Simulations of turbulent dynamos

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Pencil Code meeting 2015

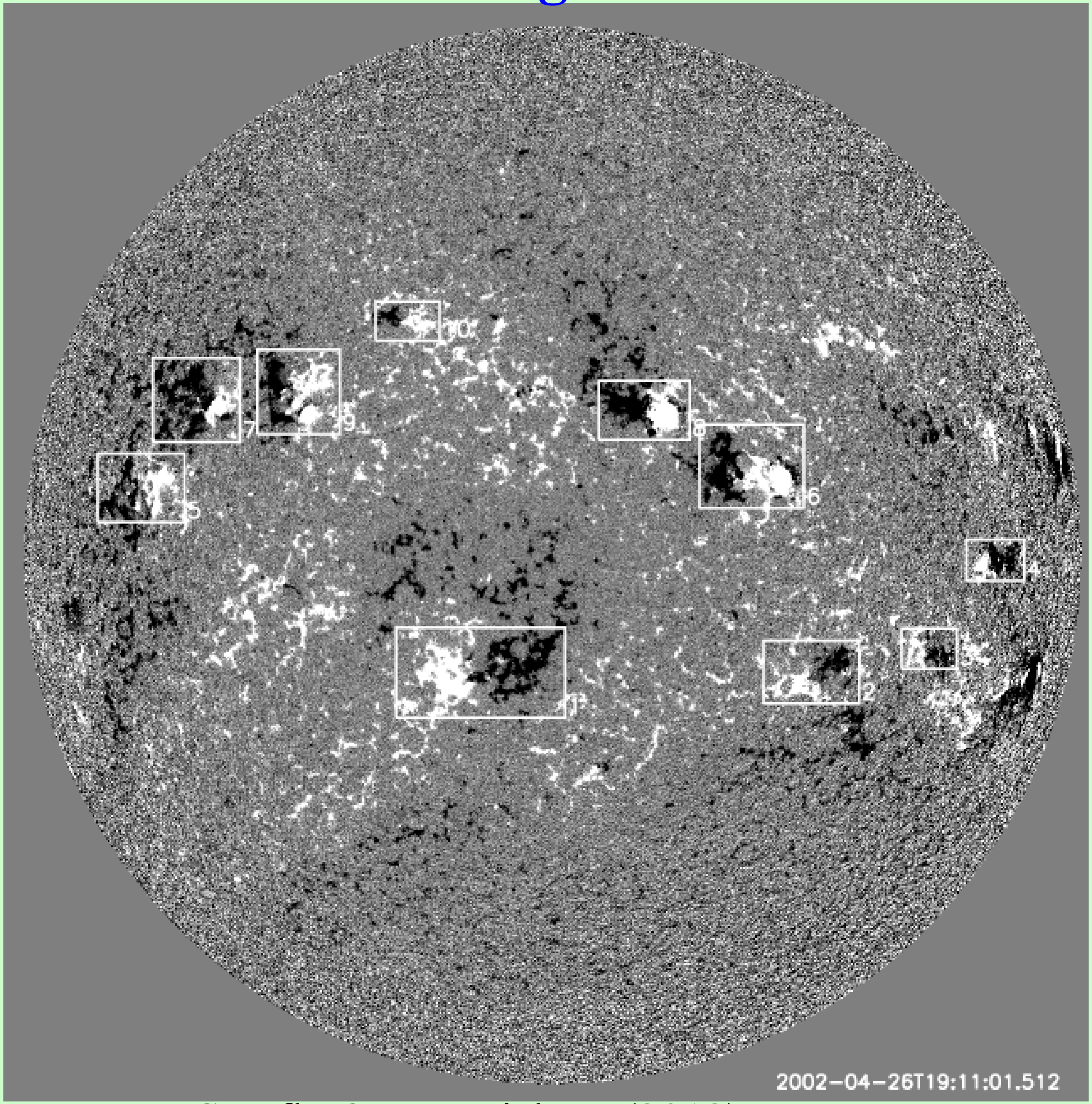
Large-scale magnetic field of Sun

Generated from the (global) large-scale dynamo operating in the solar convection zone.

Rotation: **Makes the convective motion helical (mirror-asymmetric)**

Non-uniform rotation: **Stretch the magnetic field lines.**

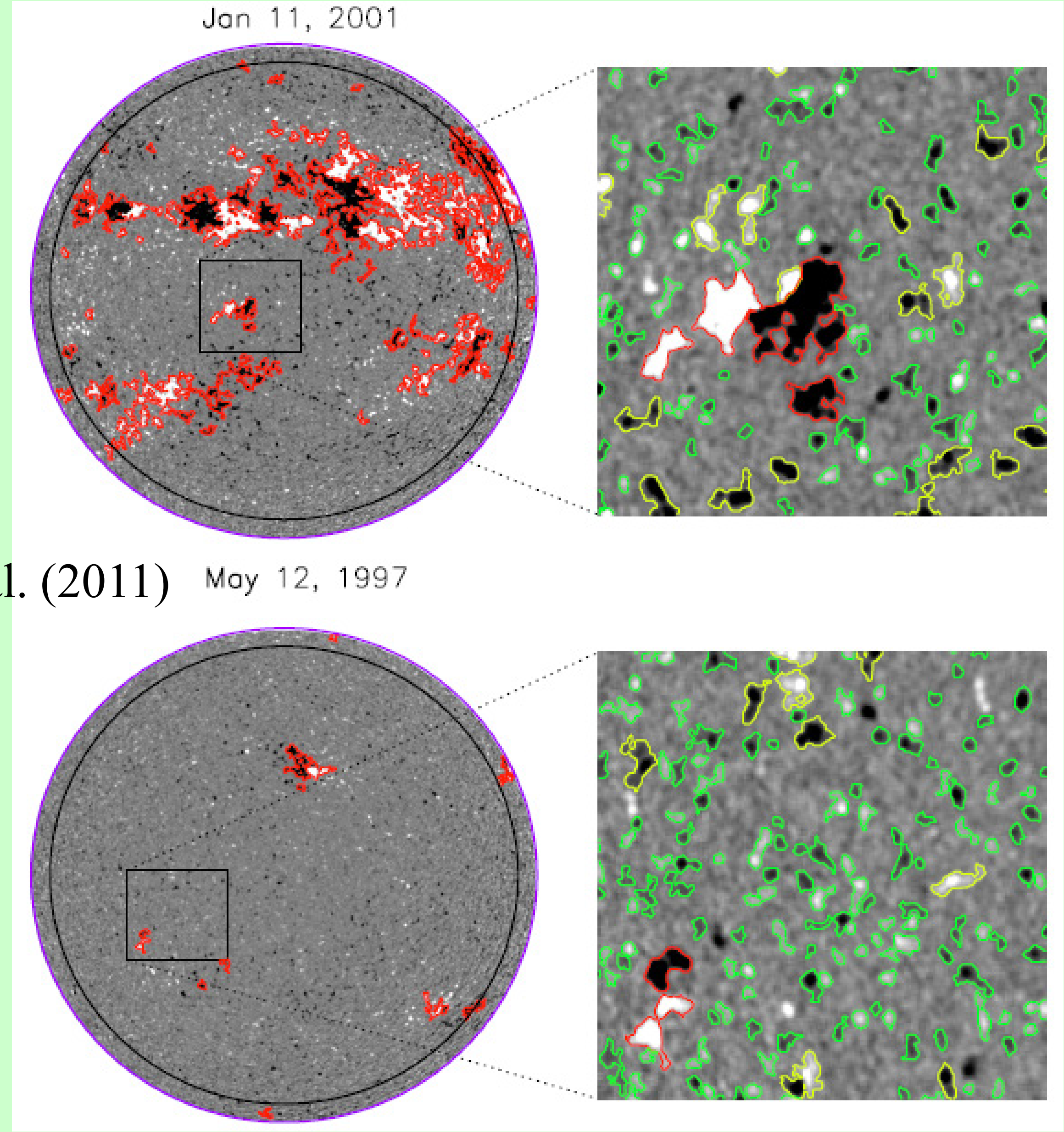
Small-scale magnetic field of Sun



Stenflo & Kosovichov (2012)

Vertical flux density map, calculated from the MDI line-of-sight magnetogram recorded on 2002 April 26.

Bipolar magnetic regions are enclosed in rectangular boxes. The gray-scale cuts are set at +100 G (white) and −100 G (black).



Lin et al. (2011)

MDI magnetogram:

Solar maximum

Solar minimum

Is it coming from the decay of active regions magnetic field (shredding and tangling of the large-scale magnetic field)? (Stenflo 2012

Observational supports of small-scale dynamo in SCZ

Small-scale magnetic field does not have solar cycle dependence, as it does not have any correlation with the large-scale global magnetic field, and it does not have any latitudinal dependence (Hagenaar et al. 2003; S´anchez Almeida 2003; Lites et al. 2008; Lites 2011).

Cyclic Variation of the Network Elements in Different Flux Ranges

Category	Flux (in Mx)	Number ratio	Flux (ratio)	Cor.
No Correlation	$(1.5–2.9) \times 10^{18}$	0.58%	6.48×10^{21} (0.05%)	–0.04
Anti-correlation	$(2.9–32.0) \times 10^{18}$	77.19%	4.72×10^{24} (37.40%)	–0.45
Transition	$(3.20–4.27) \times 10^{19}$	6.59%	1.15×10^{24} (9.08%)	–0.03
Correlation	$(4.27–38.01) \times 10^{19}$	15.65%	6.74×10^{24} (53.46%)	0.82

Jin et al. (2011)

Jin & Wang (2012)

Solar small-scale dynamo and polarity of sunspot groups

D. Sokoloff[1⋆], A. Khlystova[2†], V. Abramenko[3‡]

...ps obtained by several scientific teams, including ours, to find that the percentage of anti-Hale groups becomes indeed maximal during a solar minimum. Our interpretation is that this fact may be explained by the small-scale dynamo action inside the solar convective zone.

Small-scale dynamo/local dynamo

(Do not call it turbulent dynamo or fluctuating dynamo!)

T*hree-dimensional* velocity fields sufficiently random in space and/or time will amplify small-scale magnetic fluctuations via random stretching of the field lines (Batchelor 1950; Zel'dovich et al. 1984; Childress & Gilbert 1995).

No *net helicity* is required!

We believe that this small-scale dynamo can be studied in a homogeneous and isotropic setting. However difficult to simulate at solar parameters.

We want to setup a dynamo model which allows to excite both large-scale and small-scale dynamo, but it is computationally not very expensive!

A simple setup for dynamo simulations

$$\frac{D\boldsymbol{U}}{Dt} = -SU_x\hat{y} - c_{\rm s}^2\boldsymbol{\nabla}\ln\rho + \rho^{-1}\left[\boldsymbol{J}\times\boldsymbol{B} + \boldsymbol{\nabla}\cdot(2\rho\nu\mathsf{S})\right] + \boldsymbol{f}, \tag{2}$$

$$\frac{D\ln\rho}{Dt} = -\boldsymbol{\nabla}\cdot\boldsymbol{U}, \tag{3}$$

$$\frac{\partial\boldsymbol{A}}{\partial t} + \overline{\boldsymbol{U}}^{(S)}\cdot\boldsymbol{\nabla}\boldsymbol{A} = -SA_y\hat{x} + \boldsymbol{U}\times\boldsymbol{B} + \eta\boldsymbol{\nabla}^2\boldsymbol{A}. \tag{4}$$

Isothermal & compressible gas.
Periodic box, imposed large-scale shear,

$$\boldsymbol{f}(\boldsymbol{x},t) = \mathrm{Re}\{N\boldsymbol{f}_{\boldsymbol{k}(t)}\exp[\mathrm{i}\boldsymbol{k}(t)\cdot\boldsymbol{x} + \mathrm{i}\phi(t)]\}$$

$$\boldsymbol{f}_{\boldsymbol{k}} = \mathsf{R}\cdot\boldsymbol{f}_{\boldsymbol{k}}^{(\mathrm{nohel})} \quad \text{with} \quad \mathsf{R}_{ij} = \frac{\delta_{ij} - \mathrm{i}\sigma\epsilon_{ijk}\hat{k}_k}{\sqrt{1+\sigma^2}}$$

turbulence is generated artificially by helically forced flow.

Here $D/Dt = \partial/\partial t + (\boldsymbol{U} + \overline{\boldsymbol{U}}^{(S)})\cdot\boldsymbol{\nabla}$ is the advective time derivative, $\overline{\boldsymbol{U}}^{(S)} = (0, Sx, 0)$ with $S = \mathrm{const}$ is the

In Pencil Code

```
!  run parameters for forced helical
MHD turbulence
!
&run_pars
…..
…..
…..
/
&forcing_run_pars
  iforce='helical', force=.02, relhel=1.
  lmomentum_ff=T
/
&magnetic_run_pars
  eta=8e-3
/
&shear_run_pars
  Sshear=-.20
/
…..
…..
/
```

```
!  calculate and add forcing function
!
        select case (iforce)
        case ('2drandom_xy');     call forcing_2drandom_xy(f)
        case ('2drxy_simple');    call forcing_2drandom_xy_sim
        case ('ABC');             call forcing_ABC(f)
        case ('blobs');           call forcing_blobs(f)
        case ('chandra_kendall'); call forcing_chandra_kendall(
        case ('cktest');          call forcing_cktest(f)
        case ('diffrot');         call forcing_diffrot(f,force)
        case ('fountain', '3');   call forcing_fountain(f)
        case ('gaussianpot');     call forcing_gaussianpot(f,force)
        case ('white_noise');     call forcing_white_noise(f)
        case ('GP');              call forcing_GP(f)
        case ('irrotational');    call forcing_irro(f,force)
        case ('helical', '2');    call forcing_hel(f)
        case ('helical_both');    call forcing_hel_both(f)
        case ('helical_kprof');   call forcing_hel_kprof(f)
        case ('hel_smooth');      call forcing_hel_smooth(f)
        case ('horiz-shear');     call forcing_hshear(f)
        case ('nocos');           call forcing_nocos(f)
        case ('TG');              call forcing_TG(f)
        case ('twist');           call forcing_twist(f)
```

!**

subroutine forcing_hel(f)

!

! Add helical forcing function, using a set of precomputed wavevectors.

! The relative helicity of the forcing function is determined by the factor

! sigma, called here also relhel. If it is +1 or -1, the forcing is a fully

! helical Beltrami wave of positive or negative helicity. For |relhel| < 1

! the helicity less than maximum. For relhel=0 the forcing is nonhelical.

! The forcing function is now normalized to unity (also for |relhel| < 1).

!

$$\boldsymbol{f}(\boldsymbol{x},t)=\mathrm{Re}\{N\boldsymbol{f}_{\boldsymbol{k}(t)}\exp[\mathrm{i}\boldsymbol{k}(t)\cdot\boldsymbol{x}+\mathrm{i}\phi(t)]\}$$

! 10-apr-00/axel: coded

$$-\pi<\phi(t)\leqslant\pi$$

$$N=f_0c_{\mathrm{s}}\left(|\boldsymbol{k}|c_{\mathrm{s}}/\delta t\right)^{1/2}$$

! 3-sep-02/axel: introduced k1_f ... function if k1/=1.

! 25-sep-02/axel: preset force_ampl to unity (in case slope is not

controll

$$\boldsymbol{f}_{\boldsymbol{k}}=\mathsf{R}\cdot\boldsymbol{f}_{\boldsymbol{k}}^{(\mathrm{nohel})}\quad\text{with}\quad\mathsf{R}_{ij}=\frac{\delta_{ij}-\mathrm{i}\sigma\epsilon_{ijk}\hat{k}_k}{\sqrt{1+\sigma^2}}$$

$$|\boldsymbol{f}_{\boldsymbol{k}}|^2=1$$

! 9-no ... case |relhel| < 1.

! 23-feb-10/axel: added helicity profile with finite second derivative.

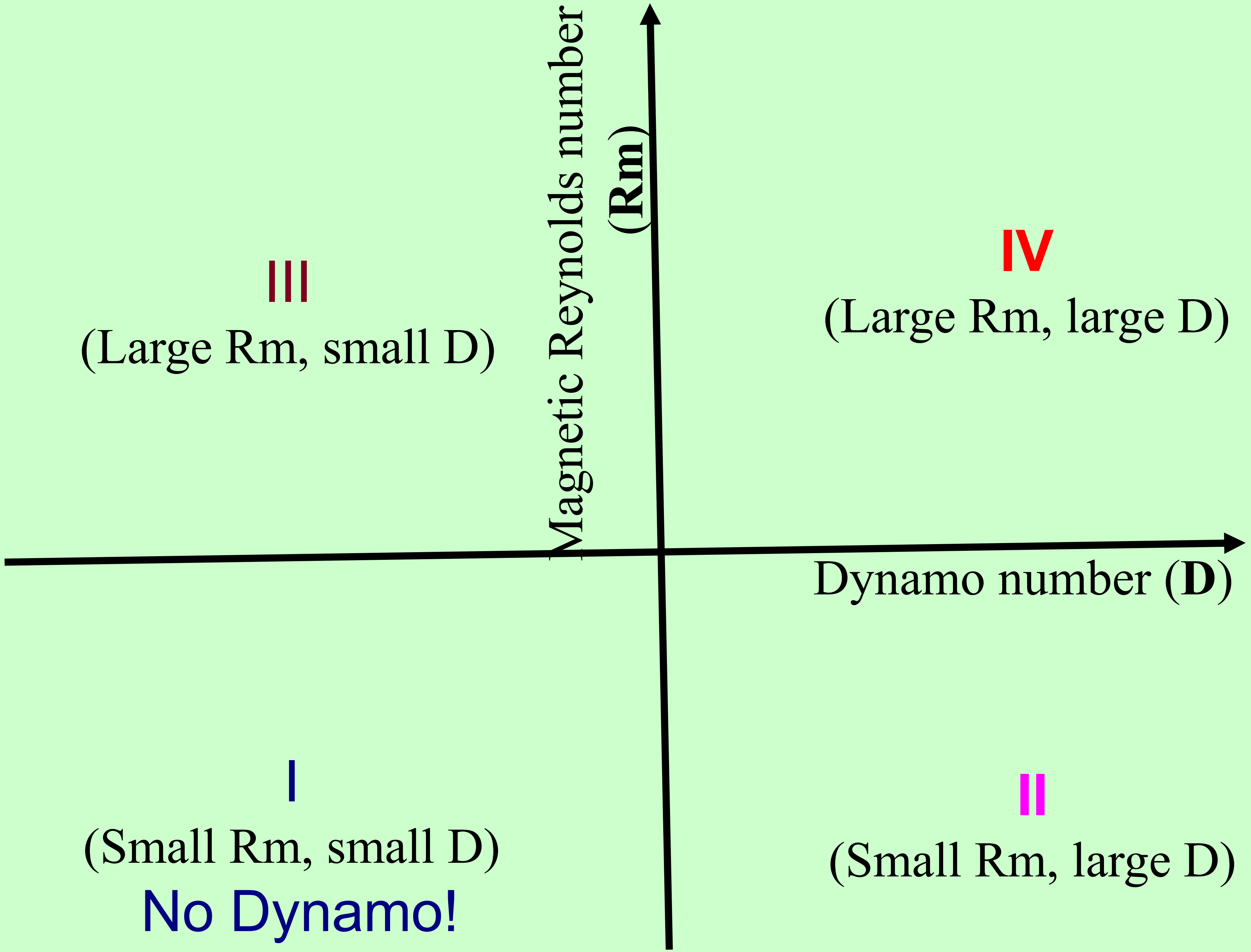
! 13- ... of f

$$\boldsymbol{f}_{\boldsymbol{k}}^{(\mathrm{nohel})}=\left(\boldsymbol{k}\times\hat{\mathrm{e}}\right)\Big/\sqrt{\boldsymbol{k}^2-\left(\boldsymbol{k}\cdot\hat{\mathrm{e}}\right)^2}$$

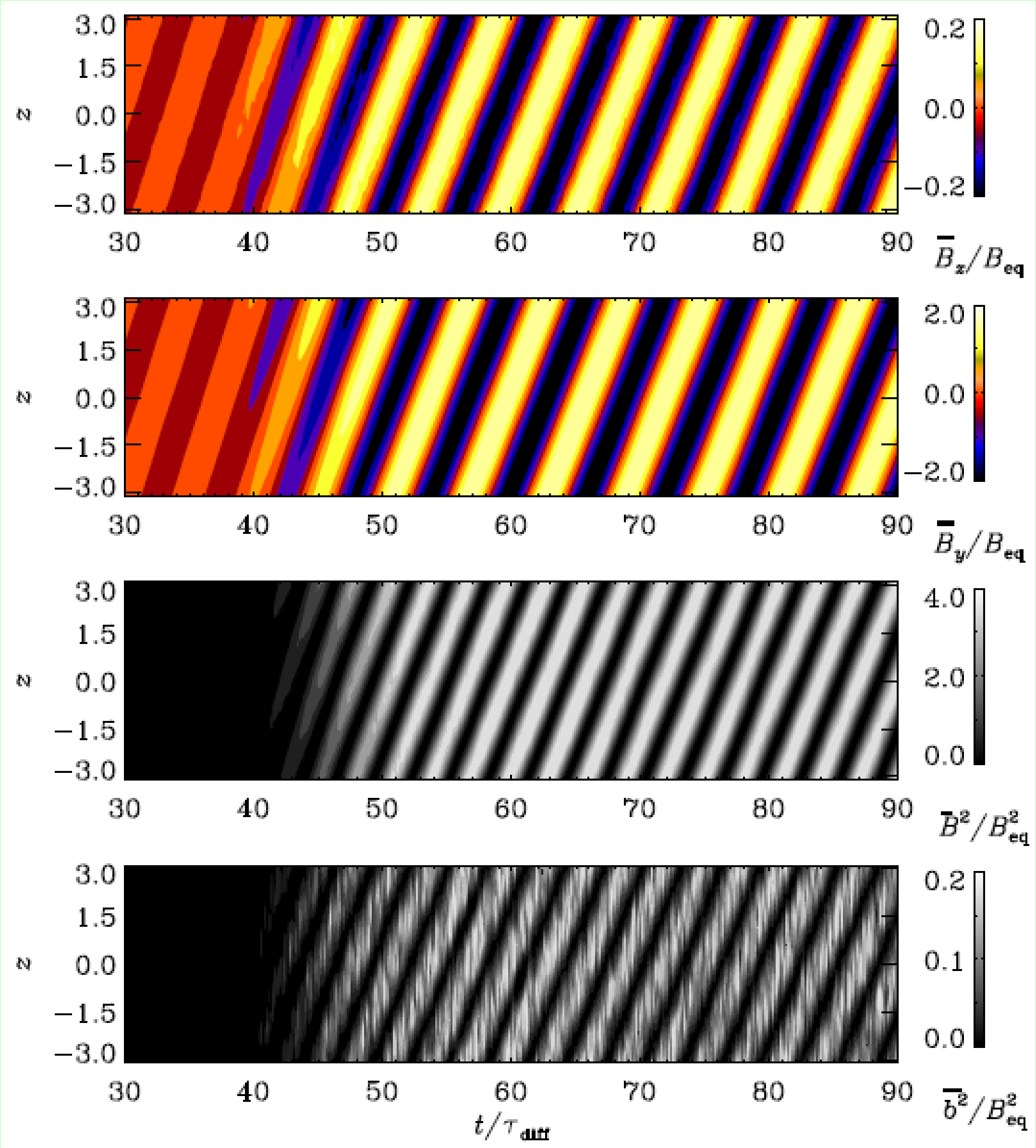
$$\boldsymbol{f}_{\boldsymbol{k}}\cdot\left(i\boldsymbol{k}\times\boldsymbol{f}_{\boldsymbol{k}}\right)^*=2\sigma k/(1+\sigma^2)$$

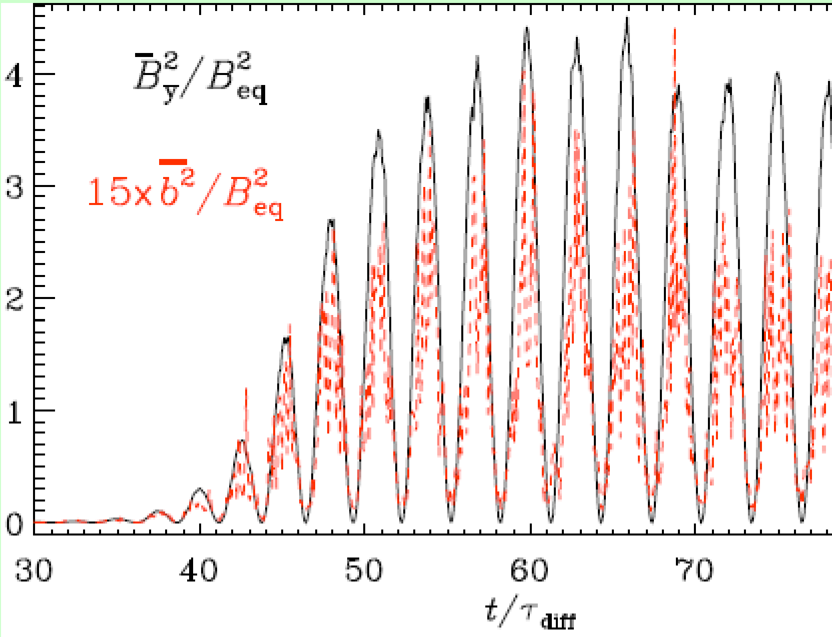
! 06- ... catable

!


Magnetic Reynolds number
(**Rm**)
Dynamo number (**D**)
III
(Large Rm, small D)
IV
(Large Rm, large D)
I
(Small Rm, small D)
No Dynamo!
II
(Small Rm, large D)

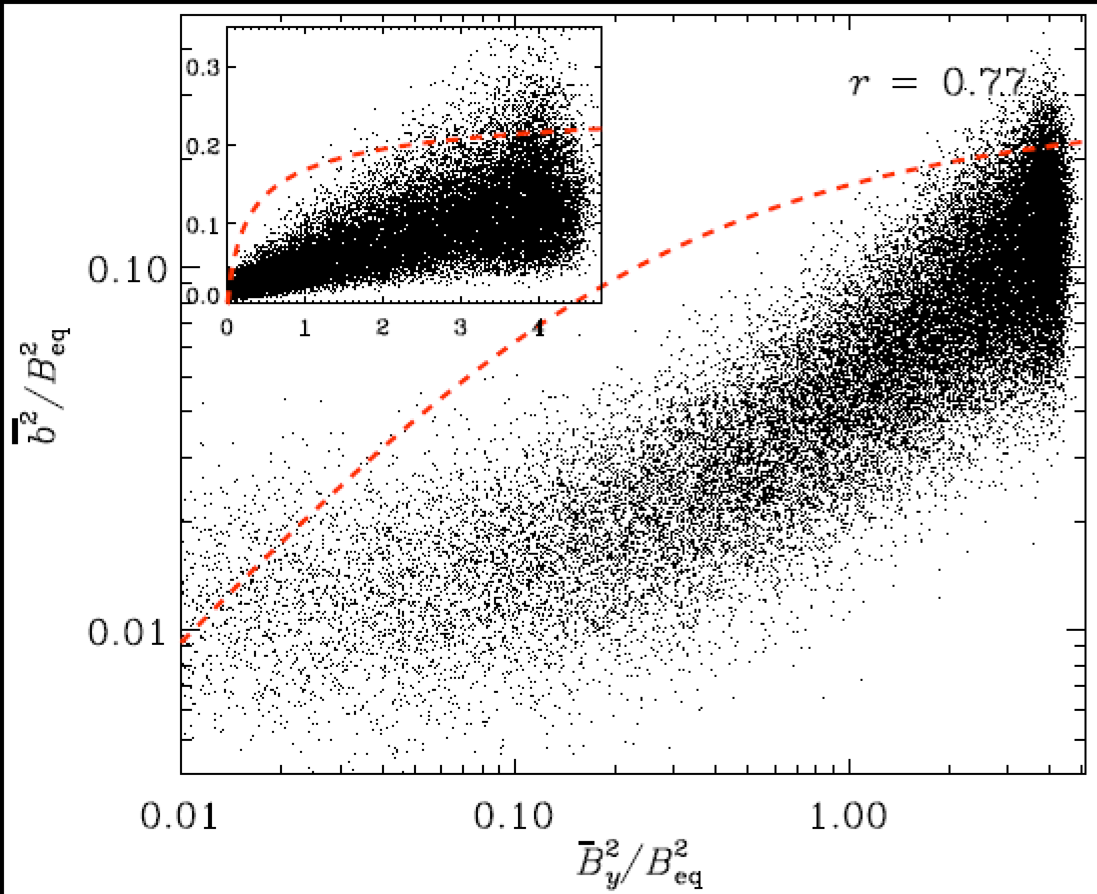
Dynamo simulations in Regime II (small Rm, large D)



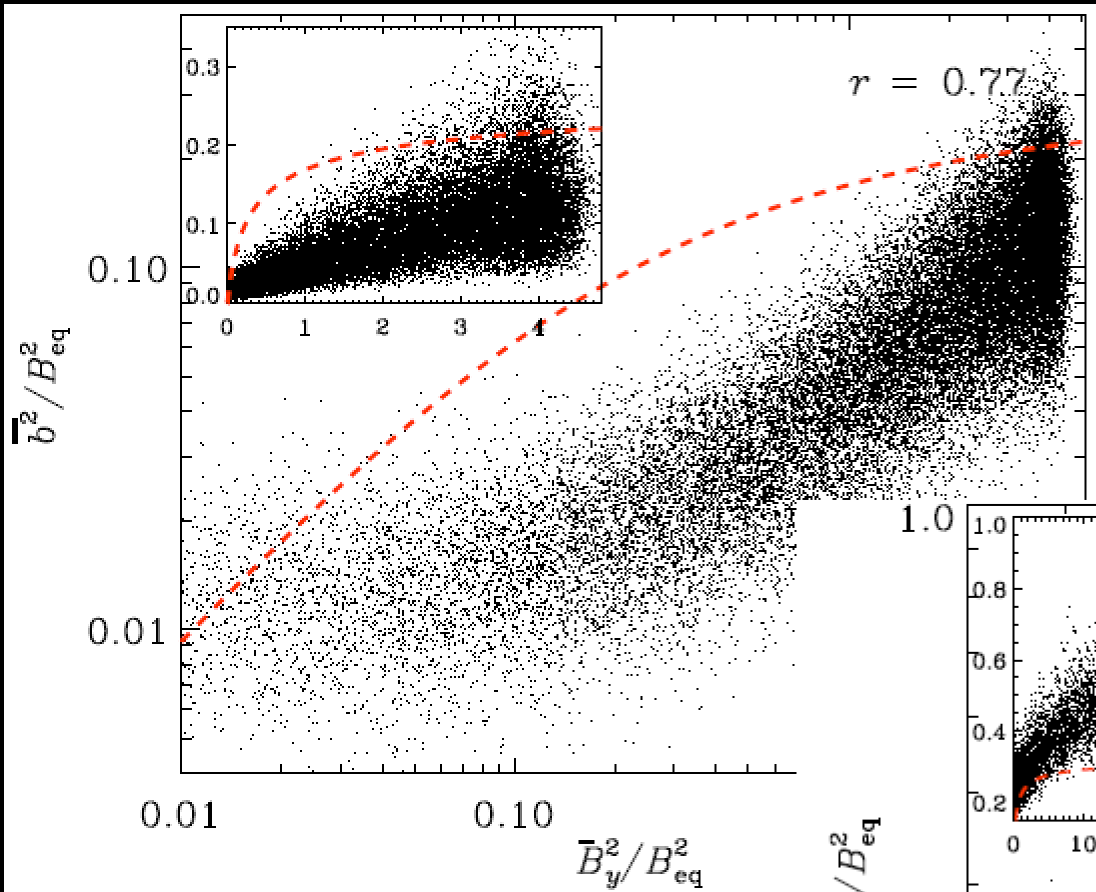


$$\overline{b^2} = \overline{B^2} - \overline{B}^2$$

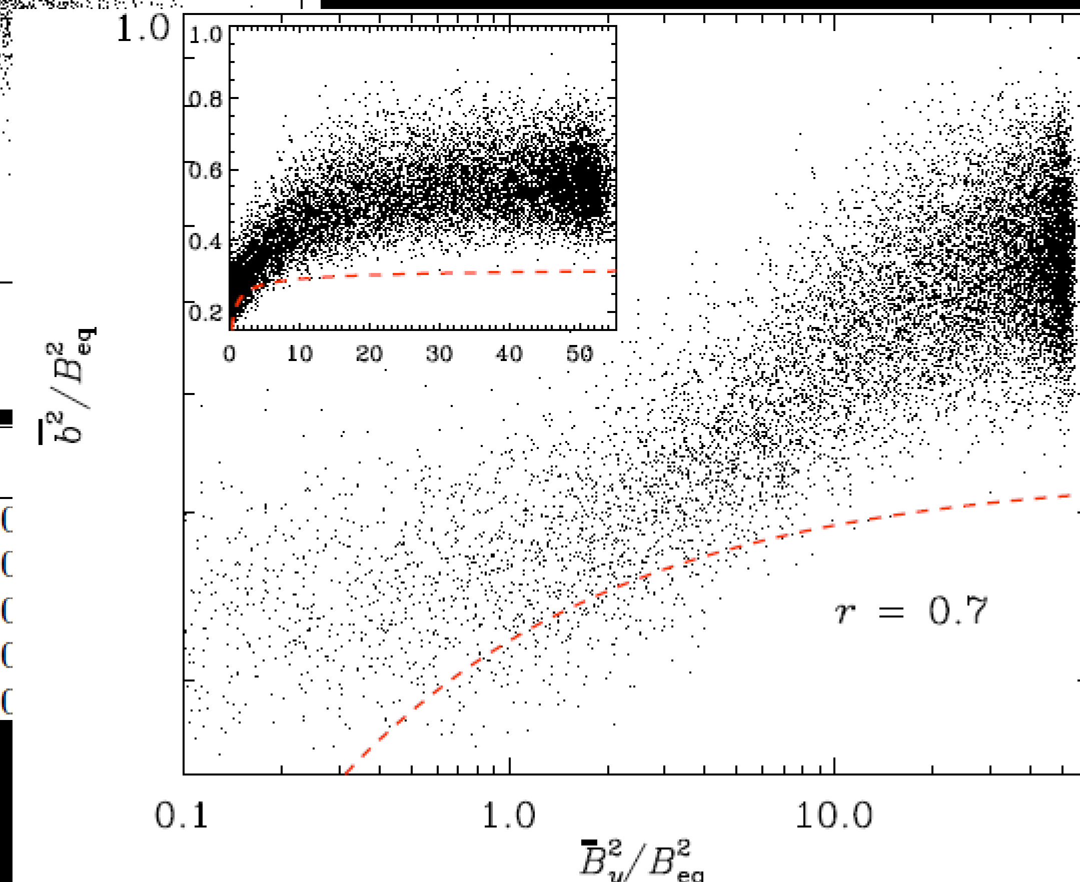
Mean is defined as xy averages

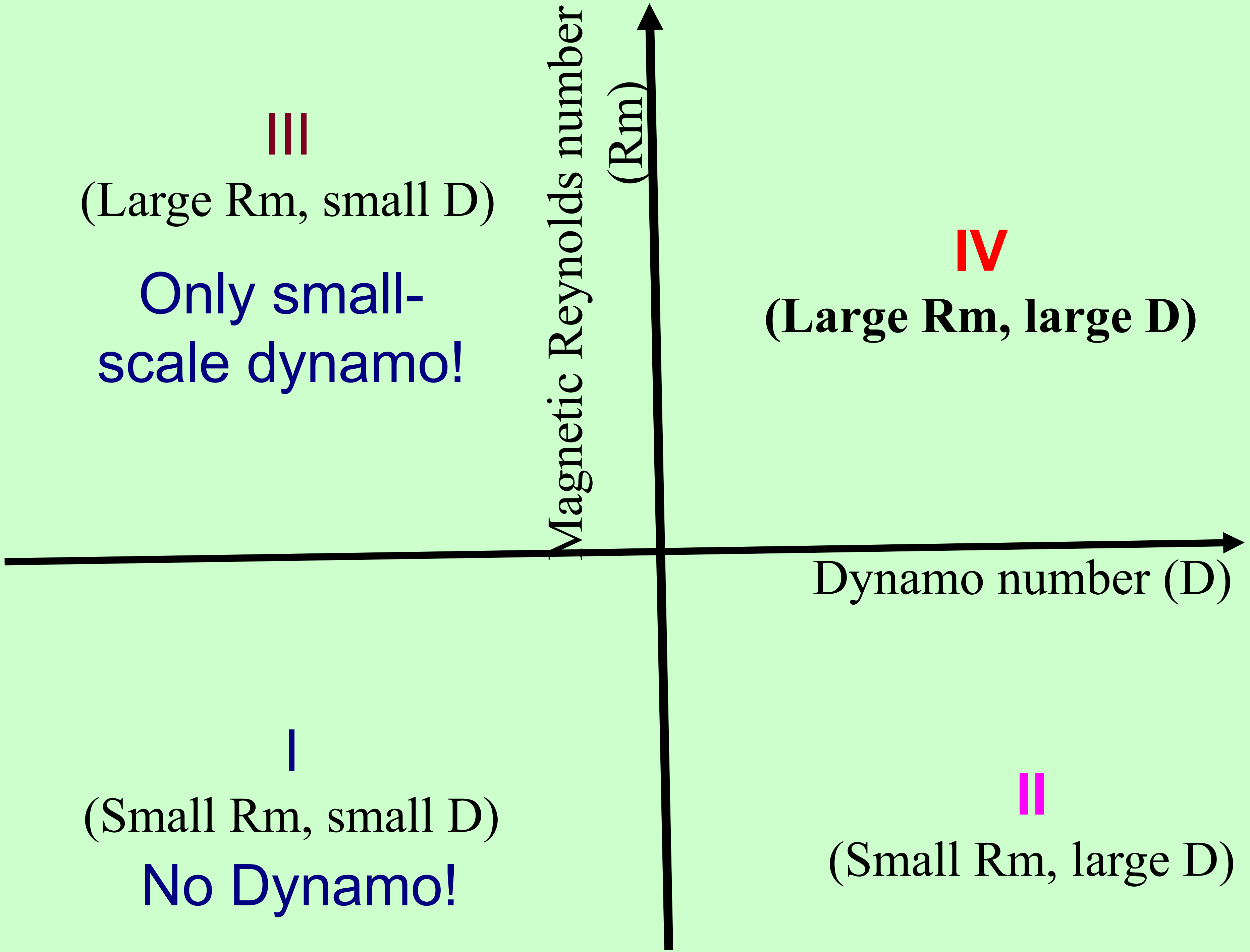


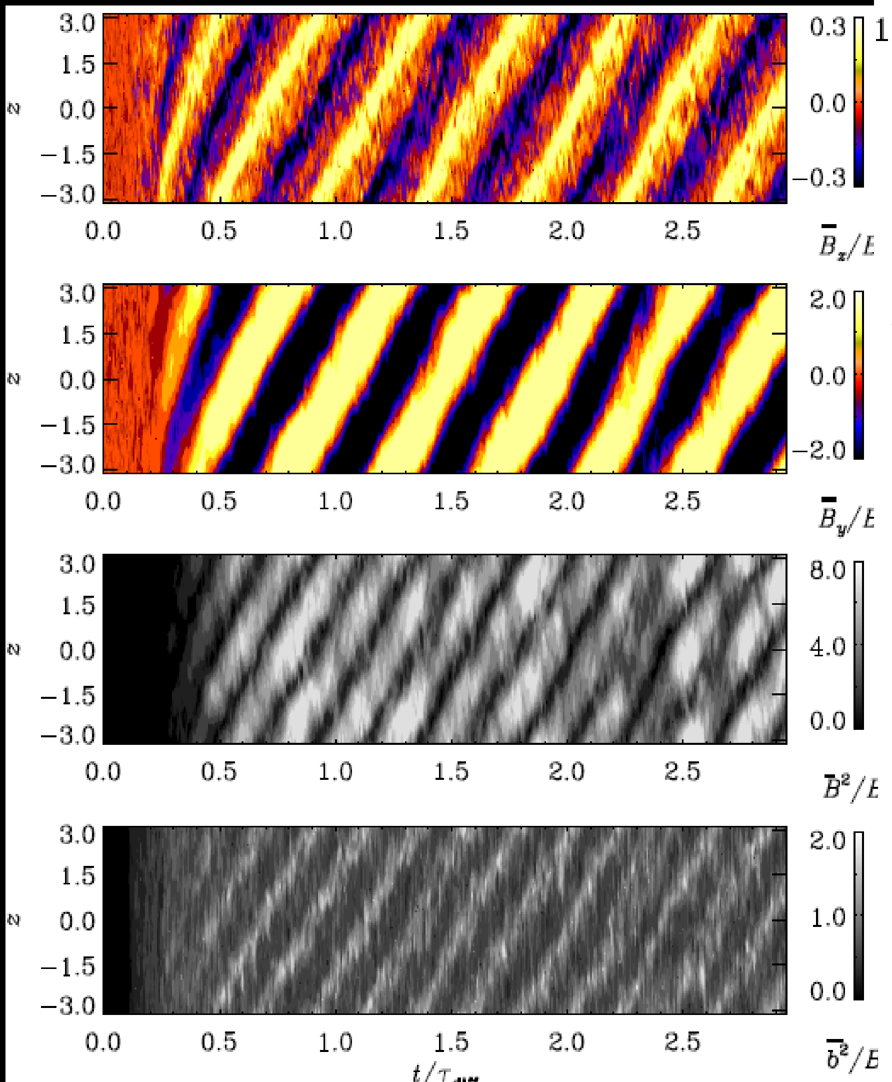
r = 0.77
0.10
0.01
0.01
0.10
1.00
0.0
0.1
0.2
0.3
0
1
2
3
4

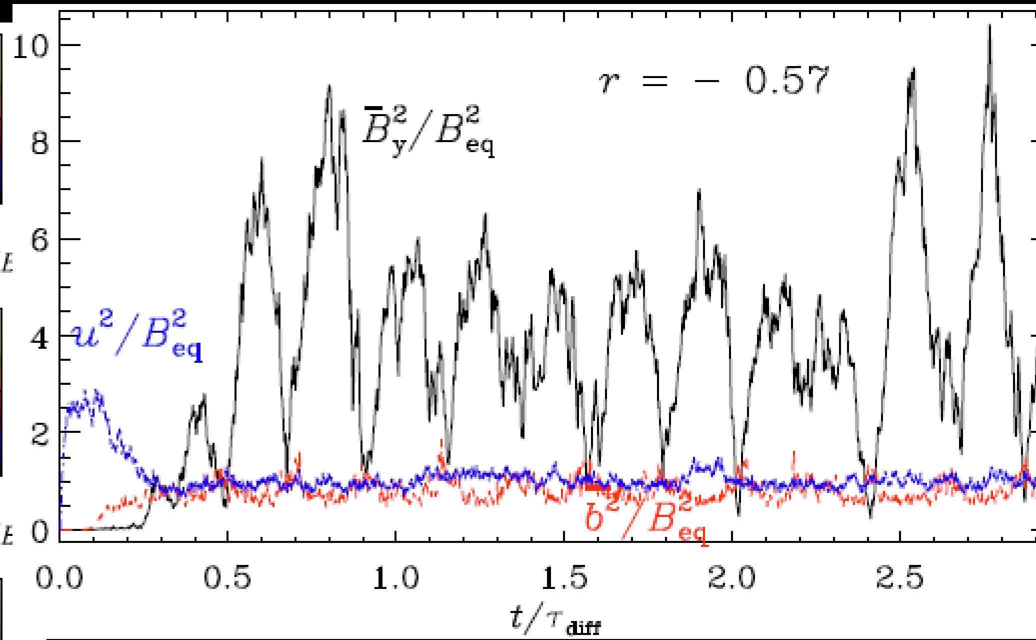


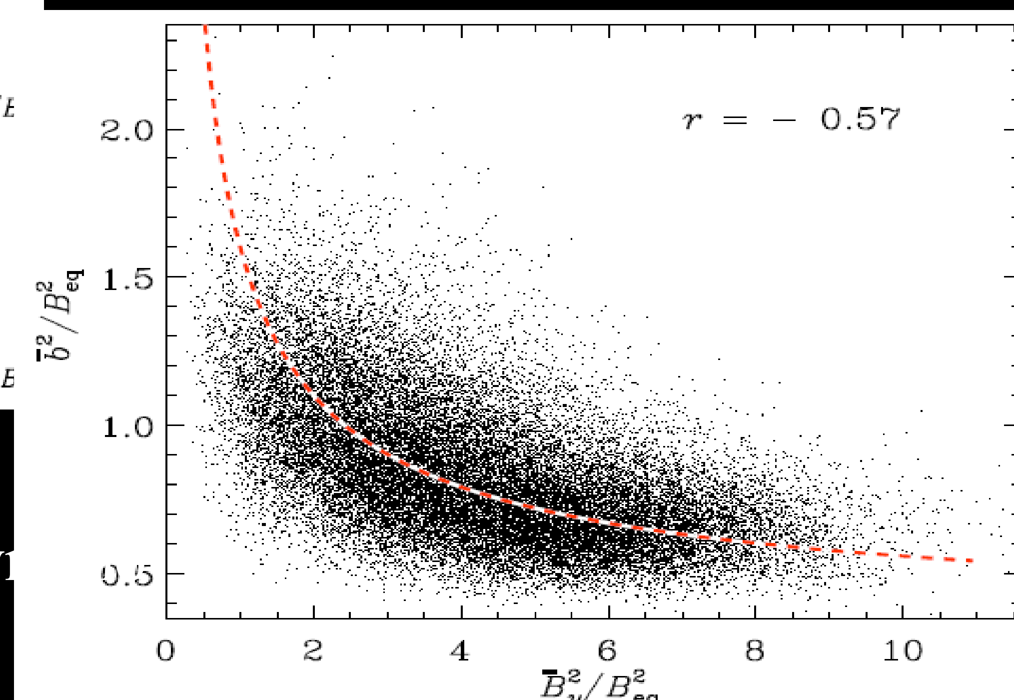
Regime	Run	grid res.	k_f	u_{rms}/c_s	R_m	P_m
I	R1	48^3	5	0.023	0.55	1.25
II	R2a	48^3	5	0.022	0.55	1.25
II	R2b	48^3	5	0.022	0.54	1.25
II	R2c	48^3	5	0.039	0.97	1.25
II	R2d	48^3	5	0.039	0.97	1.25




III
(Large Rm, small D)
Only small-scale dynamo!
Magnetic Reynolds number (Rm)
IV
(Large Rm, large D)
Dynamo number (D)
I
(Small Rm, small D)
No Dynamo!
II
(Small Rm, large D)



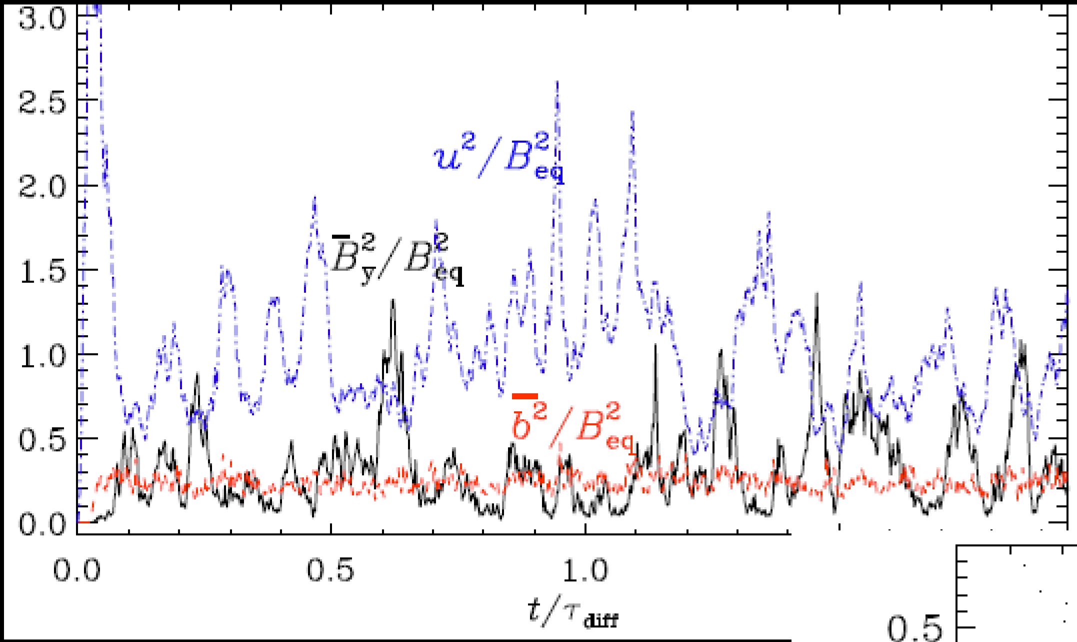




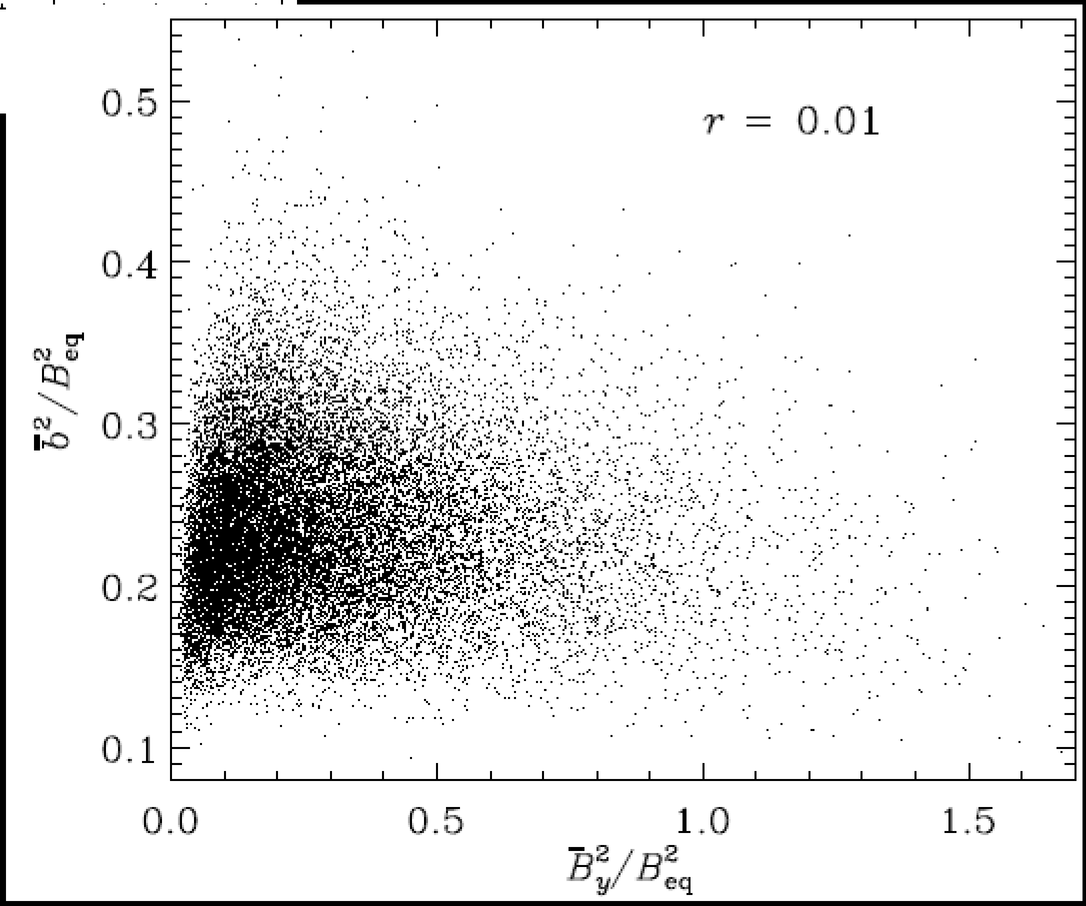
Why this anti-correlation?

Both quenching of small-scale dyn

Quenching of tangling!



Same as earlier but here large-scale dynamo is weaker



Conclusion

We have considered a simple setup of turbulent dynamo:
It captures the essential mechanism of alpha-Omega dynamo model.

When both dynamos are operating, the small-scale field is produced from both the small-scale dynamo and the tangling of the large-scale field.
In this scenario, when the large-scale field is weaker than Beq, the small-scale field is almost uncorrelated with the large-scale magnetic cycle. However, when the large-scale field > Beq, we observe an anticorrelation. This could be due to the suppression of small-scale field and the suppression of tangling.

Convection simulations in spherical geometry

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} - \mu_0 \eta \boldsymbol{J},$$

$$\frac{D \ln \rho}{Dt} = -\boldsymbol{\nabla} \cdot \boldsymbol{u},$$

$$\frac{D\boldsymbol{u}}{Dt} = \boldsymbol{g} - 2\boldsymbol{\Omega}_0 \times \boldsymbol{u} + \frac{1}{\rho}\left(\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{\nabla} \cdot 2\nu\rho\mathsf{S} - \boldsymbol{\nabla} p\right)$$

$$T\frac{Ds}{Dt} = -\frac{1}{\rho}\boldsymbol{\nabla} \cdot \left(\boldsymbol{F}^{\mathrm{rad}} + \boldsymbol{F}^{\mathrm{SGS}}\right) + 2\nu\mathsf{S}^2 + \frac{\eta\mu_0}{\rho}\boldsymbol{J}^2,$$

$$p = (\gamma - 1)\rho e, \quad e = c_{\mathrm{V}} T$$

$$\boldsymbol{F}^{\mathrm{rad}} = -K\boldsymbol{\nabla} T \quad \text{and} \quad \boldsymbol{F}^{\mathrm{SGS}} = -\chi_{\mathrm{SGS}}\rho T\boldsymbol{\nabla} s.$$

$$K(r) = K_0[n(r) + 1]$$

$$n(r) = \delta n (r/r_0)^{-15} + n_{\mathrm{ad}} - \delta n \quad n_{\mathrm{ad}} = 1.5$$

$$K_0 = (\mathcal{L}/4\pi) c_{\mathrm{V}} (\gamma - 1)(n_{\mathrm{ad}} + 1)\rho_0\sqrt{GM_\odot R_\odot}.$$

Kapyla et al. (2012, 2013, 2014)

Thank you!