asymptotic safety in perturbation theory and beyond

Daniel F Litim

University of Sussex

work with:
F Sannino
M Mojaza, F Sannino
K Falls, K Nikolakopoulos, C Rahmede
A Juettner, E Marchais

NORDITA

Probing the Fundamental Nature of Spacetime with the Renormalization Group

23-27 March 2015 | Nordita, Stockholm

probing the fundamental nature of quantum field theories

Daniel F Litim

University of Sussex

work with:
F Sannino
M Mojaza, F Sannino
K Falls, K Nikolakopoulos, C Rahmede
A Juettner, E Marchais

NORDITA

Probing the Fundamental Nature of Spacetime with the Renormalization Group

23-27 March 2015

Nordita, Stockholm

fixed points of QFTs

Gaussian

$$g_* = 0$$

anomalous dimensions

$$\eta_A = 0$$

canonical power counting

$$\{\vartheta_{\mathrm{G},n}\}$$ are known

Gaussian vs non-Gaussian

Gaussian	non-Gaussian
$g_* = 0$	$g_* \neq 0$
anomalous dimensions	anomalous dimensions
$\eta_A = 0$	$\eta_N \neq 0$
canonical power counting	**non-canonical** power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known	$\{\vartheta_n\}$ are **not** known

asymptotic freedom vs asymptotic safety

asymptotic freedom	asymptotic safety
$g_* = 0$	$g_* \neq 0$
anomalous dimensions	anomalous dimensions
$\eta_A = 0$	$\eta_N \neq 0$
canonical power counting	non-canonical power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known	$\{\vartheta_n\}$ are not known
F^{256} irrelevant !	R^{256} relevant / marginal / irrelevant ?

asymptotic safety from perturbation theory

interacting fixed point

theory with coupling α**:**

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

perturbative non-renormalisability: $A > 0$

interacting fixed point

theory with coupling α **:**

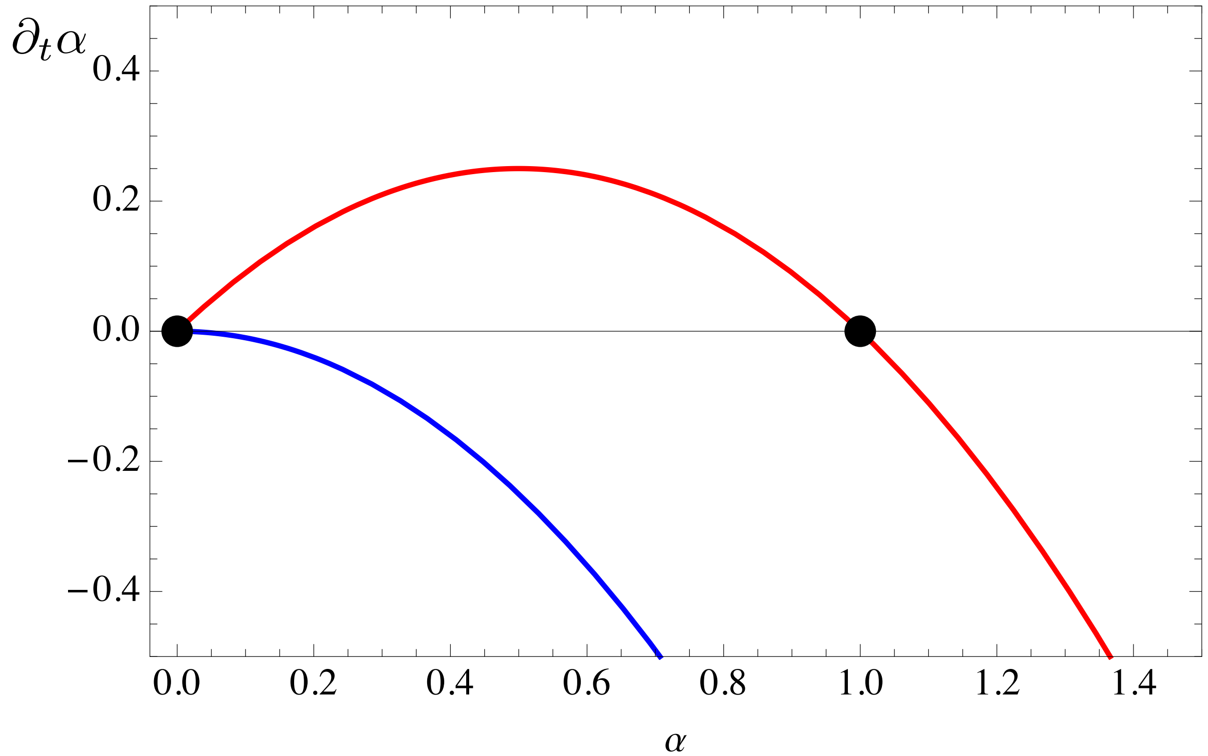
$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

if $A > 0\,,\ B > 0$ **:** **IR** **UV**



interacting fixed point

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

epsilon expansion: $\epsilon = D - D_c$

large-N expansion: many fields

perturbation theory

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

gravitons	$D = 2 + \epsilon: \quad \alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Weinberg '79 Kawai et al '90
fermions	$D = 2 + \epsilon: \quad \alpha = g_{\rm GN}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4 + \epsilon: \quad \alpha = g^2_{\rm YM}(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2 + \epsilon: \quad \alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76

non-perturbative renormalisability

$$A = \epsilon \ll 1\,, \quad B = \mathcal{O}(1) > 0$$

asymptotic safety cookbook

DL, F Sannino, JHEP1214(2014)178 arXiv:1406.2337
DL, M Mojaza, F Sannino, arXiv:1501.03061
DL, F Sannino, in prep.

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$$t = \ln \mu/\Lambda$$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$$\alpha_* \ll 1$$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \dfrac{g^2\, N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2 + C \alpha_g^3$$

$\alpha_* \ll 1$

$\alpha_* = 0$ $\alpha_g^* = B/C$

large-NF,NC (Veneziano) limit:

ϵ **continuous**

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

Veneziano '79

we consider $0 < -B \equiv -B\,(\epsilon) \ll 1$

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

however:

no perturbative UV fixed point

in gauge theories with fermionic matter

Caswell '74

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \dfrac{g^2\, N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$\alpha_* = 0$ $\alpha_g^* = B/C$

➡ **scalar fields & Yukawa couplings required**

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

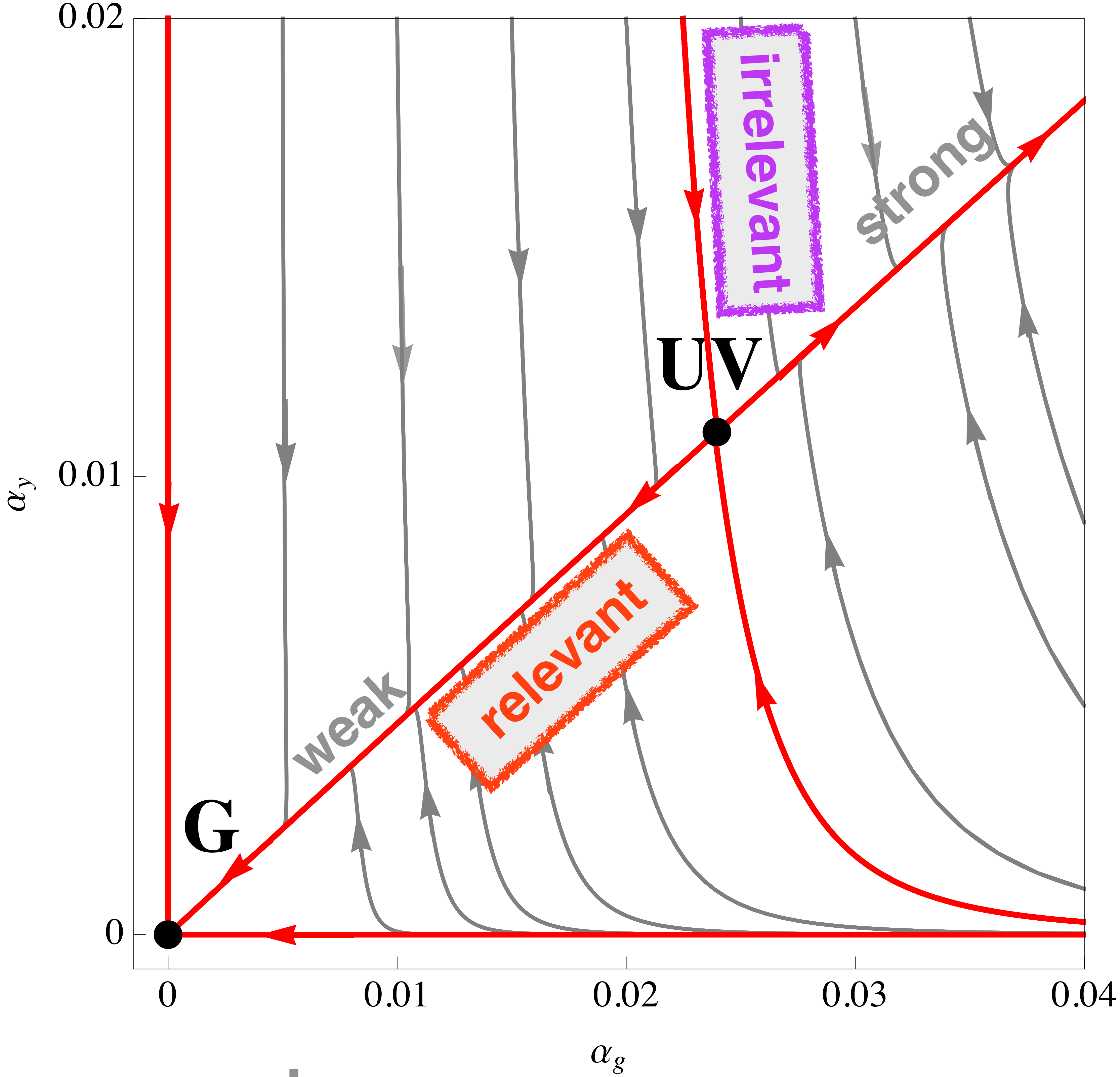
$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

→ **sensible interacting UV fixed point**

$$D\,F - C\,E > 0$$

phase diagram



UV finite theories (weak & strong)

$$\vartheta_1 \sim \mathcal{O}(\epsilon^2)$$

$$\vartheta_2 \sim \mathcal{O}(\epsilon)$$

$$\vartheta_1 > 0 > \vartheta_2$$

UV FP and scaling exponents under full perturbative control

asymptotic freedom vs asymptotic safety

asymptotic freedom		asymptotic safety	
$g_* = 0$		$g_* \neq 0$	
anomalous dimensions		anomalous dimensions	
$\eta_A = 0$		$\eta_N \neq 0$	
canonical power counting		**non-canonical** power counting	
$\{\vartheta_{G,n}\}$	are known	$\{\vartheta_n\}$	are **not** known
F^{256}	irrelevant !	R^{256}	relevant marginal irrelevant ?

asymptotic safety beyond perturbation theory

K Falls, DL, K Nikolakopoulos, C Rahmede, arXiv:1301.4191
arXiv:1410.4815
and in prep.

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

testing asymptotic safety with Ricci scalars

f(R)

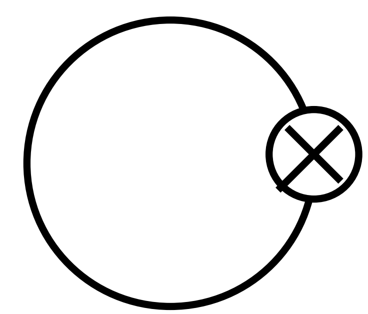
$$\Gamma_k \propto f(R)$$

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g}\, R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

technicalities: functional renormalisation

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$



here:

M Reuter hep-th/9605030

DL **hep-th/0103195**

hep-th/0312114

Falls, DL, Nikolakopoulos, Rahmede **1301.4191.pdf**

Falls, DL, Nikolakopoulos, Rahmede 1410.4815

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909

P Machado, F Saueressig 0712.0445

identifying fixed points

$$f(R) = \sum_n \lambda_n R^n$$

polynomial expansion

generating function

$$\partial_t f + 4f - 2R\, f' = I[f]$$

$$I[f] = I_0[f] + I_1[f] \cdot \partial_t f' + I_2[f] \cdot \partial_t f''$$

recursive solution of $\beta_n \equiv \partial_t \lambda_n$ $\quad \beta_{n-2} = 0$

family of FP candidates $\lambda_n = \lambda_n(\lambda_0, \lambda_1)$

`free' parameters (λ_0, λ_1)

interlude: Wilson-Fisher FP

$$u(\rho) = \sum_{n=0} \frac{\lambda_n}{n!} \rho^n \qquad \rho = \frac{1}{2}\phi^a \phi_a$$

polynomial expansion

generating function

$$\partial_t u' = -2u' + (d-2)\rho u'' - A \frac{u''}{(1+u')^2} - B \frac{3u'' + 2\rho u'''}{(1+u'+2\rho u'')^2}$$

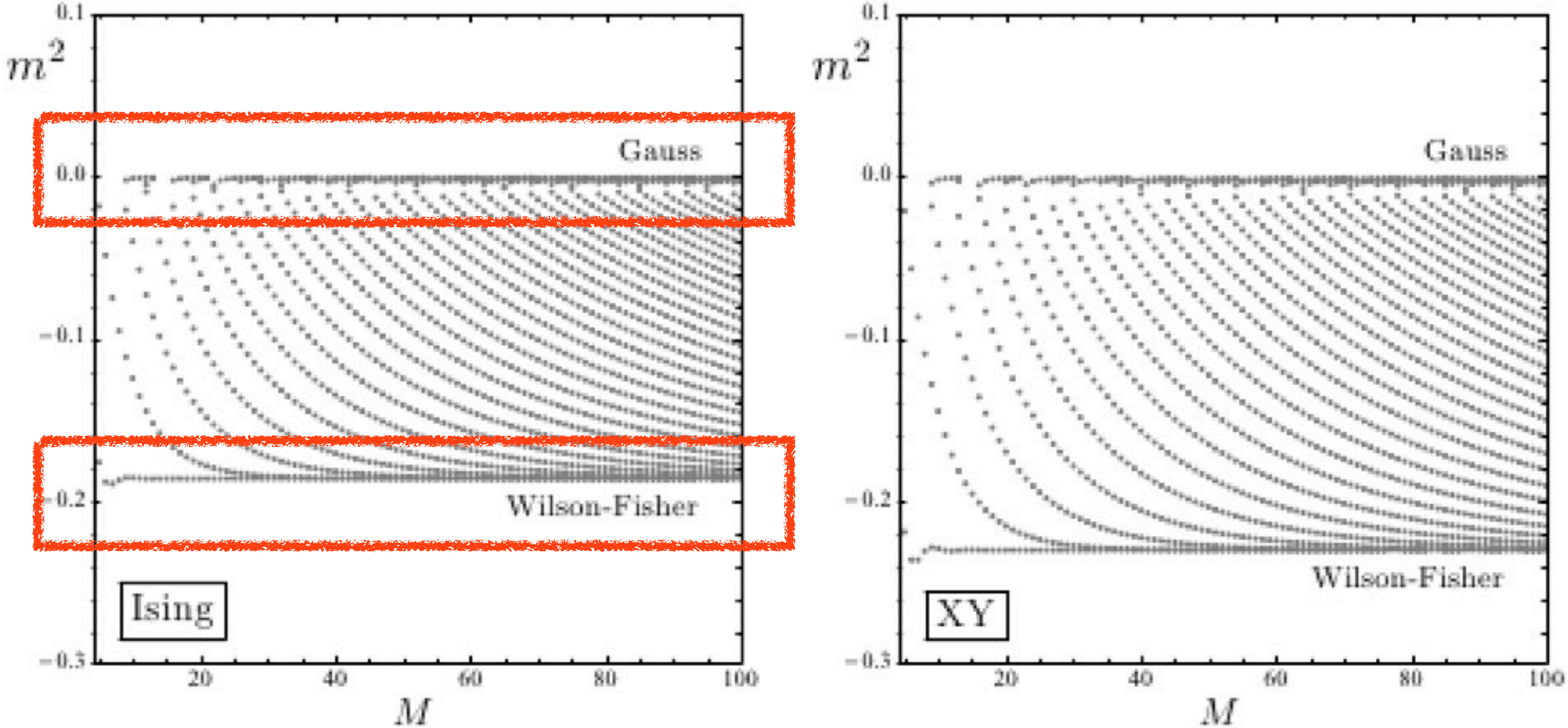
recursive solution $(\lambda_1 \equiv m^2)$

$$\lambda_n = \lambda_n(m^2)$$

Wilson-Fisher LPA

FP solutions with $\lambda_M = 0$

Juettner, DL, Marchais (in prep), arXiv:1504.00xyz



physical FP = accumulation point

Wilson-Fisher LPA

DL, hep-th/0203006

universal eigenvalues

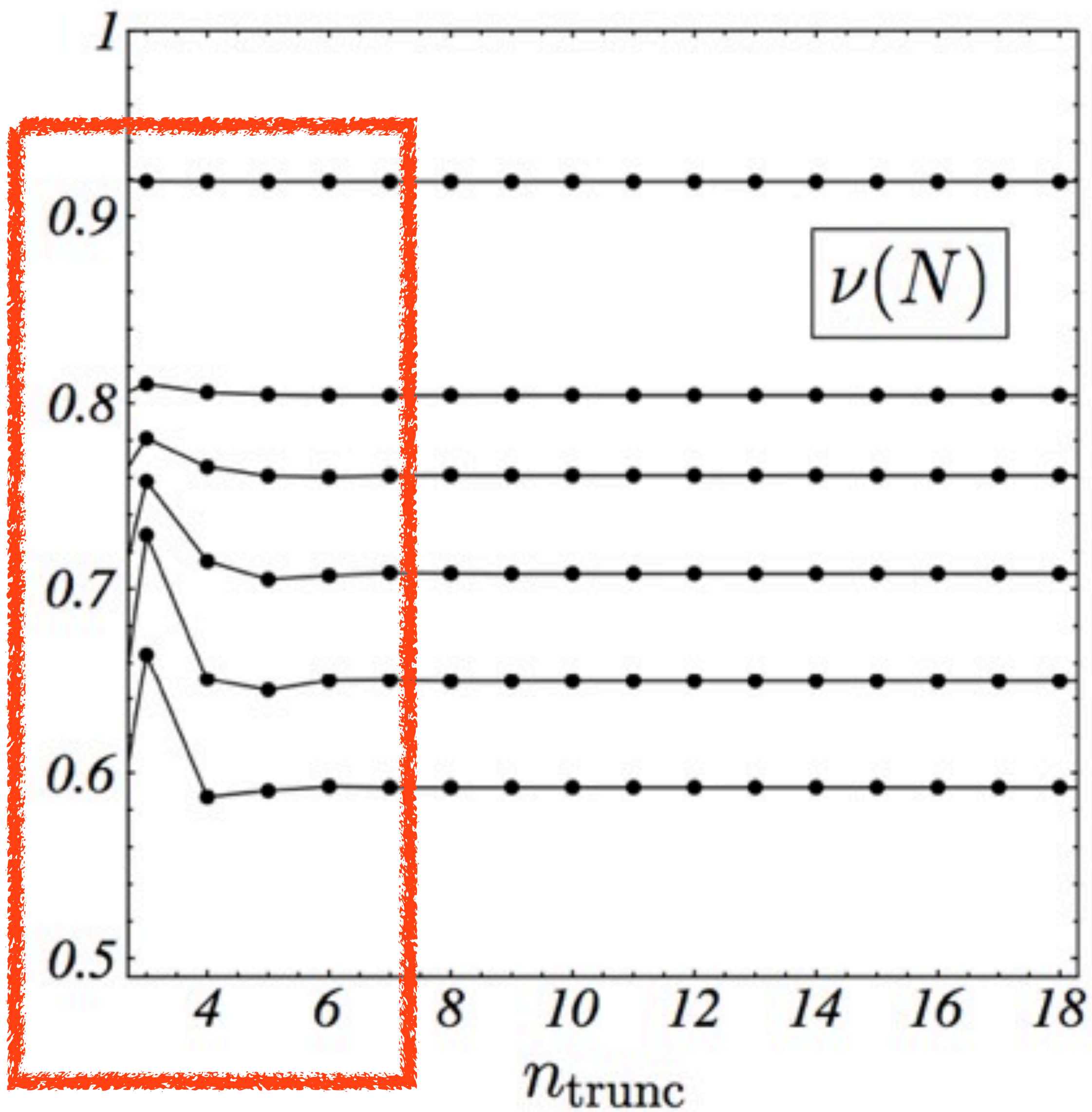


Figure 4: The exponent $\nu(N)$ as a function of N and of the order of the truncation. From top to bottom: $N = 10, 4, 3, 2, 1, 0$.

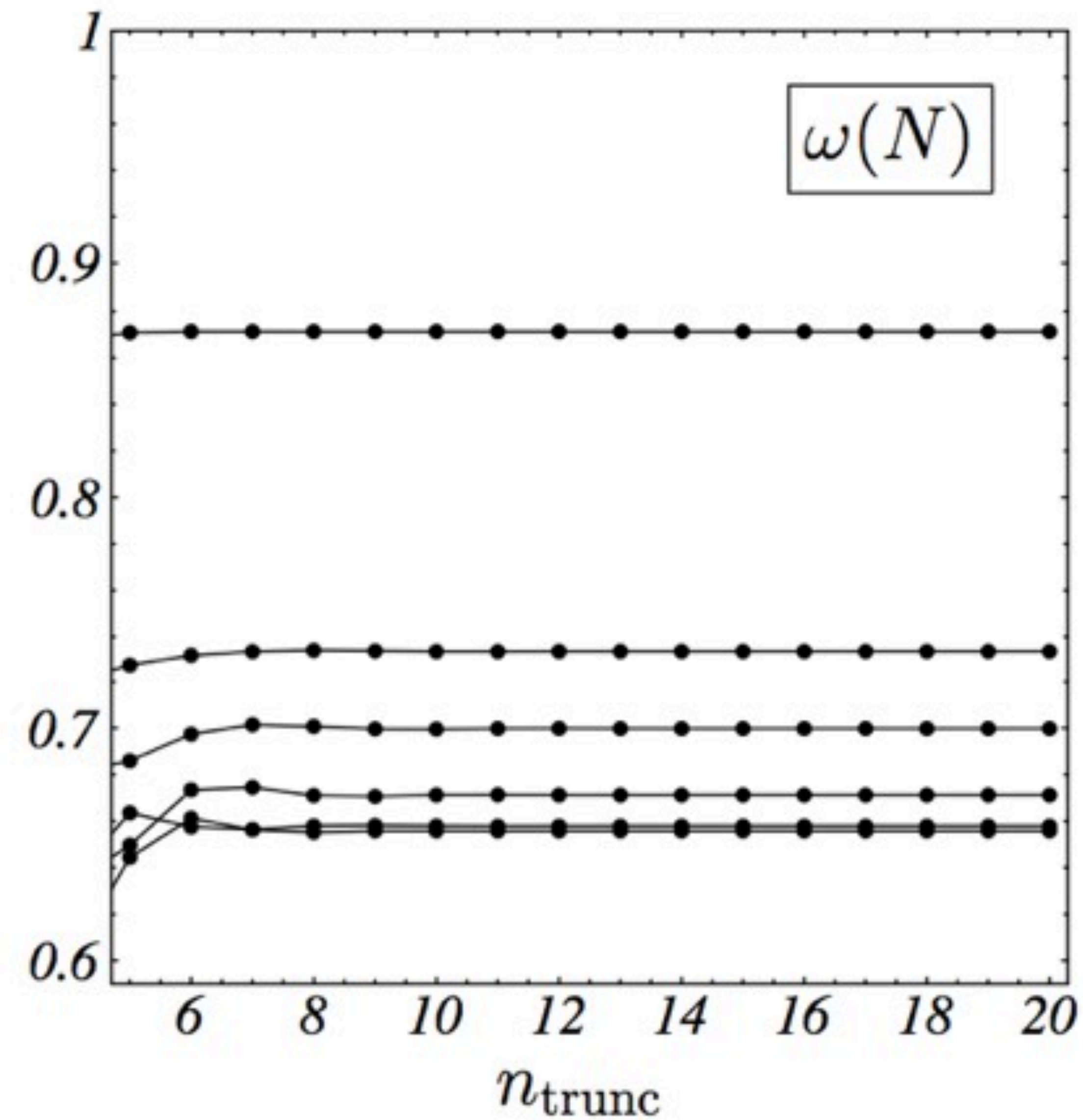


Figure 5: The eigenvalue $\omega(N)$ as a function of N and of the order of the truncation. From top to bottom: $N = 10, 4, 3, 2, 0, 1$.

f(R)

recursive solution

$$\lambda_n(\lambda_0, \lambda_1) = \frac{P_n(\lambda_0, \lambda_1)}{Q_n(\lambda_0, \lambda_1)}$$

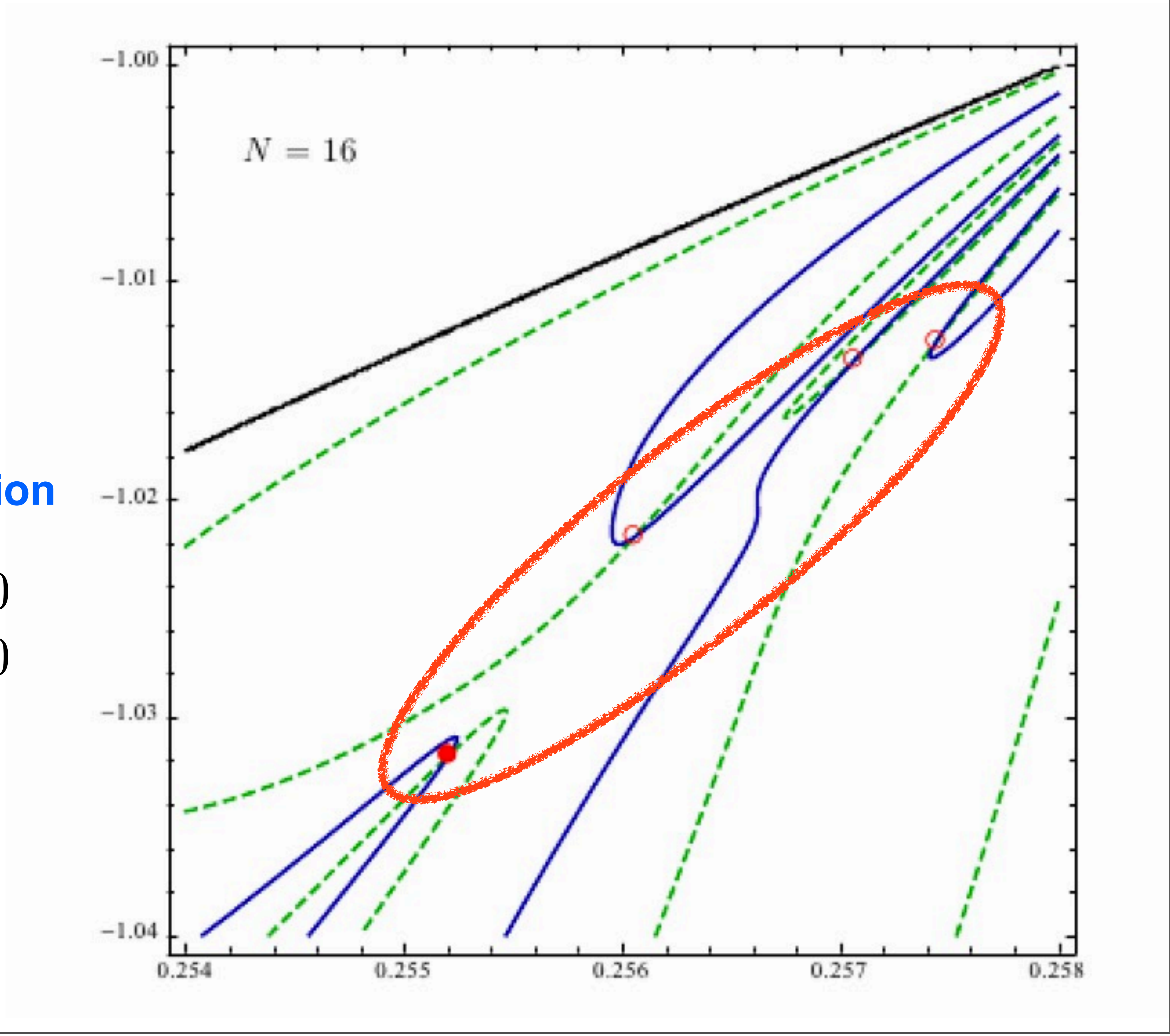
boundary condition

$$\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$$

polynomials grow large, eg.

$P_{35} \approx 45.000$ terms

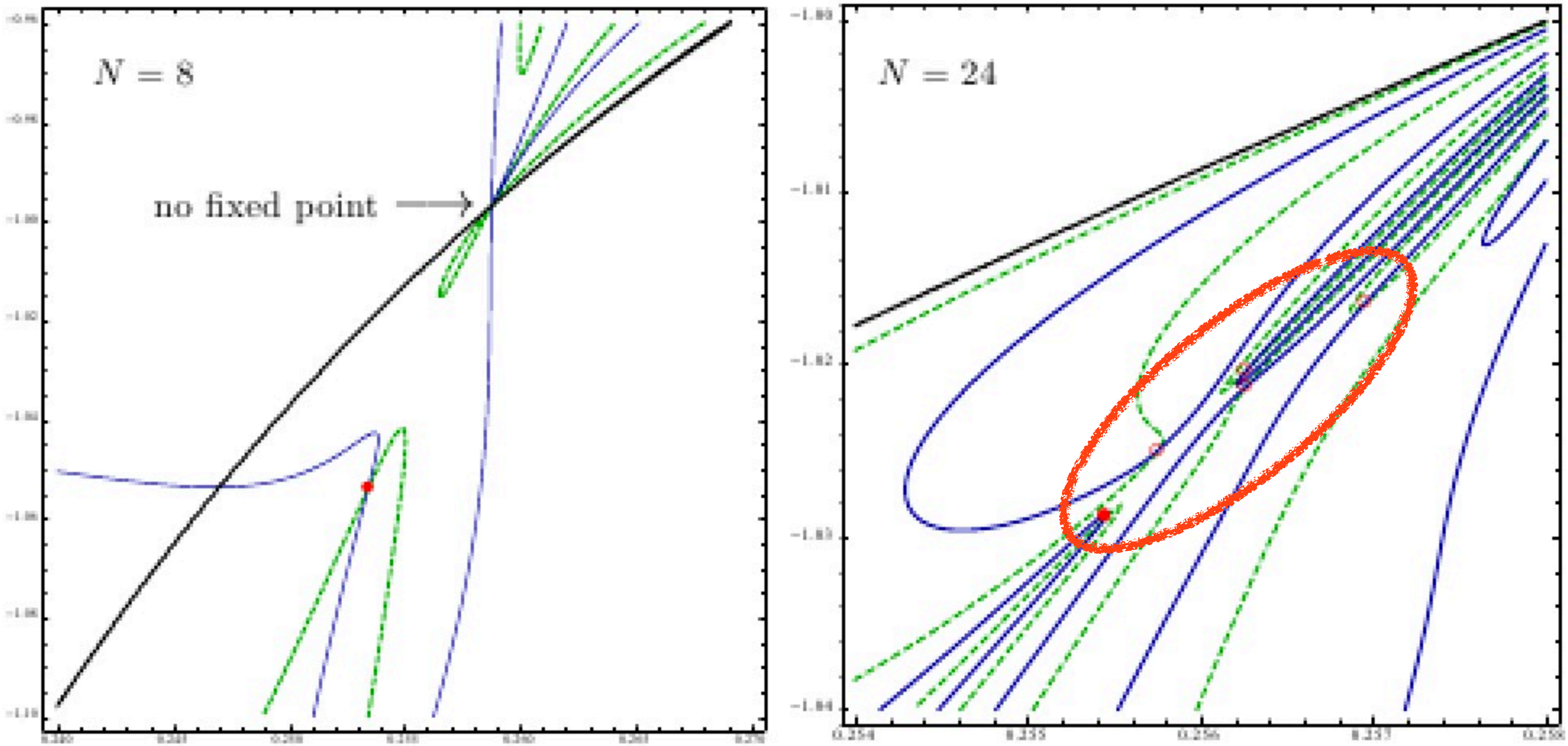
f(R)



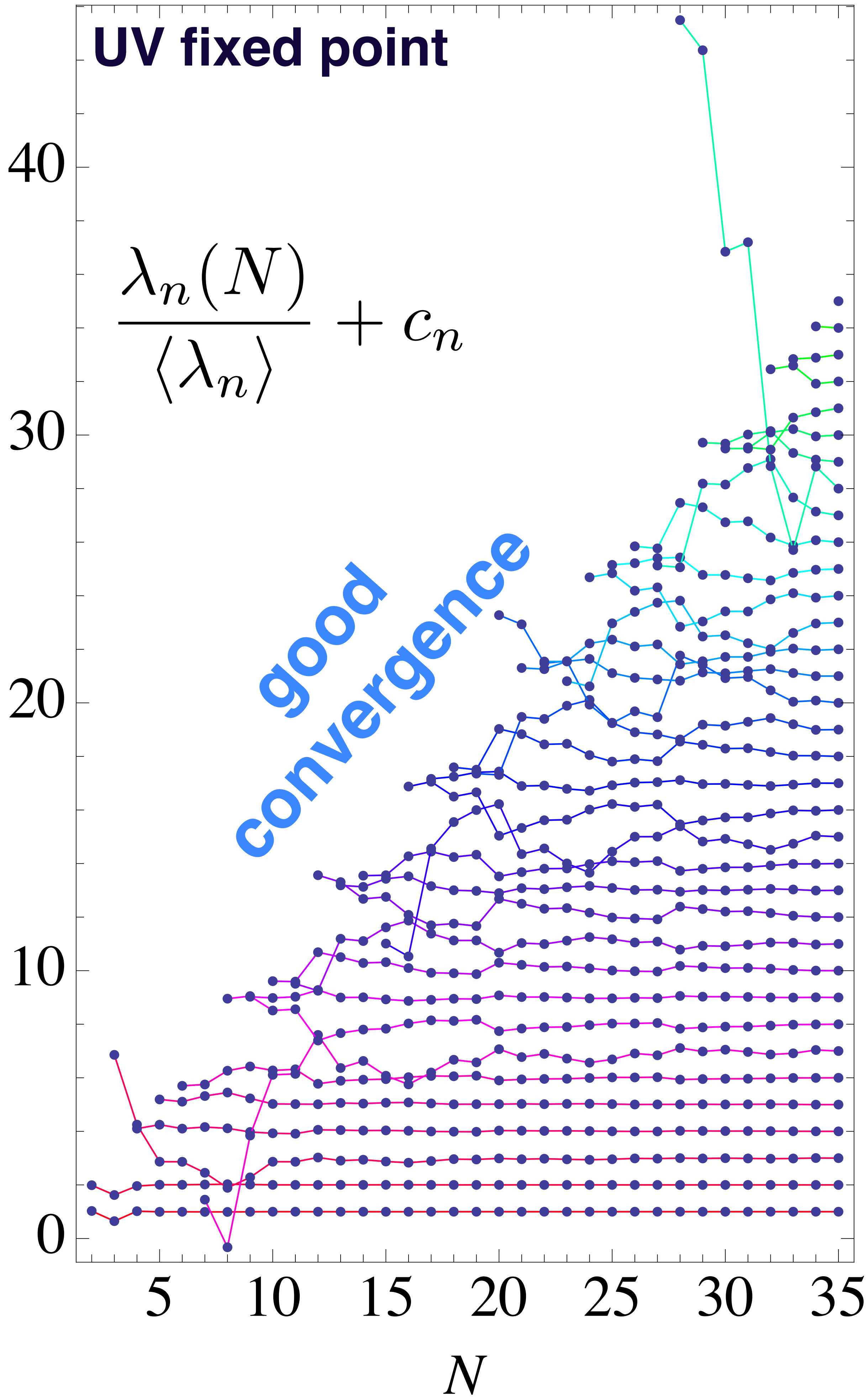
fixed point condition

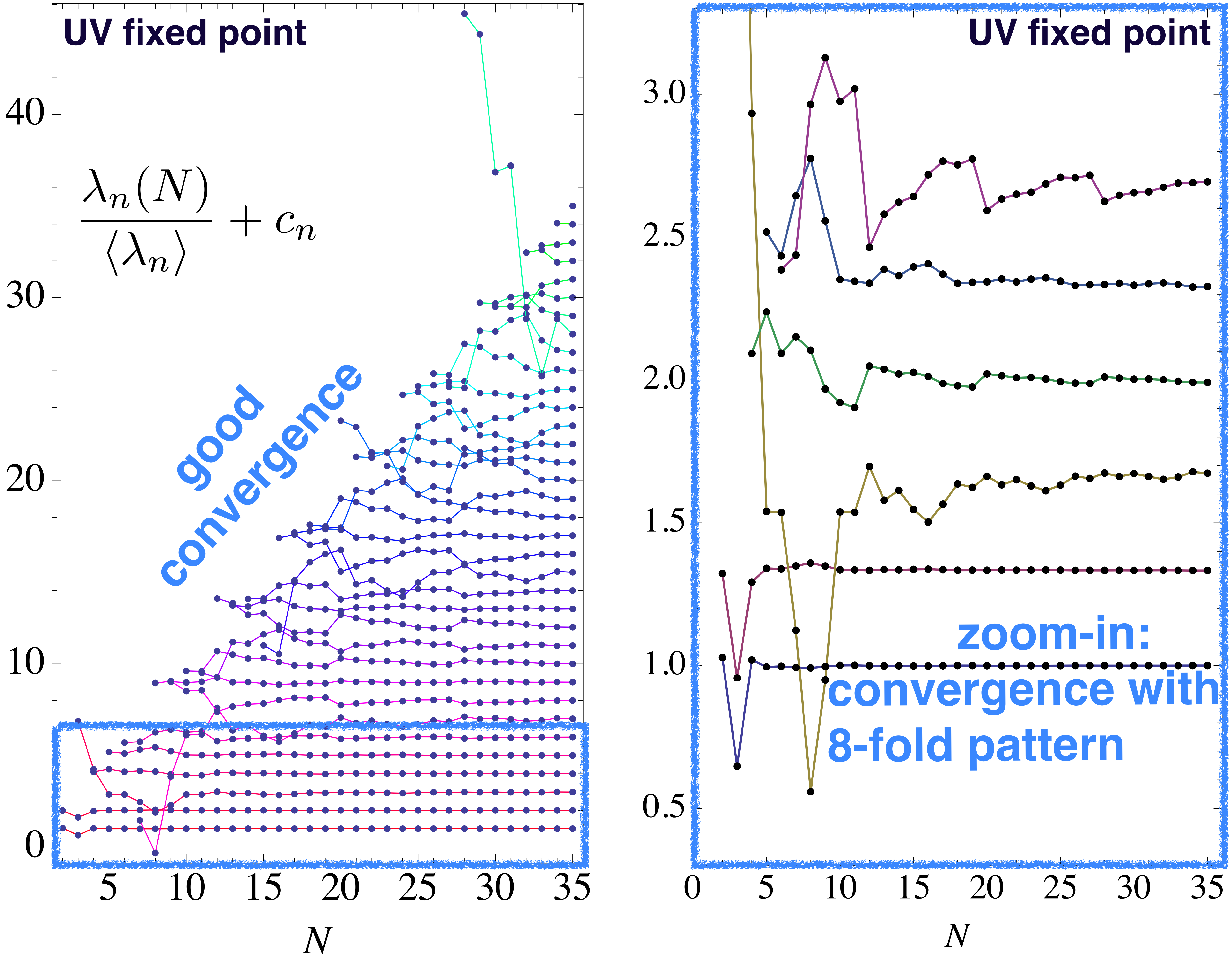
$$\lambda_N = 0$$
$$\lambda_{N+1} = 0$$

f(R)

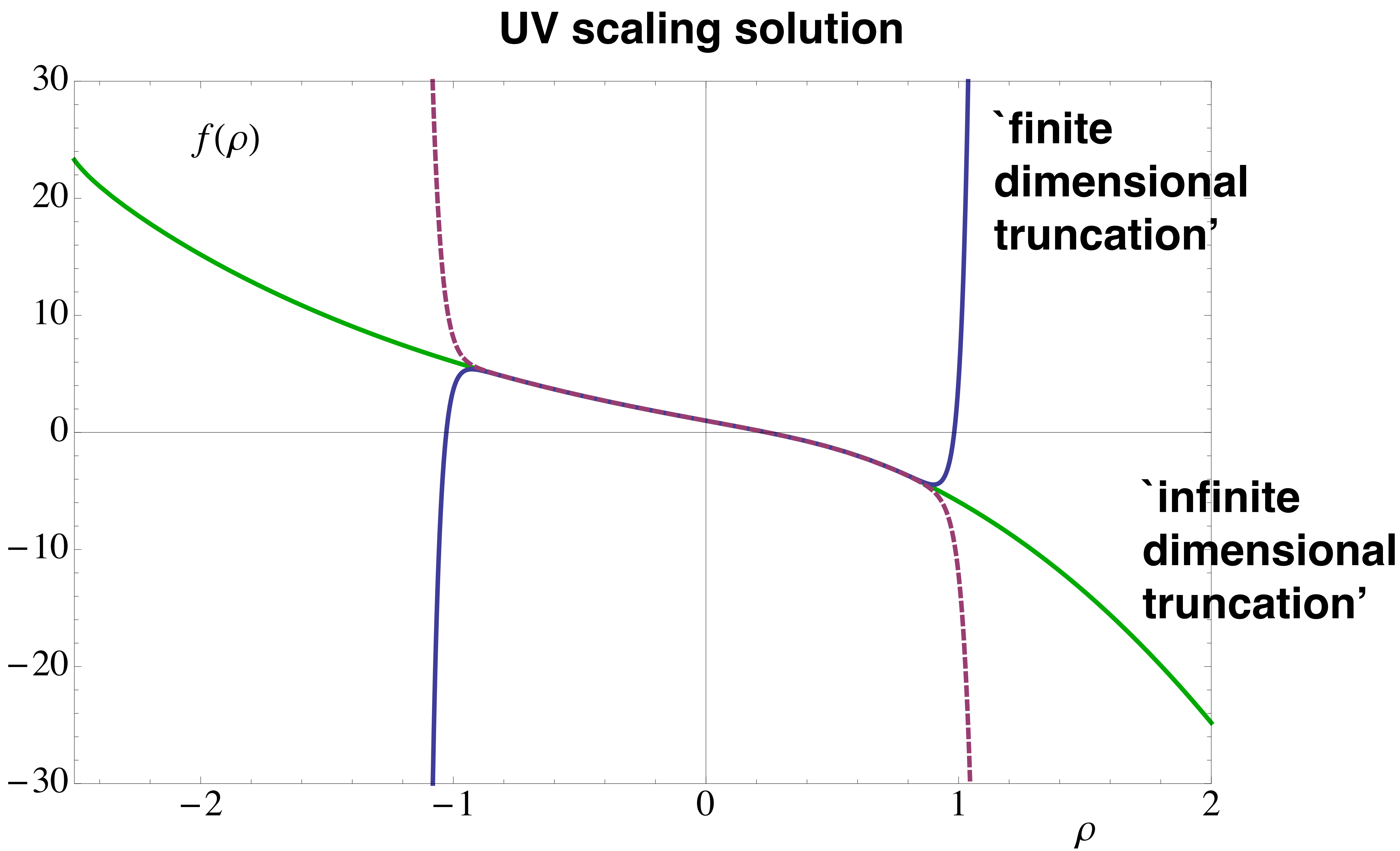


boundary condition $\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$

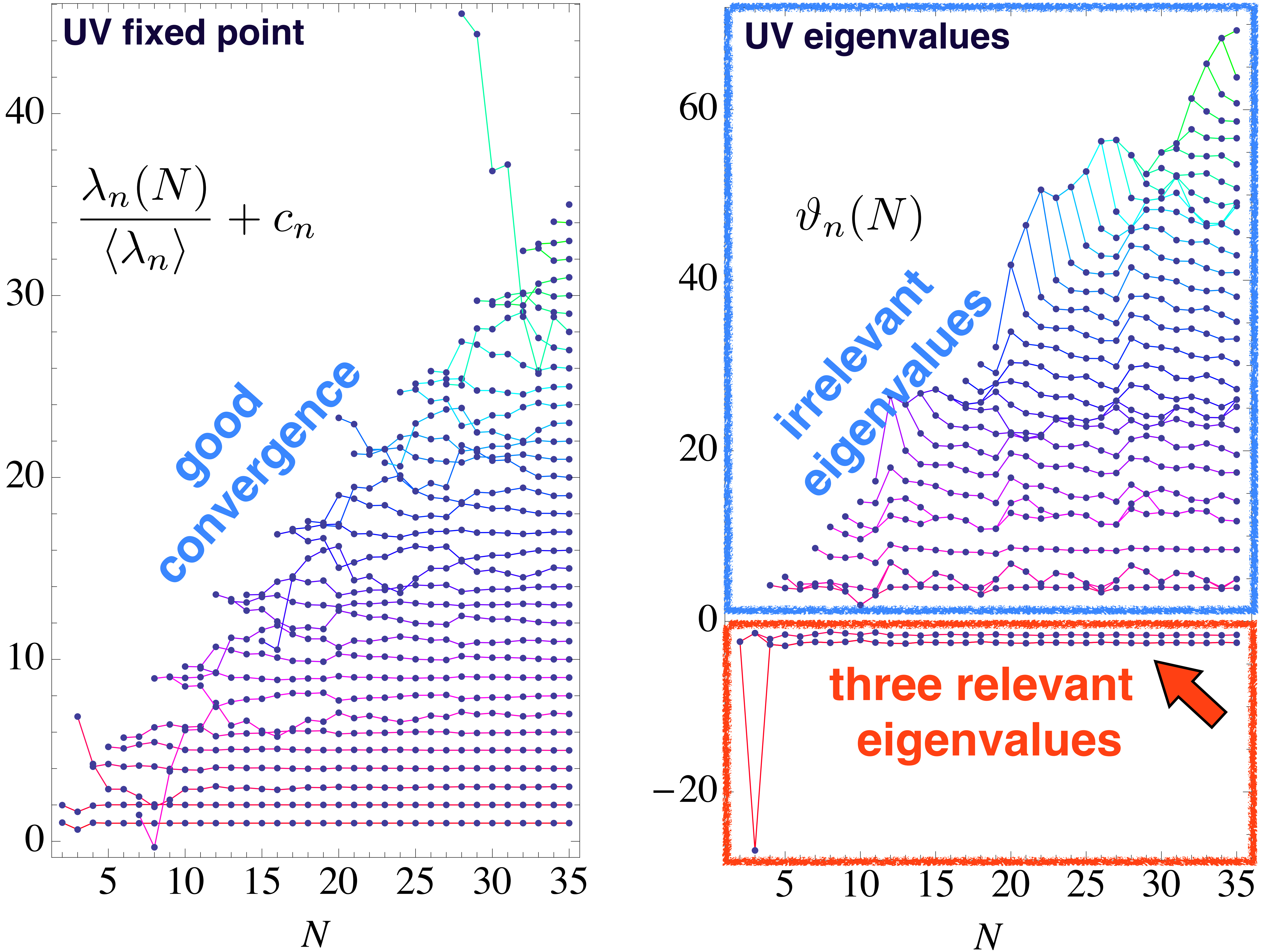

UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
0
10
20
30
40
5
10
15
20
25
30
35
N


UV fixed point
$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$
good convergence
N
UV fixed point
zoom-in: convergence with 8-fold pattern
N

f(R) quantum gravity

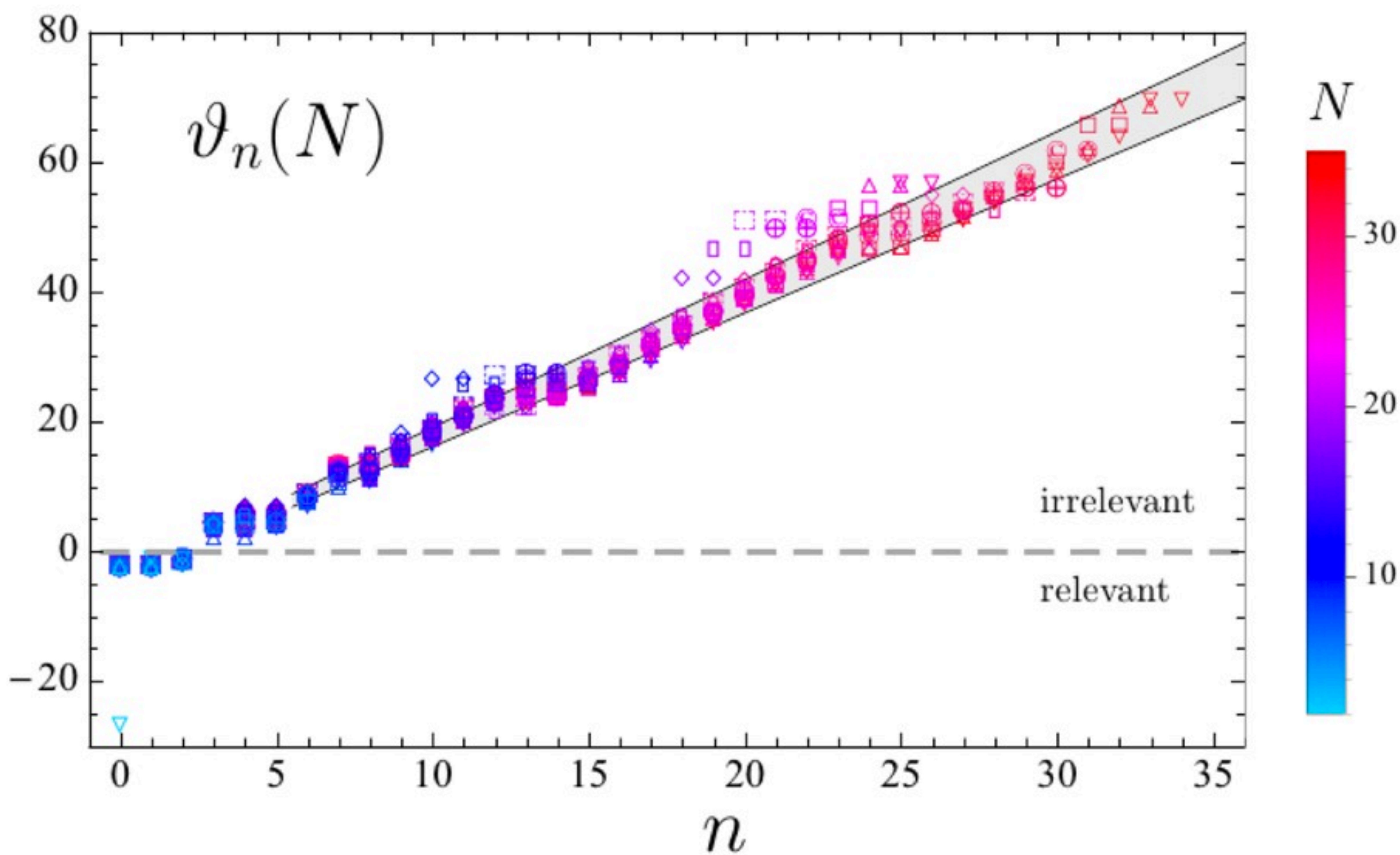


radius of convergence $\rho_c \approx 0.82 \pm 5\%$

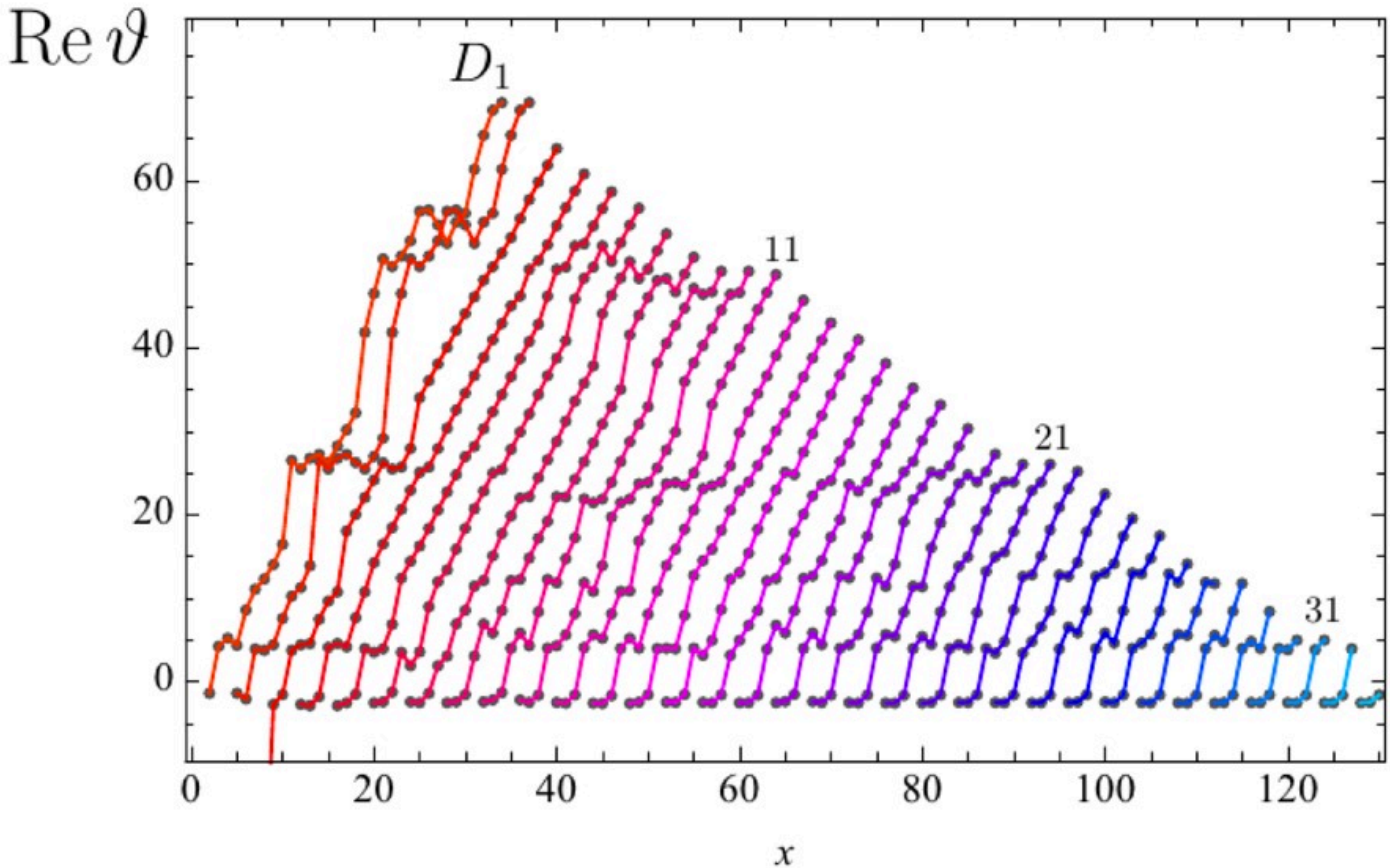


UV fixed point
$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$
good convergence
UV eigenvalues
$\vartheta_n(N)$
irrelevant eigenvalues
three relevant eigenvalues
N
N

near-Gaussian



bootstrap test



beyond Ricci scalars

f(Ricci)

$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

f(Ricci)

$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

$$\begin{aligned}\partial_t\Gamma[\bar{g},\bar{g}] =& \frac{1}{2}\mathrm{Tr}_{(2T)}\left[\frac{\partial_t\mathcal{R}_k^{h^Th^T}}{\Gamma^{(2)}_{h^Th^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\xi\xi}}{\Gamma^{(2)}_{\xi\xi}}\right] + \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma\sigma}}{\Gamma^{(2)}_{\sigma\sigma}}\right] + \frac{1}{2}\mathrm{Tr}_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{hh}}{\Gamma^{(2)}_{hh}}\right]\\ &+ \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma h}}{\Gamma^{(2)}_{\sigma h}}\right] - \mathrm{Tr}_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{C}^TC^T}}{\Gamma^{(2)}_{\bar{C}^TC^T}}\right] - \mathrm{Tr}_{(0)'}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma^{(2)}_{\bar{\eta}\eta}}\right] - \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma^{(2)}_{\bar{\lambda}\lambda}}\right]\\ &+ \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\omega\omega}}{\Gamma^{(2)}_{\omega\omega}}\right] - \mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{c}^Tc^T}}{\Gamma^{(2)}_{\bar{c}^Tc^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\zeta^T\zeta^T}}{\Gamma^{(2)}_{\zeta^T\zeta^T}}\right] + \mathrm{Tr}'_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{s}s}}{\Gamma^{(2)}_{\bar{s}s}}\right]\end{aligned}$$

f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

generating function

$$384\pi^2\left[4f + 2\rho\, z - \rho^2\,(f' + \rho\, z') + \partial_t f + \rho\, \partial_t z\right] = I[f,z](\rho)$$

$$\begin{aligned} I[f,z](\rho) =& I_0[f,z](\rho) + \partial_t z\, I_1[f,z](\rho) + \partial_t f'\, I_2[f,z](\rho) + \partial_t z'\, I_3[f,z](\rho) \\ & + \partial_t f''\, I_4[f,z](\rho) + \partial_t z''\, I_5[f,z](\rho)\ . \end{aligned}$$

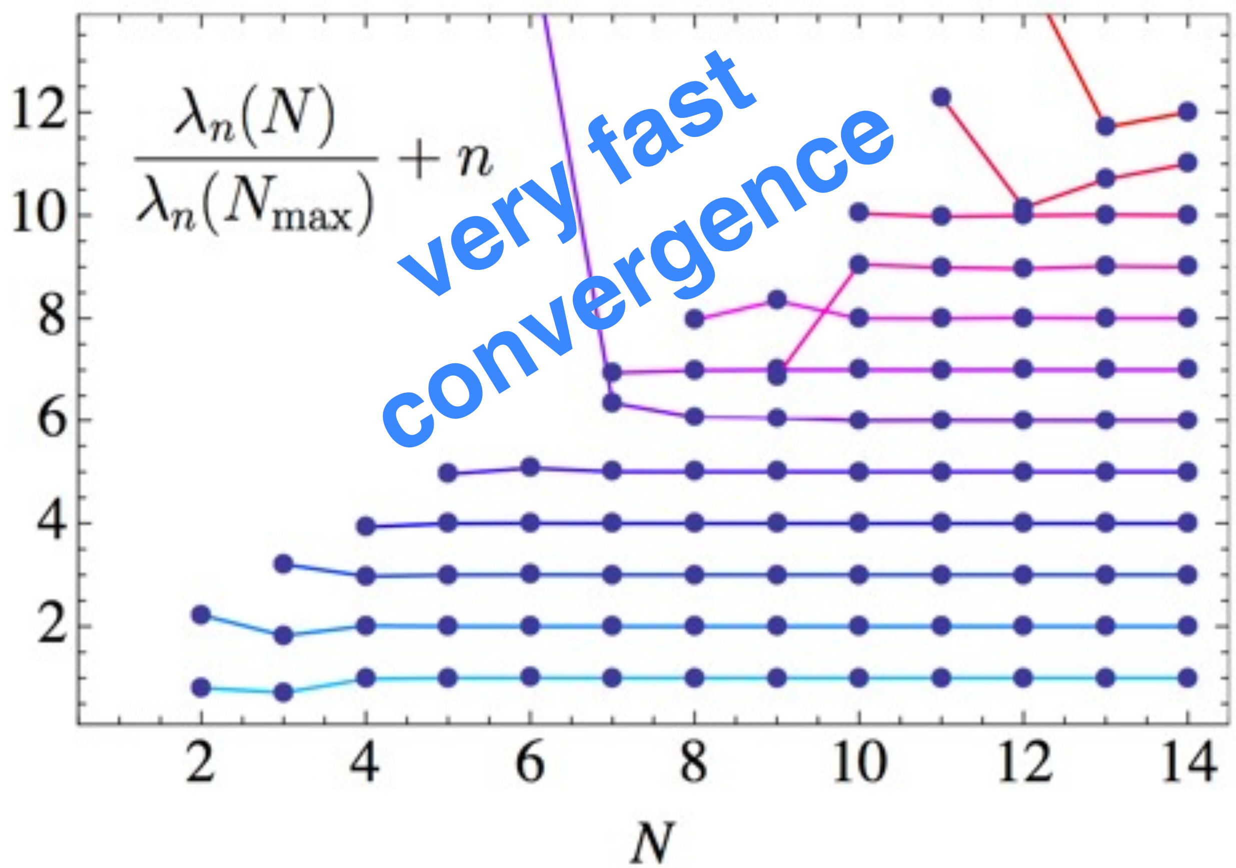
fixed points

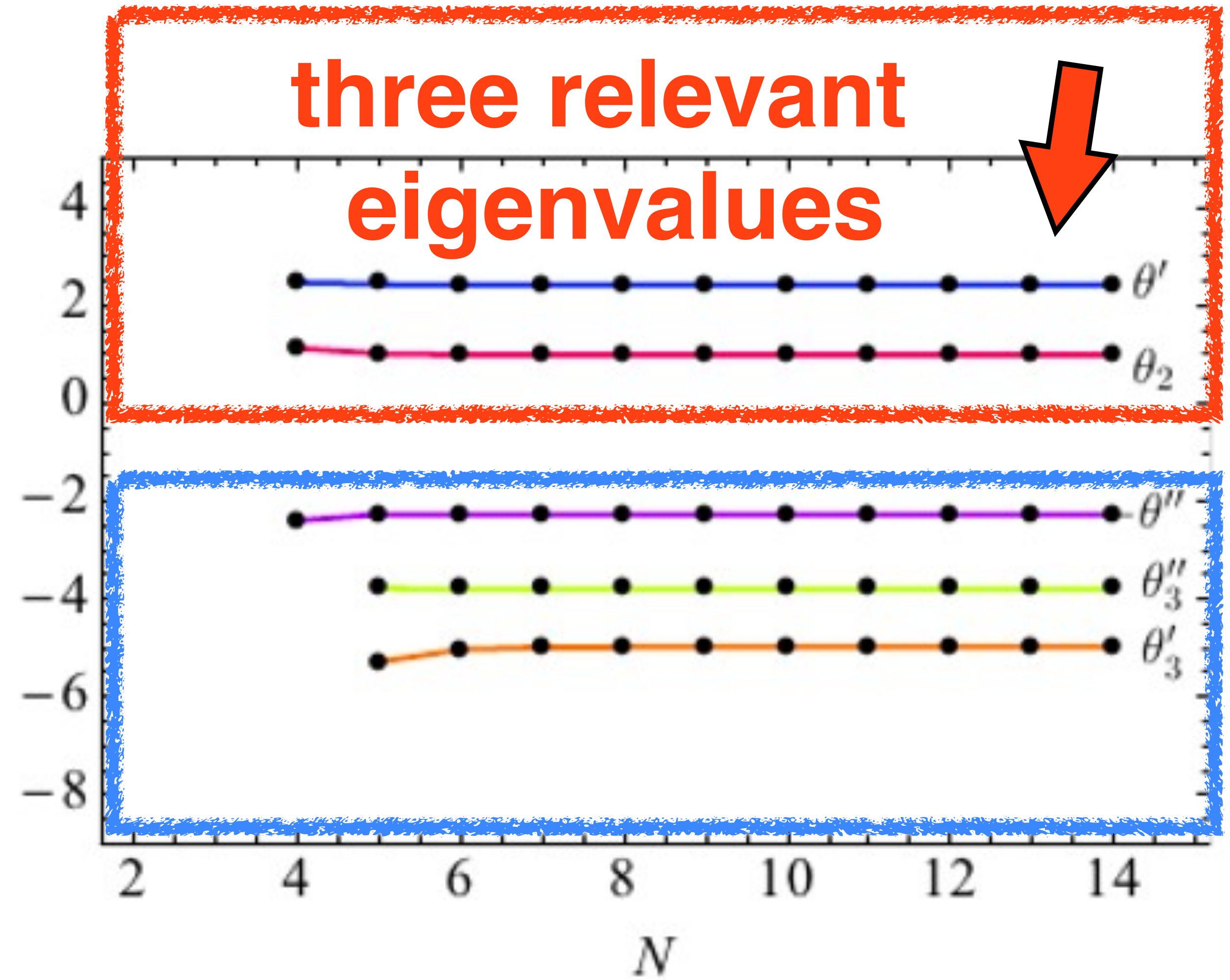
recursive solution more demanding

f(Ricci)

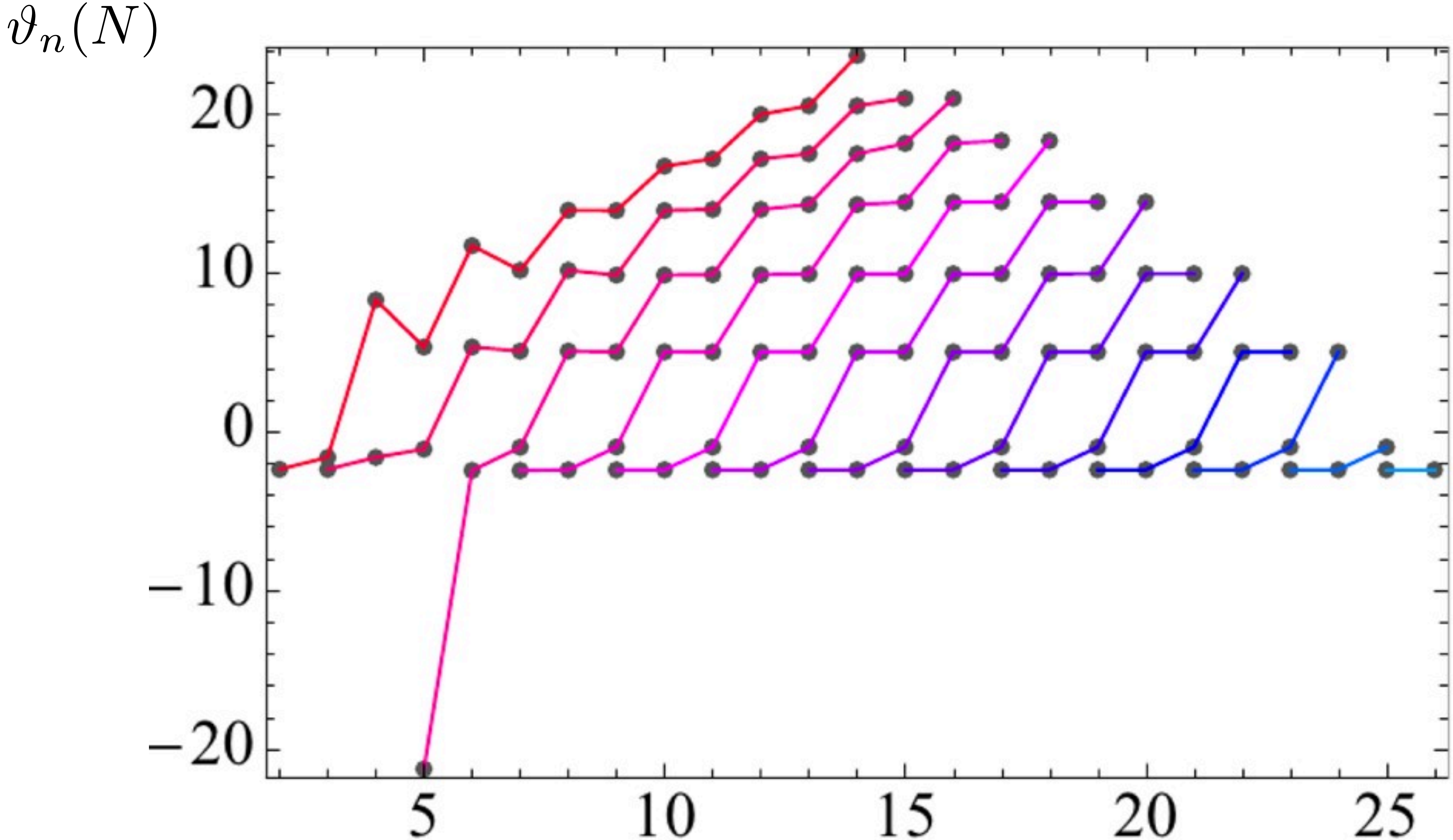
$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

results:

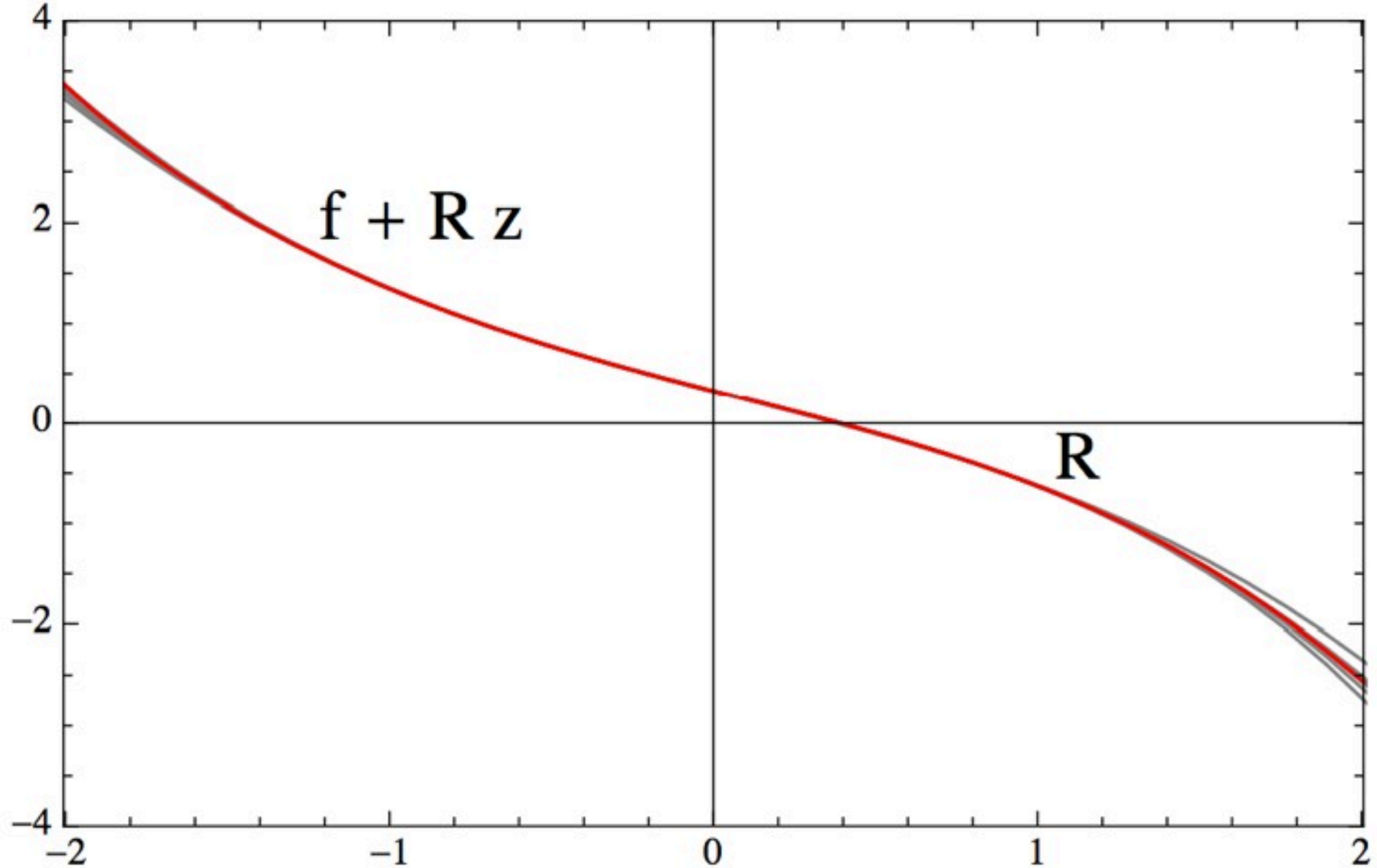




bootstrap test



f(Ricci) quantum gravity



radius of convergence $R_c \approx 2$

conclusions

QFTs beyond asymptotic freedom

4D matter-gauge theories

exact proof of existence
requires elementary scalars, fermions, vectors
no additional (super-)symmetry

4D quantum gravity

systematic non-perturbative search strategies
strong hints towards interacting UV fixed point