Asymptotic safety
and scalar-tensor theories

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Work mainly in collaboration
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see arXiv:1501.00888

Probing the Fundamental Nature of Spacetime with the Renormalization Group

Nordita: March 24, 2015

Outline

- Framework
- Truncation for the scalar-tensor model
- Gravitational sector: parameterization and gauge fixing
- Scalar-tensor models: flow equations and fixed point analysis
- Conclusions and outlook

Framework

What is the fundamental nature of gravitational interaction? Of spacetime?

To be able to really answer this question we should probably be extremely lucky!

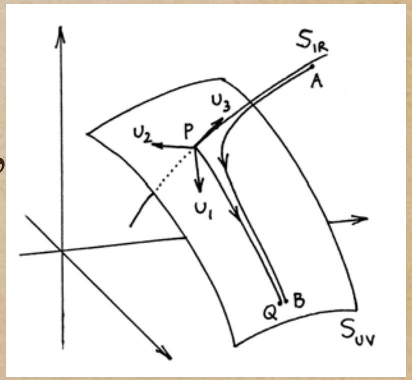
- Single fundamental theory? QFT Stringy Discrete models …
- Sequences of theories with trasmutation/generation of (some) degrees of freedom?

Criteria? Experimental input (hard) Simplicity/Beauty/Unification …

RG paradigm useful since it unifies the fundamental and effective theory point of view.

Gravity: at least classical field theory and effective field theory are good descriptions.

Simplest approach:
A gravitational QFT described by a metric and diffeomorphism symmetry, whose dynamics reveals a UV fixed point with finite dim. critical surface.



If bare action has no irrelevant operators, it is asymptotically safe.
Otherwise description is effective, originating from a more fundamental theory.

Interacting gravity-scalar field model.

Interesting at pure theoretical level, including some kind of matter, and also for possible implications in cosmology

Metric (euclidean) QFT formulation: non local effective action $\Gamma[g_{\mu\nu}, \phi]$

Asymptotic safety paradigm and FRG techniques. (Talks: Reuter, Morris, Pawloswki)

Background field formalism: $g_{\mu\nu} \, (\bar{g}_{\mu\nu}, h_{\mu\nu})$

- Issue of the double metric description / modified splitting Ward Identities.
- Choose truncations as well as coarse-graining schemes. Simple but non trivial.

Many degrees of approximations in the covariant description:
First single metric (field) description still non local truncation: $\Gamma_k = \int d^d x \sqrt{g} \; \mathcal{L}[\phi, R_{\mu\nu\lambda\sigma}]$
(Level 0)

Then maximally symmetric background (sphere), for a local "LPA" truncation,

$$\boxed{\Gamma[\phi, g_{\mu\nu}] = \int \mathrm{d}^d x \sqrt{g} \left[F(\phi, R) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]}$$

(Narain, Rahmede)

Minimal truncation

Simplest approximation: expand $F(\phi, R)$ around R=0 up to linear term, LPA truncation:

$$\boxed{\Gamma_k[\phi, g] = \int d^dx \sqrt{g} \left(V(\phi) - F(\phi) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) + S_{GF} + S_{gh}}$$

This theory contains the E-H action with a cosmological constant if we remove the scalar.

Previously studied (Narain, Percacci) (single metric, linear split) but difficult to study singular structure induced by the power expansion in the background scalar curvature R around the origin. ”Too far off-shell” !

Scalar tensor (ST)

$$\dot{v} = \frac{1}{3\pi^2} \left[\frac{f}{f - v} + \dots \right]$$

E-H

$$\dot{\lambda} \sim \frac{1}{1 - 2\lambda}$$

$$\boxed{\begin{aligned} \varphi &= \phi\, k^{-(d-2)/2} \\ \lambda &= \Lambda / k^2 \\ v \sim \varphi^d & \qquad f \sim \varphi^2 \end{aligned}}$$

Pole in the denominator —> IR singularity, for ST problem also in fixed point equation.

Spin 2 fluctuations, for $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, couple to the scalar potential via $\sqrt{g}$

$$\sqrt{g} = \sqrt{\bar{g}} \left(1 + \frac{h}{2} + \frac{h^2}{8} - \frac{1}{4} h^{\mu\nu} h_{\mu\nu} + \cdots \right) \longrightarrow \Gamma^{(2,TT)} \sim F \left(-\nabla^2 + \frac{d^2 - 3d + 4}{d(d-1)} R \right) - V$$

Gravity sector: the metric

A way to avoid this problem: use an exponential parameterization of the metric:

$$\boxed{g_{\mu\nu} = \bar{g}_{\mu\rho}(e^h)^\rho{}_\nu} \qquad \sqrt{g} = e^{h/2}\sqrt{\bar{g}} = \sqrt{\bar{g}}\left(1 + \frac{h}{2} + \frac{h^2}{8} + \cdots\right) \qquad trh = h = 2d\omega$$

Potential V couples only to the trace of the metric fluctuations.

As a change of variables the Jacobian is well defined.

We take the attitude that the metric has a non linear nature, naturally preferring the exponential parameterization.

Think about frames and vielbeins… coset space

Remark: at quantum level the off shell effective action is equivalent to other parameterizations if

- a Jacobian is taken into account
- the geometric formulation a la Vilkowisky-De Witt in considered, indeed e.g. expectations values are not trivially related, …

Remark: non linear transformation —> momentum coarse-graining qualitative different!

Gravity sector: gauge fixing and ghosts

See R. Percacci talk.

Gravity is a gauge theory: physics does not change under diffeomorphisms.

Single metric: gauge fixing and ghost terms from the lowest order quantum gauge transf.:

$$\boxed{\delta_\epsilon^{(Q)} h_{\mu\nu} = \bar\nabla_\mu \epsilon_\nu + \bar\nabla_\nu \epsilon_\mu + O(h)}$$

Decomposition of the metric and diffeomorphism generator in irreducible components:

$$h_{\mu\nu} = h^{TT}{}_{\mu\nu} + \bar\nabla_\mu \xi_\nu + \bar\nabla_\nu \xi_\mu + \bar\nabla_\mu \bar\nabla_\nu \sigma - \frac{1}{d}\bar g_{\mu\nu}\bar\nabla^2\sigma + \frac{h}{d}\bar g_{\mu\nu}$$

$$\epsilon^\mu = \epsilon^{T\mu} + \bar\nabla^\mu \frac{1}{\sqrt{-\bar\nabla^2}}\psi \; ; \qquad \bar\nabla_\mu \epsilon^{T\mu} = 0$$

Transformations:

$$\delta_{\epsilon^T}\xi^\mu = \epsilon^{T\mu} \qquad \delta_\psi \sigma = \frac{2}{\sqrt{-\bar\nabla^2}}\psi \qquad \delta_\psi h = -2\sqrt{-\bar\nabla^2}\psi$$

Gauge invariant quantities

$$s = h - \bar\nabla^2\sigma \qquad h^{TT}_{\mu\nu}$$

To adsorbe some Jacobians one can redefine: $\xi_\mu \to \xi'_\mu \;,\; \sigma \to \sigma'$

Gravity sector…

Physical gauge fixing: set to zero the gauge dependent fluctuations.
Path integral over gauge invariant fluctuations: s and $h^{TT}_{\mu\nu}$

I: $\xi'_\mu = 0 \quad , \quad h = \text{const.}$

II: $\xi'_\mu = 0 \quad , \quad \sigma' = 0$

Faddeev Popov determinants,
varying the GF conditions:

$\delta(\xi'_\mu)$	$\det\left(\sqrt{-\bar\nabla^2 - \frac{\bar R}{d}}\right)$
$\delta(h - \text{const})$	$\det(\sqrt{-\bar\nabla^2})$
$\delta(\sigma')$	$\det\left(\sqrt{-\bar\nabla^2 - \frac{\bar R}{d-1}}\right)$

The pure E-H action has a very simple Hessian using gauge invariant variables:

$$\frac{1}{2}\int dx\sqrt{\bar g}\left[\frac{1}{2}h^{TT}{}_{\mu\nu}\left(-\bar\nabla^2 + \frac{2\bar R}{d(d-1)}\right)h^{TT\mu\nu} - \frac{(d-1)(d-2)}{2d^2}s\left(-\bar\nabla^2 - \frac{\bar R}{d-1}\right)s - \frac{d-2}{4d}R\,h^2\right]$$

Eq. of motion

E-H truncation with type I cutoff and gauge fixing I (h=0).

Scalar-gravity system

Truncation with even potentials: $\Gamma_k[\phi, g] = \int d^dx \sqrt{g} \left(V(\phi) - F(\phi)R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$

Expanding also around a constant background $\phi = \bar{\phi} + \delta\phi$

Simple mixed gravity-scalar hessian: $\int dx \sqrt{g}\, \delta\phi \left[\left(V'(\bar{\phi}) - F'(\bar{\phi}) R \right) \frac{h}{2} - F'(\bar{\phi}) \frac{d-1}{d} \left(-\nabla^2 - \frac{R}{d-1} \right) s \right]$

The hessian, gauge fixed ($\xi'_\mu = 0$, $h = 0$) and for a shifted $\sigma'' = \sigma' + \cdots$ is diagonal:

$$\int dx \sqrt{\bar{g}} \left[F(\bar{\phi}) \frac{1}{4} h^{TT}{}_{\mu\nu} \left(-\bar{\nabla}^2 + \frac{2\bar{R}}{d(d-1)} \right) h^{TT\mu\nu} - \frac{(d-1)(d-2)}{4d^2} F(\bar{\phi}) \sigma'' (-\bar{\nabla}^2) \sigma'' \right.$$
$$\left. + \frac{1}{2} \delta\phi \left(-\bar{\nabla}^2 + V''(\bar{\phi}) - F''(\bar{\phi}) \bar{R} + 2 \frac{d-1}{d-2} \frac{F'(\bar{\phi})^2}{F(\bar{\phi})} \left(-\bar{\nabla}^2 - \frac{\bar{R}}{d-1} \right) \right) \delta\phi \right]$$

Wetterich equation: choose some appropriate coarse-graining cutoff operator: type I, type II, or (scalar-) pure cutoff.

We first consider a type I cutoff: $-\bar{\nabla}^2 \longrightarrow P_k(-\bar{\nabla}^2) = -\bar{\nabla}^2 + R_k(-\bar{\nabla}^2)$

Going to dimensionless quantities we can obtain the flow equations. $f(\varphi) = k^{2-d} F(\phi) \quad v(\varphi) = k^{-d} V(\phi)$

Analysis of type I flow equations

To investigate fixed point solutions in this infinite dimensional space of "couplings" we consider in d dimensions the following cases:

A. The full equations

B. The "one loop" approximation, neglecting $\dot{F}_k(\bar{\phi})$ on the r.h.s of the flow equations.

These equations have some analytic fixed point solutions (in any d):

$$\boxed{\begin{aligned} v(\varphi) &= v_0 \\ f(\varphi) &= f_0 + \frac{\xi}{2}\varphi^2 \end{aligned}}$$

	A	B
FP1	$(v_{0A}, f_{0A}, \xi = 0)$	$(v_{0B}, f_{0B}, \xi = 0)$
FP2	-	$(v_{0B}, f_{02B}, \xi > 0)$
FP3	$(v_{03}, f_{03} = 0, \xi < 0)$	

Eigenperturbations of these solutions for d=3 and d=4 cases analytically or numerically.

For example for FP1 in d=4 for case A has 4 relevant and 1 marginal directions:

$$\boxed{v_0 = 0.00396 \qquad f_0 = 0.0069}$$

$$\begin{aligned}
&\theta_1 = 4, && w_1^t = (\delta v, \delta f)_1 = (1, 0) \\
&\theta_2 = 2.553, && w_2^t = (\delta v, \delta f)_2 = (-1, 1.236) \\
&\theta_3 = 2, && w_3^t = (\delta v, \delta f)_3 = (c_{3v} + \varphi^2, c_{3f}) \\
&\theta_4 = 0.553, && w_4^t = (\delta v, \delta f)_4 = (c_{4v0} - \varphi^2, c_{4f} + 1.236\varphi^2) \\
&\theta_5 = 0, && w_5^t = (\delta v, \delta f)_5 = (c_{5v0} + c_{5v0}\varphi^2 + \varphi^4, c_{5f0} + c_{5f2}\varphi^2)
\end{aligned}$$

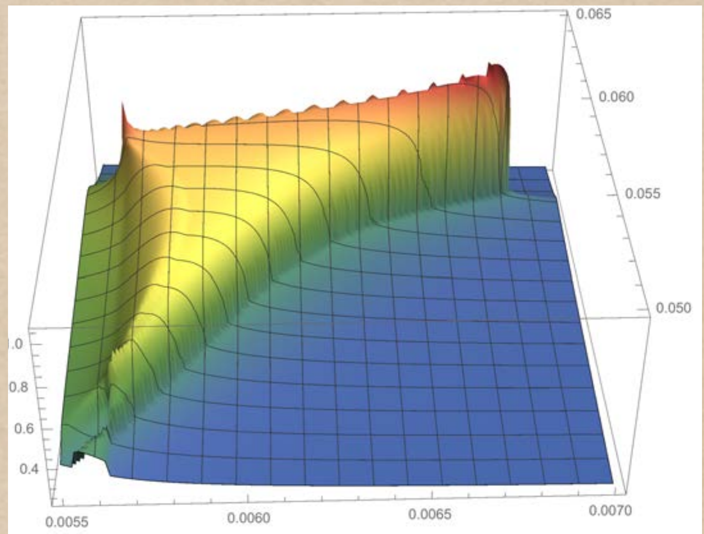
Further analysis of case B

In d=3 we expect to exists a deformation of the WF fixed point which in flat space belongs to the Ising universality class.

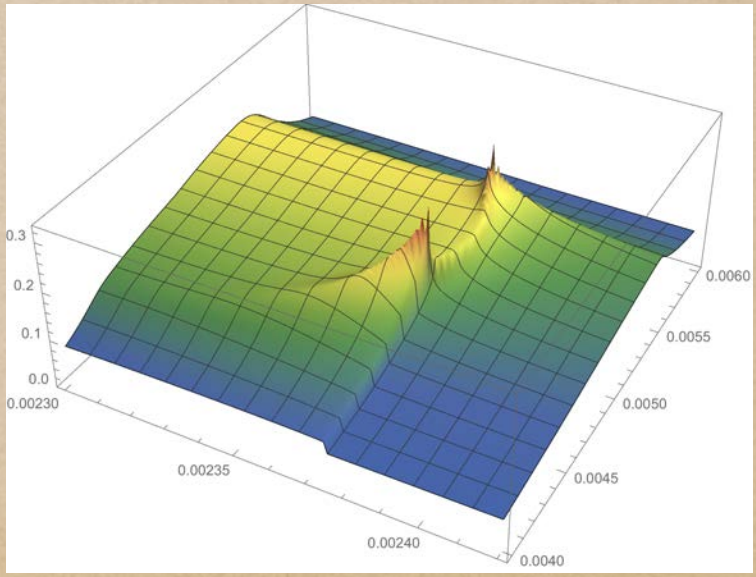
We have employed shooting methods (Morris), from the origin and the asyptotic region and various types of polynomial expansions as well.

Shooting method from the origin:

$0.0055 < v(0) < 0.0070$ and $0.050 < f(0) < 0.065.$



Three spikes corresponds to FP1, FP2, and possibly a non trivial WF solution. But this solution, (with also polynomial analysis), has the property to cross f=0 starting from f(0)>0, so that is defined as an analytic continuation.



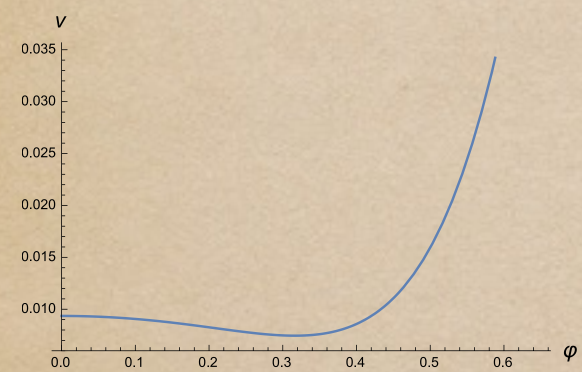
For d=4 from the shooting method give no indications that a WF type of fixed point do exist, similarly to the flat space case. In the region shown we see FP1 and FP2.

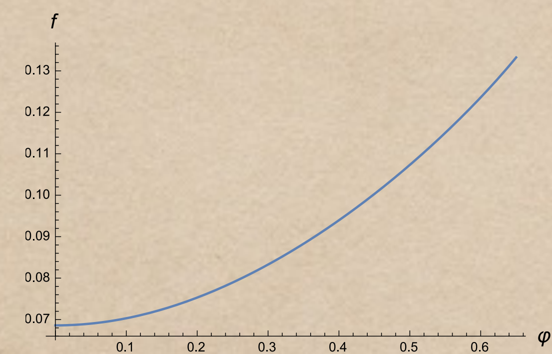
More on case A

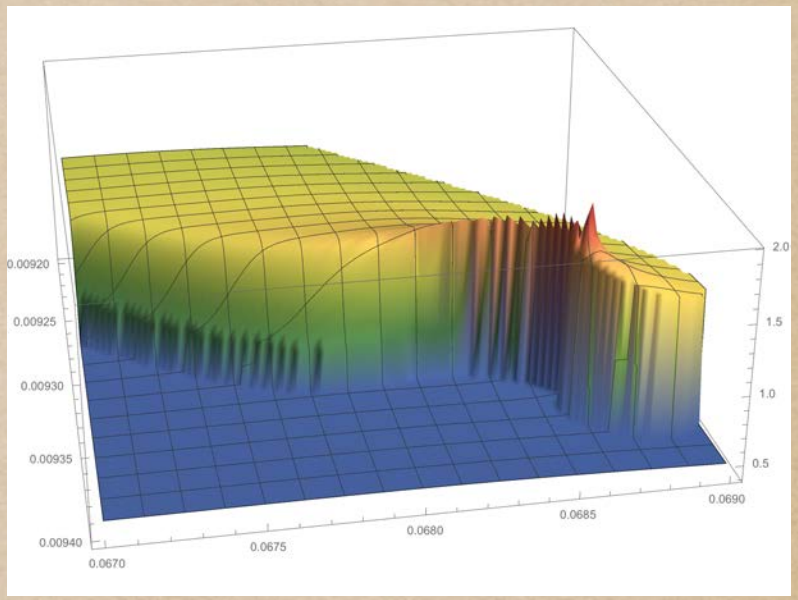
The search of a WF fixed point for these full equations was recently addressed using pseudospectral methods (based on Chebyshev polynomials). (Borchardt-Knorr)

For d=3 they show that there exist a WF-like solution, which is constructed with great precision. It has 4 relevant directions. f is always positive.

Indeed this solution can be found by shooting methods and standard polynomial expansion analysis







The full equations admit this solution, contrary to the “one loop” approximation. Schemes based on a spectrally adjusted cutoff, both split symmetries are broken.

$$\delta\phi \to \delta\phi + \delta\psi, \; \bar{\phi} \to \bar{\phi} - \delta\psi \qquad h_{\mu\nu} \to h_{\mu\nu} + \delta h_{\mu\nu}, \; \omega \to \omega + \delta\omega, \; \bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} - \delta h_{\mu\nu} - 2\bar{g}_{\mu\nu}\delta\omega$$

Background-scalar independent cutoff

Pure cutoffs (not spectrally adjusted) to respect the scalar split symmetry.
One cannot avoid instead the gravitational background dependence.

A linear cutoff $R_k(z) = \gamma k^a (k^2 - z)\theta(k^2 - z)$ (Litim) leads to more complicated equations.

We can find easily the constant analytic solution (FP1) for any d.

E.g. in d=4 and $\gamma = 1$ we have $\boxed{v_0 = 0.03314 \qquad f_0 = 0.01551}$

Spectral pattern similar to the FP1 of case A.

$$
\begin{aligned}
&\theta_1 = 4, && w_1^t = (\delta v, \delta f)_1 = (1, 0)\\
&\theta_2 = 2.273, && w_2^t = (\delta v, \delta f)_2 = (-1, 0.748)\\
&\theta_3 = 2, && w_3^t = (\delta v, \delta f)_3 = (-0.0121 + \varphi^2, 0.00774)\\
&\theta_4 = 0.273, && w_4^t = (\delta v, \delta f)_4 = (0.00382 - \varphi^2, -0.00343 + 0.748\varphi^2)\\
&\theta_5 = 0, && w_5^t = (\delta v, \delta f)_5 = (2.28 \times 10^{-4} - 0.0726\,\varphi^2 + \varphi^4, -1.97 \times 10^{-4} + 0.0464\,\varphi^2)
\end{aligned}
$$

We have to complete the search for other less trivial global solutions, also in d=3.

Other interesting cutoff we want to investigate: power like type (Morris)

Conclusions

- Scalar field interacting with gravity.
  Simple truncations may lead to difficulties to find fixed point solutions and in constructing global flows.

- The choice of how to parametrize the metric fluctuations can be useful. The exponential parametrization, being an interesting choice by itself, can help to bypass some bad features brought in by poor truncations.

- We also propose the use the so called "physical gauge fixing" related to the metric decomposition in irreducible spin components.

- Compared to a previous approach we find some analytical solutions. In d=3 they admit a WF scaling solution. We employed spectrally adjusted and also scalar-pure cutoff.

- We have also used the same approach the analyze the linear O(N) scalar model coupled to gravity. (See Percacci talk)

Outlook

- Background metric independence has to be addressed.
  This is related to the double metric framework and the msWI.

- In this framework the reduced amount of off-shellness gives less difficulties in constructing global flows from the UV to the IR.
  Needed for any phenomenological application.

- At the level of larger truncations we have started to analyze the local truncation based on a lagrangian $F_k(\rho, R)$
  Much simpler equations that in a previous work.

- Anomalous dimensions? Fermions and vectors?

- Fermions: scenario related to an old idea of R. Brout:
  an effective scalar (inflaton) generated by the condensation of fermions via gravitational interactions?

Many thanks for your attention!