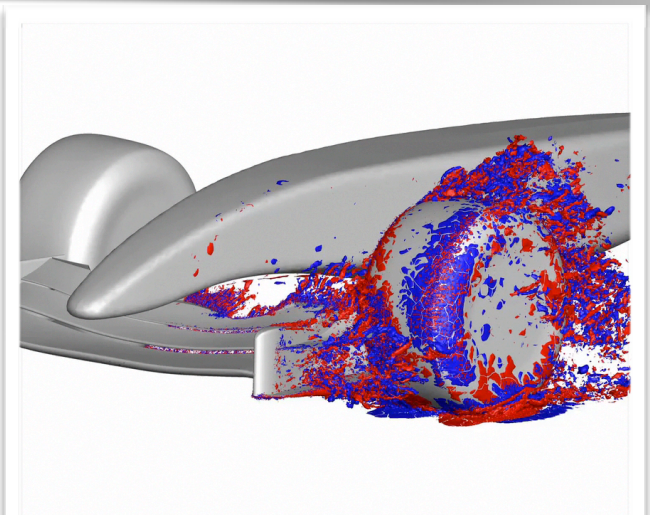
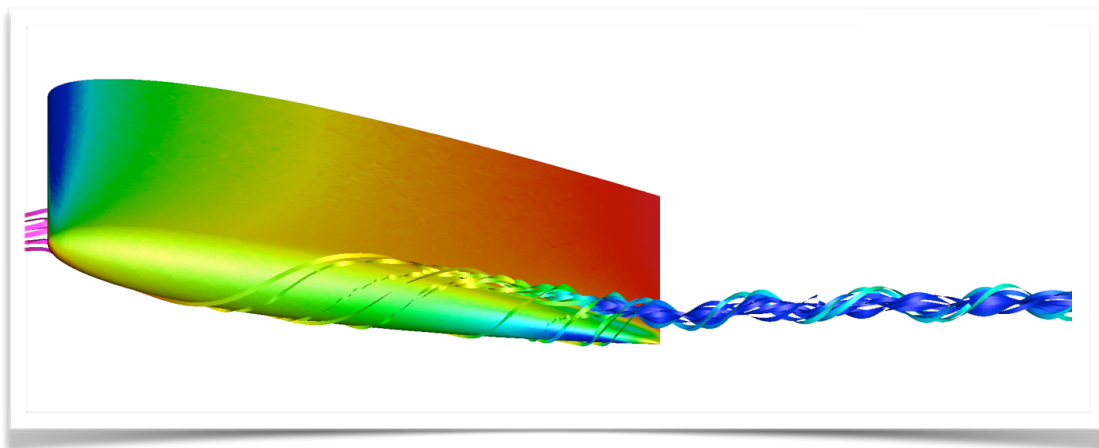
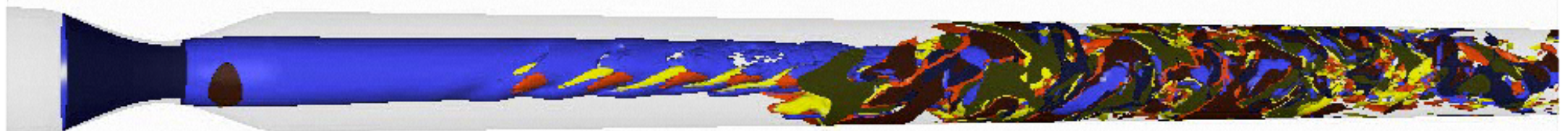


Spectral/hp element, scale resolving modelling for high Reynolds number F1 Aerodynamics

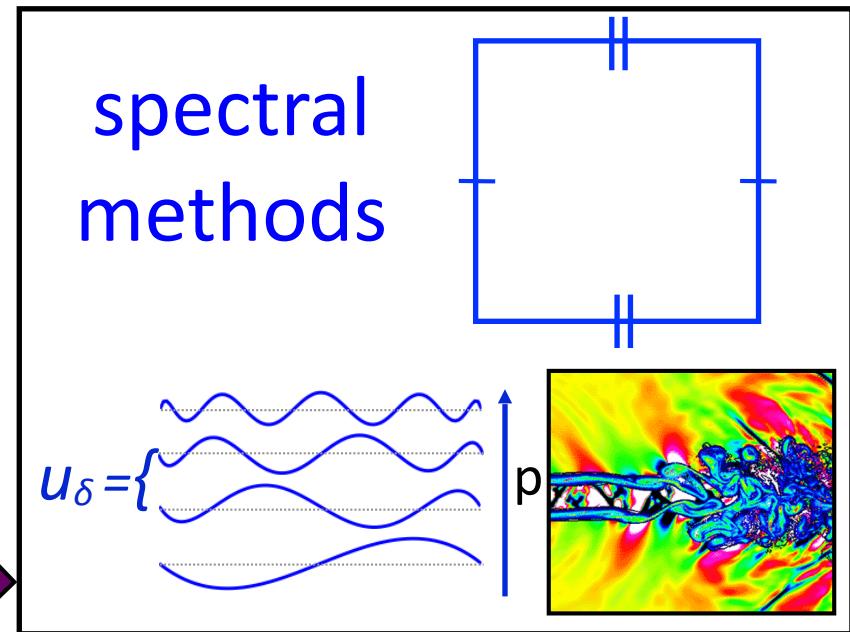
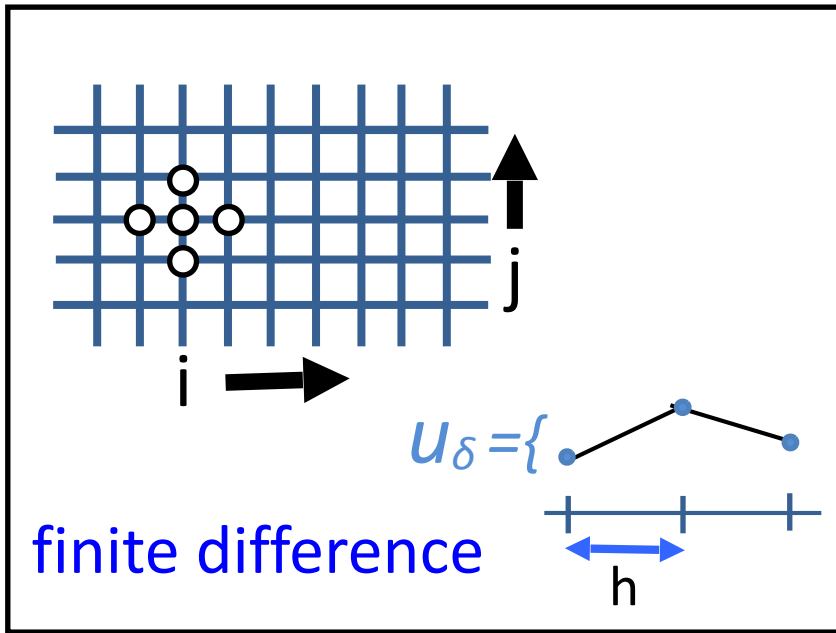
Spencer Sherwin

McLaren Racing/Royal Academy of Engineering Research Chair

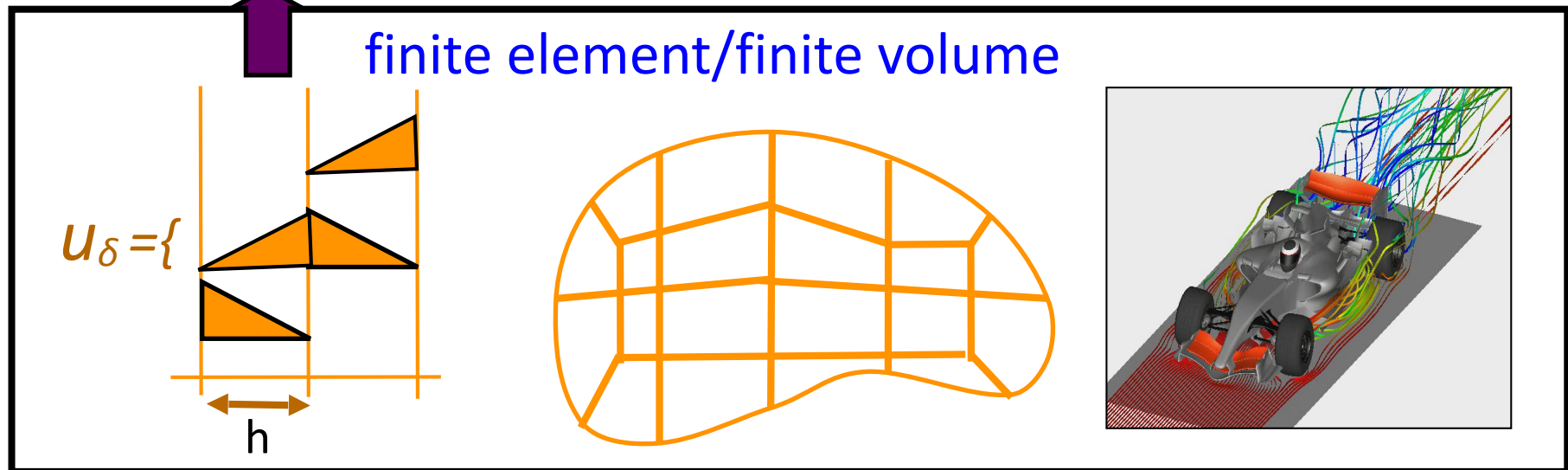


Outline

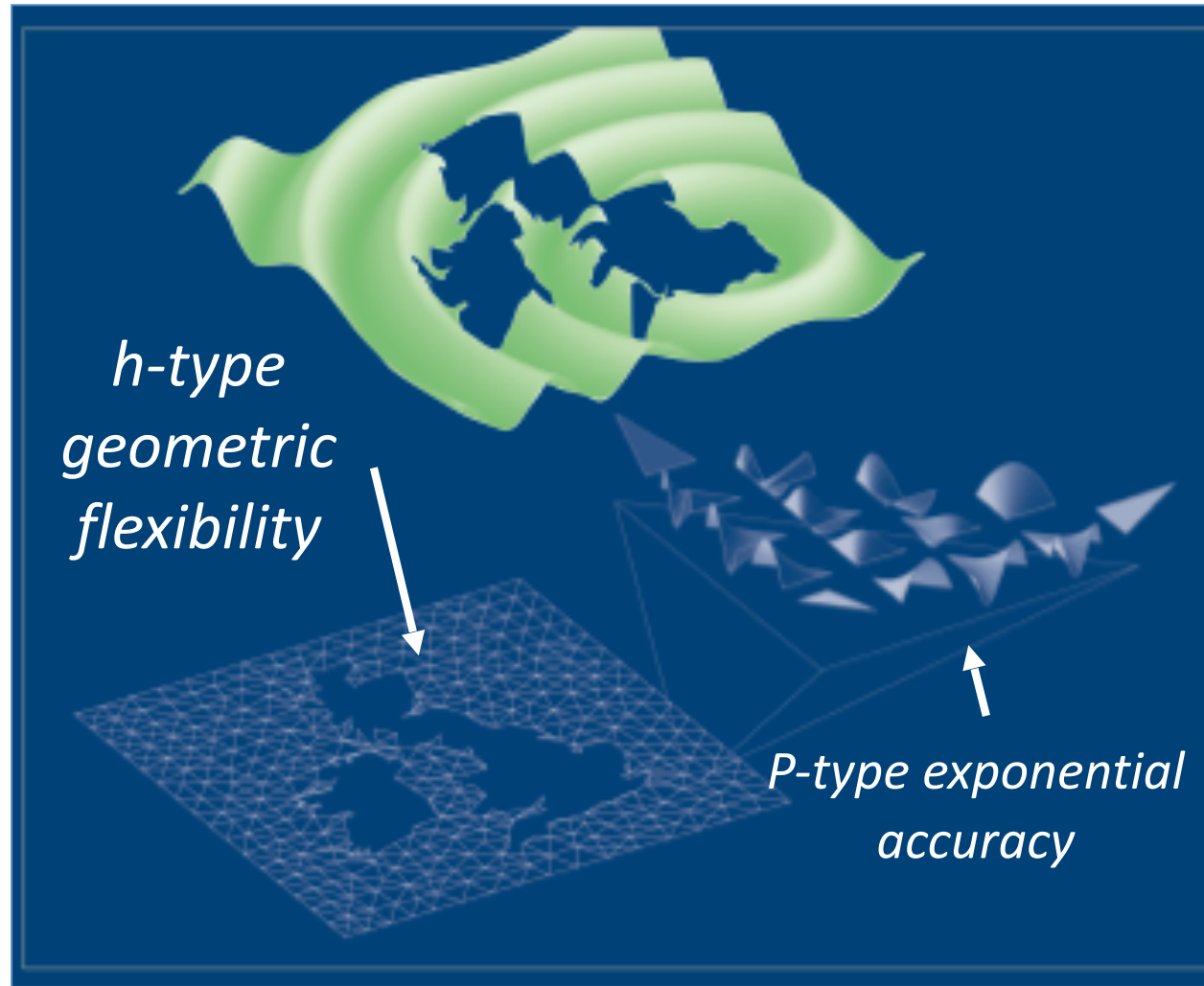
- Spectral/hp element methodology
- Motorsport in the UK
- Scale resolving simulations
 - High order meshing
 - Nodal/collocation space dealiasing
 - SW Smoothing



Spectral/hp methods



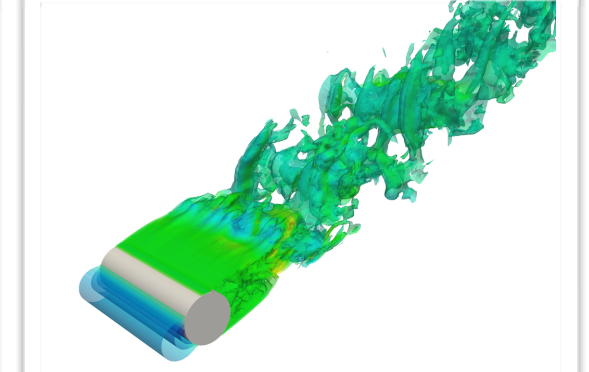
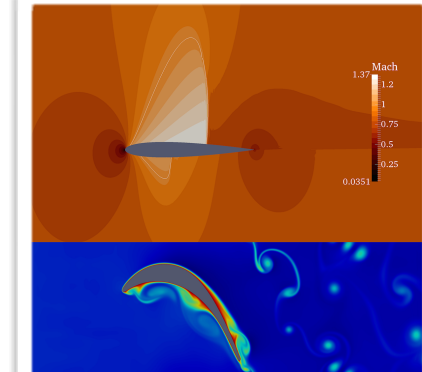
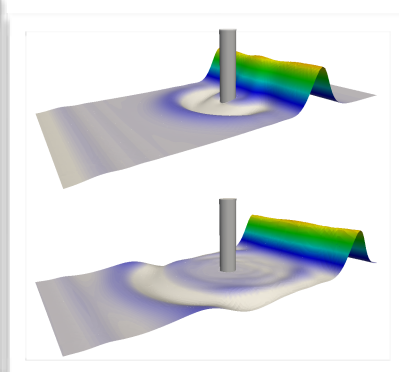
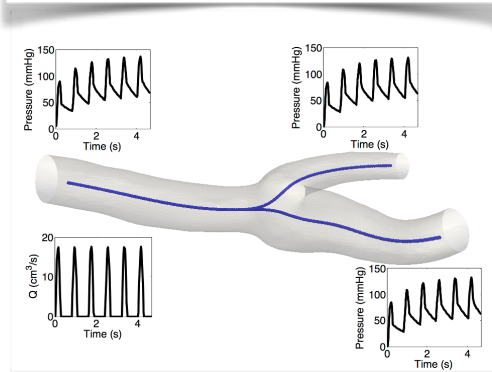
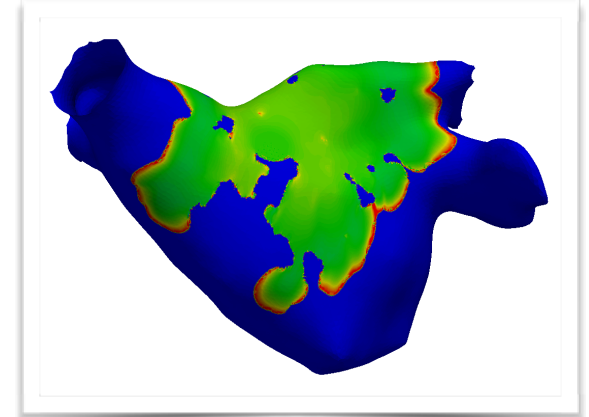
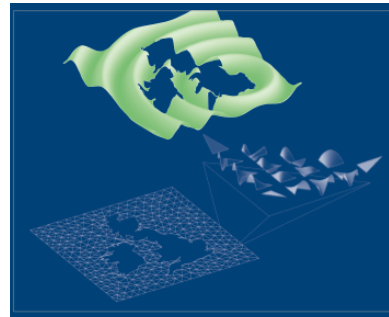
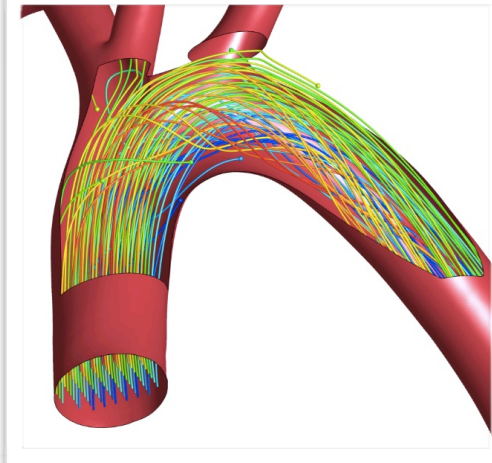
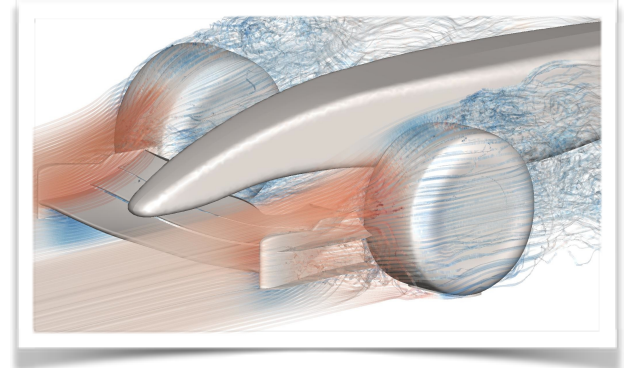
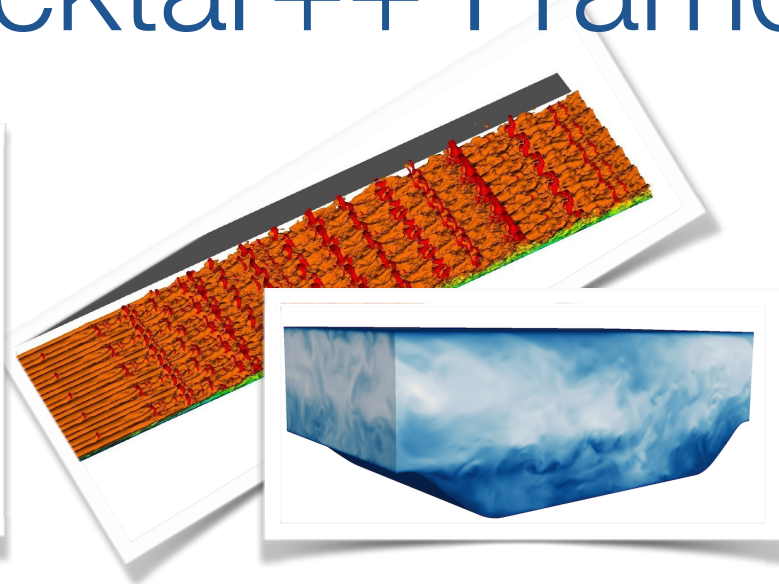
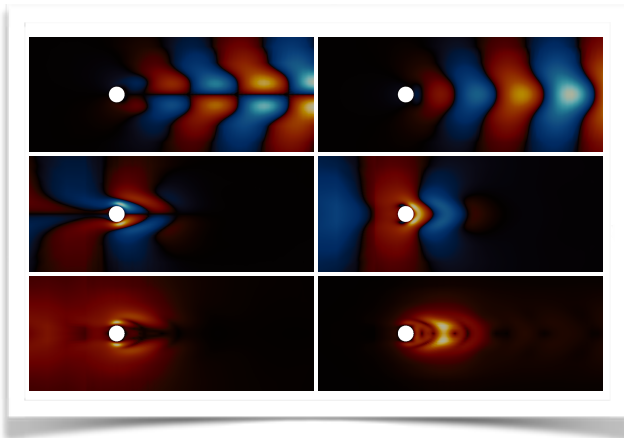
Spectral/hp element method



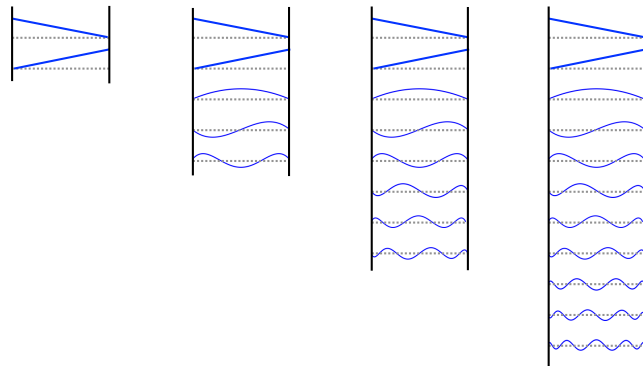
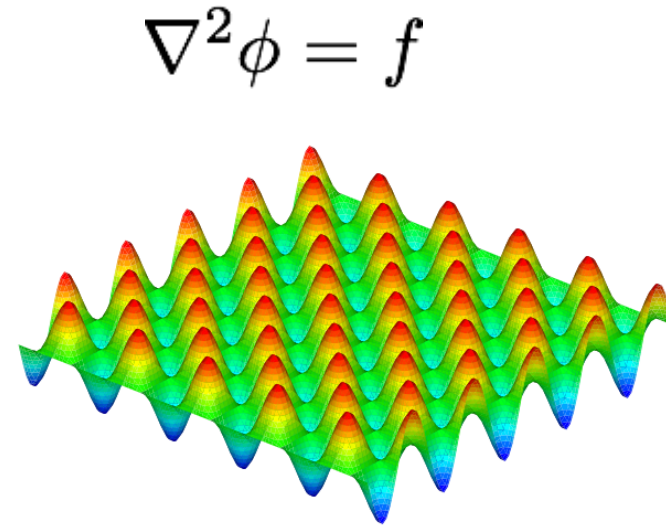
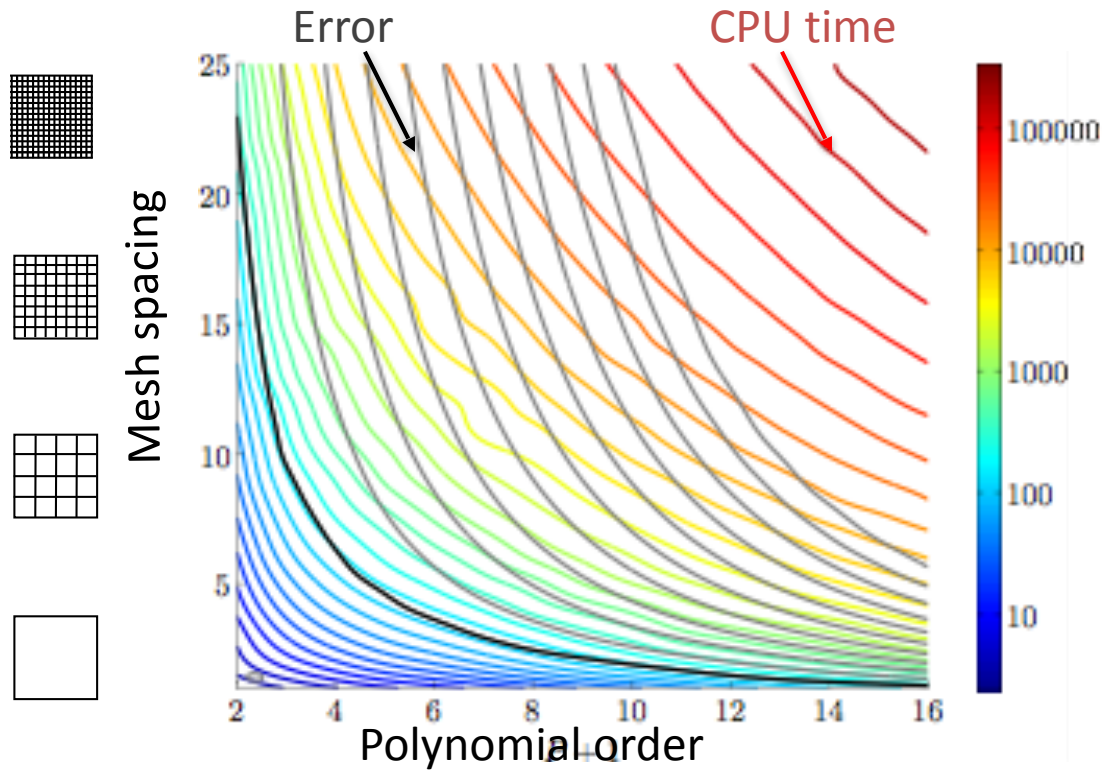
Computational cost $\sim P^4$

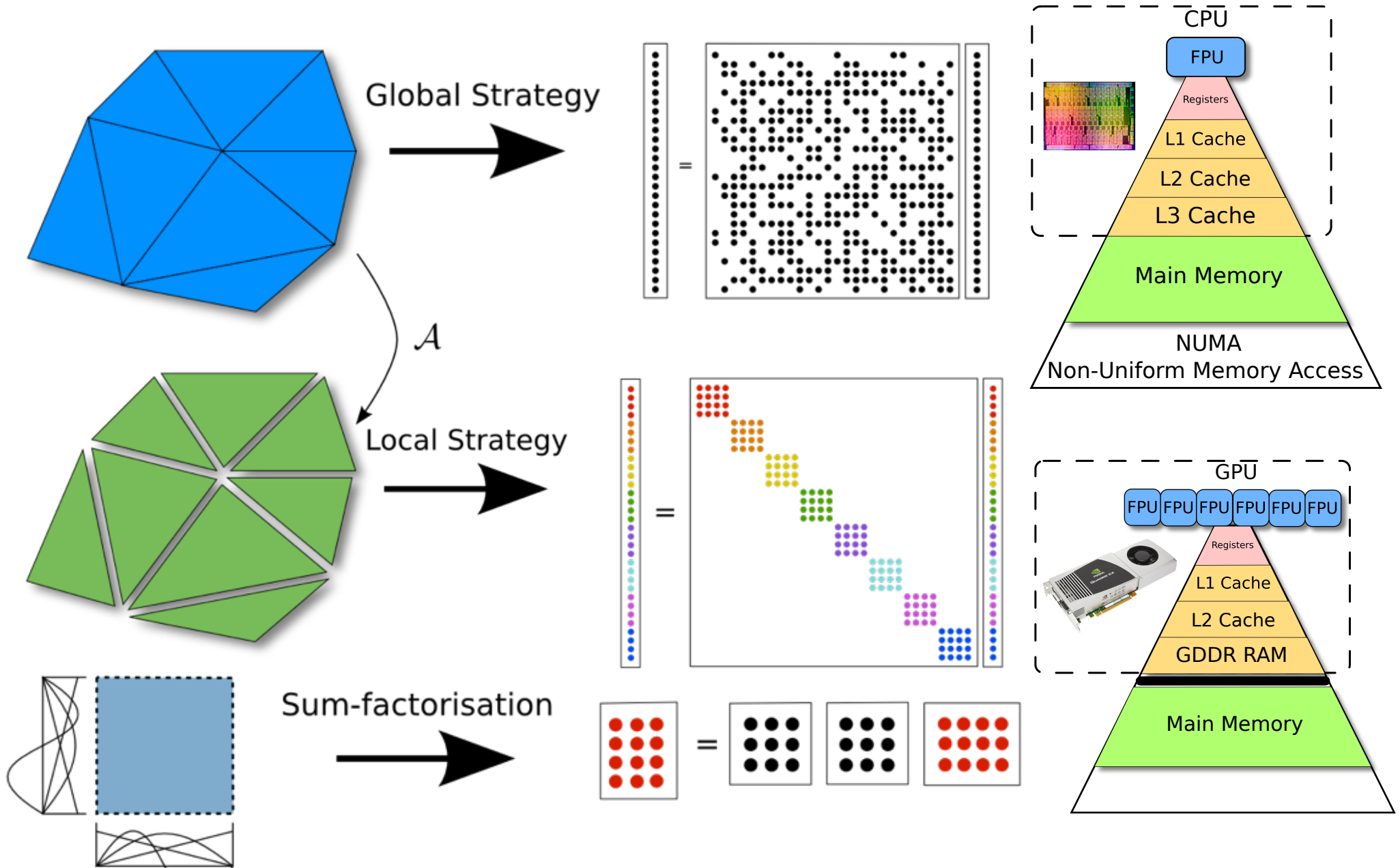
Computational error $\sim h^P$

Nektar++ Framework

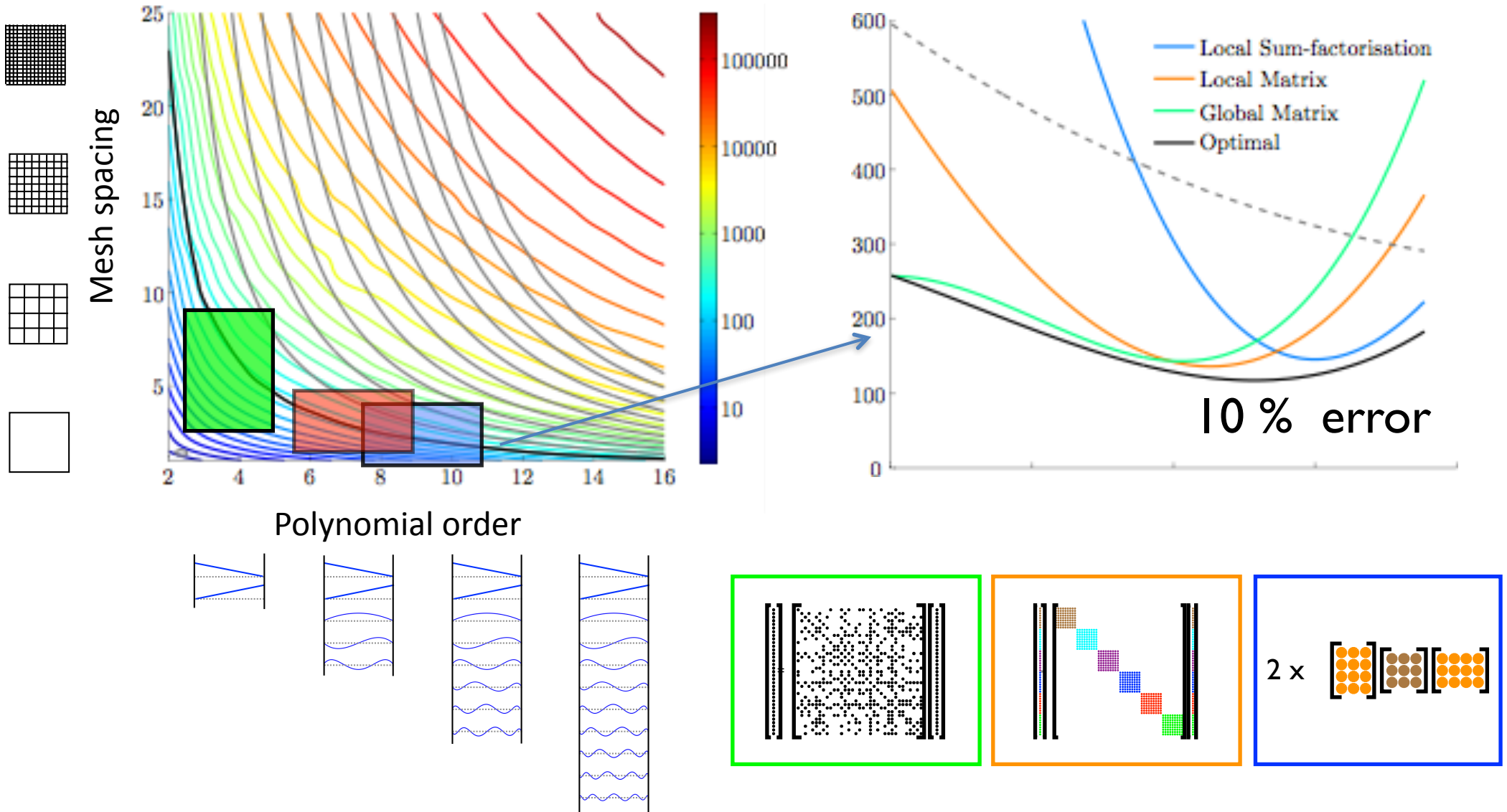


So what are the benefits?



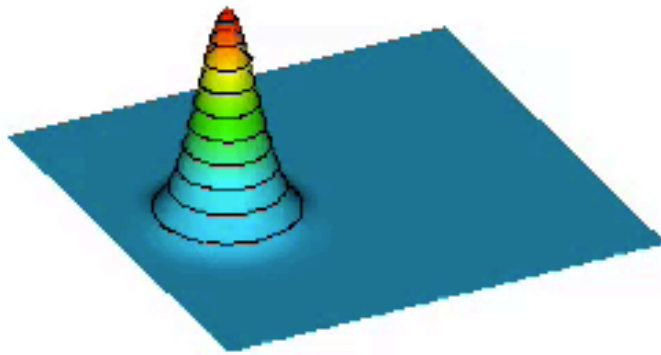


So what are the benefits?

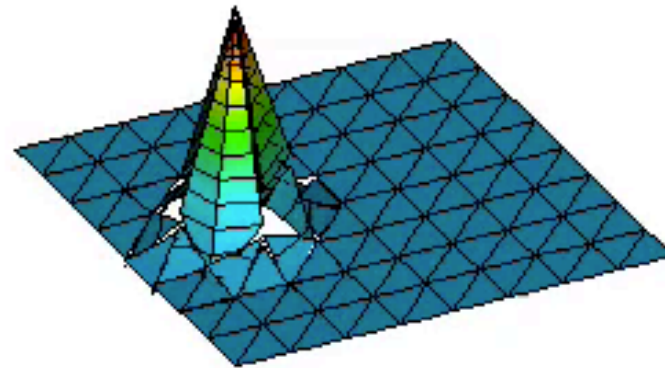


To P or not to P?

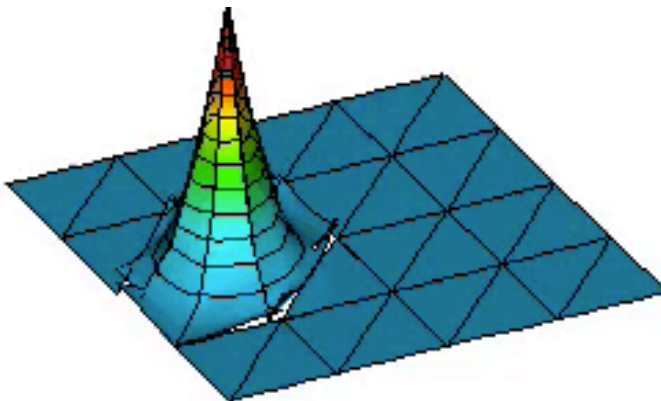
'Exact' solution



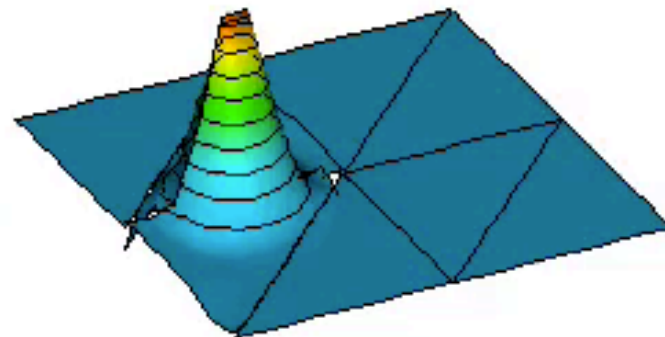
Nel=128, P=1



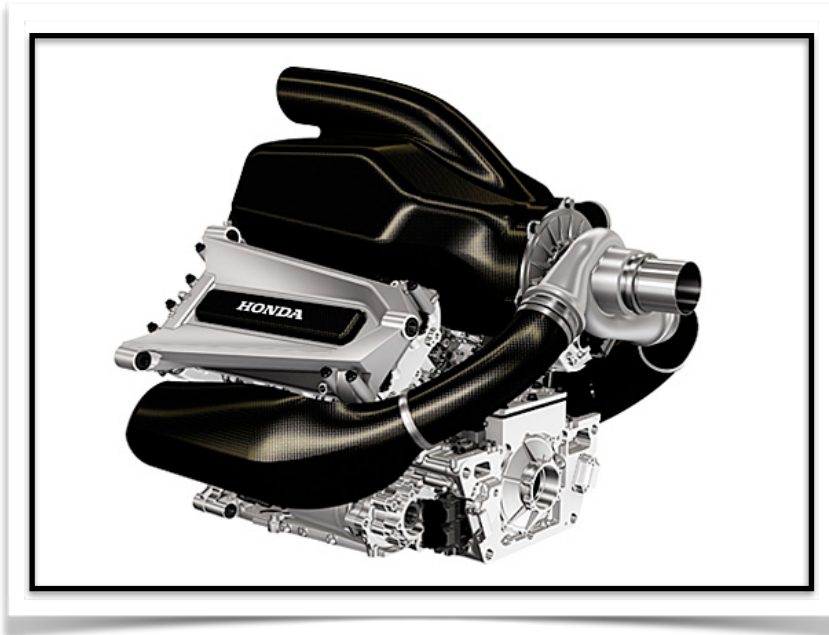
Nel=32, P=3



Nel=8, P=8



Motorsport in the UK



- 4500 companies involved in UK Motorsport and Performance Engineering sector
- 40,000 employees of which 25,000 are qualified engineers
- Annual £6 billion per annum turnover - £3.6 billion of which exported
- On average spend 30% of turnover on R&D
(compared to 15% for pharmaceutical industry)

Formula One



- Pinnacle of Motor sport
- 20 races in a year in 5 continents
- Complex set of Technical and Sporting Regulations
- 6 of 9 (8 of 11) teams are UK based winning 39 world championships since 1950
- UK F1 teams and their supply chains employ more than 5,000 people generating over £2bn in annual revenue

F1 Performance Figures



- Top Speed: >350 km/h
- Acceleration: 0 - 100 km/h: 2.3s
0 - 200 km/h: 5.0s
- Braking: 330 -100 km/h: 3.2s
'g' force 4.8g
distance 169m
- Corner forces: $\pm 4.5g$ in 0.75s
- Car weight: 691 kg (including driver & no fuel)
- Downforce: >1500 kg at 250 km/h
- Gear Changes: Number: 3300 per race
Duration: under 20ms
- Full Throttle: 70% of the time
78% of the distance



15,000 unique parts

Competitive differentiation is measured in milliseconds

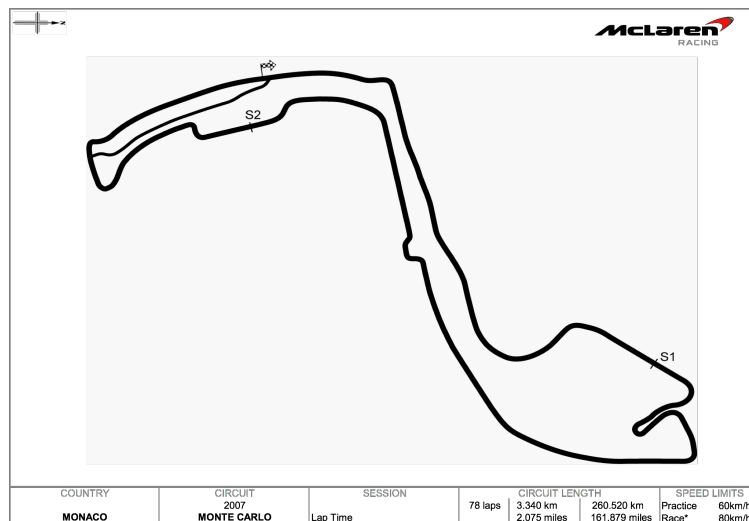
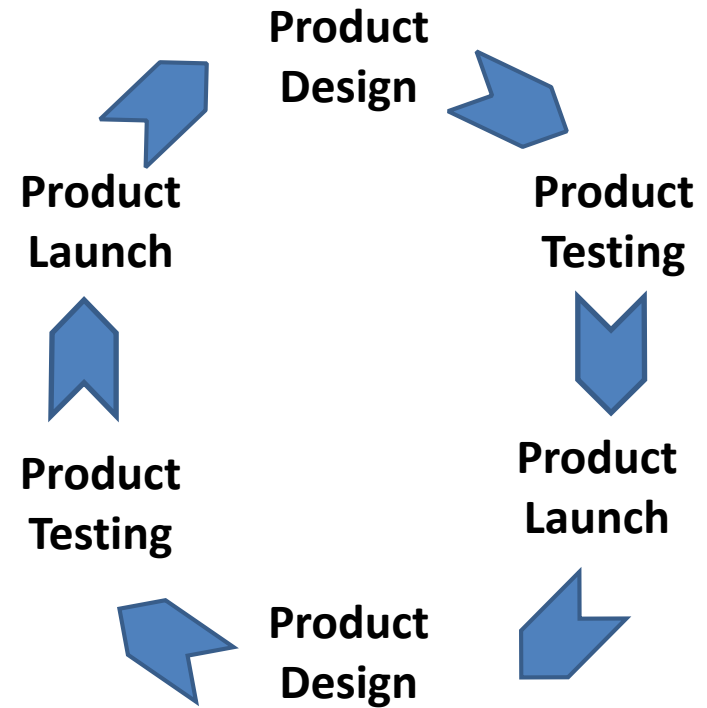
Only 2.0% performance gap between top 10 cars

Only 0.3% in top 3 cars

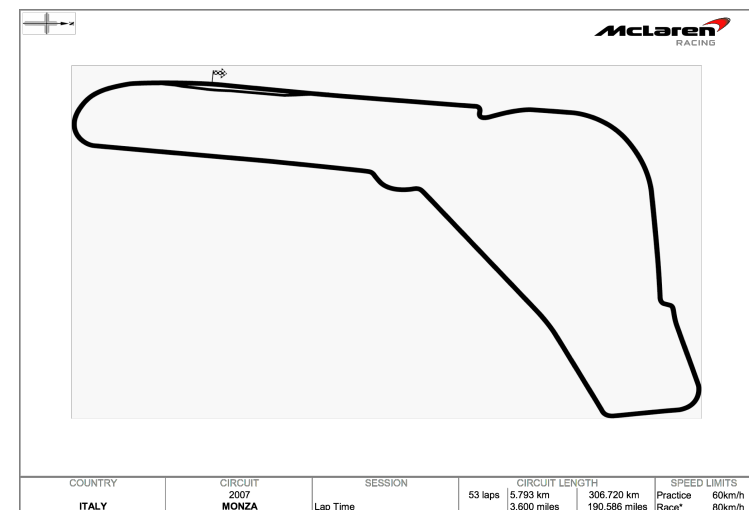
A Competitive Environment



- 6-12 month lead time to design a new car
- 12 days of testing before race 1
- Continuous product development
- Car changes every race
- Car aerodynamics optimised for each circuit



Monte Carlo



Monza

The Overall Challenge

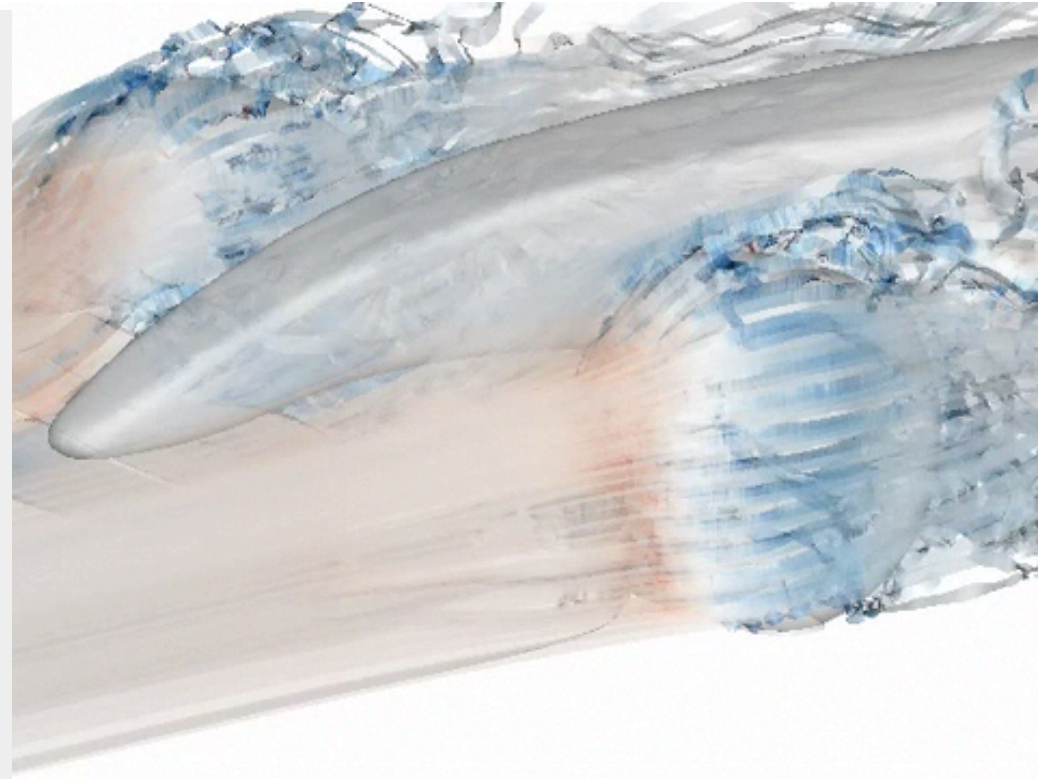
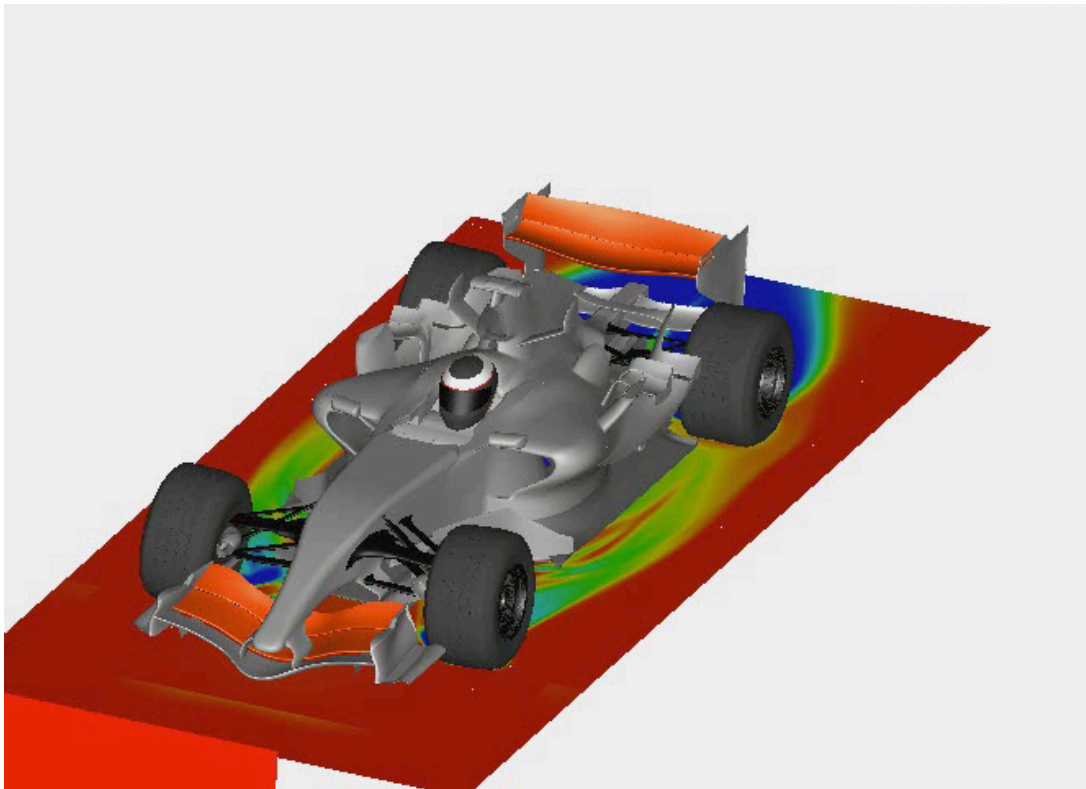


Through corner simulation requires:

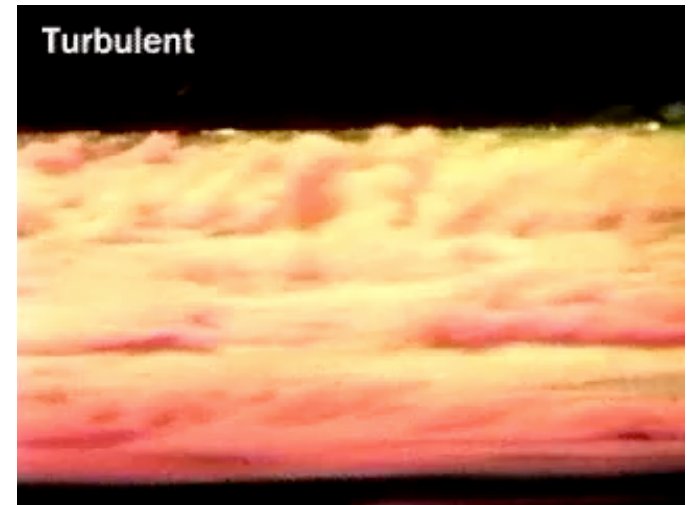
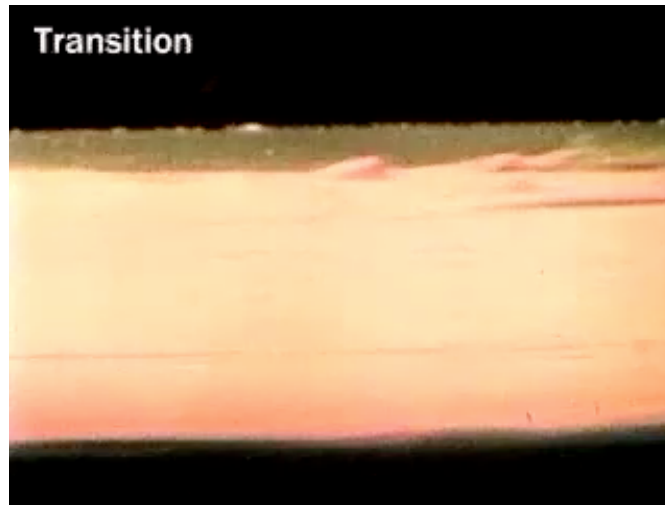
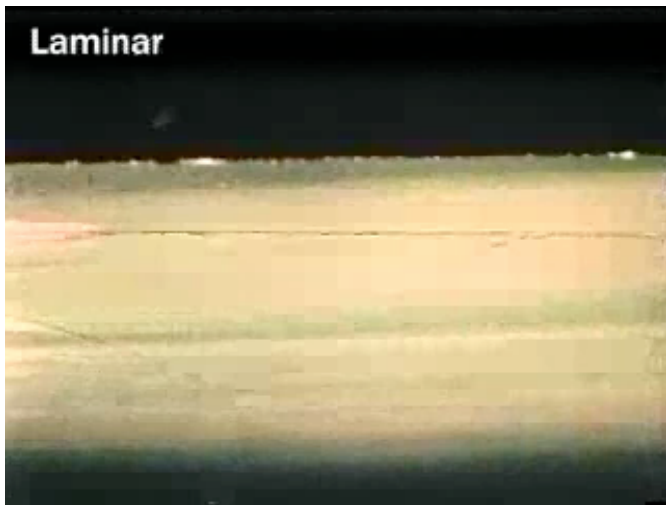
- *Roll, yaw, steer changes*
- *Ride height variation*



Transient flow modelling



Transient flow modelling



Multimedia fluid mechanics, Cambridge University Press,

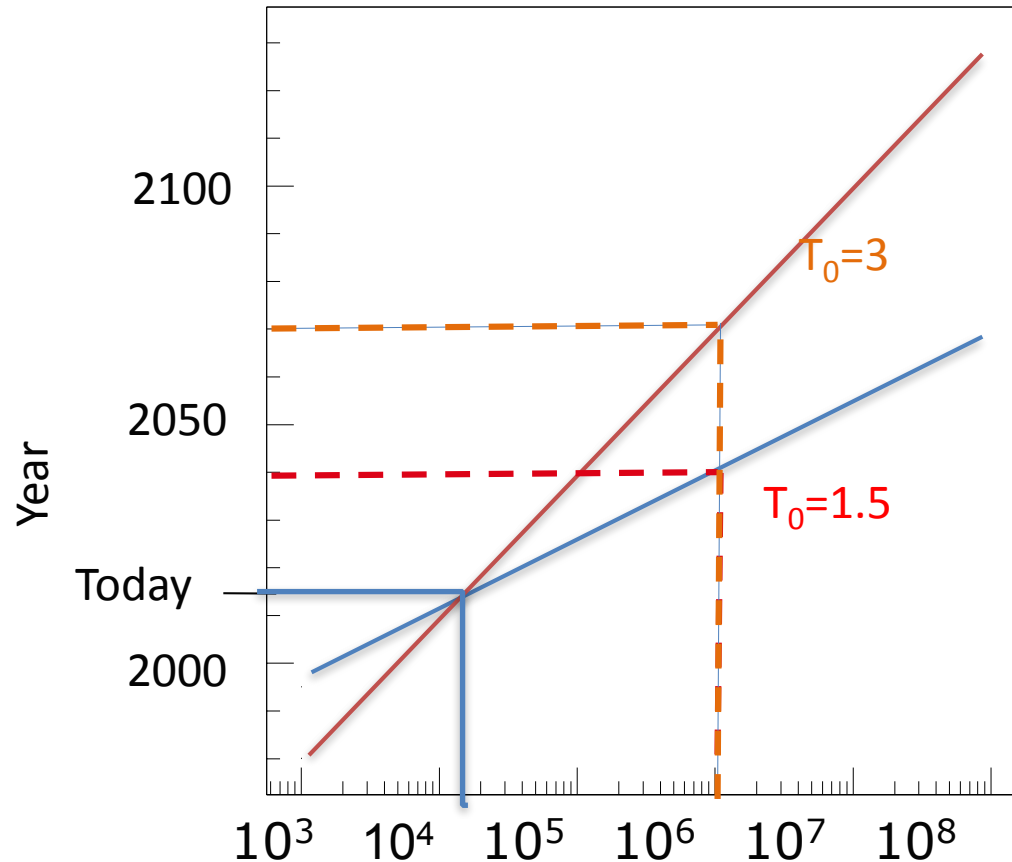
10^3 10^4 10^5 10^6 10^7 10^8



Reynolds number



Transient flow modelling



Flops required for
Reynolds No:

$$flops \propto Re^3$$

Moore's Law

$$\frac{flops_t}{flops_{T_0}} = 2^{t/T_0}$$



Reynolds number

Spectral/hp for high Reynolds number applications

- High order meshing
- Nodal/collocation space dealiasing
- SVV Smoothing

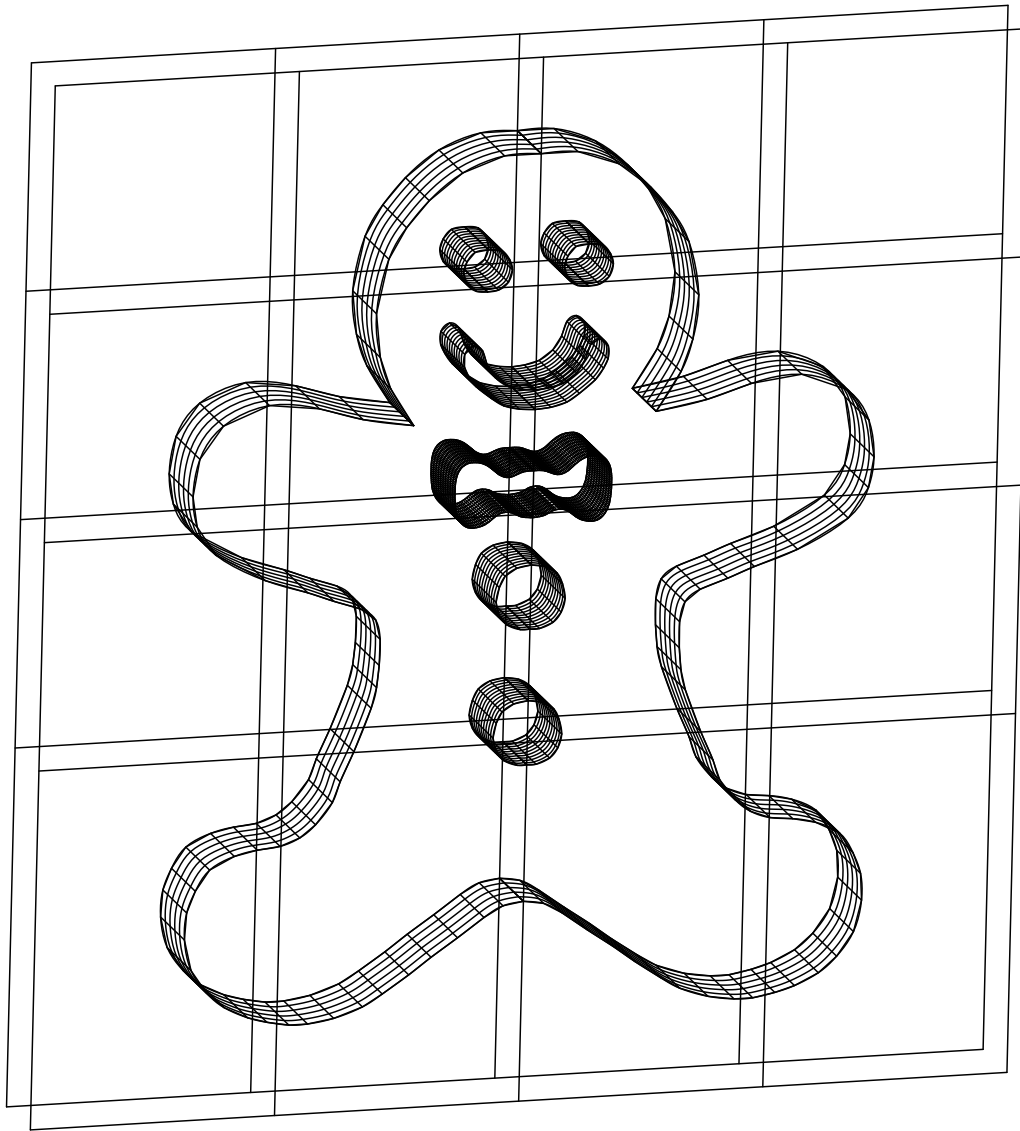
High order Mesh Generation

or

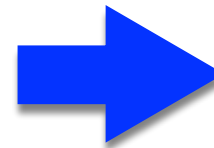
Mesh Modifications for High Order Methods

Dave Moxey & Joaquim Peiro

High-order mesh generation (1)

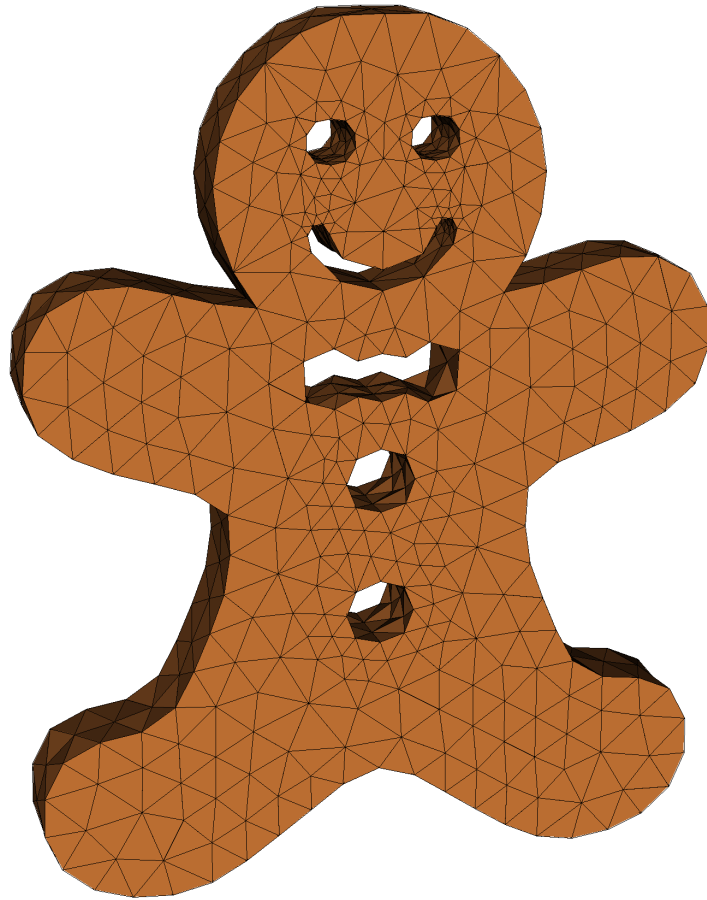


B-Rep

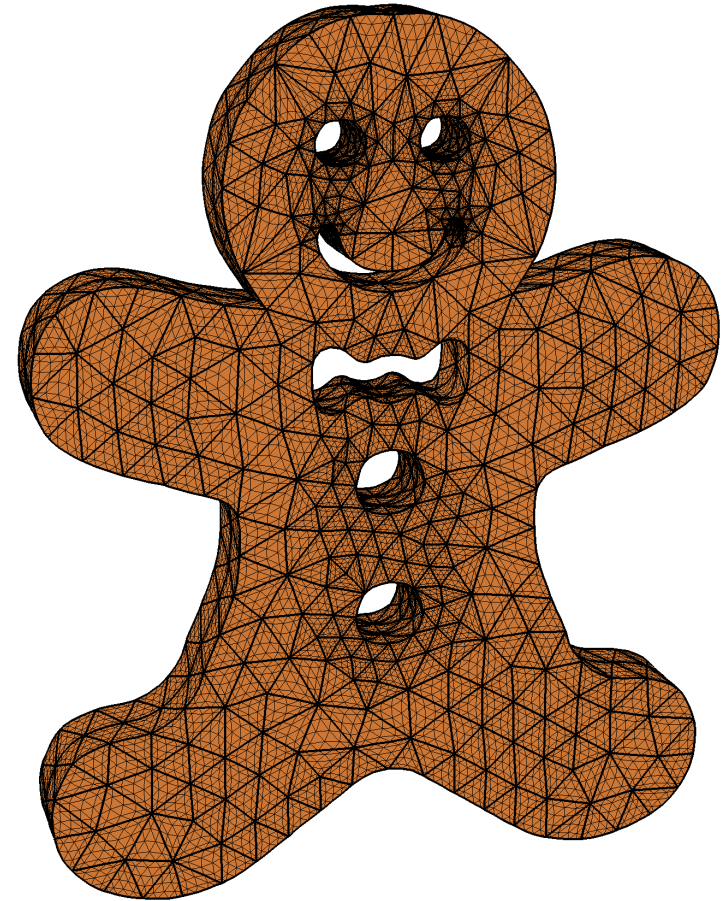
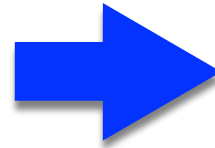


Linear mesh

High-order mesh generation (2)

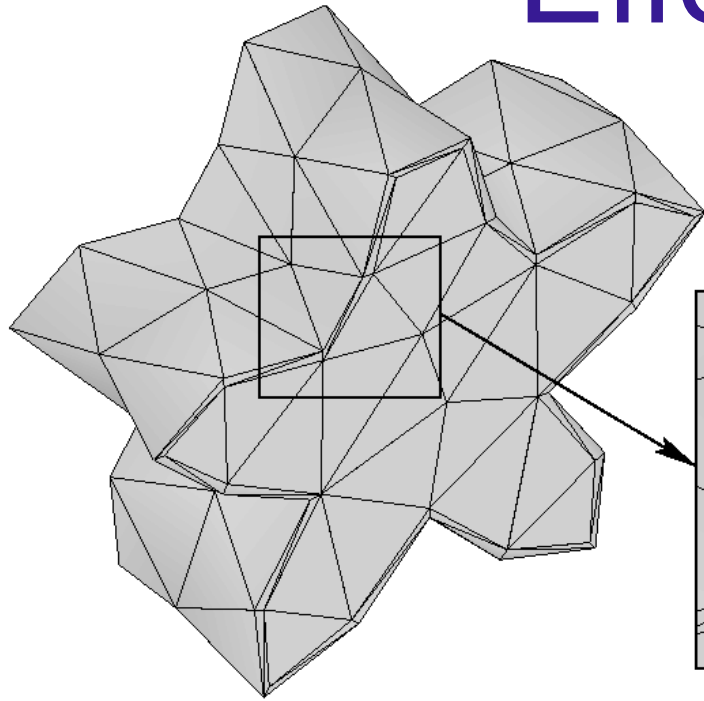


Linear mesh

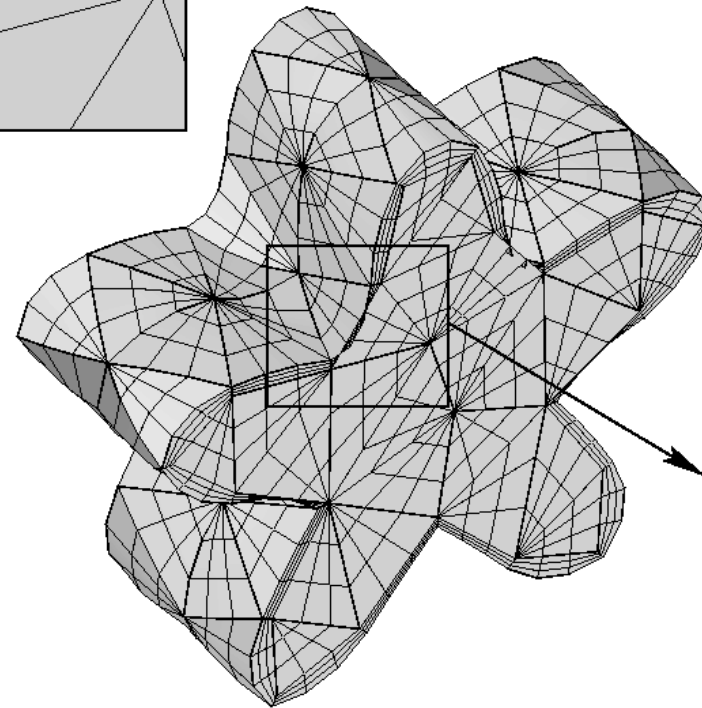
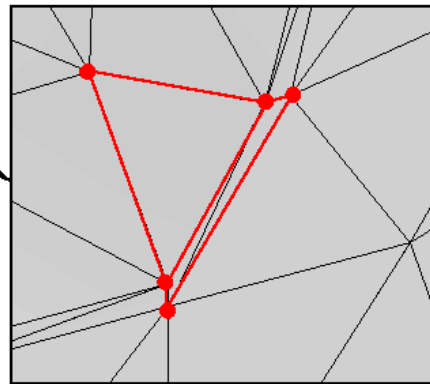


High-order mesh

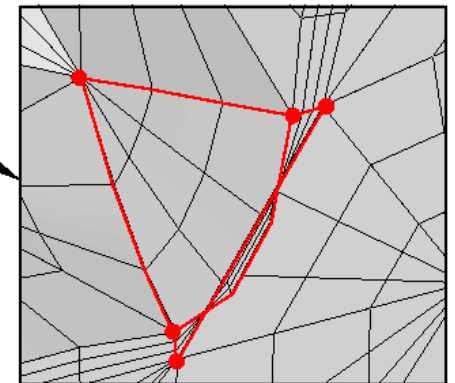
Effect of curvature



VALID ELEMENT



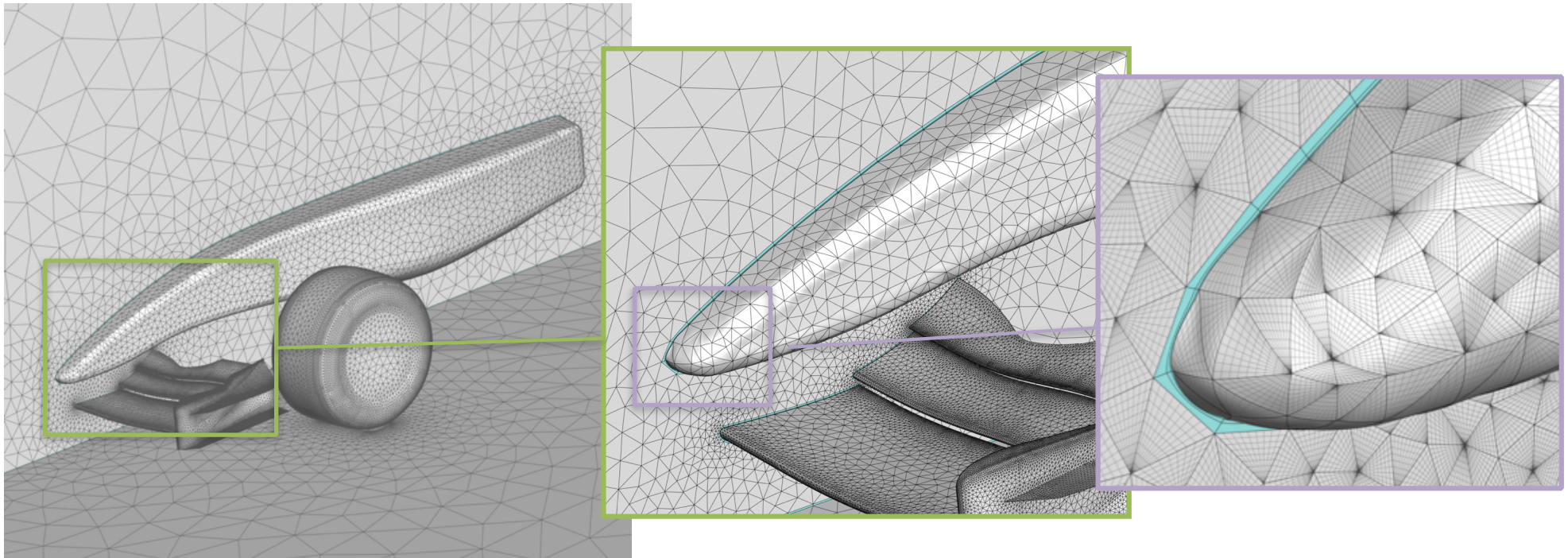
INVALID ELEMENT



Producing meshes for high-Re simulations

Viscous flows → boundary layer around walls.

- From the surface triangulation, we generate a prismatic boundary layer (better mesh quality).
- Rest of the volume is constructed using tetrahedra.



Producing meshes for high-Re simulations

Viscous flows → boundary layer around walls.

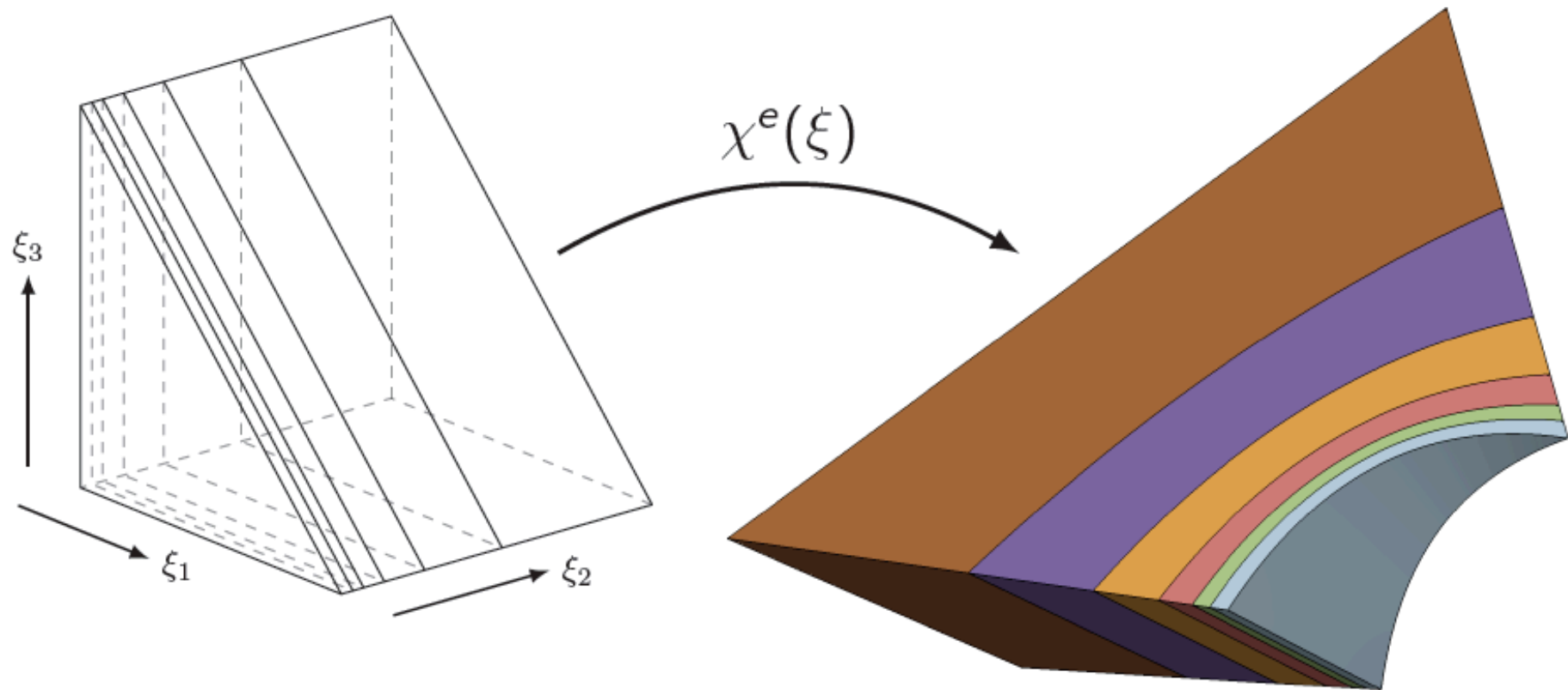
- From the surface triangulation, we generate a prismatic boundary layer (better mesh quality).
- Rest of the volume is constructed using tetrahedra.

For high Reynolds number simulations:

- Require an extremely thin boundary layer ($y^+ \sim 1$)
- Must not contain invalid elements.

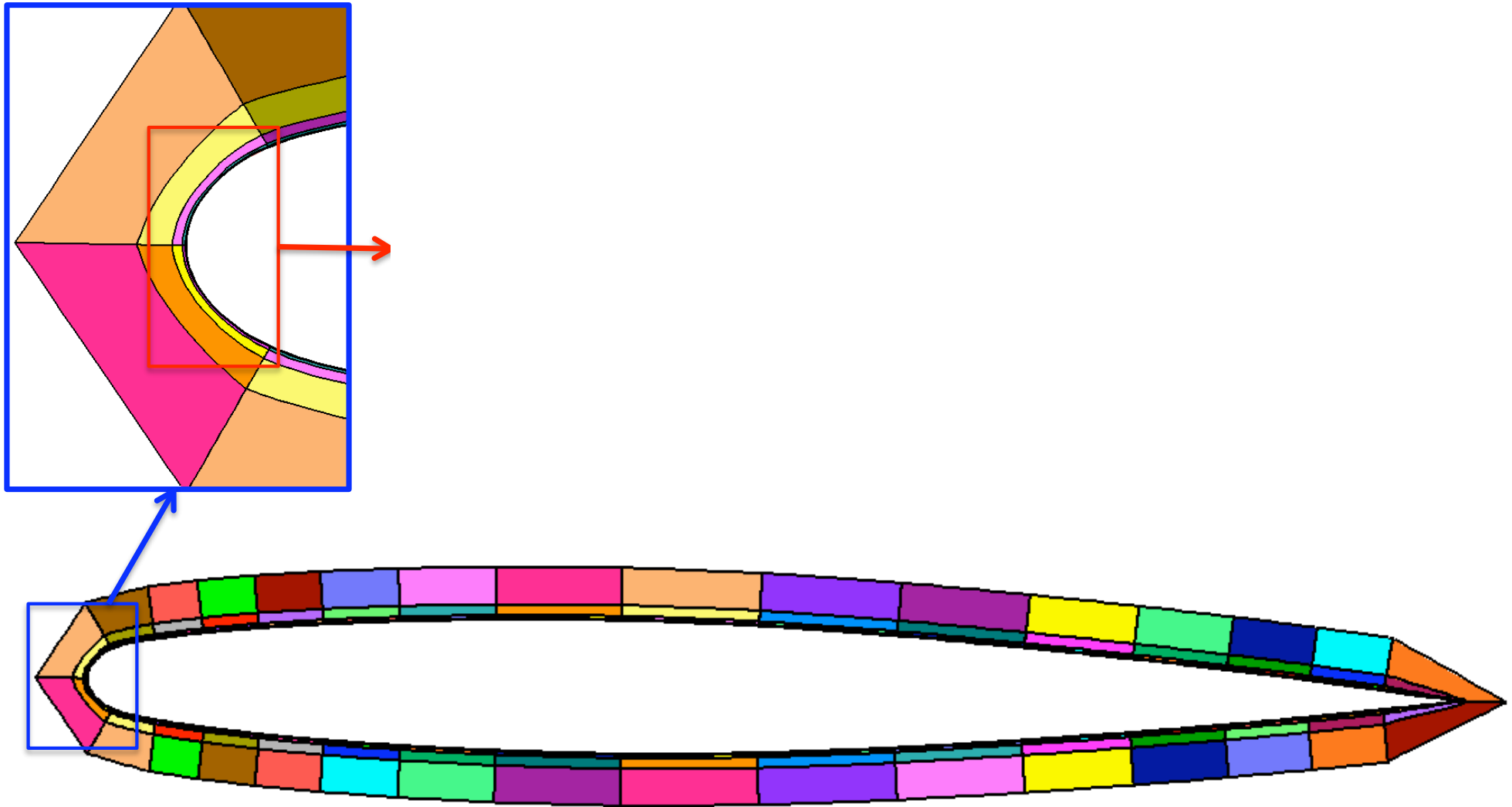
Refine a **valid** coarse prismatic mesh to produce a **valid** mesh of triangular prisms or tetrahedra.

Boundary-layer mesh generation



Subdivide the master element to generate
a boundary-layer mesh

Proof of Concept

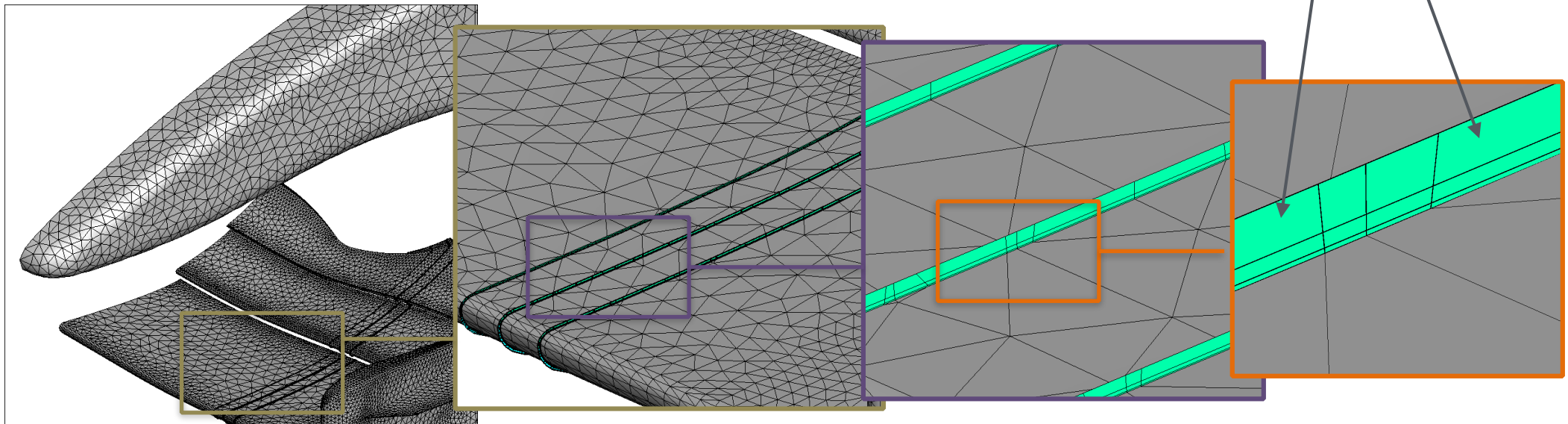
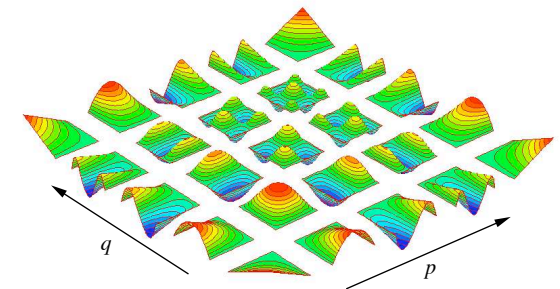
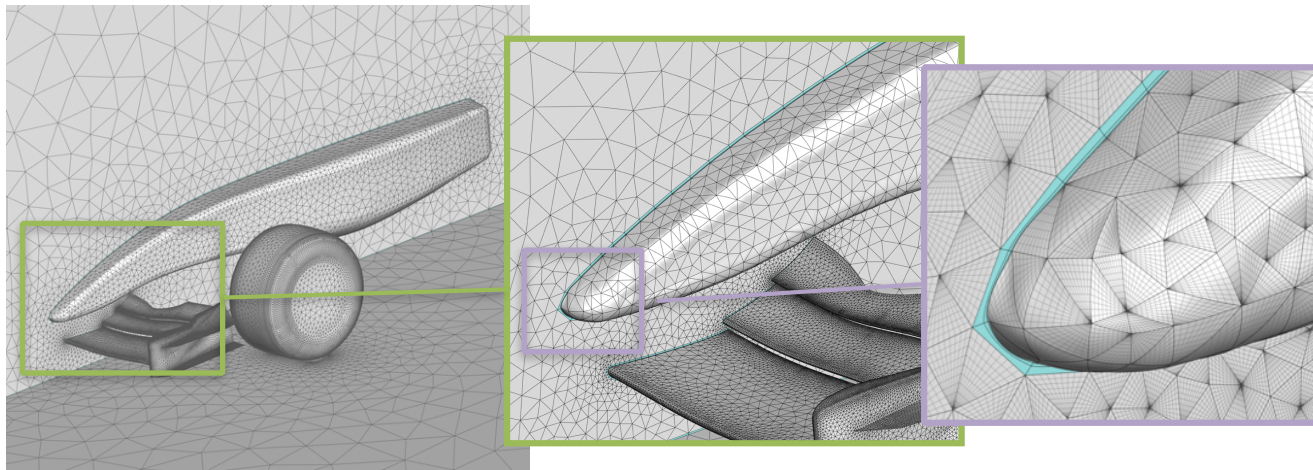


7 layers of refinement

IDIHOM

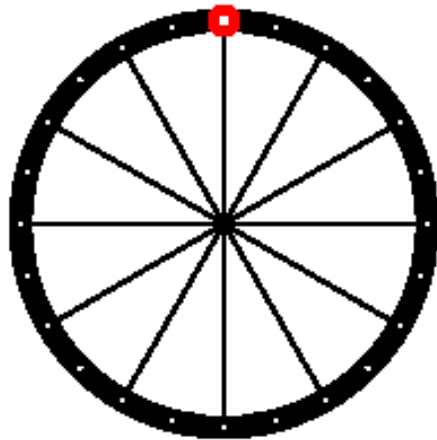
Application to F1 Geometry

Meshing



Spectral/hp for high Reynolds number applications

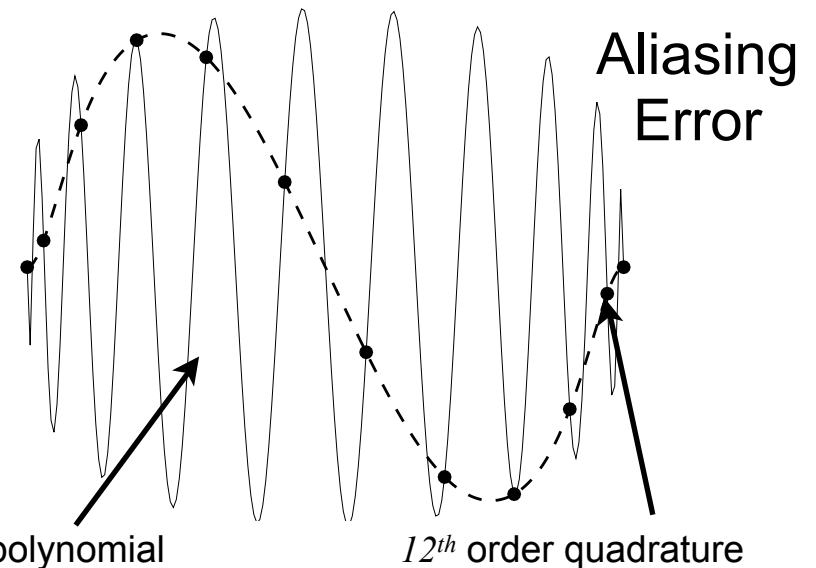
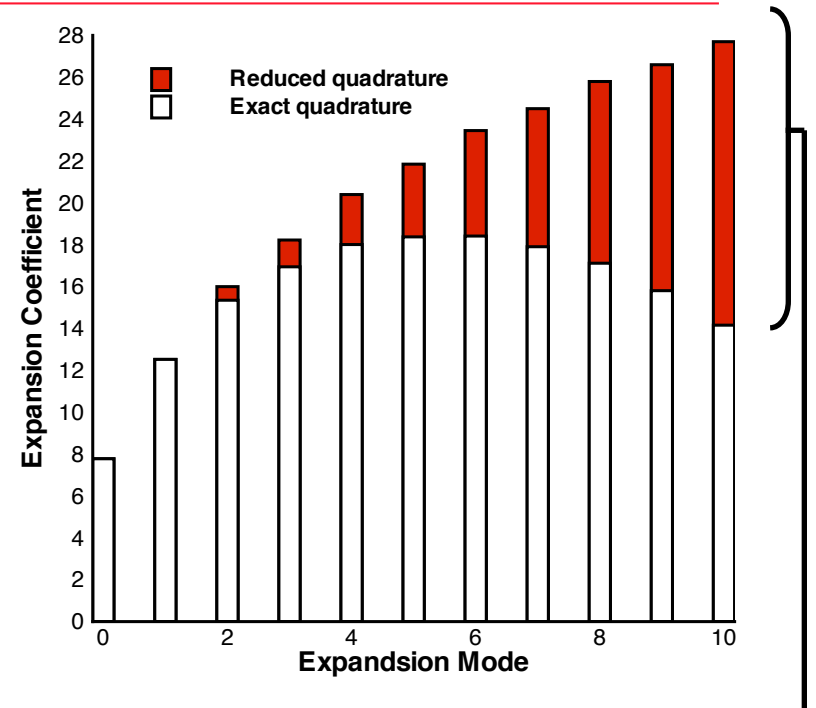
- High order meshing
- Nodal/collocation space dealiasing
- SVV Smoothing



Example:
$$u(\xi) = \sum_{i=0}^{10} \phi_i(\xi)$$

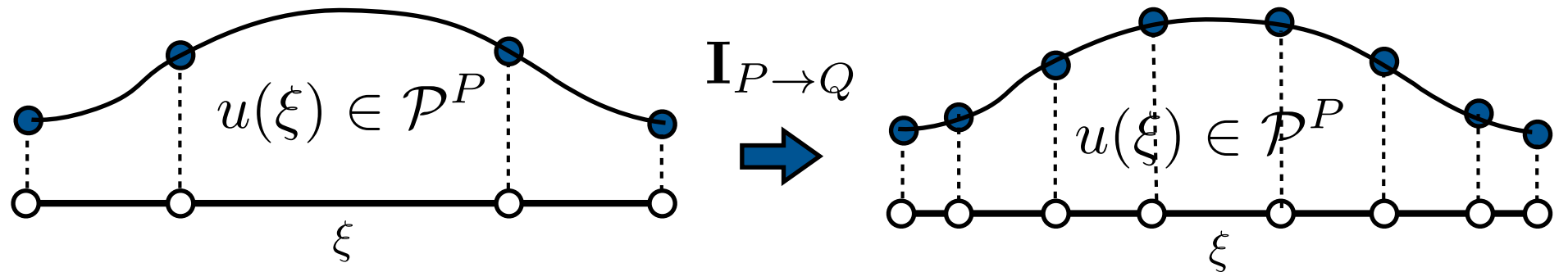
Galerkin projection of u^2 using:

- $Q = 17$ – exact Quadrature
- $Q = 12$ – sufficient for integrating 20th degree polynomials

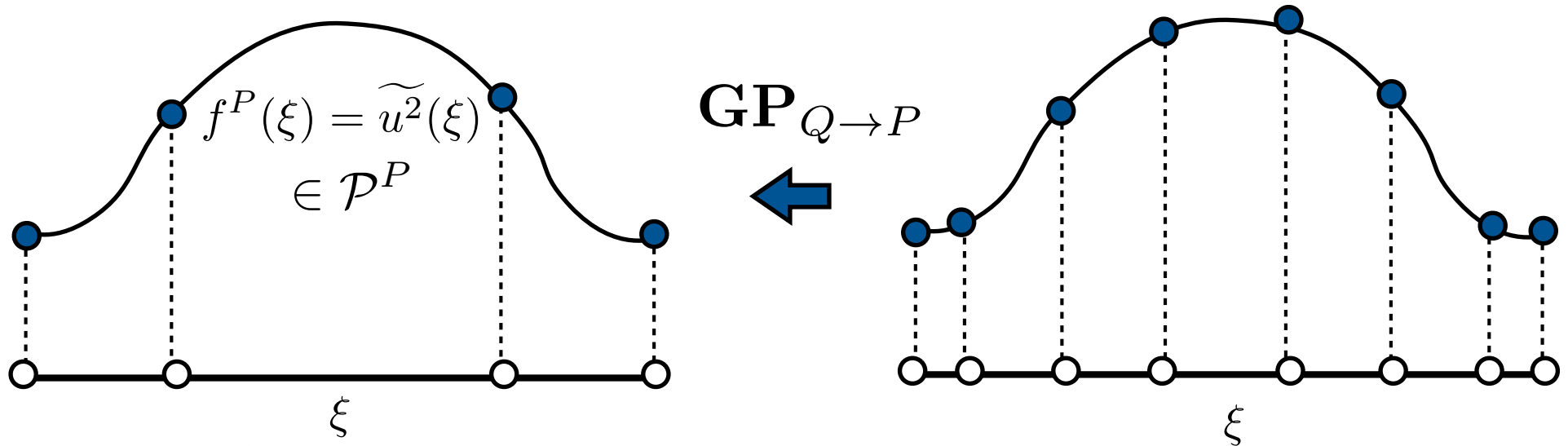


Example from Kirby & Karniadakis, *J. Comp. Phys* (2003)

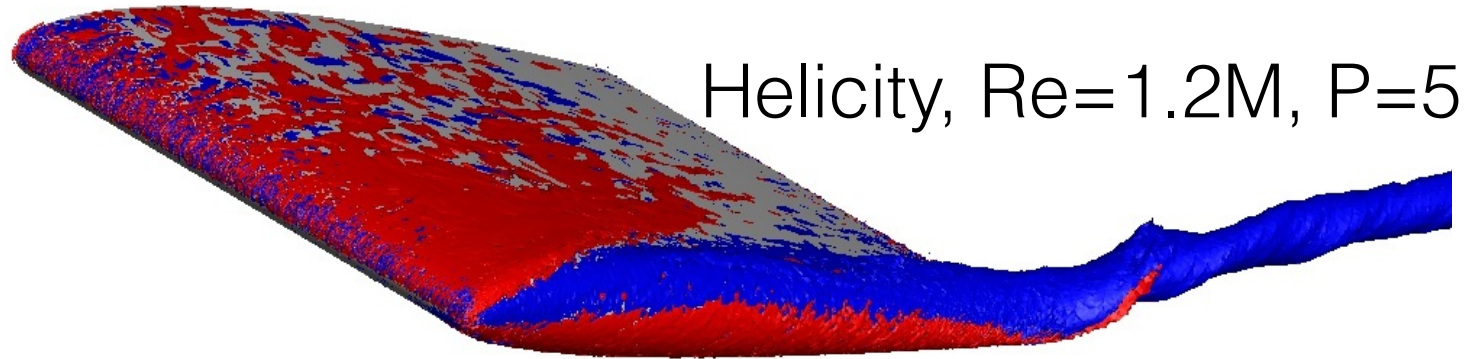
Overview of nodal projection of u^2



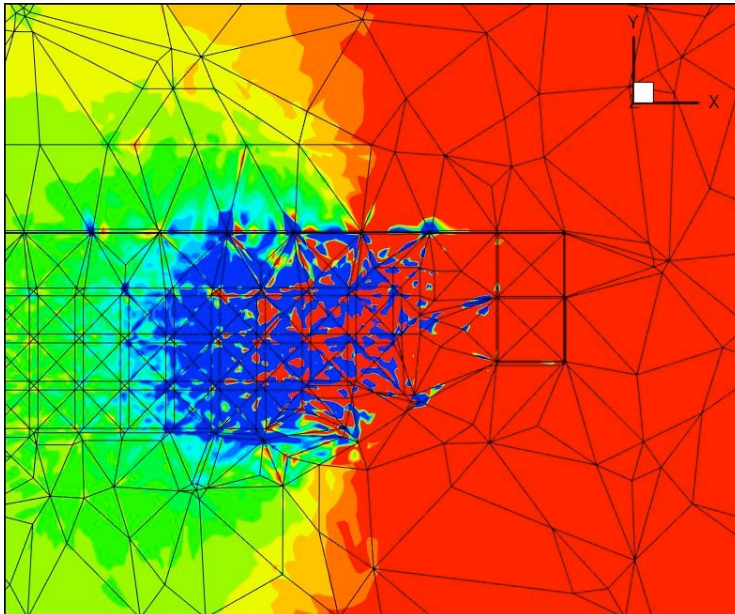
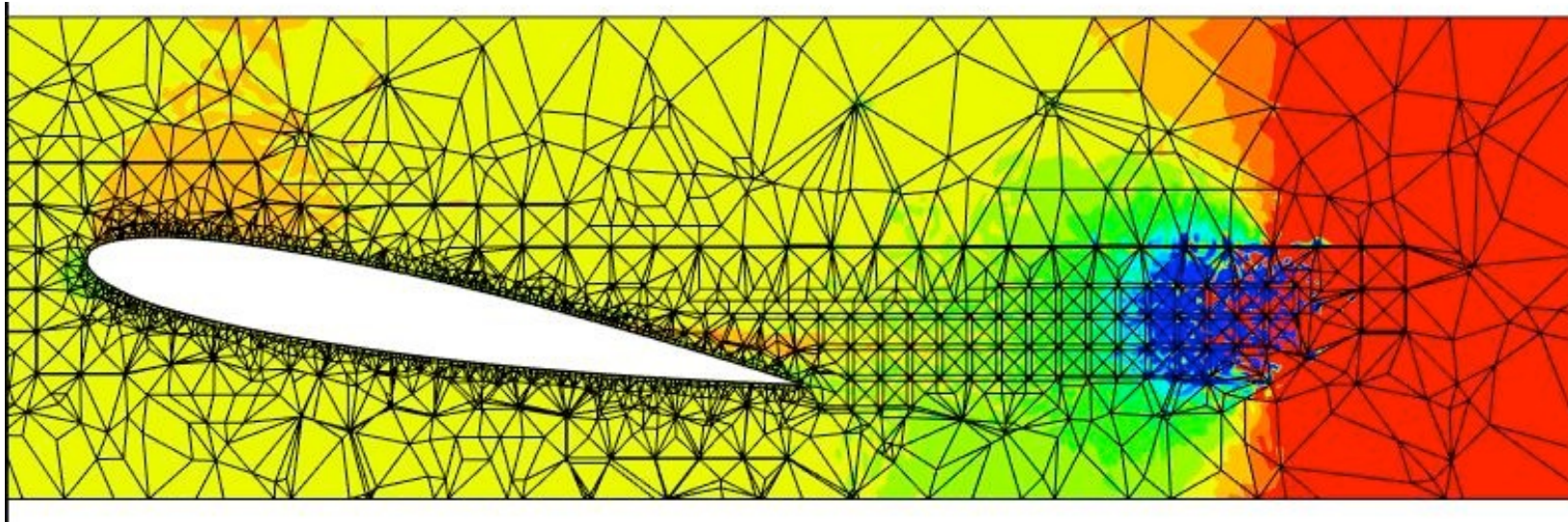
$$GP[i, j] = \frac{1}{w_i^P} \mathbf{I}_{p \rightarrow Q}^T[i, j] w_j^Q$$



Aliasing error in boundary layer



$Re=500K, P=7$ Instability



Spectral/hp for high Reynolds number applications

- High order meshing
- Nodal/collocation space dealiasing
- SVV Smoothing

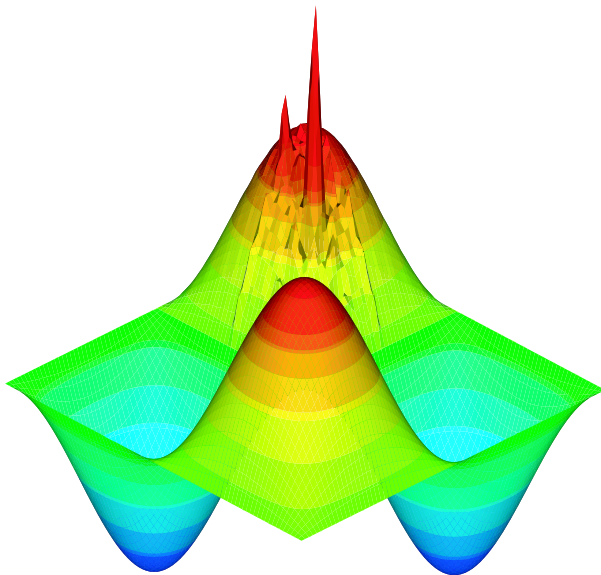
Stabilisation through smoothing

- Filtering often used in Finite Difference Methods
- Legendre Filtering used by Fischer in Nek5000
- Spectral Vanishing Viscosity is a temporal smoothing/filtering
 - Used by Pasquetti, Stiller for High Re Simulation

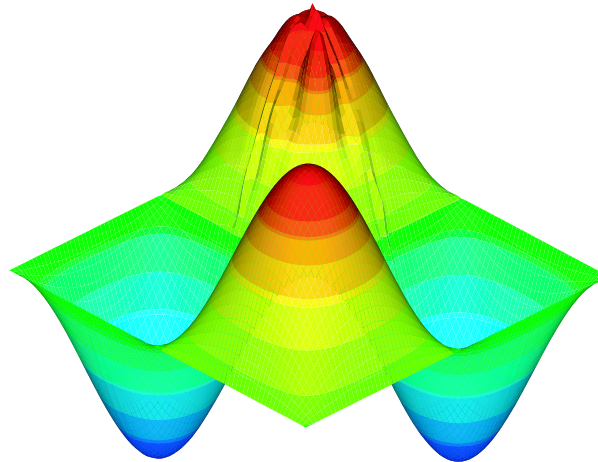
Spectral vanishing viscosity

Tadmor, (89) Maday, Kaber & Tadmor (93) :

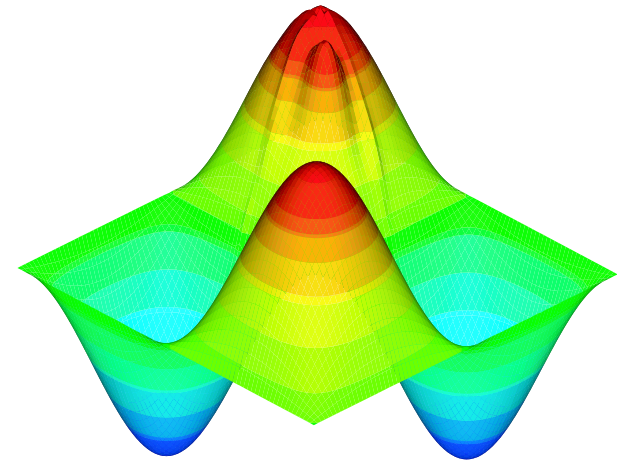
$$\frac{\partial}{\partial t} u(x, t) + \frac{\partial}{\partial x} \left[\frac{u^2(x, t)}{2} \right] = \epsilon \frac{\partial}{\partial x} \left[Q_\epsilon \frac{\partial u}{\partial x} \right] \quad \hat{Q}_k = e^{-\frac{(k-N)^2}{(k-P_{cut})^2}}, \quad k > P_{cut}.$$



No SVV

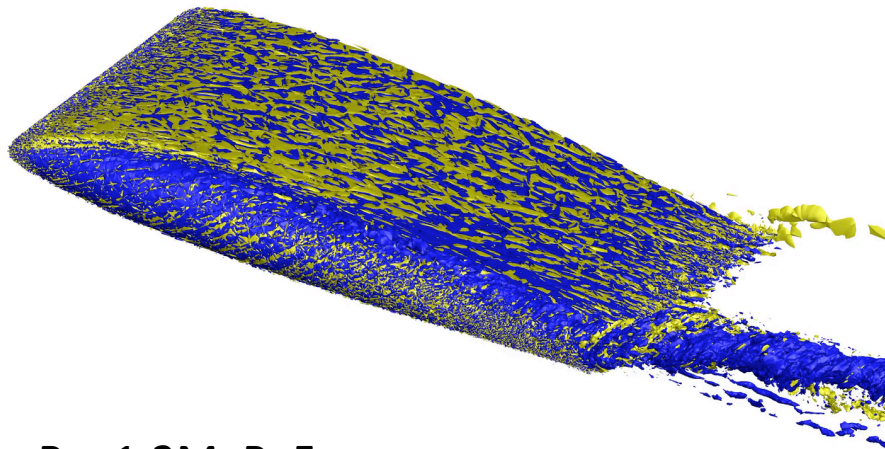
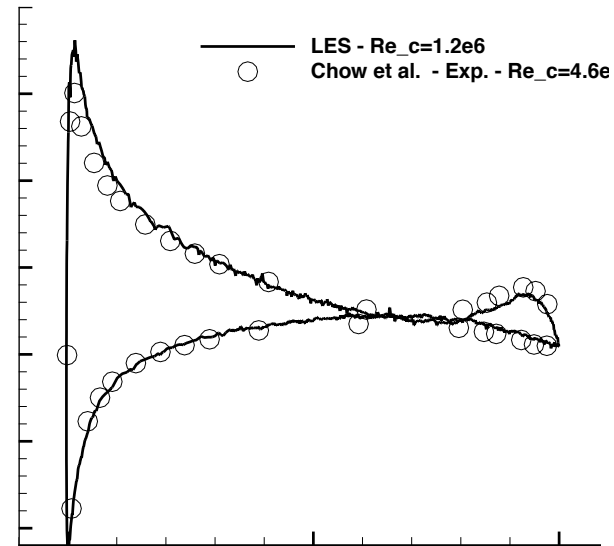
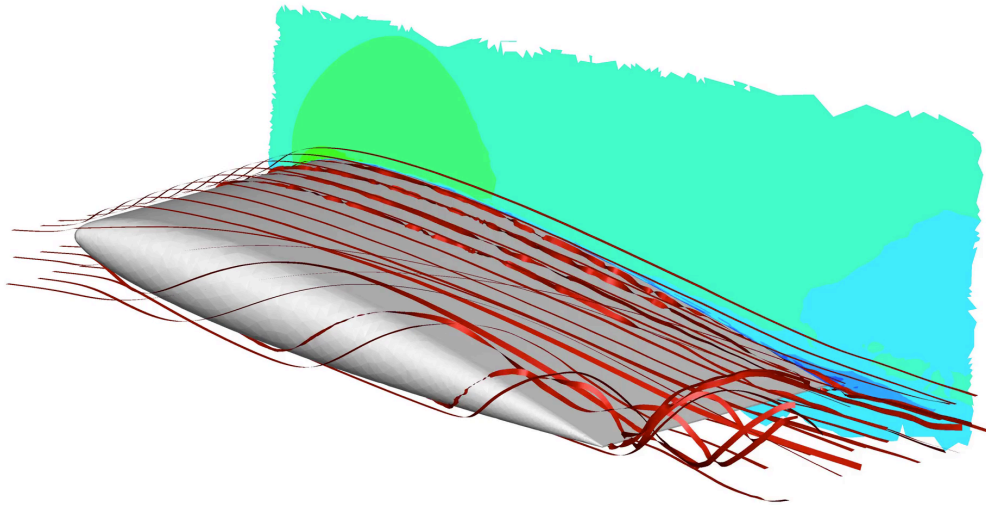


$P_{cut} = 7,$
 $\epsilon_{SVV} = 0.1$

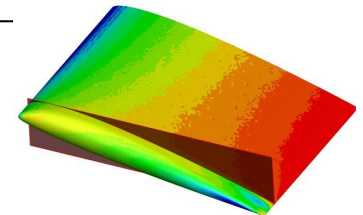
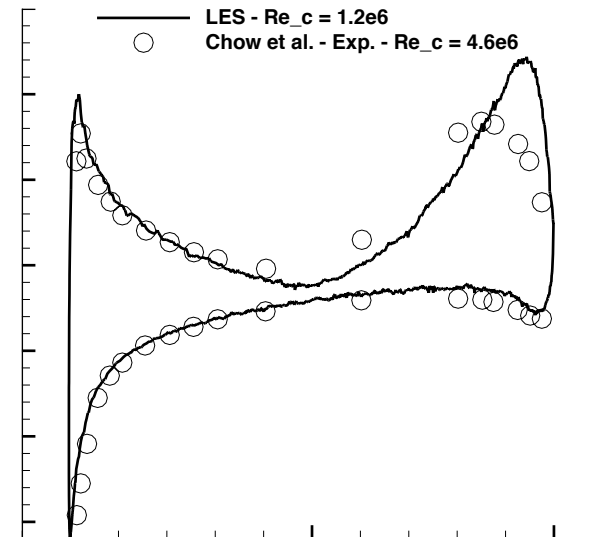


$P_{cut} = 3,$
 $\epsilon_{SVV} = 0.1$

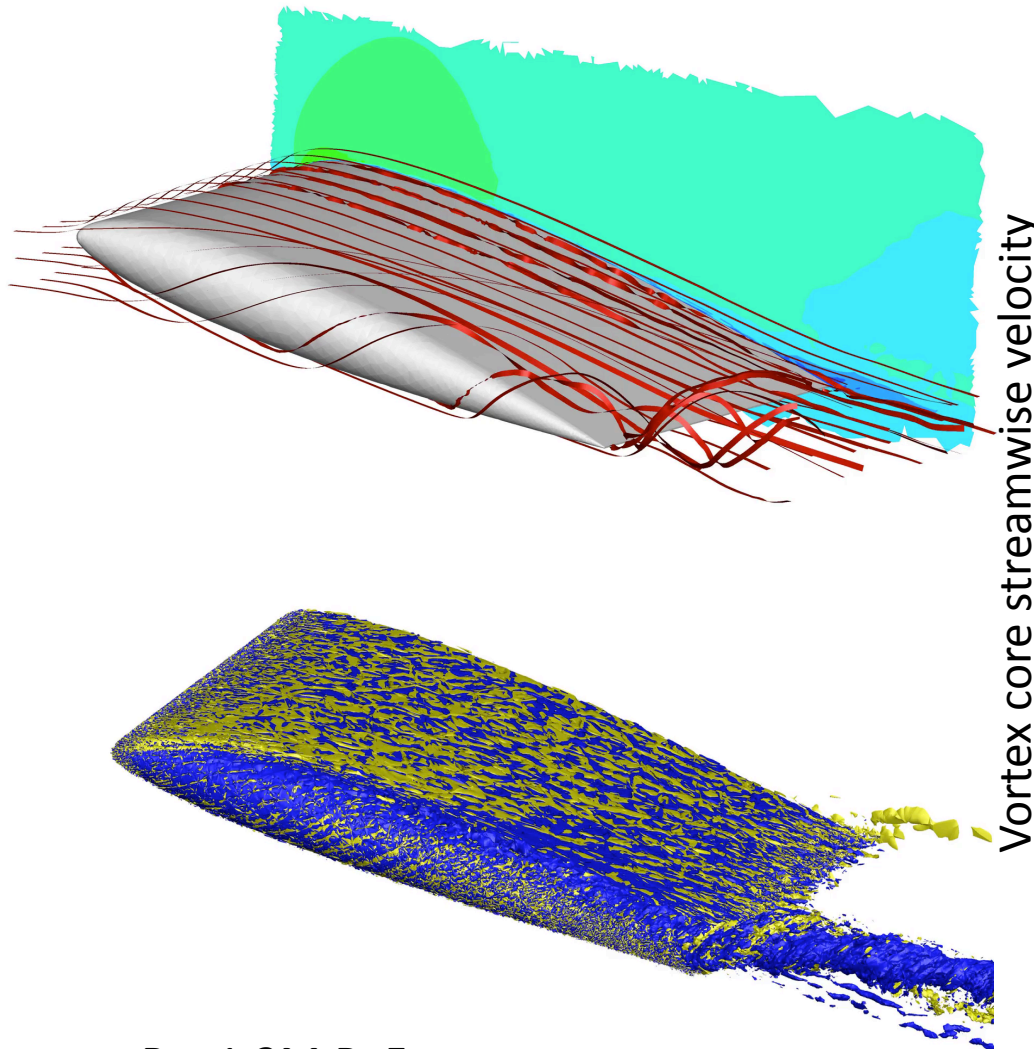
Comparison with Commercial Tools



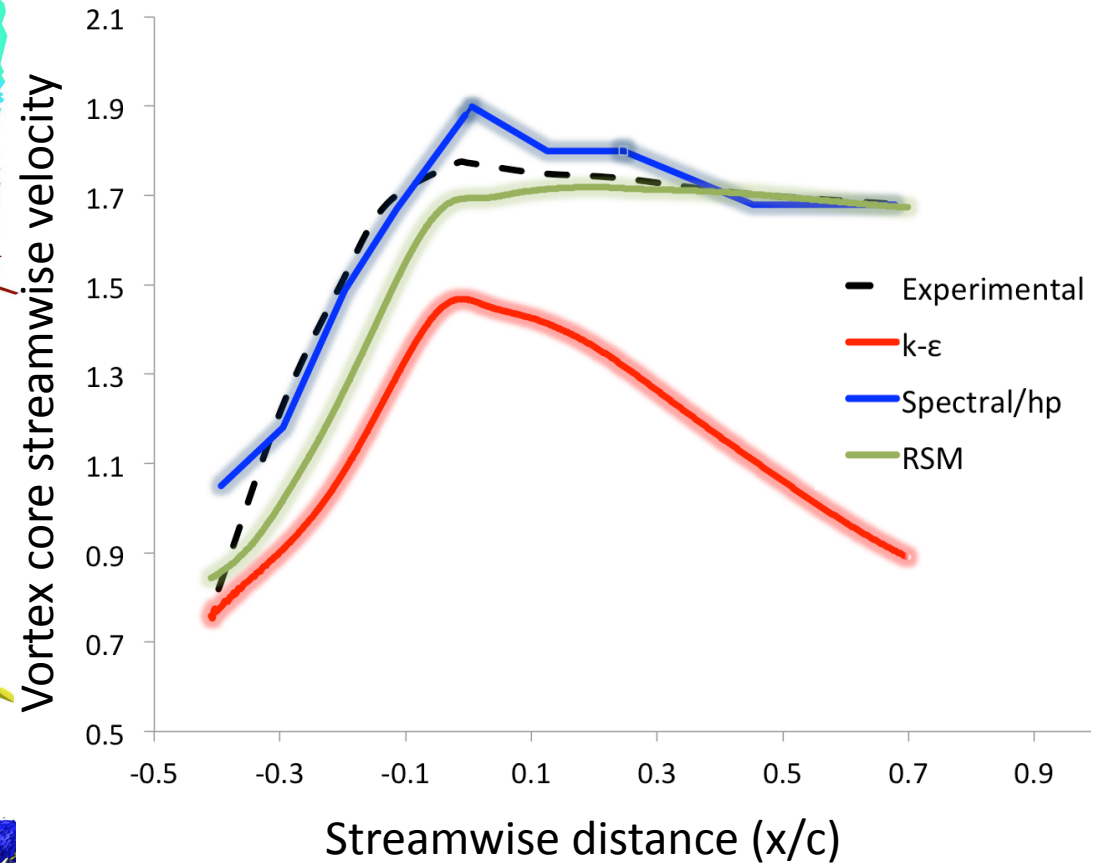
Re=1.2M, P=5

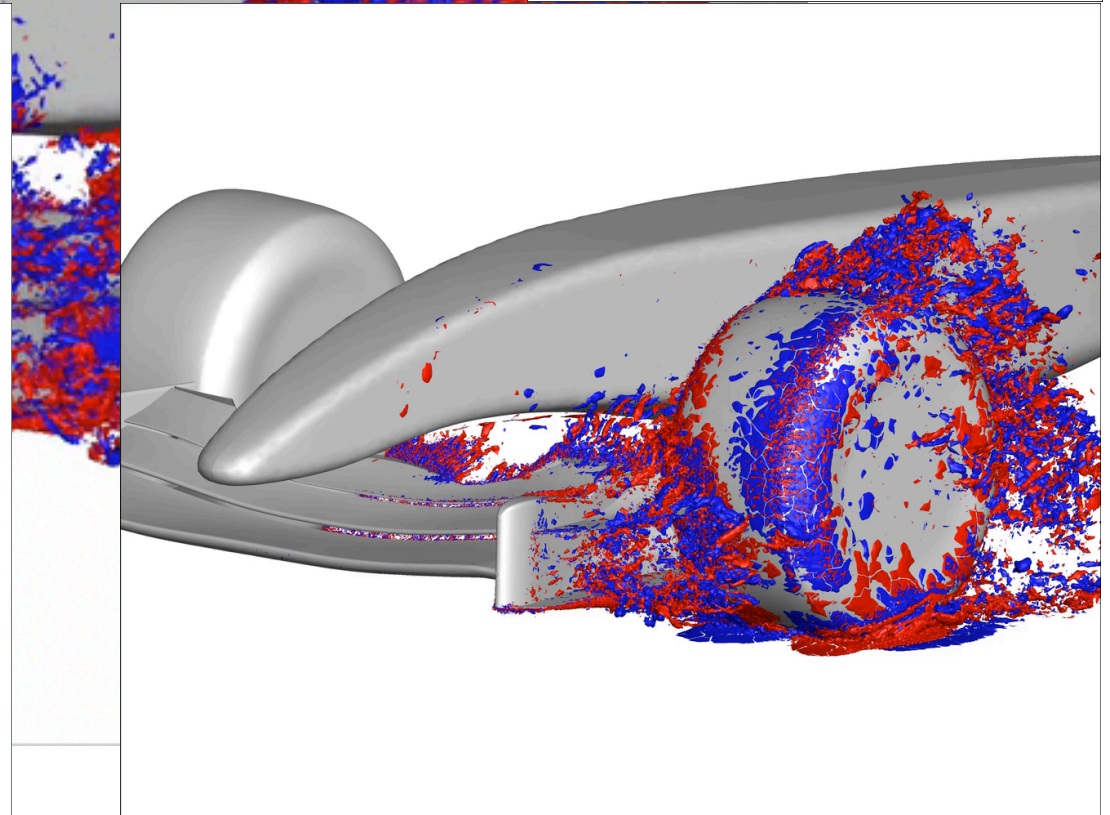
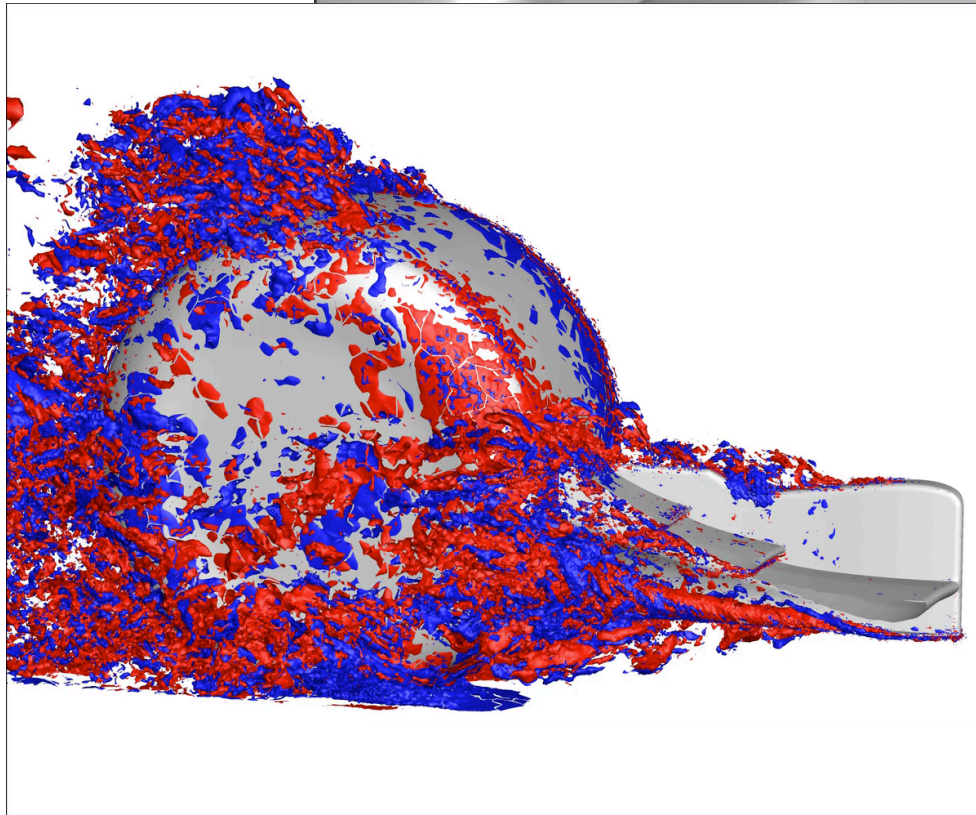
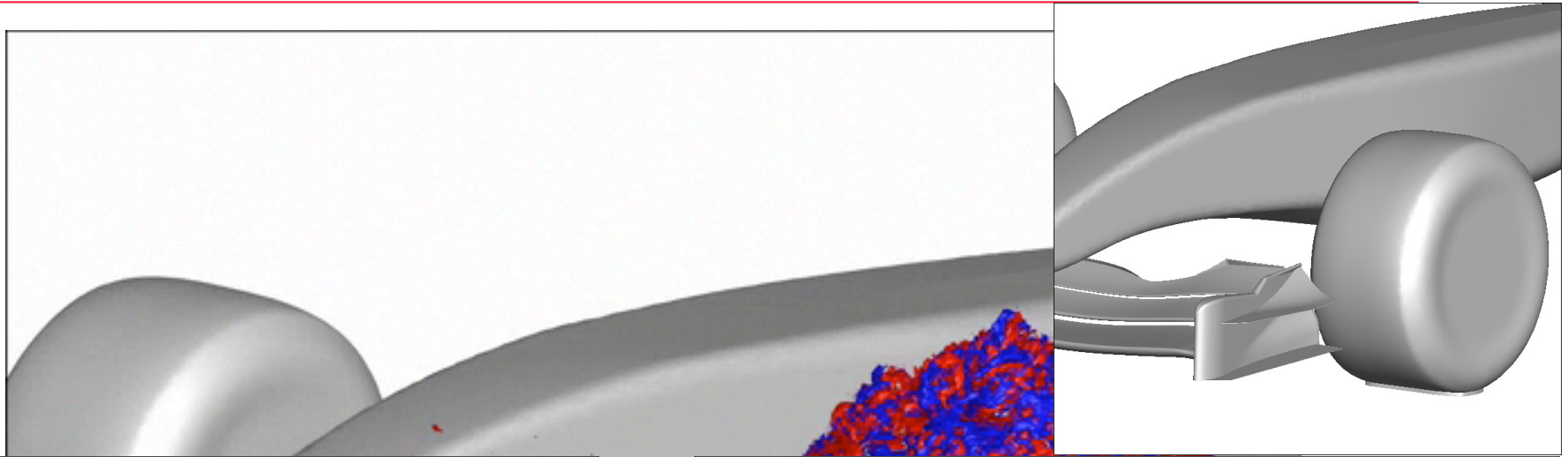


Comparison with Commercial Tools



Re=1.2M,P=5





Summary

- High order compact discretisations such as spectral/hp element methods provide a suitable discretisation for current/emerging HPC hardware
- High accuracy transient flow modelling is an enabling technology for high-end engineering such as automotive and aeronautical sectors.