Correlation Hunting

Generalizing Hanbury Brown - Twiss

with Jordan Cotler; also Aram Harrow, Maulik Parikh

- From the perspective of quantum theory, *interference* arises from the possibility of getting from the same initial state to the same final state through several distinct paths.

- The probability for the overall process is then the square of the sum of amplitudes, which differs from the sum of the squares.

$$|\,A + B\,|^2 \;=\; |A|^2 + |B|^2 + (AB^* + A^*B)$$

interference term

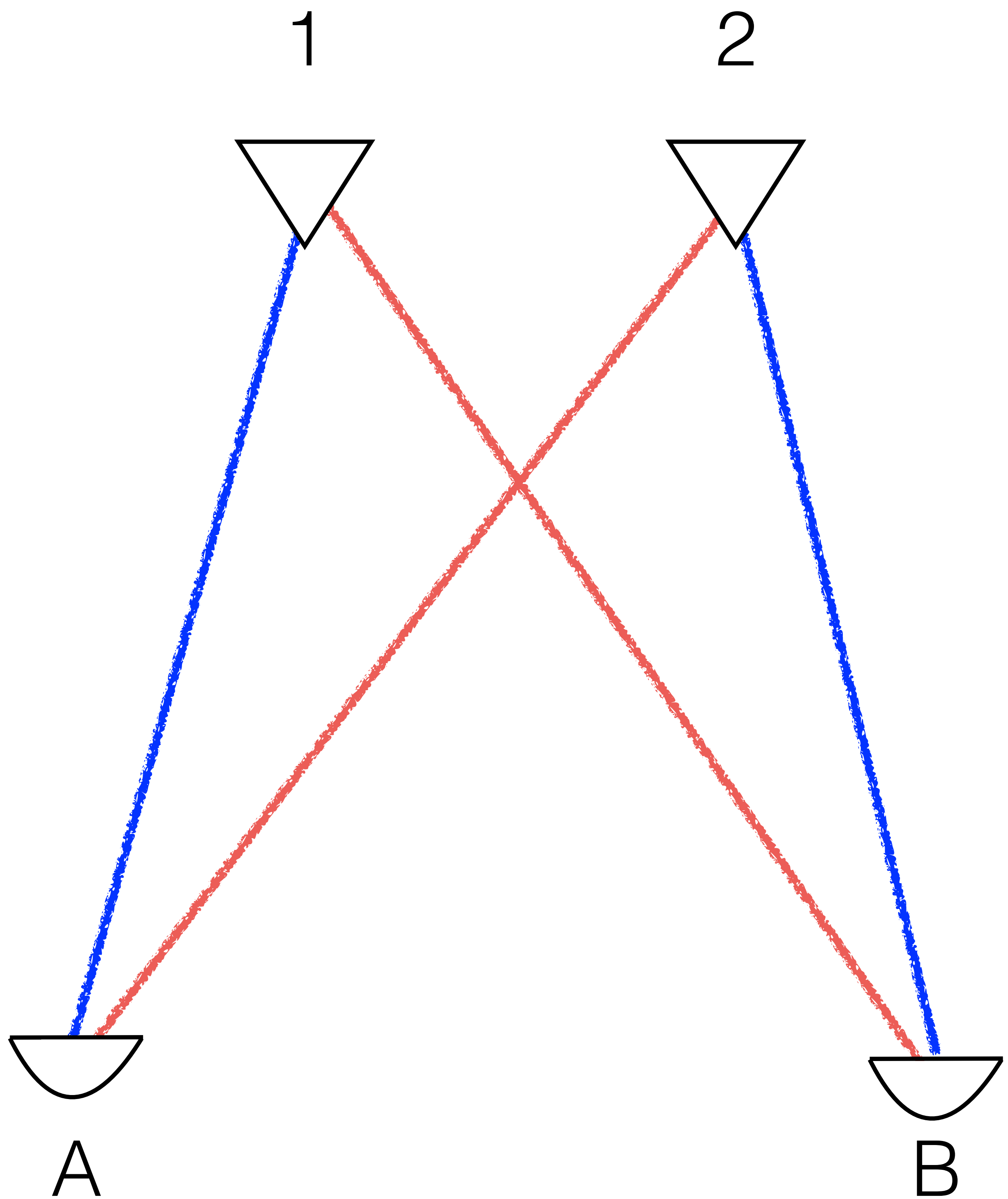
- Interference depends on the relative phase between the different branches (as well as on their magnitudes).
- For that reason, interference usually requires coherence of the sources.

Hanbury Brown - Twiss

- The radio astronomer Hanbury Brown, and his associate Twiss, in 1949 discovered a new and fascinating kind of interference effect, that does not require coherence.

- They used it to measure the diameter of several stars, including Sirius.

- In later years, their basic idea has found many other applications, ranging from heavy ion collisions to condensed matter.



the basic HBT setup

- Here one considers the probability for triggering of two detectors A, B, due to reception of emissions from two sources 1, 2.

- The two sources can be different parts of the star, or whatever.

- There are two distinct *processes* involved (red and blue, above), and they interfere:

$$|\,D_{1A}D_{2B} + D_{2A}D_{1B}\,|^2$$

$$D_{1A}D_{2B}D^*_{1B}D^*_{2A} + \text{c.c.}$$

- Note that in this interference term random phases associated with the emission events cancels out!
- The interference term depends only on the relative phase, which is essentially geometrical.
- As one varies the distance between the detectors, one gets positive or negative interference. The distance between maxima reflects the separation of the sources.

- For a single extended source, such as a star, the contrast will wash out at large detector separations.

- The rate with which that happens reflects the angular size of the source, and can be used to measure it.

Polarization

- It was implicit, in our preceding discussion, that the detector “couldn’t reveal” where its photon came from. (The final states must overlap.)
- If the photons have orthogonal polarizations, for example, they will not interfere.
- For unpolarized sources, this simply halves the HBT effect.

- The question naturally arises: If the emitters *do* have non-trivial polarization properties, can we access them?

- For example: If we have two very nearby sources, that emit in orthogonal polarizations, can we resolve them?

- Unadorned HBT won’t serve here, but (as we’ll see) a simple refinement accesses much more information, and does the job.

Polarization Projections

$$\begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \Pi_A \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} \gamma^* & \delta^* \end{pmatrix} \Pi_B \begin{pmatrix} \gamma \\ \delta \end{pmatrix} |D_{1A}|^2 |D_{2B}|^2 +$$

$$\begin{pmatrix} \gamma^* & \delta^* \end{pmatrix} \Pi_A \begin{pmatrix} \gamma \\ \delta \end{pmatrix} \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \Pi_B \begin{pmatrix} \alpha \\ \beta \end{pmatrix} |D_{2A}|^2 |D_{1B}|^2$$

$$+ \begin{pmatrix} \gamma^* & \delta^* \end{pmatrix} \Pi_A \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \Pi_B \begin{pmatrix} \gamma \\ \delta \end{pmatrix} D_{1A} D_{2B} D^*_{2A} D^*_{1B} + \text{c.c.}$$

Density Matrices

$$\mathrm{Tr}\,\Pi_A\pi_1\,\mathrm{Tr}\,\Pi_B\pi_2\;|D_{1A}|^2|D_{2B}|^2 + \mathrm{Tr}\,\Pi_A\pi_2\,\mathrm{Tr}\,\Pi_B\pi_1\;|D_{2A}|^2|D_{1B}|^2$$

$$+ \quad \mathrm{Tr}\,\Pi_A\pi_1\Pi_B\pi_2\;D_{1A}D_{2B}D^*_{2A}D^*_{1B} + \text{c.c.}$$

$$\left(\quad = \boxed{\mathrm{Tr}\,\Pi_A\pi_1\Pi_B\pi_2\;D_{1A}D_{2B}D^*_{2A}D^*_{1B} + \mathrm{Tr}\,\Pi_A\pi_2\Pi_B\pi_1\;D^*_{1A}D^*_{2B}D_{2A}D_{1B}\right)}$$

- We solve our model problem, by letting ∏ interpolate between the otherwise orthogonal polarizations.

- More generally, protocols using a sequence of ∏s can significantly enhance our perception of the sources (quantum state tomography).

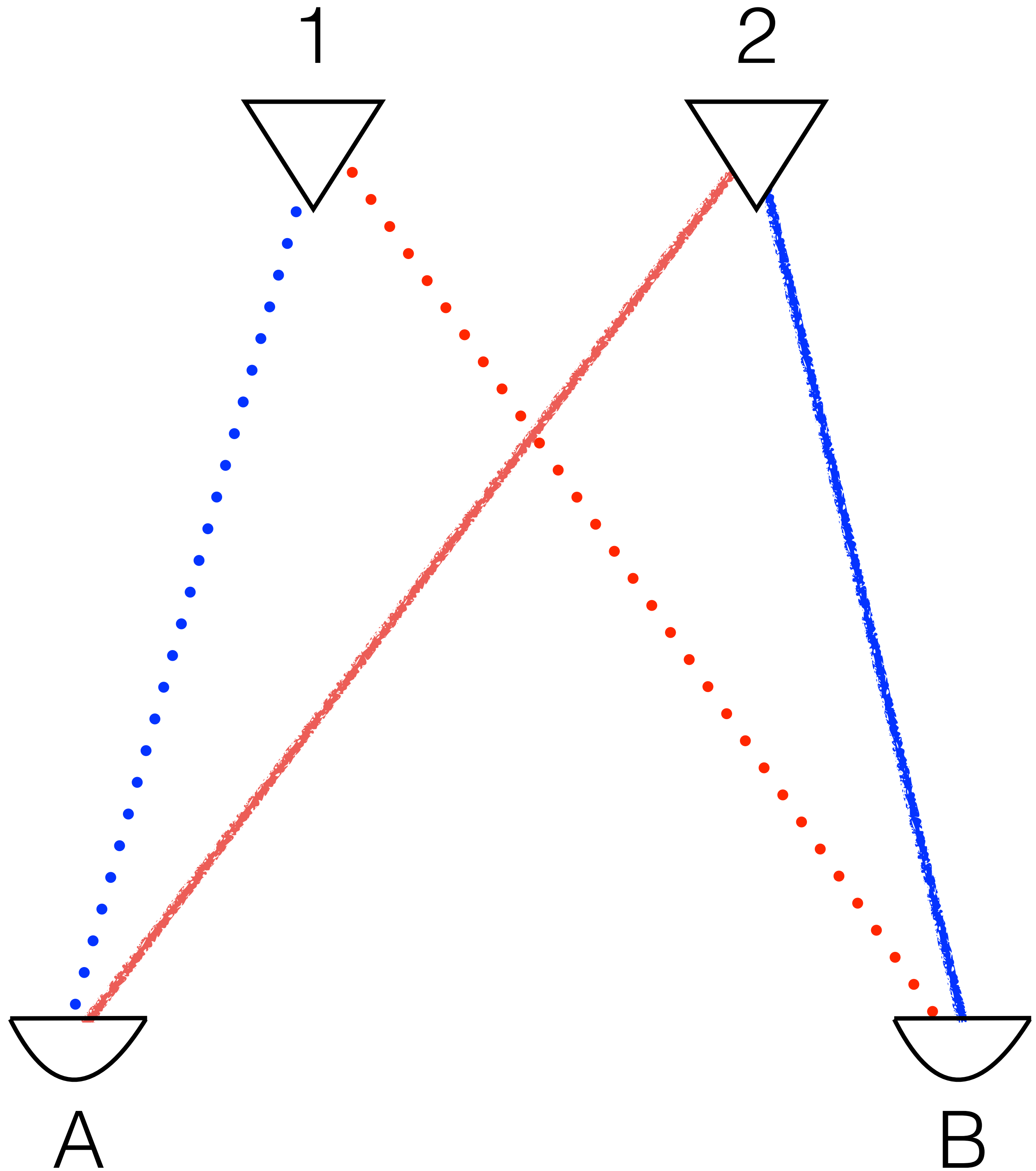
Entanglement

- So far, we have used selective *projection* to get interference between non-identical emissions.

- A more general and powerful technique exploits *entanglement* of the detectors.

- As an extreme example, let us consider that one of our emitters emits bosons B, while the other emits fermions F.

- A detector that receives a boson goes into state **B**, a detector that receives a fermion goes into state **F**.


1
2
A
B

We would like to get interference between the terms in

$$S_{1A}\, D_{2B} |\, \mathbf{FB} \rangle \;+\; D_{2A}\, S_{1B}\, |\mathbf{BF}\rangle$$

Unfortunately, they are orthogonal.

- Following a similar philosophy to our polarization example, we change the state basis - and erase information - to access interference.

$$
\begin{aligned}
& S_{1A}\, D_{2B} |\, \mathbf{FB}\rangle \;+\; D_{2A}\, S_{1B}\, |\mathbf{BF}\rangle \\
= \quad & \frac{1}{2}(S_{1A}\, D_{2B} \;+\; D_{2A}\, S_{1B})(|\, \mathbf{FB}\rangle \;+\; |\mathbf{BF}\rangle) \\
+ \quad & \frac{1}{2}(S_{1A}\, D_{2B} \;-\; D_{2A}\, S_{1B})(|\, \mathbf{FB}\rangle \;-\; |\mathbf{BF}\rangle)
\end{aligned}
$$

So if we project on the entangled state

$$\frac{1}{\sqrt{2}}(|\mathbf{FB}\rangle \;+\; |\mathbf{BF}\rangle)$$

we measure

$$|S_{1A} D_{2B} \;+\; D_{2A} S_{1B}|^2$$

- This consideration shows*, through a definite physical effect, that superselection cannot be interpreted locally.

More Entanglement

- The projectors Π_A, Π_B encode density matrices for the detectors A, B. When their states are entangled, however, the density matrix of the entire system will not factorize, and we will need to employ a system density matrix, in the form

$$(\Pi_A)^{\alpha_1}_{\alpha_2} (\Pi_B)^{\beta_1}_{\beta_2} \rightarrow \mathbf{\Pi}^{\alpha_1 \beta_1}_{\alpha_2 \beta_2}$$

- We should also allow for the interesting possibility of entanglement in the emitters. That is accommodated according to

$$(\pi_1)^{\alpha_1}_{\alpha_2} \, (\pi_2)^{\beta_1}_{\beta_2} \;\rightarrow\; \boldsymbol{\pi}^{\alpha_1 \beta_1}_{\alpha_2 \beta_2}$$

- With these notations, we can generalize our master formula, in the form

$$\mathbf{\Pi}^{\alpha_1\beta_1}_{\alpha_2\beta_2}\,\boldsymbol{\pi}^{\alpha_2\beta_2}_{\alpha_1\beta_1}\,|D_{1A}|^2|D_{2B}|^2 + \mathbf{\Pi}^{\alpha_1\beta_1}_{\alpha_2\beta_2}\,\boldsymbol{\pi}^{\beta_2\alpha_2}_{\beta_1\alpha_1}\,|D_{2A}|^2|D_{1B}|^2$$

$$+\mathbf{\Pi}^{\alpha_1\beta_1}_{\alpha_2\beta_2}\,\boldsymbol{\pi}^{\beta_2\alpha_2}_{\alpha_1\beta_1}\;D_{1A}D_{2B}D^*_{2A}D^*_{1B} + \mathbf{\Pi}^{\alpha_1\beta_1}_{\alpha_2\beta_2}\,\boldsymbol{\pi}^{\alpha_2\beta_2}_{\beta_1\alpha_1}\;D^*_{1A}D^*_{2B}D_{2A}D_{1B}$$

There is a duality between sources and detectors.

- By comparing experimental data with this expression and determining whether it is consistent with factorization* of **π** we become sensitive to entanglement between the emitters.

- This effect, or its dual, could be used as a probe for proposed exotic states of matter that feature long-range entanglement.

- In that application, we should use designed sources, and consider the matter as “detector”.

Implementations and a Variation

- Tools for operations with polarization are very well developed (phase shifters, filters).

- We can obtain entangled detector (or source) states, in principle, with spatial swaps

$$\widehat{S} = \frac{1}{\sqrt{2}} \left(|A\rangle \otimes |B\rangle + |B\rangle \otimes |A\rangle \right) \left(\langle A| \otimes \langle B| \right) + \frac{1}{\sqrt{2}} \left(|A\rangle \otimes |B\rangle - |B\rangle \otimes |A\rangle \right) \left(\langle B| \otimes \langle A| \right)$$
$$+ |A\rangle\langle A| \otimes |A\rangle\langle A| + |B\rangle\langle B| \otimes |B\rangle\langle B|$$

$$\frac{1}{\sqrt{2}} \left(|\mathbb{F}\rangle|A\rangle \otimes |\mathbb{B}\rangle|B\rangle + |\mathbb{F}\rangle|B\rangle \otimes |\mathbb{B}\rangle|A\rangle \right)$$

$$\boxed{\frac{1}{\sqrt{2}}(S_{1A}D_{2B}+D_{2A}S_{1B})|\mathbf{F}\rangle|A\rangle\otimes|\mathbf{B}\rangle|B\rangle} + \frac{1}{\sqrt{2}}(S_{1A}D_{2B}-D_{2A}S_{1B})|\mathbf{F}\rangle|B\rangle\otimes|\mathbf{B}\rangle|A\rangle$$

- Another promising idea is to interfere photons emitted at different *times*.   This can be done by storing and “forgetting”.

- Impressive tools for storing photons, while preserving their quantum state, are emerging:

Storage of Light in Atomic Vapor

D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin
Phys. Rev. Lett. **86**, 783 – Published 29 January 2001

ABSTRACT

We report an experiment in which a light pulse is effectively decelerated and trapped in a vapor of Rb atoms, stored for a controlled period of time, and then released on demand. We accomplish this "storage of light" by dynamically reducing the group velocity of the light pulse to zero, so that the coherent excitation of the light is reversibly mapped into a Zeeman (spin) coherence of the Rb vapor.

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Potential Applications

- Spinning or magnetized stars (polarization)
- Cosmic masers (coherence, polarization, entanglement)
- With photon storage: structure of atmospheres

- Cosmic microwave background

Hanbury Brown-Twiss interferometry and second-order correlations of inflaton quanta

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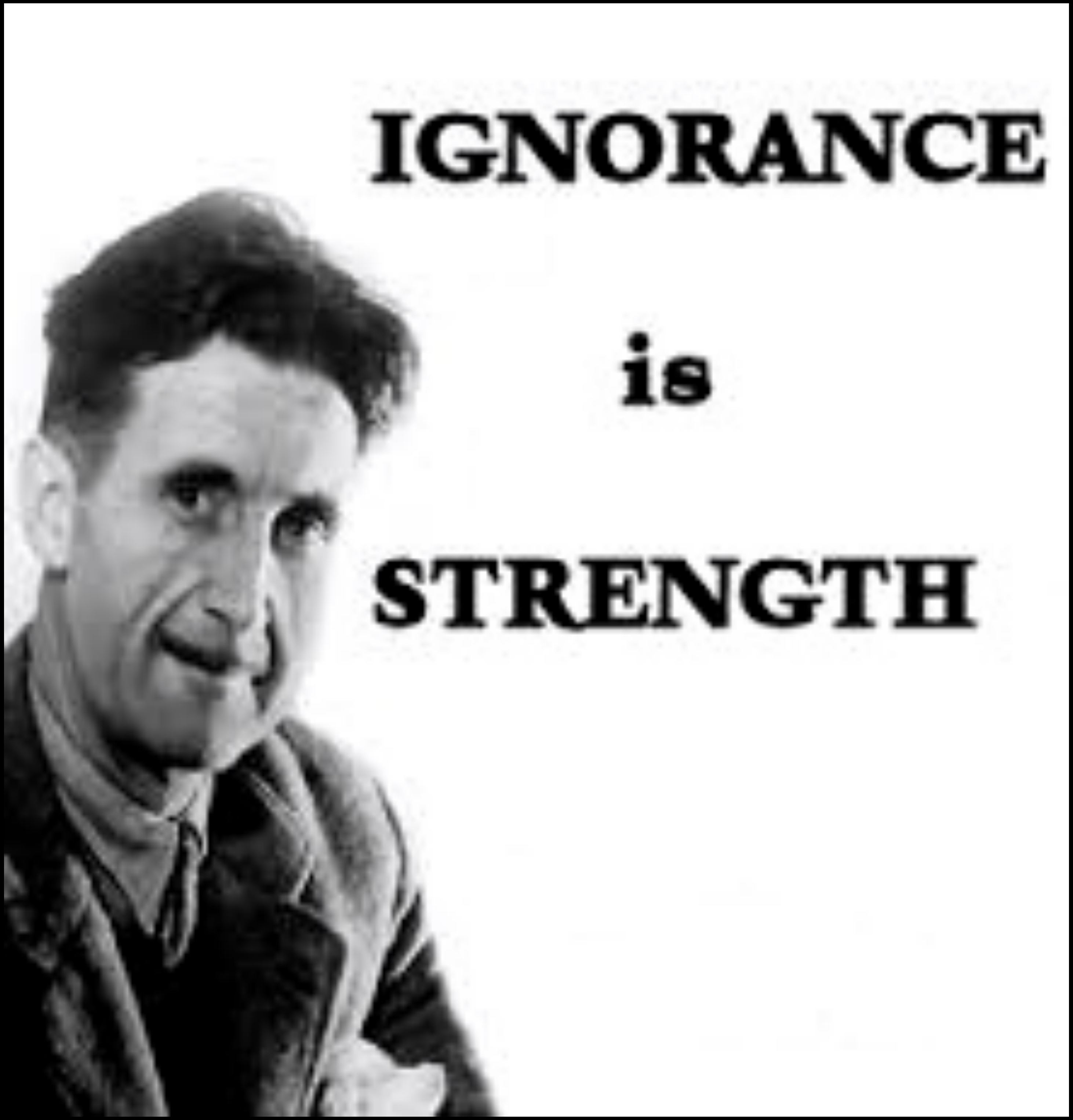
Abstract

The quantum theory of optical coherence is applied to the scrutiny of the statistical properties of the relic inflaton quanta. After adapting the description of the quantized scalar and tensor modes of the geometry to the analysis of intensity correlations, the normalized degrees of first-order and second-order coherence are computed in the concordance paradigm and are shown to encode faithfully the statistical properties of the initial quantum state. The strongly bunched curvature phonons are not only super-Poissonian but also super-chaotic. Testable inequalities are derived in the limit of large angular scales and can be physically interpreted in the light of the tenets of Hanbury Brown-Twiss interferometry. The quantum mechanical results are compared and contrasted with different situations including the one where intensity correlations are the result of a classical stochastic process. The survival of second-order correlations (not necessarily related to the purity of the initial quantum state) is addressed by defining a generalized ensemble where super-Poissonian statistics is an intrinsic property of the density matrix and turns out to be associated with finite volume effects which are expected to vanish in the thermodynamic limit.

Summary

- *We can gain new information by intelligent erasure of other (potential) information.*
- Or as George Orwell put it:


IGNORANCE
is
STRENGTH