Fermions, Gravity & Chiral Symmetry

Holger Gies

Helmholtz Institute Jena & TPI, Friedrich-Schiller-Universität Jena

HELMHOLTZ GEMEINSCHAFT
Helmholtz-Institut Jena

& Astrid Eichhorn New J.Phys. 13 (2011) 125012,
& Stefan Lippoldt Phys.Rev. D87 (2013) 104026,
Phys.Rev. D89 (2014) 6, 064040, Phys.Lett. B (2015) [arXiv:1502.00918]

Probing the Fundamental Nature of Spacetime with the Renormalization Group, Nordita, March 23-27 2015

- Gravity

- Fermions:

- Chiral Symmetry:

- Gravity

  . . . see workshop title

- Fermions:

  . . . most of the luminous matter in the universe

- Chiral Symmetry:

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- Chiral Symmetry:
  - symmetry of all known fermions
  - protects mass $\Longrightarrow$ light fermions
  - non-chiral fermions expected to be heavy
  - chiral symmetry breaking (χSB): by strong & EW force

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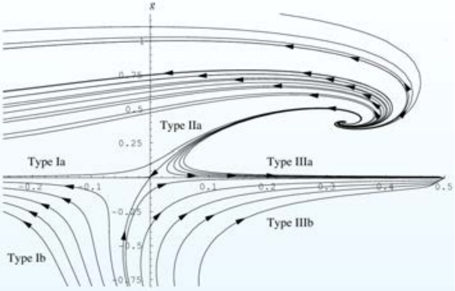
...what about gravity?

Asymptotically safe metric quantum gravity

- e.g., flow in Einstein-Hilbert theory space:

(Reuter'96)

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^D x \sqrt{g}(-R + 2\bar{\Lambda}_k)$$



$\Longrightarrow$ so far remarkably robust

(Dou,Percacci'97)

(Souma'99)

(Lauscher,Reuter'01'02)

(Reuter,Saueressig'01)

(Niedermaier'02)

(Litim'03; Pawlowski'03)

(Codello,Percacci'06)

(Codello,Percacci,Rahmede'07'08)

(Machado,Saueressig'07)

(Benedetti,Machado,Saueressig'09)

(Eichhorn,Gies,Scherer'09)

(Benedetti,Groh,Machado,Saueressig'10)

(Donkin,Pawlowksi'12)

(. . . . .)

Criteria for Asymptotic Safety

(WEINBERG'76'79)

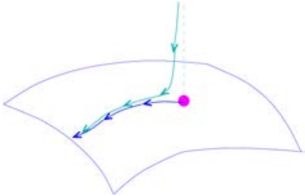
- Existence of a (non-Gaußian) fixed point g_*

$$\beta(g_*) = 0$$

- # relevant directions $\dim \mathcal{S} < \infty$

$$\{\Theta^I\} = \text{spectr}\left(-\frac{\partial \beta_i}{\partial g_j}\bigg|_{g_*} \right)$$

$$\dim \mathcal{S} = \#(\Theta^I > 0)$$



$\Longrightarrow$ Predictivity

Asymptotically safe quantum gravity in nature

- testable physical predictions

  . . . nice to have . . .

  (TALKS: A. BONANNO, A CODELLO)

- consistency with existing observations

  . . . “matter matters” . . .

Consistency with observations

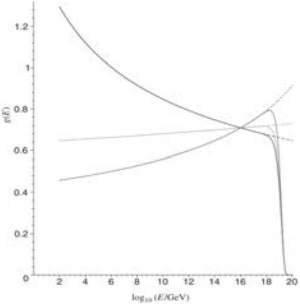
- Gravity & gauge sectors:
  - nonabelian asymptotic freedom preserved (or enhanced)

    (Wilczek,Robinson'06)

    (Pietrykowski'07;Toms'07'10;Ebert,Plefka,Rodigast'08)

    (Tang,Wu'08;Daum,Harst,Reuter'10)

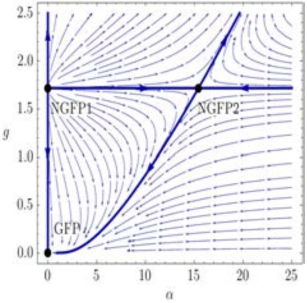
    (Folkerts,Litim,Pawlowski'12)



  - abelian sector: (Harst,Reuter'11)

    similarly UV controlled (NGFP1)

    or even higher predictivity (NGFP2)



Consistency with observations

- Gravity & matter sectors:

Properties/Existence of gravity fixed point depend on matter content

(Percacci,Perini'03)

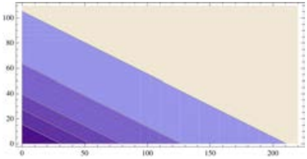
(Codello,Percacci,Rahmede'09;Vacca,Zanusso'10; Donà,Percacci'13)

one-loop:

(Donà,Eichhorn,Percacci'13)

$$g_* = \frac{12\pi}{46 + 4N_V - 2N_D - N_S}$$

$$\lambda_* = \frac{3}{4}\frac{2 + 2N_V - 4N_D + N_S}{31 + 4N_V - 2N_D - N_S}$$



Consistency with observations

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(PERCACCI,PERINI'03)

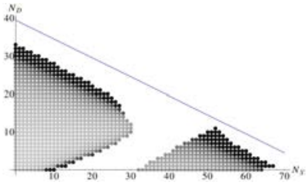
(CODELLO,PERCACCI,RAHMEDE'09;VACCA,ZANUSSO'10; DONÀ,PERCACCI'13)

(DONÀ,EICHHORN,PERCACCI'13)

refined analysis:

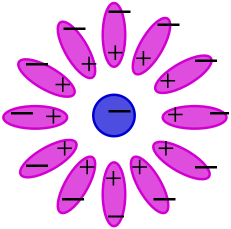
using bi-metric input

(CODELLO,D'ODORICO,PAGANI'13)



Similarity to many-flavor QCD

- charge screening:



- β function

$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_D\right)\frac{g^4}{16\pi^2} - 2\left(\frac{34{N_c}^3 + 3N_D - 13{N_c}^2 N_D}{3N_c}\right)\frac{g^6}{(16\pi^2)^2} + \dots$$

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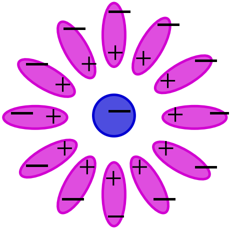

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$$\text{for } N_D > \frac{34{N_c}^3}{13{N_c}^2 - 3} \overset{\mathrm{SU(3)}}{\simeq} 8.05$$

Many-flavor QCD

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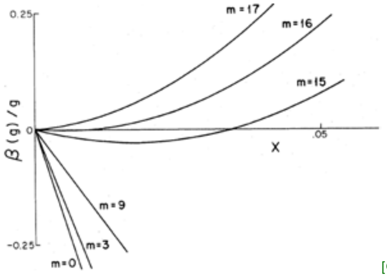
- e.g., SU(3): IR fixed point α_* (CASWELL'74; BANKS&ZAKS'82)



[CASWELL@PHYS.REV.LETT.33:244,1974]

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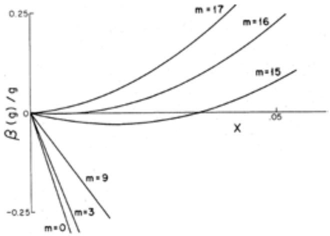
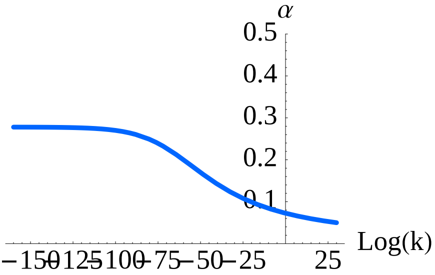
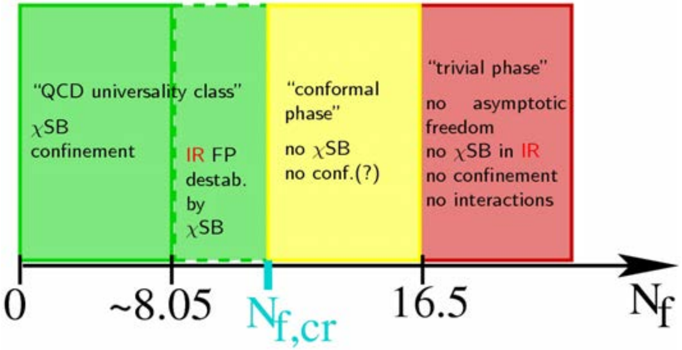
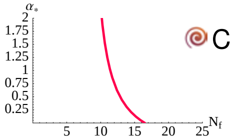
- e.g. $N_D = 14$ $\Longrightarrow$ IR fixed point





Many-flavor QCD





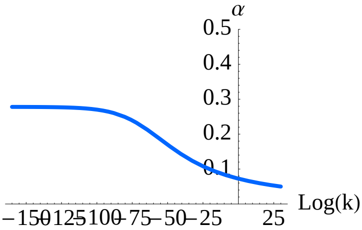
- Caswell-Banks-Zaks fixed point destabilized by χSB

$\Longrightarrow$ conformal phase $\neq$ real life QCD

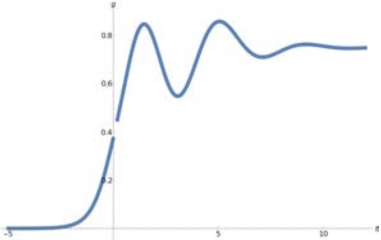
Many flavor QCD vs. Gravity

- UV to IR flow:

many-flavor QCD



Gravity

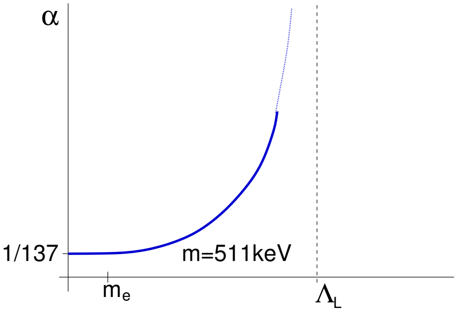


$\Longrightarrow$ check status of chiral symmetry in UV regime!

Comparison to QED

- coupling α grows towards UV (GELL-MANN,LOW'54)

. . . Landau-Pole (LANDAU'55)



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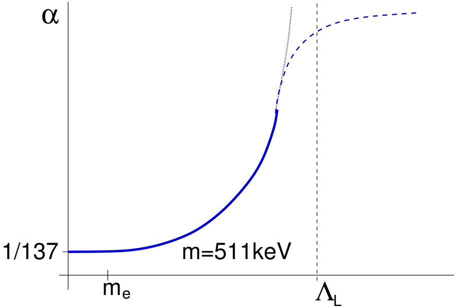

- . . . even if UV was controlled . . .

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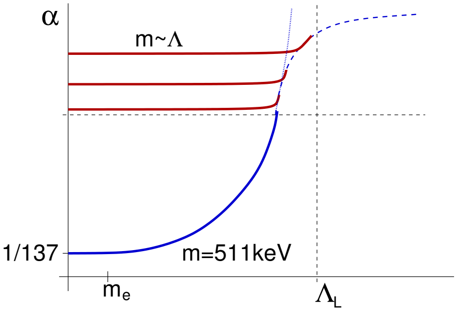


- χSB leads to heavy fermions $m \sim \Lambda$ if $\alpha > \alpha_{cr}$ (MIRANSKY'85)

$\Longrightarrow$ Strong QED and "real" QED not on a line of constant physics

(GOCKELER ET AL.'98; HG,JAECKEL'04)

Chiral Symmetry Breaking (χSB) in fermionic systems

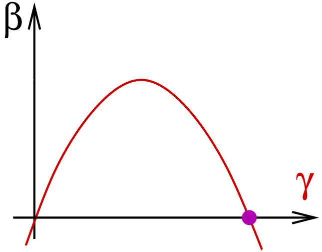
- e.g., Gross-Neveu model in 3 dimensions:

$$\Gamma_k = \int d^3x \; \bar{\psi} i \partial\!\!\!/ \, \psi + \frac{1}{2} \bar{\gamma} (\bar{\psi}\psi)^2 + \ldots \quad , \quad [\bar{\gamma}] = -1$$

- dim'less coupling $\gamma = k \, \bar{\gamma}$

$$\partial_t \gamma = \gamma - c \, \gamma^2$$

- UV fixed point $\gamma_* = 1/c$
- critical exponent $\Theta = 1$



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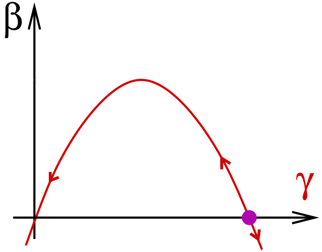

$\Longrightarrow$ asymptotically safe

(Gawedzki, Kupiainen'85; Rosenstein, Warr, Park'89; de Calan et al.'91; Braun,HG,Scherer'11; Janssen,HG'12;)

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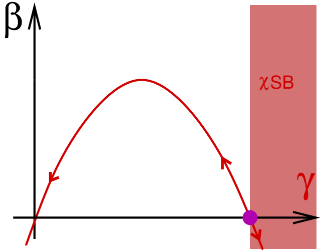


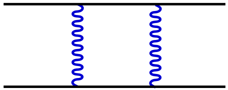
$\Longrightarrow$ fixed point = critical point γ_{cr} of χSB quantum phase transition

Interaction-induced χSB

- e.g., gauge interactions:

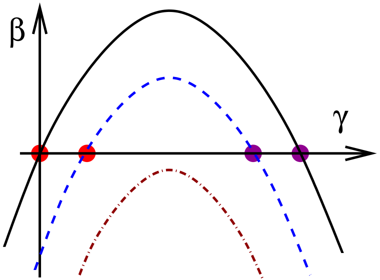
$$S_{\text{int}} \sim \int d^d x \, \dots \, \bar{g} \, \bar{\psi} \not{A} \psi$$

- gauge-boson exchange:



- coupling flow:

$$\partial_t \gamma = \gamma - c \, \gamma^2 - c_G \, g^4$$



$\Longrightarrow$ fixed-point collision: dynamical χSB for $g^2 > g_{\text{cr}}^2$

(HG,Jaeckel'04; Braun,HG'06; Braun,HG,Scherer'11; Janssen, HG'12; Mitter,Pawlowski,Strodthoff'14)

Gravity-induced χSB?

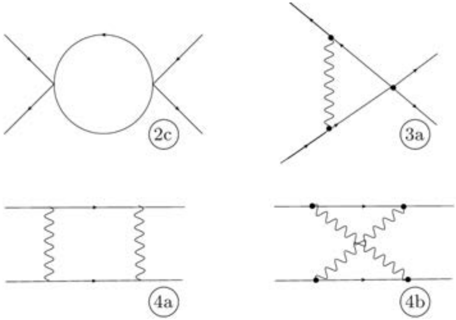
(Eichhorn,HG'11)

- chiral fermions and quantum gravity, $\mathrm{SU}(N_D)_R \times \mathrm{SU}(N_D)_L$ in $D = 4$:

$$\Gamma_{\mathrm{int},k} = \int d^4x \sqrt{g}\, \frac{1}{2} \int d^4x \sqrt{g} \left[\bar{\lambda}_- (V - A) + \bar{\lambda}_+ (V + A) \right]$$

$$V = \left(\bar{\psi}^i \gamma_\mu \psi^i\right) \left(\bar{\psi}^j \gamma^\mu \psi^j\right), \quad A = -\left(\bar{\psi}^i \gamma_\mu \gamma^5 \psi^i\right) \left(\bar{\psi}^j \gamma^\mu \gamma^5 \psi^j\right)$$

- Diagrammar:



cancellations!

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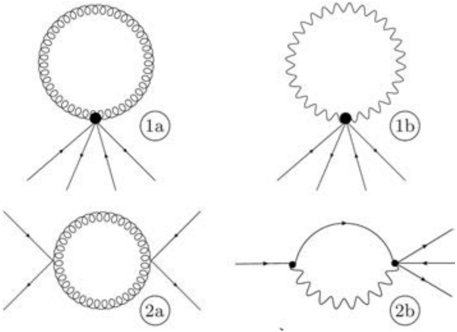

specific to gravity!

Gravity-induced χSB?

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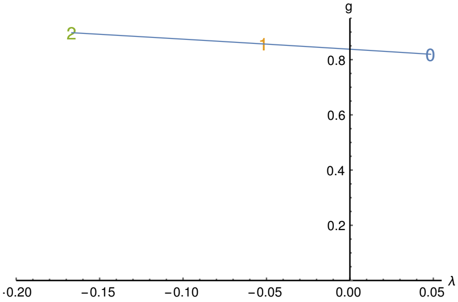
- RG flow, e.g. λ_+:

$$\partial_t \lambda_+ \quad = \quad 2\lambda_+ + c\lambda_+^2 + b_{G\lambda_+} \frac{g\lambda_+}{(1 - 2\lambda)^2} + c_G \frac{g^2}{(1 - 2\lambda)^3}$$

  - attractive gravitational binding $\sim c_G > 0$
  - effective repulsive gravity-fermion interplay $\sim b_{G\lambda_+} > 0$
  - gravitational decoupling for $\lambda < 0$

Gravity-induced χSB?

- N_D dependence of gravitational fixed point (DONÀ,EICHHORN,PERCACCI'13)



  - e.g., one-loop

$\Longrightarrow$ attraction, repulsion, decoupling @ work

Gravity-induced χSB?

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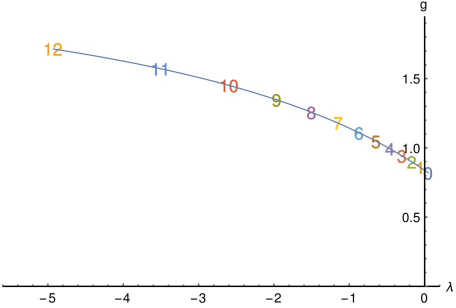

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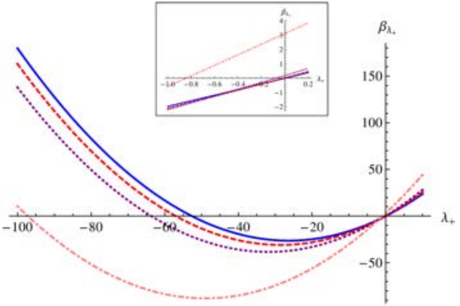
Gravity-induced χSB?

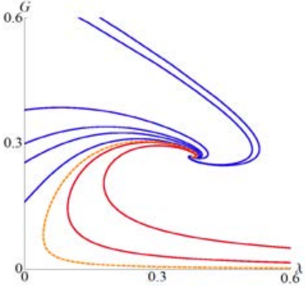
- no fixed-point collision from gravity fluctuations (EICHHORN,HG'13)



- $D = 4$: repulsive gravity-fermion interplay dominates
- existence of light fermions compatible with asymptotic safety
- non-Gaußianity: $\Longrightarrow$ shifted Gaußian fixed point
- so far no constraint on matter

Asymptotic Safety as a quantum phase transition

- dS/AdS quantum phase transition



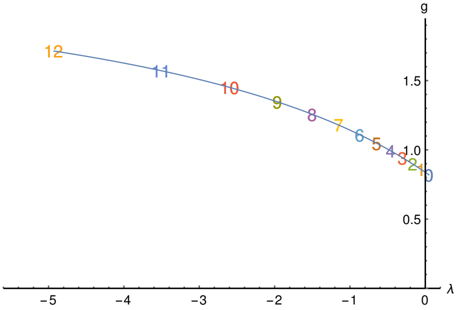
- spacetime structure in the fixed-point regime: (LAUSCHER,REUTER'05)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\bar{\Lambda} g_{\mu\nu} \quad \rightarrow R = \frac{2d}{d-2} \bar{\Lambda}$$

. . . . . . Einstein-Hilbert . . .)

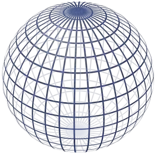
Spacetime in fixed point regime

- spacetime curvature in fixed-point regime: matter dependent:

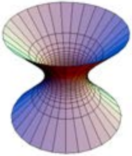


$\Longrightarrow$ Chiral symmetry needs to be studied on curved background.

Gravitational Catalysis



- positive curvature screens IR fluctuations



- negative curvature enhances IR fluctuations

- effective dimensional reduction of Dirac spectrum on H^d (GORBAR'08)

$$d + 1 \rightarrow 1 + 1$$

$\Longrightarrow$ negative curvature supports χSB (BUCHBINDER,KIRILLOVA'89)

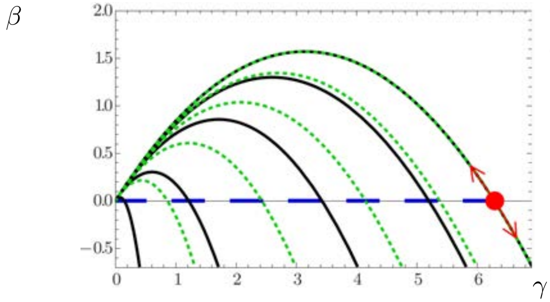
(INAGAKI,MUTO,ODINTSOV'93'97; WACHS,WIPF'93; ELIZALDE ET AL.'96; MIELE,VITALE'96'98; GORBAR,GUSYNIN'07)

opposite effect for bosons (BENEDETTI'14)

Gravitational Catalysis á la RG

- e.g., Gross-Neveu model in $D = 3$: (HG,LIPPOLDT'13)

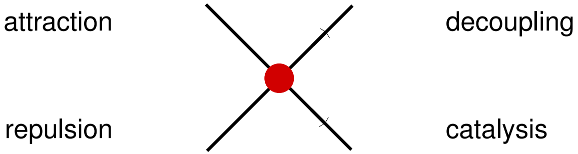
$$\partial_t \gamma = \beta_\gamma = \gamma - N_D \frac{\gamma^2}{\pi} \left(1 + \frac{|R|}{24k^2}\right), \quad R < 0$$



- AdS
- Lobachevsky plane

- χSB even for the weakest attraction (for fixed $R < 0$)
- fermion mass, e.g., $m_{\mathrm{f}} \sim k_{\mathrm{UV}}$ (weak coupling, large fixed $R < 0$)

Gravity-induced χSB, Part II?



- Simple model for gravity-induced χSB?
  - $D = 4$: flavor singlet channels: $(V \pm A)$

    catalysis in vector-channels not yet studied

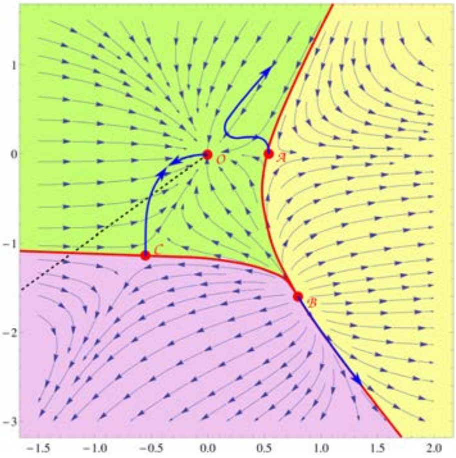
  - $D = 3$: gravity preserves $\mathrm{U}(2N_D)$ symmetry

    singlet channels: $(\bar{\psi}\gamma_{45}\psi)^2, \quad (\bar{\psi}\gamma_{\mu}\psi)^2$

    no gravity-induced attraction in singlet channel

Gravity-induced χSB, Part II?

- $D = 3$: fixed-point collision in mixed channel expected
  $\Longrightarrow$ symmetry breaking pattern: $U(2N_D) \to U(N_D) \times U(N_D)$



e.g., QED_3

(Braun,HG,Janssen,Roscher'14)

…as difficult as $D = 4$

Gravity-induced χSB, Part II

- a generic one-channel model ($D = 3$):

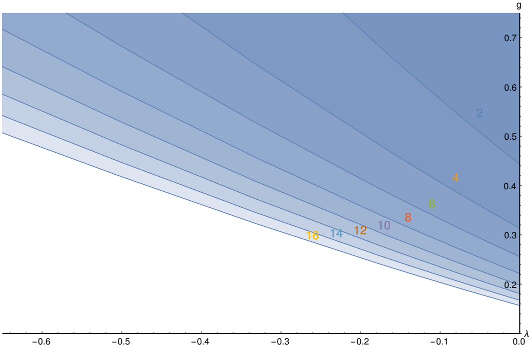
$$\partial_t \gamma = \beta_\gamma = \gamma - N_D \frac{\gamma^2}{\pi}\left(1 + \frac{|R|}{24k^2}\right) - c_G \frac{g^2}{(1-2\lambda)^3}, \quad R < 0$$

Gravity-induced χSB, Part II

- a generic one-channel model ($D = 3$) near the fixed point:

$$\partial_t \gamma = \beta_\gamma = \gamma - N_D \frac{\gamma^2}{\pi} (1 - c_{\text{Cat}} \lambda) - c_G \frac{g^2}{(1-2\lambda)^3}, \quad \lambda < 0$$

- fixed-point collision $\Longrightarrow$ χSB for N_D flavors, e.g. $c_{\text{Cat}} = \frac{1}{4}$, $c_G = 2$



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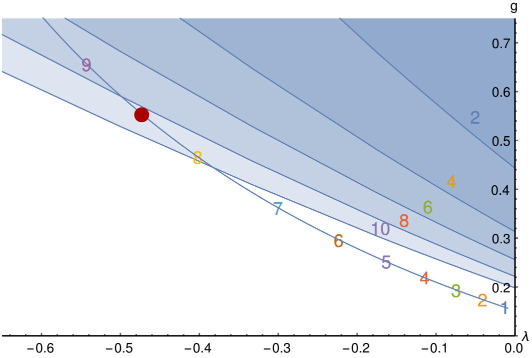

$N_{D,\text{cr}} \simeq 8.5$

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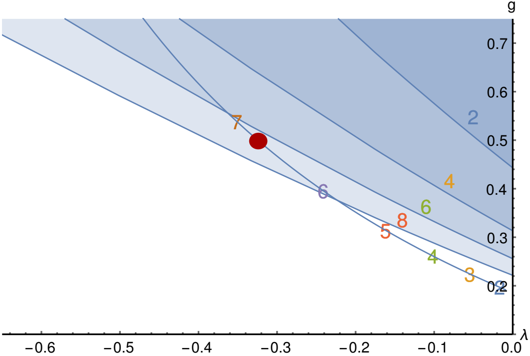

$N_{D,\text{cr}} \simeq 6.7$

Gravity-induced χSB, Part II

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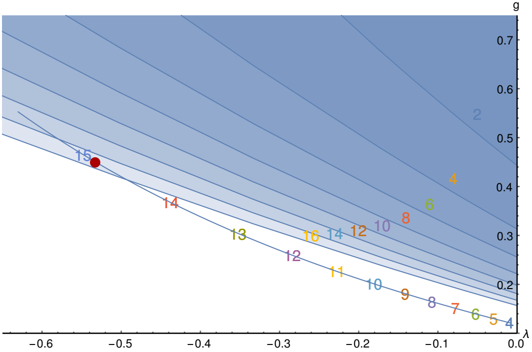
- gravity fixed point for $N_S = 4$, $N_V = 4$,



$N_{D,\text{cr}} \simeq 15$

Gravity-induced χSB

- details matter
  - Fierz completeness
  - finite $\mathcal{O}(1)$ corrections vs. large-N_D expansion
  - residual interactions of scalar, gauge, etc. sectors (EICHHORN'12)

    shifted Gaußian fixed point
  - anomalous dimensions
  - bimetric treatment

    ...weakens link between decoupling and catalysis
  - depends on quantum degrees of freedom

    ...e.g., catalysis irrelevant for unimodular gravity

    (EICHHORN'13'15)

Gravity, Fermions & Chiral Symmetry

- mechanisms of gravity-induced χSB

  ... (not only) put a constraint on N_D

- gravity-induced χSB + asymptotically safe gravity

  ... yields a dynamical theory of N_D

  - Given a UV theory with $N_{D,\mathrm{UV}} > N_{D,\mathrm{cr}}$
  - χSB $\Longrightarrow$ $N_{D,\mathrm{light}} < N_{D,\mathrm{cr}}$

    ... depending on breaking patter

    CAVE: Goldstone bosons!

  - Observation: $N_{D,\mathrm{SM}} \leq N_{D,\mathrm{light}}$

Is there a theory of the discrete numbers of nature?

- N_D ✓

  gravity + fermions + χSB

- N_V (✓)

  YM theories + Higgs mechanism

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- N_S ?

Is there a theory of the discrete numbers of nature?

- N_D ✓

gravity + fermions + χSB

- N_V (✓)

YM theories + Higgs mechanism

- N_S ?

- D ?

Why $D = 4$?

$N_O = 3$

$$N_O = 3$$

$N_O = 3$

Thanks for all your efforts!!!*

* uncontroversial statement

Hen or Egg?

- quantum fields $\hat{=}$ representations of Lorentz group
  - fundamental (nontrivial) building block: spin $\frac{1}{2}$
  - ...perhaps all fields are made of fermions
  - ...perhaps also the graviton

$\Longrightarrow$ fermions first! (Hebecker,Wetterich'03; Diakonov'11)

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  - metric is needed beforehand

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$\Longrightarrow$ metric first! ...or Dirac matrices first?

Fluctuating fermions and metric gravity?

- fermions: $\mathcal{D}\bar{\psi}\mathcal{D}\psi$; gravity: $\mathcal{D}g$, $\mathcal{D}e$, $\mathcal{D}\Gamma, \ldots$?

(CAPPOVILLA,JACOBSON,DELL'91; DAUM,REUTER'10; HARST,REUTER'12; DONÀ,PERCACCI'12)

- guiding principle: simplicity (LIPPOLDT,HG'14'15)
  - symmetries preserving the Clifford algebra: $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$
  - Diff's: $g'_{\mu\nu} = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\lambda}{\partial x'^\nu} g_{\rho\lambda}$
  - spin-base transformation: $\mathcal{S} \in \mathrm{SL}(4, \mathbb{C})$ (SCHRÖDINGER'32; BARGMANN'32)

$$\psi \to \mathcal{S}\psi, \quad \bar{\psi} \to \bar{\psi}\mathcal{S}^{-1}$$

  - most general Dirac matrix transformation (PAULI'36)

$$\gamma_\mu \to \gamma'_\mu = \frac{\partial x^\rho}{\partial x'^\mu}\mathcal{S}\gamma_\rho\mathcal{S}^{-1}$$

- spin-base group factors out of functional integral (HG,LIPPOLDT'14)

$\Longrightarrow$ metric-based quantum gravity with fermions ("as usual")