# Fanning out of the *f*-mode in the presence of nonuniform magnetic fields

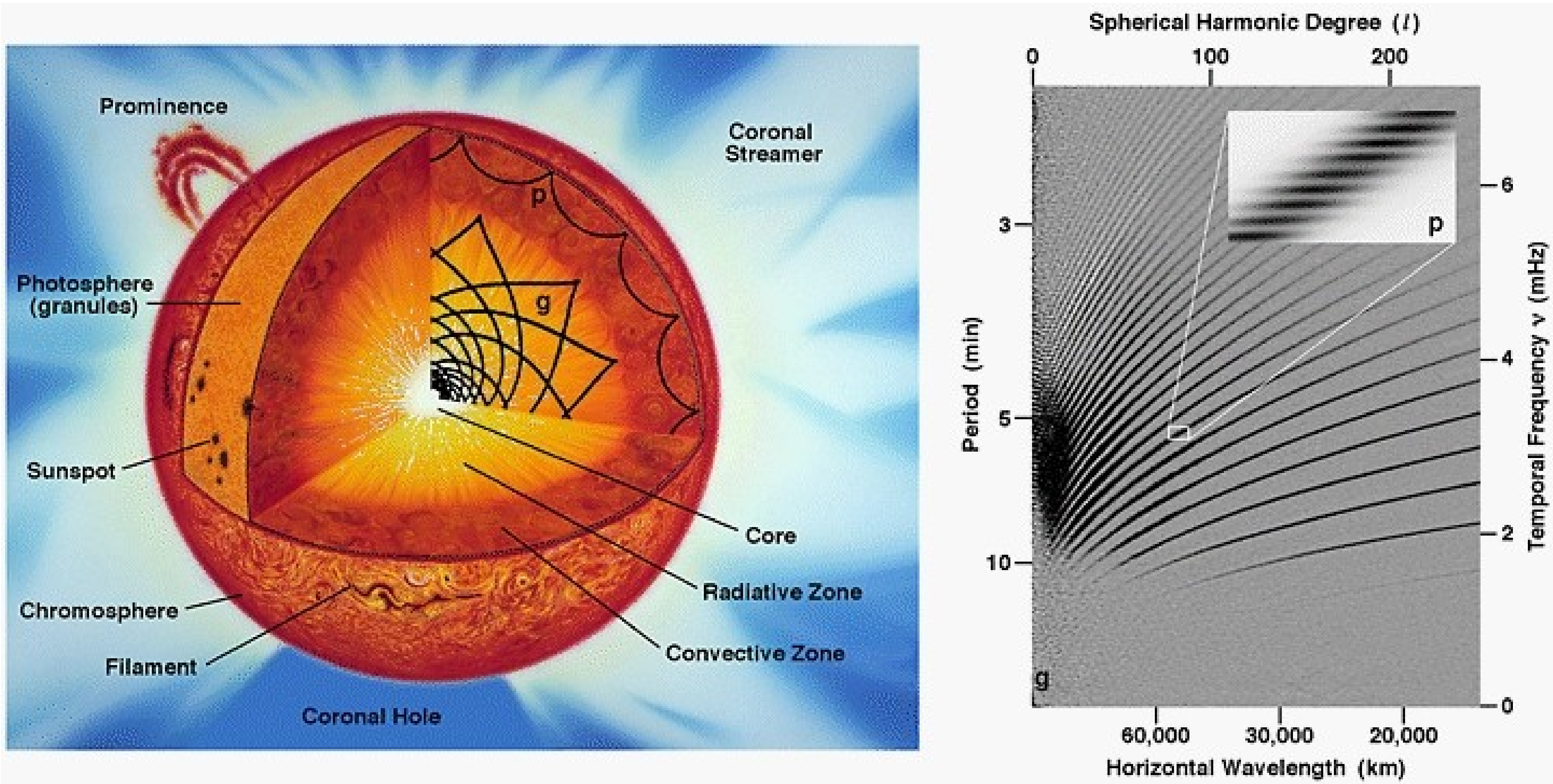
Nishant K. Singh
NORDITA

With
Axel Brandenburg, S. M. Chitre, Matthias Rheinhardt, Harsha Raichur

Simulations: `https://code.google.com/p/pencil-code/`
Observations: `SDO/HMI`

`Pencil Code User Meeting, Trondheim, 14 MAY 2015`

# The Sun and Helioseismology




The Sun supports a wide variety of waves

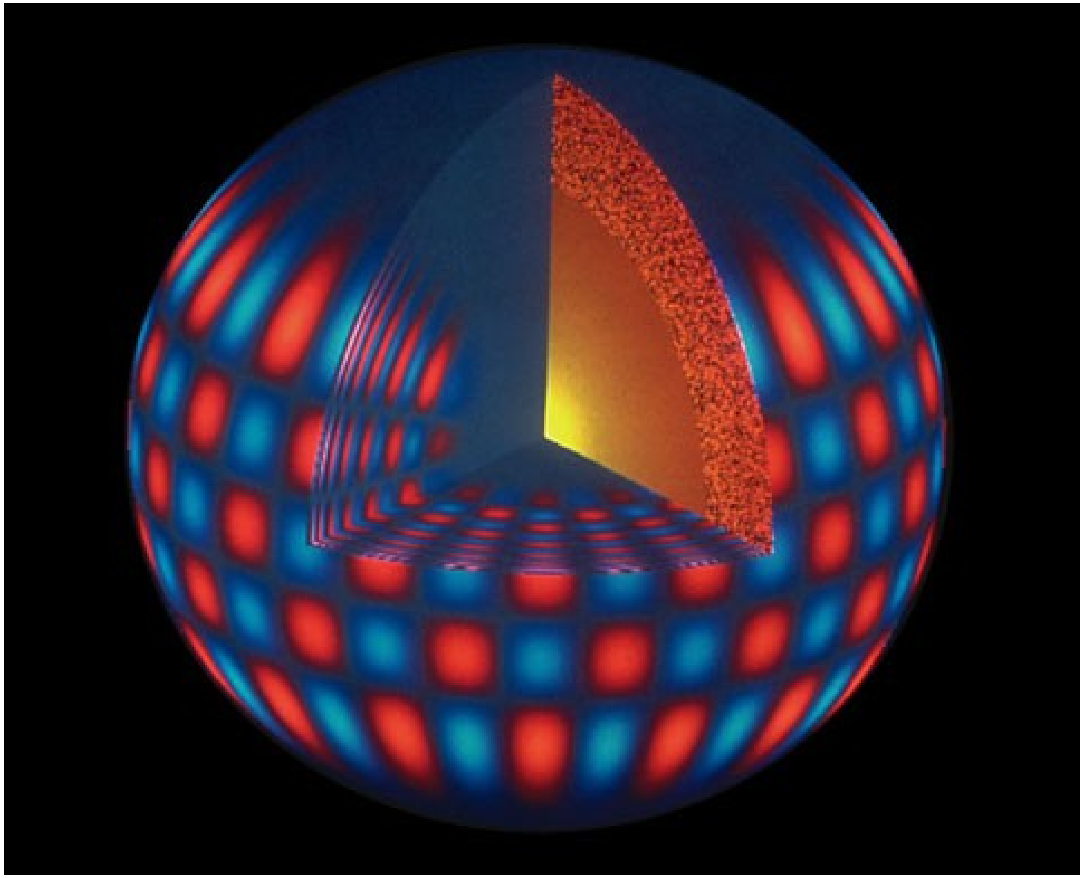
Helioseismology: science studying the wave oscillations in the Sun

Goal is to determine the physical conditions in the Solar interior



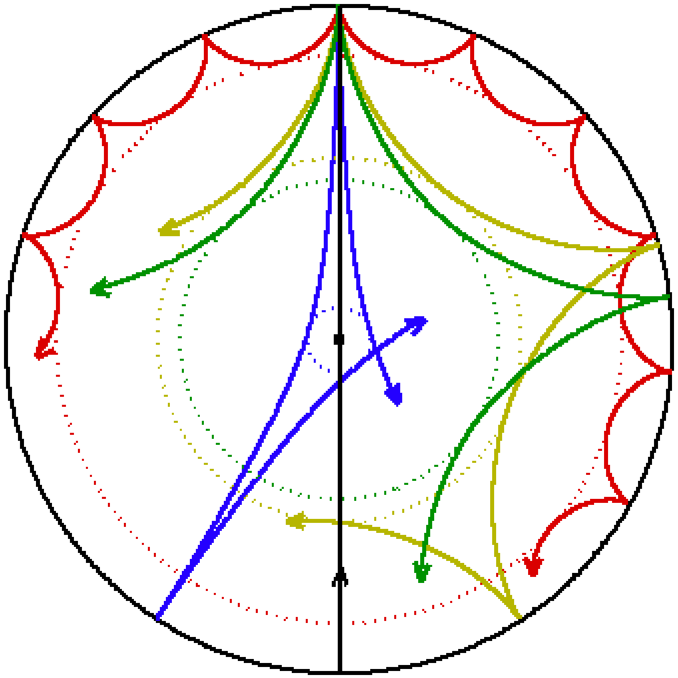
Figure Source: SOHO image gallery

# Oscillations



*About 250,000 out of 10 million expected modes have been identified*

Figure Source: NOAO



Internal ray paths of acoustic waves

- Refraction of waves
- Low *l*'s or large wavelengths penetrate to deeper layers

$$r_{\rm min} = \frac{c_{\rm s}}{\omega}\sqrt{\ell(\ell+1)}$$

# The modes: dispersion relations

- *g*-modes → Gravity waves (Buoyancy driven)

$$\omega_{\mathrm{BV}}^2 = \frac{g}{c_p}\frac{ds}{dz}$$

  - In stably stratified medium
  - Expected to exist in the core of the Sun
  - Not yet observed in Sun !

- *p*-modes → Sound waves (Pressure driven)

$$\omega^2 = \omega_c^2 + c_{\mathrm{s}}^2\left(k_x^2 + k_z^2\right)$$

(Isothermal)

  - Excited by convective motions
  - Observed and most extensively used
  - The Sun is transparent to acoustic radiation
  - Energy travel time through the Sun ~ few hrs

- *f*-mode → Surface waves (On interface)

$$\omega_{\mathrm{f}}^2 = gk_x\,\frac{1-q}{1+q}$$

with $q < 1$

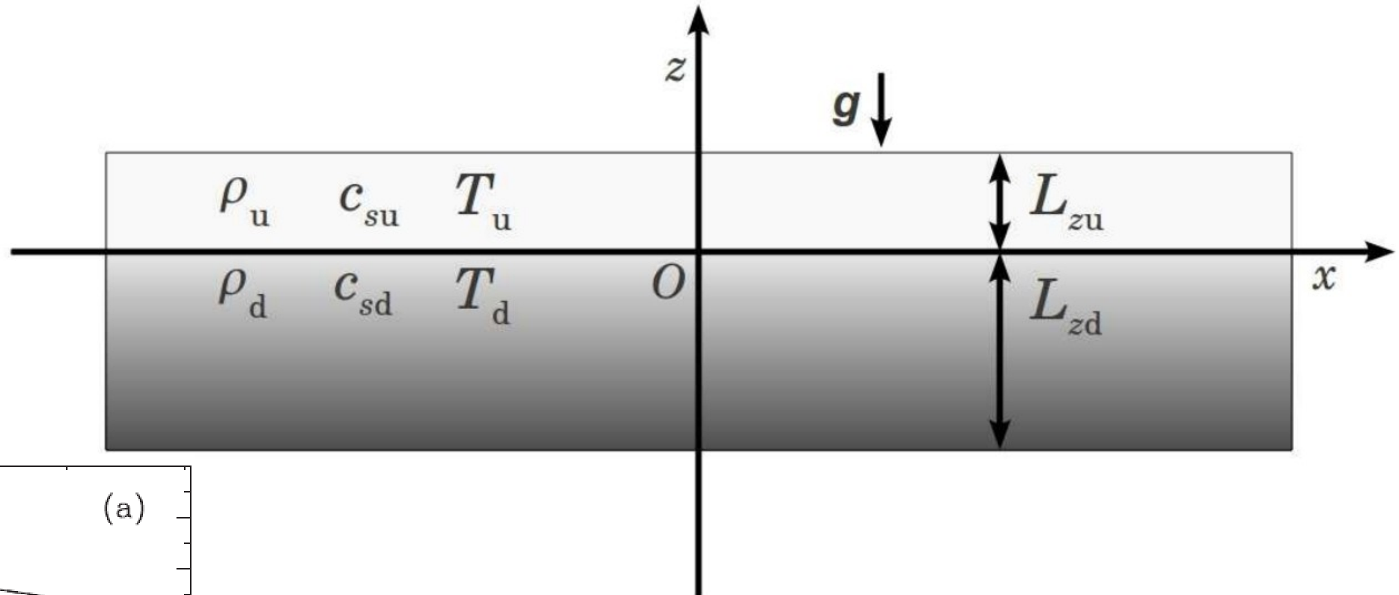
  - Water (surface) waves; dispersive
  - Observed on solar surface
  - Traditionally thought to be of less diagnostic value
  - Significant frequency shifts are detected
  - Shifts could tell us about the inhomogeneous structure

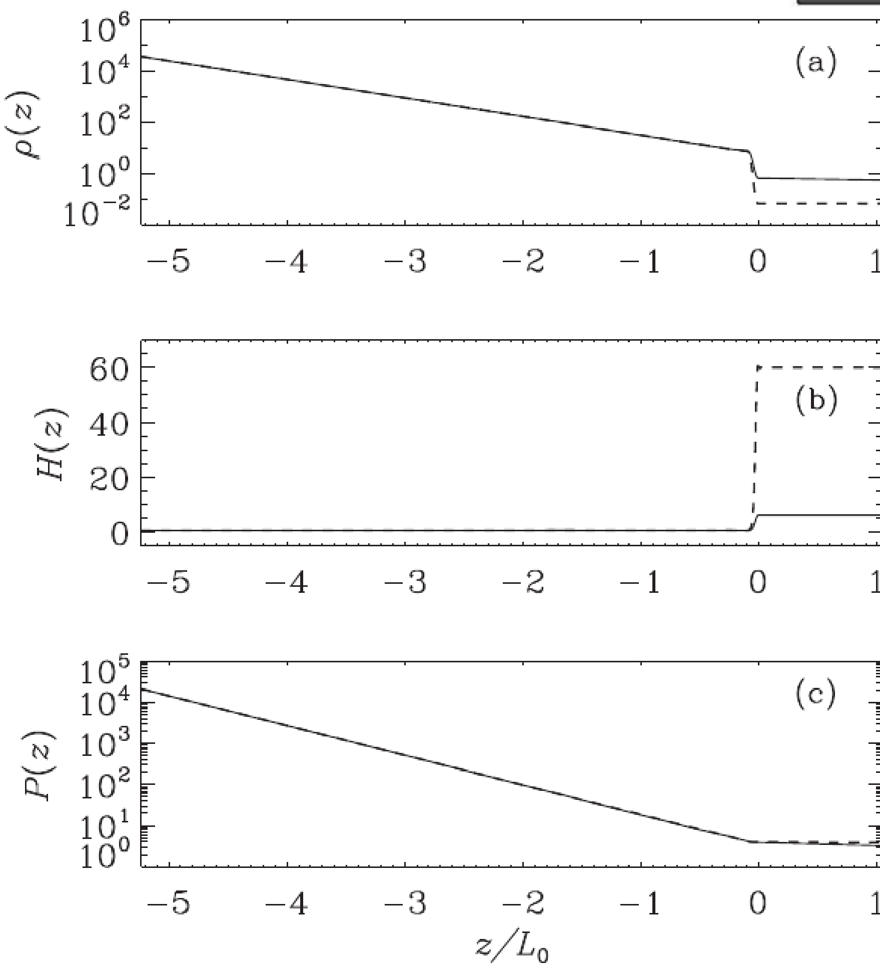


See, e.g., lecture notes by J. Christensen-Dalsgaard

# Our model setup and initial conditions

- 2-D Cartesian section
- Stably stratified under $\boldsymbol{g}$
- Piecewise isothermal
- Interface at $z = 0$
- Bulk ($z < 0$)
- Atmosphere ($z > 0$)




- Sharp jump at the interface:

$$q = \rho_{\rm u}(0)/\rho_{\rm d}(0) = c_{\rm sd}^2/c_{\rm su}^2$$

- Background density:

$$\rho_{\rm d,u}(z) = \rho_{\rm d,u}(0)\exp(-z/H_{\rm d,u})$$

- Scale height:

$$H_{\rm d,u} = (c_p - c_v)T_{\rm d,u}/g$$

$$L_0 = \gamma\, H_{\rm d}$$



For details: Singh et al, MNRAS (2015)

# Wave excitation and measurements

- Weak stochastic forcing ***f*** in a homogeneous and isotropic fashion
- Stirring scale << the box dimensions (we chose, $k_f/k_1 = 20$)
- No effect seen when restricted the forcing to lower subdomain
- We measure the vertical velocity at the interface $z$=0, $u_z(x, 0, t)$
- Fourier transforming this in $x$ and $t$, we get $\hat{u}_z(k_x, \omega)$

## Useful dimensionless quantities

$$\mathrm{Re} = \frac{u_{\mathrm{rms,d}}}{\nu k_{\mathrm{f}}} \qquad \mathrm{Ma} = \frac{u_{\mathrm{rms,d}}}{c_{s\mathrm{d}}} \qquad \mathcal{F} = f_0\,\mathrm{Rc} = \frac{f_0\,c_{s\mathrm{d}}}{\nu\,k_{\mathrm{f}}}$$

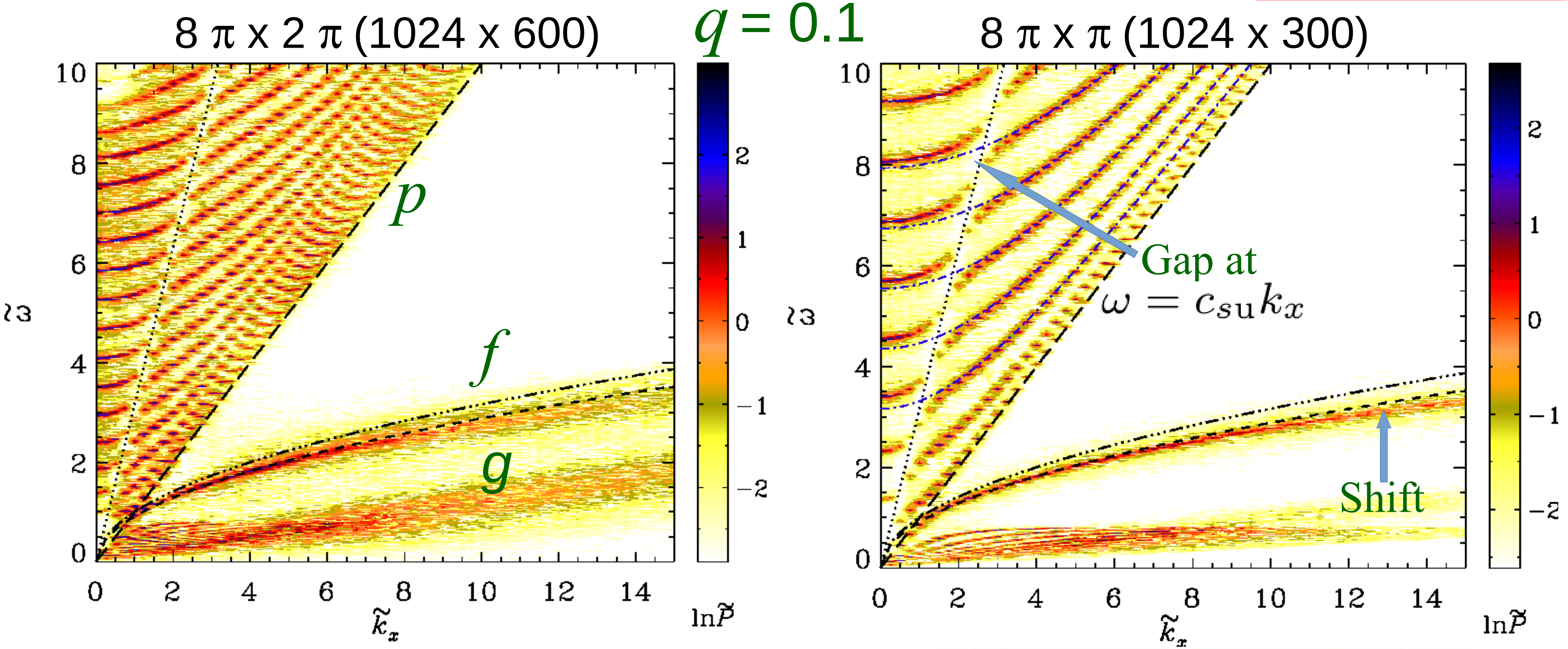
We define,

$$\widetilde{k}_x = k_x L_0, \quad \widetilde{\omega} = \frac{\omega}{\omega_0}, \quad \text{where} \quad \omega_0 = \frac{g}{c_{s\mathrm{d}}}$$

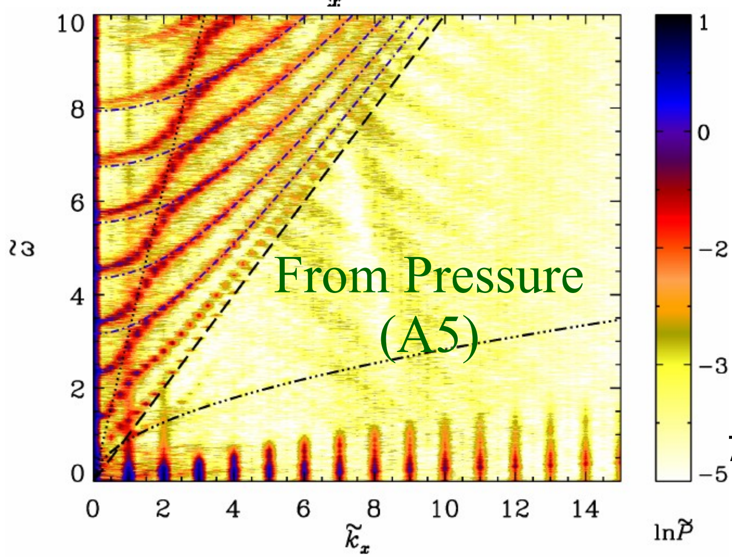
$$\widetilde{P}(k_x,\omega) = \frac{|\hat{u}_z|}{\mathcal{D}^2} = \frac{|\hat{u}_z|}{L_0^2}\,\frac{c_{s\mathrm{d}}^2}{u_{\mathrm{rms,d}}^2}$$

# Non-magnetic case

power_xy
(Matthias)

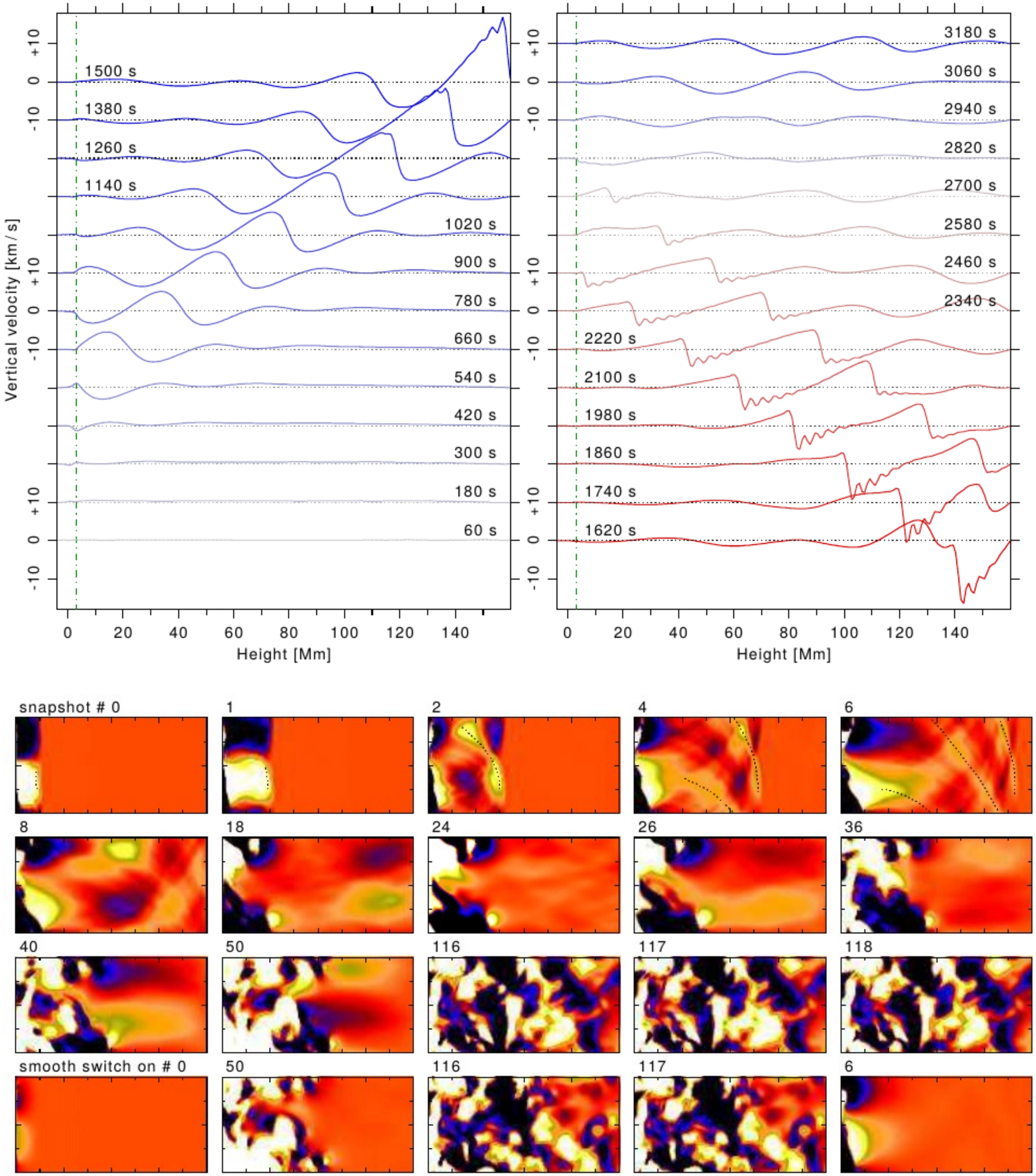



- Second family of $g$-modes due to hot layer above
  Hindman & Zweibel (94); Wagner & Schmitz (07)



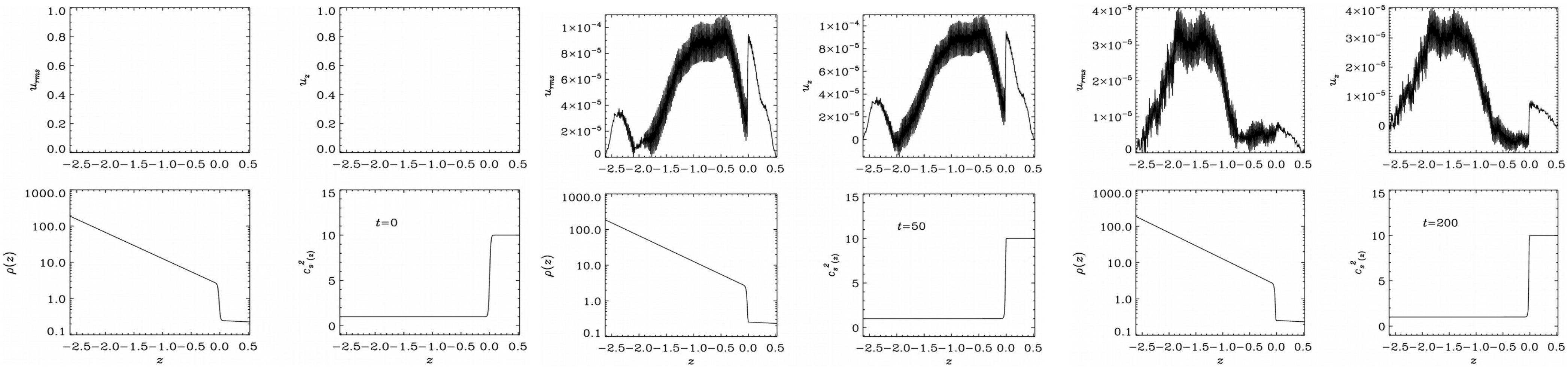

- Avoided crossings => "*a*-mode"
- Acoustic mode localized in the upper hot layer
- Discussed/seen in more realistic setup of HZ94
- Non-detection in solar context =>
  *lower limit on chromospheric temperature ?*

# Switch-on effects: Bourdin (2014)

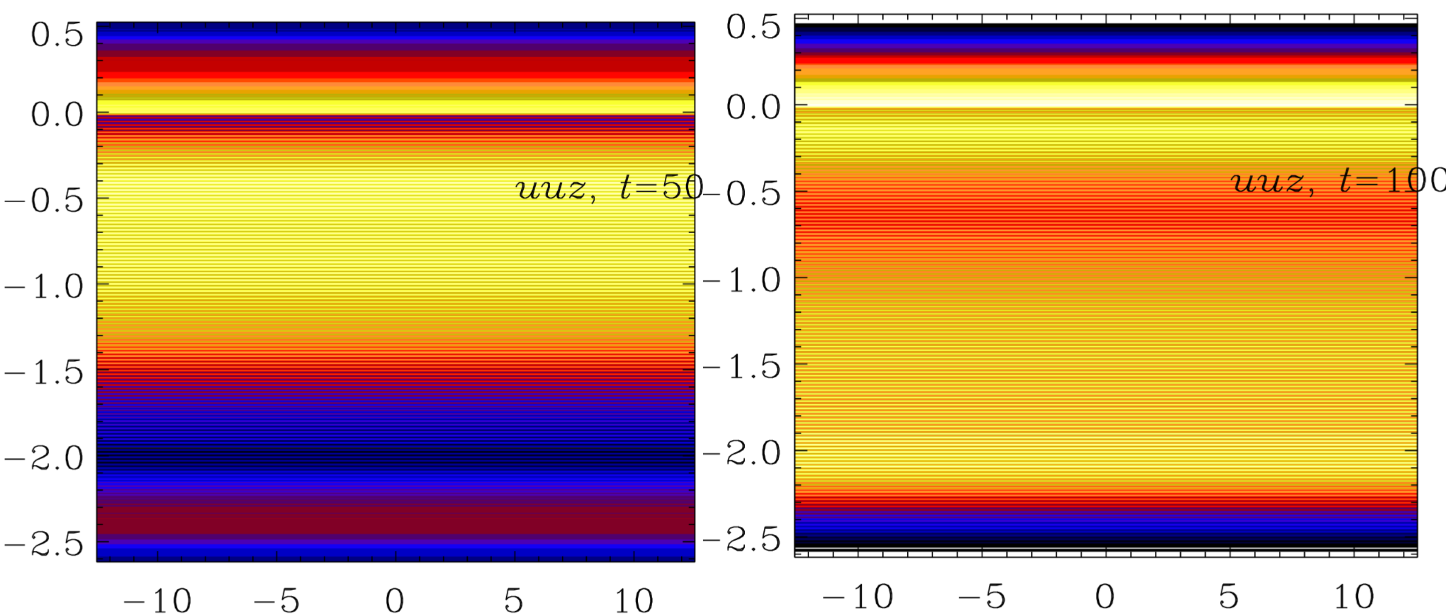


# Experiment (Philippe and Matthias)

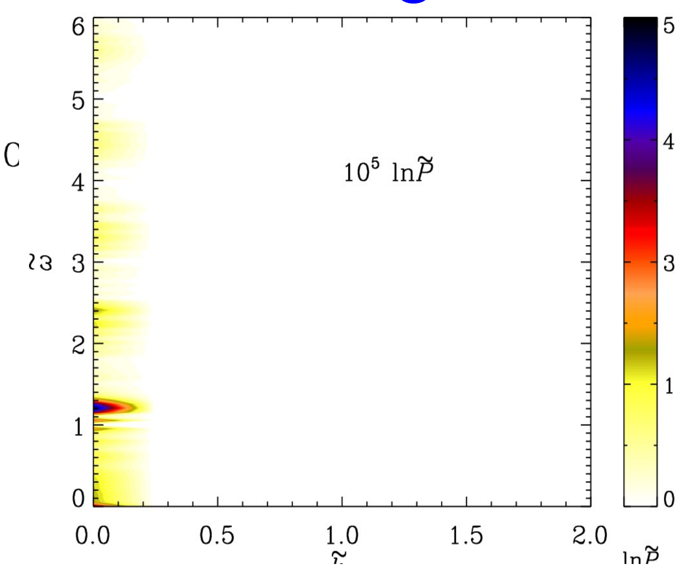
*t*=0 *t*=50 *t*=200



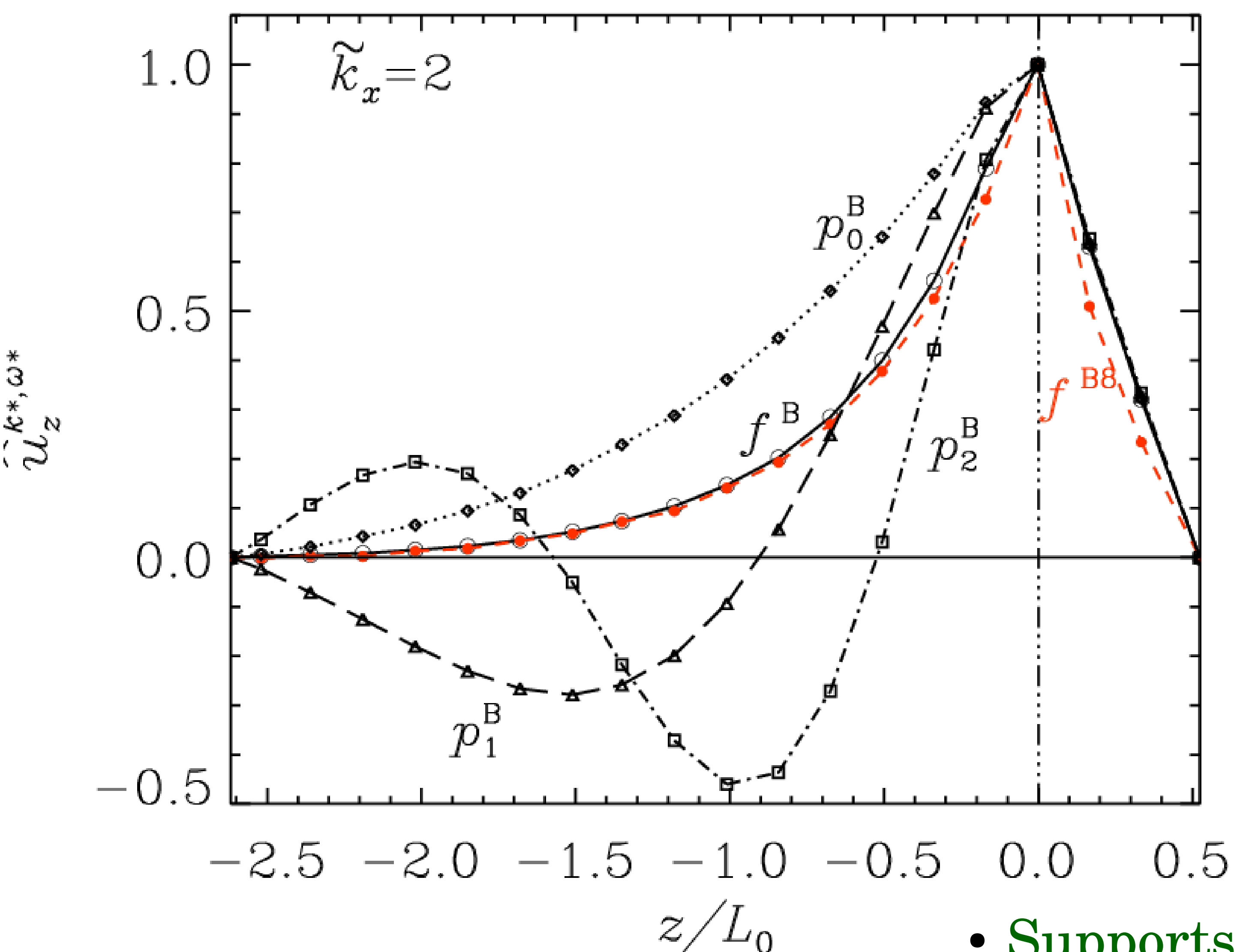

Time




k-omega



# Eigenfunctions ($f$ AND $p_0$) and Duvall's Law




- Duvall's Law:

$$k_z = \frac{\pi(n_D + \alpha)}{L_{z\mathrm{d}}}$$

$$n_D = 1, 2, 3, \ldots$$

Best Fit: $\alpha = 3/2$

- These simulations:

$$k_z = \frac{\pi(n + 1/2)}{L_{z\mathrm{d}}}$$

$$n = 0, 1, 2, \ldots$$

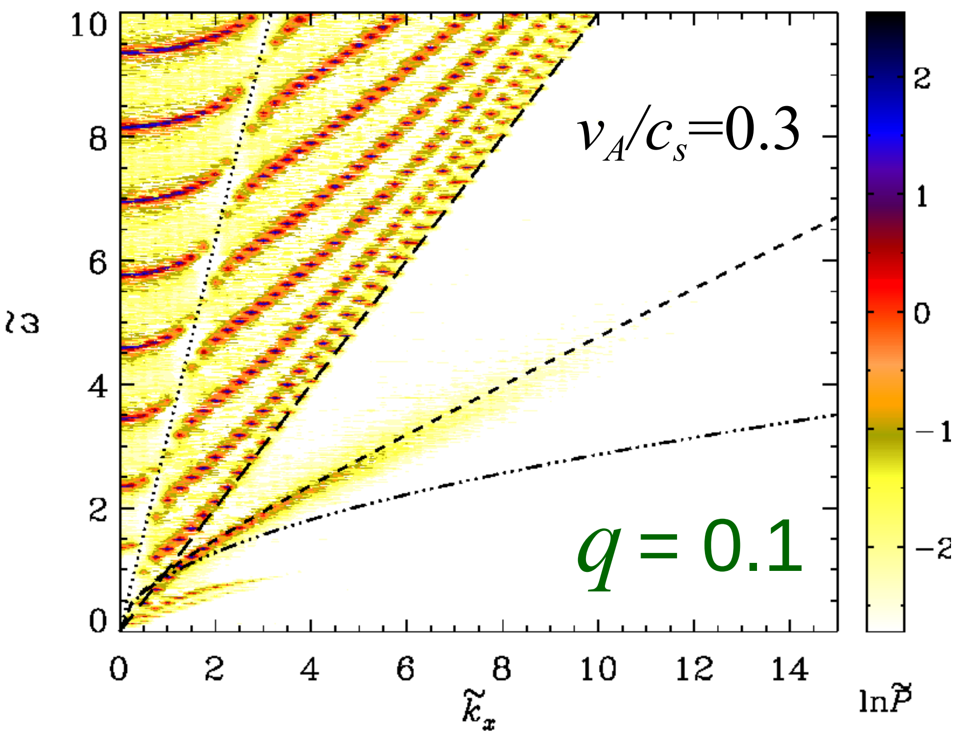
- Supports quarter waves as discussed in earlier works



See some discussions in: Leibacher & Stein (81); Stein & Leibacher (81); Duvall (82)

# Horizontal magnetic field

- We impose a uniform horizontal magnetic field in the entire domain
- Alfven speeds in the two subdomains: $v_{\mathrm{A}x\,\mathrm{d,u}} = \dfrac{B_x}{\sqrt{\mu_0 \rho_{\mathrm{d,u}}}}$




- Phase speed at a magnetic interface (no gravity):

$$c_{\mathrm{fm}}^2 = \frac{\rho_{\mathrm{u}} v_{\mathrm{A}x\mathrm{u}}^2 + \rho_{\mathrm{d}} v_{\mathrm{A}x\mathrm{d}}^2}{\rho_{\mathrm{u}} + \rho_{\mathrm{d}}}$$

$$= \frac{2\rho_{\mathrm{u}} v_{\mathrm{A}x\mathrm{u}}^2}{\rho_{\mathrm{d}} + \rho_{\mathrm{u}}} = \frac{2\rho_{\mathrm{d}} v_{\mathrm{A}x\mathrm{d}}^2}{\rho_{\mathrm{d}} + \rho_{\mathrm{u}}}$$

Kruskal & Schwarzschild 1954
Miles & Roberts 1989

- Dispersion relation in the presence of gravity:

$$\omega_{\mathrm{fm}}^2 = c_{\mathrm{fm}}^2 k_x^2 + g k_x \frac{1-q}{1+q}$$



Chandrasekhar 1961; Miles et al 1992

# Vertical/oblique magnetic field

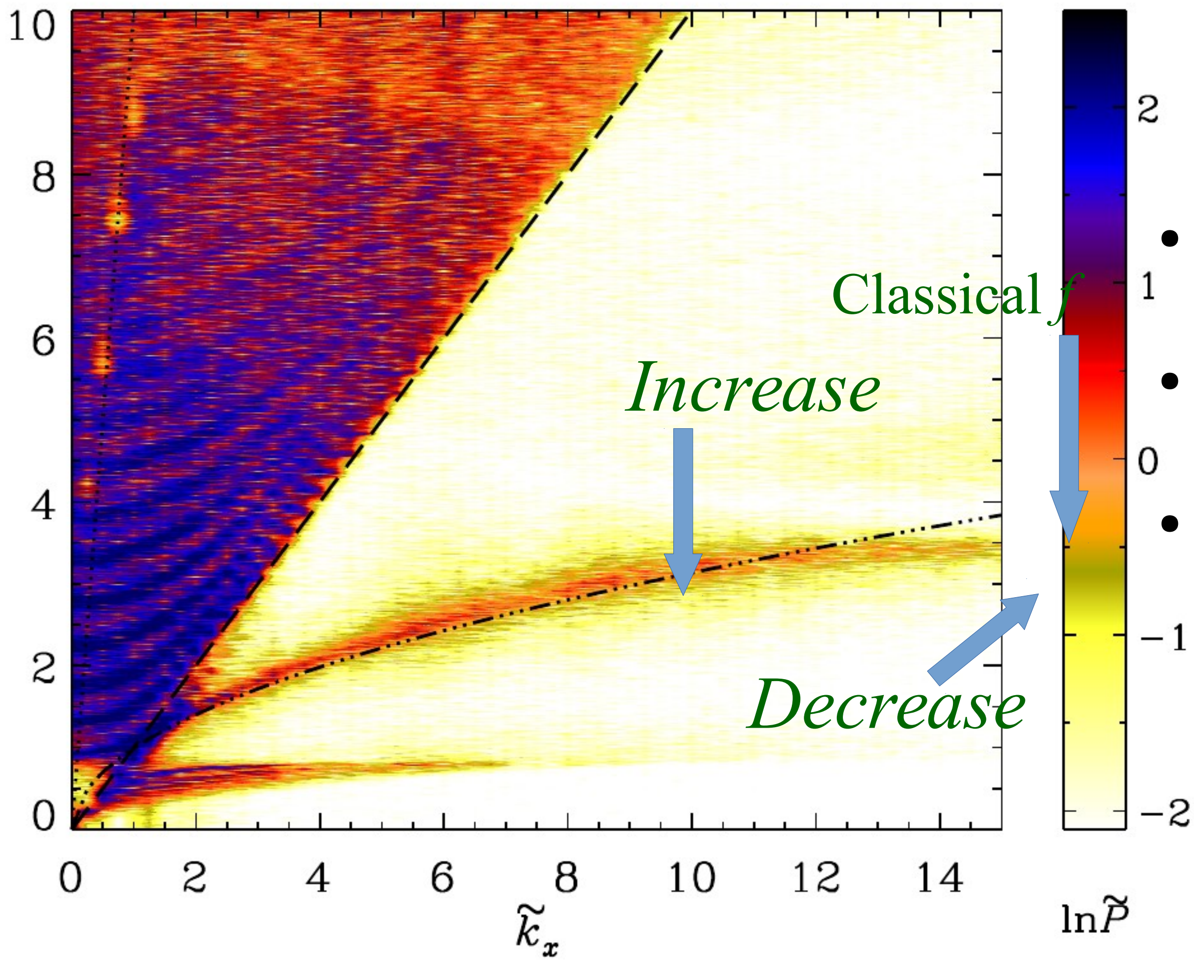
- We impose a uniform magnetic field in the entire domain
- Alfven speeds (vertical component) in the two subdomains

$$v_{\mathrm{A}z\,\mathrm{d,u}} = \frac{B_{z0}}{\sqrt{\mu_0 \rho_{\mathrm{d,u}}}}$$

## Vertical

$q = 0.01$

$v_A/c_s = 0.12$

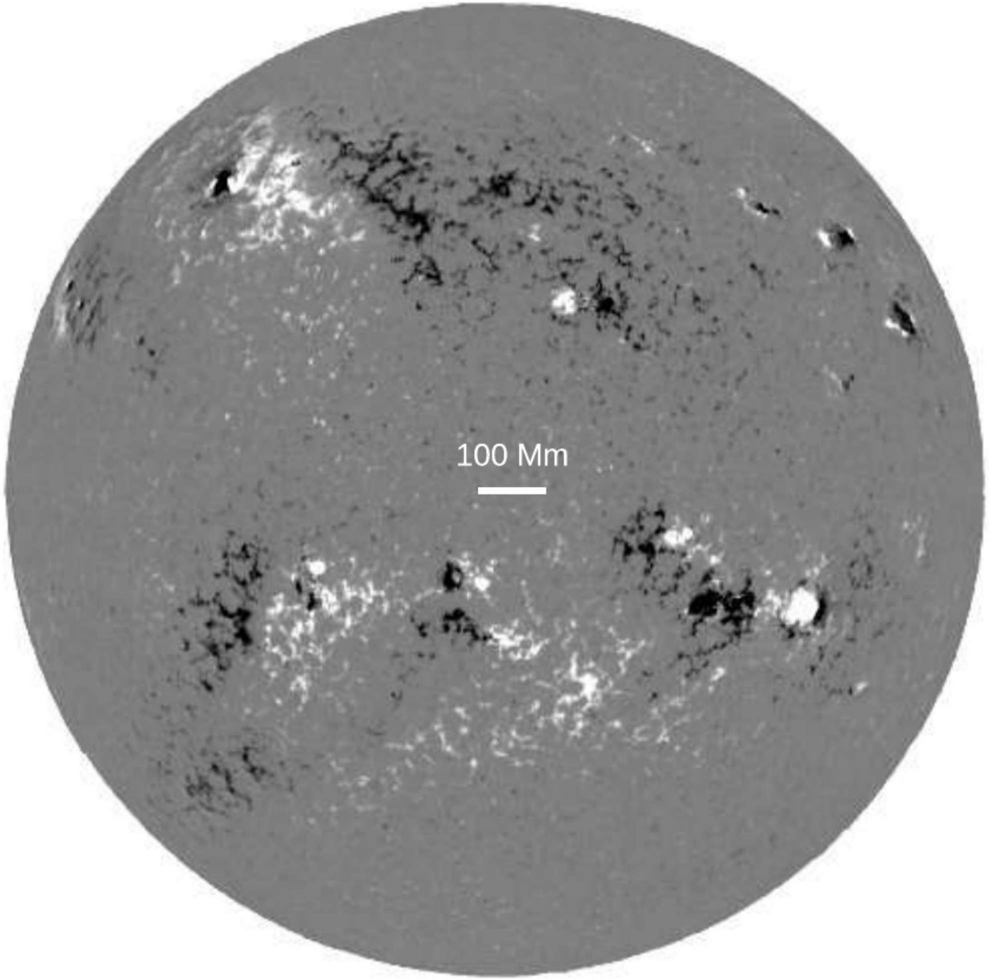



Some discussions:

- Libbrecht et al (90)
- Fernandes et al (92)
- Hindman & Zweibel (94)

# *Non-uniform magnetic field*
# *Fanning of the f-mode*

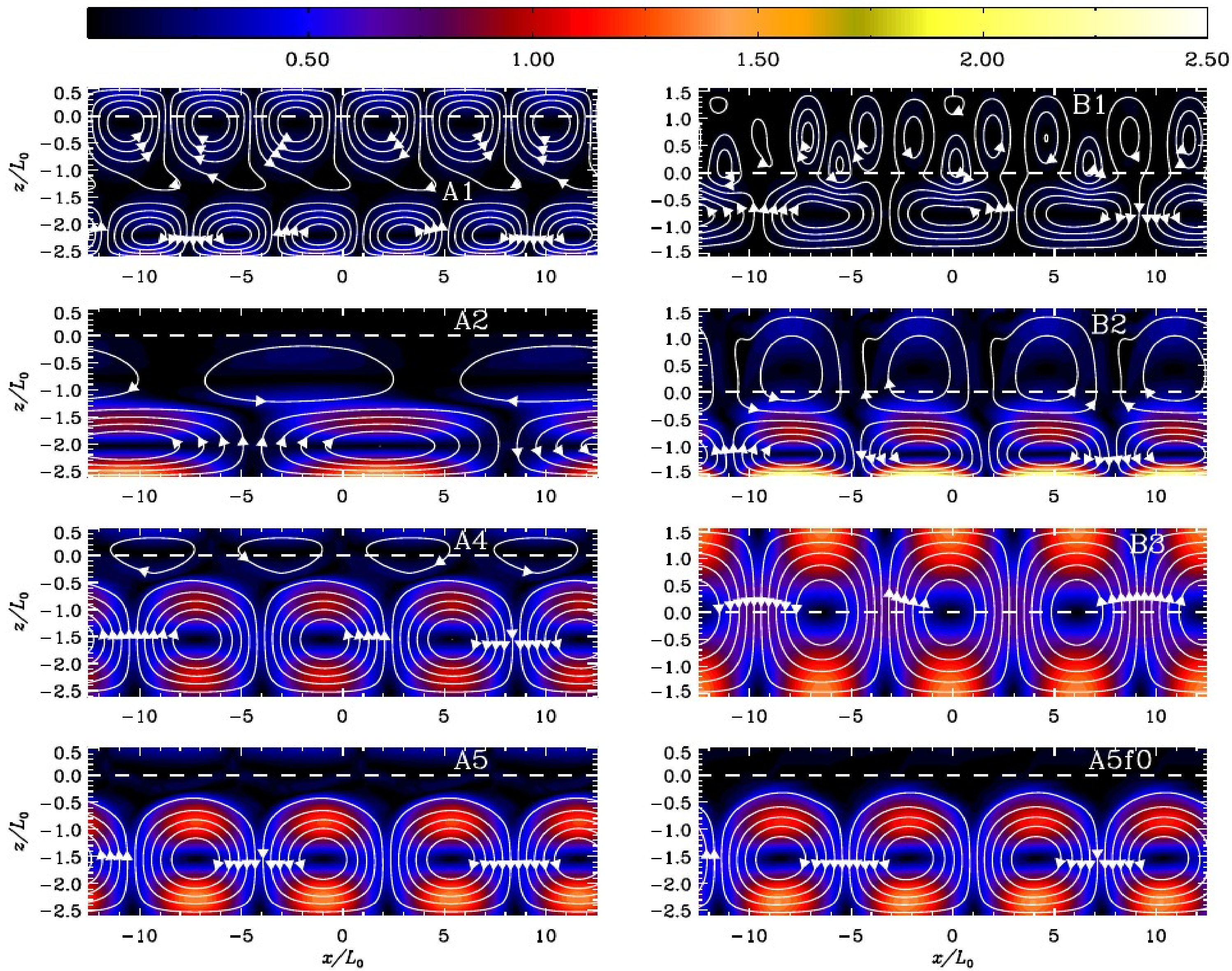
*Seeking motivation from an active phase of the Sun*




Magnetogram (MDI)

# Background magnetic field: nonuniform

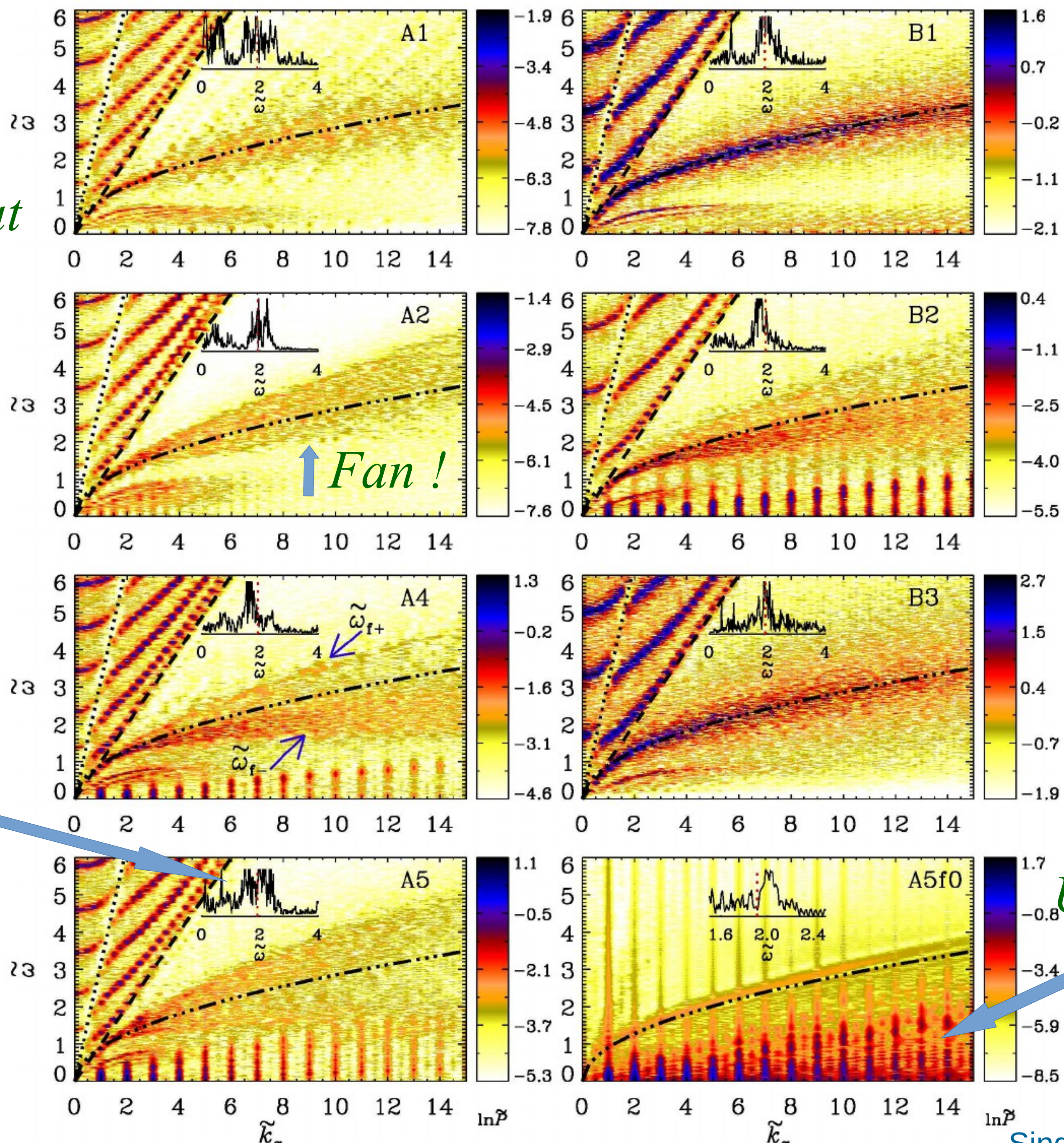
- Consider harmonically varying magnetic field in space
- Force the induction equation with $\mathcal{E}_{0y} = \hat{\mathcal{E}}_0 \cos(k_x^B x) \cos(k_z^B z)$



# Fanning out of the *f*-mode: a precursor transient?

Inset:
*f-mode profile at*
$\widetilde{k}_x = 4.5$

*Nearly top hat*

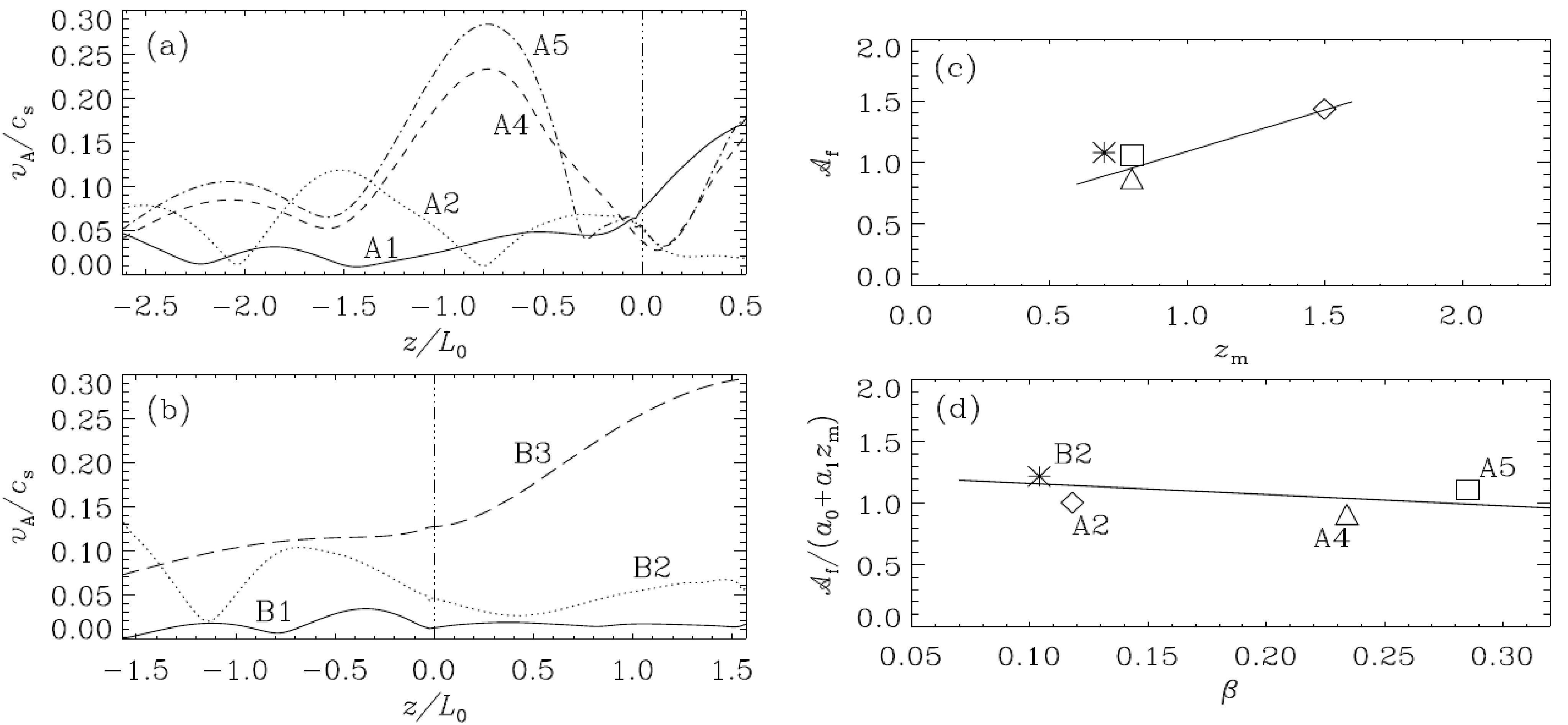



*Unstable modes*



Singh et al, ApJL (2014)

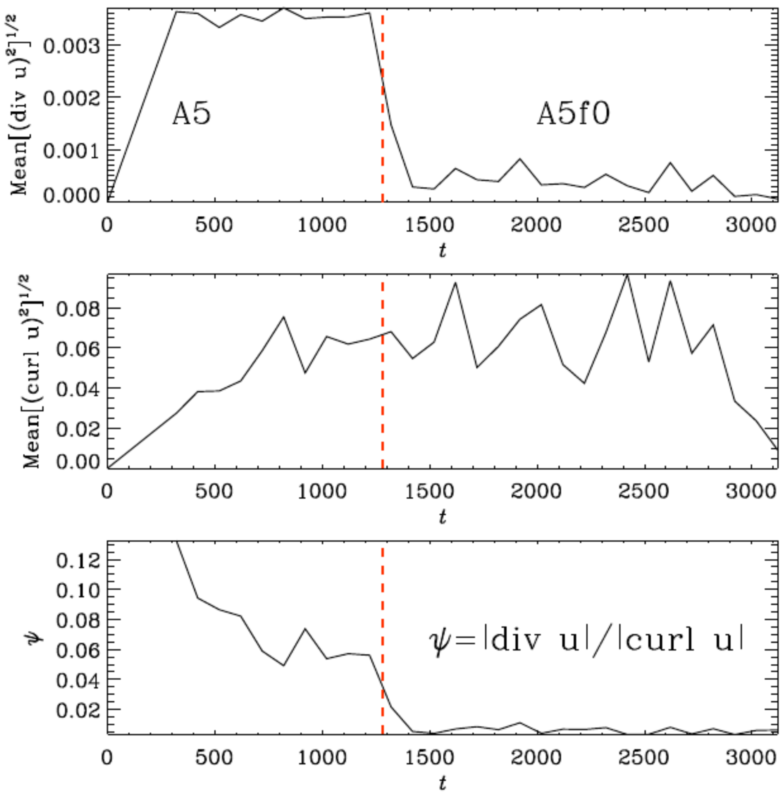
# Subsurface concentration and asymmetry




$$v_A(z) = \sqrt{\frac{\langle B^2 \rangle_x(z)}{\mu_0 \rho(z)}}, \quad \beta = \left(\max_{z \le 0} v_A(z)\right)/c_{sd}$$

# Vertical stripes: unstable eigenmodes

- Persistent after switching off hydrodynamic forcing (A5f0)
- Can be identified to indicate at least one *unstable* eigenmode
- The ratio $|\nabla \cdot \boldsymbol{u}|/|\nabla \times \boldsymbol{u}|$ → everywhere small for A5f0
- Velocity field of the stripes thus close to *solenoidal*
- Their occurrence and amplitude depend strongly on $\boldsymbol{B}_0$
- Notice the difference of *f*-mode in A5 and A5f0: *Fan disappears !*



# Implications for the Sun

- For the Sun : $$\gamma H_{\rm d} \approx 0.5\,{\rm Mm}$$

$H_{\rm d}$ is the pressure scale height

- The fan is best observed at

$$\widetilde{k}_x \gtrsim 3 \ \text{and} \ 1 \lesssim \widetilde{\omega} \lesssim 3$$

- For Sun with radius $R = 700$ Mm, this corresponds to

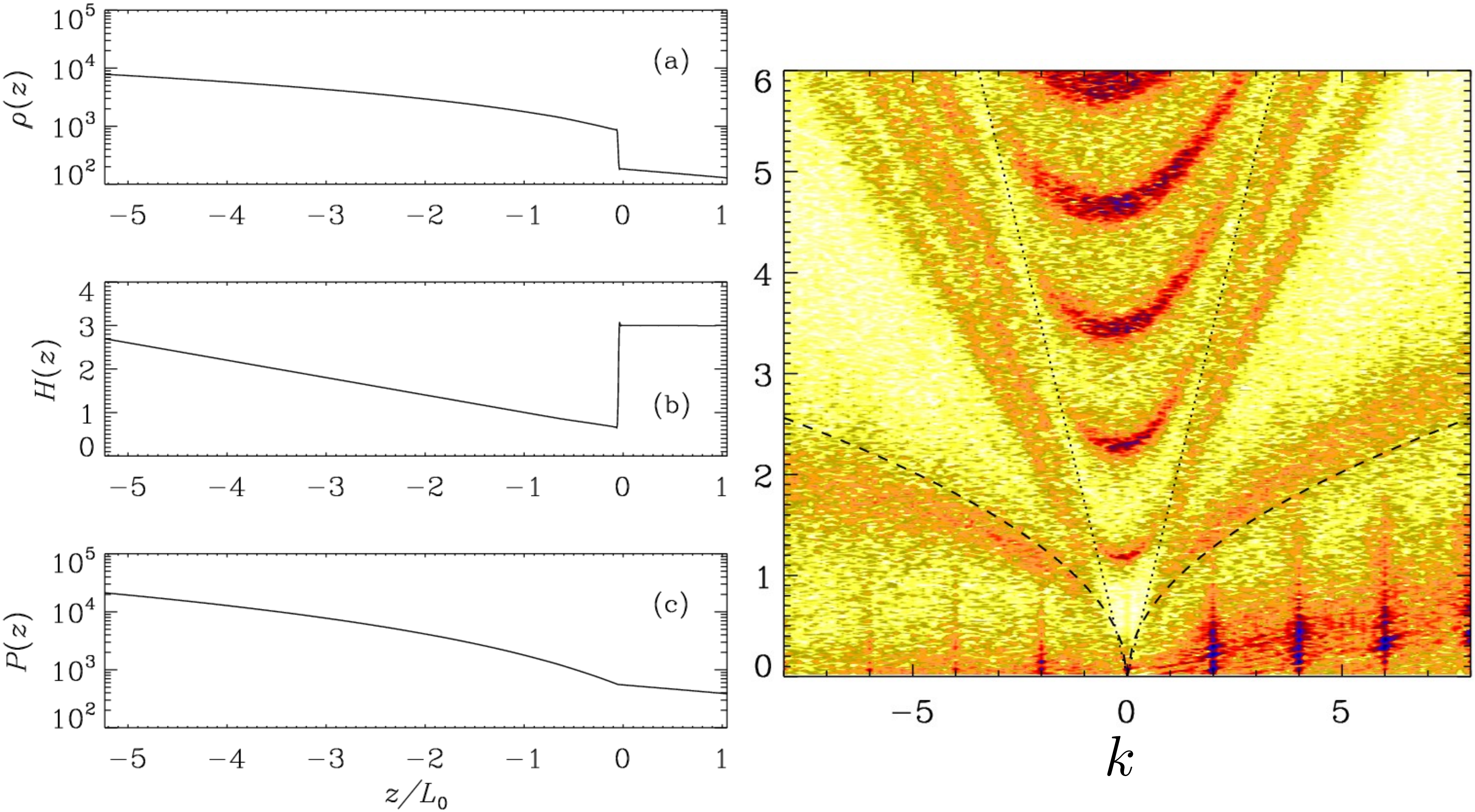
$$k_x \gtrsim 6\,{\rm Mm}^{-1}, \text{ or } \ell > 4200$$

- Cadence requirement : less than a minute



- Next: Do more realistic models help?

# Isentropic stratification with sub-surface shear



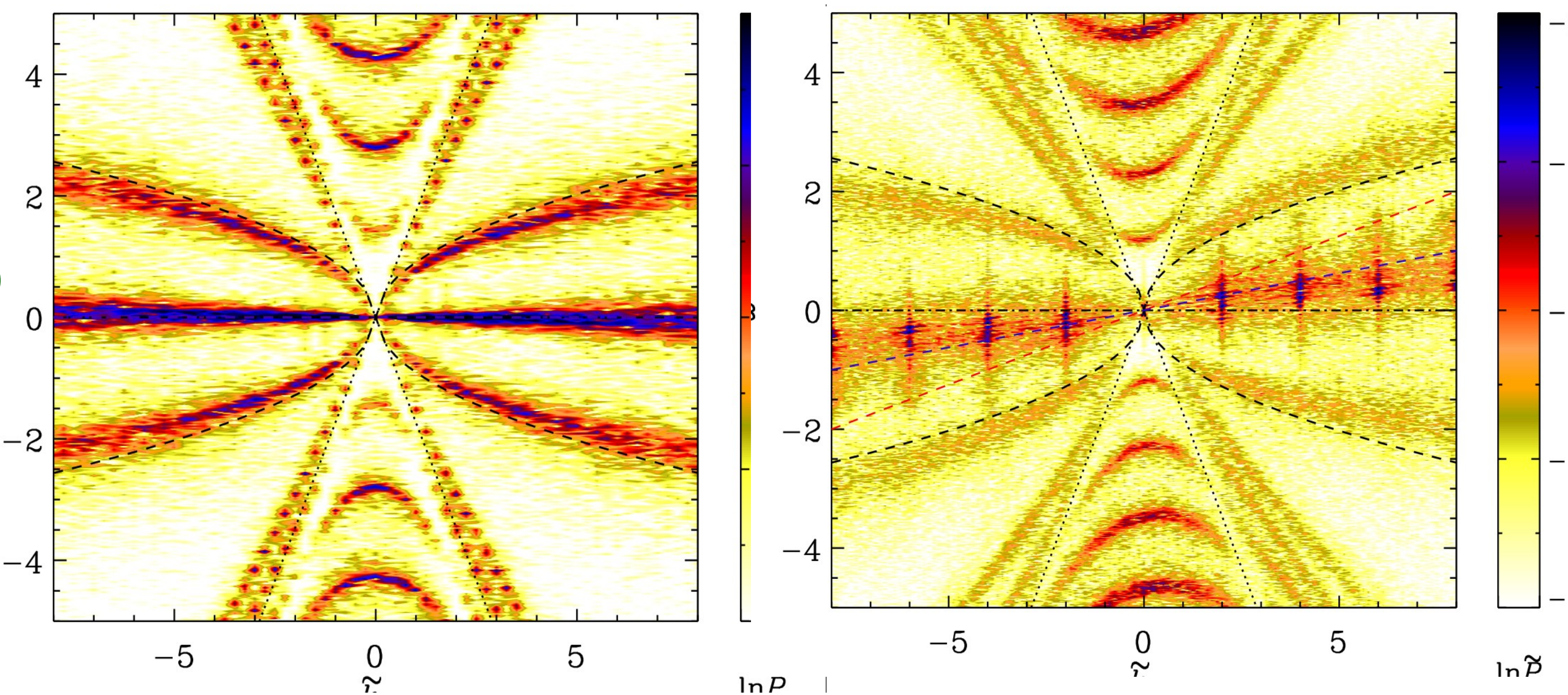

Notice: asymmetry of the *f*-mode, fan and *a*-mode

# Preliminary experiments (with/out shear)

Shear

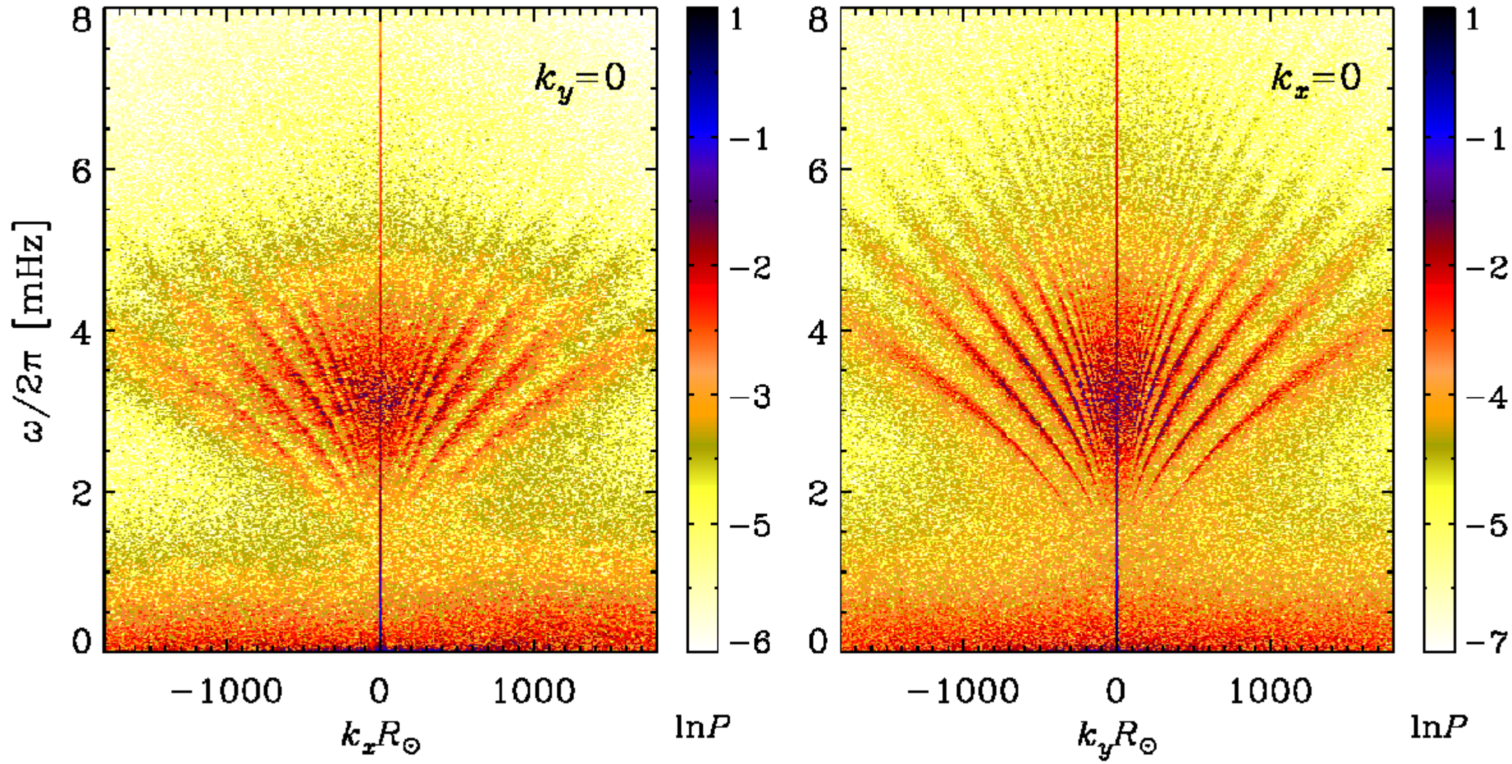
No Shear

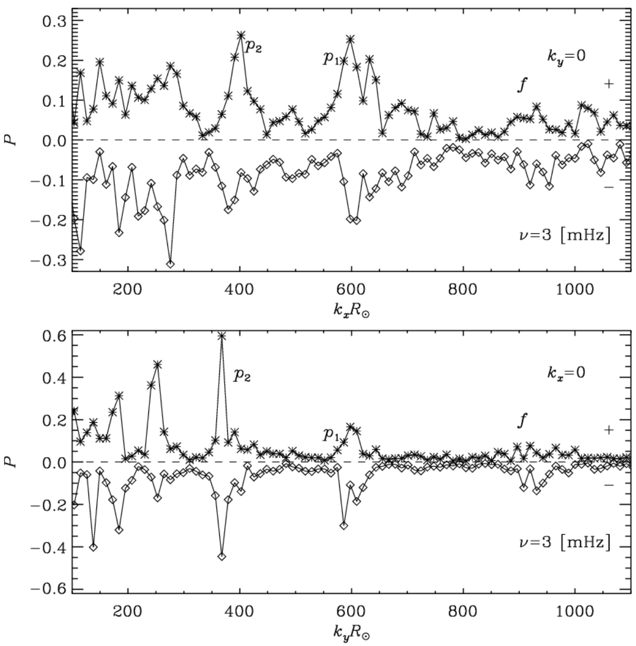
Omega




Horizontal wavenumber

# Preliminary analysis using HMI data

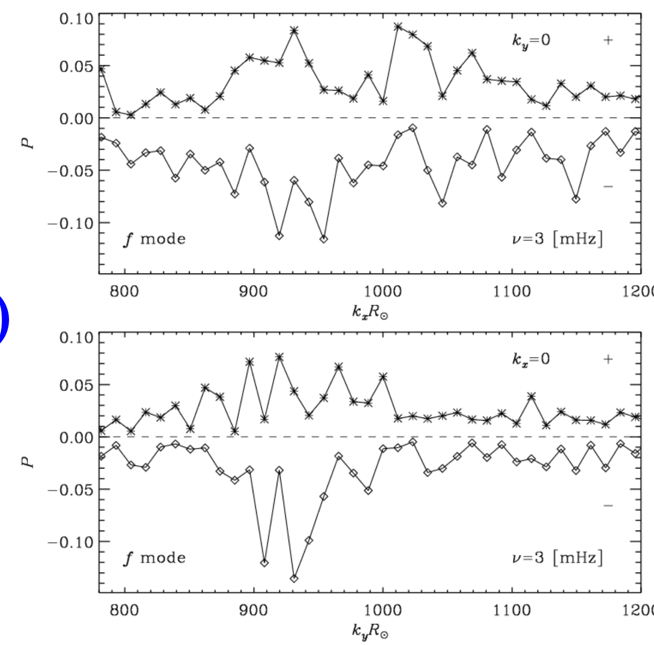



1024 x 1024 patch

24 hours of data (2010)

Line profiles at 3 mHz



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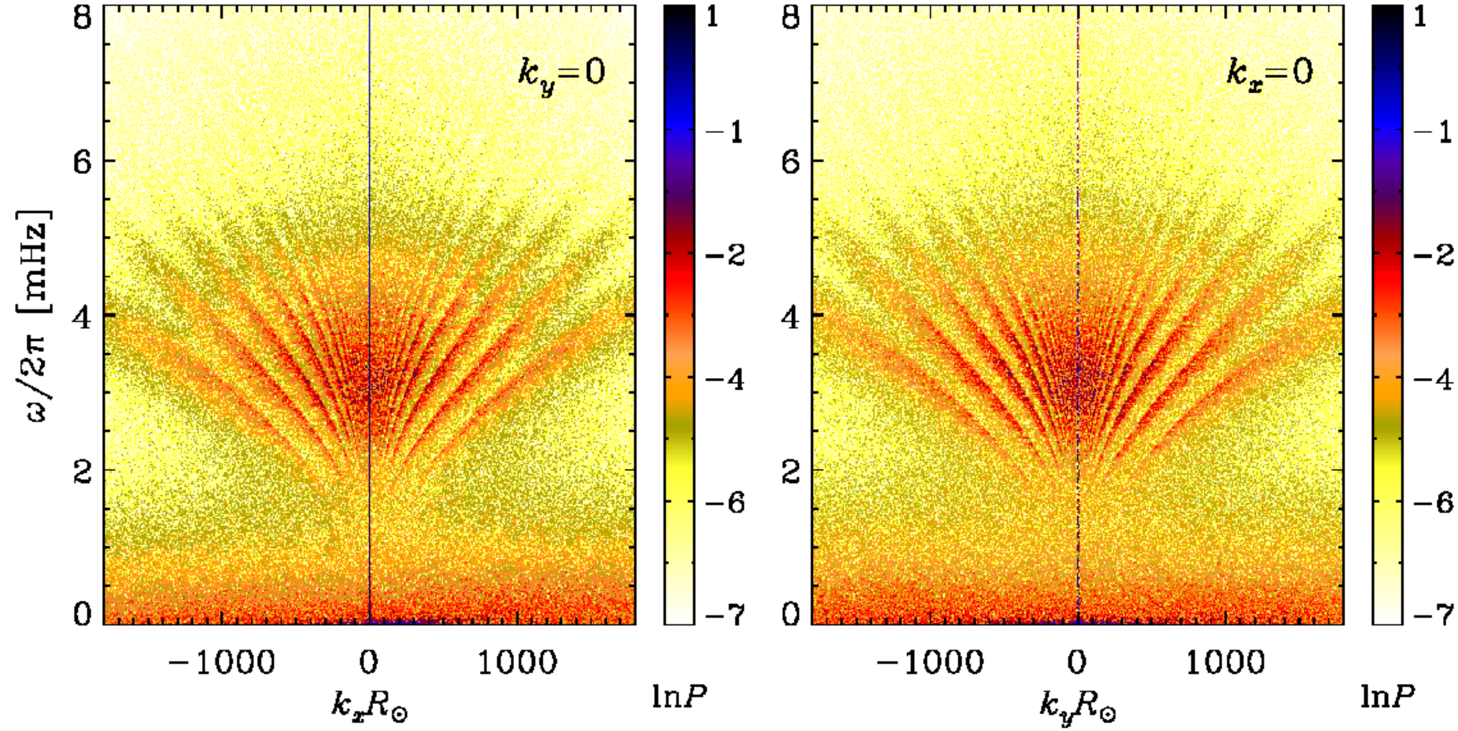
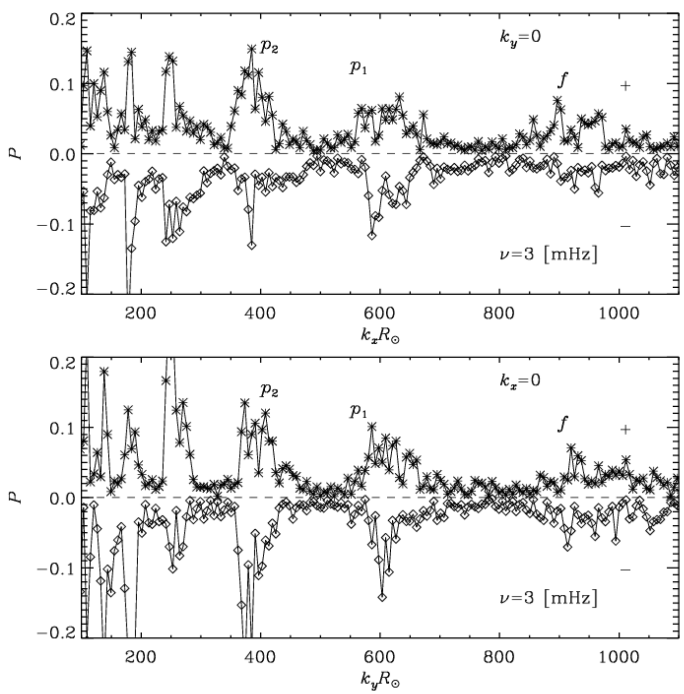
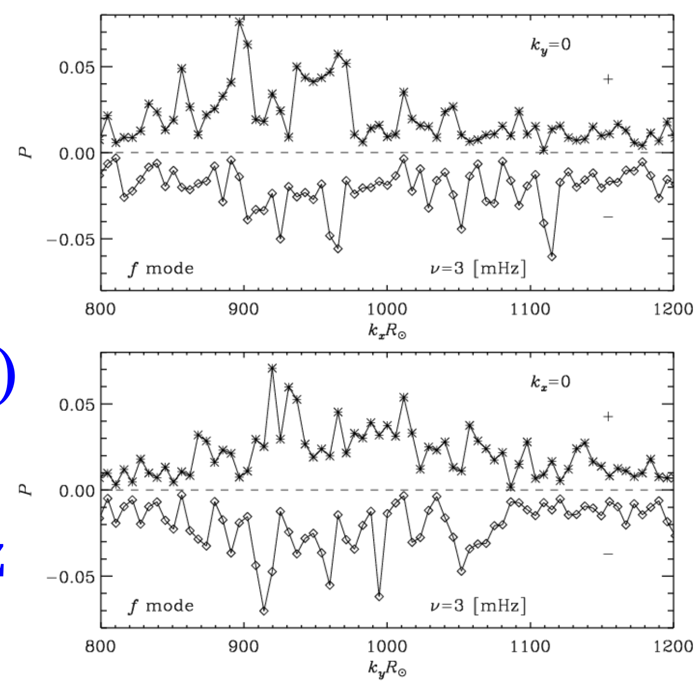



2048 x 2048 patch

24 hours of data (2010)

Line profiles at 3 mHz



# Conclusions

- (Uniform) Horizontal and vertical imposed magnetic fields have distinguishing effects
- Goal: To identify diagnostic signatures of nonuniform ***B***-field
- Fanning out of the *f*-mode and pattern of vertical stripes seen
- Fanning and its asymmetry depend on the strength and the location of field concentration beneath the interface

*Measurements of such fanning and its asymmetry with respect to the classical f-mode have the potential to determine the subsurface flux concentrations in the Sun*

→ *Precursors to the flux emergence at the surface ?*