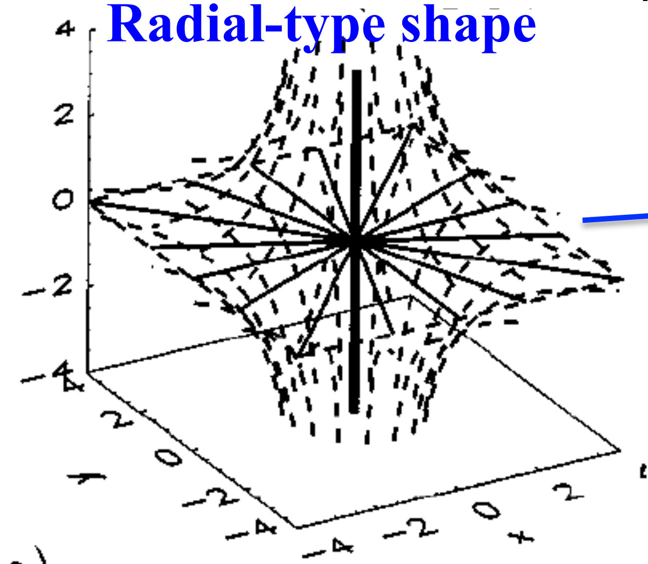
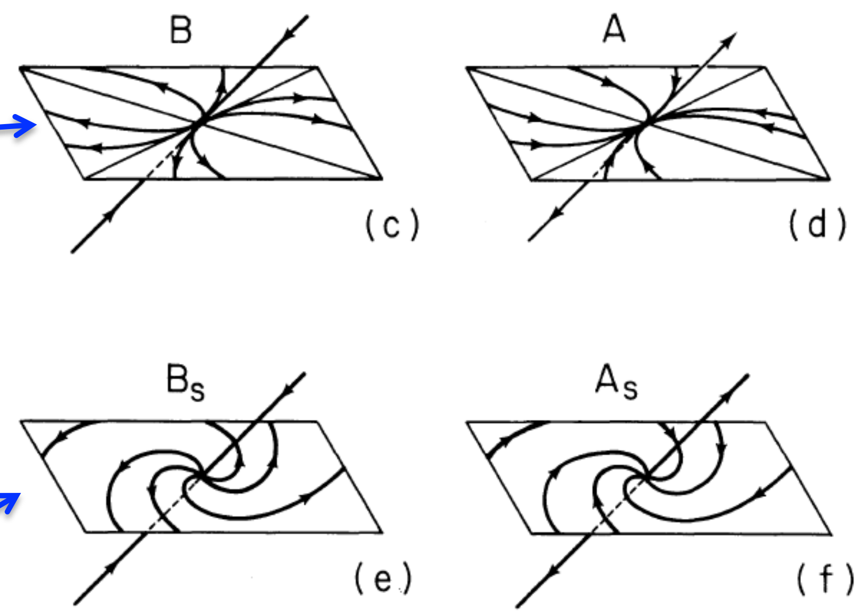
What is magnetic null?

|B|=0 Topology

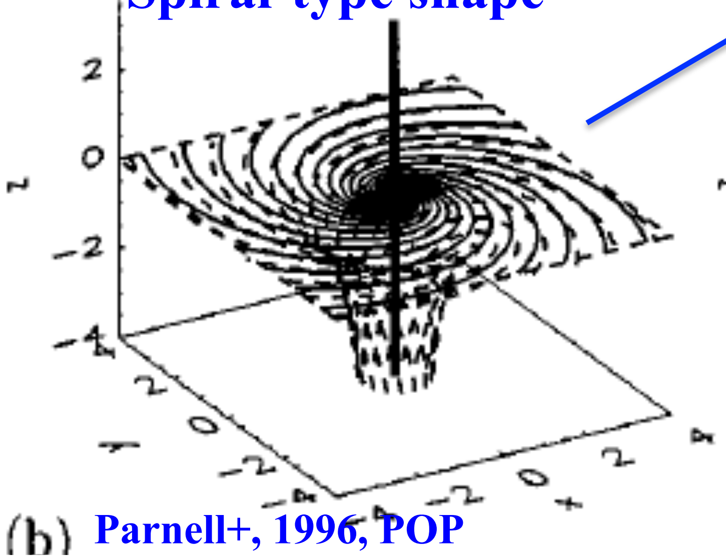
Radial-type shape



3D null



Spiral-type shape

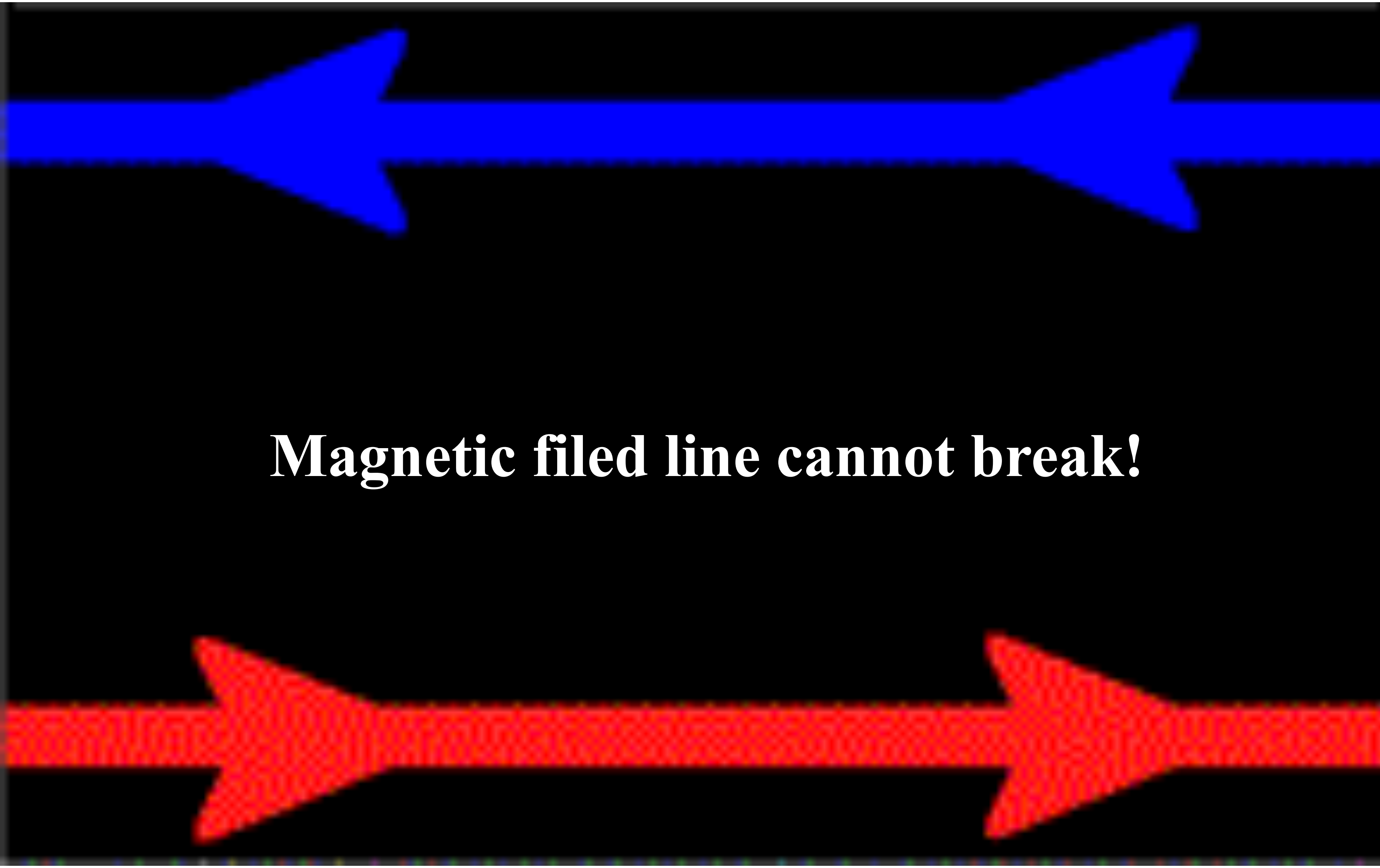


Parnell+, 1996, POP

2D null

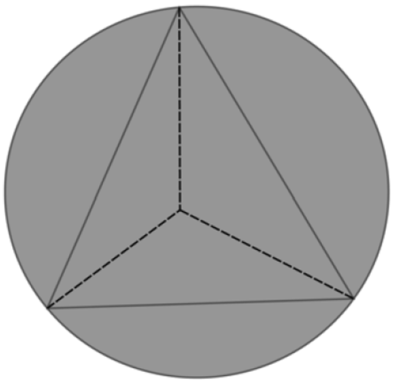
Lau+Finn, 1990, APJ

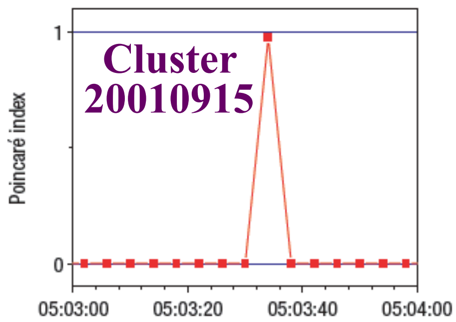
Why study magnetic null?



How to find magnetic null?

Poincare index

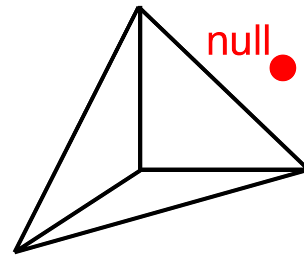




Xiao+ 2006NatPys; He+ 2008JGR
Hu+ 2008SciChina; Deng+ 2009JGI
Dunlop+ 2009PRL; Wendel+ 2013J

$$I(x,y,z) = \frac{1}{4\pi}\iint_S \frac{\cos(\boldsymbol{r},\boldsymbol{n})}{r^2}\mathrm{d}S$$

First-order Taylor expansion (FOTE)

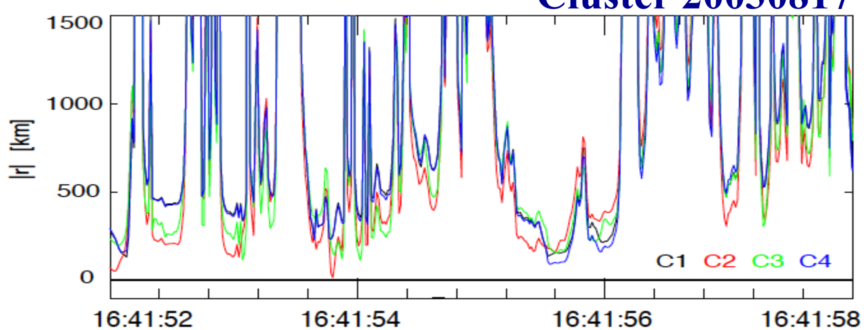


Fu+ 2015JGR

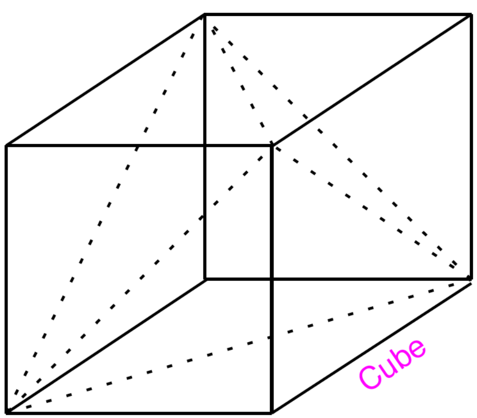
$$\mathbf{B}(r) = \nabla\mathbf{B}\cdot\mathbf{r}$$

$$\nabla\mathbf{B} = \begin{bmatrix} \Delta B_{x2} & \Delta B_{x3} & \Delta B_{x4} \\ \Delta B_{y2} & \Delta B_{y3} & \Delta B_{y4} \\ \Delta B_{z2} & \Delta B_{z3} & \Delta B_{z4} \end{bmatrix} \begin{bmatrix} \Delta x_2 & \Delta x_3 & \Delta x_4 \\ \Delta y_2 & \Delta y_3 & \Delta y_4 \\ \Delta z_2 & \Delta z_3 & \Delta z_4 \end{bmatrix}^{-1}$$

Cluster 20030817



Trilinear method



Haynes+Parnell 2007PoP

$a = f_{000}$,

$b = f_{100} - f_{000}$,

$c = f_{010} - f_{000}$,

$d = f_{110} - f_{100} - f_{010} + f_{000}$,

$e = f_{001} - f_{000}$,

$f = f_{101} - f_{100} - f_{001} + f_{000}$,

$g = f_{011} - f_{010} - f_{001} + f_{000}$,

$h = f_{111} - f_{110} - f_{101} - f_{011} +$

$$f(x,y,z) = a + bx + cy + dxy + ez + fxz + gyz + hxyz.$$

Summary

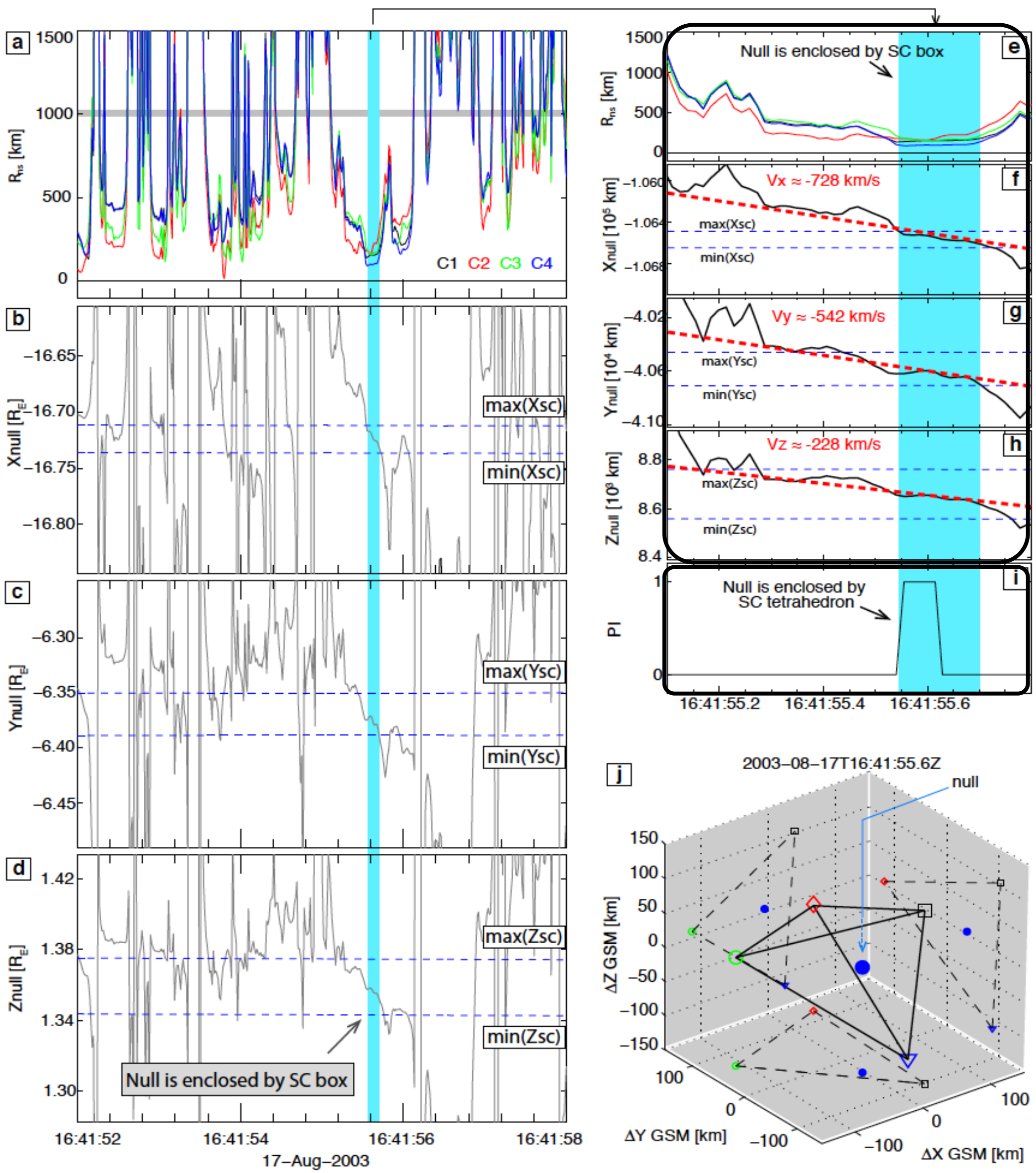
All three methods assume the field is linear

Poincare: detect whether a null is enclosed

Trilinear: eight-point data (only in simulation)

FOTE: detect a null both inside and outside

FOTE method VS PI method



Cluster measurements on 20030817

Null position from FOTE method

Results from PI method

min (Xsc), max(Xsc), min (Ysc), max(Ysc), min (Zsc), max(Zsc) form a spacecraft box

FOTE results: Null is inside spacecraft box	PI results: Null is inside sc tetrahedron

Consistent !

Advantage of FOTE 1: data resolution

null-type identification

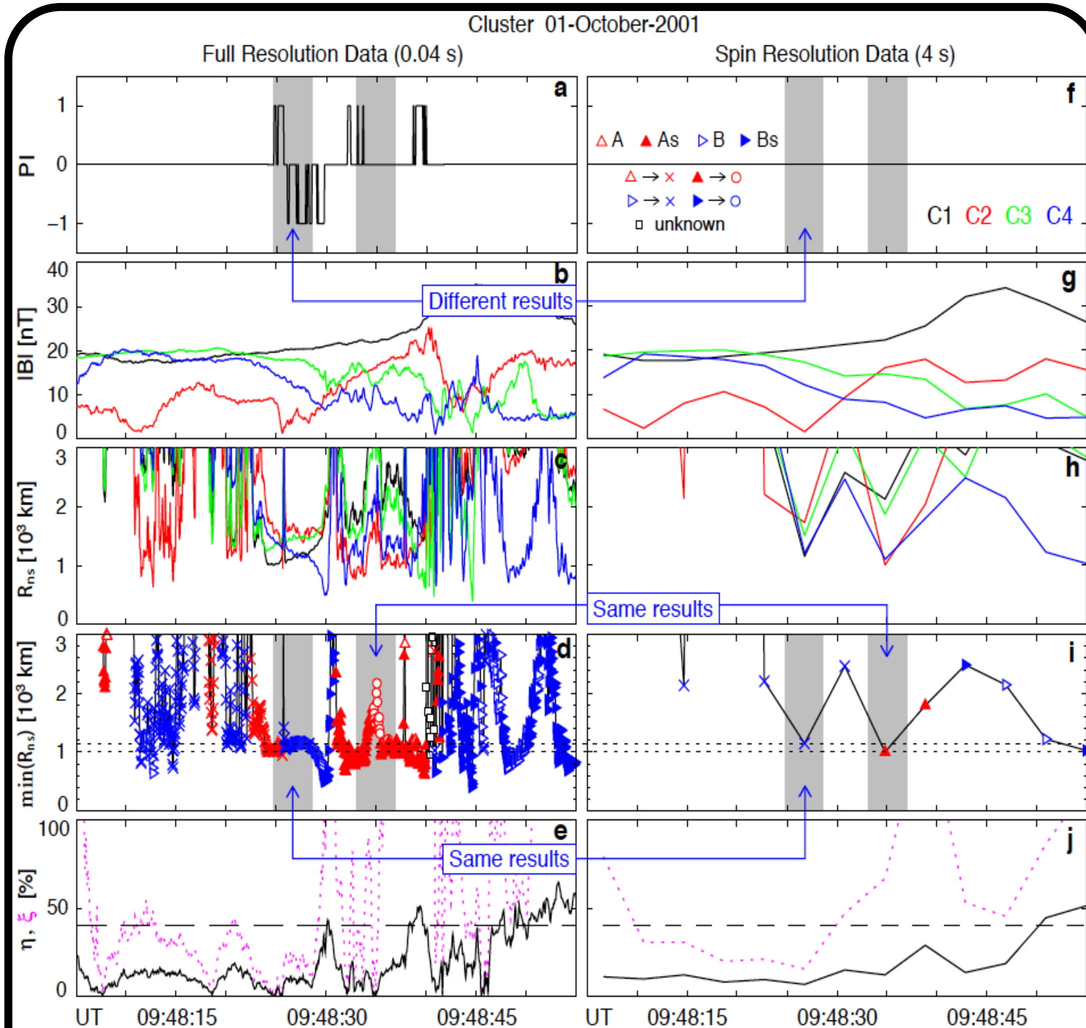
λ_1	λ_2	λ_3	Null type	Labeling system	Dimension
0	$+\lambda$	$-\lambda$	X	×	2D
0	$+i\lambda$	$-i\lambda$	O	○	
$-\lambda_1$	$-\lambda_2$	$+(\lambda_1+\lambda_2)$	A	△	3D
$+\lambda_1$	$+\lambda_2$	$-(\lambda_1+\lambda_2)$	B	▷	
$+\lambda_1$	$-\lambda_1/2+i\lambda_2$	$-\lambda_1/2-i\lambda_2$	As	▲	
$-\lambda_1$	$\lambda_1/2+i\lambda_2$	$\lambda_1/2-i\lambda_2$	Bs	▶	

PI method:
Find nulls only in high-res data

FOTE method:
Find nulls in both data sets

Nulls should not disappear because of non-physical reasons

Conclusion:
FOTE is not affected by data resolution

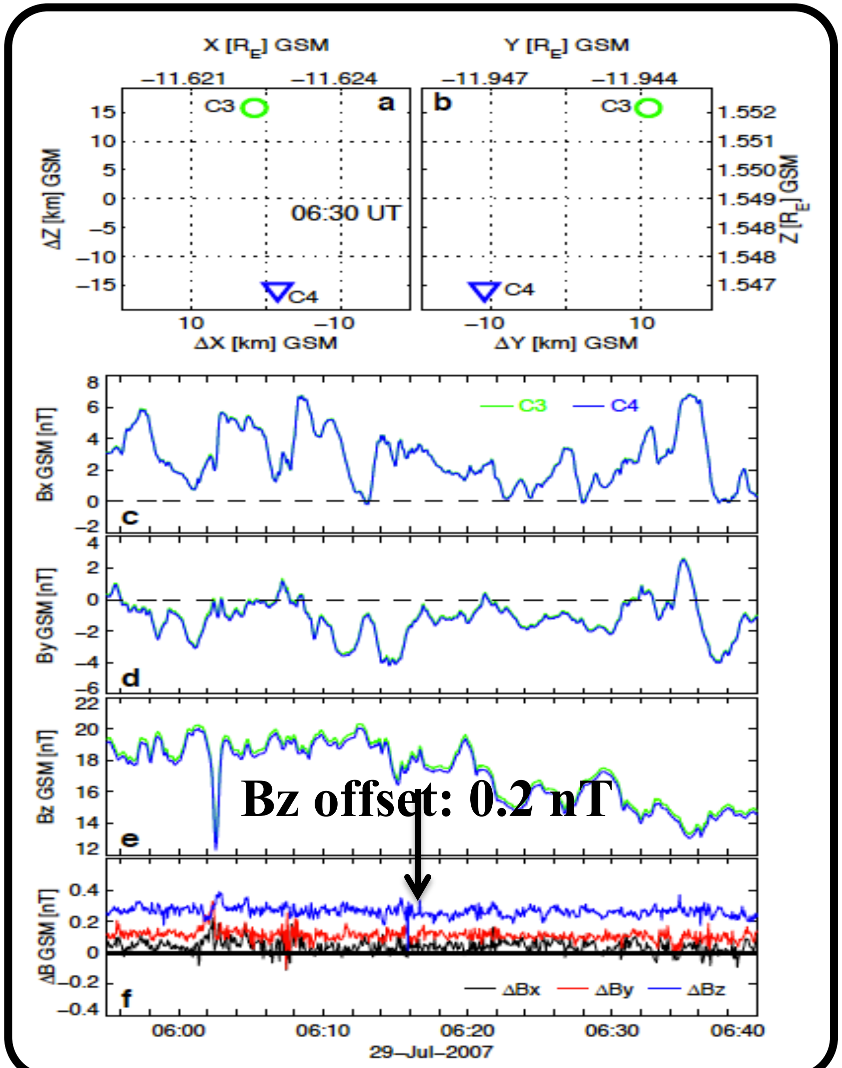


$$\eta \equiv |\nabla \cdot \mathbf{B}| / |\nabla \times \mathbf{B}| \qquad \xi \equiv |(\lambda_1 + \lambda_2 + \lambda_3)| / |\lambda|_{max}$$

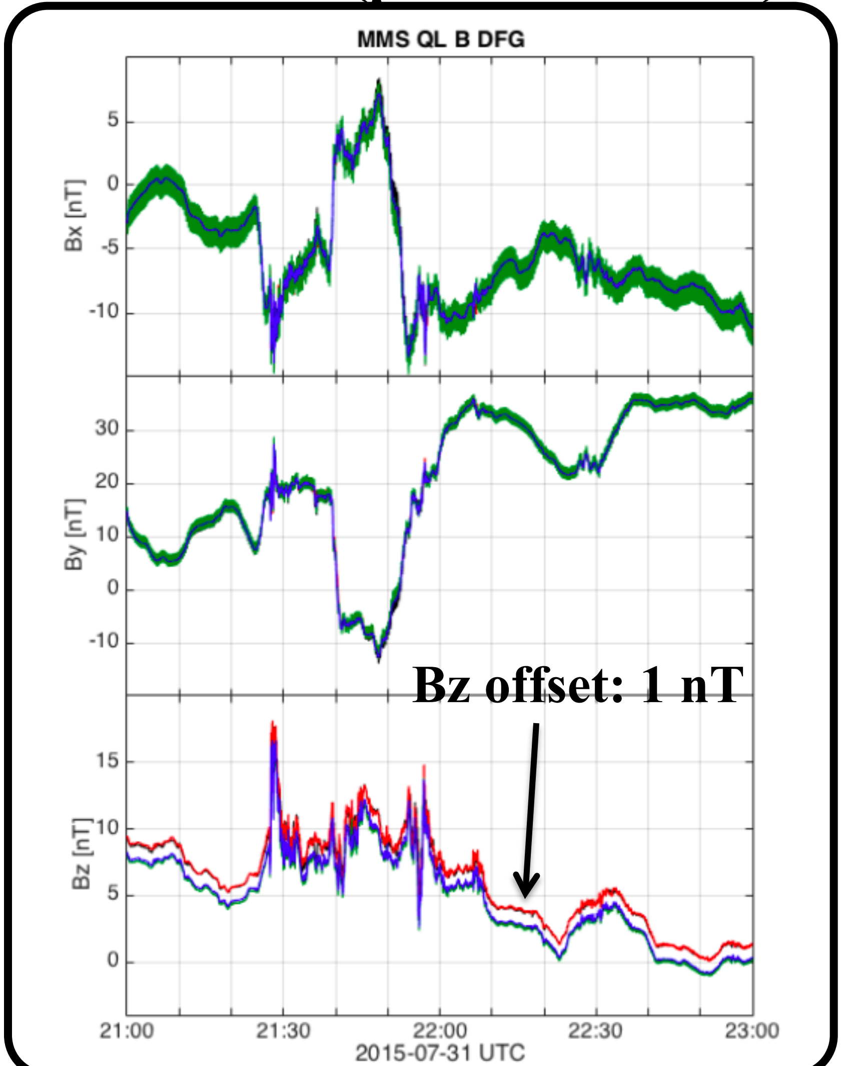
Advantage of FOTE 2: Bz offset

What is Bz offset?

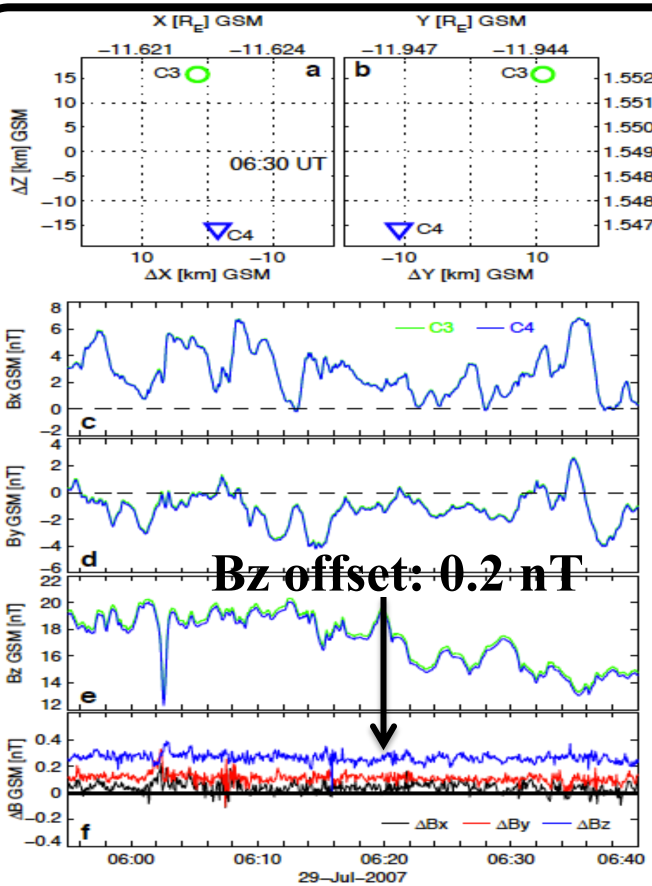
Cluster data



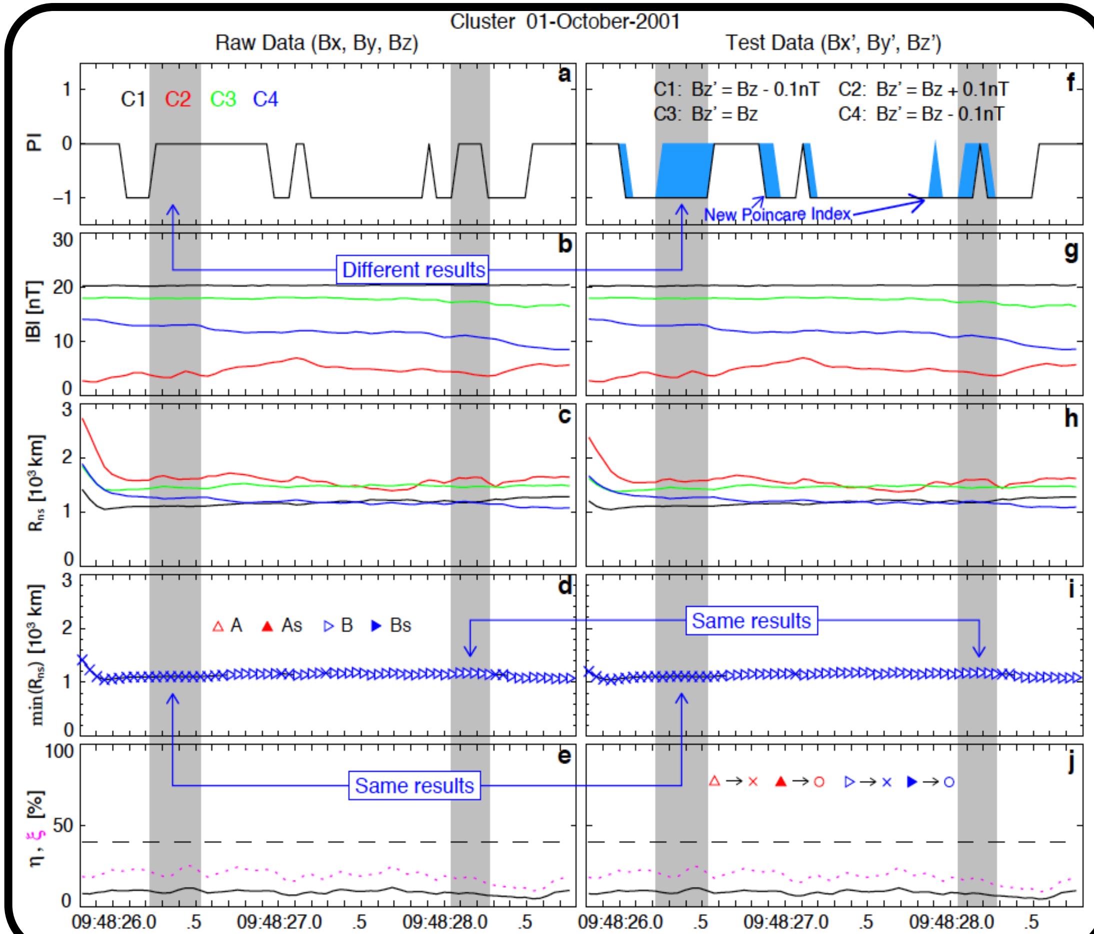
MMS data (pre-calibration)



Advantage of FOTE 2: Bz offset



Very common for MMS mission



Conclusion:

FOTE is not affected by Bz offset

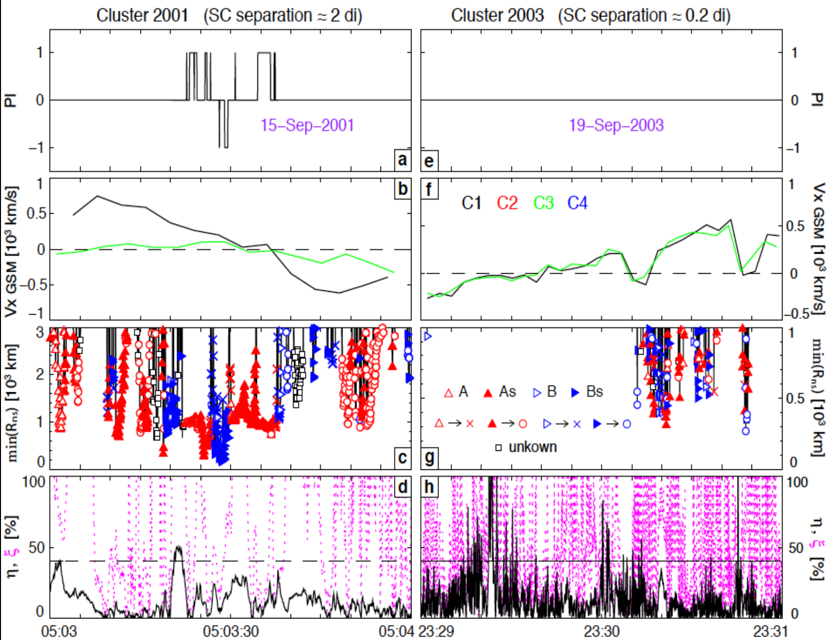
PI method: Find new nulls when adding offset (0.1 nT)

FOTE method: Same results when adding Bz offset

The result should not change due to 0.1 nT offset

Advantage of FOTE 3: SC separation

Cluster observation



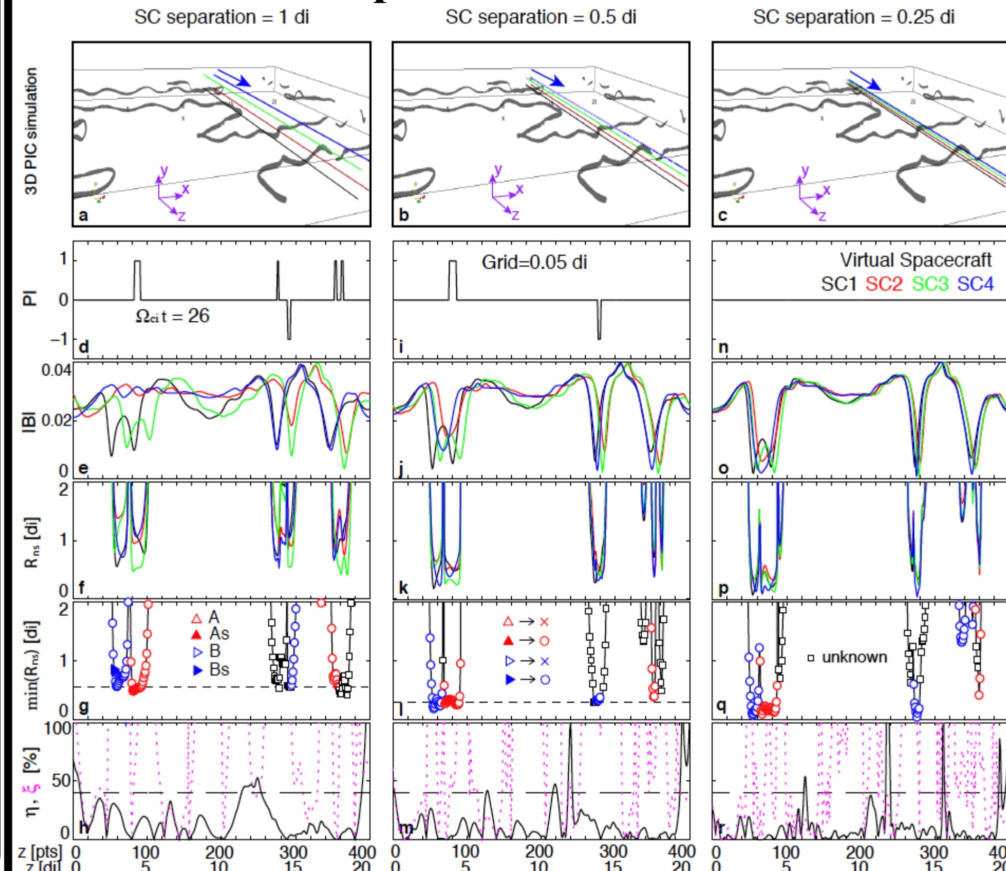
Cluster2001 data
(separation=2 di)

Cluster2003 data
(separation=0.2 di)

SC separation=2 di: FOTE find null; PI find null

SC separation=0.2di: FOTE find null; PI find no null

3D implicit PIC simulation



SC separation=1 di: FOTE find null; PI find null

SC separation=0.5 di: FOTE find null; PI find null

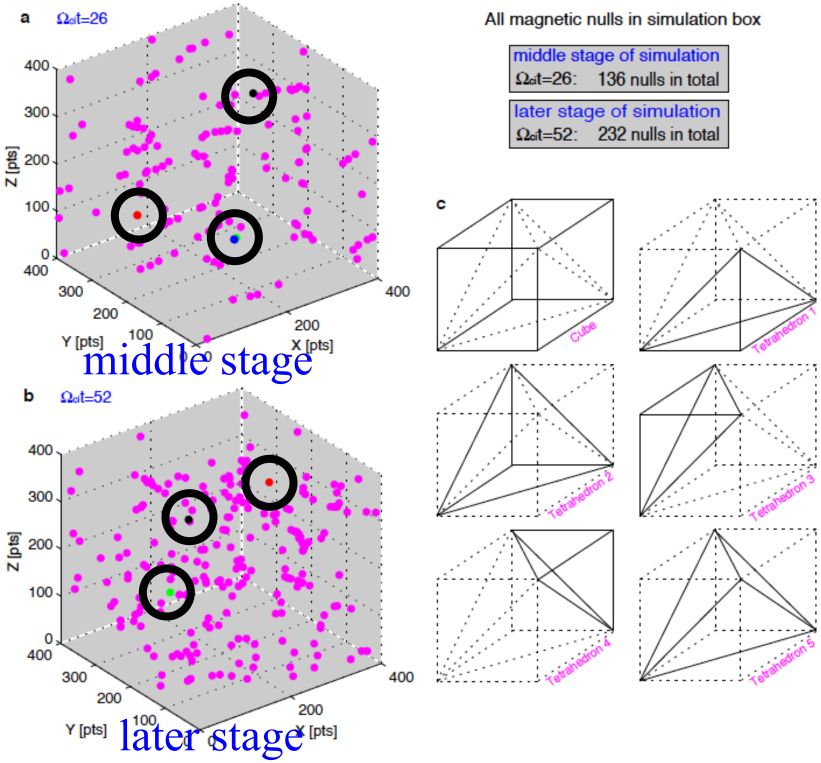
SC separation=0.25 di: FOTE find null; PI find no null

Conclusion:

FOTE is not affected by SC separation

How accurate is FOTE? (finding null)

Data for analysis



3D PIC simulation [Olshevsky+, 2013PRL]

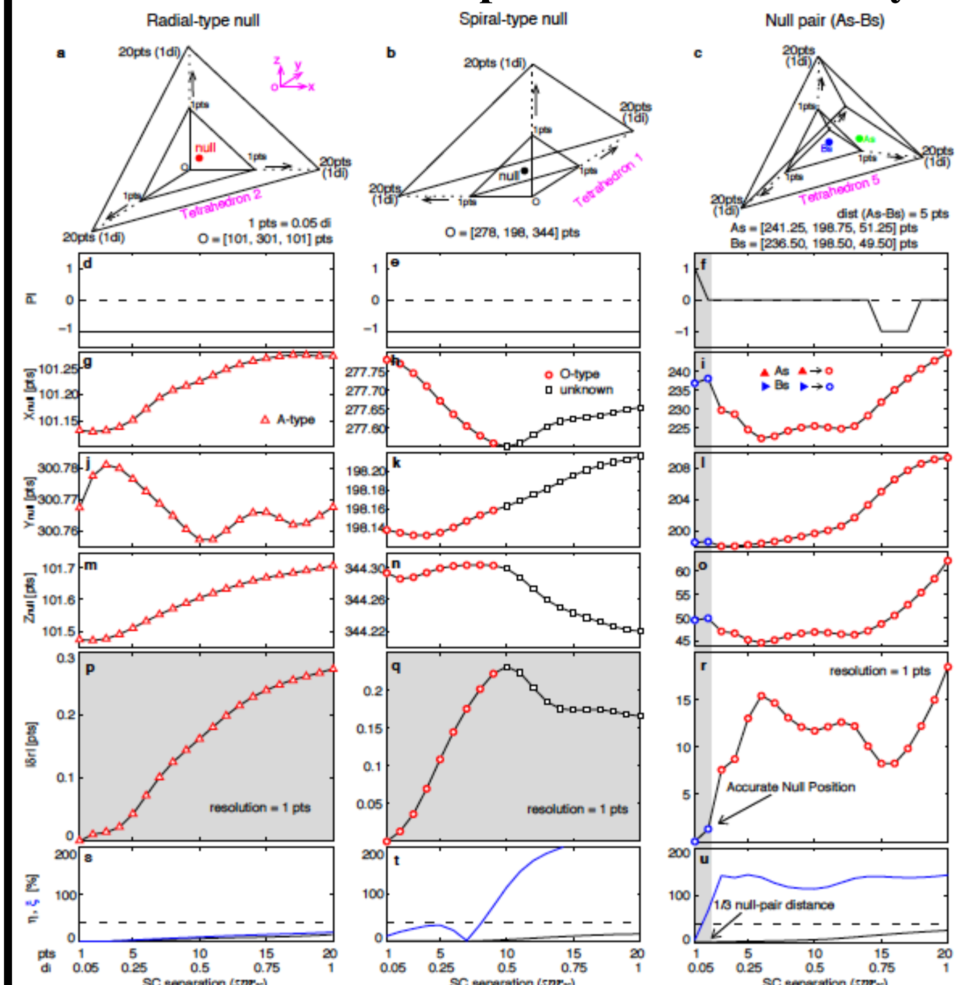
Divide each cube to five tetrahedrons to find null

Middle stage of simulation: 136 nulls in total

Later stage of simulation: 232 nulls in total

Both η and ξ increase with SC separation

Influence of SC separation on accuracy

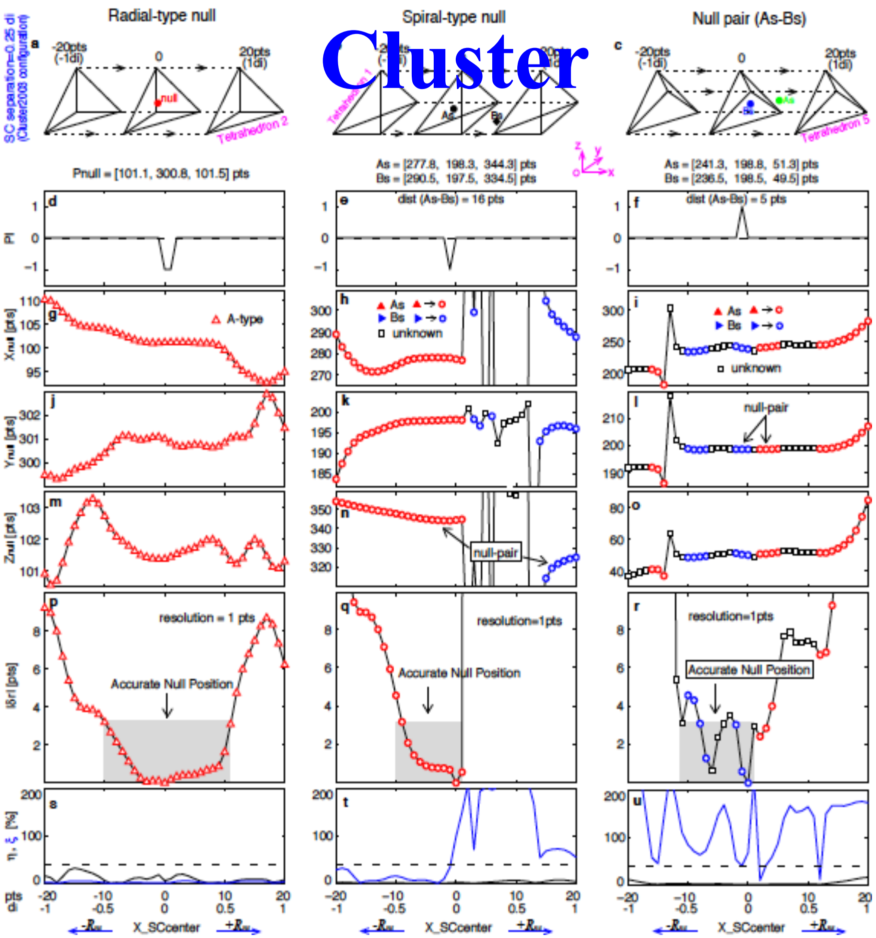


Isolated null: FOTE is accurate when $spr_{sc}<1d_i$

Null-pair: FOTE is accurate when $spr_{sc}<1/3D_{np}$

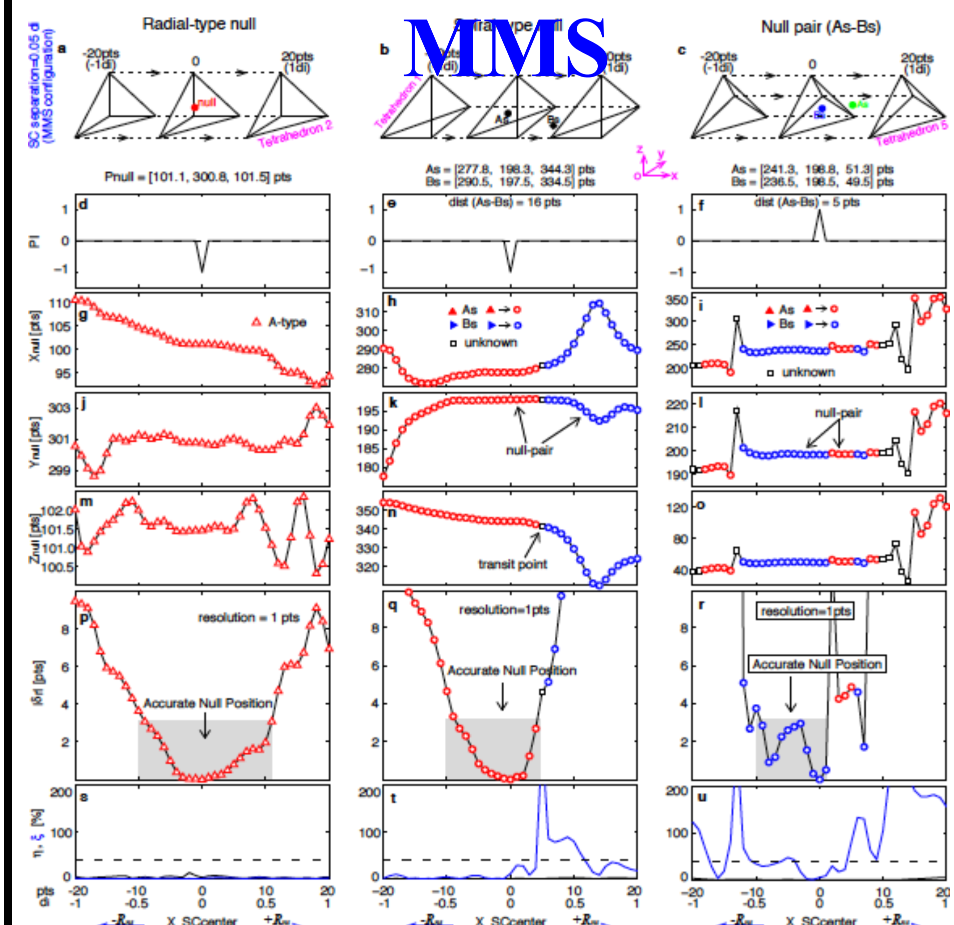
How accurate is FOTE? (finding null)

Influence of null-SC distance on accuracy



Isolated null: accurate at $R_{ns}<0.5d_i$
Null-pair (outbound): accurate at $R_{ns}<0.5d_i$
Null-pair (inbound): inaccurate

Influence of null-SC distance on accuracy



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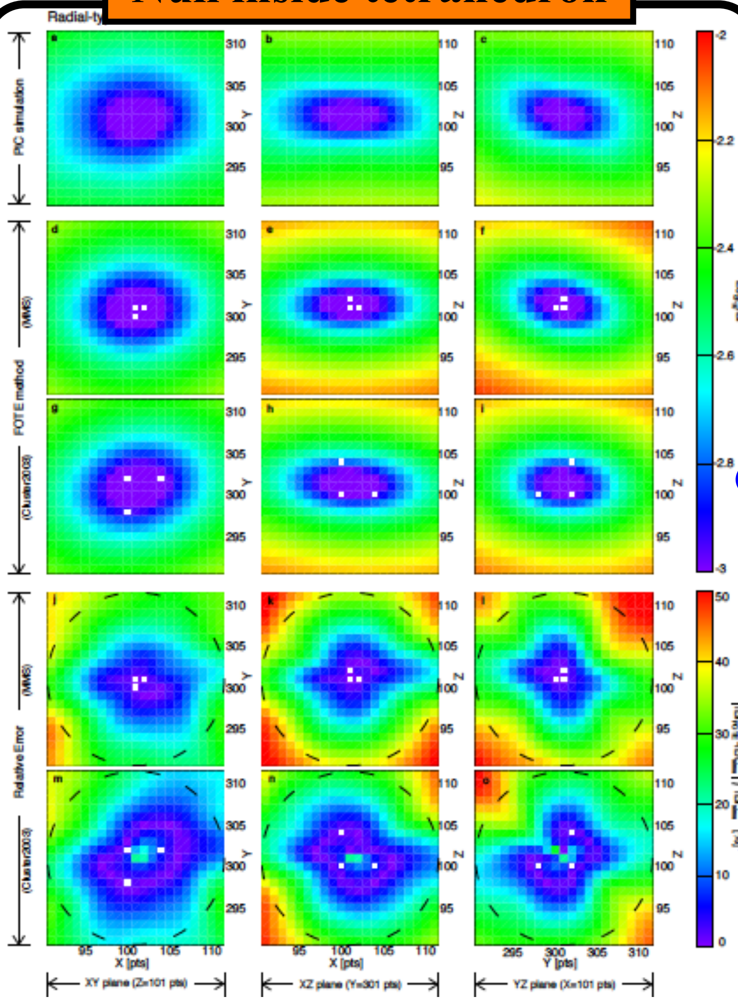
How accurate is FOTE? (topology)

Null inside tetrahedron



MMS

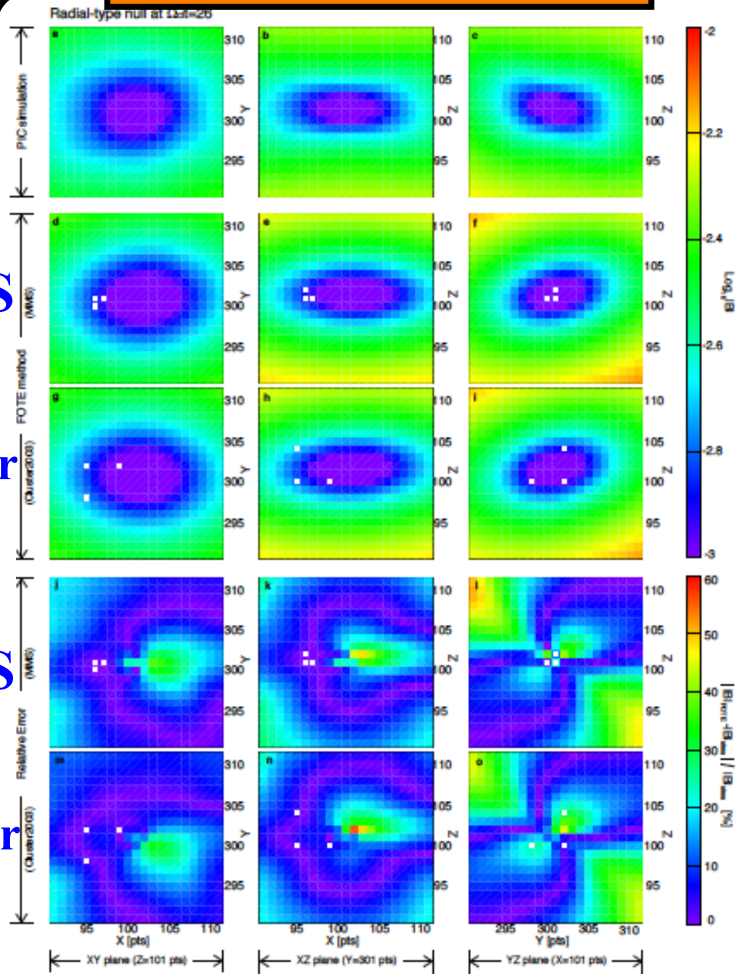
Cluster

MMS

Cluster

Accurate (*err*<30%)
in the nearby $0.5d_i$ domain

Null outside tetrahedron



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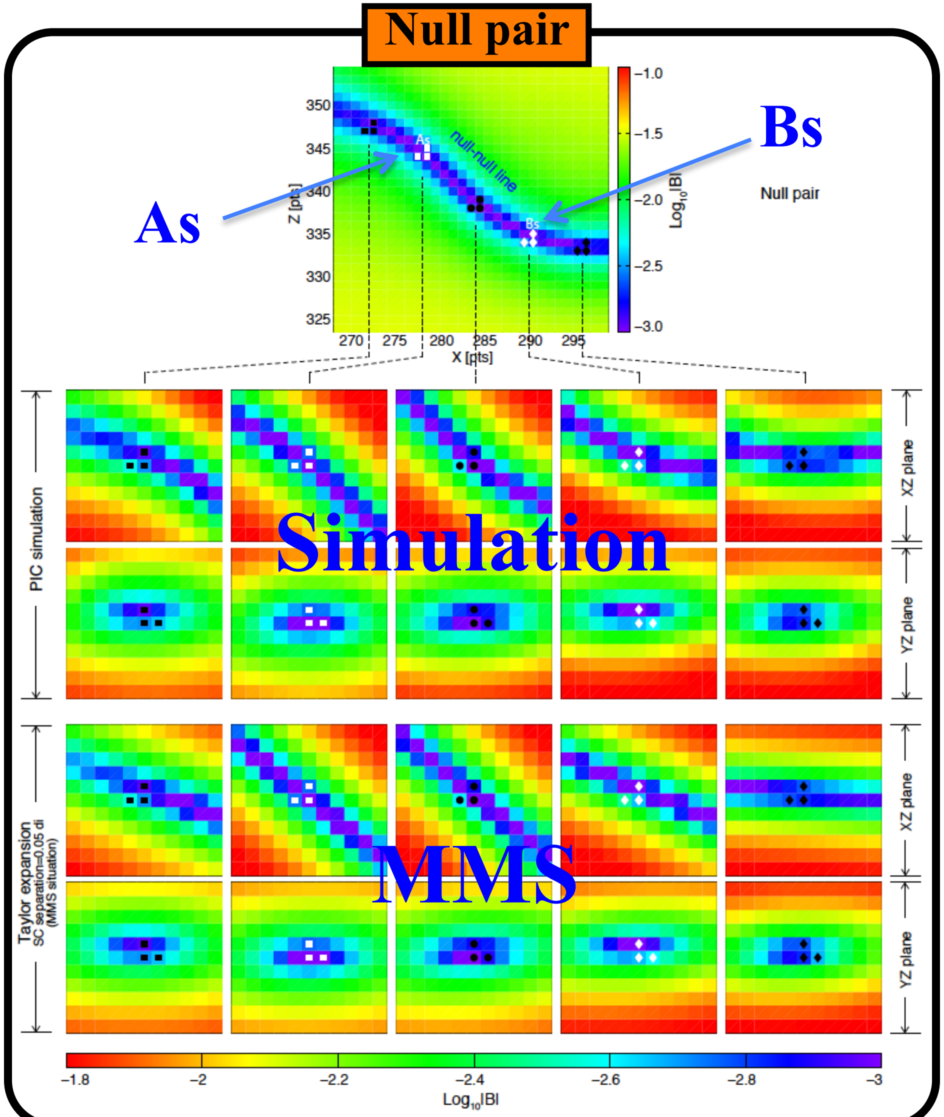
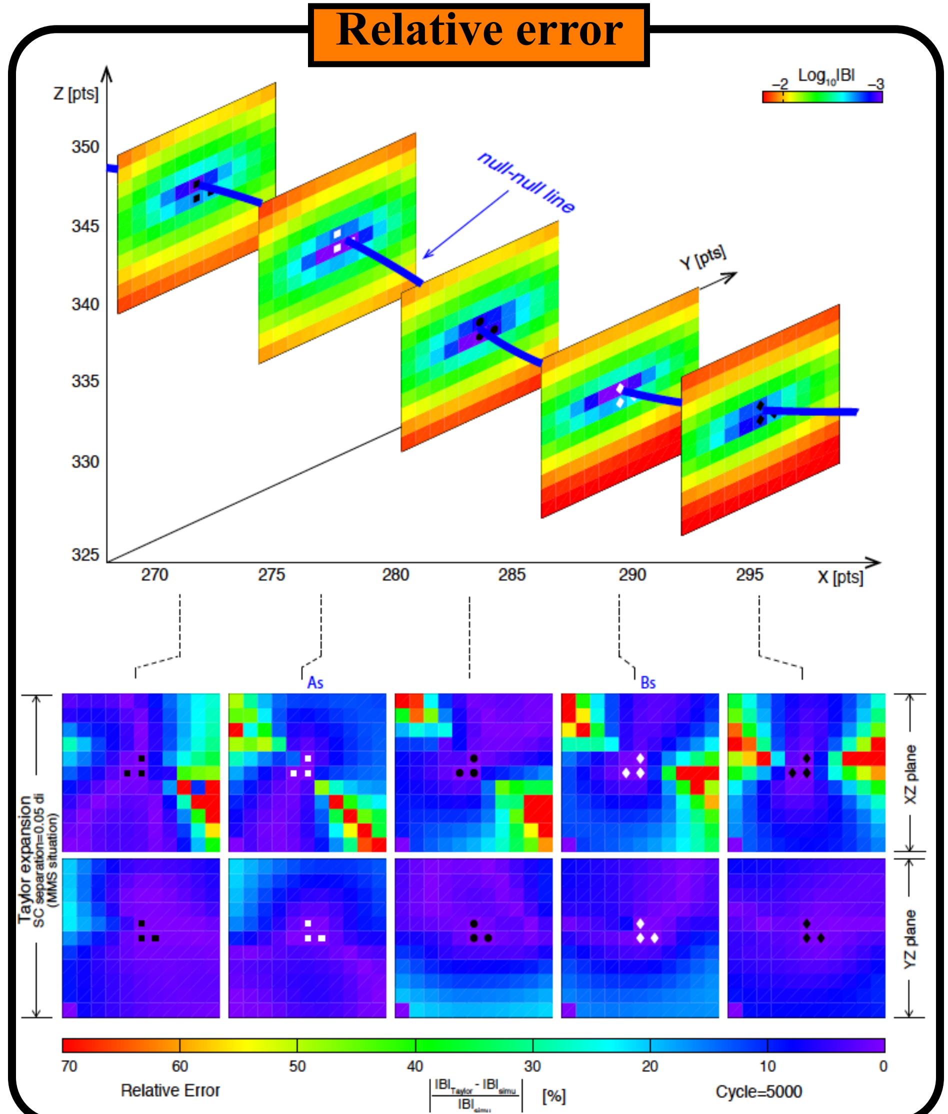
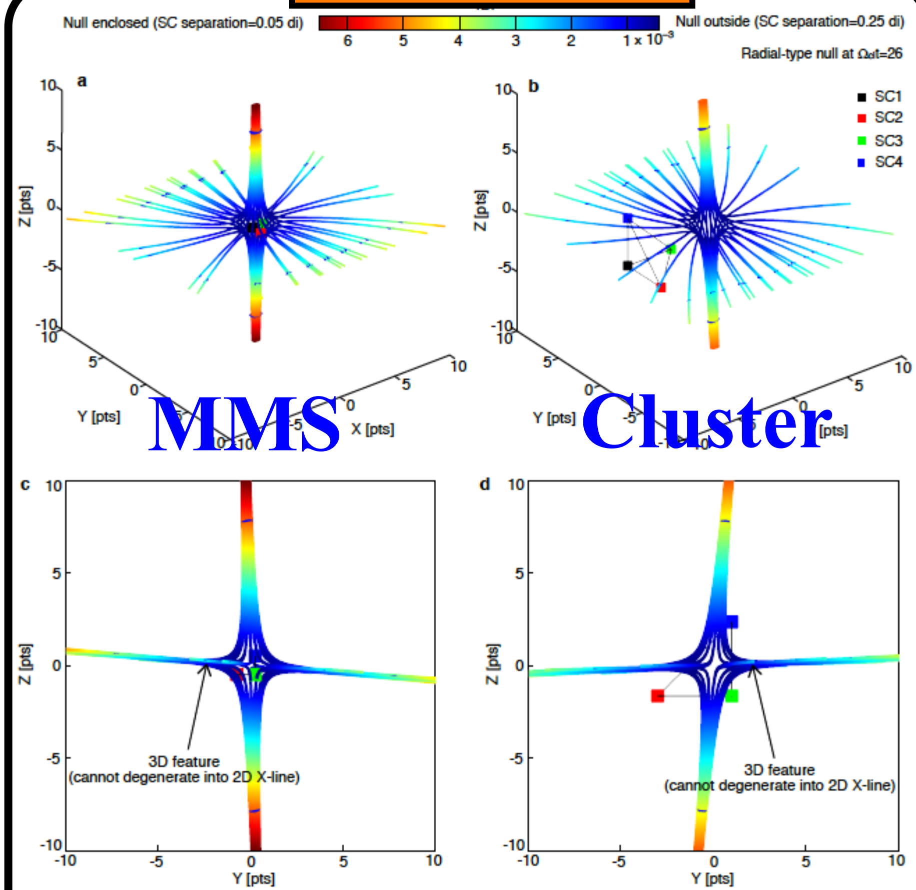




Accurate (err<20%) in the nearby $0.25d_i$ domain; Not good (err>70%) along the null-null line

Reconstruct null topology

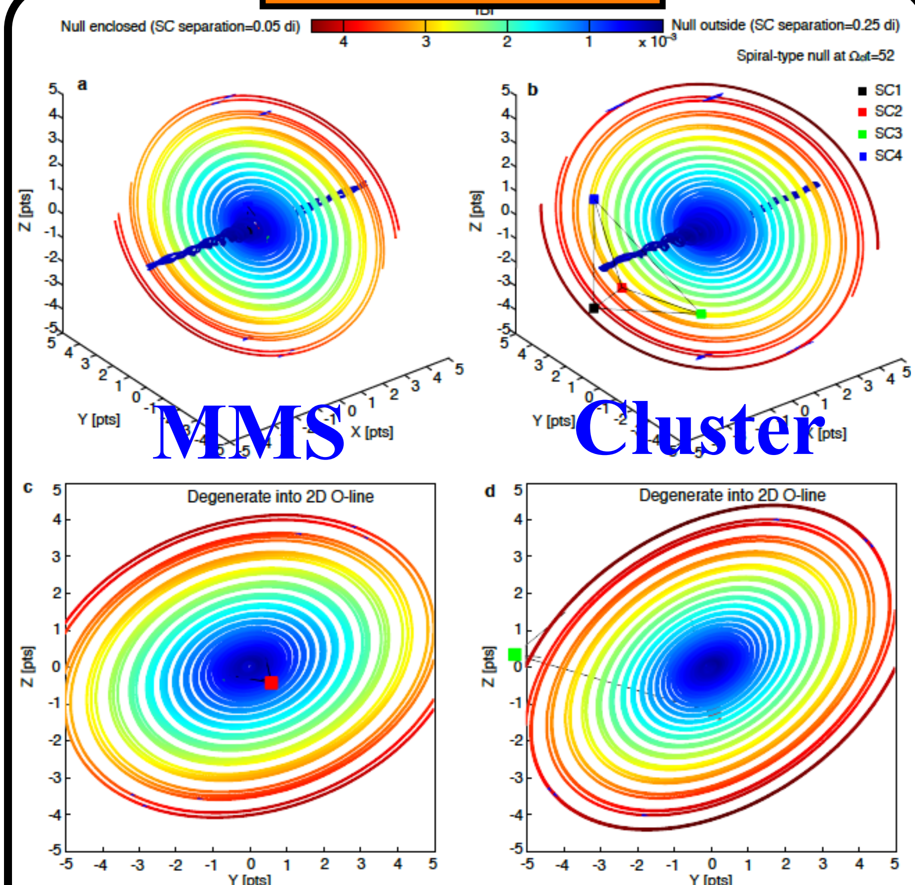
Radial-type null



SC separation=$0.05d_i$
Null is enclosed

SC separation=$0.25d_i$
Null is outside

Spiral-type null



SC separation=$0.05d_i$
Null is enclosed

SC separation=$0.25d_i$
Null is outside

Trace and inverse-trace a few points around null
Consistent well with theoretic prediction

Large- and small-tetrahedron give similar topology
Null outside and null inside give similar topology

Why MMS needs the FOTE method?

- **Where to find electron diffusion region?**

Apply the FOTE method to MMS data:
If the null-SC distance is smaller than 1 d_e associated with the flow velocity reversal, an electron diffusion region probably is encountered.

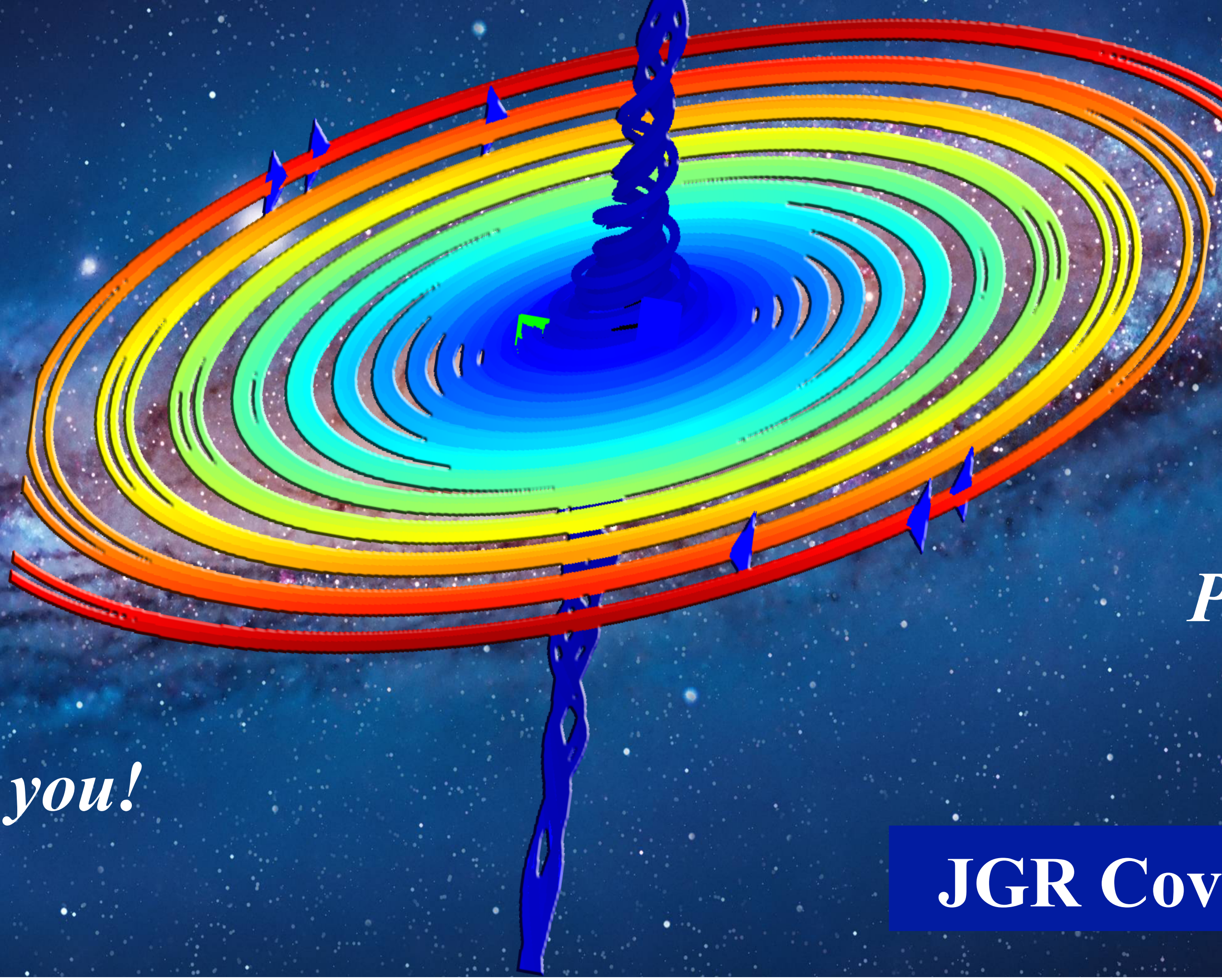
Coclusions

1. We develop a new method (FOTE) to find magnetic nulls and reconstruct field topology

2. We quantitively test this method, and compare it with the Poincare index and trilinear method

3. This method is useful for MMS mission, particularly for finding electron diffusion region

Fu H.S., A. Vaivads, Y.V. Khotyaintsev, V. Olshevsky, M. André, J.B. Cao, S.Y. Huang, A. Retinò, and G. Lapenta (2015),
How to find magnetic nulls and reconstruct field topology with MMS data?,
J. Geophys. Res. Space Physics, 120, 3758–3782.



Poster

Thank you!

JGR Cover Paper

Why use linear approximation?

1. So far, all the methods for finding nulls are based on linear approximation, including Poincare index, Trilinear method.

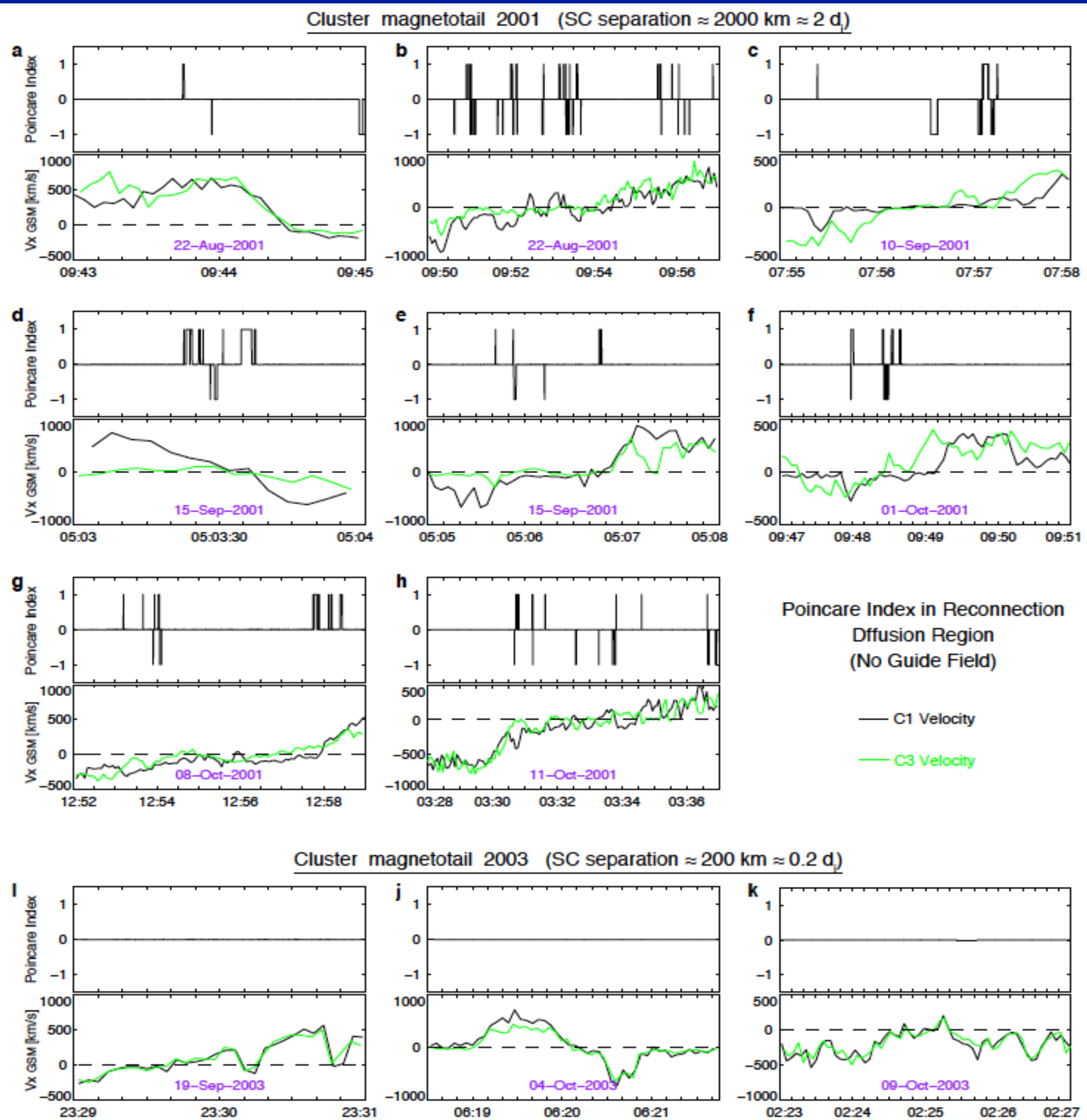
2. For MMS mission, the linear approximation should be OK, because the SC separation is very small.

3. We have already used two parameters to qualify this approximation. If they are small, the results are reliable.

4. We have tested the approximation on simulation data, and find it works well. Notice: the simulation data is nonlinear.

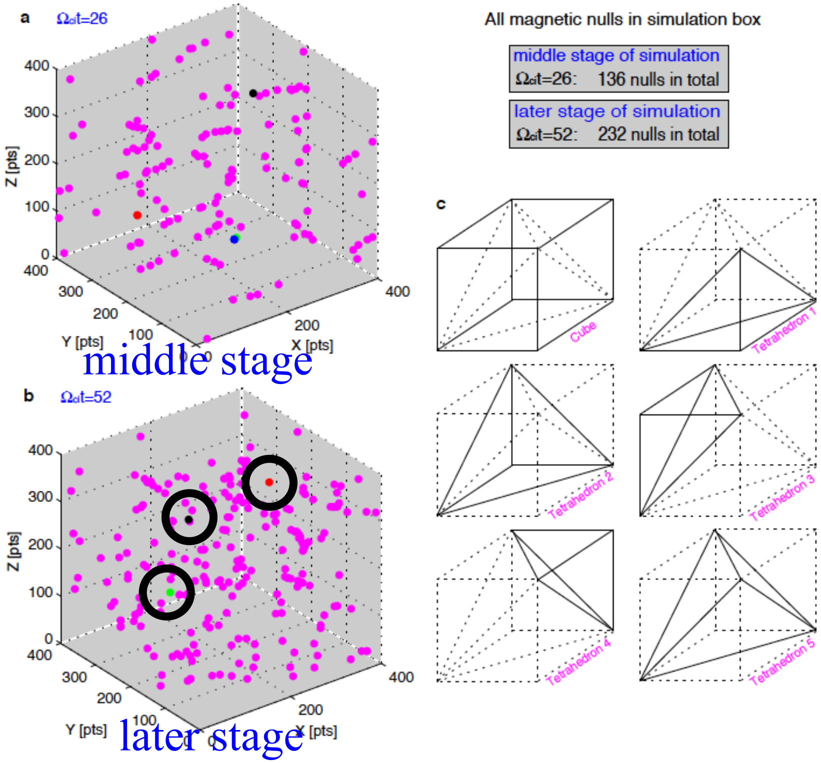
5. We have used the method to reconstruct the field topology, and find it agree very well with theoretic predictions.

All reconnection events measured by Cluster in 2001 and 2003 (Poincare index)



Accuracy of FOTE for finding null (later stage)

Data for analysis



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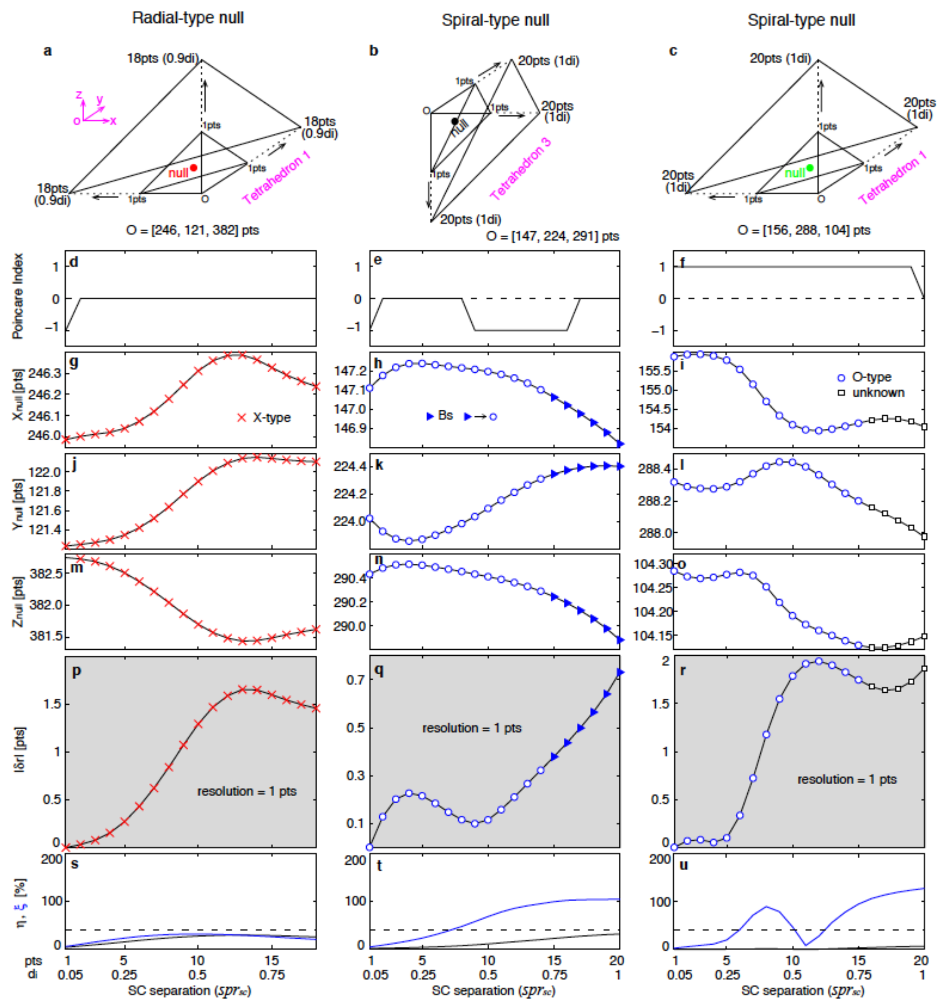
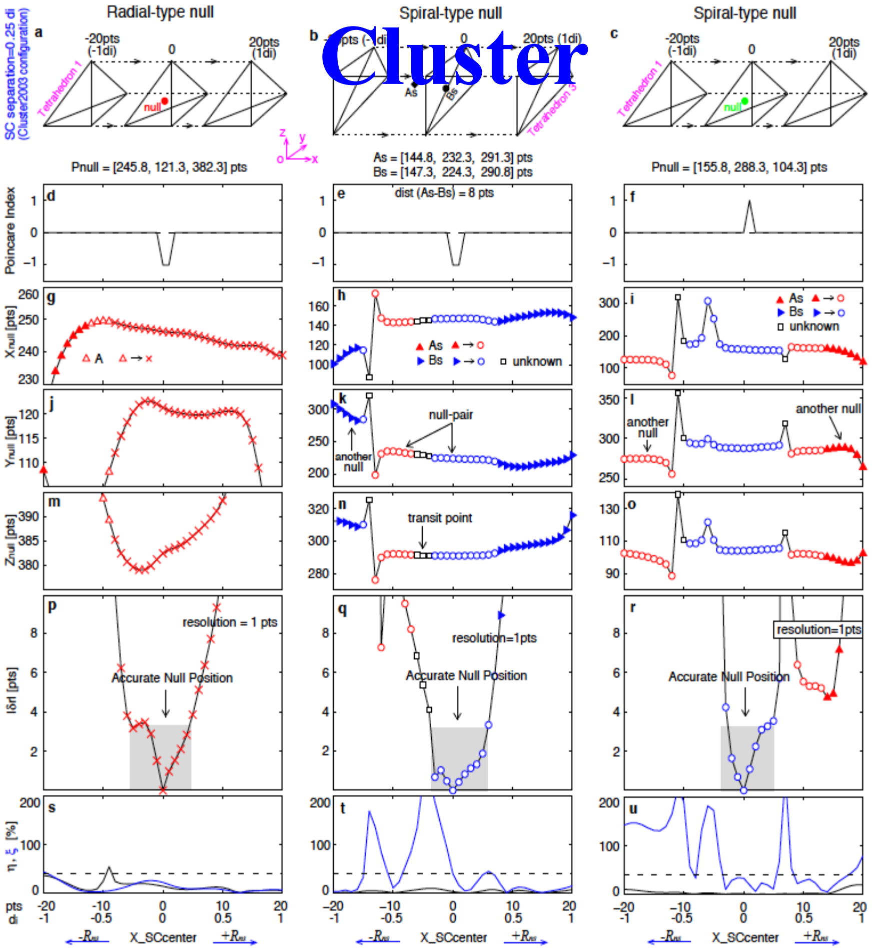
Influence of SC separation on accuracy



Isolated null: FOTE is accurate when spr_{sc}<$1d_i$

Accuracy of FOTE for finding null (later stage)

Influence of null-SC distance on accuracy

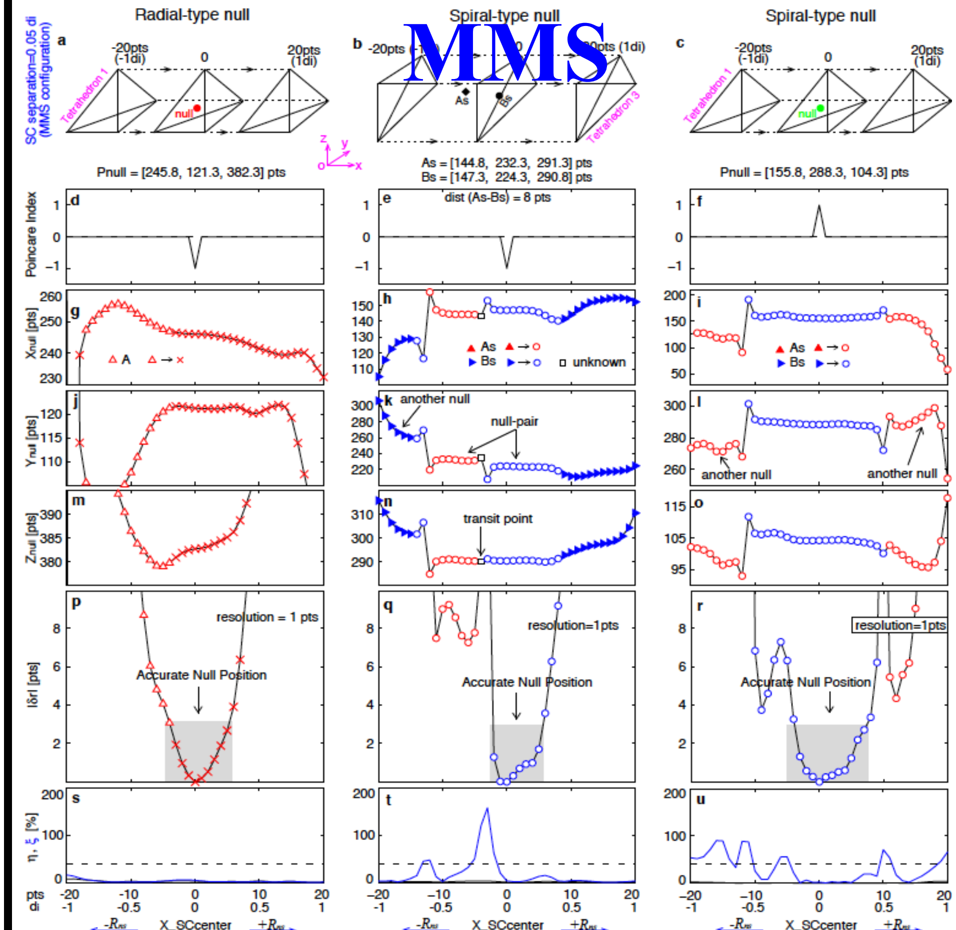


Isolated null: accurate at $R_{ns}<0.25d_i$

Null-pair (outbound): accurate at $R_{ns}<0.25d_i$

Null-pair (inbound): inaccurate

Influence of null-SC distance on accuracy



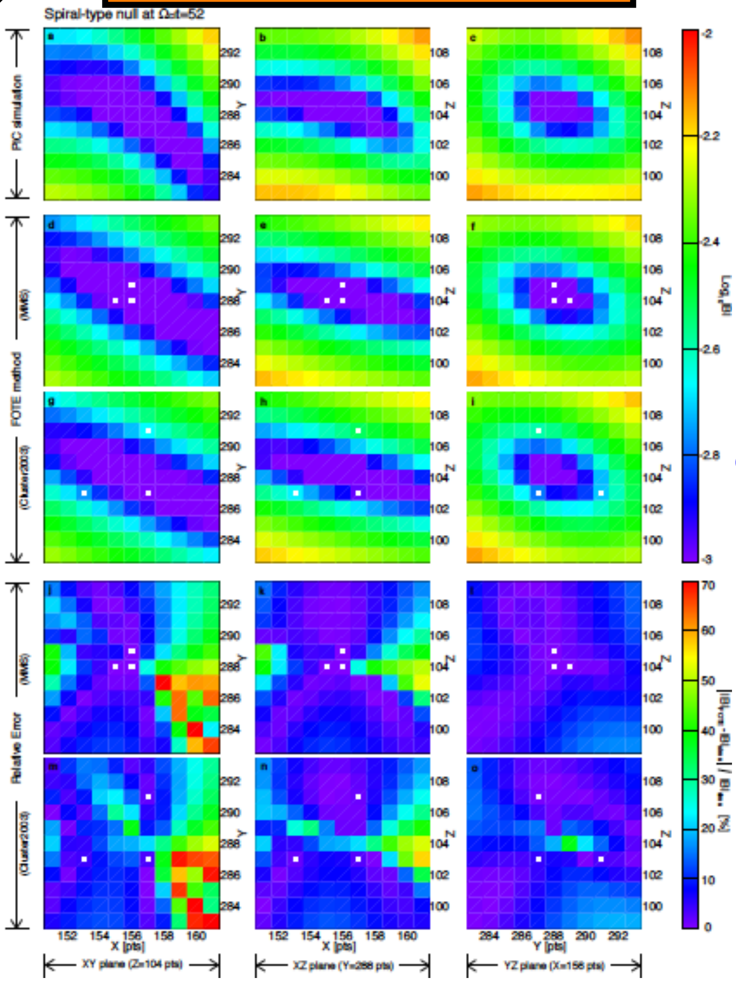
Isolated null: accurate at $R_{ns}<0.25d_i$

Null-pair (outbound): accurate at $R_{ns}<0.25d_i$

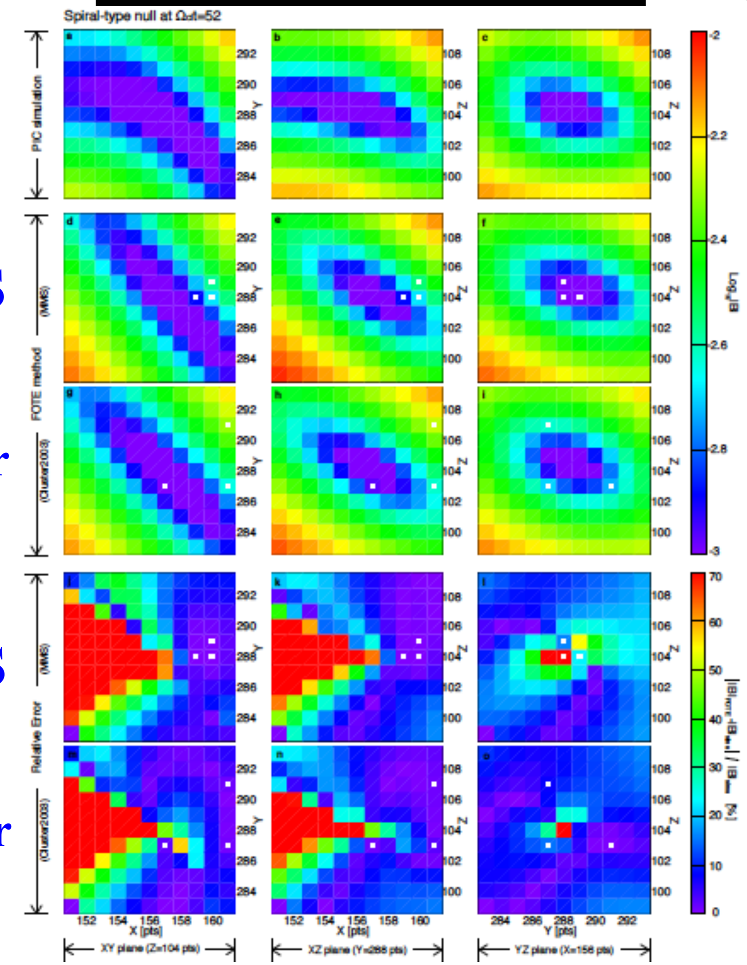
Null-pair (inbound): accurate at $R_{ns}<1/4D_{np}$

Accuracy of topology: spiral null

Null inside tetrahedron



Null outside tetrahedron



MMS

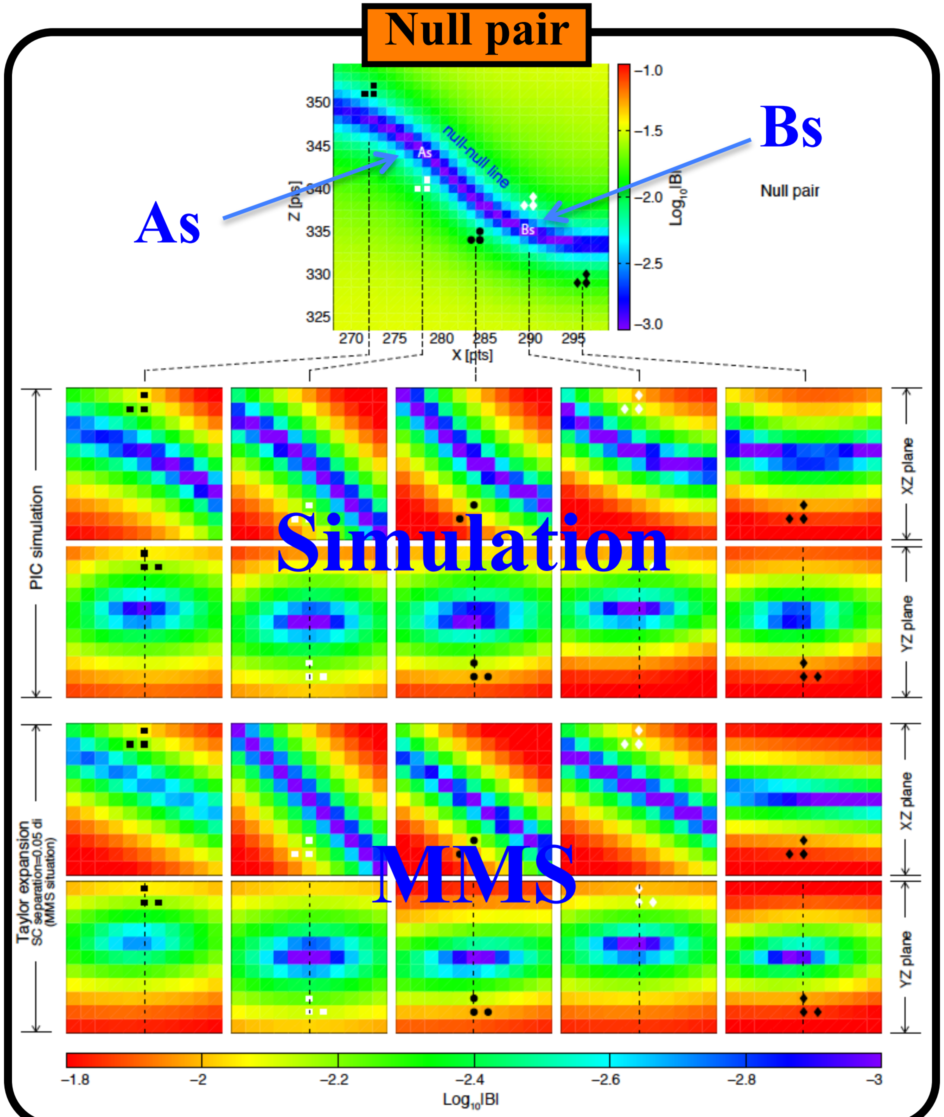
Cluster

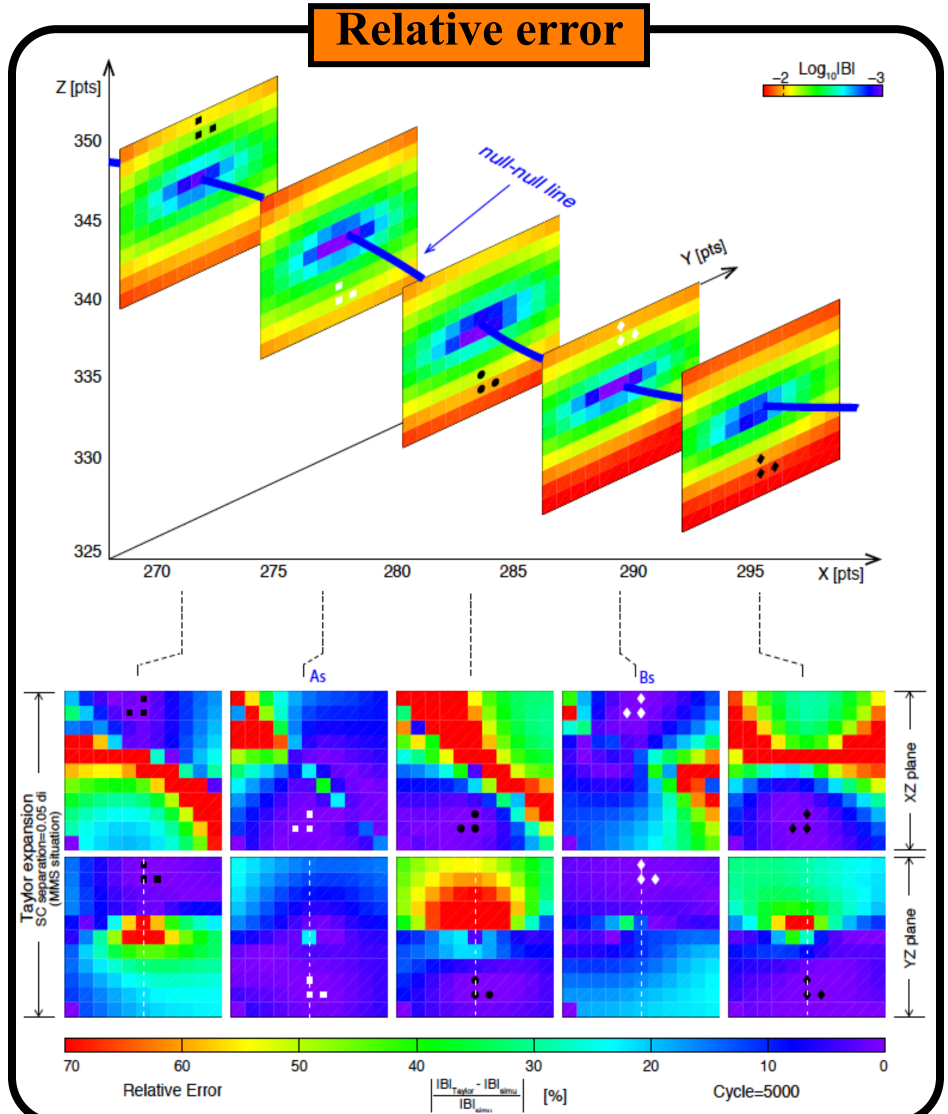
MMS

Cluster

Accurate (*err*<20%) in the nearby $0.25d_i$ domain; Not good (*err*>70%) along the spine

Accuracy of topology: null-pair (outside)





Accurate (err<20%) in the nearby $0.25d_i$ domain; Not good (err>70%) along the null-null line

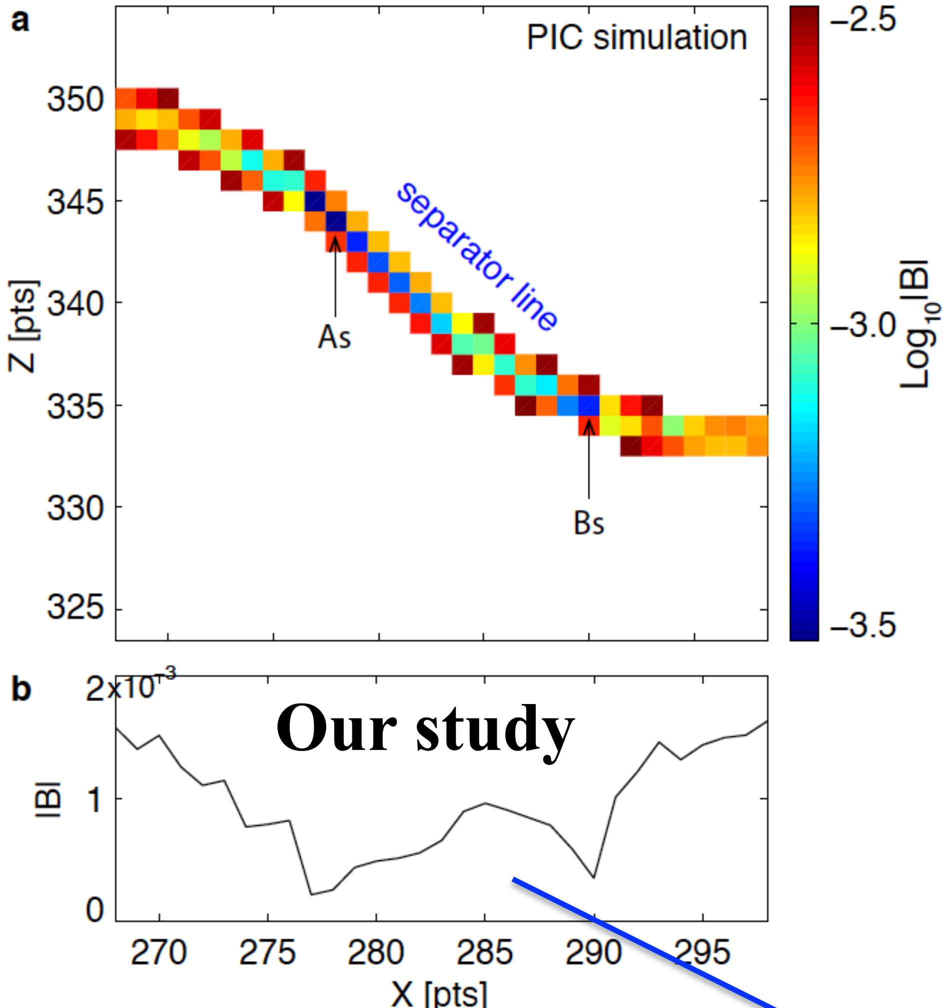
Accuracy of FOTE for finding null: summary

		Weakly chaotic field	Strongly chaotic field
Influence of SC separation (null is enclosed)		IN: accurate at $spr_{sc}<1d_i$ NP: accurate at $spr_{sc}<1/3D_{np}$	IN: accurate at $spr_{sc}<1d_i$
		η and ξ both increase with spr_{sc}	
Influence of null-SC distance	$spr_{sc}=0.25d_i$ (Cluster)	IN: accurate at $R_{ns}<0.5d_i$ NP (outbound): accurate at $R_{ns}<0.5d_i$ NP (inbound): inaccurate	IN: accurate at $R_{ns}<0.25d_i$ NP (outbound): accurate at $R_{ns}<0.25d_i$ NP (inbound): inaccurate
		η is always small, ξ is small for IN and NP (outbound)	
	$spr_{sc}=0.05d_i$ (MMS)	IN: accurate at $R_{ns}<0.5d_i$ NP (outbound): accurate at $R_{ns}<0.5d_i$ NP (inbound): accurate at $R_{ns}<1/4D_{np}$	IN: accurate at $R_{ns}<0.25d_i$ NP (outbound): accurate at $R_{ns}<0.25d_i$ NP (inbound): accurate at $R_{ns}<1/4D_{np}$
		η is always small, ξ is small for IN and NP (outbound)	

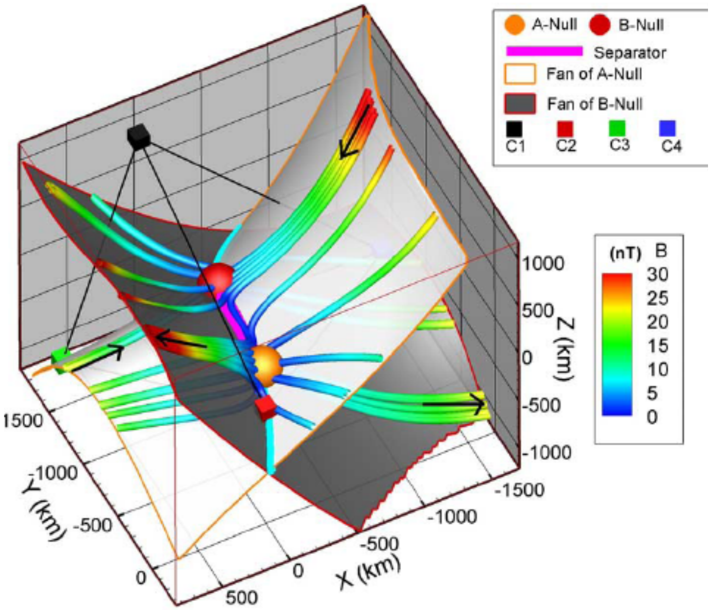
Accuracy of FOTE for topology: summary

		Null is enclosed	Null not enclosed
Radial-type & weakly chaotic field	spr_{sc}=0.05d_i (MMS)	accurate (*err*<30%) in the nearby 0.5d_i domain	
	spr_{sc}=0.25d_i (Cluster)		
Spiral-type & strongly chaotic field	spr_{sc}=0.05d_i (MMS)	accurate (*err*<20%) in the nearby 0.25d_i domain not good (*err*>70%) along the spine or null-null line	
	spr_{sc}=0.25d_i (Cluster)		
Null pair & weakly chaotic field	spr_{sc}=0.05d_i (MMS)		

Detail of null-null line (separator line)

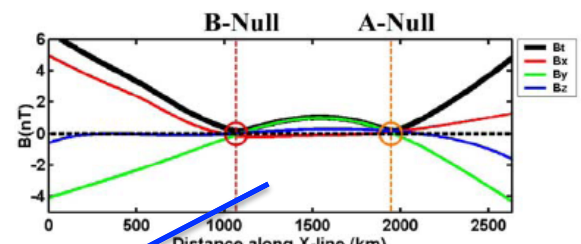


Our study



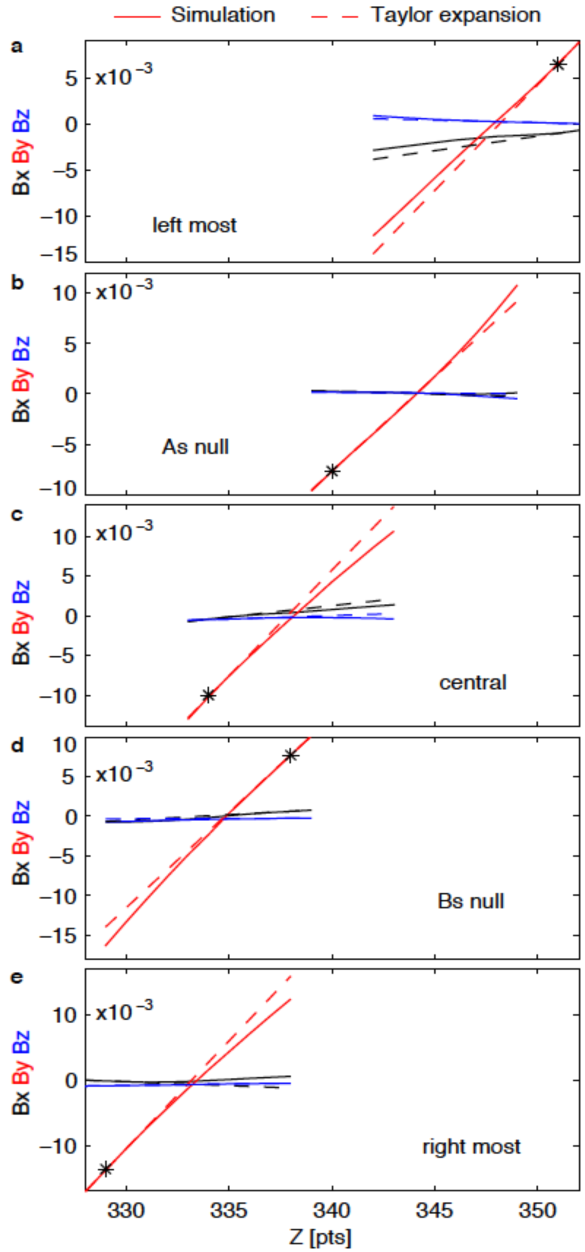
a

He+ 2008GRL



Consistent !

Absolute error at null-null line (separator line)

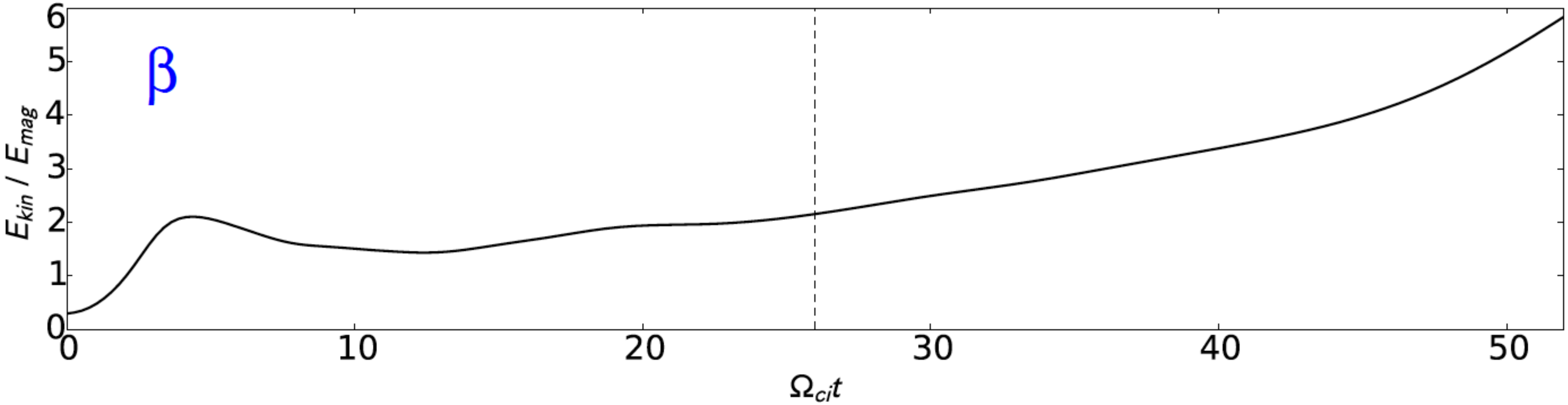


The relative error at separator line is large

The absolute error at separator line is small

Large relative error is attributed to weak field

Plasma beta in simulation box



The plasma beta in simulation box is comparable to that in solar wind, solar corona, and the Earth's magnetosphere, meaning that the test results of simulations can be applied to *in-situ* spacecraft measurements, such as Cluster and the MMS mission.