Tensor Models, from Random Trees to Brownian Spheres

Vincent Rivasseau

Laboratoire de Physique théorique
Université Paris-Sud
and Perimeter Institute

Probing the Fundamental Nature of Spacetime
with the Renormalization Group
Nordita, Stockholm, March 26 2015

Tensor Track, I

It can be seen as

- a generalization of random matrices and of non commutative field theory
- a revival of the dynamical triangulation program
- an improvement of group field theory
- possibly a "pregeometric completion" for asymptotic safety scenarios.

Tensor Track, I

It can be seen as

- a generalization of random matrices and of non commutative field theory
- a revival of the dynamical triangulation program
- an improvement of group field theory
- possibly a "pregeometric completion" for asymptotic safety scenarios.

Tensor Track, I

It can be seen as

- a generalization of random matrices and of non commutative field theory
- a revival of the dynamical triangulation program
- an improvement of group field theory
- possibly a "pregeometric completion" for asymptotic safety scenarios.

Tensor Track, I

It can be seen as

- a generalization of random matrices and of non commutative field theory
- a revival of the dynamical triangulation program
- an improvement of group field theory
- possibly a "pregeometric completion" for asymptotic safety scenarios.

Tensor Track, I

It can be seen as

- a generalization of random matrices and of non commutative field theory
- a revival of the dynamical triangulation program
- an improvement of group field theory
- possibly a "pregeometric completion" for asymptotic safety scenarios.

Tensor Track, I

It can be seen as

- a generalization of random matrices and of non commutative field theory
- a revival of the dynamical triangulation program
- an improvement of group field theory
- possibly a "pregeometric completion" for asymptotic safety scenarios.

Tensor Track, II

Quantum field theory of space time, not on space-time, which can incorporate several interesting features

- background independence
- sum over topologies
- renormalizability
- asymptotic freedom
- constructive (i.e. non-perturbative) analysis

Tensor Track, II

Quantum field theory of space time, not on space-time, which can incorporate several interesting features

- background independence
- sum over topologies
- renormalizability
- asymptotic freedom
- constructive (i.e. non-perturbative) analysis

Tensor Track, II

Quantum field theory of space time, not on space-time, which can incorporate several interesting features

- background independence
- sum over topologies
- renormalizability
- asymptotic freedom
- constructive (i.e. non-perturbative) analysis

Tensor Track, II

Quantum field theory of space time, not on space-time, which can incorporate several interesting features

- background independence
- sum over topologies
- renormalizability
- asymptotic freedom
- constructive (i.e. non-perturbative) analysis

Tensor Track, II

Quantum field theory of space time, not on space-time, which can incorporate several interesting features

- background independence
- sum over topologies
- renormalizability
- asymptotic freedom
- constructive (i.e. non-perturbative) analysis

Tensor Track, II

Quantum field theory of space time, not on space-time, which can incorporate several interesting features

- background independence
- sum over topologies
- renormalizability
- asymptotic freedom
- constructive (i.e. non-perturbative) analysis

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\mathrm{Regge}} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\,\delta(\sigma_{D-2})$$

Equilateral => $S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

The exact correspondence is

$$\begin{aligned} \ln N &= \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G}\,, \\ \ln \lambda &= \frac{D}{16\pi G} vol(\sigma_{D-2}) \Big(\pi (D-1) - (D+1) \arccos \frac{1}{D} \Big) - 2\Lambda\, vol(\sigma_D) \end{aligned}$$

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\rm Regge} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\, \delta(\sigma_{D-2})$$

Equilateral => $S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

The exact correspondence is

$$\ln N = \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G} ,$$

$$\ln \lambda = \frac{D}{16\pi G} vol(\sigma_{D-2}) \Big(\pi(D-1) - (D+1) \arccos \frac{1}{D} \Big) - 2\Lambda\, vol(\sigma_D)$$

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\mathrm{Regge}} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\, \delta(\sigma_{D-2})$$

Equilateral => $S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

The exact correspondence is

$$\ln N = \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G},$$

$$\ln \lambda = \frac{D}{16\pi G} vol(\sigma_{D-2}) \Big(\pi(D-1) - (D+1) \arccos \frac{1}{D} \Big) - 2\Lambda\, vol(\sigma_D)$$

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\mathrm{Regge}} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\,\delta(\sigma_{D-2})$$

Equilateral => $S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

The exact correspondence is

$$\ln N = \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G},$$

$$\ln \lambda = \frac{D}{16\pi G} vol(\sigma_{D-2})\Big(\pi(D-1) - (D+1)\arccos\frac{1}{D}\Big) - 2\Lambda\, vol(\sigma_D)$$

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\rm Regge} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\, \delta(\sigma_{D-2})$$

Equilateral => $S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

The exact correspondence is

$$\begin{aligned} \ln N &= \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G}, \\ \ln \lambda &= \frac{D}{16\pi G} vol(\sigma_{D-2}) \Big(\pi(D-1) - (D+1) \arccos \frac{1}{D} \Big) - 2\Lambda\, vol(\sigma_D) \end{aligned}$$

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\text{Regge}} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\, \delta(\sigma_{D-2})$$

Equilateral => $S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

The exact correspondence is

$$\ln N = \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G},$$

$$\ln \lambda = \frac{D}{16\pi G} vol(\sigma_{D-2}) \Big(\pi(D-1) - (D+1) \arccos \frac{1}{D} \Big) - 2\Lambda\, vol(\sigma_D)$$

Tensor Models and Quantum Gravity

Tensor models = Equilateral Regge calculus

$$S_{\rm Regge} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\, \delta(\sigma_{D-2})$$

Equilateral $=> S = \kappa_1 Q_{D-2} - \kappa_2 Q_D$

On the dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence amplitudes are $A_G(N) = \lambda^n N^F$ the amplitudes of tensor models.

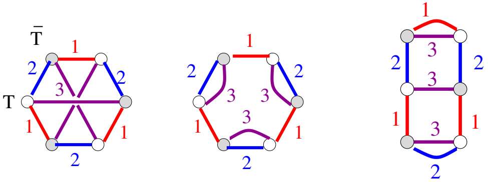
The exact correspondence is

$$\begin{aligned} \ln N &= \frac{vol(\sigma_{D-2})}{8G} = \frac{a_D}{G} \,, \\ \ln \lambda &= \frac{D}{16\pi G} vol(\sigma_{D-2}) \Big(\pi(D-1) - (D+1) \arccos \frac{1}{D} \Big) - 2\Lambda\, vol(\sigma_D) \end{aligned}$$

Tensor Theory Space

Gurau + followers: rank-D complex random tensors have $U(N)^{\otimes D}$ invariance

Invariants = interactions = observables are d-regular edge-colored bipartite graphs



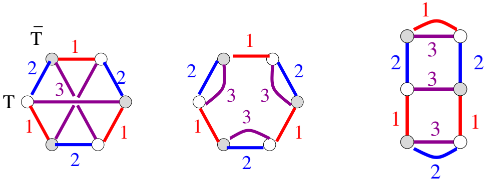
Number of connected ($\simeq$ single trace) invariants increases fast, eg at rank 3

$$Z_3^c(n) = 1, 3, 7, 26, 97, 624, 4163...$$

Tensor Theory Space

Gurau + followers: rank-D complex random tensors have $U(N)^{\otimes D}$ invariance

Invariants = interactions = observables are d-regular edge-colored bipartite graphs



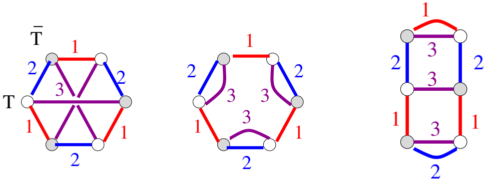
Number of connected ($\simeq$ single trace) invariants increases fast, eg at rank 3

$$Z_3^c(n) = 1, 3, 7, 26, 97, 624, 4163...$$

Tensor Theory Space

Gurau + followers: rank-D complex random tensors have $U(N)^{\otimes D}$ invariance

Invariants = interactions = observables are d-regular edge-colored bipartite graphs



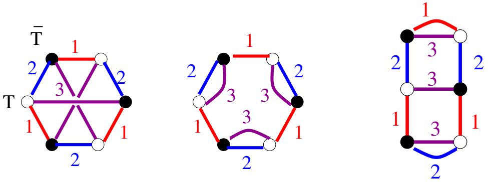
Number of connected ($\simeq$ single trace) invariants increases fast, eg at rank 3

$$Z_3^c(n) = 1, 3, 7, 26, 97, 624, 4163...$$

Tensor Theory Space

Gurau + followers: rank-D complex random tensors have $U(N)^{\otimes D}$ invariance

Invariants = interactions = observables are d-regular edge-colored bipartite graphs



Number of connected ($\simeq$ single trace) invariants increases fast, eg at rank 3

$$Z_3^c(n) = 1, 3, 7, 26, 97, 624, 4163...$$

Feynman Graphs

Vertices are D-regular edge-colored bipartite graphs

Feynman graphs are $(D+1)$-regular edge-colored bipartite graphs

They triangulate all manifolds plus a restricted class of quasi-manifolds

Feynman Graphs

Vertices are D-regular edge-colored bipartite graphs

Feynman graphs are $(D+1)$-regular edge-colored bipartite graphs

They triangulate all manifolds plus a restricted class of quasi-manifolds

Feynman Graphs

Vertices are D-regular edge-colored bipartite graphs

Feynman graphs are $(D+1)$-regular edge-colored bipartite graphs

They triangulate all manifolds plus a restricted class of quasi-manifolds

Feynman Graphs

Vertices are D-regular edge-colored bipartite graphs

Feynman graphs are $(D+1)$-regular edge-colored bipartite graphs

They triangulate all manifolds plus a restricted class of quasi-manifolds

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants × polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories ≃ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants × polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories ≃ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants × polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants × polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants $\times$ polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants × polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants × polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants $\times$ polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

Tensor Field Theories

Field theory is not exactly local. Fields propagate => renormalization group

Tensorial invariance plays the role of locality in usual QFT

Hence it could be slightly broken at propagator level

Tensor theory space = quasi-invariants = invariants $\times$ polynomial functions of the indices

$\sum_{i,j,k} \bar{T}_{ijk} T_{ijk}$ invariant; $\sum_{i,j,k} (i+j+k) \bar{T}_{ijk} T_{ijk}$ quasi-invariant

tensor field theory = invariant interaction + quasi invariant propagator (Ben Geloun, R.)

tensor models/tensor field theories $\simeq$ matrix models/NC field theories (Grosse-Wulkenhaar)

field theoretic methods apply (RG (Ben Geloun, Samary, Carrozza...), FRGE (Benedetti, Ben Geloun, Eichhorn, Koslowski, Oriti...) constructive techniques (Delepouve, Gurau, R...)

group field theory conditions can be added (Carrozza, Oriti, R...)

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T}, \bar{\mathbf{T}}), \qquad (1)$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \; \exp\Big(-N^{\alpha} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \qquad (2)$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T}, \bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^\alpha S(\mathbf{T}, \bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T},\bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T},\bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \; \exp\Big(-N^{\alpha} S(\mathbf{T},\bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T}, \bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \; \exp\Big(-N^{\alpha} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T}, \bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \; \exp\Big(-N^{\alpha} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T}, \bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \; \exp\Big(-N^{\alpha} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T},\bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T},\bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \; \exp\Big(-N^{\alpha} S(\mathbf{T},\bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i\in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T},\bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i\in I} \lambda_i B_i(\mathbf{T},\bar{\mathbf{T}}), \tag{1}$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \, \exp\Big(-N^{\alpha} S(\mathbf{T},\bar{\mathbf{T}})\Big). \tag{2}$$

Initial Gurau's $1/N$ expansion required:

- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i\in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.

Relax third requirement => enhanced models with new large N behaviors.

1/N Expansion

Model = action S, partition function Z, free energy F:

$$S(\mathbf{T},\bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} + \sum_{i \in I} \lambda_i B_i(\mathbf{T},\bar{\mathbf{T}}), \qquad (1)$$

$$F = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{\alpha} S(\mathbf{T},\bar{\mathbf{T}})\Big). \qquad (2)$$

Initial Gurau's $1/N$ expansion required:

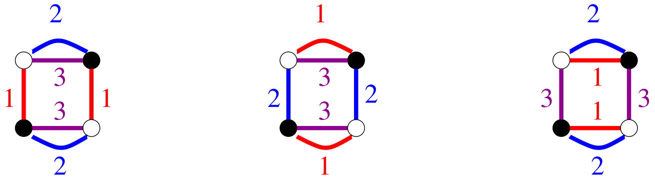
- free energy bounded as $|F| \leq N^K$,
- infinitely many graphs are leading order (LO)
- coupling constants $\{\lambda_i\}_{i \in I}$ independent of N.

The two first criteria uniquely select $\alpha = D - 1$.
Relax third requirement => enhanced models with new large N behaviors.

Rank Three Quartic Tensor Models

Three quartic invariants $\mathcal{B}_i = \mathrm{Tr} M_i^2$ at rank 3, all melonic, M's being the partial traces

$$[M_1]_{jk} := \sum_{l,m=1}^{N} T_{jlm} \bar{T}_{klm}, \quad [M_2]_{jk} := \sum_{l,m=1}^{N} T_{ljm} \bar{T}_{lkm}, \quad [M_3]_{jk} := \sum_{l,m=1}^{N} T_{lmj} \bar{T}_{lmk}$$



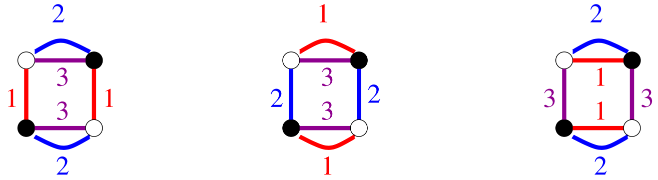
Simple color-symmetric quartic model

$$Z = \int dT d\bar{T} e^{-\frac{1}{2} N^2 \bar{T}.T - \frac{\lambda}{2} N^2 \sum_{i=1}^{3} \mathcal{B}_i(\bar{T},T)}$$

Rank Three Quartic Tensor Models

Three quartic invariants $\mathcal{B}_i = \mathrm{Tr} M_i^2$ at rank 3, all melonic, M's being the partial traces

$$[M_1]_{jk} := \sum_{l,m=1}^{N} T_{jlm} \bar{T}_{klm}, \quad [M_2]_{jk} := \sum_{l,m=1}^{N} T_{ljm} \bar{T}_{lkm}, \quad [M_3]_{jk} := \sum_{l,m=1}^{N} T_{lmj} \bar{T}_{lmk}$$



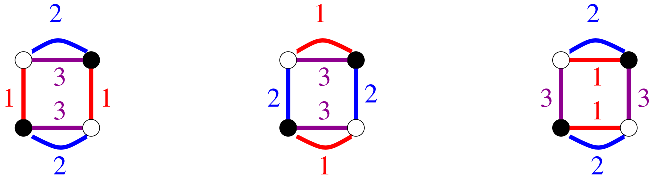
Simple color-symmetric quartic model

$$Z = \int dT d\bar{T} e^{-\frac{1}{2} N^2 \bar{T}.T - \frac{\lambda}{2} N^2 \sum_{i=1}^{3} \mathcal{B}_i(\bar{T},T)}$$

Rank Three Quartic Tensor Models

Three quartic invariants $\mathcal{B}_i = \mathrm{Tr} M_i^2$ at rank 3, all melonic, M's being the partial traces

$$[M_1]_{jk} := \sum_{l,m=1}^{N} T_{jlm} \bar{T}_{klm}, \quad [M_2]_{jk} := \sum_{l,m=1}^{N} T_{ljm} \bar{T}_{lkm}, \quad [M_3]_{jk} := \sum_{l,m=1}^{N} T_{lmj} \bar{T}_{lmk}$$



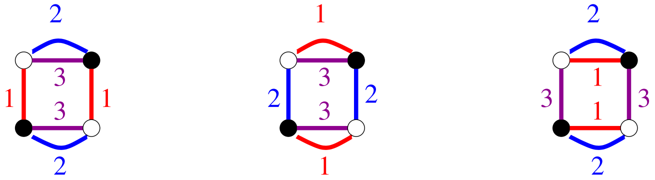
Simple color-symmetric quartic model

$$Z = \int dT d\bar{T} e^{-\frac{1}{2} N^2 \bar{T}.T - \frac{\lambda}{2} N^2 \sum_{i=1}^{3} \mathcal{B}_i(\bar{T}, T)}$$

Rank Three Quartic Tensor Models

Three quartic invariants $\mathcal{B}_i = \mathrm{Tr} M_i^2$ at rank 3, all melonic, M's being the partial traces

$$[M_1]_{jk} := \sum_{l,m=1}^{N} T_{jlm} \bar{T}_{klm}, \quad [M_2]_{jk} := \sum_{l,m=1}^{N} T_{ljm} \bar{T}_{lkm}, \quad [M_3]_{jk} := \sum_{l,m=1}^{N} T_{lmj} \bar{T}_{lmk}$$



Simple color-symmetric quartic model

$$Z = \int dT d\bar{T} e^{-\frac{1}{2} N^2 \, \bar{T}.T - \frac{\lambda}{2} N^2 \sum_{i=1}^{3} \mathcal{B}_i(\bar{T}, T)}$$

Intermediate Field Representation

Intermediate field representation introduces matrix fields σ_1, σ_2, σ_3, using

$$e^{-\frac{\lambda}{2}N^2 \,\mathrm{Tr} M_j^2} = \int d\sigma_j e^{-\frac{1}{2}\mathrm{Tr}\sigma_j^2 + i\sqrt{\lambda}N \mathrm{Tr}\sigma_j}$$

$$Z = \int d\sigma_1 d\sigma_2 d\sigma_3 e^{-\frac{1}{2}\sum_j \mathrm{Tr}\sigma_j^2 - \mathrm{Tr}^{\otimes}\log\left[1\otimes 1\otimes 1 + i\sqrt{\lambda}/N(\sigma_1\otimes 1\otimes 1 + 1\otimes\sigma_2\otimes 1 + 1\otimes 1\otimes\sigma_3)\right]}$$

It simplifies the expansion, melonic graphs $\rightarrow$ trees and builds a bridge between tensor and matrix models.

Intermediate Field Representation

Intermediate field representation introduces matrix fields σ_1, σ_2, σ_3, using

$$e^{-\frac{\lambda}{2}N^2 \mathrm{Tr} M_j^2} = \int d\sigma_j e^{-\frac{1}{2}\mathrm{Tr}\sigma_j^2 + i\sqrt{\lambda}N \mathrm{Tr}\sigma_j}$$

$$Z = \int d\sigma_1 d\sigma_2 d\sigma_3 e^{-\frac{1}{2}\sum_j \mathrm{Tr}\sigma_j^2 - \mathrm{Tr}^{\otimes} \log\left[1\otimes 1\otimes 1 + i\sqrt{\lambda}/N(\sigma_1\otimes 1\otimes 1 + 1\otimes\sigma_2\otimes 1 + 1\otimes 1\otimes\sigma_3)\right]}$$

It simplifies the expansion, melonic graphs $\to$ trees and builds a bridge between tensor and matrix models.

Intermediate Field Representation

Intermediate field representation introduces matrix fields σ_1, σ_2, σ_3, using

$$e^{-\frac{\lambda}{2} N^2 \, \mathrm{Tr} M_j^2} = \int d\sigma_j e^{-\frac{1}{2}\mathrm{Tr}\sigma_j^2 + i\sqrt{\lambda} N \mathrm{Tr}\sigma_j}$$

$$Z = \int d\sigma_1 d\sigma_2 d\sigma_3 e^{-\frac{1}{2}\sum_j \mathrm{Tr}\sigma_j^2 - \mathrm{Tr}^{\otimes} \log\left[1\otimes1\otimes1 + i\sqrt{\lambda}/N(\sigma_1\otimes1\otimes1 + 1\otimes\sigma_2\otimes1 + 1\otimes1\otimes\sigma_3)\right]}$$

It simplifies the expansion, melonic graphs $\to$ trees and builds a bridge between tensor and matrix models.

Intermediate Field Representation

Intermediate field representation introduces matrix fields σ_1, σ_2, σ_3, using

$$e^{-\frac{\lambda}{2} N^2 \operatorname{Tr} M_j^2} = \int d\sigma_j e^{-\frac{1}{2}\operatorname{Tr}\sigma_j^2 + i\sqrt{\lambda} N \operatorname{Tr}\sigma_j}$$

$$Z = \int d\sigma_1 d\sigma_2 d\sigma_3 e^{-\frac{1}{2}\sum_j \operatorname{Tr}\sigma_j^2 - \operatorname{Tr}^{\otimes} \log\left[1\otimes 1\otimes 1 + i\sqrt{\lambda}/N(\sigma_1\otimes 1\otimes 1 + 1\otimes\sigma_2\otimes 1 + 1\otimes 1\otimes\sigma_3)\right]}$$

It simplifies the expansion, melonic graphs $\rightarrow$ trees and builds a bridge between tensor and matrix models.

Rank Four Quartic Tensor Models

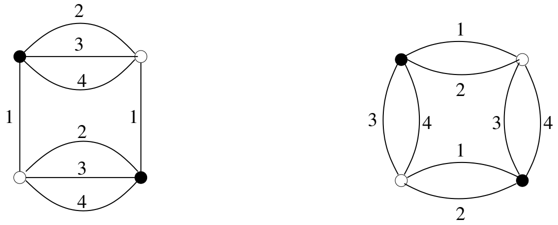


Figure: Quartic invariants at rank 4.

$$B_{\mathcal{C}_1}(\overline{\mathbf{T}},\mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n'_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n_2 n_3 n_4}$$

and three similar formulae for $B_{\mathcal{C}_2}$, $B_{\mathcal{C}_3}$ and $B_{\mathcal{C}_4}$.

$$B_{\mathcal{C}_{12}}(\overline{\mathbf{T}},\mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n'_2 n_3 n_4}$$

and two similar formulae for $B_{\mathcal{C}_{13}}$, $B_{\mathcal{C}_{14}}$

Rank Four Quartic Tensor Models

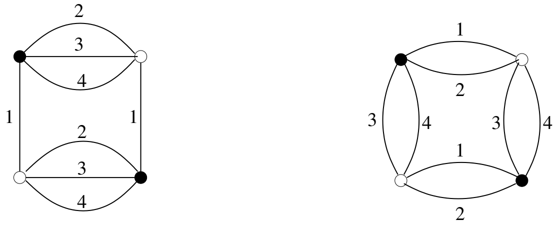


Figure: Quartic invariants at rank 4.

$$B_{\mathcal{C}_1}(\overline{\mathbf{T}}, \mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n'_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n_2 n_3 n_4}$$

and three similar formulae for $B_{\mathcal{C}_2}$, $B_{\mathcal{C}_3}$ and $B_{\mathcal{C}_4}$.

$$B_{\mathcal{C}_{12}}(\overline{\mathbf{T}}, \mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n'_2 n_3 n_4}$$

and two similar formulae for $B_{\mathcal{C}_{13}}$, $B_{\mathcal{C}_{14}}$

Enhanced Models

(joint work with V. Bonzom and T. Delepouve, arXiv 1502.01365.)

Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \, e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^3 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \qquad (3)$$

Enhanced (maximally rescaled) general quartic model at rank 4

$$d\mu_{max} = d\mu_0 \, e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^4 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \qquad (4)$$

Intermediate matrix fields, σ_i, $i = 1, \cdots, 4$ and σ_{1j}, $j = 2, \cdots, 4$.

Enhanced Models

(joint work with V. Bonzom and T. Delepouve, arXiv 1502.01365.)

Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \; e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^3 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \quad (3)$$

Enhanced (maximally rescaled) general quartic model at rank 4

$$d\mu_{max} = d\mu_0 \; e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^4 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \quad (4)$$

Intermediate matrix fields, σ_i, $i = 1, \cdots, 4$ and σ_{1j}, $j = 2, \cdots, 4$.

Enhanced Models

(joint work with V. Bonzom and T. Delepouve, arXiv 1502.01365.)

Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \, e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \, -\lambda' N^3 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \, . \quad (3)$$

Enhanced (maximally rescaled) general quartic model at rank 4

$$d\mu_{max} = d\mu_0 \, e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \, -\lambda' N^4 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \, . \quad (4)$$

Intermediate matrix fields, σ_i, $i = 1, \cdots, 4$ and σ_{1j}, $j = 2, \cdots, 4$.

Enhanced Models

(joint work with V. Bonzom and T. Delepouve, arXiv 1502.01365.)

Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \; e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^3 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \tag{3}$$

Enhanced (maximally rescaled) general quartic model at rank 4

$$d\mu_{max} = d\mu_0 \; e^{-\lambda N^3 \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^4 \sum_{j=2}^4 B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \tag{4}$$

Intermediate matrix fields, σ_i, $i = 1, \cdots, 4$ and σ_{1j}, $j = 2, \cdots, 4$.

Enhanced Models

(joint work with V. Bonzom and T. Delepouve, arXiv 1502.01365.)

Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \; e^{-\lambda N^3 \sum_{i=1}^{4} B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^3 \sum_{j=2}^{4} B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \tag{3}$$

Enhanced (maximally rescaled) general quartic model at rank 4

$$d\mu_{max} = d\mu_0 \; e^{-\lambda N^3 \sum_{i=1}^{4} B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; -\lambda' N^4 \sum_{j=2}^{4} B_{\mathcal{C}_{1j}}(\overline{\mathbf{T}},\mathbf{T})} \; . \tag{4}$$

Intermediate matrix fields, σ_i, $i = 1, \cdots, 4$ and σ_{1j}, $j = 2, \cdots, 4$.

Leading Order Maps

Leading order maps of this enhanced model (in the IF representation) are planar, and made of trees of monocolored edges which connect bicolored planar disks. The latter can touch only at single vertices hence do not form closed chains, thus displaying a "cactus" structure.

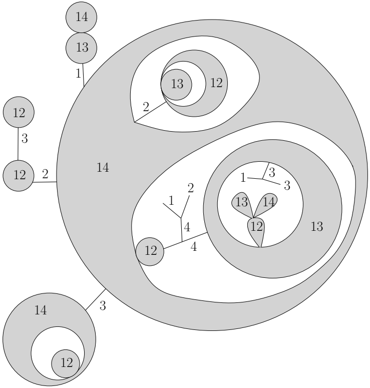


Figure: Grey areas are connected components of given color types. A bicolored connected component can be attached to another one on a single vertex, without forming cycles of such components.

Universality

Induction: A *tree of necklaces* of type $\{p_1, \ldots, p_n, p_{n+1}\}$ is obtained from a tree of necklaces of type $\{p_1, \ldots, p_n\}$ by removing any edge of color i and replacing it with the necklace of size p_{n+1} open on an edge of color i (preserving bipartiteness).

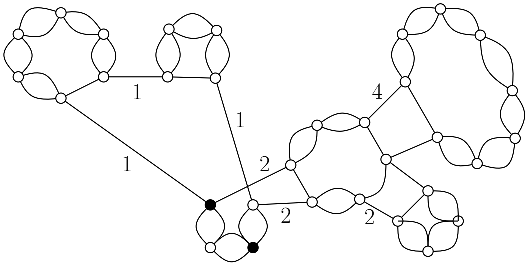


Figure: Trees of necklaces

The data $\{p_1, \ldots, p_n\}$ do not capture the full structure of the observable. It only records the sizes of the necklaces which are inserted one after the other one. It is sufficient for enumeration of the leading order contributions.

Universality

Induction: A *tree of necklaces* of type $\{p_1, \ldots, p_n, p_{n+1}\}$ is obtained from a tree of necklaces of type $\{p_1, \ldots, p_n\}$ by removing any edge of color i and replacing it with the necklace of size p_{n+1} open on an edge of color i (preserving bipartiteness).

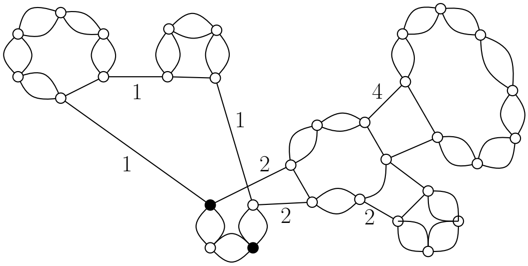


Figure: Trees of necklaces

The data $\{p_1, \ldots, p_n\}$ do not capture the full structure of the observable. It only records the sizes of the necklaces which are inserted one after the other one. It is sufficient for enumeration of the leading order contributions.

Enhancement of trees of necklaces

Let us denote a generic tree of necklaces by $\mathcal{L}$. If it is of type $\{p_1, \ldots, p_n\}$, the enhancement it requires to contribute at large N is

$$\omega(\mathcal{L}) = \sum_{k=1}^{n} (2 + p_k) - 3(n-1) = 3 - n + \sum_{k=1}^{n} p_k . \tag{5}$$

Generalized model has measure

$$d\mu(\mathbf{T}, \overline{\mathbf{T}}) = \exp\Big(-\sum_{\mathcal{L}} \lambda_{\mathcal{L}} \, N^{\omega(\mathcal{L})} \, B_{\mathcal{L}}(\mathbf{T}, \overline{\mathbf{T}})\Big) \; d\mu_0(\mathbf{T}, \overline{\mathbf{T}}). \tag{6}$$

where the sum in the exponential is over a finite number of trees of necklaces.

Enhancement of trees of necklaces

Let us denote a generic tree of necklaces by $\mathcal{L}$. If it is of type $\{p_1, \dots, p_n\}$, the enhancement it requires to contribute at large N is

$$\omega(\mathcal{L}) = \sum_{k=1}^{n} (2 + p_k) - 3(n-1) = 3 - n + \sum_{k=1}^{n} p_k . \tag{5}$$

Generalized model has measure

$$d\mu(\mathbf{T}, \overline{\mathbf{T}}) = \exp\Big(- \sum_{\mathcal{L}} \lambda_{\mathcal{L}} \, N^{\omega(\mathcal{L})} \, B_{\mathcal{L}}(\mathbf{T}, \overline{\mathbf{T}}) \Big) \; d\mu_0(\mathbf{T}, \overline{\mathbf{T}}). \tag{6}$$

where the sum in the exponential is over a finite number of trees of necklaces.

Enhancement of trees of necklaces

Let us denote a generic tree of necklaces by $\mathcal{L}$. If it is of type $\{p_1, \dots, p_n\}$, the enhancement it requires to contribute at large N is

$$\omega(\mathcal{L}) = \sum_{k=1}^{n} (2 + p_k) - 3(n-1) = 3 - n + \sum_{k=1}^{n} p_k . \tag{5}$$

Generalized model has measure

$$d\mu(\mathbf{T}, \overline{\mathbf{T}}) = \exp\Big(- \sum_{\mathcal{L}} \lambda_{\mathcal{L}} \, N^{\omega(\mathcal{L})} \, B_{\mathcal{L}}(\mathbf{T}, \overline{\mathbf{T}}) \Big) \; d\mu_0(\mathbf{T}, \overline{\mathbf{T}}). \tag{6}$$

where the sum in the exponential is over a finite number of trees of necklaces.

Factorization

Theorem

Let us denote the expectation of the necklace of size p as

$$C_p = \frac{N^{2+p}}{N^4} \left\langle B^{(p)}_{12}(\mathbf{T}, \overline{\mathbf{T}}) \right\rangle = \frac{N^{2+p}}{N^4} \frac{\int d\mu(\mathbf{T}, \overline{\mathbf{T}}) B^{(p)}_{12}(\mathbf{T}, \overline{\mathbf{T}})}{\int d\mu(\mathbf{T}, \overline{\mathbf{T}})} . \tag{7}$$

Then the expectation of any tree of necklaces $\mathcal{L}_{\{p_1, \dots, p_n\}}$ factorizes in the large N limit like

$$\frac{N^{\omega(\mathcal{L}_{\{p_1, \dots, p_n\}})}}{N^4} \langle \mathcal{L}_{\{p_1, \dots, p_n\}}(\mathbf{T}, \overline{\mathbf{T}}) \rangle = \prod_{k=1}^{n} C_{p_k} . \tag{8}$$

Factorization

Theorem

Let us denote the expectation of the necklace of size p as

$$C_p = \frac{N^{2+p}}{N^4} \left\langle B_{12}^{(p)}(\mathbf{T}, \overline{\mathbf{T}}) \right\rangle = \frac{N^{2+p}}{N^4} \frac{\int d\mu(\mathbf{T}, \overline{\mathbf{T}}) B_{12}^{(p)}(\mathbf{T}, \overline{\mathbf{T}})}{\int d\mu(\mathbf{T}, \overline{\mathbf{T}})}. \qquad (7)$$

Then the expectation of any tree of necklaces $\mathcal{L}_{\{p_1,\dots,p_n\}}$ factorizes in the large N limit like

$$\frac{N^{\omega(\mathcal{L}_{\{p_1,\dots,p_n\}})}}{N^4} \langle \mathcal{L}_{\{p_1,\dots,p_n\}}(\mathbf{T}, \overline{\mathbf{T}}) \rangle = \prod_{k=1}^{n} C_{p_k}. \qquad (8)$$

Factorization

Theorem

Let us denote the expectation of the necklace of size p as

$$C_p = \frac{N^{2+p}}{N^4} \left\langle B_{12}^{(p)}(\mathbf{T},\overline{\mathbf{T}}) \right\rangle = \frac{N^{2+p}}{N^4} \frac{\int d\mu(\mathbf{T},\overline{\mathbf{T}}) B_{12}^{(p)}(\mathbf{T},\overline{\mathbf{T}})}{\int d\mu(\mathbf{T},\overline{\mathbf{T}})}. \quad (7)$$

Then the expectation of any tree of necklaces $\mathcal{L}_{\{p_1,\dots,p_n\}}$ factorizes in the large N limit like

$$\frac{N^{\omega(\mathcal{L}_{\{p_1,\dots,p_n\}})}}{N^4} \left\langle \mathcal{L}_{\{p_1,\dots,p_n\}}(\mathbf{T},\overline{\mathbf{T}}) \right\rangle = \prod_{k=1}^{n} C_{p_k}. \quad (8)$$

Schwinger-Dyson equations at leading order

Schwinger-Dyson equation

$$C_p = \sum_{k=0}^{p-1} C_k \, C_{p-k-1} + \sum_{j \geq 1} j \, \partial_j V(C_1, C_2, C_3, \dots) \, C_{j+p-1}. \tag{9}$$

where V is some polynomial, and C_p is the number of maps with root vertex of degree p. The quadratic term corresponds, as usual for equations *à la Tutte*, to the case where the root edge is a bridge.

The second term extends the length of the boundary face from p to $p + j - 1$, which is also usual for planar maps. However, it here comes with a more complicated weight $j\partial_j V(C_1, C_2, \dots)$, due to the *branching process*.

Schwinger-Dyson equations at leading order

Schwinger-Dyson equation

$$C_p = \sum_{k=0}^{p-1} C_k \, C_{p-k-1} + \sum_{j \geq 1} j \, \partial_j V(C_1, C_2, C_3, \dots) \; C_{j+p-1}. \tag{9}$$

where V is some polynomial, and C_p is the number of maps with root vertex of degree p. The quadratic term corresponds, as usual for equations *à la Tutte*, to the case where the root edge is a bridge.

The second term extends the length of the boundary face from p to $p+j-1$, which is also usual for planar maps. However, it here comes with a more complicated weight $j\partial_j V(C_1, C_2, \dots)$, due to the *branching process*.

Schwinger-Dyson equations at leading order

Schwinger-Dyson equation

$$C_p = \sum_{k=0}^{p-1} C_k \, C_{p-k-1} + \sum_{j\geq 1} j \, \partial_j V(C_1, C_2, C_3, \dots) \; C_{j+p-1}. \tag{9}$$

where V is some polynomial, and C_p is the number of maps with root vertex of degree p. The quadratic term corresponds, as usual for equations *à la Tutte*, to the case where the root edge is a bridge.

The second term extends the length of the boundary face from p to $p + j - 1$, which is also usual for planar maps. However, it here comes with a more complicated weight $j\partial_j V(C_1, C_2, \dots)$, due to the *branching process*.

Schwinger-Dyson equations at leading order

This equation was found and studied for multi-trace matrix models.
Free energy behaves like $(g - g_c)^{2-\gamma}$, where g is the "cosmological constant", g_c is the radius of convergence of the generating function for $(C_p)_{p\geq 1}$ and γ is the *entropy exponent*. It classifies the various universality classes which can be achieved.

- planar components critical while branching process sub-critical $=> \gamma = -1/2$ (QG2)
- planar components sub-critical while branching process critical $=> \gamma = +1/2$ (BP)
- planar components and branching critical $=> \gamma = +1/3$ (baby universes)
- Tuning more couplings $=> \gamma = p/(n+m+1)$, where $p \leq n$ and m are integers.

Schwinger-Dyson equations at leading order

This equation was found and studied for multi-trace matrix models.
Free energy behaves like $(g - g_c)^{2-\gamma}$, where g is the "cosmological constant", g_c is the radius of convergence of the generating function for $(C_p)_{p\geq 1}$ and γ is the *entropy exponent*. It classifies the various universality classes which can be achieved.

- planar components critical while branching process sub-critical $=> \gamma = -1/2$ (QG2)
- planar components sub-critical while branching process critical $=> \gamma = +1/2$ (BP)
- planar components and branching critical $=> \gamma = +1/3$ (baby universes)
- Tuning more couplings $=> \gamma = p/(n + m + 1)$, where $p \leq n$ and m are integers.

Schwinger-Dyson equations at leading order

This equation was found and studied for multi-trace matrix models.
Free energy behaves like $(g - g_c)^{2-\gamma}$, where g is the "cosmological constant", g_c is the radius of convergence of the generating function for $(C_p)_{p \geq 1}$ and γ is the *entropy exponent*. It classifies the various universality classes which can be achieved.

- planar components critical while branching process sub-critical $=> \gamma = -1/2$ (QG2)
- planar components sub-critical while branching process critical $=> \gamma = +1/2$ (BP)
- planar components and branching critical $=> \gamma = +1/3$ (baby universes)
- Tuning more couplings $=> \gamma = p/(n + m + 1)$, where $p \leq n$ and m are integers.

Schwinger-Dyson equations at leading order

This equation was found and studied for multi-trace matrix models.
Free energy behaves like $(g - g_c)^{2-\gamma}$, where g is the "cosmological constant", g_c is the radius of convergence of the generating function for $(C_p)_{p\geq 1}$ and γ is the *entropy exponent*. It classifies the various universality classes which can be achieved.

- planar components critical while branching process sub-critical $=> \gamma = -1/2$ (QG2)
- planar components sub-critical while branching process critical $=> \gamma = +1/2$ (BP)
- planar components and branching critical $=> \gamma = +1/3$ (baby universes)
- Tuning more couplings $=> \gamma = p/(n + m + 1)$, where $p \leq n$ and m are integers.

Schwinger-Dyson equations at leading order

This equation was found and studied for multi-trace matrix models.
Free energy behaves like $(g - g_c)^{2-\gamma}$, where g is the "cosmological constant", g_c is the radius of convergence of the generating function for $(C_p)_{p\geq 1}$ and γ is the *entropy exponent*. It classifies the various universality classes which can be achieved.

- planar components critical while branching process sub-critical $=> \gamma = -1/2$ (QG2)
- planar components sub-critical while branching process critical $=> \gamma = +1/2$ (BP)
- planar components and branching critical $=> \gamma = +1/3$ (baby universes)
- Tuning more couplings $=> \gamma = p/(n + m + 1)$, where $p \leq n$ and m are integers.

Schwinger-Dyson equations at leading order

This equation was found and studied for multi-trace matrix models.
Free energy behaves like $(g - g_c)^{2-\gamma}$, where g is the "cosmological constant", g_c is the radius of convergence of the generating function for $(C_p)_{p\geq 1}$ and γ is the *entropy exponent*. It classifies the various universality classes which can be achieved.

- planar components critical while branching process sub-critical $=> \gamma = -1/2$ (QG2)
- planar components sub-critical while branching process critical $=> \gamma = +1/2$ (BP)
- planar components and branching critical $=> \gamma = +1/3$ (baby universes)
- Tuning more couplings $=> \gamma = p/(n+m+1)$, where $p \leq n$ and m are integers.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)
Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T},\bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T},\bar{\mathbf{T}}), \quad (10)$$

$$W = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T},\bar{\mathbf{T}})\Big). \quad (11)$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)

Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T}, \bar{\mathbf{T}}), \quad (10)$$

$$W = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \quad (11)$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)
Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T}, \bar{\mathbf{T}}), \qquad (10)$$

$$W = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \qquad (11)$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)
Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T}, \bar{\mathbf{T}}), \tag{10}$$

$$W = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \tag{11}$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)
Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T}, \bar{\mathbf{T}}), \quad (10)$$

$$W = \log Z = \log \int \prod_{n} \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \quad (11)$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)
Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T}, \bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T}, \bar{\mathbf{T}}), \quad (10)$$

$$W = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T}, \bar{\mathbf{T}})\Big). \quad (11)$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Enhancing More Invariants

(joint work with V. Bonzom and L. Lionni)
Consider any fixed invariant $\mathcal{B}$ and the model

$$S(\mathbf{T},\bar{\mathbf{T}}) = \bar{\mathbf{T}} \cdot_{\mathcal{D}} \mathbf{T} - \lambda N^{\beta} \mathcal{B}(\mathbf{T},\bar{\mathbf{T}}), \tag{10}$$

$$W = \log Z = \log \int \prod_n \frac{d\bar{T}_n dT_n}{2\imath\pi} \exp\Big(-N^{D-1} S(\mathbf{T},\bar{\mathbf{T}})\Big). \tag{11}$$

- For any pairing of the arguments of $\mathcal{B}$ there is an associated IF representation
- There exists a maximal β (enhancement) such that the $1/N$ expansion still exists.
- β can be computed in terms of the number of faces in an optimal pairing of $\mathcal{B}$ (ie a pairing with maximal number of faces)
- The leading order (LO) graphs for the $1/N$ expansion always include the infinite family made of all the trees in the IF representation of an optimal pairing. There can be also LO graphs with loops, if optimal pairing not unique.

Conclusion

- Simple tensor models can interpolate between melonic world (BP) and brownian spheres (BS).
- The technique of IF is interesting (tensor models = multi-matrix models coupled in new ways).
- Tensor field theories of this type should allow to explore the frontier between asymptotically free tensor field theories and asymptotically safe non-commutative field theories (as for $N_f/N_c \simeq 11/2$, no SUSY here...).
- Any tensor invariant interaction can be enhanced but no general description yet of LO (ie which loops are involved).

Conclusion

- Simple tensor models can interpolate between melonic world (BP) and brownian spheres (BS).
- The technique of IF is interesting (tensor models = multi-matrix models coupled in new ways).
- Tensor field theories of this type should allow to explore the frontier between asymptotically free tensor field theories and asymptotically safe non-commutative field theories (as for $N_f/N_c \simeq 11/2$, no SUSY here...).
- Any tensor invariant interaction can be enhanced but no general description yet of LO (ie which loops are involved).

Conclusion

- Simple tensor models can interpolate between melonic world (BP) and brownian spheres (BS).
- The technique of IF is interesting (tensor models = multi-matrix models coupled in new ways).
- Tensor field theories of this type should allow to explore the frontier between asymptotically free tensor field theories and asymptotically safe non-commutative field theories (as for $N_f/N_c \simeq 11/2$, no SUSY here...).
- Any tensor invariant interaction can be enhanced but no general description yet of LO (ie which loops are involved).

Conclusion

- Simple tensor models can interpolate between melonic world (BP) and brownian spheres (BS).
- The technique of IF is interesting (tensor models = multi-matrix models coupled in new ways).
- Tensor field theories of this type should allow to explore the frontier between asymptotically free tensor field theories and asymptotically safe non-commutative field theories (as for $N_f/N_c \simeq 11/2$, no SUSY here...).
- Any tensor invariant interaction can be enhanced but no general description yet of LO (ie which loops are involved).

Conclusion

- Simple tensor models can interpolate between melonic world (BP) and brownian spheres (BS).
- The technique of IF is interesting (tensor models = multi-matrix models coupled in new ways).
- Tensor field theories of this type should allow to explore the frontier between asymptotically free tensor field theories and asymptotically safe non-commutative field theories (as for $N_f/N_c \simeq 11/2$, no SUSY here...).
- Any tensor invariant interaction can be enhanced but no general description yet of LO (ie which loops are involved).