EFT of Gravity and Cosmology

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Probing the fundamental nature of spacetime with the RG

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work with:

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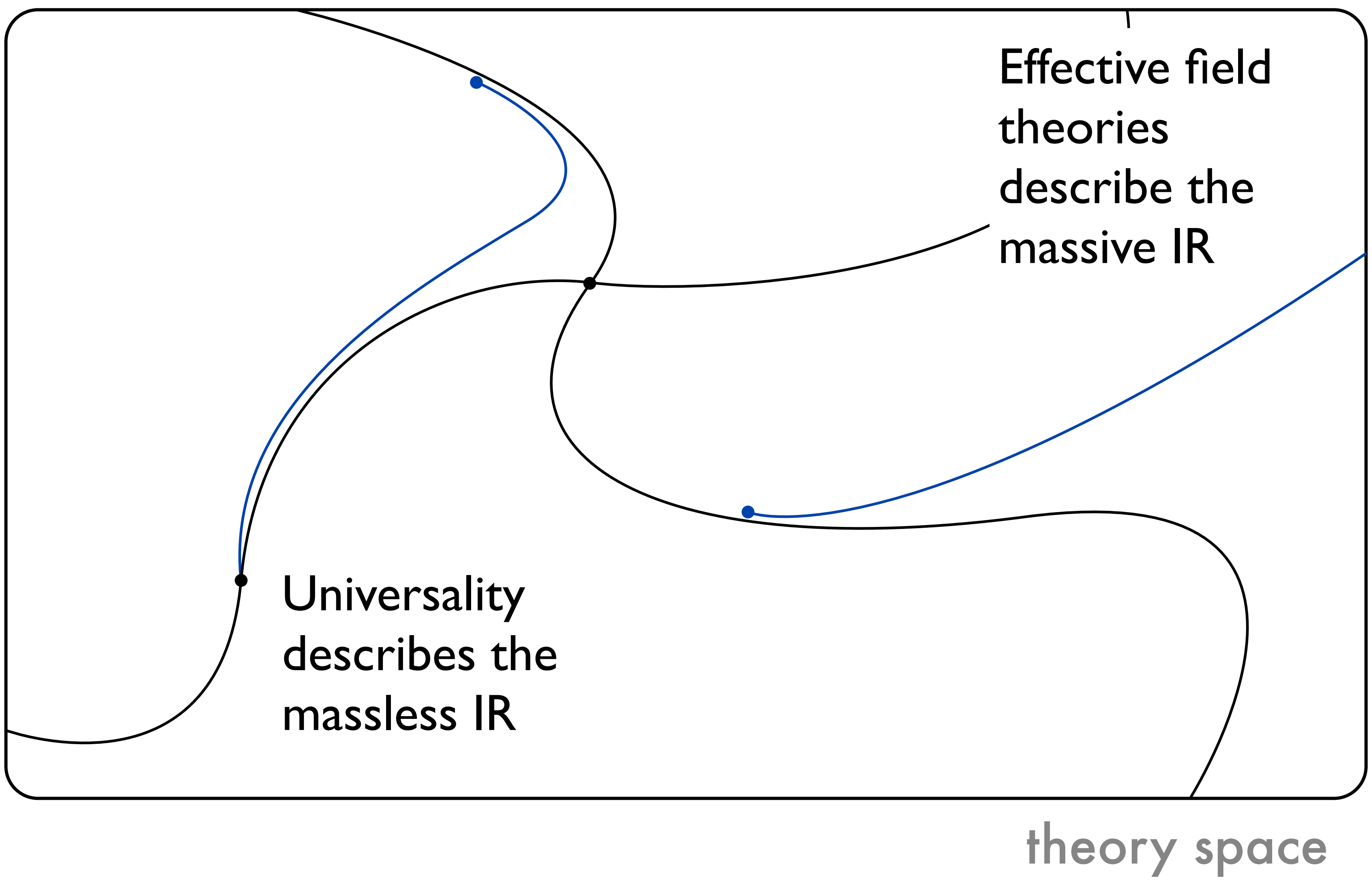
R Percacci, A Tonero, L Rachwal
R K Jain

Outline of the talk

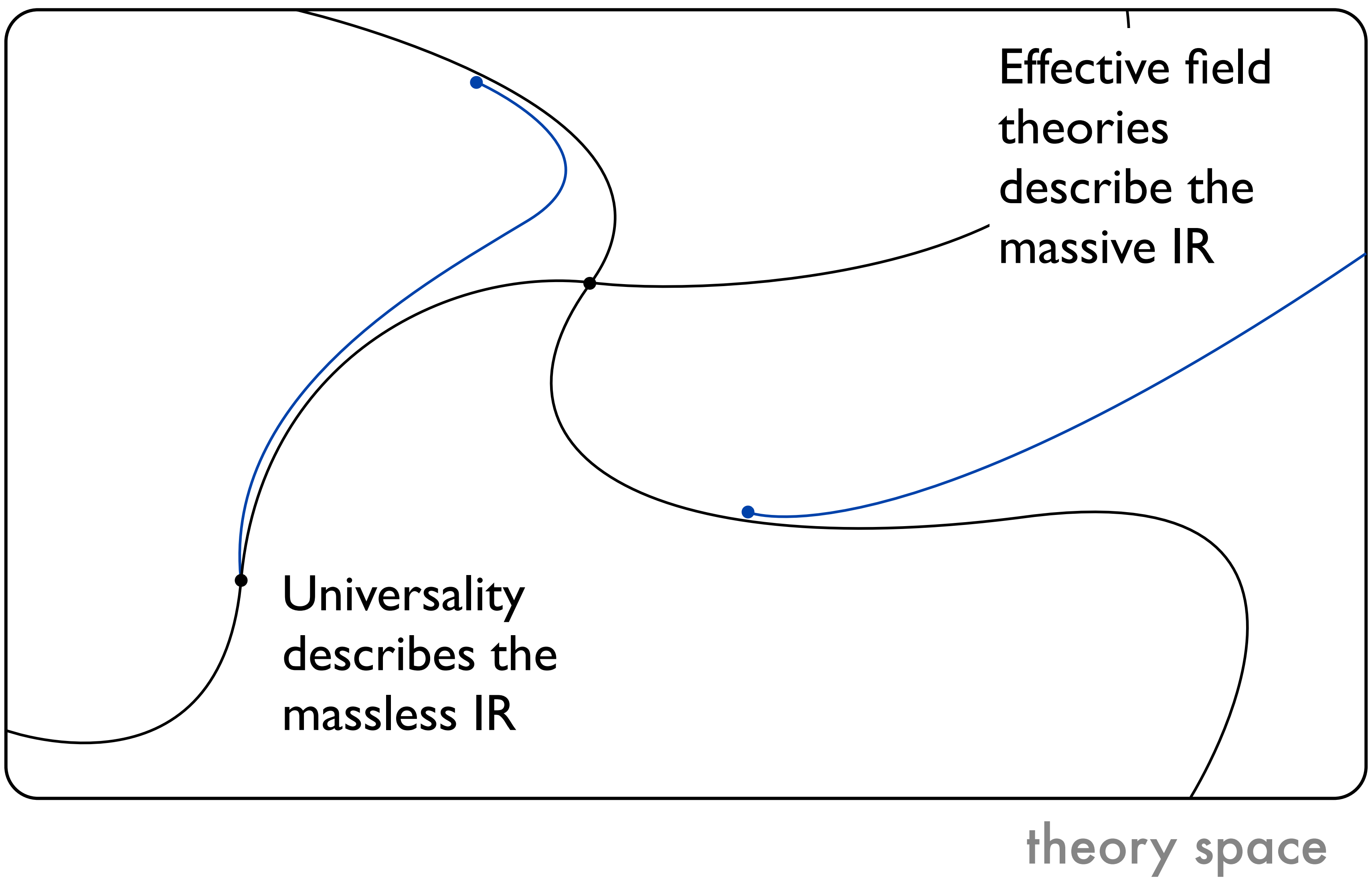
- Effectivity vs Universality
- Example: EFT of Pions (CPT)
- Covariant EFT of Gravity in three lines
- Renormalization in different schemes
- Phenomenological parameters and their estimation
- Adding matter
- LO quantum corrections
- Marginally deformed Starobinsky
- Effective Friedmann equations

Effectivity vs Universality



Two main reasons why mathematical modeling of nature actually works

Effectivity vs Universality



Massive IR lies in the broken phase (G to G/H)

Characteristic large scale M at which G is broken

The theory of pions (CPT)

- Low energy QCD can be described by an EFT of pions
- Symmetry braking pattern $SU(2)_L \times SU(2)_R \to SU(2)_V$
- M is the pion decay constant $F_\pi \sim 10^2\,\mathrm{MeV}$
- Phenomenological parameters l_i to be fixed by experiments ($i = 1, 2$)
- Renormalization and scale dependence

$$l_i(\mu_1) = l_i(\mu_2) + \frac{\gamma_i}{(4\pi)^2} \log \frac{\mu_1}{\mu_2}$$

- Low energy expansion of physical quantities

$$A(E) = \frac{E^2}{M^2} \left(1 + C_1 l_1 \frac{E^2}{M^2} + C_2 l_2 \frac{E^2}{M^2} \right) + \ldots$$

EFT of Gravity

- The theory of small fluctuations of the metric

$$g_{\mu\nu} \to g_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M} h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19}\,\mathrm{GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

EFT of Gravity

$$S_{eff}[g] = \int d^4x\sqrt{g}\left[M^4 c_0 + M^2(-R) + c_{2,1}R^2 + c_{2,2}\mathrm{Ric}^2 + c_{2,3}\mathrm{Riem}^2 + \frac{1}{M^2}c_{3,1}R^3 + ...\right]$$

EFT of Gravity

$$S_{eff}[g] = \int d^4x\sqrt{g}\left[M^4 \underbrace{c_0}_{\mathcal{R}^0} + M^2 \underbrace{(-R)}_{\mathcal{R}} + \underbrace{c_{2,1}R^2 + c_{2,2}\mathrm{Ric}^2 + c_{2,3}\mathrm{Riem}^2}_{\mathcal{R}^2} + \frac{1}{M^2}\underbrace{c_{3,1}R^3 + ...}_{\mathcal{R}^3}\right]$$

EFT of Gravity

$$S_{eff}[g] = \int d^4x\sqrt{g}\left[M^4 \underbrace{c_0}_{\mathcal{R}^0} + M^2 \underbrace{(-R)}_{\mathcal{R}} + \underbrace{c_{2,1}R^2 + c_{2,2}\mathrm{Ric}^2 + c_{2,3}\mathrm{Riem}^2}_{\mathcal{R}^2} + \frac{1}{M^2}\underbrace{c_{3,1}R^3 + ...}_{\mathcal{R}^3}\right]$$

$$\equiv M^2\left[I_1[g] + \frac{1}{M^2}I_2[g] + \frac{1}{M^4}I_3[g] + ...\right]$$

$$m^2 = -2\Lambda$$

Covariant EFT of Gravity

$$e^{-\Gamma[g]} = \int_{1PI} \mathcal{D}h_{\mu\nu}\, e^{-S_{eff}[g+\frac{1}{M}h]}$$

$$= \int_{1PI} \mathcal{D}h_{\mu\nu}\, e^{-M^2\left\{I_1[g+\frac{1}{M}h]+\frac{1}{M^2}I_2[g+\frac{1}{M}h]+\ldots\right\}}$$

EFT: saddle point expansion in $\frac{1}{M^2}$

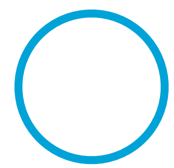
Covariant EFT of Gravity

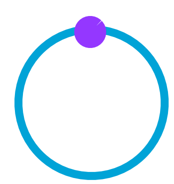
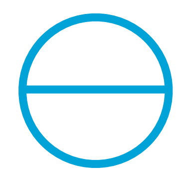
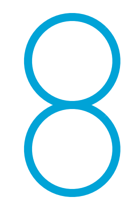
$$
\begin{aligned}
\Gamma[g] \;&=\; I_1[g] && \text{CT} \\
&+ \frac{1}{M^2}\left\{ I_2[g] + \frac{1}{2}\mathrm{Tr}\log I_1^{(2)}[g] \right\} && \text{LO} \\
&+ \frac{1}{M^4}\left\{ I_3[g] + \frac{1}{2}\mathrm{Tr}\left[\left(I_1^{(2)}[g] \right)^{-1} I_2^{(2)}[g] \right] + \text{2-loops with } I_1[g] \right\} && \text{NLO} \\
&+ \ldots && \text{NNLO}
\end{aligned}
$$

EFT: saddle point expansion in $\frac{1}{M^2}$

Covariant EFT of Gravity

$\Gamma = \; \bullet_{I_1}$ — CT

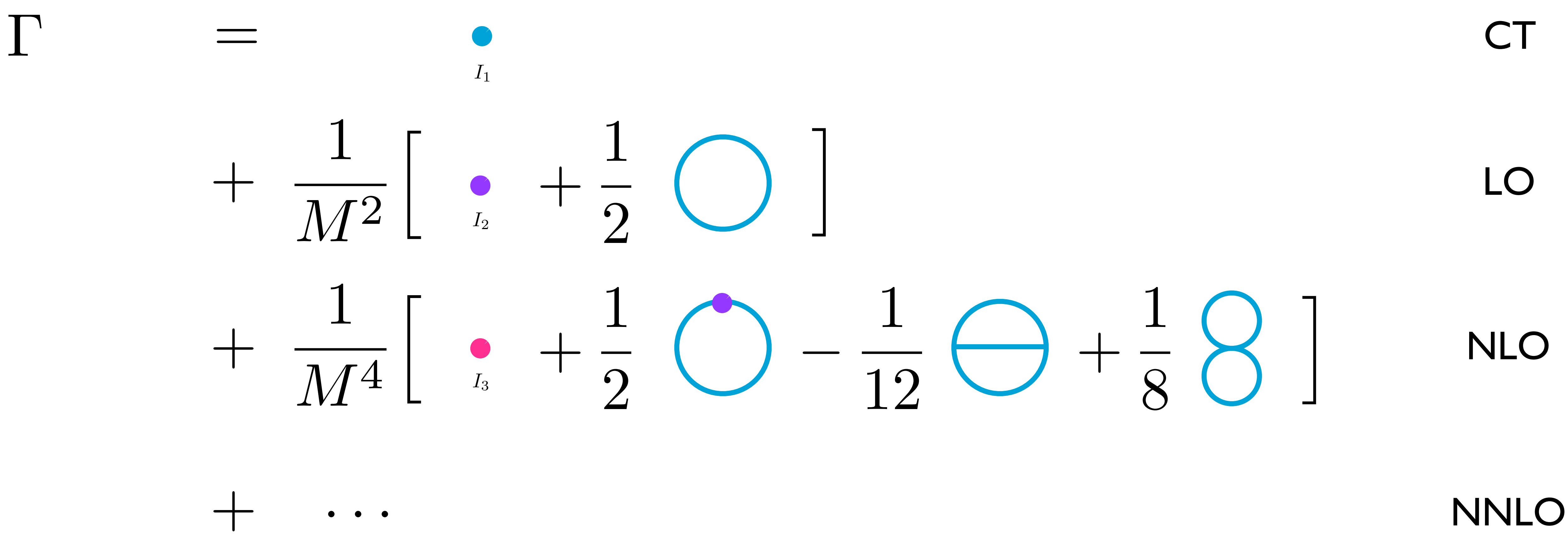
$+ \frac{1}{M^2}\Big[\; \bullet_{I_2} + \frac{1}{2}$  $\Big]$ — LO

$+ \frac{1}{M^4}\Big[\; \bullet_{I_3} + \frac{1}{2}$  $- \frac{1}{12}$  $+ \frac{1}{8}$  $\Big]$ — NLO

$+ \; \cdots$ — NNLO

Covariant EFT of Gravity

The EFT recipe in three lines



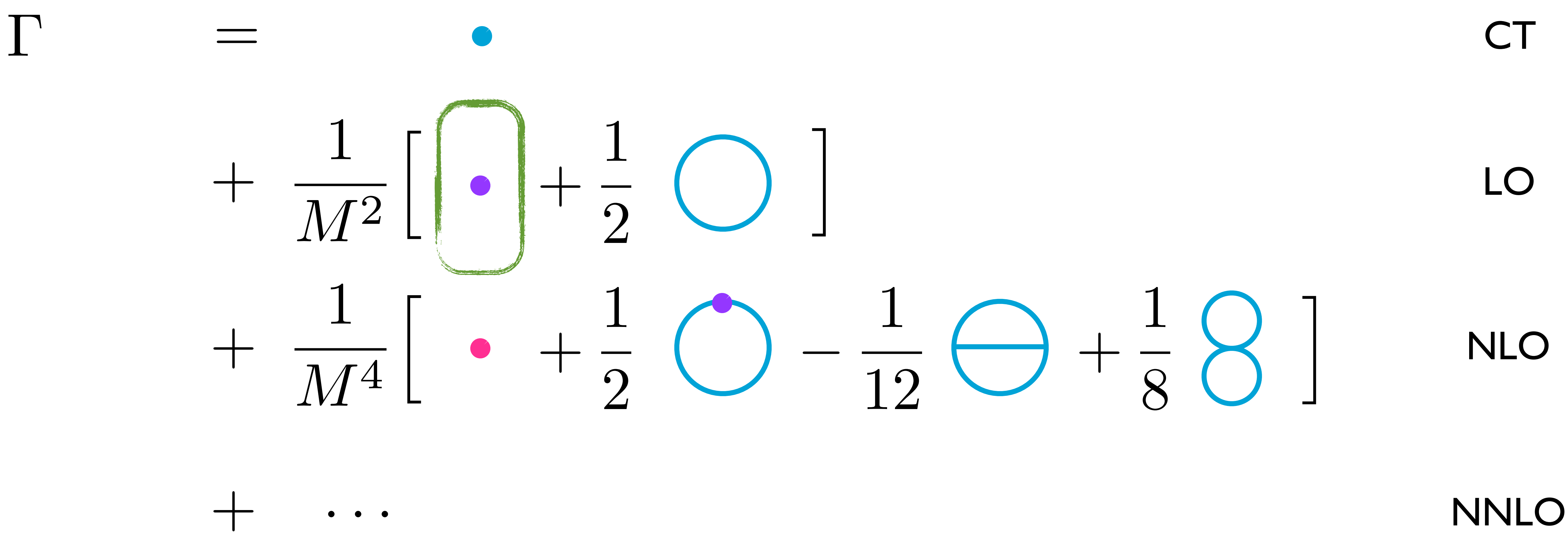
1) the general lagrangian of order E^2 is to be used both at tree level and in loop diagrams

2) the general lagrangian of order $E^{n\geq 4}$ is to be used at tree level and as an insertion in loop diagrams

3) the renormalization program is carried out order by order

Covariant EFT of Gravity

What do we already know?

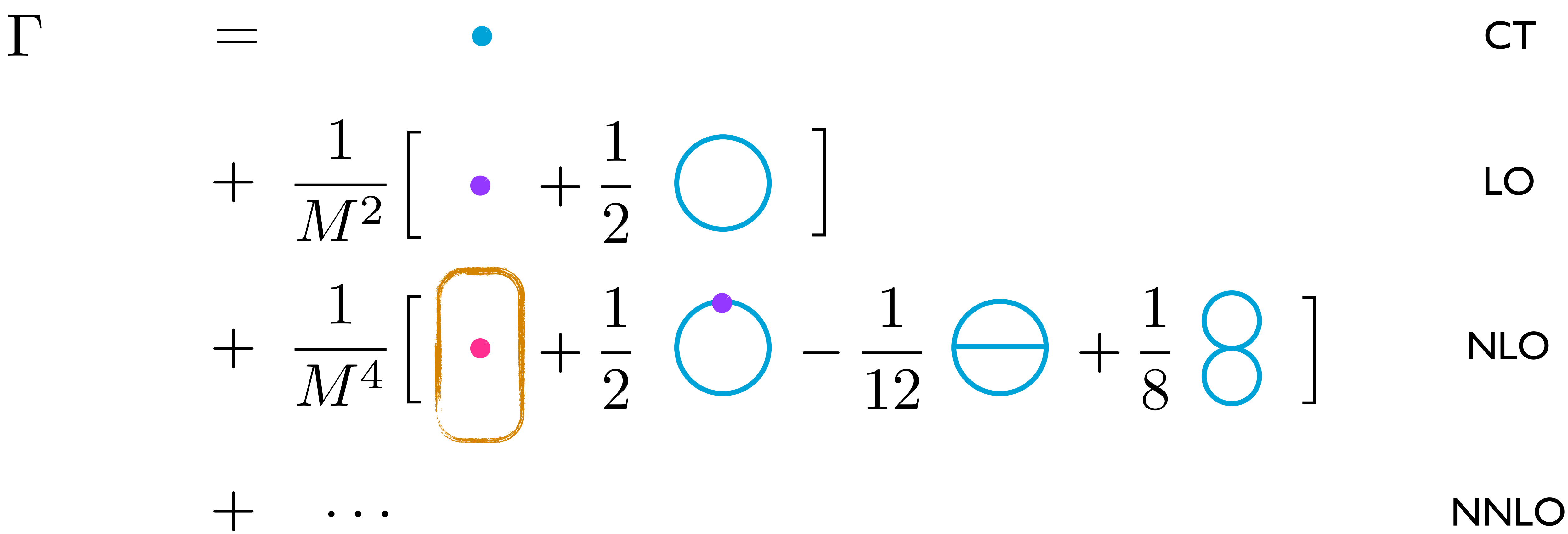


UV divergencies and renormalization

G. 't Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A 20 (1974) 69
G. W. Gibbons, S. W. Hawking and M. J. Perry, Nucl. Phys. B 138 (1978) 141
S. M. Christensen and M. J. Duff, Nucl. Phys. B 170 (1980)

Covariant EFT of Gravity

What do we already know?

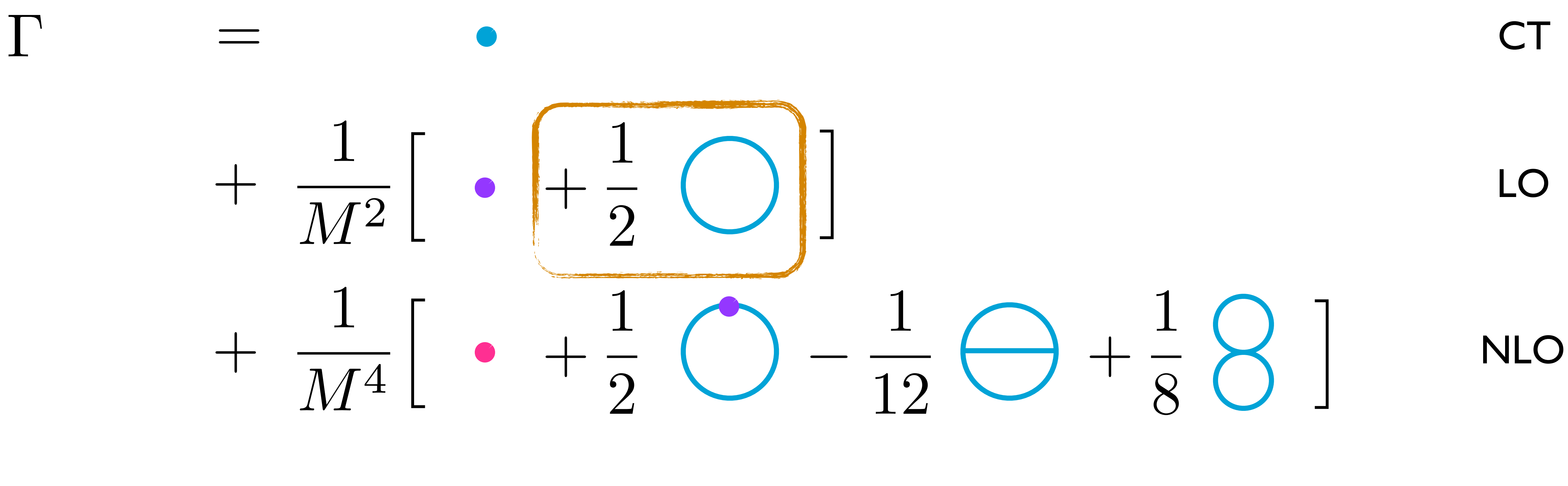


Two loops UV divergencies

M.H. Goroff and A. Sagnotti, Nucl.Phys.B266, 709 (1986)
A. E. M. van de Ven, Nucl. Phys. B378, 309 (1992)

Covariant EFT of Gravity

What do we already know?



$+ \cdots$ NNLO

Finite LO terms

Leading logs

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

A. C., J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502 (2015) 050

Conformal anomaly

S. Deser, M. J. Duff and C. J. Isham, Nucl. Phys. B 111, 45 (1976)

R.J. Riegert, Phys. Lett. B 134 (1984) 56

Four graviton vertex in Minkowski space

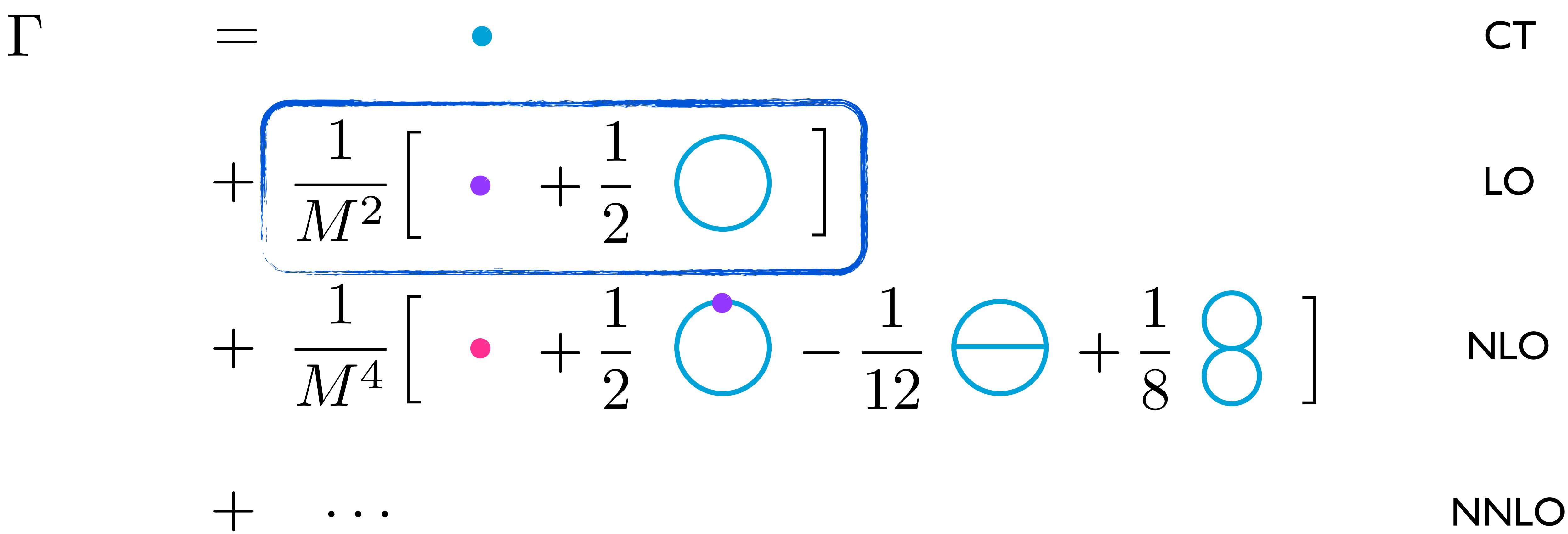
D. C. Dunbar and P. S. Norridge, Nucl. Phys. B 433, 181 (1995)

Curvature square terms

A. C. and R. K. Jain, in preparation

Covariant EFT of Gravity

LOQG: the only QG we will ever observe!



Even if we have a fundamental theory its is generally difficult to compute phenomenological parameters...

Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

- $$= \frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{g}\left[\Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4\log\frac{\Lambda_{UV}^2}{m^2} + \left(-\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2\log\frac{\Lambda_{UV}^2}{m^2}\right)R + \left(\frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\Box R\right)\log\frac{\Lambda_{UV}^2}{m^2}\right]$$

measured
+ phenomenological
parameters

Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

- $$= \frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{g}\left[\Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4\log\frac{\Lambda_{UV}^2}{m^2} + \left(-\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2\log\frac{\Lambda_{UV}^2}{m^2}\right)R + \left(\frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\Box R\right)\log\frac{\Lambda_{UV}^2}{m^2}\right]$$

measured
+ phenomenological
parameters

Dimensional regularization

$$\log\Lambda_{UV}^2 \to \frac{1}{\epsilon} \qquad \Lambda_{UV}^2 \to 0 \qquad \Lambda_{UV}^4 \to 0$$

Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

- $$= \frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{g}\left[\Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4\log\frac{\Lambda_{UV}^2}{m^2} + \left(-\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2\log\frac{\Lambda_{UV}^2}{m^2}\right)R + \left(\frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\Box R\right)\log\frac{\Lambda_{UV}^2}{m^2}\right]$$

measured
+ phenomenological
parameters

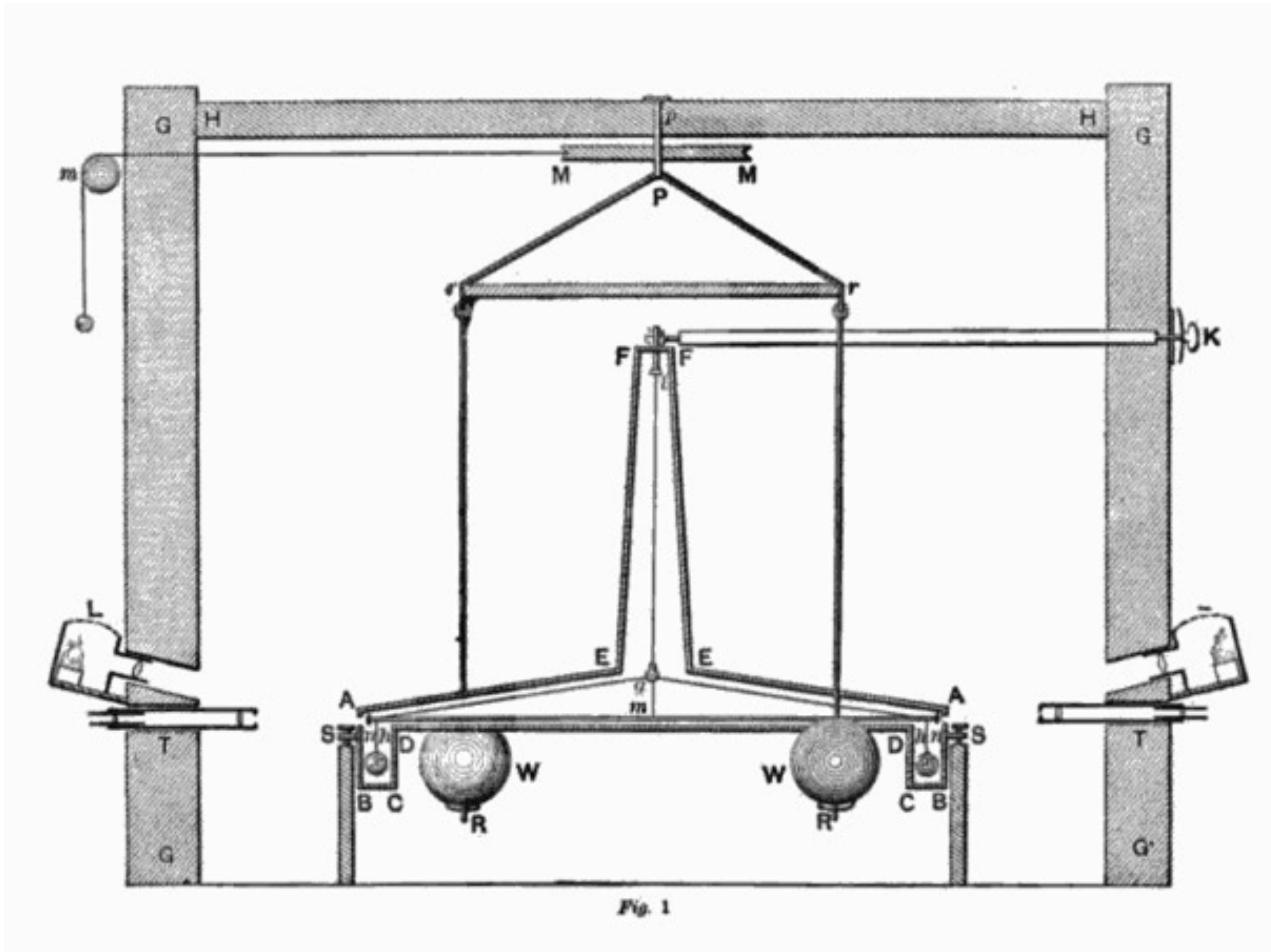
Dimensional regularization

$$\log\Lambda_{UV}^2 \to \frac{1}{\epsilon} \qquad \Lambda_{UV}^2 \to 0 \qquad \Lambda_{UV}^4 \to 0$$

G runs if there is a mass scale involved also
in dimensional regularization [Kirill's talk]

$$c_i(\mu_1) = c_i(\mu_2) + \frac{\gamma_i}{(4\pi)^2}\log\frac{\mu_1}{\mu_2} \qquad \mu\partial_\mu c_i = \frac{\gamma_i}{(4\pi)^2}$$

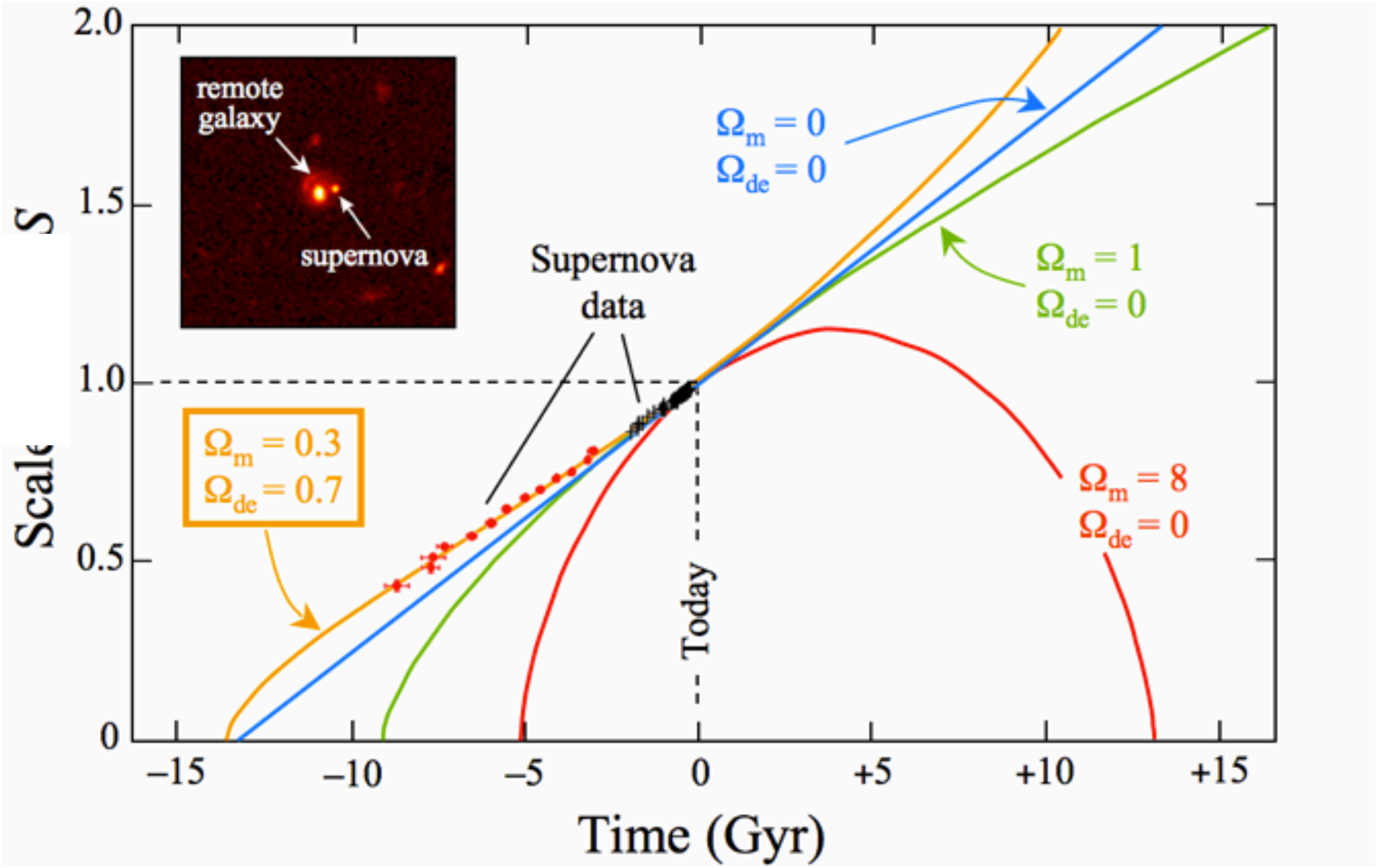
Phenomenological parameters



$$G = 6.67428 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$$

Cavendish 1797 (1% off best value!)

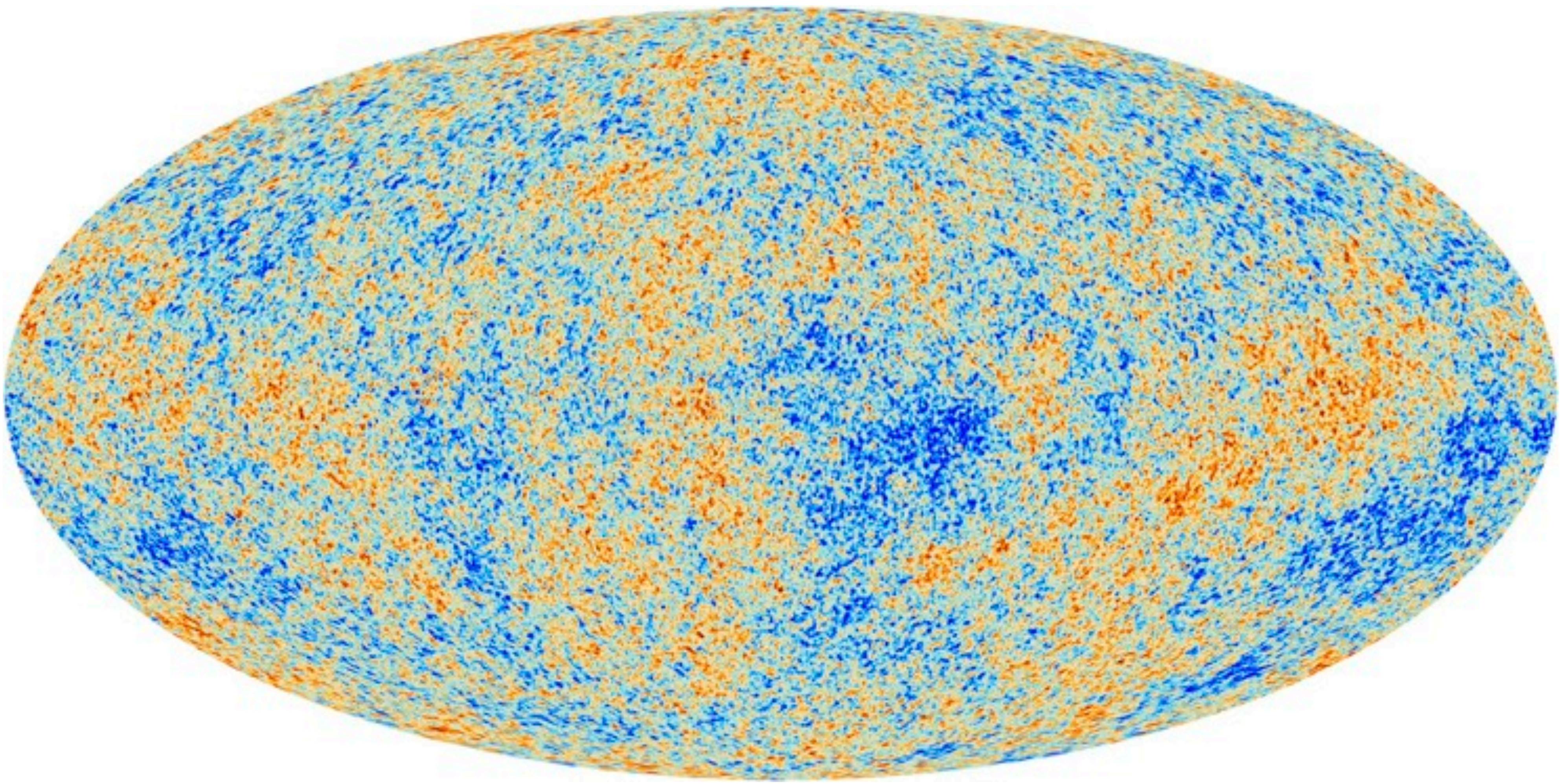
Phenomenological parameters



$$\Lambda = 10^{-47}\,\mathrm{GeV}^4$$

Supernova Cosmology Project

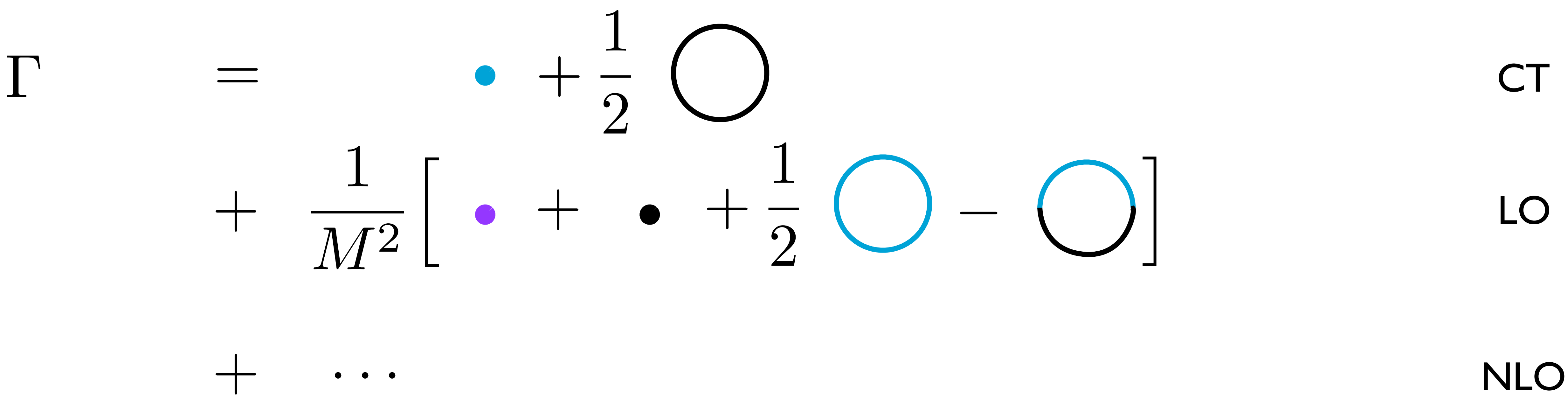
Phenomenological parameters



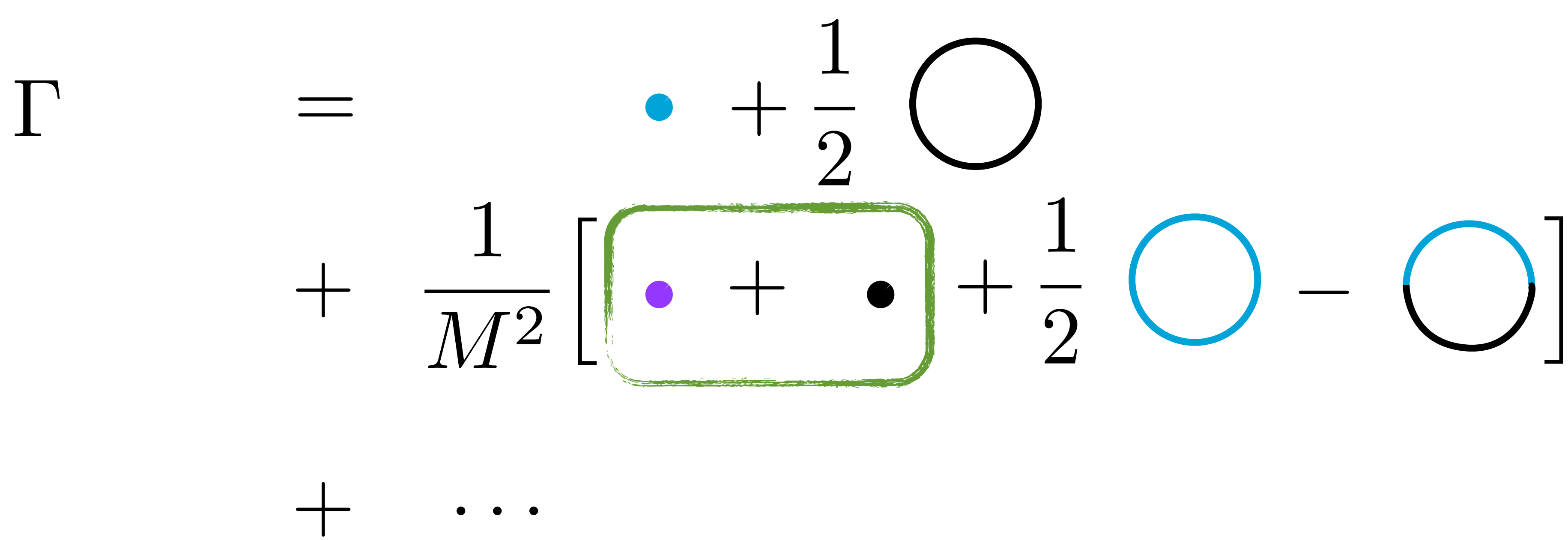
$\xi \equiv c_{R^2}$ $\xi(k_*) \sim 10^9$ $k_* \equiv 0.05\,\mathrm{Mpc}^{-1} \sim 10^{-40}\mathrm{GeV}$

Planck mission
[Alfio's talk]

Adding matter



Adding matter



CT

LO

NLO

UV divergencies and renormalization with matter

Scalars

A. O. Barvinsky, A. Y. .Kamenshchik and I. P. Karmazin, Phys. Rev. D 48 (1993) 3677

...

Gauge

S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006)

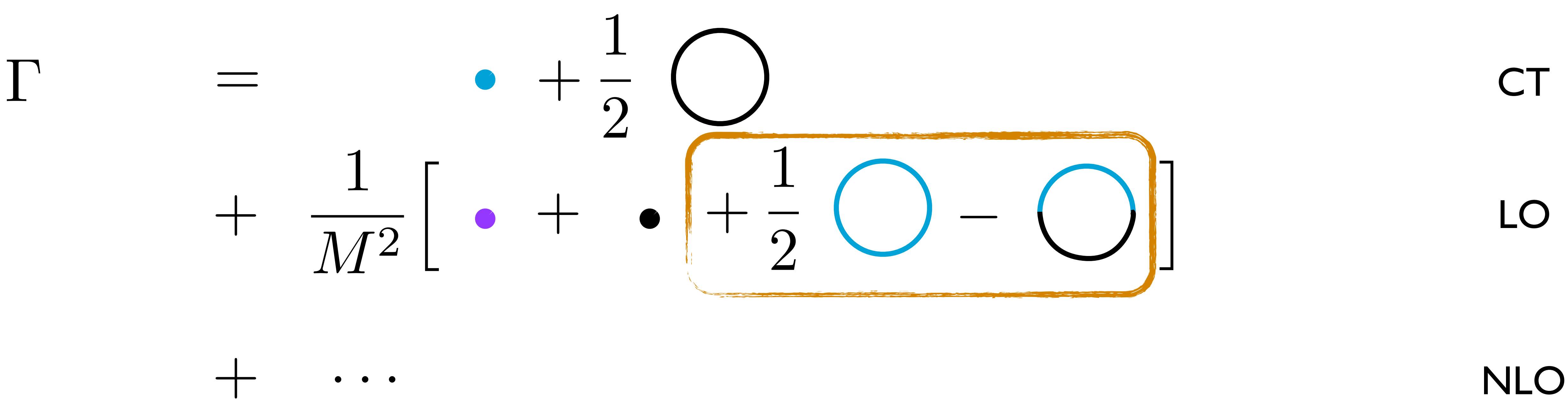
...

Yukawa

A. Rodigast and T. Schuster, Phys. Rev. Lett. 104, 081301 (2010)

...

Adding matter



Finite LO terms with matter

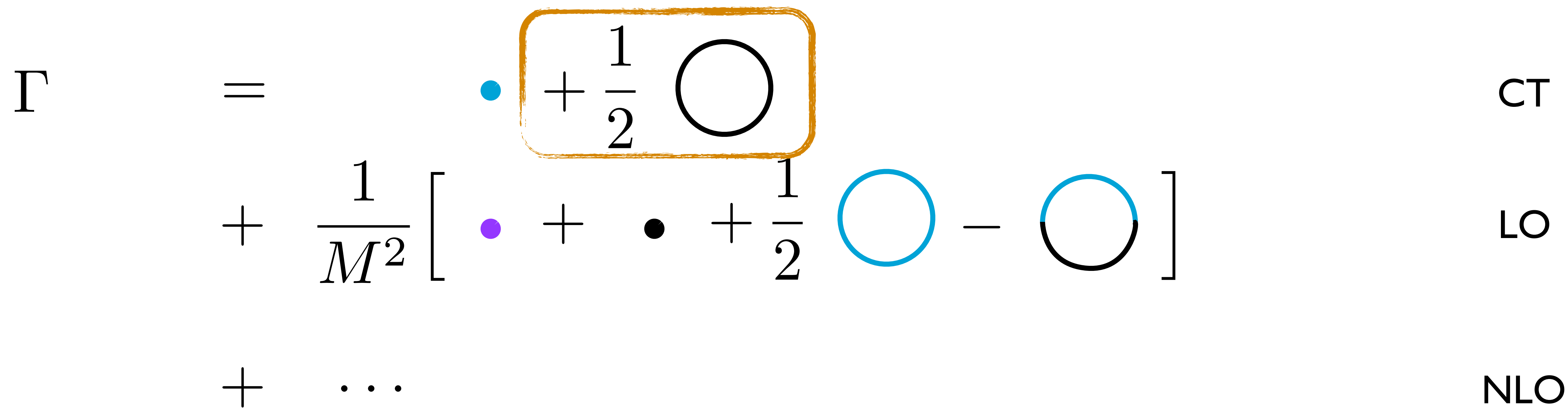
Flat space corrections to Newton's potential

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)
N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein (2003b) Phys. Rev. D 67
I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

Covariant leading logs

A C, R Percacci, L Rachwal and A Tonero in preparation

Adding matter



Matter induced effective action

Integrating out matter

$$\frac{1}{2} \bigcirc = -\frac{1}{2}\frac{1}{(4\pi)^2}\sum_{n=\frac{d}{2}+1}\frac{1}{m^{2n-d}}B_{2n}$$

Local heat kernel coefficients

B6

Gilkey, PB
A. O. Barvinsky and G. A. Vilkovisky, Phys. Rept. 119 (1985) 1.
I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1.
Groh, Kai Saueressig, Frank Zanusso, Omar

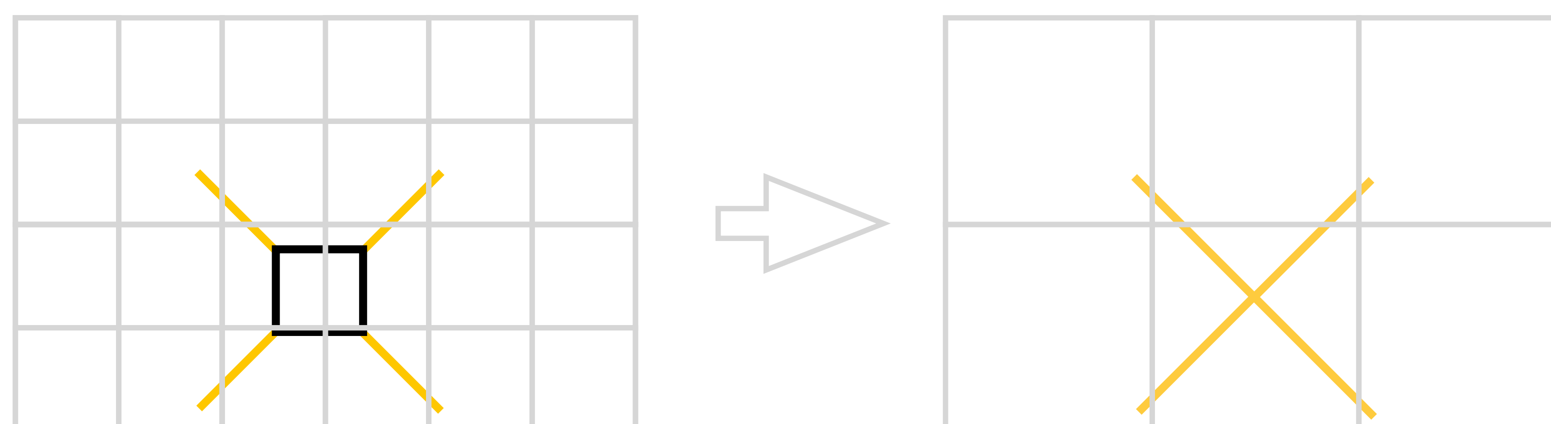
B8

Amsterdamski, P
Ven, AEM Van de

Matter induced effective action:
expandable in inverse powers of the (lightest) particle masse

Integrating out matter

$$\frac{1}{2} \bigcirc = -\frac{1}{2}\frac{1}{(4\pi)^2}\frac{1}{m^2}B_6 + \dots$$



As in QED when we integrate out electrons

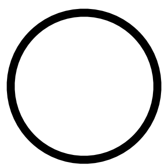
Integrating out matter

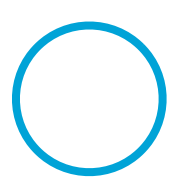
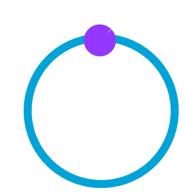
$$
\begin{aligned}
\frac{1}{2}\bigcirc = -\frac{1}{2}\frac{1}{(4\pi)^2}\frac{1}{m^2}\int d^4x\sqrt{g}\,\Bigg[&\frac{1}{336}R\Box R+\frac{1}{840}R_{\mu\nu}\Box R^{\mu\nu} \\
&+\frac{1}{1296}R^3-\frac{1}{1080}RR_{\mu\nu}R^{\mu\nu}-\frac{4}{2835}R^{\nu}_{\mu}R^{\alpha}_{\nu}R^{\mu}_{\alpha} \\
&+\frac{1}{945}R_{\mu\nu}R_{\alpha\beta}R^{\mu\alpha\nu\beta}+\frac{1}{1080}RR_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}+\frac{1}{7560}R_{\mu\nu}R^{\mu\alpha\beta\gamma}R^{\nu}{}_{\alpha\beta\gamma} \\
&+\frac{17}{45360}R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\mu\nu}-\frac{1}{1620}R^{\alpha\ \beta}_{\ \mu\ \nu}R^{\mu\ \nu}_{\ \gamma\ \delta}R^{\gamma\ \delta}_{\ \alpha\ \beta}\Bigg]+\ldots
\end{aligned}
$$

Changes how cubic terms are suppressed:

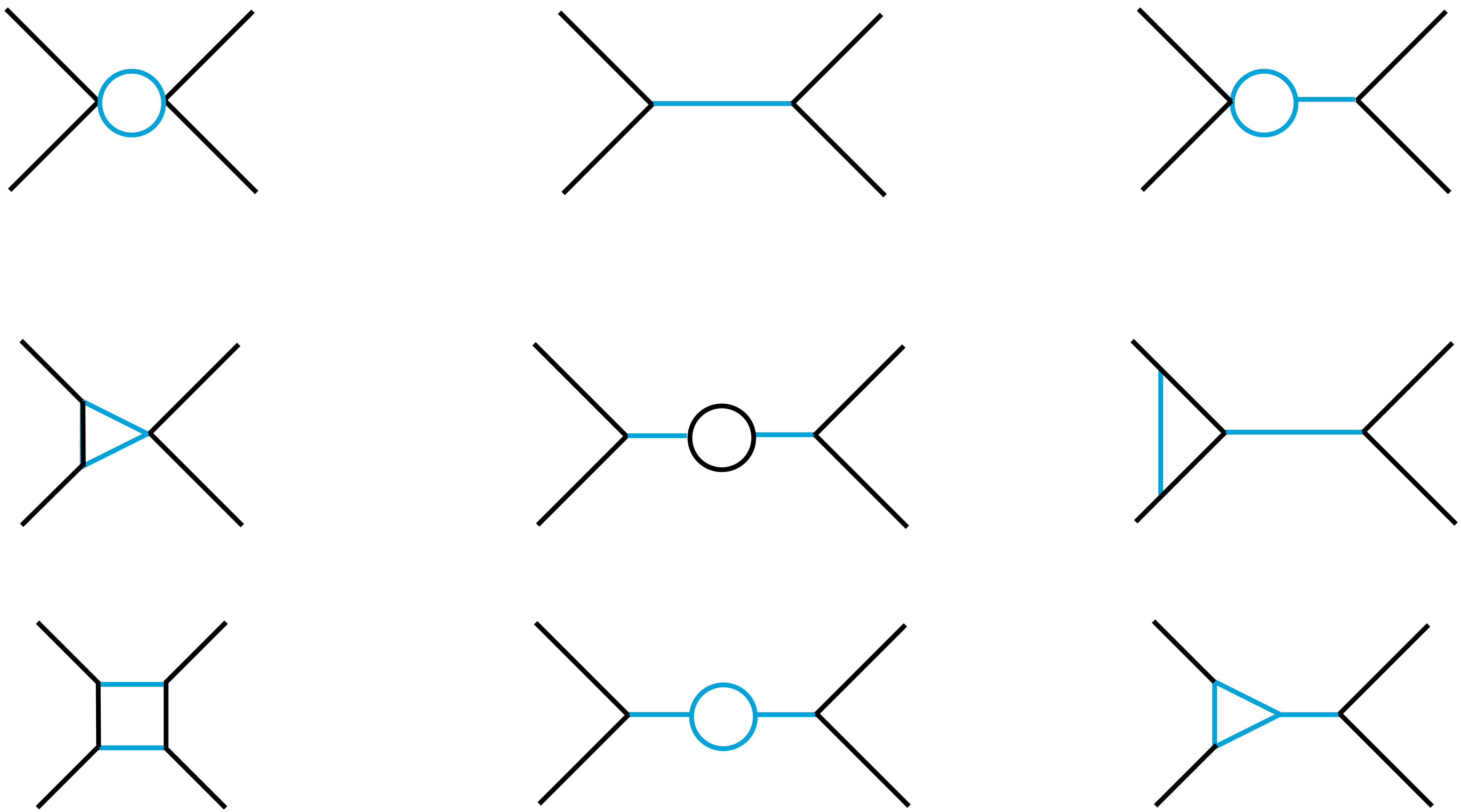
$$
\frac{1}{M^4}\to\frac{1}{m^2M^2}
$$

Integrating out matter

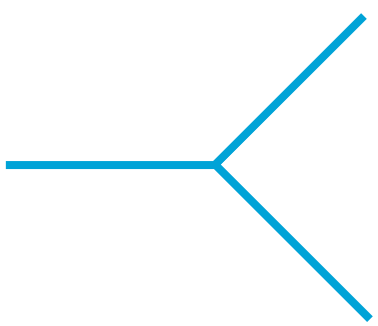
$$\frac{1}{2}$$  $$= -\frac{1}{2}\frac{1}{(4\pi)^2}\frac{1}{m^2}\int d^4x\sqrt{g}\left[\frac{1}{336}R\Box R + \frac{1}{840}R_{\mu\nu}\Box R^{\mu\nu} + \frac{1}{1296}R^3 - \frac{1}{1080}RR_{\mu\nu}R^{\mu\nu} - \frac{4}{2835}R^{\nu}_{\mu}R^{\alpha}_{\nu}R^{\mu}_{\alpha} + \frac{1}{945}R_{\mu\nu}R_{\alpha\beta}R^{\mu\alpha\nu\beta} + \frac{1}{1080}RR_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \frac{1}{7560}R_{\mu\nu}R^{\mu\alpha\beta\gamma}R^{\nu}{}_{\alpha\beta\gamma} + \frac{17}{45360}R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\mu\nu} - \frac{1}{1620}R^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu}R^{\mu}{}_{\gamma}{}^{\nu}{}_{\delta}R^{\gamma}{}_{\alpha}{}^{\delta}{}_{\beta}\right] + \ldots$$

The same can be applied to  and with some modifications to  and thus find 

Corrections to Newton's interaction



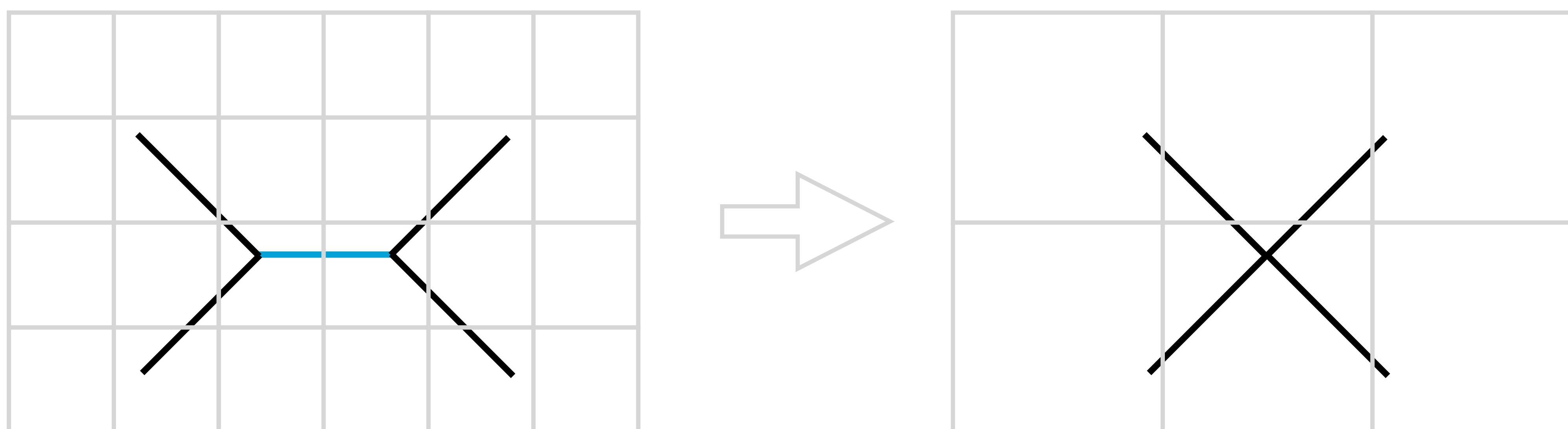
Corrections to Newton's interaction



$$
\begin{aligned}
= \; & \frac{i\kappa}{2}\Big(P_{\alpha\beta,\gamma\delta}\Big[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\Big] \\
& +2q_\lambda q_\sigma \Big[I^{\lambda\sigma,}{}_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} - I^{\lambda\mu,}{}_{\alpha\beta} I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta}\Big] \\
& +\Big[q_\lambda q^\mu \Big(\eta_{\alpha\beta} I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}{}_{\alpha\beta}\Big) + q_\lambda q^\nu \Big(\eta_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}{}_{\alpha\beta}\Big) \\
& -q^2\Big(\eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta}\Big) - \eta^{\mu\nu} q^\lambda q^\sigma \left(\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma}\right)\Big] \\
& +\Big[2q^\lambda \Big(I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma}(k-q)^\mu + I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma}(k-q)^\nu \\
& -I^{\sigma\nu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu\Big) \\
& +q^2\Big(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\nu} + I_{\alpha\beta,\sigma}{}^{\nu} I^{\sigma\mu,}{}_{\alpha\delta}\Big) + \eta^{\mu\nu} q^\lambda q_\sigma \Big(I_{\alpha\beta,\lambda\rho} I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma,}{}_{\alpha\beta}\Big)\Big] \\
& +\Big\{\left(k^2+(k-q)^2\right)\Big(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\nu} + I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\mu} - \frac{1}{2}\eta^{\mu\nu}P_{\alpha\beta,\gamma\delta}\Big) \\
& -\Big(k^2\eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} + (k-q)^2\eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta}\Big)\Big\}\Big)
\end{aligned}
$$

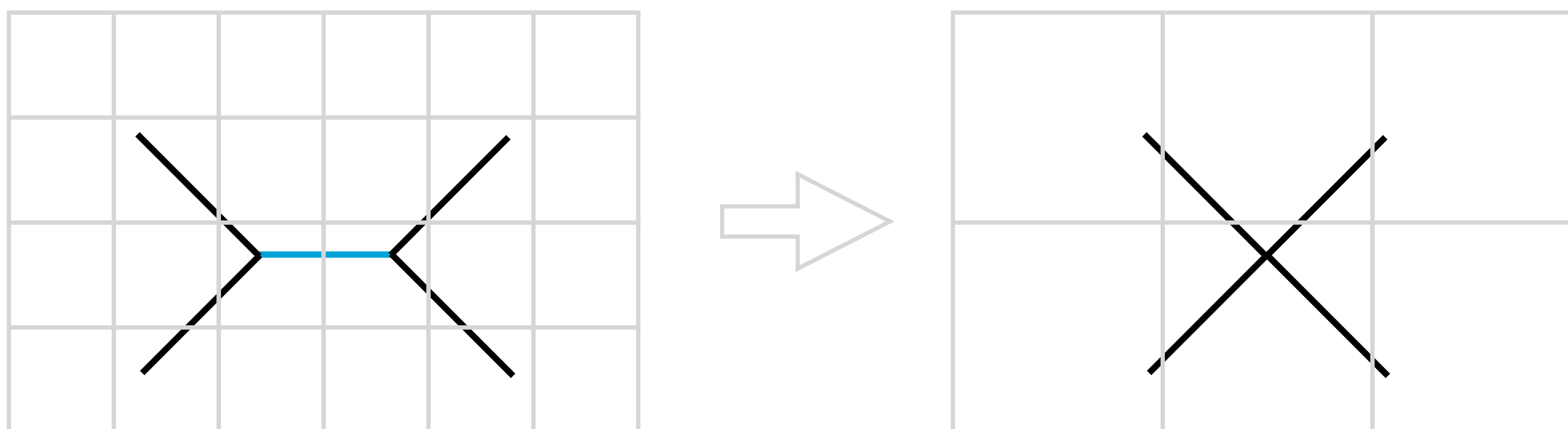
the truth behind Feynman diagrams...

Corrections to Newton's interaction



$$V = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{c^2 r} + b\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

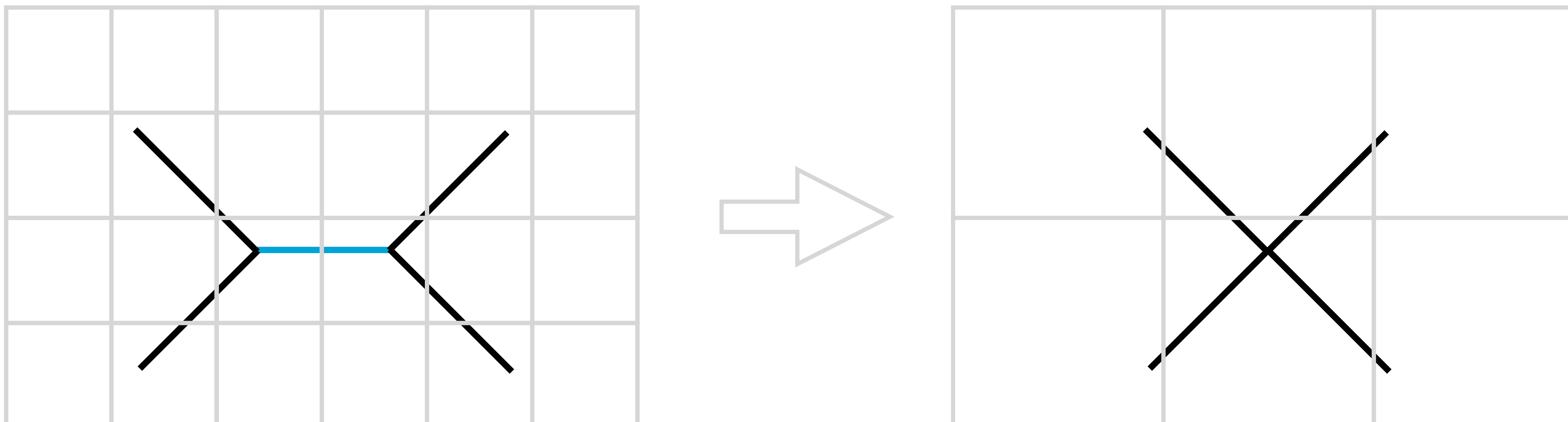
Corrections to Newton's interaction



$$V = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{c^2 r} + b\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

$$[G] = \frac{\mathrm{m}^3}{\mathrm{Kg\ s}^2} \qquad [\hbar] = \frac{\mathrm{m}^2\ \mathrm{Kg}}{\mathrm{s}} \qquad [c] = \frac{\mathrm{m}}{\mathrm{s}}$$

Corrections to Newton's interaction

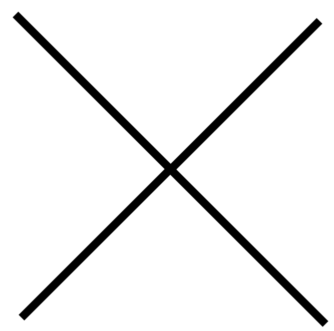


$$V = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{c^2 r} + \frac{41}{10\pi}\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

Leading quantum corrections to Newton's potential

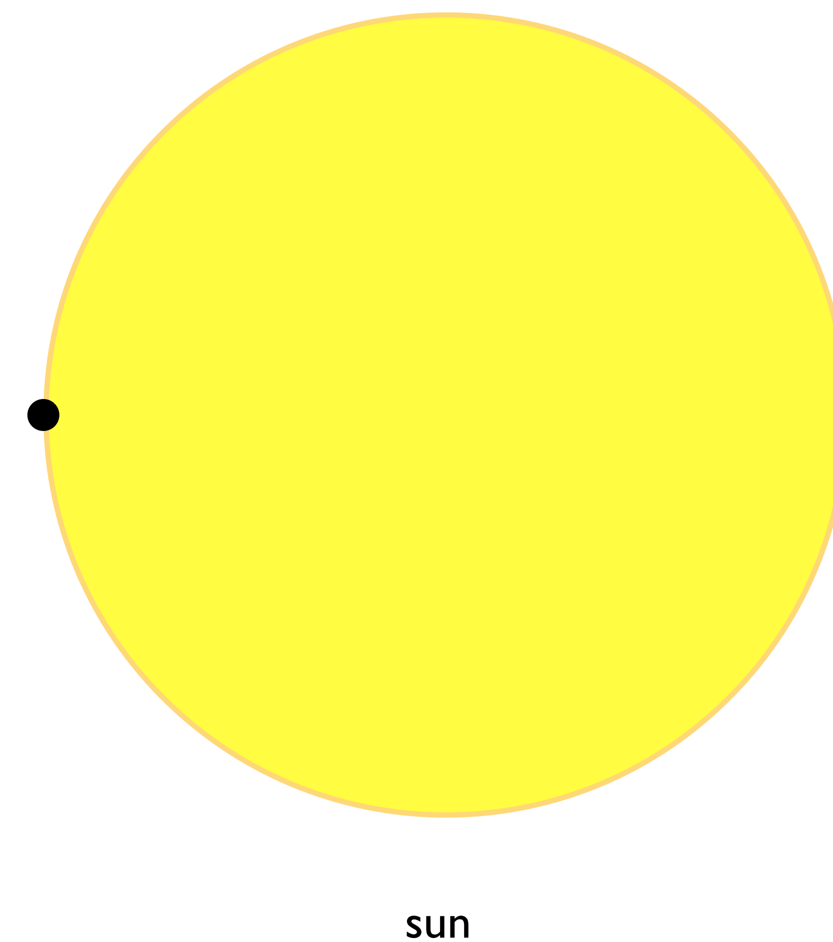
J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

Corrections to Newton's interaction



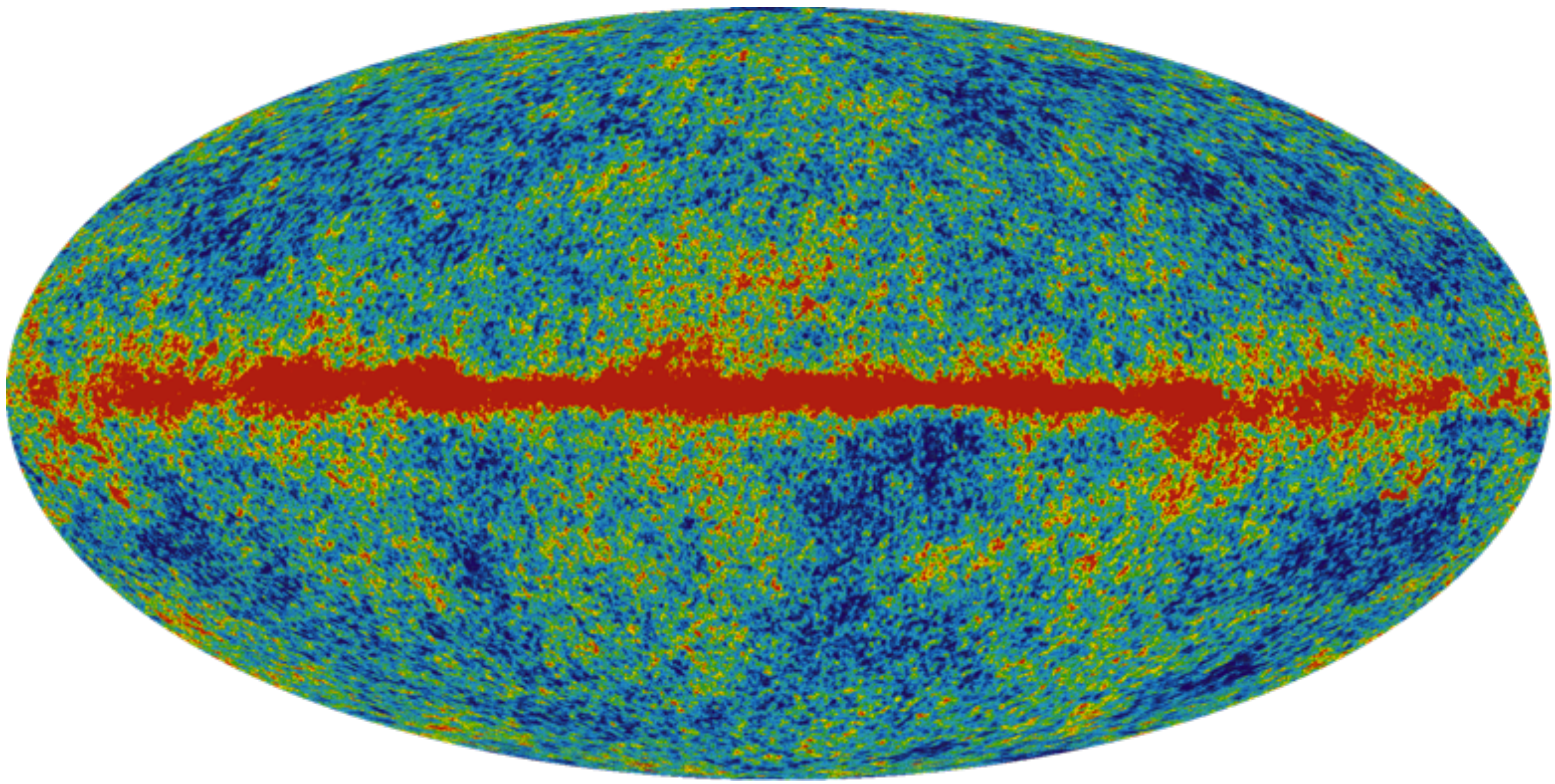
$$\frac{GM_\odot}{c^2 r_\odot} \sim 10^{-6}$$

$$\frac{G\hbar}{c^3 r_\odot^2} \sim 10^{-88}$$



Leading quantum corrections to Newton's law are incredibly small!

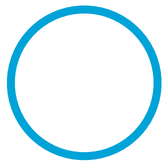
Can we ever observe quantum gravity effects?



Maybe there are some inside here...

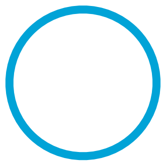
LO quantum corrections

Non-analytical vs non-local

- $+\dfrac{1}{2}$  $= -\dfrac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\left[\alpha\log\dfrac{R}{\mu^2}+\beta\log\dfrac{-\Box}{\mu^2}\right]R+\ldots$

LO quantum corrections

Non-analytical vs non-local

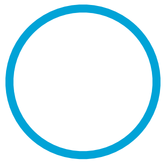
- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\left[\alpha\log\frac{R}{\mu^2}+\beta\log\frac{-\Box}{\mu^2}\right]R+\ldots$

Obtain finite part from beta functions (physical running)

$$2R\,\partial_R = -\mu\,\partial_\mu \qquad\qquad 2q^2\,\partial_{q^2} = -\mu\,\partial_\mu$$

LO quantum corrections

Non-analytical vs non-local

 $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\left[\alpha\log\frac{R}{\mu^2}+\beta\log\frac{-\Box}{\mu^2}\right]R+\ldots$

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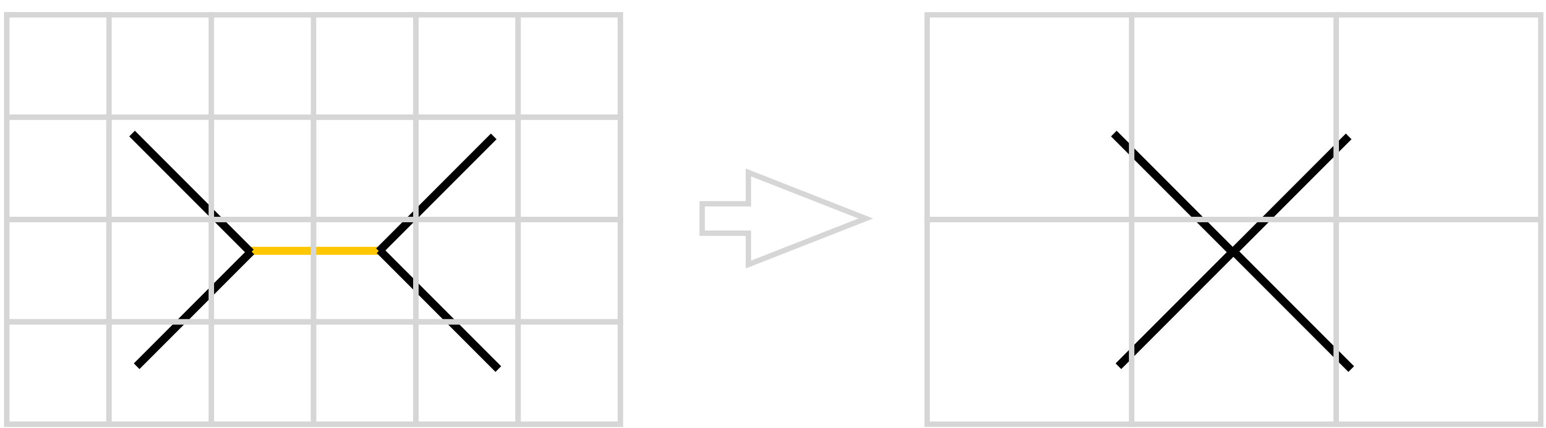
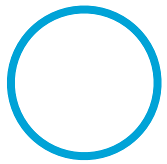


As in QED

LO quantum corrections

Non-analytical vs non-local

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\left[\alpha\log\frac{R}{\mu^2}+\beta\log\frac{-\Box}{\mu^2}\right]R+\ldots$

Obtain finite part from beta functions (physical running)

$$2R\,\partial_R = -\mu\,\partial_\mu \qquad\qquad 2q^2\,\partial_{q^2} = -\mu\,\partial_\mu$$

$$\alpha = \beta \qquad\qquad \alpha = \frac{C}{4(4\pi)^2}$$

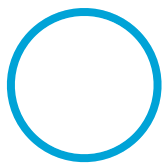
A topology must be chosen...

$$C = \frac{N_S}{72} \qquad \text{minimally coupled scalars}$$

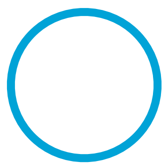
$$C = \frac{1}{4} \qquad \text{EFT gravity}$$

$$C = \frac{5}{36} \qquad \text{HDG}$$

Marginally deformed Starobinsky

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\,h\left(\frac{R}{\mu^2}\right)R + \ldots$

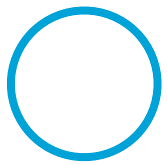
Marginally deformed Starobinsky

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\,h\left(\frac{R}{\mu^2}\right)R + \dots$

One loop flow equation

$$R\,\partial_R h = -\frac{C}{2(4\pi)^2} \qquad\qquad h(R/\mu^2) = \log R/\mu^2$$

Marginally deformed Starobinsky

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,R\,h\left(\frac{R}{\mu^2}\right)R + \ldots$

One loop flow equation

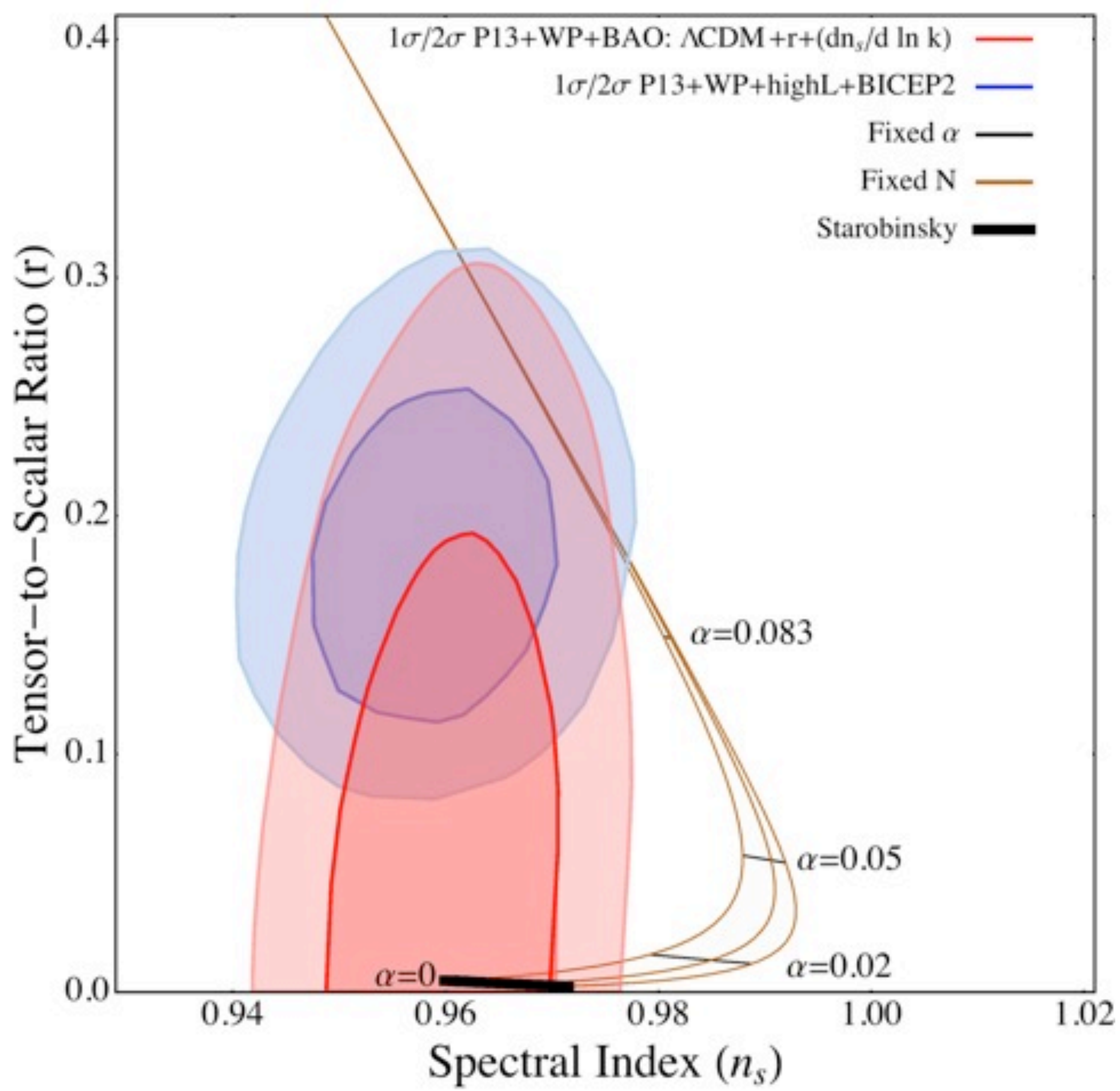
$$R\,\partial_R h = -\frac{C}{2(4\pi)^2} \qquad\qquad h(R/\mu^2) = \log R/\mu^2$$

RG improved flow equation

$$R\,\partial_R h = -\frac{C}{2(4\pi)^2}h \qquad\qquad h(R) = h(R_0)\left(\frac{R}{R_0}\right)^{-\frac{C}{2(4\pi)^2}}$$

$$C > 0 \qquad\qquad \alpha = \frac{C}{4(4\pi)^2} \qquad\qquad \alpha > 0$$

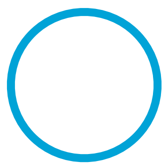
Marginally deformed Starobinsky (L)



Leading quantum corrections to tensor-to-scalar ratio

A. C, J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502, 050 (2015)

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^{d/2}} \int d^d x \sqrt{g}\, \mathrm{tr}\, \mathcal{R}\, \gamma_i \left(\frac{-\Box}{m^2} \right) \mathcal{R} + \ldots$

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

$$\gamma_i\left(\frac{X}{m^2}\right) \equiv \lim_{\Lambda_{UV}\to\infty} \int_{1/\Lambda_{UV}^2}^{\infty} \frac{ds}{s} s^{-d/2+2} \left[f_i(sX) - f_i(0) \right]$$

Non-local heat kernel structure functions

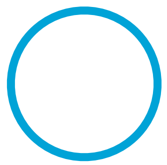
Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163

I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1

A. Codello and O. Zanusso, J. Math. Phys. 54 (2013) 013513

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$

$$-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$$

Explicit form for the structure functions

$$\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2}\int_0^1 d\xi\,\left[\frac{1}{u} + \xi(1-\xi)\right]^2 \log\left[1 + u\,\xi(1-\xi)\right]$$

$$\gamma_R(u) = -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32}\int_0^1 d\xi\,\Big\{\frac{2}{u^2} + \frac{4}{u}\left[1+\xi(1-\xi)\right] - 1 + 2\xi(2-\xi)(1-\xi^2)\Big\}\log\left[1 + u\,\xi(1-\xi)\right]$$

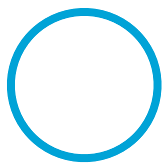
$$\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\,\left[\frac{1}{u} - \frac{1}{2} + \xi(1-\xi)\right]\log\left[1 + u\,\xi(1-\xi)\right]$$

$$\gamma_U(u) = -\frac{1}{2}\int_0^1 d\xi\,\log\left[1 + u\,\xi(1-\xi)\right]$$

$$\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\,\left[\frac{1}{u} + \xi(1-\xi)\right]\log\left[1 + u\,\xi(1-\xi)\right]$$

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\left[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\left(\frac{-\Box}{m^2}\right)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\left(\frac{-\Box}{m^2}\right)R\right.$
$\left. -\frac{1}{6}R\gamma_{RU}\left(\frac{-\Box}{m^2}\right)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\left(\frac{-\Box}{m^2}\right)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\left(\frac{-\Box}{m^2}\right)\mathbf{\Omega}^{\mu\nu}\right]$

Large energy expansion $u \gg 1$

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

$$\gamma_R(u) = -\frac{u}{336} + \frac{11u^2}{30240} - \frac{19u^3}{332640} + O(u^4)$$

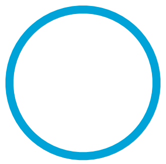
$$\gamma_{RU}(u) = \frac{u}{30} - \frac{u^2}{280} + \frac{u^3}{1890} + O(u^4)$$

$$\gamma_U(u) = -\frac{u}{12} + \frac{u^2}{120} - \frac{u^3}{840} + O(u^4)$$

$$\gamma_\Omega(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R - \frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

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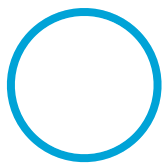
$$\gamma_\Omega(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

match with local heat kernel

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$
$-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

Low energy expansion $u \ll 1$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60}\log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120}\log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6}\log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_U(u) = 1 - \frac{1}{2}\log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12}\log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$u \equiv \frac{-\Box}{m^2}$$

Cosmological effective action (L)

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \xi\int d^4x\sqrt{-g}\,R^2 + \int d^4x\sqrt{-g}\,RF(\Box)R$$

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$$F(\Box) = \alpha\log\frac{-\Box}{m^2}$$

$$+\,\beta\frac{m^2}{-\Box}$$

$\alpha,\ \beta,\ \gamma,\ \delta$
are calculable constants depending on matter content

$$+\,\gamma\frac{m^2}{-\Box}\log\frac{-\Box}{m^2}$$

$$+\,\delta\frac{m^4}{(-\Box)^2}$$

$$+\,...$$

Cosmological effective action (L)

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \xi\int d^4x\sqrt{-g}\,R^2 + \int d^4x\sqrt{-g}\,RF(\Box)R$$

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Leading logs
J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014)

$$+\,\beta\frac{m^2}{-\Box}$$

Non-local cosmology
S. Deser and R. P. Woodard, Phys. Rev. Lett. 99, 111301 (2007)

$\alpha,\ \beta,\ \gamma,\ \delta$
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$$+\,\gamma\frac{m^2}{-\Box}\log\frac{-\Box}{m^2}$$

$$+\,\delta\frac{m^4}{(-\Box)^2}$$

Non-local gravity and dark energy
M. Maggiore and M. Mancarella, Phys. Rev. D 90, 023005 (2014).

$$+\,\ldots$$

Effective non-local cosmology
A. C. and K. J. Jain in preparation

Effective Friedmann equations (L)

Local gravity @ LO

$$H^2 + 96\pi\xi G\left[2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

Effective Friedmann equations (L)

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Pure Starobinsky gravity is exactly solvable

$$2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 = 0$$

$$H = 0$$

$$t + C_2 = \int \frac{dH}{C_1\sqrt{H} - 2H^2}$$

$$H = \frac{1}{2t}$$

Effective Friedmann equations (L)

Local gravity @ LO

$$H^2 + 96\pi\xi G\left[2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

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$$H = 0$$

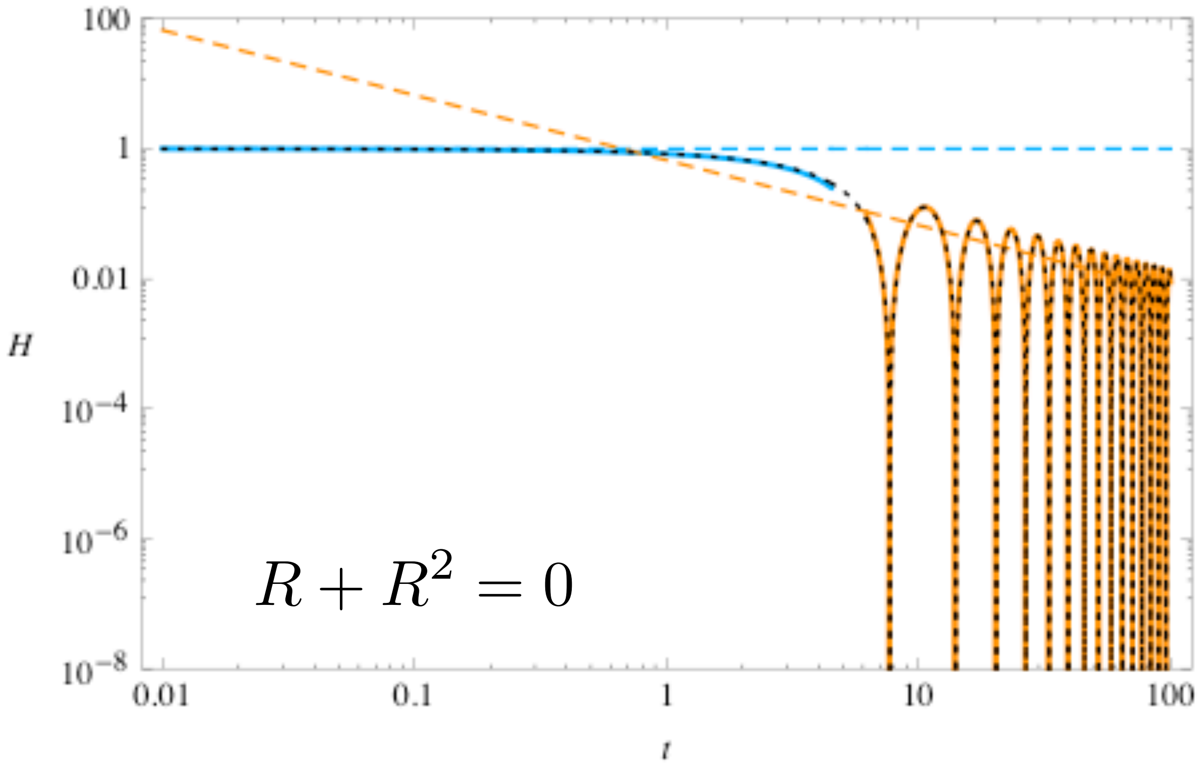
$$t + C_2 = \int \frac{dH}{C_1\sqrt{H} - 2H^2}$$

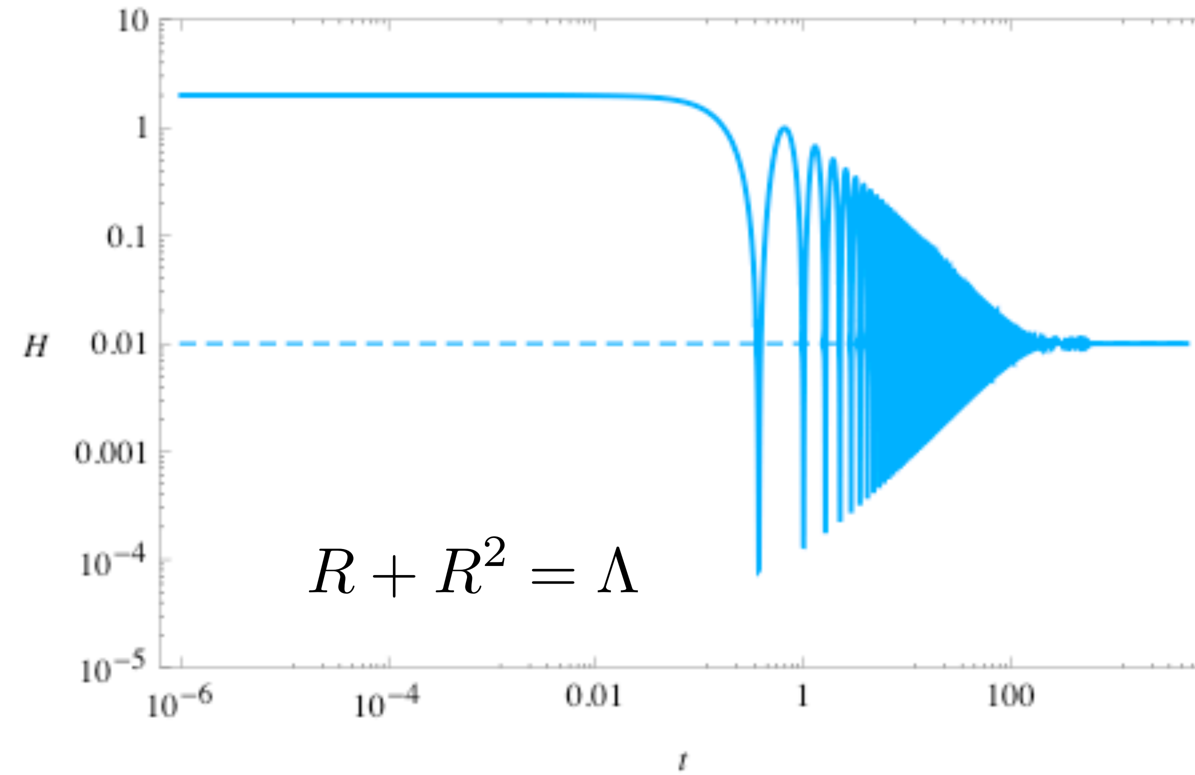
$$H = \frac{1}{2t}$$

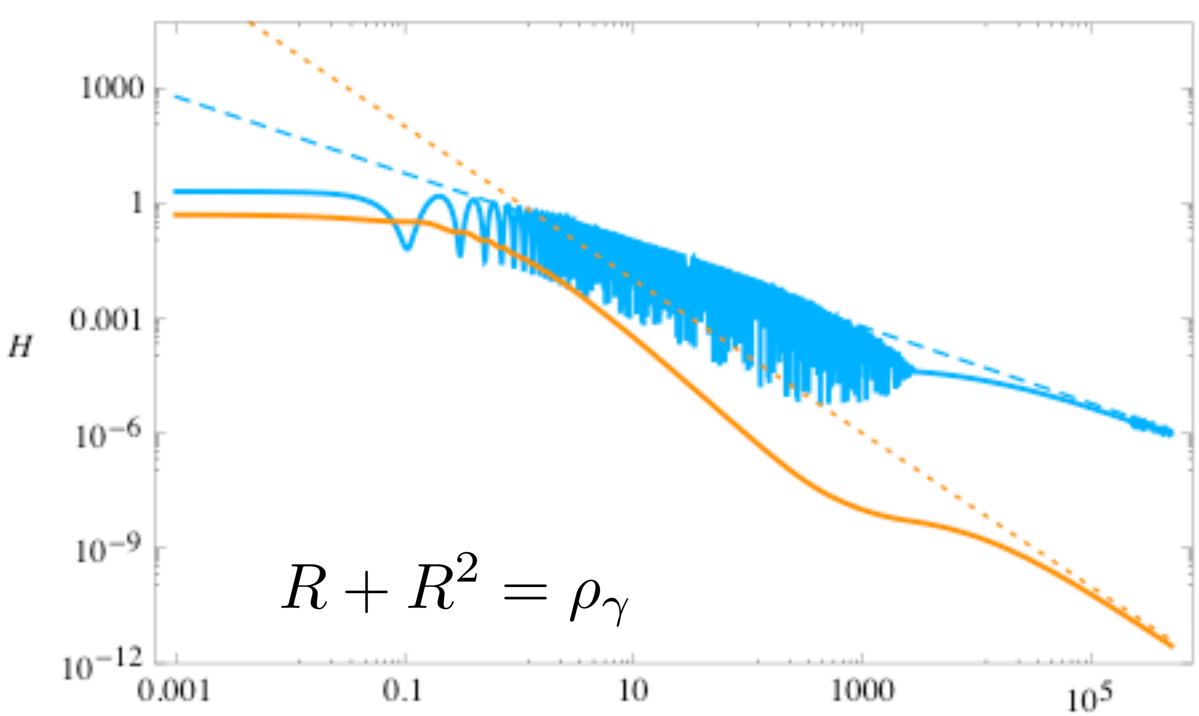
$$R + R^{n-1}\delta R = -8\pi G\, T$$

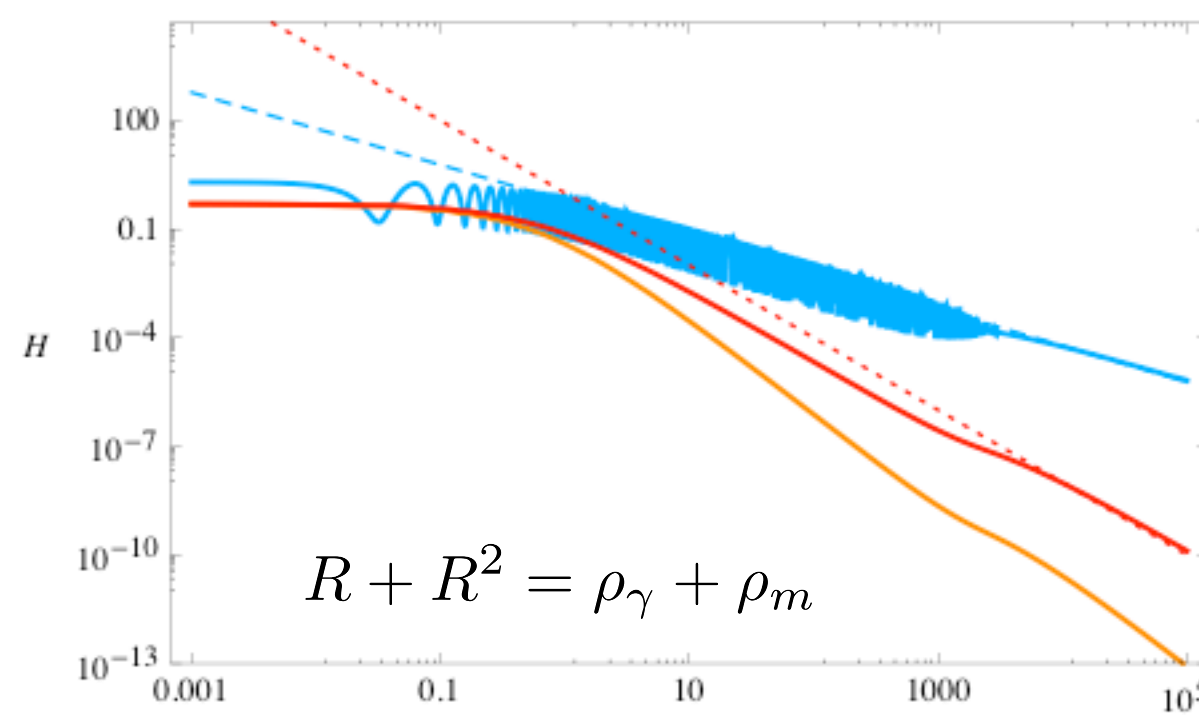
$$T = 0 \qquad \Rightarrow \qquad R = 0$$

Effective Friedmann equations (L)









Effective Friedmann equations (L)

$$H^2(t) + 16\pi G\alpha \left\{ -\left[\frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt'\, a^{\frac{3}{2}}(t')R(t')\, L_-(t-t') \right.$$
$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt'\, L_-(t-t')\, \frac{d}{dt'}\left[a^{\frac{3}{2}}(t')R(t') \right] \right\} = \frac{8\pi G}{3}\rho(t)$$

How to interpret non-local terms?

Effective Friedmann equations (L)

$$H^2(t) + 16\pi G\alpha \left\{ -\left[\frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt'\, a^{\frac{3}{2}}(t')R(t')\, L_-(t-t') \right.$$
$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt'\, L_-(t-t')\, \frac{d}{dt'}\left[a^{\frac{3}{2}}(t')R(t') \right] \right\} = \frac{8\pi G}{3}\rho(t)$$

How to interpret non-local terms?

$$L \equiv \log \frac{-\Box}{\mu^2}$$

$$L(t-t') = \int_0^\infty ds \left[\frac{1}{\mu^2+s} - G(t-t';\sqrt{s}) \right]$$

$$G_-(t-t';m) = \frac{\theta(t-t')}{\sqrt{a^3(t)a^3(t')}} \frac{\sin m(t-t')}{m} + O(\dot{a})$$

$$L_-(t-t') = -2 \lim_{\epsilon\to 0} \left[\frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')\log\mu\,\epsilon \right]$$

Effective Friedmann equations (L)

$$H^2(t) + 16\pi G\alpha \left\{ -\left[\frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt'\, a^{\frac{3}{2}}(t')R(t')\, L_-(t-t') + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt'\, L_-(t-t')\, \frac{d}{dt'} \left[a^{\frac{3}{2}}(t')R(t') \right] \right\} = \frac{8\pi G}{3}\rho(t)$$

Try an iterative solution...

$$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$$

$$a(t) = (t/t_0)^{1/2}$$

$$a(t) = (t/t_0)^{2/3}$$

Effective Friedmann equations (L)

PRELIMINARY RESULTS

$$H^2(t) + 16\pi G\alpha \left\{ - \left[\frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt'\, a^{\frac{3}{2}}(t') R(t')\, L_-(t-t') \right.$$

$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt'\, L_-(t-t')\, \frac{d}{dt'} \left[a^{\frac{3}{2}}(t') R(t') \right] \right\} = \frac{8\pi G}{3}\rho(t)$$

Try an iterative solution...

$$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$$

no effect

$$a(t) = (t/t_0)^{1/2}$$

$$a(t) = (t/t_0)^{2/3}$$

Effective Friedmann equations (L)

$$H^2(t) + 16\pi G\alpha \left\{ -\left[\frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}}\right] \int dt'\, a^{\frac{3}{2}}(t')R(t')\, L_-(t-t') \right.$$
$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt'\, L_-(t-t')\, \frac{d}{dt'}\left[a^{\frac{3}{2}}(t')R(t')\right] \right\} = \frac{8\pi G}{3}\rho(t)$$

Try an iterative solution...

$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$ no effect

$a(t) = (t/t_0)^{1/2}$ no effect

$a(t) = (t/t_0)^{2/3}$

Effective Friedmann equations (L)

$$H^2(t) + 16\pi G\alpha \left\{ -\left[\frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt'\, a^{\frac{3}{2}}(t') R(t')\, L_-(t-t') \right.$$
$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt'\, L_-(t-t')\, \frac{d}{dt'} \left[a^{\frac{3}{2}}(t') R(t') \right] \right\} = \frac{8\pi G}{3}\rho(t)$$

Try an iterative solution...

$$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$$ no effect

$$a(t) = (t/t_0)^{1/2}$$ no effect

$$a(t) = (t/t_0)^{2/3}$$ some effect

$$H^2(t) + \frac{256\pi G\alpha}{3t^4}\left[\log \mu t + \log\left(\frac{t}{t_0} - 1\right) + \frac{2}{3}\left(\frac{t}{t_0} - 1\right) \right] = \frac{8\pi G}{3}\rho_m(t_0)\left(\frac{t_0}{t}\right)^2$$

... log R dominant a early times while log box at late times

Conclusions and Outlook

- Compute all LO terms
- Renormalization of NLO
- Conformal anomaly contribution
- Apply to cosmology
- Apply to stars/black holes
- Add the SM and constrain BSM
- Connection with high energy quantum gravity
- Falsify!

Thank you