The Return of Newton-Cartan space-time

The Spacetime Odyssey Continues

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based on work with:

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and

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1311.4794 (PRD) & 1311.6471 (JHEP)

Outline

- Why Newton-Cartan (NC) ? (-> non-relativistic space-time)
  - holography,
  - field theory
  - gravity

- What is NC (& its torsionful generalization TNC) geometry ?
  - NC from gauging the Bargmann algebra

- What theory of gravity does one get when making TNC dynamical ?
  - connection to Horava-Lifshitz gravity
  - perspectives (cosmology)

- Outlook

Motivation (Holography)

AdS/CFT has been very successful tool in studying strongly coupled (conformal) relativistic systems
-> holography beyond original AdS-setup ?

- How general is holographic paradigm ?
  (nature of quantum gravity, black hole physics, cosmology)

- Examples of potentially holographic descriptions based on non-AdS space-times: Lifshitz, Schrödinger, warped AdS3 (Kerr/CFT), flat space-time.

- simplest example appears to be Lifshitz spacetimes

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

characterized by anisotropic (non-relativistic) scaling between time and space

$$t \to \lambda^z t\,, \qquad \vec{x} \to \lambda \vec{x}$$

[Kachru,Liu,Mulligan]

* introduced originally to study strongly coupled systems with critical exponent z

Motivation (Holography) cont'd

- for standard AdS setup: boundary geometry is Riemannian just like the bulk geometry

- not generic: in beyond-AdS holography bdry. geometry typically non-Riemannian

Christensen,Hartong,Rollier,NO (1311)
Hartong,Kiritsis,NO (1409)

-> need new approach: prime (simplest) example to gain traction = Lifshitz
(lessons can subsequently be applied to other cases)

by making the resulting non-Riemannian geometry dynamical one gains access to
other bulk theories of gravity (than those based on Riemannian gravity)
- apply holography (e.g. HL gravity)
- interesting in their own right

Different Holographic setups

bulk: Riemannian gravity (GR) | non-Riemmanian gravity (e.g. HL gravity)

AdS | non-AdS | non-AdS

boundary: CFT | NR-FT with scaling | NR-FT with scaling

coupling to:

Riemannian geometry | non-Riemannian

Motivation (Field Theory)

- in relativistic FT: very useful to couple to background (Riemannian) geometry
  -> compute EM tensors, study anomalies, Ward identities, etc.

- background field methods for systems with non-relativistic (NR) symmetries require NC geometry (with torsion)

  -> there is full space-time diffeomorphism invariance when coupling to the right background fields

- Recent examples
  * Son's approach to the effective field theory for the FQHE [Son, 2013], [Geracie, Son, Wu, Wu, 2014]

  * non-relativistic (NR) hydrodynamics [Jensen,2014]

Motivation (Gravity)

- interesting to make NC geometry dynamical
  -> "new" theories of gravity

Hartong,Kiritsis,NO (1504)

will see: | dynamical Newton-Cartan (NC) = Horava-Lifshitz (HL) gravity |

⇒ natural geometric framework with full diffeomorphism invariance
& possibly non-trivial consequences for HL gravity

such theories of gravity interesting as
- other bulk theories of gravity in holographic setups
- effective theories

- novel theories of gravity to examine cosmology
- effective theories for inflation ?

Newton-Cartan makes Galilean local

- NC geometry originally introduced by Cartan to geometrize Newtonian gravity

⇨ both Einstein's and Newton's theories of gravity admit geometrical formulations which are diffeomorphism invariant

- NC originally formulated in "metric" formulation
  more recently: vielbein formulation (shows underlying sym. principle better)
  Andringa,Bergshoeff,Panda,de Roo

Riemannian geometry: tangent space is Poincare invariant

Newton-Cartan geometry: tangent space is Bargmann (central ext. Gal.) invariant

- gives geometrical framework and extension to include torsion
i.e. as geometry to which non-relativistic field theories can couple
(boundary geometry in holographic setup is non-dynamical)

* will next consider dynamical (torsional) Newton-Cartan

From Poincare to GR

- make Poincare local (i.e. gauge the translations and rotations)

vielbein → ; spin connection →

$$A_\mu = P_a e^a_\mu + \frac{1}{2} J_{ab} \omega_\mu{}^{ab}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu \,, A_\nu] = P_a R_{\mu\nu}{}^a(P) + \frac{1}{2} J_{ab} R_{\mu\nu}{}^{ab}(J)$$

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu \,, \Lambda] \,, \qquad \Lambda = \xi^\mu A_\mu + \frac{1}{2} J_{ab} \lambda^{ab}$$

$R_{\mu\nu}{}^a(P) = 0$ {

spin connection expressed in terms of vielbein

$\delta A_\mu \to \delta e^a_\mu = \mathcal{L}_\xi e^a_\mu + e^b_\mu \lambda_b{}^a$

covariant derivative defined via vielbein postulate

$R_{\mu\nu}{}^{ab}(J)$ = Riemann curvature 2-form

- GR is a diff invariant theory whose tangent space invariance group is the Poincaré group

* Einstein equivalence principle -> local Lorentz invariance

Gauging the Bargmann algebra

Galilean

$$H, P_a, J_{ab}, G_a \quad N$$

Bargmann

(Galilean algebra is c to infinity limit of Poincare)

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$[P_a, G_b] = N\delta_{ab}$$

gauge Bargmann and impose curvature constraints

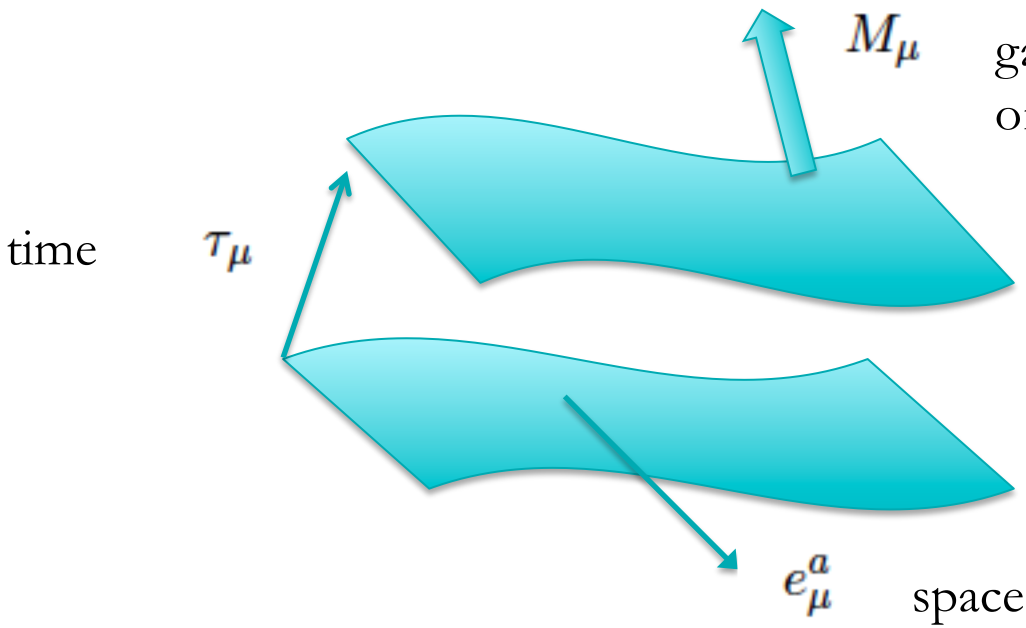
$$R_{\mu\nu}(H) = R_{\mu\nu}{}^a(P) = R_{\mu\nu}(N) = 0.$$

independent fields: $\tau_\mu,\ e_\mu^a,\ m_\mu$

$$h_{\mu\nu} = e_\mu^a e_\nu^b \delta_{ab}$$

= gauge fields of Hamiltonian, spatial translations and central charge

Newton-Cartan geometry



gauge field of central extension of Galilean algebra (Bargmann)

spatial metric:

$$h_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab}$$

NC geometry = no torsion → $\tau_\mu = \partial_\mu t$ notion of absolute time

TTNC geometry = twistless torsion → $\tau_\mu = \text{HSO}$ preferred foliation in equal time slices

TNC geometry no condition on τ_μ

- in TTNC: torsion measured by $a_\mu = \mathcal{L}_{\hat{v}} \tau_\mu$
  geometry on spatial slices is Riemannian

Adding torsion to NC

Christensen,Hartong,Rollier,NO
Hartong,Kiritsis,NO/Hartong,NO
Bergshoeff,Hartong,Rosseel

- inverse vielbeins (v^μ, e^μ_a)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e^a_\mu = 0\,, \qquad e^\mu_a \tau_\mu = 0\,, \qquad e^\mu_a e^b_\mu = \delta^b_a$$

can build Galilean boost-invariants

$$\hat{v}^\mu = v^\mu - h^{\mu\nu} M_\nu\,,$$
$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tau_\mu M_\nu - \tau_\nu M_\mu\,,$$
$$\tilde{\Phi} = -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu\,,$$

-introduce Stueckelberg scalar chi
(to ensure N-invariance):

$$M_\mu = m_\mu - \partial_\mu \chi.$$

→ affine connection of TNC (inert under G,J,N)

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma} \left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu} \right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\frac{1}{2} \hat{v}^\rho (\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$

analogue of metric compatibility

torsion in NC (recent activity)

- NC introduced in problem of FQH Son (1306)

- TNC first observed as bdry geometry in z=2 Lifshitz holography & generalized to large class with general z Christensen,Hartong,Rollier,NO (1311) Hartong,Kiritsis,NO (1409)

- TTNC introduced in FQH Geracie,Son,Wu,Wu(1407))

- TNC from gauging Schrödinger algebra Bergshoeff,Hartong,Rosseel (1409)
- TNC from gauging Bargmann (with torsion) Hartong,NO (1504)

- coupling of non-relativistic field theories to TNC (independent of holography) Jensen (1408) Hartong,Kiritsis,NO (1409)

- TNC related to warped geometry that couples to 2D WCFT Hofmann,Rollier (1411)

- other approaches Banerjee,Mitra,Mukherjee (1407), Brauner,Endlich,Monin,Penco(1407) Bekaert,Morand (1412)

- recent activity using NC/TNC in CM (strongly-correlated electron system, FQH) Gromov,Abanov][Moroz,Hoyos][Geracie,Son] [Wu,Wu],[Geracie,Golkar,Roberts] ,….

- (T)NC from non-rel limits Jensen,Karch (1412) , Bergshoeff,Rosseel,Zojer (1505)

intermezzo: geodesics on NC space-time

- worldline action of non-rel particle of mass m on NC background

$$S = \int d\lambda L = \frac{m}{2} \int d\lambda \frac{\bar{h}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{\tau_\rho \dot{x}^\rho}$$

[Kuchar], [Bergshoeff et al]

- gives the geodesic equation with NC connection $\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\lambda} \frac{dx^\rho}{d\lambda} = 0$,

* reduces to Newton's law $\frac{d^2 x^i}{dt^2} + \delta^{ij} \partial_j \Phi = 0$,

provided we take

$$M_t = \partial_t M + \Phi\,, \qquad M_i = \partial_i M\,,$$

for flat NC space-time: zero Newtonian potential

symmetries of flat NC = conformal Killing vectors (spanning Lifshitz) + extra

Dynamical Newton-Cartan geometry

so far: (T)NC geometry was non-dynamial:
- what happens when we allow it to fluctuate ?

• what is the theory of gravity that incorporates local Galilean symmetry ?
(Einstein equivalence principle, but applied to Galilean instead of Lorentz)

recently shown that: [Hartong,NO]

• dynamical NC geometry = projectable HL gravity

• dynamical TTNC geometry = non-projectable HL gravity

* Horava-Lifshitz gravity was originally introduced as non-Lorentz invariant and renormalizable UV completion of gravity
- phenomenologically viable ?
- interesting theoretically as alternate bulk gravity theories relevant to i) holography for strongly coupled non-relativistic systems
ii) alternate theories in cosmology

NC/TTNC gravity

TNC geometry is a natural geometrical framework underlying HL gravity

- NC quantities combine into: $g_{\mu\nu} = -\tau_\mu \tau_\nu + \hat{h}_{\mu\nu}$
- ADM parametrization of metric used in HL gravity:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

relation:

$$\tau_\mu \sim \text{lapse} \quad , \quad \hat{h}_{\mu\nu} \sim \text{spatial metric} \quad , \quad m_\mu \sim \text{shift} + \text{Newtonian potential}$$

some features:

- khronon field of BPS appears naturally $\tau_\mu = \psi \partial_\mu \tau$ Blas,Pujolas,,Sibiryakov (2010)

  NC (no torsion): $N = N(t)$ projectable HL gravity

  TTNC: $N = N(t, x)$ non-projectable HL gravity
- U(1) extension of HMT emerges naturally as Bargmann U(1)
- new perspective (via chi field) on nature of U(1) symmetry

Horava,Melby-Thompson,(2010)

Effective actions reproduce HL

- covariant building blocks:

- extrinsic curvature: $\hat{h}_{\nu\rho}\nabla_\mu \hat{v}^\rho = -K_{\mu\nu}$ spatial curvature $\check{R}_{\mu\nu\sigma}{}^\rho$.

- covariant derivative, torsion vector a_μ , inverse spatial metric $h^{\mu\nu}$

- tangent space invariant integration measure $e = \det(\tau_\mu, e^a_\nu)$

-> construct all terms that are relevant or marginal (up to dilatation weight d+z)

- in 2+1 dimensions for $1 < z \leq 2$

$$S = \int d^3x e \left[C\left(h^{\mu\rho}h^{\nu\sigma}K_{\mu\nu}K_{\rho\sigma} - \lambda\left(h^{\mu\nu}K_{\mu\nu}\right)^2\right) - \mathcal{V}\right]$$

kinetic terms (2nd order)

potential:

$$-\mathcal{V} = 2\Lambda + c_1 h^{\mu\nu}a_\mu a_\nu + c_2\mathcal{R} + \delta_{z,2}\left[c_{10}\left(h^{\mu\nu}a_\mu a_\nu\right)^2 + c_{11}h^{\mu\rho}a_\mu a_\rho \nabla_\nu\left(h^{\nu\sigma}a_\sigma\right)\right.$$
$$\left. + c_{12}\nabla_\nu\left(h^{\mu\rho}a_\rho\right)\nabla_\mu\left(h^{\nu\sigma}a_\sigma\right) + c_{13}\mathcal{R}^2 + c_{14}\mathcal{R}\nabla_\mu\left(h^{\mu\nu}a_\nu\right) + c_{15}\mathcal{R}h^{\mu\nu}a_\mu a_\nu\right]$$

Perspectives

new perspectives on HL:
- different vacuum (flat NC space-time): reexamine issues with HL gravity
- IR effective theory for non-relativistic field theories
- insights into non-relativistic quantum gravity corner of $(\hbar, G_N, 1/c)$ cube ?

- relevance for cosmology ?

alternate theories of gravity in cosmological scenarios, effective theories for inflation

e.g. aspects of HL cosmology:
- dark matter shows up as integration constant
- bouncing and cyclic universes
- scale-invariant perturbations (z=3, in D=4) without inflation

Outlook

- revisit HL gravity using TNC language-> implications for HL cosmology ?

- gauging other groups: e.g. Caroll group > ulra-relativistic gravity [Hartong]

- applications to non-rel. hydrodynamics (also relevant for cosmo):
  fluid/gravity: black branes with zero/non-zero particle no. ? Galilean perfect fluids
  [Hoyos,Kim,Oz] [Jensen]

- NC supergravity, NC in string theory
  [Bergshoeff et al]

- employ similar techniques to Schrödinger, warped AdS, flat space holography
  [Andrade,Keeler,Peach,Ross,].[Hofman,Rollier][Armas,Blau,Hartong(in progress)]

- adding charge (Maxwell in bulk),
  adding other exponents (hyperscaling, matter scaling)
  [Kiritsis,Goutereaux][Gath,Hartong,Monteiro,NO]
  [Khveshchenko][Karch][Hartnoll,Karch]

- effective TNC theories and their coupling to matter (e.g. QH-effect)
  [Son] et al