Gravitational Condensate Stars: What's the (Quantum) Matter with 'Black Holes'?

Negative Pressure & Surface Tension of the Interior Schwarzschild Solution

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➔ Class. Quant. Grav. (2015)

Proc. Natl. Acad. Sci., 101, 9545 (2004)

Review& Microphysics: EFT of Gravity:

Acta Phys. Pol. B 41, 2031 (2010)

Classical Black Holes

Schwarzschild Metric (1916)

$$ds^2 = -dt^2\, f(r) + \frac{dr^2}{h(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$f(r) = 1 - \frac{2GM}{r} = h(r)$$

Classical Singularities:

- $r = 0$: Infinite Tidal Forces, Breakdown of Gen. Rel.
- $r \equiv R_S = 2GM \quad (c = 1)$: Event Horizon, Infinite Blueshift, Change of sign of f, h

Trapping of light inside the horizon is what makes a black hole

BLACK

The $r = R_S$ singularity is purely kinematic, removable by a coordinate transformation iff $\hbar = 0$

And iff $T_{\mu\nu} = 0$ on the horizon

Black Holes and Entropy

- A fixed classical solution usually has no entropy :
  (What is the "entropy" of the Coulomb potential $\Phi = Q/r$?)
  … But if matter/radiation disappears into the black hole,
  what happens to its entropy? (Only M, J, Q remain)
- Horizon area A (which always increases) a kind of "entropy"?
  To get units of entropy need to divide A by (length)^2
  … But there is no fixed length scale in classical Gen. Rel.
- Planck length $\ell_{Pl}^2 = \hbar G/c^3$ involves $\hbar$
- Bekenstein suggested $S_{BH} = \gamma\, k_B\, A/L_{Pl}^2$ with $\gamma \sim O(1)$
- Hawking (1974) argued black holes emit thermal radiation at

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

Apparently then the classical Smarr relation $dE = \kappa\, dA/8\pi G$
becomes first law, $dE = T_H\, dS_{BH}$ fixes $\gamma = 1/4$
(multiply & divide by $\hbar$) *But …*

A few problems remained …

- Hawking Effect involves trans-Planckian frequencies
- $S_{BH} \propto A$ is *non-extensive* and *HUGE*
- In the classical limit $T_H \rightarrow 0$ (cold) but $S_{BH} \rightarrow \infty$ (?)
- $E \propto T^{-1}$ implies negative heat capacity

$$\frac{dE}{dT} << 0 \quad \Rightarrow \text{highly unstable}$$

Equilibrium Thermodynamics cannot be applied

- Information Paradox: Where does the information go?

(Pure states → Mixed States? *Unitarity* ?)

- What is the statistical interpretation of S_{BH} ?

Boltzmann asks: $S = k_B \ln W$??

Black 'Holes'… or Not

Black Holes believed 'inevitable' in General Relativity but

- Difficulties reconciling Black Holes with Quantum Mechanics
  - Hawking Temperature & the 'Trans-Planckian Problem'
  - Entropy & the Second Law of Thermodynamics
  - Negative Heat Capacity & the 'Information Paradox'
- Singularity Theorems assume Trapped Surface and Energy Conditions: Strong Energy Condition

$$\rho + \sum_{i=1}^{3} p_i \geq 0$$

Violated by Quantum Fields, e.g. by Casimir Effect & Hadron 'Bag', Cosmological Dark Energy, Inflation V(ϕ):

Negative Pressure $\boxed{p_i = -\rho < 0}$ Effective **Repulsion**

Static, Spherical Symmetry

- 2 Metric Fns.

$$f(r),\ \ h(r) \equiv 1 - \frac{2Gm(r)}{r}$$

← Misner-Sharp Mass

- 3 Stress Tensor Fns.

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p_{\perp} & 0 \\ 0 & 0 & 0 & p_{\perp} \end{pmatrix}$$

- 2 Einstein Eqs.

$$\frac{dm}{dr} = 4\pi\, r^2 \rho \qquad \frac{h}{2f}\frac{df}{dr} = \frac{Gm}{r^2} + 4\pi G p r$$

- 1 Conservation Eq.

$$\nabla_{\mu} T^{\mu}_{r} = \frac{dp}{dr} + \frac{\rho + p}{2f}\frac{df}{dr} + \frac{2\,(p - p_{\perp})}{r} = 0$$

Buchdahl Bound (1959)

Assuming classical Einstein eqs. &

- Static Killing time: $K^\mu \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial t}$
- Spherical Symmetry:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{h(r)} + r^2\,d\Omega^2$$

- Isotropic Pressure: $p_i = p(r)$
- Positive Monotonically Decreasing Density: $\frac{d\rho}{dr} \leq 0$
- Metric Continuity at Surface of Star r=R
- Then $R > \frac{9}{8} R_s = \frac{9}{4} GM$

or the pressure must diverge in the Interior
before horizon is reached

Schwarzschild Interior (1916)

- Importance of Buchdahl Bound is Something happens Before the Event Horizon is even Reached
- Bound is Saturated by Schwarzschild Interior Soln.
- Constant Density $\rho(r) = \bar{\rho} \equiv \dfrac{3M}{4\pi R^3}$
- Solve for Pressure *p(r)*:

$$p(r) + \bar{\rho} = 2\,\bar{\rho}\,\frac{\sqrt{1 - R_s/R}}{D}$$

$$f(r) = D^2/4 \quad h(r) = 1 - R_s r^2/R^3$$

$$D \equiv 3\sqrt{1 - R_s/R} - \sqrt{1 - R_s\, r^2/R^3}$$

Schwarzschild Interior

- **Constant Density** $m(r) = \frac{4\pi}{3}\bar{\rho} r^3 = \frac{M}{R^3} r^3$

$$\rho' = 0$$

$$h(r) = 1 - H^2 r^2$$

Saturates Buchdahl Bound

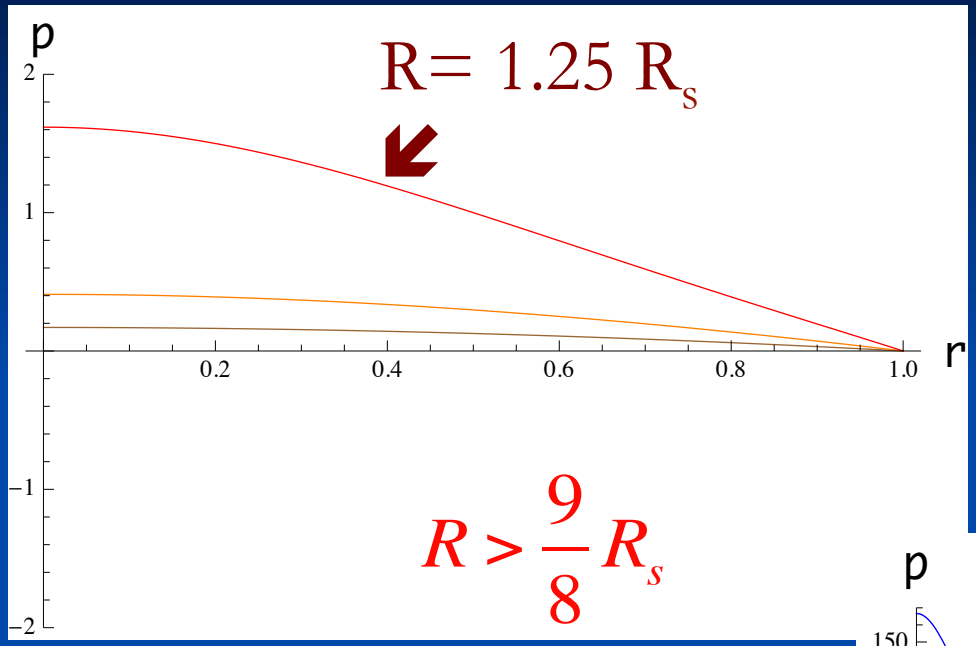
$$H^2 = \frac{8\pi G}{3}\bar{\rho} = \frac{2GM}{R^3}$$

- **Pressure** $p(r) = \bar{\rho}\left[\frac{\sqrt{1-H^2r^2} - \sqrt{1-H^2R^2}}{3\sqrt{1-H^2R^2} - \sqrt{1-H^2r^2}}\right]$

- **Diverges at** $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$ iff $R < \frac{9}{8}R_s = \frac{9}{4}GM$

- **Pressure becomes negative for** $0 < r < R_0$

Interior Pressure

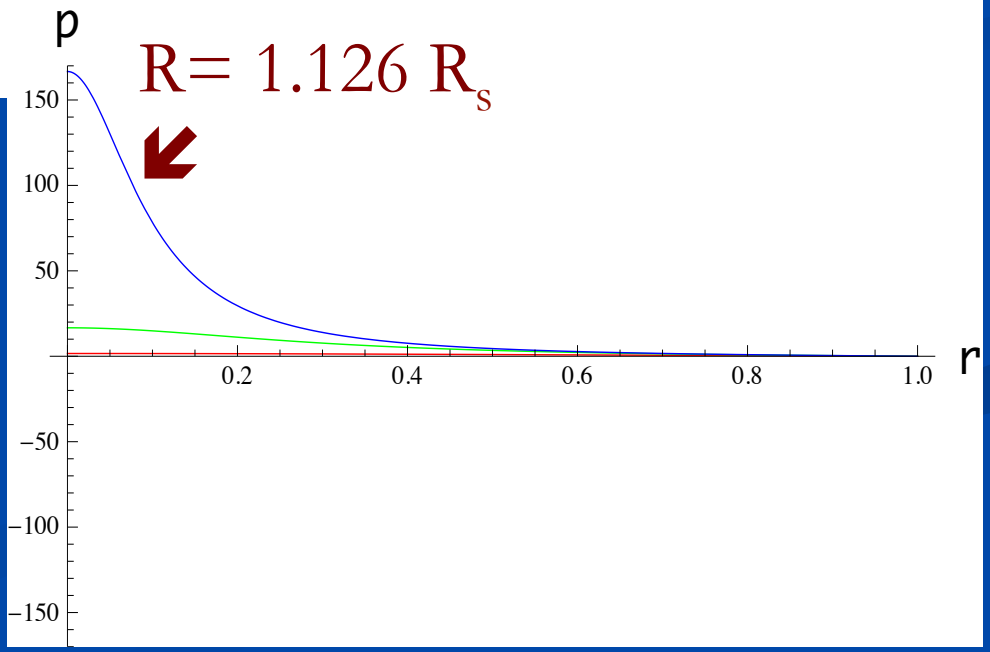


R= 1.25 R_s

$$R > \frac{9}{8} R_s$$

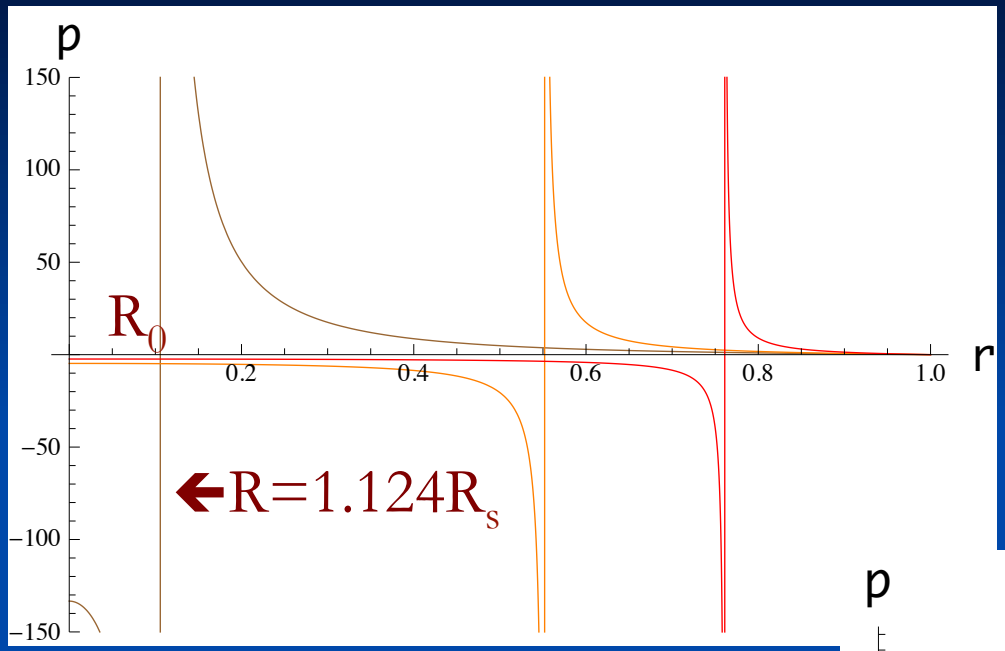
As R → 9/8 R_s from above

central pressure diverges



R= 1.126 R_s

Interior Pressure

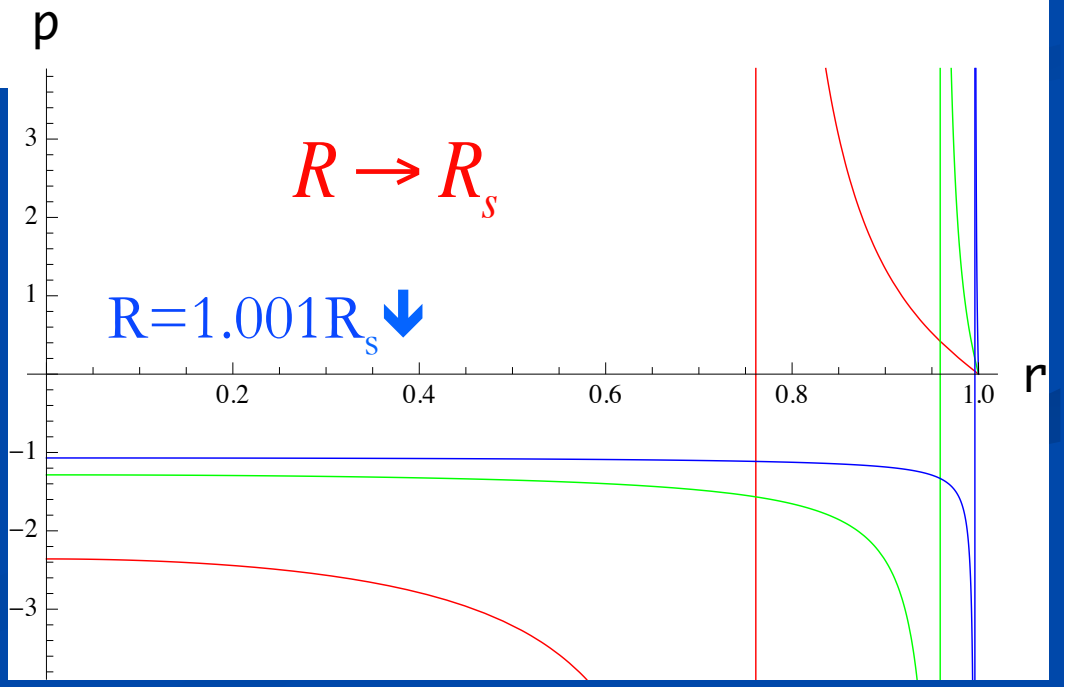


$$R < \frac{9}{8} R_s$$

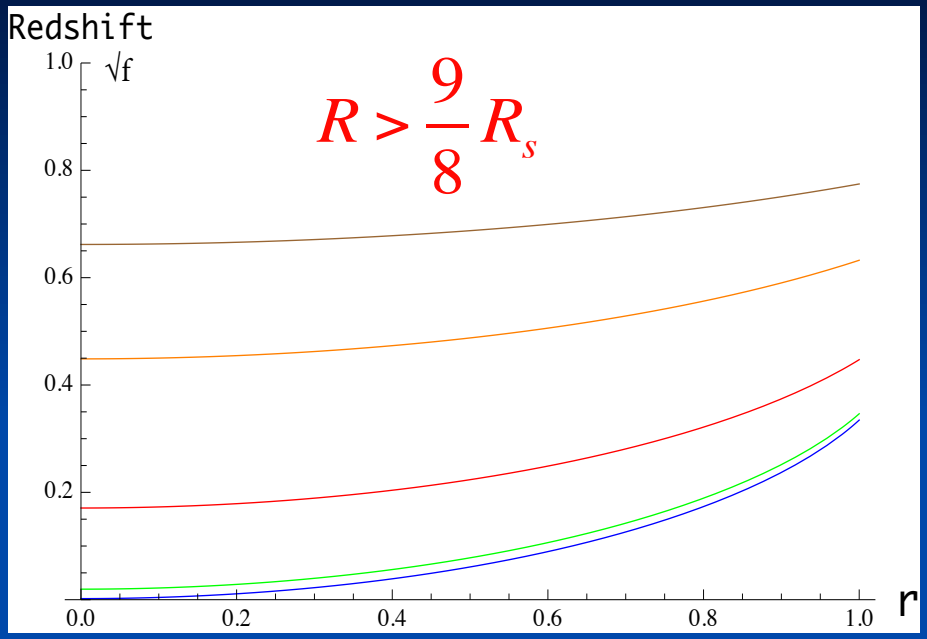
Negative Pressure soln. opens up for $R < R_0$

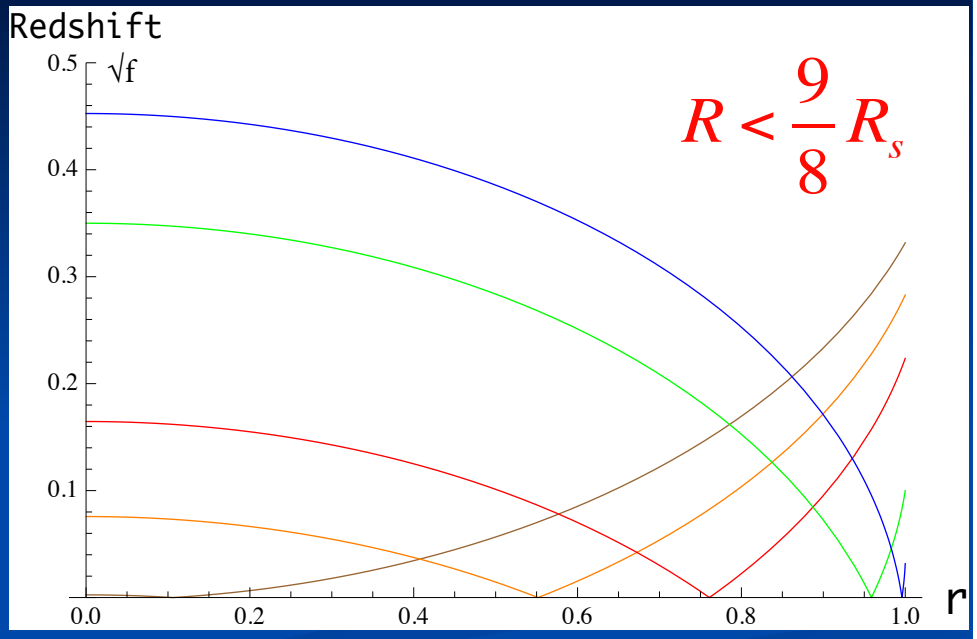
As $R \to R_s$ from above $R_0 \to R_s$ from below and negative pressure region fills entire interior with

$p = -\rho$, $w = -1$



Interior Redshift





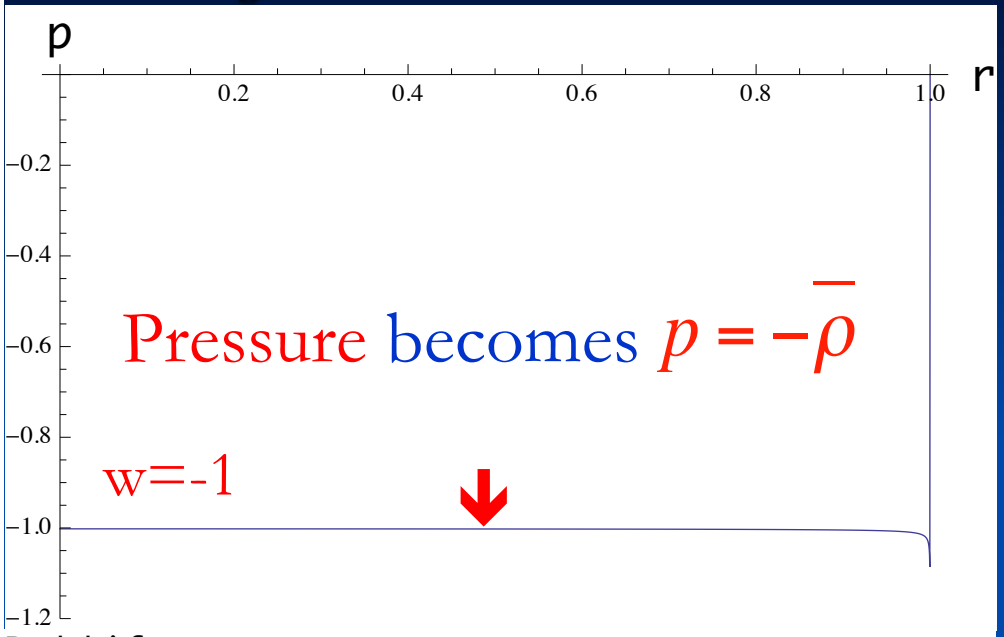
$$f(r) = \frac{1}{4} D^2 \geq 0$$ Non-negative (no horizon)

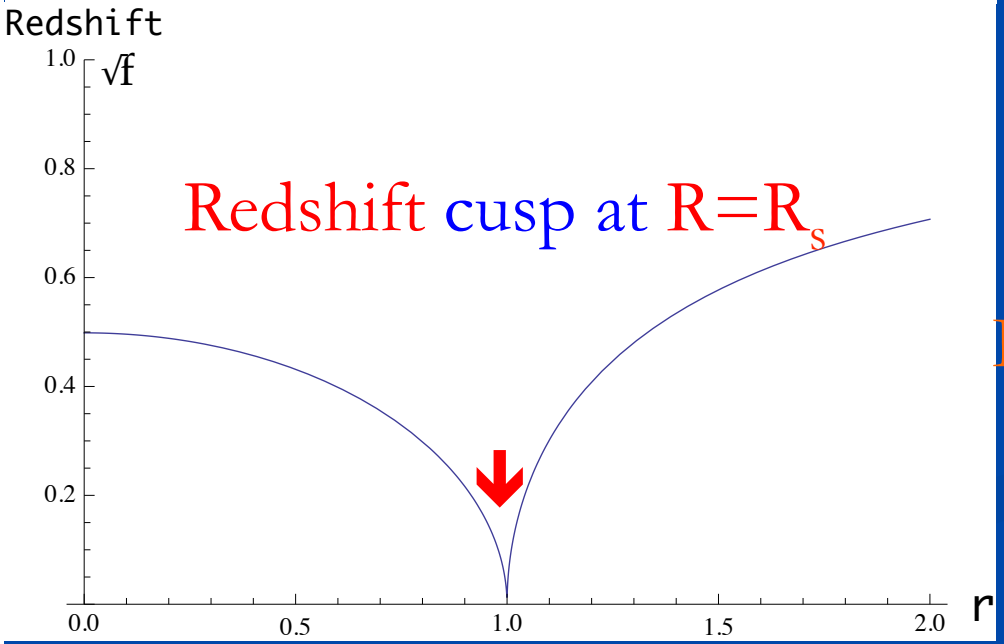
$$D \equiv 3\sqrt{1 - R_s/R} - \sqrt{1 - R_s\, r^2/R^3}$$

vanishes at same radius $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$ where p diverges

Redshift $\sqrt{f(r)} = \frac{1}{2}|D|$ has cusp-like behavior

$R=R_s$ Limit is Grav. Condensate Star (2001-4)





No divergence in p

$$p = -\bar{\rho}$$

$$h(r) = 1 - H^2 r^2$$

$$f(r) = \frac{1}{4}\, h(r)$$

$$H = 1/R_s$$

but non-analytic cusp

Discontinuity (classically)

Interior is de Sitter space in (modified) static coordinates

(Time runs slower inside)

Komar Mass-Energy Flux (1959-62)

$$\frac{1}{G}\frac{d}{dr}\left(r^2\kappa\right) = 4\pi\sqrt{\frac{f}{h}}\, r^2\left(\rho + p + 2p_\perp\right)$$

$$\kappa(r) = \frac{1}{2}\sqrt{\frac{h}{f}}\,\frac{df}{dr} \to \frac{GM}{r^2}$$ Surface Gravity

Total Mass: Compare Gauss' Law

$$M = 4\pi\int_0^{R_s} dr\,\sqrt{\frac{f}{h}}\, r^2\left(\rho + p + 2p_\perp\right)$$

Transverse Pressure

Cusp in Redshift produces Transverse Pressure

$$r\frac{d}{dr}\left[(p+\bar{\rho})f^{\frac{1}{2}}\right] = 2\,(p_{\perp}-p)f^{\frac{1}{2}}$$

Localized at $r = R_0$

$$8\pi\sqrt{\frac{f}{h}}\;r^2\,(p_{\perp}-p) = \frac{8\pi}{3}\,\bar{\rho}\,{R_0}^3\,\delta(r-R_0)$$

Integrable Surface Energy

$$E_s = \frac{8\pi}{3}\,\bar{\rho}\,{R_0}^3 = 2M\left(\frac{R_0}{R}\right)^3 \rightarrow 2M$$

$$\boxed{M = E_v + E_s}$$

Surface Tension

Discontinuity in Surface Gravities

$$\kappa_\pm \equiv \lim_{r \to R_0^\pm} \kappa(r) = \pm \frac{4\pi G}{3} \bar{\rho} R_0$$

$$\Delta\kappa \equiv \kappa_+ - \kappa_- = \frac{R_s R_0}{R^3} \to \frac{1}{R_s}$$

is (redshifted) surface tension

$$\boxed{\tau_s = \frac{E_s}{2A} = \frac{\Delta\kappa}{8\pi G}} \to \frac{1}{8\pi G R_s} > 0$$

Interior is not analytic continuation of exterior

First Law

Classical Mechanical Conservation of Energy

$$dM = dE_v + \tau_s \, dA$$

Gibbs Relation

$$p + \rho = s\,T + \mu\,N = 0$$

Schw. Interior Soln. in $R \to R_s$ Limit describes a **Zero Entropy/Zero Temperature Condensate**

Discontinuity in κ implies non-analytic behavior

No horizon, Truly Static, t is a Global Time

Surface Area is Surface Area not Entropy

Surface Gravity is Surface Tension not Temperature

Gravitational Vacuum Condensates

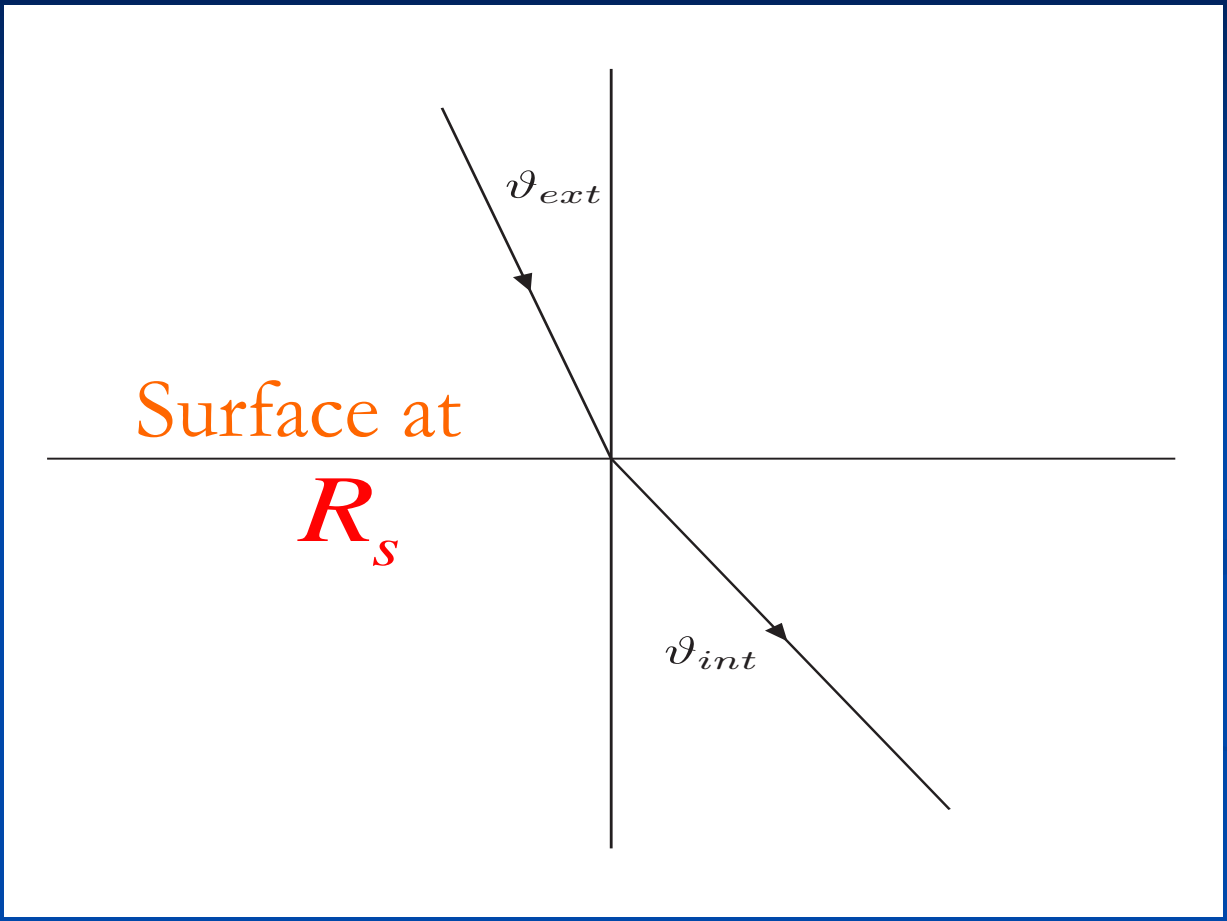
- Gravity is a theory of spin-2 bosons
- Its interactions are attractive
- The interactions become strong near $r = R_S$
- Energy of any scalar order parameter must couple to gravity with the vacuum eq. of state,
$$p_V = -\rho_V = -V(\phi)$$
- Relativistic Entropy Density s is (for $\mu = 0$),
$$Ts = p + \rho = 0 \text{ if } p = -\rho$$
- Zero entropy density for a single macroscopic quantum state, $k_B \ln \Omega = 0$ for $\Omega = 1$
- This eq. of state violates the energy condition, $\rho + 3p \geq 0$ (if $\rho_V > 0$) needed to prove the classical singularity theorems
- Dark Energy acts as a repulsive core

A GBEC phase transition can stabilize a high density, compact cold stellar remnant to further gravitational collapse

Predicted by EFT of Quantum Conformal Anomaly

Scalar Mode in Gravity—Realized in Schwarzschild Soln.

Refraction of Null Rays at Surface



Solve Geodesic Eq.

➔ Snell's Law

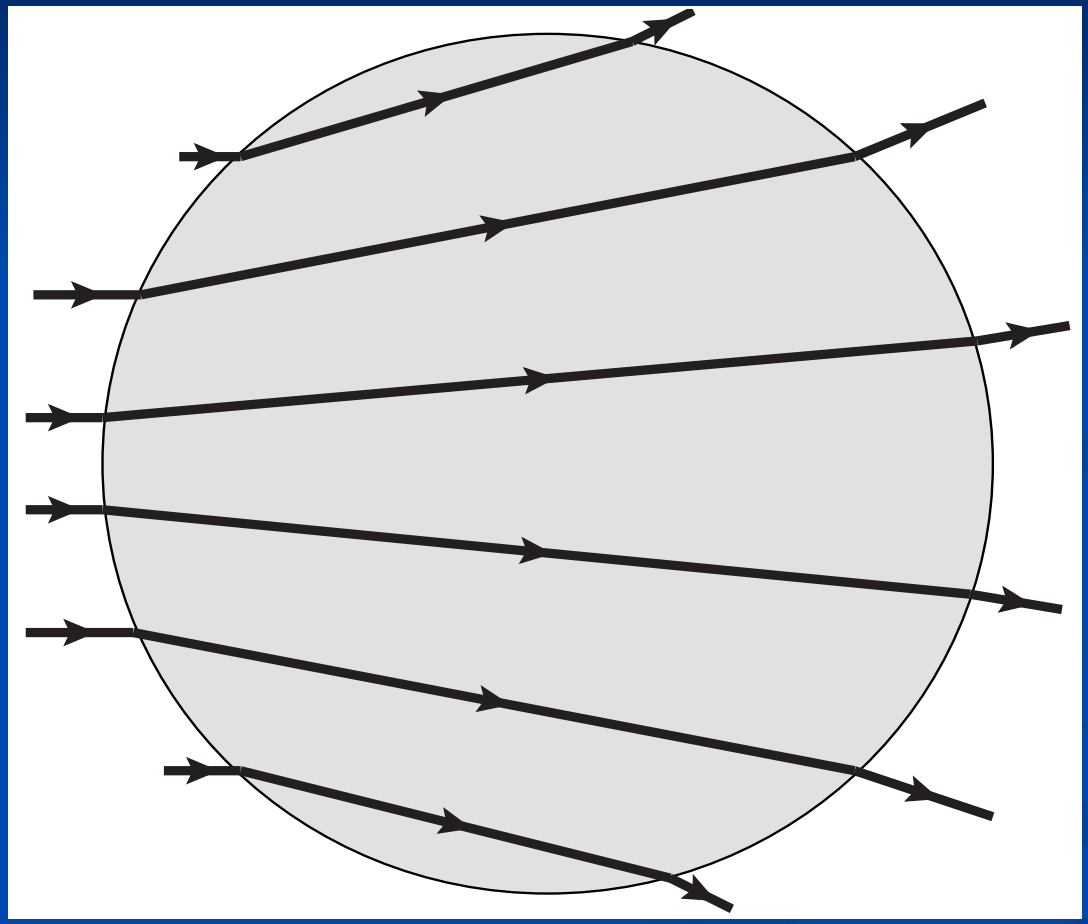
n < 1

Impact Parameter b

$$\sin\vartheta_{ext}\sqrt{1+\frac{4R_s^2}{b^2}} = \sin\vartheta_{int}\sqrt{1+\frac{R_s^2}{b^2}}$$

Defocusing of Null Rays

No Horizon ➔ Light Rays Penetrate Interior



Completely Different Imaging from a Black Hole

Surface Oscillations

$$dM = dE_v + \tau_s \, dA$$

- Energy Minimized by minimizing A for fixed Volume
- Surface Tension acts as a restoring force
- Surface Oscillations are **Stable**
- Surface Normal Modes are **Discrete**
- Characteristic Frequency

$$\omega \sim \frac{c}{R_s} = 101.5 \left(\frac{M_\odot}{M}\right) \text{ kHz}$$

- **Discrete Gravitational Wave Spectrum**
- Striking Signature for LIGO/VIRGO for

$$M \sim 10^{2-3} M_\odot$$

Summary

- Buchdahl Bound ➔ Interior Pressure Divergence Develops **before** Event Horizon Forms for $R > \frac{9}{8} R_s = \frac{9}{4} GM$
- Constant Density Interior Schwarzschild Solution
  Saturates Bound & illustrates the generic behavior
- Infinite Redshift at the Central Pressure Divergence
- Pressure Singularity is **Integrable**
- Implies Formation of a **δ-fn.** Surface & **Surface Tension**
- & a Non-Singular (de Sitter $p = -\rho$ 'BH' Interior
- Area term is **Mechanical Surface Energy not Entropy**
- QM, Unitarity ✓ **No 'Information Paradox'**
- **Condensate Star** $p = -\rho$ **negative pressure**
  already realized/inherent in Classical General Relativity

Observations are Coming

- High resolution sub-mm **Very Large Baseline Imaging** (VLBI) will zero in on event horizon of black hole Sgr A* (or gravastar surface) in the center of our galaxy
- Maxima of X-ray Continuum Thermal Spectra from Accretion Disk may be able to determine **Innermost Stable Circular Orbit** (ISCO) of candidate black holes
- X-Ray Fe Line Spectra Doppler Shifts will allow measurement of velocities and **test rotating Kerr solution, no-hair theorem** in external geometry
- **Gravitational Waves** expected first detection by Advanced LIGO II will observe inspiral and black hole merger events
- **Millisec. Pulsar Timing Arrays** also may detect **GW's**

Testing BH or Surface Physics is getting closer

Gravitational Vacuum Condensate Stars

Gravastars as Astrophysical Objects

- Cold, Dark, Compact, Arbitrary M, J
- Accrete Matter just like a black hole
- But matter does not disappear down a 'hole'
- Relativistic Surface Layer can re-emit radiation
- Can support Electric Currents, Large Magnetic Fields
- Possibly more efficient central engine for Gamma Ray Bursters, Jets, UHE Cosmic Rays
- Formation should be a violent phase transition converting gravitational energy and baryons into HE leptons and entropy
- Interior could be completely non-singular **dynamical condensate**
- Dark Energy as Condensate -- Finite Size effect of boundary conditions at the horizon ➔ Implications for Cosmology

- Gravitational Condensate Stars resolve all 'black hole' paradoxes
- **Discrete Grav. Wave Signatures of Surface Modes**
- **Astrophysics** of gravastars & 'no-hair' **testable** by mm VLBI, X-rays, ISCO's, GWaves in this decade (Sgr A* mm **imaging** by Event Horizon Telescope)

Spacetime Odyssey to discover
what spacetime is continues
BH's and Dark Energy are the keys