



# EFT of Gravity and Cosmology

Alessandro Codello

*Probing the fundamental  
nature of spacetime with the RG*  
NORDITA, Stockholm 23-27 March 2015

**work with:**

J Joergensen, F Sannino, O Svendsen

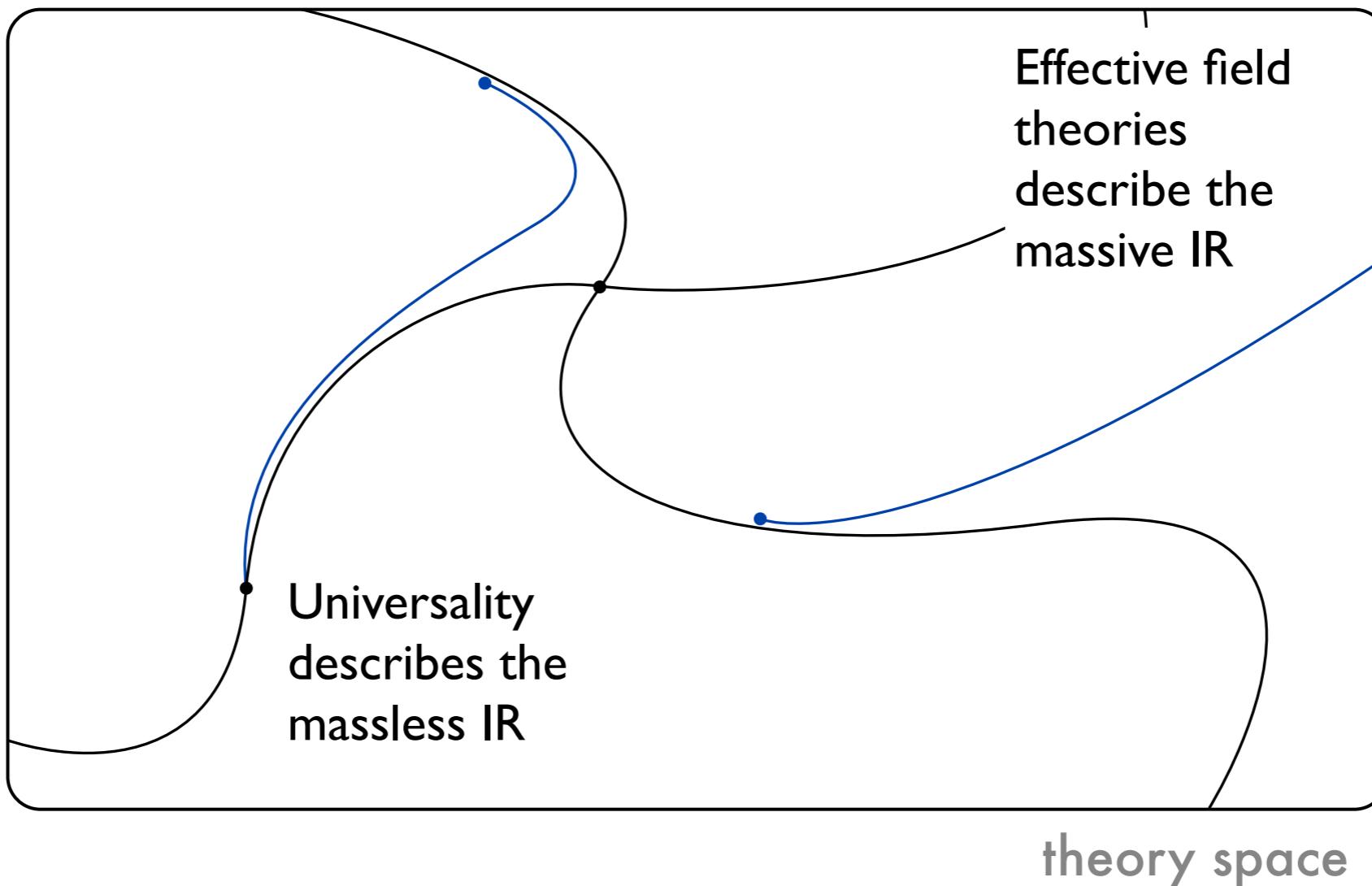
R Percacci, A Tonero, L Rachwal

R K Jain

# Outline of the talk

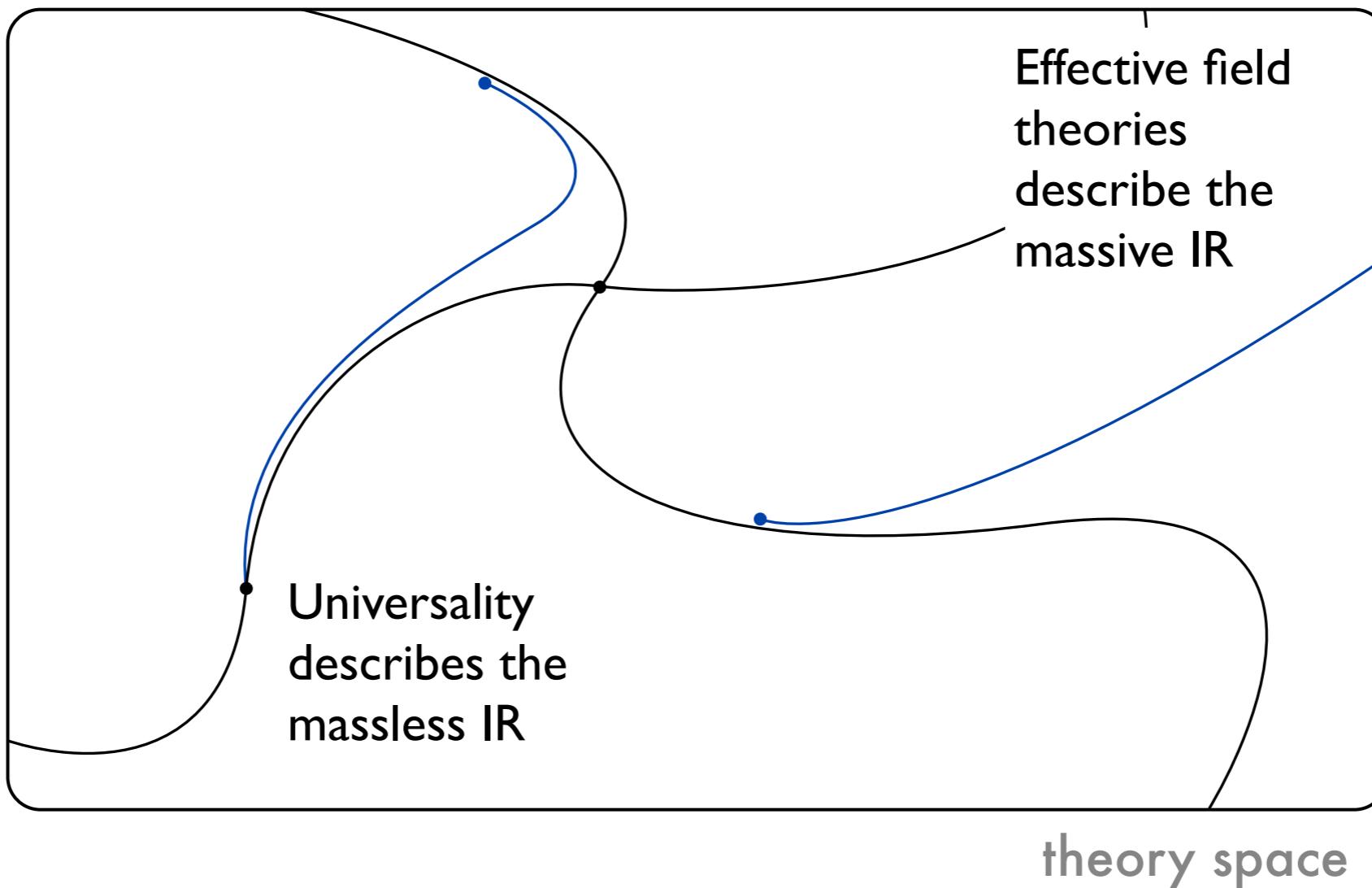
- Effectivity vs Universality
- Example: EFT of Pions (CPT)
- Covariant EFT of Gravity in three lines
- Renormalization in different schemes
- Phenomenological parameters and their estimation
- Adding matter
- LO quantum corrections
- Marginally deformed Starobinsky
- Effective Friedmann equations

# Effectivity vs Universality



Two main reasons why mathematical modeling of nature actually works

# Effectivity vs Universality



Massive IR lies in the broken phase ( $G$  to  $G/H$ )

Characteristic large scale  $M$  at which  $G$  is broken

# The theory of pions (CPT)

- Low energy QCD can be described by an EFT of pions
- Symmetry breaking pattern  $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$
- $M$  is the pion decay constant  $F_\pi \sim 10^2$  MeV
- Phenomenological parameters  $l_i$  to be fixed by experiments ( $i = 1, 2$ )
- Renormalization and scale dependence

$$l_i(\mu_1) = l_i(\mu_2) + \frac{\gamma_i}{(4\pi)^2} \log \frac{\mu_1}{\mu_2}$$

- Low energy expansion of physical quantities

$$A(E) = \frac{E^2}{M^2} \left( 1 + C_1 l_1 \frac{E^2}{M^2} + C_2 l_2 \frac{E^2}{M^2} \right) + \dots$$

# EFT of Gravity

- The theory of small fluctuations of the metric

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M} h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19} \text{ GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

# EFT of Gravity

$$S_{eff}[g] = \int d^4x \sqrt{g} \left[ M^4 c_0 + M^2 (-R) + c_{2,1} R^2 + c_{2,2} \text{Ric}^2 + c_{2,3} \text{Riem}^2 + \frac{1}{M^2} c_{3,1} R^3 + \dots \right]$$

# EFT of Gravity

$$S_{eff}[g] = \int d^4x \sqrt{g} \left[ M^4 \underbrace{c_0}_{\mathcal{R}^0} + M^2 \underbrace{(-R)}_{\mathcal{R}} + \underbrace{c_{2,1} R^2 + c_{2,2} \text{Ric}^2 + c_{2,3} \text{Riem}^2}_{\mathcal{R}^2} + \frac{1}{M^2} \underbrace{c_{3,1} R^3}_{\mathcal{R}^3} + \dots \right]$$

# EFT of Gravity

$$S_{eff}[g] = \int d^4x \sqrt{g} \left[ M^4 \underbrace{c_0}_{\mathcal{R}^0} + M^2 \underbrace{(-R)}_{\mathcal{R}} + \underbrace{c_{2,1}R^2 + c_{2,2}\text{Ric}^2 + c_{2,3}\text{Riem}^2}_{\mathcal{R}^2} + \frac{1}{M^2} \underbrace{c_{3,1}R^3}_{\mathcal{R}^3} + \dots \right]$$
$$\equiv M^2 \left[ I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + \dots \right] \quad m^2 = -2\Lambda$$

# Covariant EFT of Gravity

$$\begin{aligned} e^{-\Gamma[g]} &= \int_{1PI} \mathcal{D}h_{\mu\nu} e^{-S_{eff}[g + \frac{1}{M}h]} \\ &= \int_{1PI} \mathcal{D}h_{\mu\nu} e^{-M^2 \left\{ I_1[g + \frac{1}{M}h] + \frac{1}{M^2} I_2[g + \frac{1}{M}h] + \dots \right\}} \end{aligned}$$

EFT: saddle point expansion in  $\frac{1}{M^2}$

# Covariant EFT of Gravity

$$\Gamma[g] = I_1[g] \quad \text{CT}$$

$$+ \frac{1}{M^2} \left\{ I_2[g] + \frac{1}{2} \text{Tr} \log I_1^{(2)}[g] \right\} \quad \text{LO}$$

$$+ \frac{1}{M^4} \left\{ I_3[g] + \frac{1}{2} \text{Tr} \left[ \left( I_1^{(2)}[g] \right)^{-1} I_2^{(2)}[g] \right] + \text{2-loops with } I_1[g] \right\} \quad \text{NLO}$$

$$+ \dots \quad \text{NNLO}$$

EFT: saddle point expansion in  $\frac{1}{M^2}$

# Covariant EFT of Gravity

$$\Gamma = \begin{array}{c} \text{CT} \\ \text{I}_1 \\ + \frac{1}{M^2} \left[ \text{I}_2 + \frac{1}{2} \text{O} \right] \\ + \frac{1}{M^4} \left[ \text{I}_3 + \frac{1}{2} \text{O} - \frac{1}{12} \text{S} + \frac{1}{8} \text{E} \right] \\ + \dots \end{array}$$

The equation shows the expansion of the Covariant EFT of Gravity. The terms are labeled by their order in the inverse mass  $M$ : CT (Counterterm) at order 0, LO (Loop Order) at order 1, NLO (Next-to-Loop Order) at order 2, and NNLO (Next-to-Next-to-Loop Order) at order 3. The diagrams are represented by colored dots and circles:

- $\text{I}_1$ : A single blue dot.
- $\text{I}_2$ : A purple dot with a vertical line through it.
- $\text{O}$ : A blue circle.
- $\text{I}_3$ : A pink dot with a vertical line through it.
- $\text{S}$ : A blue circle with a horizontal line through it.
- $\text{E}$ : A blue circle with a diagonal line through it.

# Covariant EFT of Gravity

## The EFT recipe in three lines

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ I_1 + \frac{1}{2} \text{LO} \right] + \frac{1}{M^4} \left[ I_2 + \frac{1}{2} \text{NLO} \right] + \dots$$

  
 $I_1$   
  
 $I_2$   
  
 $I_3$

- 1) the general lagrangian of order  $E^2$  is to be used both at tree level and in loop diagrams
  - 2) the general lagrangian of order  $E^{n \geq 4}$  is to be used at tree level and as an insertion in loop diagrams
  - 3) the renormalization program is carried out order by order

# Covariant EFT of Gravity

What do we already know?

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ \text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[ \text{NLO} - \frac{1}{12} \text{NNLO} + \frac{1}{8} \text{NNLO} \right] + \dots$$

The diagram shows the expansion of the effective action  $\Gamma$ . The first term is a blue dot. The second term is  $\frac{1}{M^2}$  times the sum of a green box containing a purple dot and a blue circle divided by 2. The third term is  $\frac{1}{M^4}$  times the sum of a pink dot, a blue circle divided by 2, a blue circle divided by 12, and a blue circle divided by 8. The ellipsis indicates higher-order terms.

## UV divergencies and renormalization

G. 't Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A 20 (1974) 69

G. W. Gibbons, S. W. Hawking and M. J. Perry, Nucl. Phys. B 138 (1978) 141

S. M. Christensen and M. J. Duff, Nucl. Phys. B 170 (1980)

# Covariant EFT of Gravity

What do we already know?

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ \text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[ \text{NLO} + \frac{1}{2} \text{NNLO} - \frac{1}{12} \text{UV} + \frac{1}{8} \text{UV} \right] + \dots$$

The diagram shows the expansion of the effective action  $\Gamma$ . The first term is a single blue dot labeled 'CT'. The second term is  $\frac{1}{M^2}$  times a sum of two diagrams: a purple dot and a blue circle divided by a horizontal line. The third term is  $\frac{1}{M^4}$  times a sum of three diagrams: a red box containing a pink dot, a blue circle with a purple dot inside, and a blue circle divided by a horizontal line. The fourth term is indicated by three dots. The labels 'LO', 'NLO', and 'NNLO' are placed to the right of the corresponding terms.

Two loops UV divergencies

M.H. Goroff and A. Sagnotti, Nucl.Phys.B266, 709 (1986)  
A. E. M. van de Ven, Nucl. Phys. B378, 309 (1992)

# Covariant EFT of Gravity

What do we already know?

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ \text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[ \text{NLO} + \frac{1}{2} \text{NNLO} \right] + \dots$$

The diagram shows the four-point vertex  $\Gamma$  as a sum of contributions. The first term is labeled "CT". The second term is labeled "LO" and contains a blue dot and a blue circle. The third term is labeled "NLO" and contains a purple dot, a pink circle, a blue circle, a blue circle with a horizontal line, and a blue circle with the number 8. The fourth term is labeled "NNLO" and contains three dots.

Finite LO terms

Leading logs

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

A. C., J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502 (2015) 050

Conformal anomaly

S. Deser, M. J. Duff and C. J. Isham, Nucl. Phys. B 111, 45 (1976)

R.J. Riegert, Phys. Lett. B 134 (1984) 56

Four graviton vertex in Minkowski space

D. C. Dunbar and P. S. Norridge, Nucl. Phys. B 433, 181 (1995)

Curvature square terms

A. C. and R. K. Jain, in preparation

# Covariant EFT of Gravity

LOQG: the only QG we will ever observe!

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ \text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[ \text{NLO} - \frac{1}{12} \text{NNLO} \right] + \dots$$

The diagram shows the expansion of the effective coupling  $\Gamma$ . The first term is a blue dot labeled "CT". The second term is a blue box containing a purple dot and a blue circle, labeled "LO". The third term is a blue box containing a pink dot, a blue circle, a blue circle with a horizontal line, and a blue circle with a vertical line, labeled "NLO". The fourth term is a blue box containing three dots and three circles, labeled "NNLO". Ellipses indicate higher-order terms.

Even if we have a fundamental theory its is generally difficult to compute phenomenological parameters...

# Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

- $= \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left[ \Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4 \log \frac{\Lambda_{UV}^2}{m^2} + \left( -\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2 \log \frac{\Lambda_{UV}^2}{m^2} \right) R + \left( \frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\square R \right) \log \frac{\Lambda_{UV}^2}{m^2} \right]$ 

measured  
+ phenomenological  
parameters

# Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

$$\bullet = \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left[ \Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4 \log \frac{\Lambda_{UV}^2}{m^2} \right. \\ \left. + \left( -\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2 \log \frac{\Lambda_{UV}^2}{m^2} \right) R \right. \\ \left. + \left( \frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\square R \right) \log \frac{\Lambda_{UV}^2}{m^2} \right] \quad \begin{matrix} \text{measured} \\ + \text{phenomenological} \\ \text{parameters} \end{matrix}$$

Dimensional regularization

$$\log \Lambda_{UV}^2 \rightarrow \frac{1}{\epsilon} \quad \Lambda_{UV}^2 \rightarrow 0 \quad \Lambda_{UV}^4 \rightarrow 0$$

# Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

- $= \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left[ \Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4 \log \frac{\Lambda_{UV}^2}{m^2} + \left( -\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2 \log \frac{\Lambda_{UV}^2}{m^2} \right) R + \left( \frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\square R \right) \log \frac{\Lambda_{UV}^2}{m^2} \right]$  measured + phenomenological parameters

Dimensional regularization

$$\log \Lambda_{UV}^2 \rightarrow \frac{1}{\epsilon} \quad \Lambda_{UV}^2 \rightarrow 0 \quad \Lambda_{UV}^4 \rightarrow 0$$

G runs if there is a mass scale involved also  
in dimensional regularization [Kirill's talk]

$$c_i(\mu_1) = c_i(\mu_2) + \frac{\gamma_i}{(4\pi)^2} \log \frac{\mu_1}{\mu_2} \quad \mu \partial_\mu c_i = \frac{\gamma_i}{(4\pi)^2}$$

# Phenomenological parameters

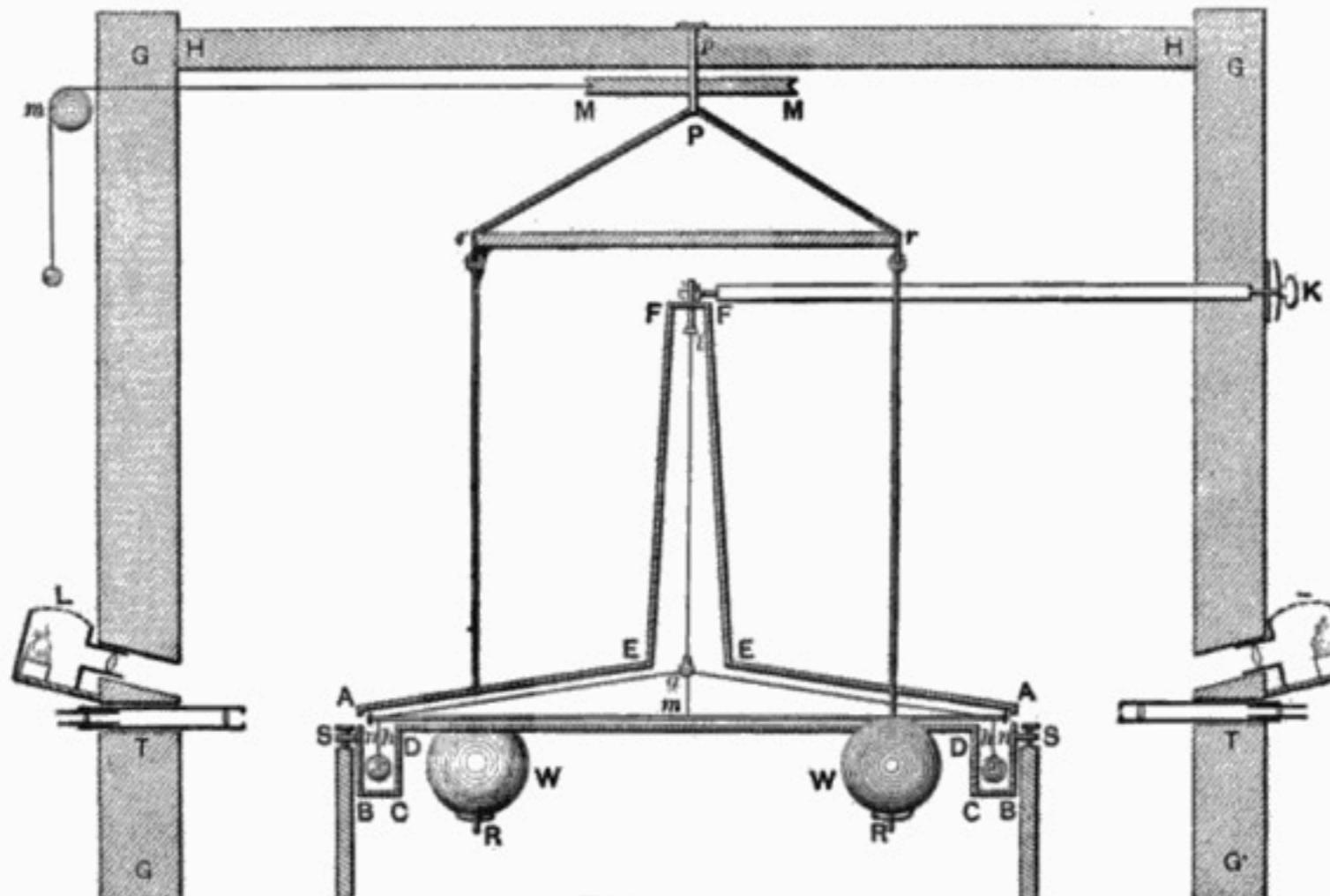
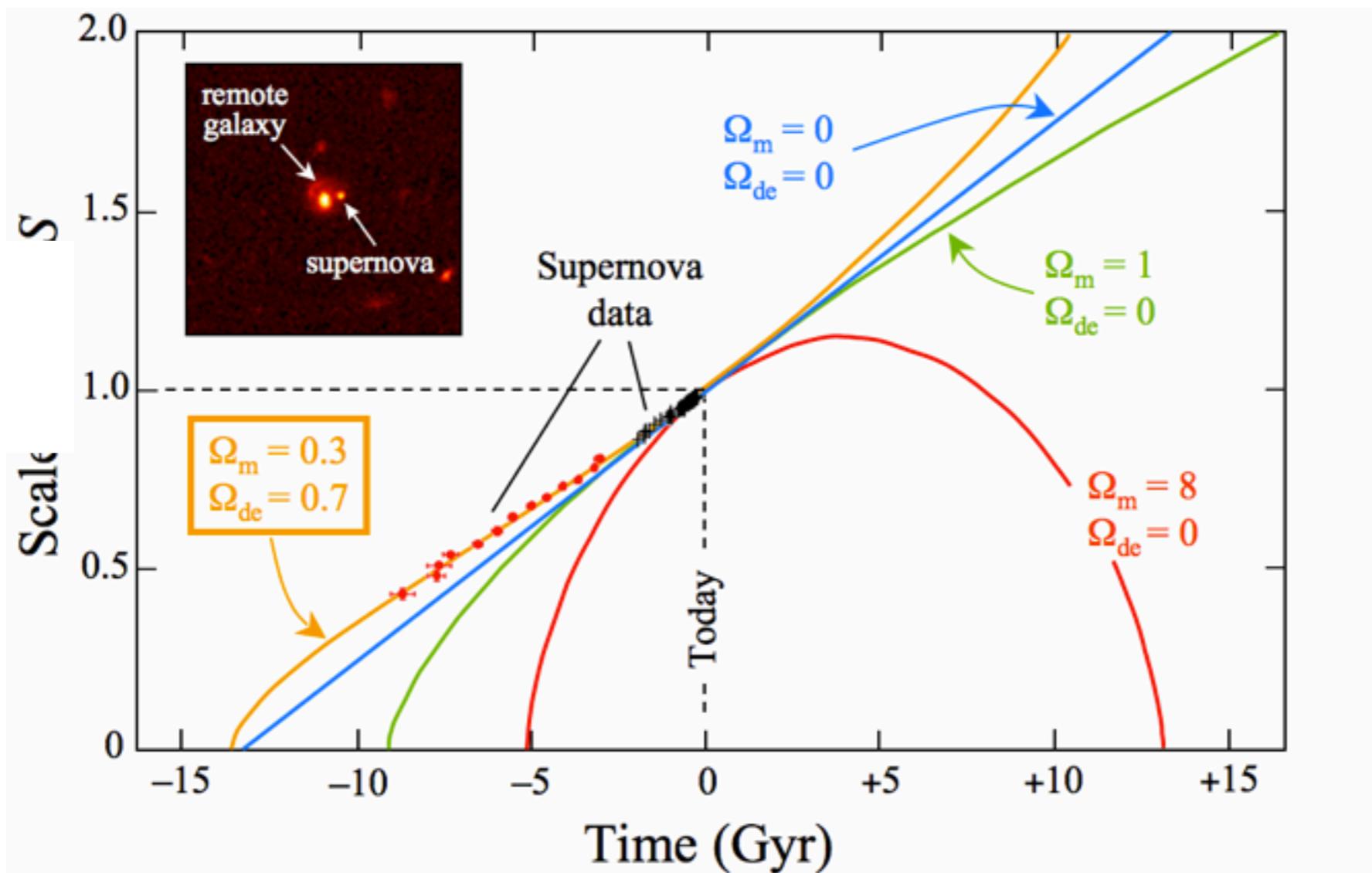


Fig. 1

$$G = 6.67428 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$$

Cavendish 1797 (1% off best value!)

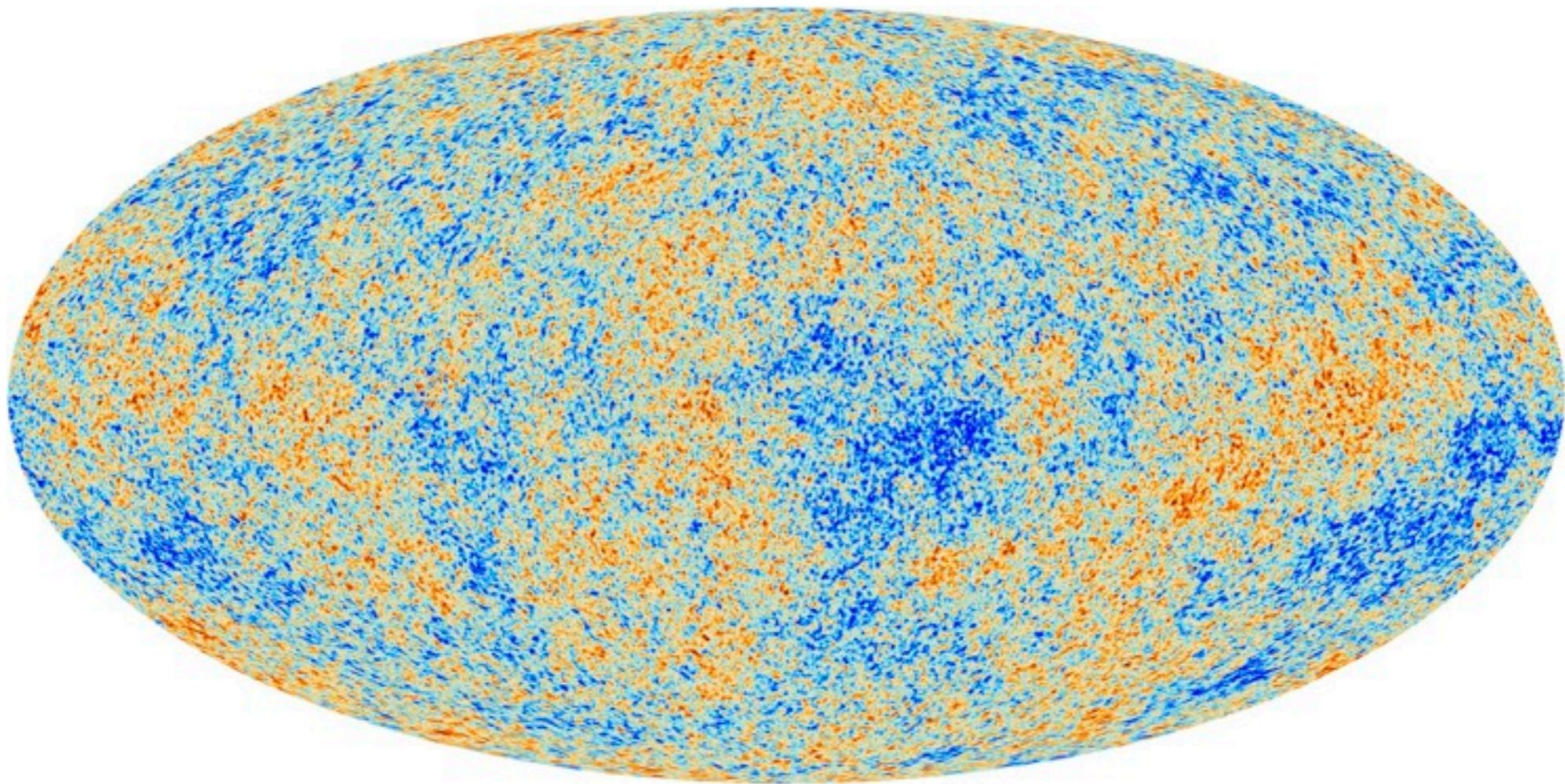
# Phenomenological parameters



$$\Lambda = 10^{-47} \text{ GeV}^4$$

Supernova Cosmology Project

# Phenomenological parameters



$$\xi \equiv c_{R^2}$$

$$\xi(k_*) \sim 10^9$$

$$k_* \equiv 0.05 \text{ Mpc}^{-1} \sim 10^{-40} \text{ GeV}$$

Planck mission  
[Alfio's talk]

# Adding matter

$$\Gamma = \bullet + \frac{1}{2} \text{O} \quad \text{CT}$$

$$+ \frac{1}{M^2} \left[ \bullet + \bullet + \bullet + \frac{1}{2} \text{O} - \text{O} \right] \quad \text{LO}$$

$$+ \dots \quad \text{NLO}$$

# Adding matter

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ \text{LO} + \frac{1}{2} \text{NLO} \right]$$

The equation shows the expansion of the effective action  $\Gamma$ . The first term is labeled "CT". The second term is  $\frac{1}{M^2}$  times the sum of two parts: "LO" (represented by a green box containing a purple dot and a black dot) and "NLO" (represented by a blue circle minus a black circle). The "NLO" term is preceded by a plus sign.

## UV divergencies and renormalization with matter Scalars

A. O. Barvinsky, A. Y. Kamenshchik and I. P. Karmazin, Phys. Rev. D 48 (1993) 3677

...

## Gauge

S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006)

...

## Yukawa

A. Rodigast and T. Schuster, Phys. Rev. Lett. 104, 081301 (2010)

...

# Adding matter

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[ \text{LO} + \dots \right]$$

The diagram shows the definition of the function  $\Gamma$ . It is equal to a constant term labeled "CT" plus one over  $M^2$  times a bracketed expression. The bracketed expression contains a term labeled "LO" (Low Order) and a series of ellipses. The "LO" term consists of a blue dot, a purple dot, and a black dot connected by a horizontal line. Above this line is a blue circle, and below it is a black circle. A blue circle is also positioned to the right of the line. To the left of the bracketed expression is a blue dot, followed by a plus sign, a purple dot, another plus sign, and a black dot. Below the bracketed expression is a plus sign followed by three dots.

## Finite LO terms with matter

Flat space corrections to Newton's potential

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein (2003b) Phys. Rev. D 67

I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

## Covariant leading logs

A C, R Percacci, L Rachwal and A Tonero in preparation

# Adding matter

## Matter induced effective action

# Integrating out matter

$$\frac{1}{2} \bigcirc = -\frac{1}{2} \frac{1}{(4\pi)^2} \sum_{n=\frac{d}{2}+1} \frac{1}{m^{2n-d}} B_{2n}$$

Local heat kernel coefficients

**B6**

Gilkey, PB

A. O. Barvinsky and G. A. Vilkovisky, Phys. Rept. 119 (1985) 1.

I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1.

Groh, Kai Saueressig, Frank Zanuso, Omar

**B8**

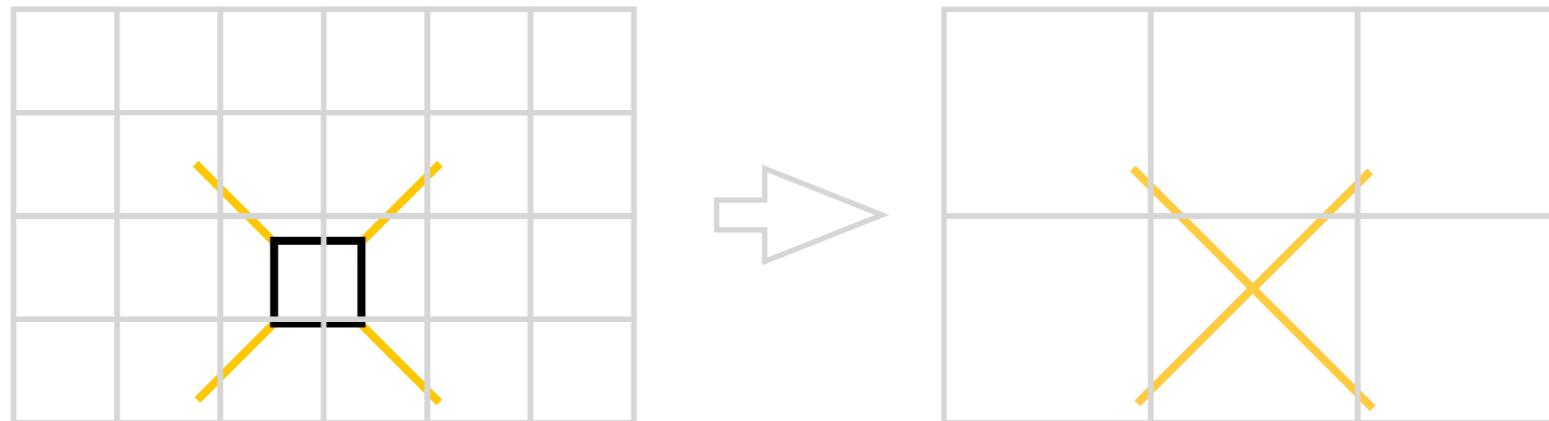
Amsterdamski, P

Ven, AEM Van de

Matter induced effective action:  
expandable in inverse powers of the (lightest) particle masse

# Integrating out matter

$$\frac{1}{2} \text{ } \bigcirc = -\frac{1}{2} \frac{1}{(4\pi)^2} \frac{1}{m^2} B_6 + \dots$$



As in QED when we integrate out electrons

# Integrating out matter

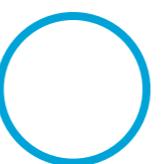
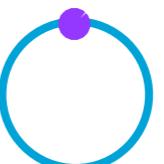
$$\frac{1}{2} \bigcirc = -\frac{1}{2} \frac{1}{(4\pi)^2} \frac{1}{m^2} \int d^4x \sqrt{g} \left[ \frac{1}{336} R \square R + \frac{1}{840} R_{\mu\nu} \square R^{\mu\nu} \right. \\ \left. + \frac{1}{1296} R^3 - \frac{1}{1080} R R_{\mu\nu} R^{\mu\nu} - \frac{4}{2835} R_\mu^\nu R_\nu^\alpha R_\alpha^\mu \right. \\ \left. + \frac{1}{945} R_{\mu\nu} R_{\alpha\beta} R^{\mu\alpha\nu\beta} + \frac{1}{1080} R R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \frac{1}{7560} R_{\mu\nu} R^{\mu\alpha\beta\gamma} R_\alpha^\nu R_\beta^\gamma \right. \\ \left. + \frac{17}{45360} R_{\mu\nu}^{\alpha\beta} R_{\alpha\beta}^{\gamma\delta} R_{\gamma\delta}^{\mu\nu} - \frac{1}{1620} R_\mu^{\alpha\beta} R_\nu^{\mu\nu} R_\gamma^{\delta\alpha} R_\beta^{\gamma\delta} \right] + \dots$$

Changes how cubic terms are suppressed:

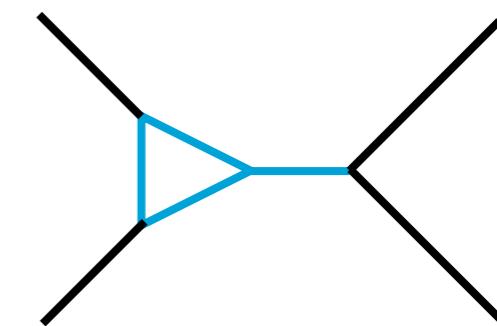
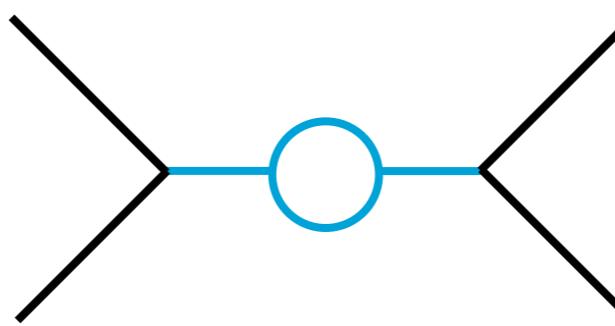
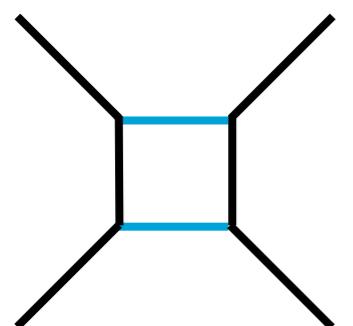
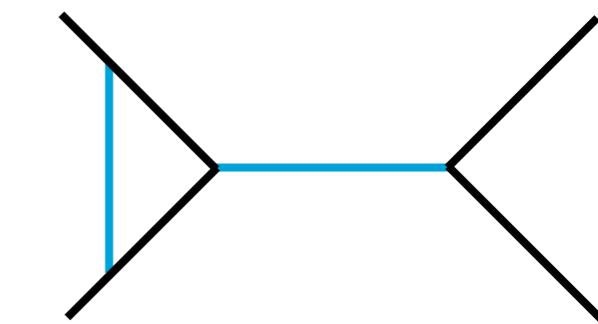
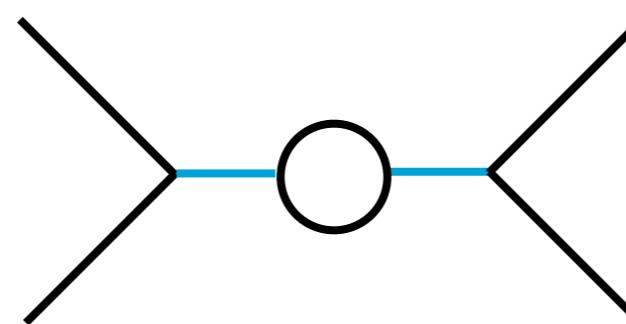
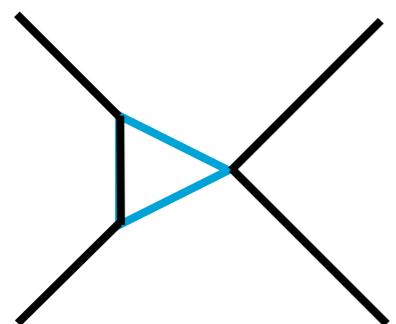
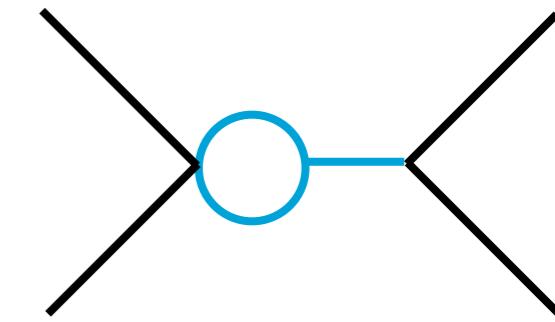
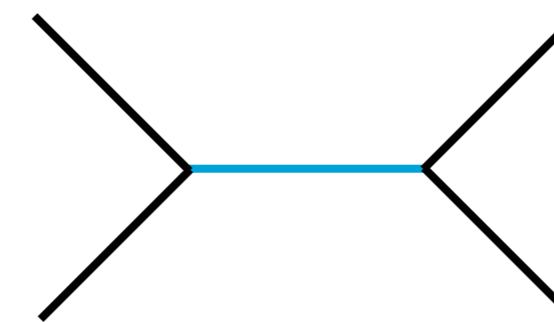
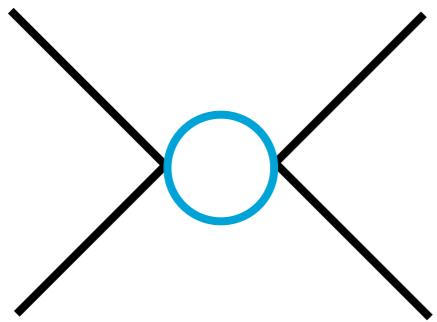
$$\frac{1}{M^4} \rightarrow \frac{1}{m^2 M^2}$$

# Integrating out matter

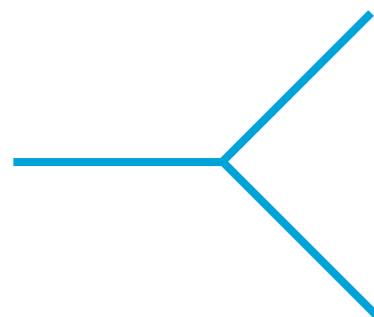
$$\frac{1}{2} \bigcirc = -\frac{1}{2} \frac{1}{(4\pi)^2} \frac{1}{m^2} \int d^4x \sqrt{g} \left[ \frac{1}{336} R \square R + \frac{1}{840} R_{\mu\nu} \square R^{\mu\nu} \right. \\ \left. + \frac{1}{1296} R^3 - \frac{1}{1080} R R_{\mu\nu} R^{\mu\nu} - \frac{4}{2835} R_\mu^\nu R_\nu^\alpha R_\alpha^\mu \right. \\ \left. + \frac{1}{945} R_{\mu\nu} R_{\alpha\beta} R^{\mu\alpha\nu\beta} + \frac{1}{1080} R R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \frac{1}{7560} R_{\mu\nu} R^{\mu\alpha\beta\gamma} R_\alpha^\nu R_\beta^\gamma \right. \\ \left. + \frac{17}{45360} R_{\mu\nu}^{\alpha\beta} R_{\alpha\beta}^{\gamma\delta} R_{\gamma\delta}^{\mu\nu} - \frac{1}{1620} R_\mu^\alpha R_\nu^\beta R_\gamma^\mu R_\delta^\nu \right] + \dots$$

The same can be applied to  and with some modifications to  and thus find 

# Corrections to Newton's interaction



# Corrections to Newton's interaction

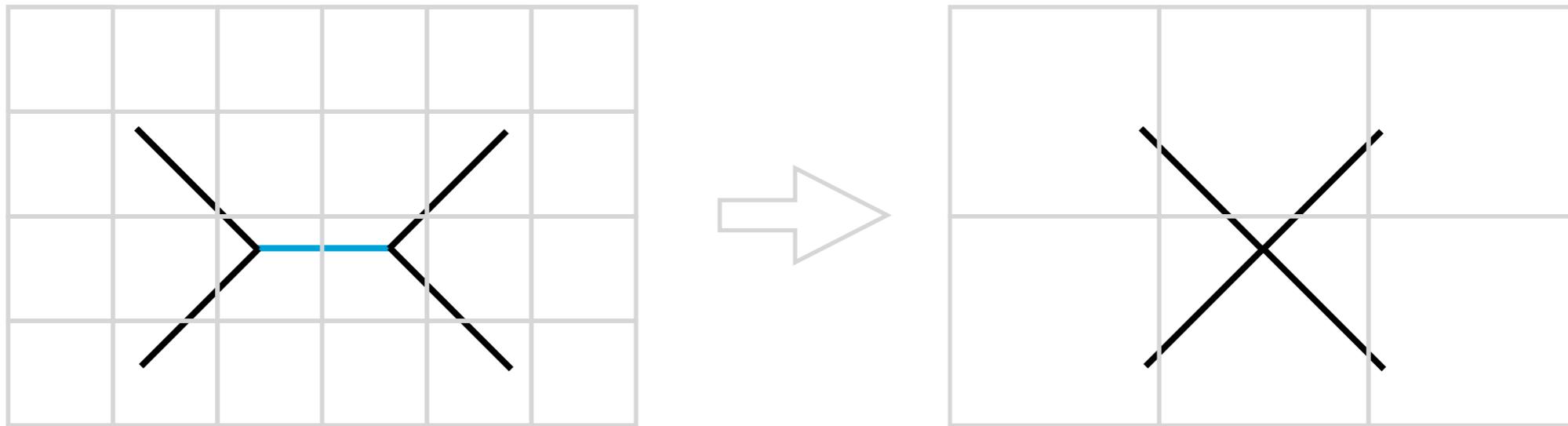


=

$$\begin{aligned}
 & \frac{i\kappa}{2} \left( P_{\alpha\beta,\gamma\delta} \left[ k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right] \right. \\
 & + 2q_\lambda q_\sigma \left[ I^{\lambda\sigma,}_{\alpha\beta} I^{\mu\nu,}_{\gamma\delta} + I^{\lambda\sigma,}_{\gamma\delta} I^{\mu\nu,}_{\alpha\beta} - I^{\lambda\mu,}_{\alpha\beta} I^{\sigma\nu,}_{\gamma\delta} - I^{\sigma\nu,}_{\alpha\beta} I^{\lambda\mu,}_{\gamma\delta} \right] \\
 & + \left[ q_\lambda q^\mu \left( \eta_{\alpha\beta} I^{\lambda\nu,}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}_{\alpha\beta} \right) + q_\lambda q^\nu \left( \eta_{\alpha\beta} I^{\lambda\mu,}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}_{\alpha\beta} \right) \right. \\
 & - q^2 \left( \eta_{\alpha\beta} I^{\mu\nu,}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu,}_{\alpha\beta} \right) - \eta^{\mu\nu} q^\lambda q^\sigma (\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma}) \Big] \\
 & + \left[ 2q^\lambda \left( I^{\sigma\nu,}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\mu + I^{\sigma\mu,}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\nu \right. \right. \\
 & \quad \left. \left. - I^{\sigma\nu,}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu,}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu \right) \right. \\
 & + q^2 \left( I^{\sigma\mu,}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu,}_{\alpha\delta} \right) + \eta^{\mu\nu} q^\lambda q_\sigma \left( I_{\alpha\beta,\lambda\rho} I^{\rho\sigma,}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma,}_{\alpha\beta} \right) \Big] \\
 & + \left\{ (k^2 + (k-q)^2) \left( I^{\sigma\mu,}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu,}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2} \eta^{\mu\nu} P_{\alpha\beta,\gamma\delta} \right) \right. \\
 & \quad \left. - \left( k^2 \eta_{\gamma\delta} I^{\mu\nu,}_{\alpha\beta} + (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu,}_{\gamma\delta} \right) \right\}
 \end{aligned}$$

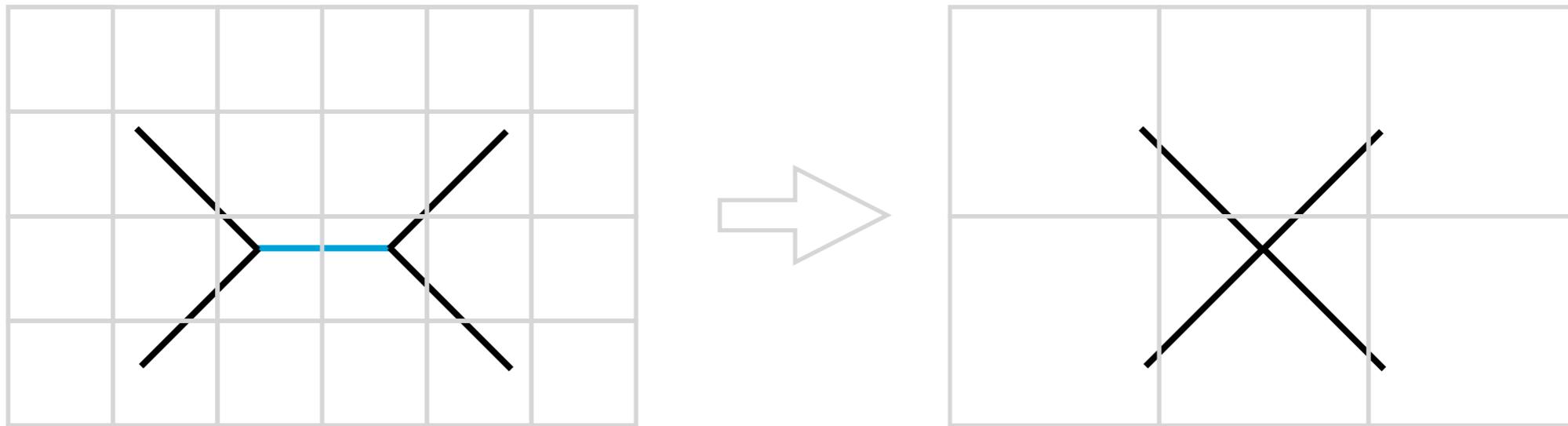
the truth behind Feynman diagrams...

# Corrections to Newton's interaction



$$V = -\frac{GMm}{r} \left[ 1 + a \frac{G(M+m)}{c^2 r} + b \frac{G\hbar}{c^3 r^2} + \dots \right]$$

# Corrections to Newton's interaction



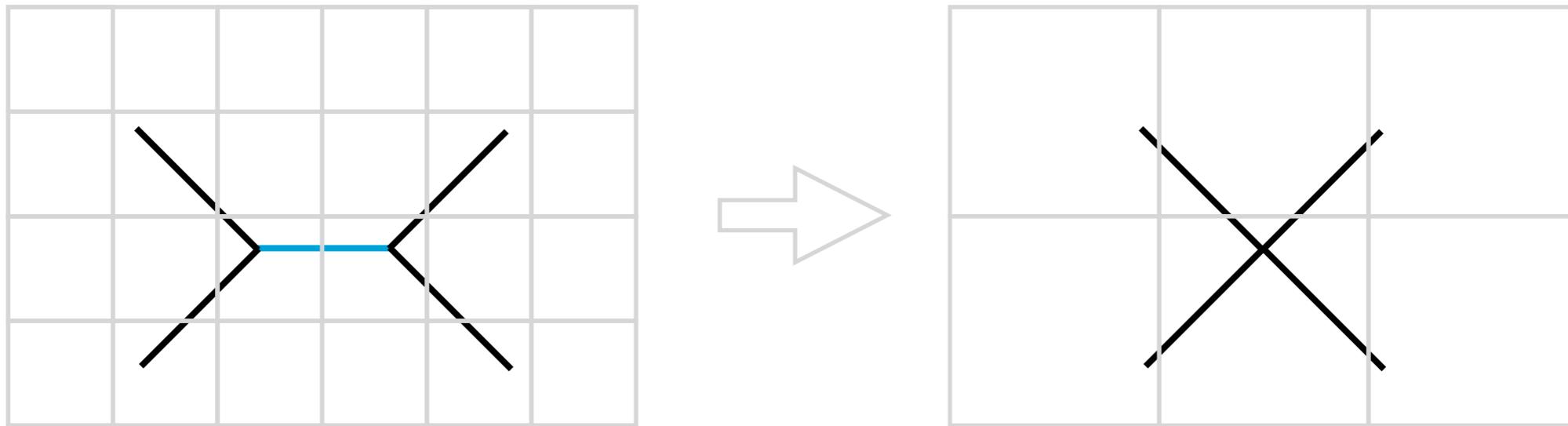
$$V = -\frac{GMm}{r} \left[ 1 + a \frac{G(M+m)}{c^2 r} + b \frac{G\hbar}{c^3 r^2} + \dots \right]$$

$$[G] = \frac{\text{m}^3}{\text{Kg s}^2}$$

$$[\hbar] = \frac{\text{m}^2 \text{ Kg}}{\text{s}}$$

$$[c] = \frac{\text{m}}{\text{s}}$$

# Corrections to Newton's interaction

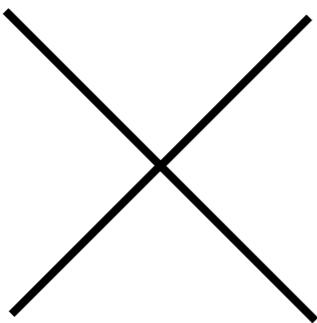


$$V = -\frac{GMm}{r} \left[ 1 + 3\frac{G(M+m)}{c^2 r} + \frac{41}{10\pi} \frac{G\hbar}{c^3 r^2} + \dots \right]$$

Leading quantum corrections to Newton's potential

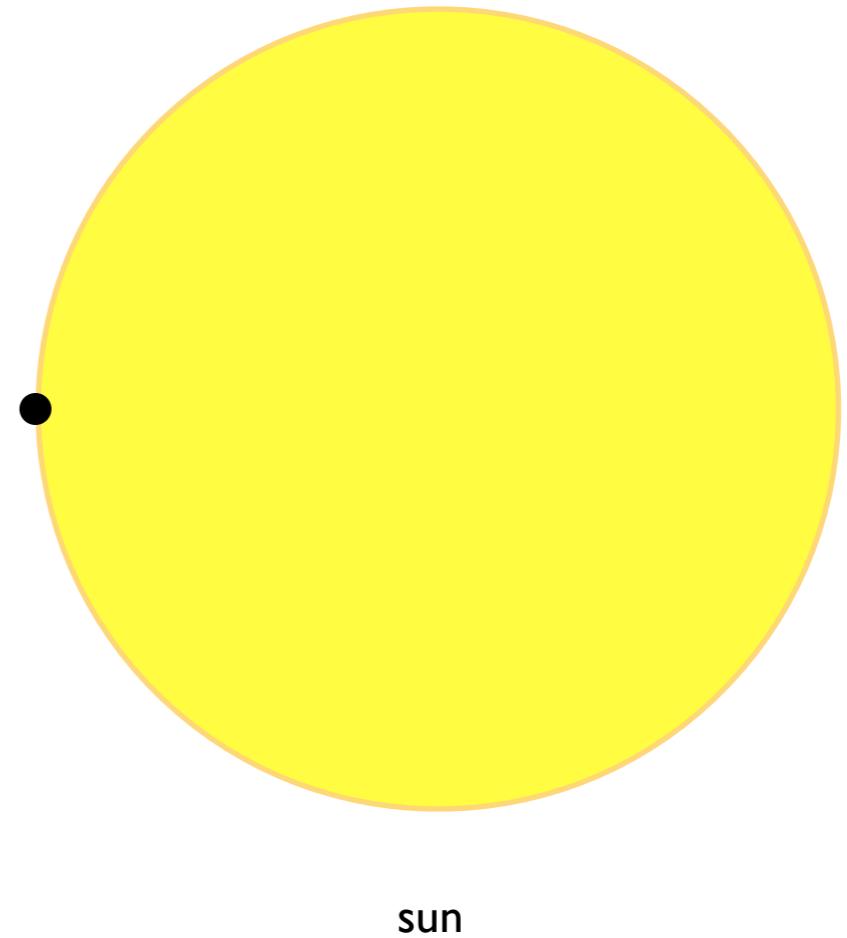
J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

# Corrections to Newton's interaction



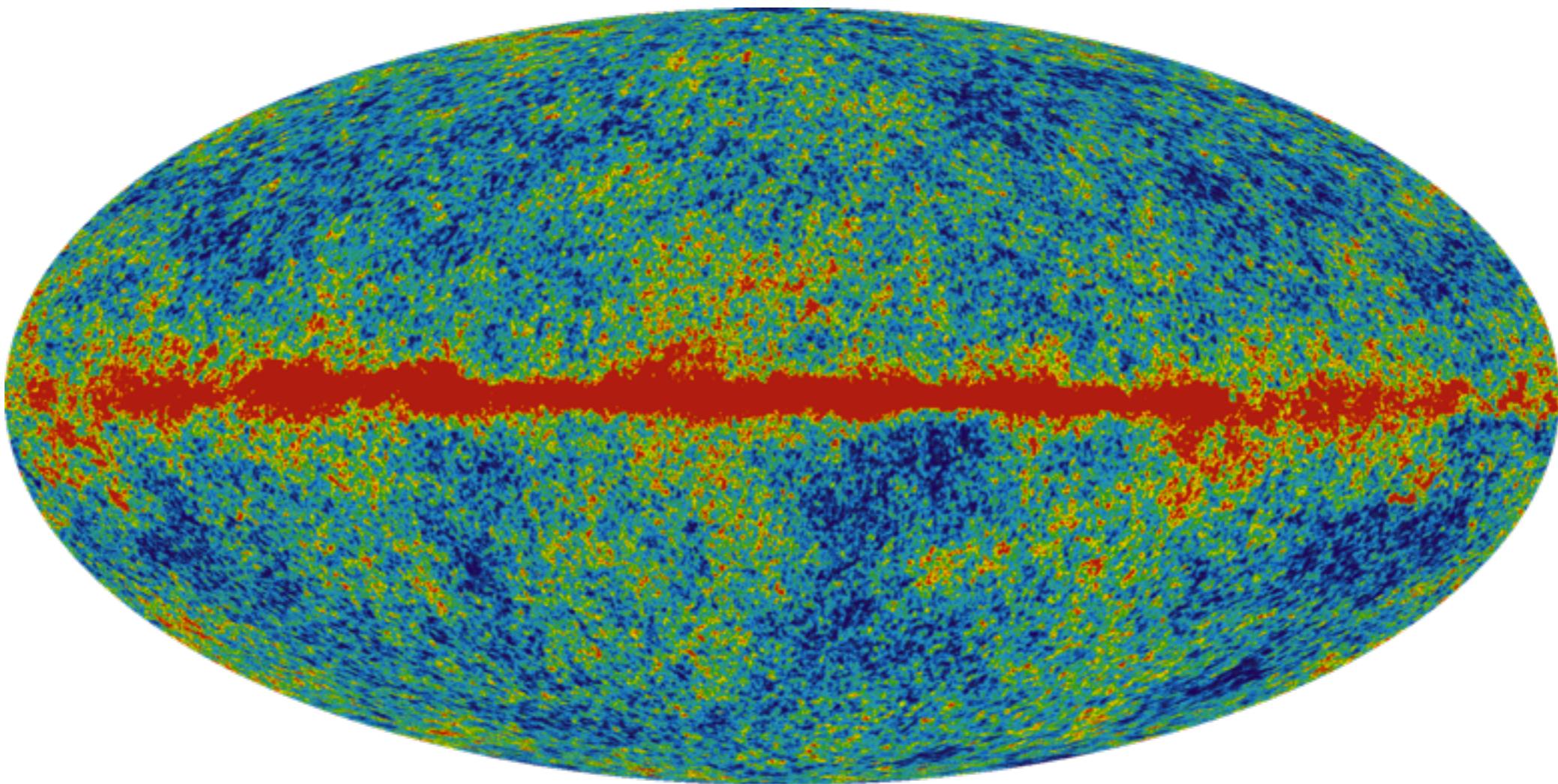
$$\frac{GM_{\odot}}{c^2 r_{\odot}} \sim 10^{-6}$$

$$\frac{G\hbar}{c^3 r_{\odot}^2} \sim 10^{-88}$$



Leading quantum corrections to Newton's law are incredibly small!

# Can we ever observe quantum gravity effects?



Maybe there are some inside here...

# LO quantum corrections

Non-analytical vs non-local

- $+ \frac{1}{2} \circlearrowleft = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} R \left[ \alpha \log \frac{R}{\mu^2} + \beta \log \frac{-\square}{\mu^2} \right] R + \dots$

# LO quantum corrections

Non-analytical vs non-local

- $+ \frac{1}{2} \text{ } \bigcirc = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} R \left[ \alpha \log \frac{R}{\mu^2} + \beta \log \frac{-\square}{\mu^2} \right] R + \dots$

Obtain finite part from beta functions (physical running)

$$2R \partial_R = -\mu \partial_\mu \quad 2q^2 \partial_{q^2} = -\mu \partial_\mu$$

# LO quantum corrections

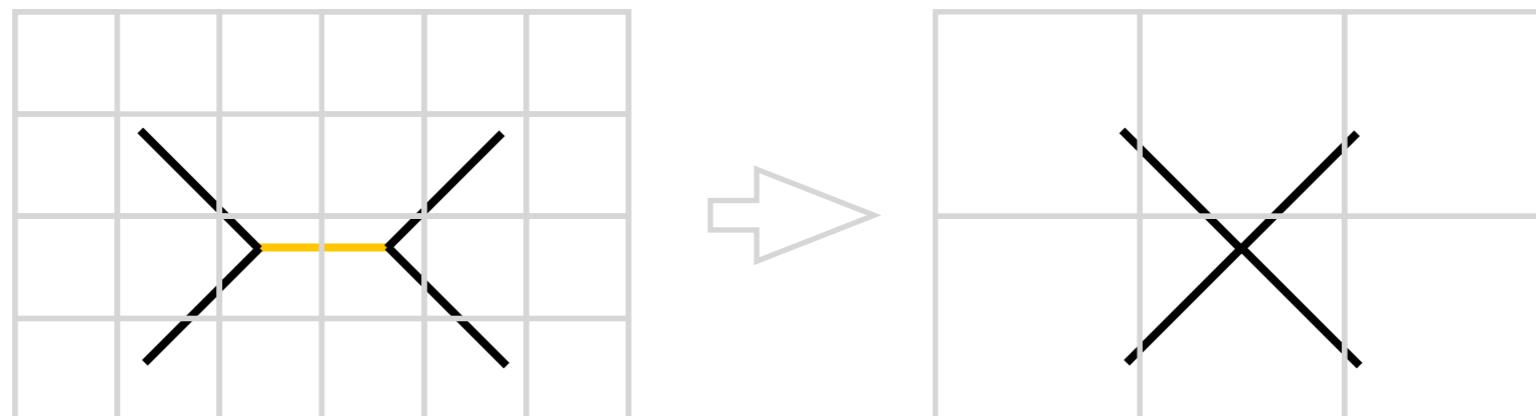
Non-analytical vs non-local

- $+ \frac{1}{2} \textcolor{cyan}{\bigcirc} = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} R \left[ \alpha \log \frac{R}{\mu^2} + \beta \log \frac{-\square}{\mu^2} \right] R + \dots$

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# LO quantum corrections

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Obtain finite part from beta functions (physical running)

$$2R \partial_R = -\mu \partial_\mu$$

$$2q^2 \partial_{q^2} = -\mu \partial_\mu$$

$$\alpha = \beta$$

$$\alpha = \frac{C}{4(4\pi)^2}$$

A topology must be chosen...

$$C = \frac{N_S}{72} \quad \text{minimally coupled scalars}$$

$$C = \frac{1}{4} \quad \text{EFT gravity}$$

$$C = \frac{5}{36} \quad \text{HDG}$$

# Marginally deformed Starobinsky

- $+ \frac{1}{2} \text{ } \bigcirc = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} R h\left(\frac{R}{\mu^2}\right) R + \dots$

# Marginally deformed Starobinsky

- $+ \frac{1}{2} \circ = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} R h\left(\frac{R}{\mu^2}\right) R + \dots$

One loop flow equation

$$R \partial_R h = -\frac{C}{2(4\pi)^2}$$

$$h(R/\mu^2) = \log R/\mu^2$$

# Marginally deformed Starobinsky

- $+ \frac{1}{2} \bigcirc = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} R h\left(\frac{R}{\mu^2}\right) R + \dots$

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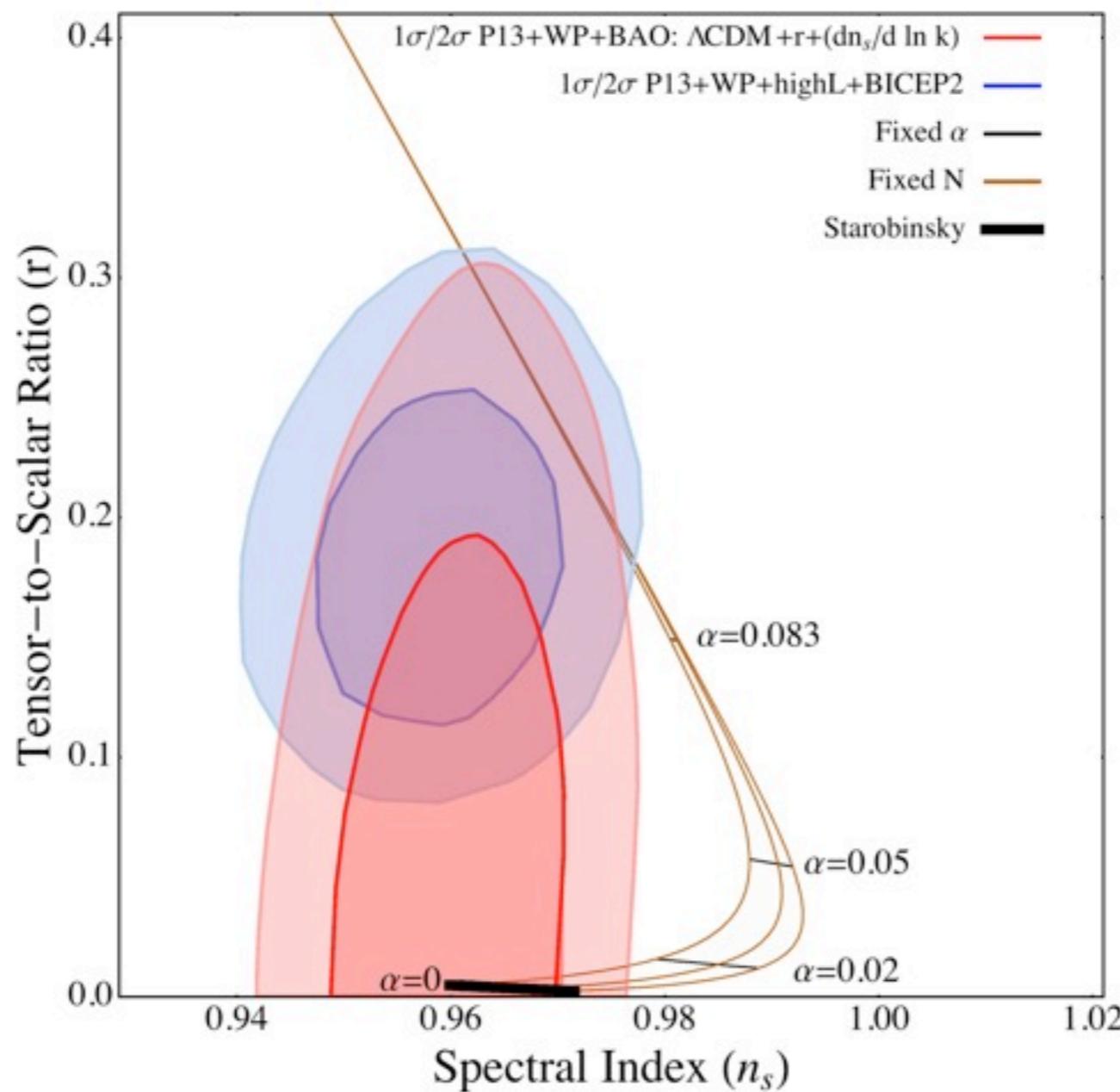
$$R \partial_R h = -\frac{C}{2(4\pi)^2} \quad h(R/\mu^2) = \log R/\mu^2$$

RG improved flow equation

$$R \partial_R h = -\frac{C}{2(4\pi)^2} h \quad h(R) = h(R_0) \left(\frac{R}{R_0}\right)^{-\frac{C}{2(4\pi)^2}}$$

$$C > 0 \quad \alpha = \frac{C}{4(4\pi)^2} \quad \alpha > 0$$

# Marginally deformed Starobinsky (L)



Leading quantum corrections to tensor-to-scalar ratio

A. C. J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502, 050 (2015)

# Curvature expansion

- $+ \frac{1}{2} \circ = -\frac{1}{2(4\pi)^{d/2}} \int d^d x \sqrt{g} \operatorname{tr} \mathcal{R} \gamma_i \left( \frac{-\square}{m^2} \right) \mathcal{R} + \dots$

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

$$\gamma_i \left( \frac{X}{m^2} \right) \equiv \lim_{\Lambda_{UV} \rightarrow \infty} \int_{1/\Lambda_{UV}^2}^{\infty} \frac{ds}{s} s^{-d/2+2} [f_i(sX) - f_i(0)]$$



## Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163

I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1

A. Codello and O. Zanusso, J. Math. Phys. 54 (2013) 013513

## Non-local heat kernel structure functions

# Curvature expansion

- $+ \frac{1}{2} \circlearrowleft = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[ \mathbf{1} R_{\mu\nu} \gamma_{Ric} \left( \frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left( \frac{-\square}{m^2} \right) R \right.$   
 $\left. - \frac{1}{6} R \gamma_{RU} \left( \frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left( \frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \boldsymbol{\Omega}_{\mu\nu} \gamma_\Omega \left( \frac{-\square}{m^2} \right) \boldsymbol{\Omega}^{\mu\nu} \right]$

Explicit form for the structure functions

$$\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2} \int_0^1 d\xi \left[ \frac{1}{u} + \xi(1-\xi) \right]^2 \log [1 + u \xi(1-\xi)]$$

$$\begin{aligned} \gamma_R(u) = & -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32} \int_0^1 d\xi \left\{ \frac{2}{u^2} + \frac{4}{u} [1 + \xi(1-\xi)] \right. \\ & \left. - 1 + 2\xi(2-\xi)(1-\xi^2) \right\} \log [1 + u \xi(1-\xi)] \end{aligned}$$

$$\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2} \int_0^1 d\xi \left[ \frac{1}{u} - \frac{1}{2} + \xi(1-\xi) \right] \log [1 + u \xi(1-\xi)]$$

$$\gamma_U(u) = -\frac{1}{2} \int_0^1 d\xi \log [1 + u \xi(1-\xi)]$$

$$\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2} \int_0^1 d\xi \left[ \frac{1}{u} + \xi(1-\xi) \right] \log [1 + u \xi(1-\xi)]$$

$$u \equiv \frac{-\square}{m^2}$$

# Curvature expansion

- $+ \frac{1}{2} \circlearrowleft = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[ \mathbf{1} R_{\mu\nu} \gamma_{Ric} \left( \frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left( \frac{-\square}{m^2} \right) R \right.$   
 $\left. - \frac{1}{6} R \gamma_{RU} \left( \frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left( \frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \boldsymbol{\Omega}_{\mu\nu} \gamma_\Omega \left( \frac{-\square}{m^2} \right) \boldsymbol{\Omega}^{\mu\nu} \right]$

Large energy expansion  $u \gg 1$

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

$$\gamma_R(u) = -\frac{u}{336} + \frac{11u^2}{30240} - \frac{19u^3}{332640} + O(u^4)$$

$$\gamma_{RU}(u) = \frac{u}{30} - \frac{u^2}{280} + \frac{u^3}{1890} + O(u^4)$$

$$\gamma_U(u) = -\frac{u}{12} + \frac{u^2}{120} - \frac{u^3}{840} + O(u^4)$$

$$\gamma_\Omega(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

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# Curvature expansion

- $+ \frac{1}{2} \circ = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[ \mathbf{1} R_{\mu\nu} \gamma_{Ric} \left( \frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left( \frac{-\square}{m^2} \right) R \right.$   
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Large energy expansion  $u \gg 1$

match with  
local heat  
kernel

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

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# Curvature expansion

- $+ \frac{1}{2} \circ = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[ \mathbf{1} R_{\mu\nu} \gamma_{Ric} \left( \frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left( \frac{-\square}{m^2} \right) R \right.$   
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Low energy expansion  $u \ll 1$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60} \log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120} \log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6} \log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_U(u) = 1 - \frac{1}{2} \log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12} \log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$u \equiv \frac{-\square}{m^2}$$

# Cosmological effective action ( $L$ )

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \xi \int d^4x \sqrt{-g} R^2 + \int d^4x \sqrt{-g} RF(\square)R$$

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$$F(\square) = \alpha \log \frac{-\square}{m^2}$$

$$+ \beta \frac{m^2}{-\square}$$

$\alpha, \beta, \gamma, \delta$

are calculable constants  
depending on matter  
content

$$+ \gamma \frac{m^2}{-\square} \log \frac{-\square}{m^2}$$

$$+ \delta \frac{m^4}{(-\square)^2}$$

+ ...

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**Leading logs**

J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014)

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**Non-local cosmology**

S. Deser and R. P. Woodard, Phys. Rev. Lett. 99, 111301 (2007)

+ ...

**Effective non-local cosmology**

A. C. and K. J. Jain in preparation

**Non-local gravity and dark energy**

M. Maggiore and M. Mancarella, Phys. Rev. D 90, 023005 (2014).

# Effective Friedmann equations (L)

Local gravity @ LO

$$H^2 + 96\pi\xi G \left[ 2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 \right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

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Pure Starobinsky gravity is exactly solvable

$$2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 = 0$$

$$H = 0$$

$$t + C_2 = \int \frac{dH}{C_1\sqrt{H} - 2H^2}$$

$$H = \frac{1}{2t}$$

# Effective Friedmann equations (L)

Local gravity @ LO

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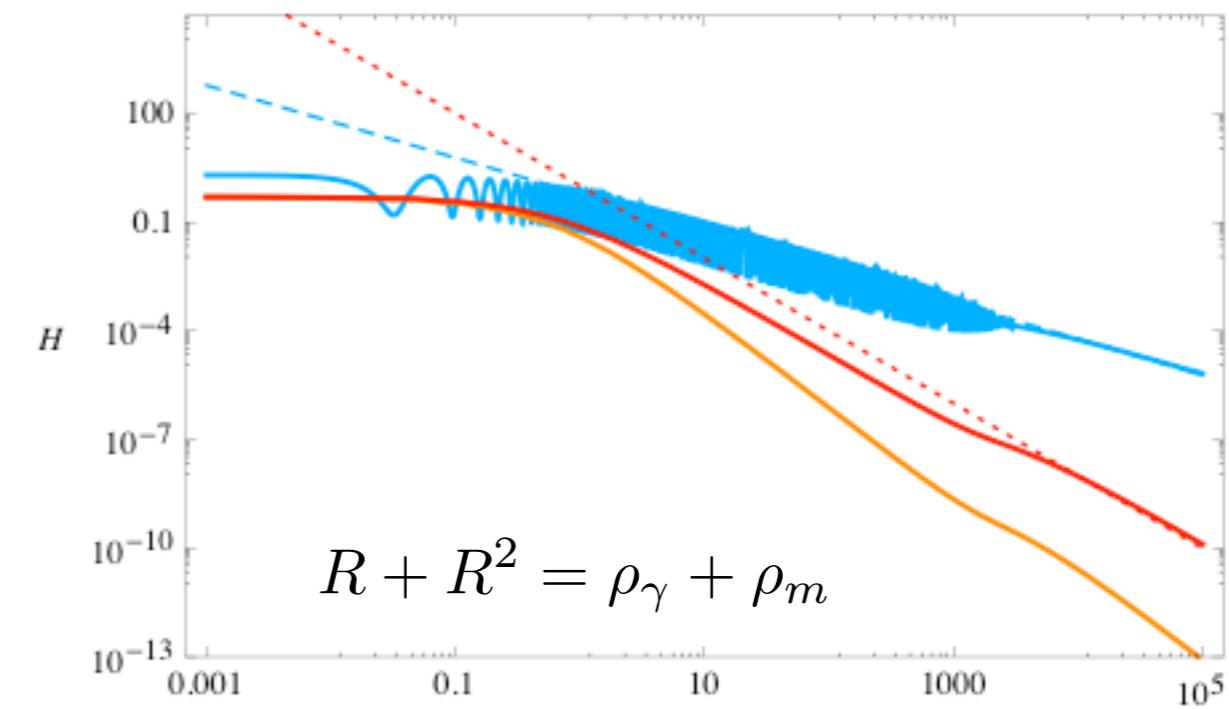
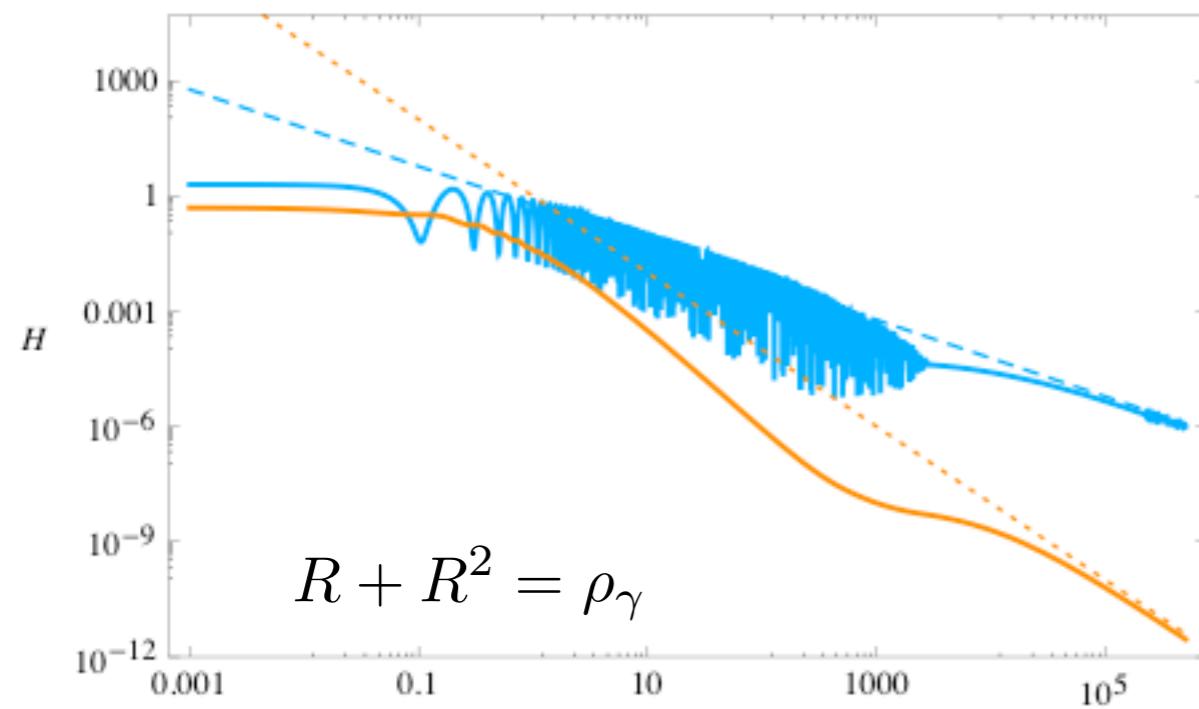
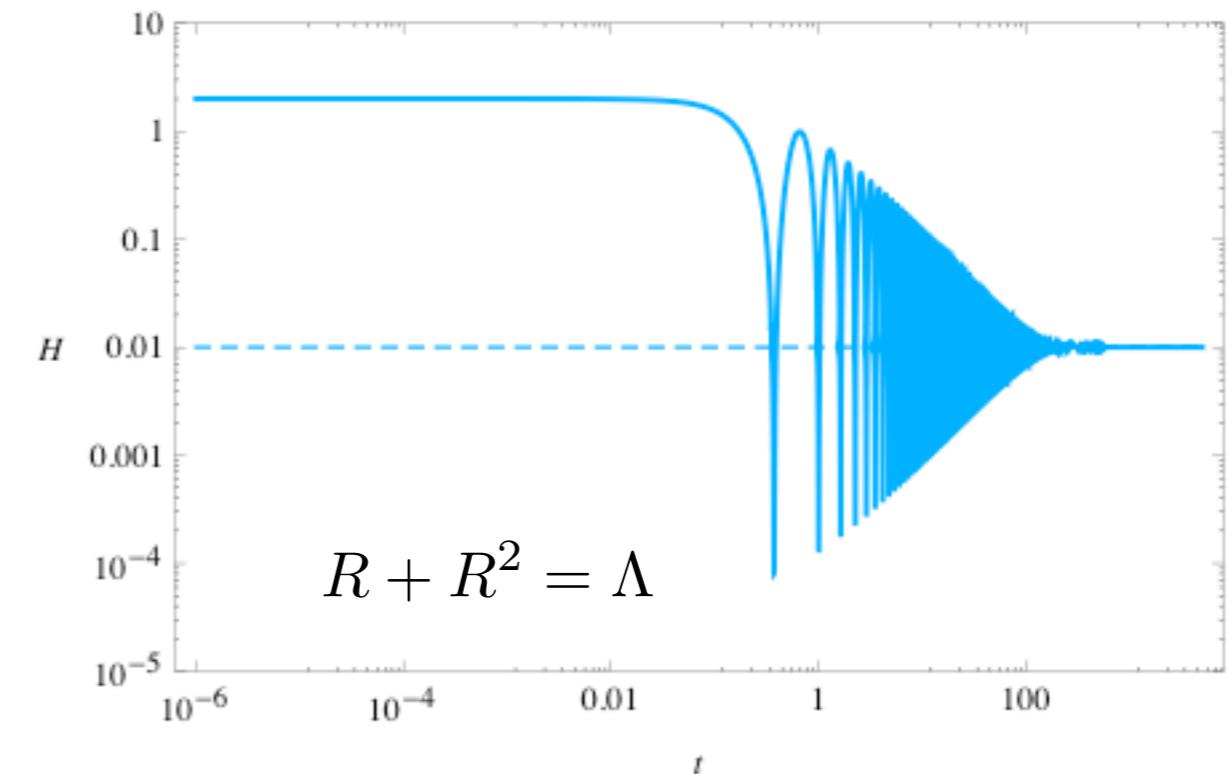
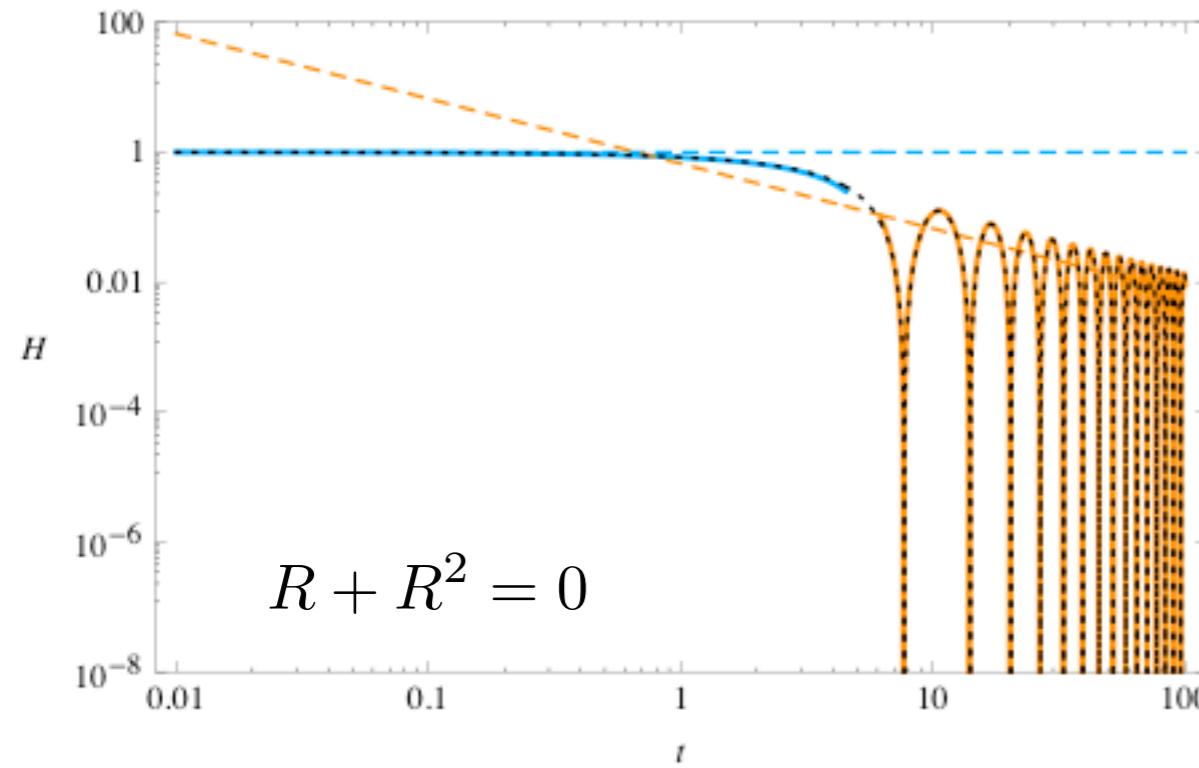
$$t + C_2 = \int \frac{dH}{C_1\sqrt{H} - 2H^2}$$

$$H = \frac{1}{2t}$$

$$R + R^{n-1}\delta R = -8\pi G T$$

$$T = 0 \quad \Rightarrow \quad R = 0$$

# Effective Friedmann equations (L)



# Effective Friedmann equations (L)

$$H^2(t) + 16\pi G \alpha \left\{ - \left[ \frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt' a^{\frac{3}{2}}(t') R(t') L_-(t-t') \right.$$
$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt' L_-(t-t') \frac{d}{dt'} \left[ a^{\frac{3}{2}}(t') R(t') \right] \right\} = \frac{8\pi G}{3} \rho(t)$$

How to interpret non-local terms?

# Effective Friedmann equations (L)

$$H^2(t) + 16\pi G \alpha \left\{ - \left[ \frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt' a^{\frac{3}{2}}(t') R(t') L_-(t-t') \right.$$

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How to interpret non-local terms?

$$L \equiv \log \frac{-\square}{\mu^2}$$

$$L(t-t') = \int_0^\infty ds \left[ \frac{1}{\mu^2 + s} - G(t-t'; \sqrt{s}) \right]$$

$$G_-(t-t'; m) = \frac{\theta(t-t')}{\sqrt{a^3(t)a^3(t')}} \frac{\sin m(t-t')}{m} + O(\dot{a})$$

$$L_-(t-t') = -2 \lim_{\epsilon \rightarrow 0} \left[ \frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t') \log \mu \epsilon \right]$$

# Effective Friedmann equations (L)

$$H^2(t) + 16\pi G \alpha \left\{ - \left[ \frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt' a^{\frac{3}{2}}(t') R(t') L_-(t-t') \right.$$
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Try an iterative solution...

$$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}} t}$$

$$a(t) = (t/t_0)^{1/2}$$

$$a(t) = (t/t_0)^{2/3}$$

# Effective Friedmann equations (L)

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$$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}} t}$$
no effect

$$a(t) = (t/t_0)^{1/2}$$
no effect

$$a(t) = (t/t_0)^{2/3}$$

# Effective Friedmann equations (L)

$$H^2(t) + 16\pi G \alpha \left\{ - \left[ \frac{\ddot{a}(t)}{a^2(t)\sqrt{a(t)}} + 2\frac{H^2(t)}{a(t)\sqrt{a(t)}} \right] \int dt' a^{\frac{3}{2}}(t') R(t') L_-(t-t') \right.$$

$$\left. + 2\frac{H(t)}{a(t)\sqrt{a(t)}} \int dt' L_-(t-t') \frac{d}{dt'} \left[ a^{\frac{3}{2}}(t') R(t') \right] \right\} = \frac{8\pi G}{3} \rho(t)$$

Try an iterative solution...

$$a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}} t}$$
no effect

$$a(t) = (t/t_0)^{1/2}$$
no effect

$$a(t) = (t/t_0)^{2/3}$$
some effect

$$H^2(t) + \frac{256\pi G \alpha}{3t^4} \left[ \log \mu t + \log \left( \frac{t}{t_0} - 1 \right) + \frac{2}{3} \left( \frac{t}{t_0} - 1 \right) \right] = \frac{8\pi G}{3} \rho_m(t_0) \left( \frac{t_0}{t} \right)^2$$

... log R dominant at early times while log box at late times

# Conclusions and Outlook

- Compute all LO terms
- Renormalization of NLO
- Conformal anomaly contribution
- Apply to cosmology
- Apply to stars/black holes
- Add the SM and constrain BSM
- Connection with high energy quantum gravity
- Falsify!

Thank you