Adding Condensation to the Smoluchowski Equation

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2015/05/13

Outline

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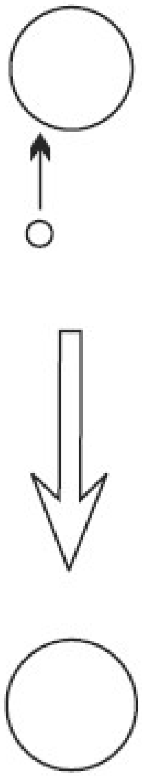
- Cloud droplet growth by condensation
- Conclusion

Background



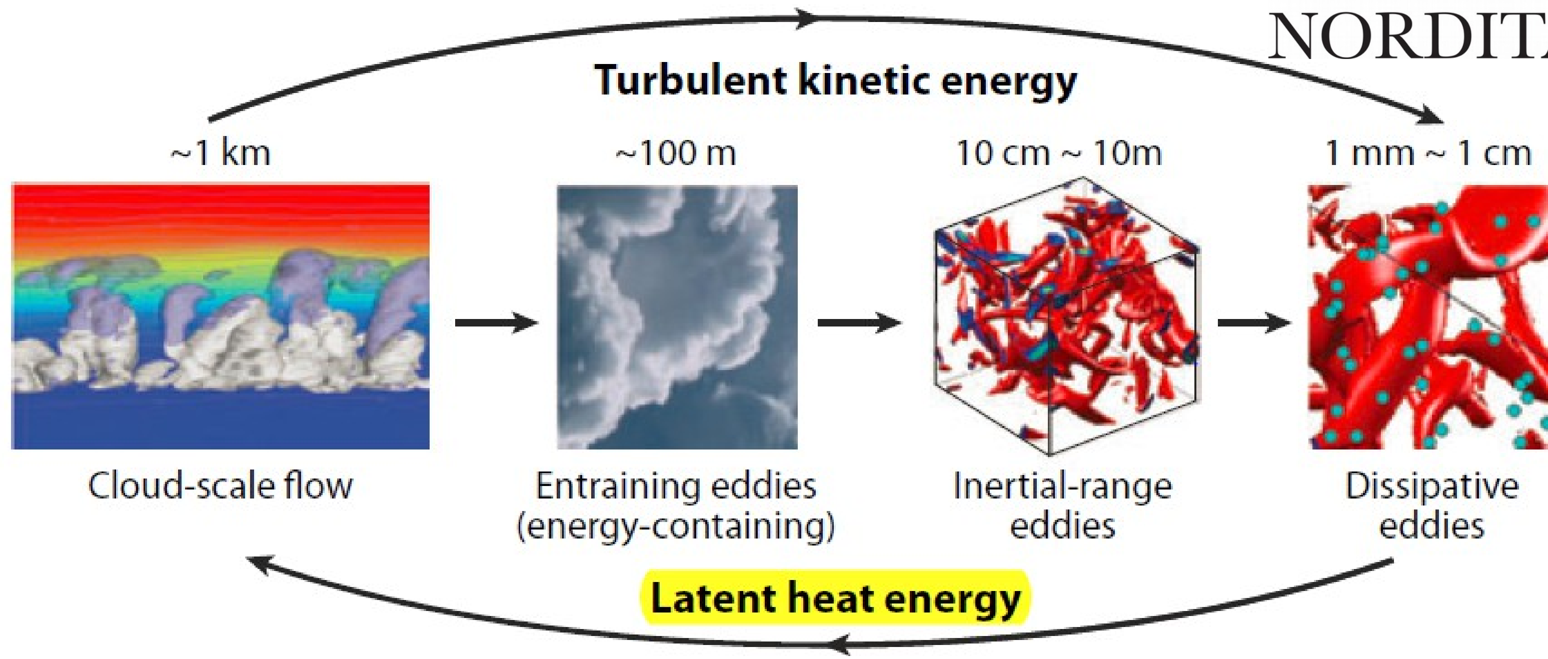
Collision-
coalescence

2011, Cambridge , Lamb

$$\Gamma^{g}_{12} = \pi(a_i + a_j)^2 \left\langle | v_i^T - v_j^T | \right\rangle$$

1980, Hall

Gravitational Collision-Coalescence



Multiscale interactions in atmospheric clouds.

2013, AFM, Grabowski

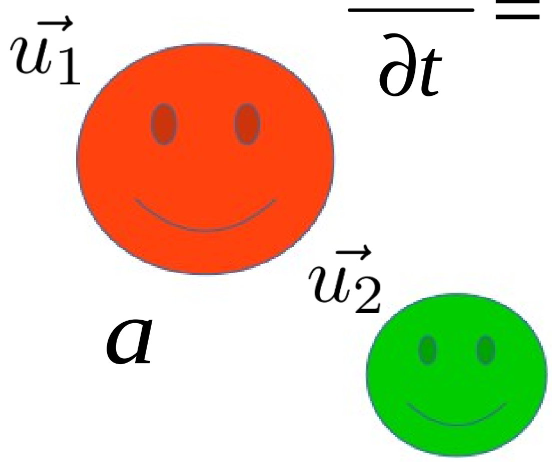
Turbulent Collision-Coalescence

Bottleneck:

Rapid growth of cloud droplets in the size range **15–40 µm** in radius , **neither** the **diffusional mechanism nor** the **gravitational collision-coalescence mechanism** is effective.

2013, AFM, Grabowski

Cloud droplet growth by coagulation

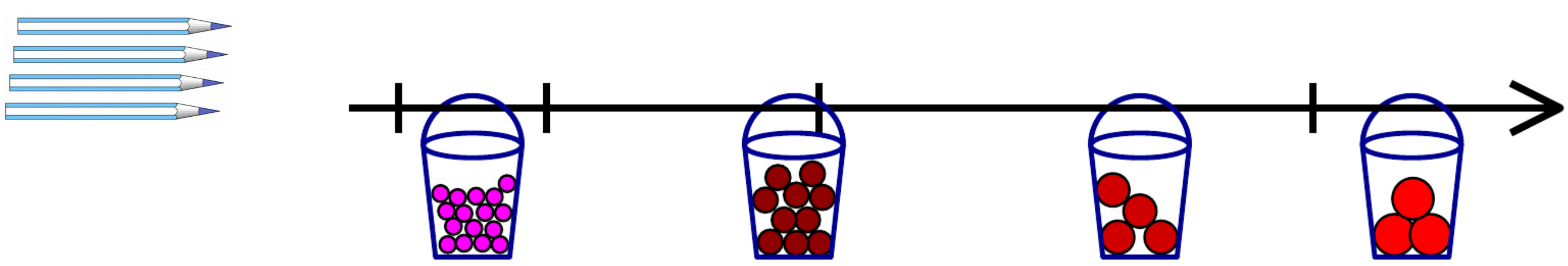


$$\frac{\partial n_k}{\partial t} = -\nabla \cdot n_k \mathbf{u}_k + \tfrac{1}{2} \sum_{i+j=k} K_{ij} n_i n_j - n_k \sum_{i=1}^{N} K_{ik} n_i + D \nabla^2 n_k$$

$$n_k = n_k(\mathrm{m}, \mathrm{t})$$:Number density distribution function

$$K_{ij} = \pi(\mathrm{a}_i + \mathrm{a}_j)^2 \left\langle |\, \mathrm{u}_i - u_j \,| \right\rangle$$ **:Turbulent Collision Kernel**

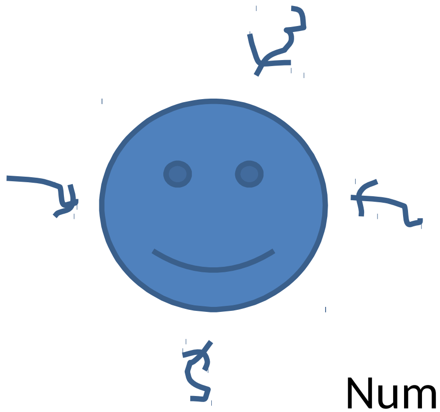
$$D = 1 \times 10^{-5} m^2 / s$$ **:Diffusion coefficient**



Mass bins, mass space

2004, Johanse

Cloud droplet growth by condensation (Basic Model)



$$a\frac{da}{at} = GS \qquad G = 5\times10^{-11}\, m^2/s$$

$$a \sim t^{1/2}$$

2011, Lamb

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Number conservation: $$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial m}(\lambda n)$$

$$\frac{\partial}{\partial t} n_k = -(\lambda_k n_k - \lambda_{k-1} n_{k-1})$$

Mass decay/growth rate: $$\lambda_k = \frac{\dot{m}}{m}$$

$$\dot{m} = 4\pi\rho_w a^2 \frac{da}{dt} = 4\pi\rho_w a G S$$

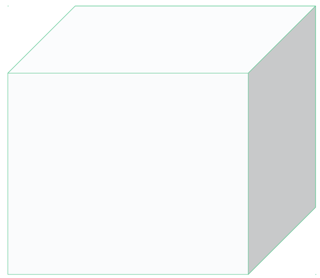
$$m = \frac{4\pi}{3}\rho_w a^3$$

$$\lambda = \frac{3GS}{a^2}$$

Cloud droplet growth by condensation (0D)

Experimental setup

$L = 0.4\,m$



Initial distribution:

$$n_k = n_0 \exp\left[-(a_0 - a_k)^2/2\sigma^2\right]$$

$$n_0 = 1 \times 10^8 m^{-3}$$

$$a_0 = 2 \times 10^{-6} m$$

$$\sigma = 1 \times 10^{-6} m$$

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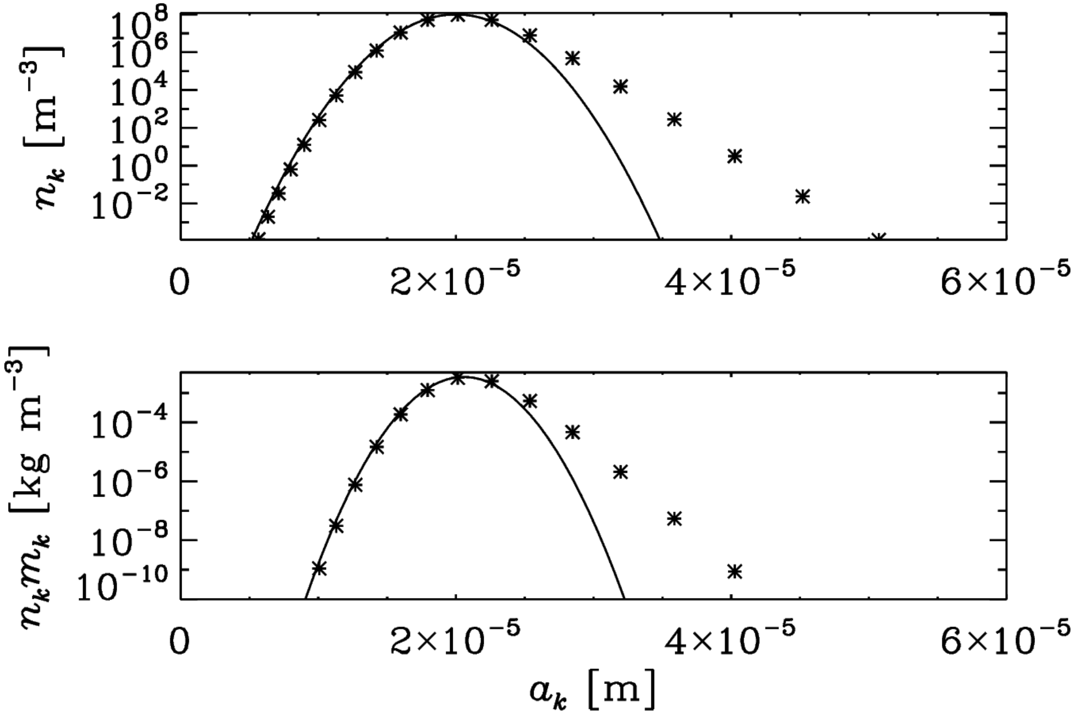
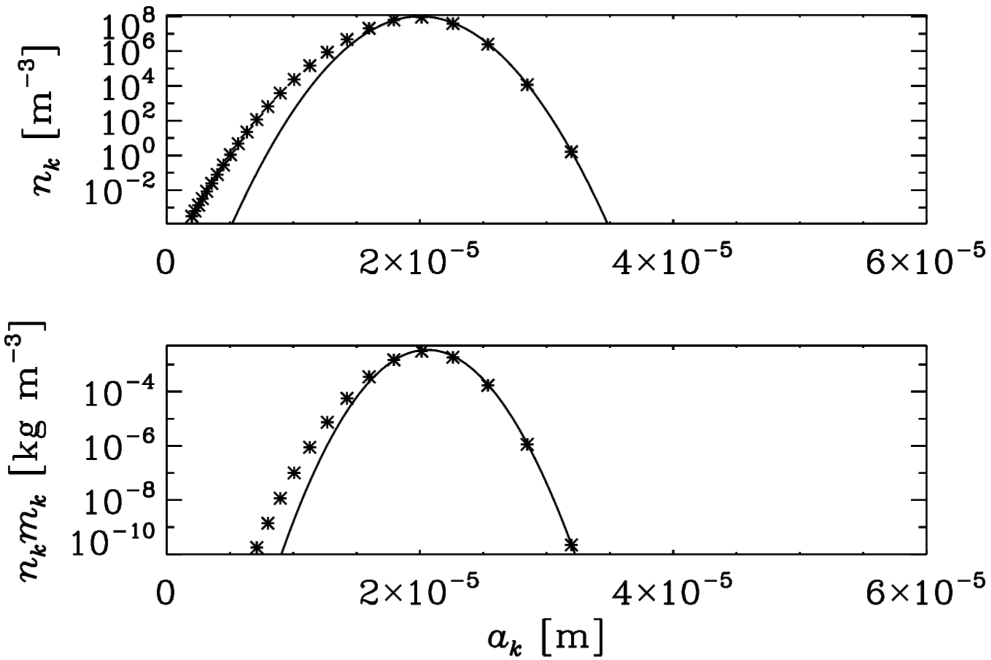
Cloud droplet growth by condensation (0D)

$$s = constant$$

Model test:



$$s > 0$$



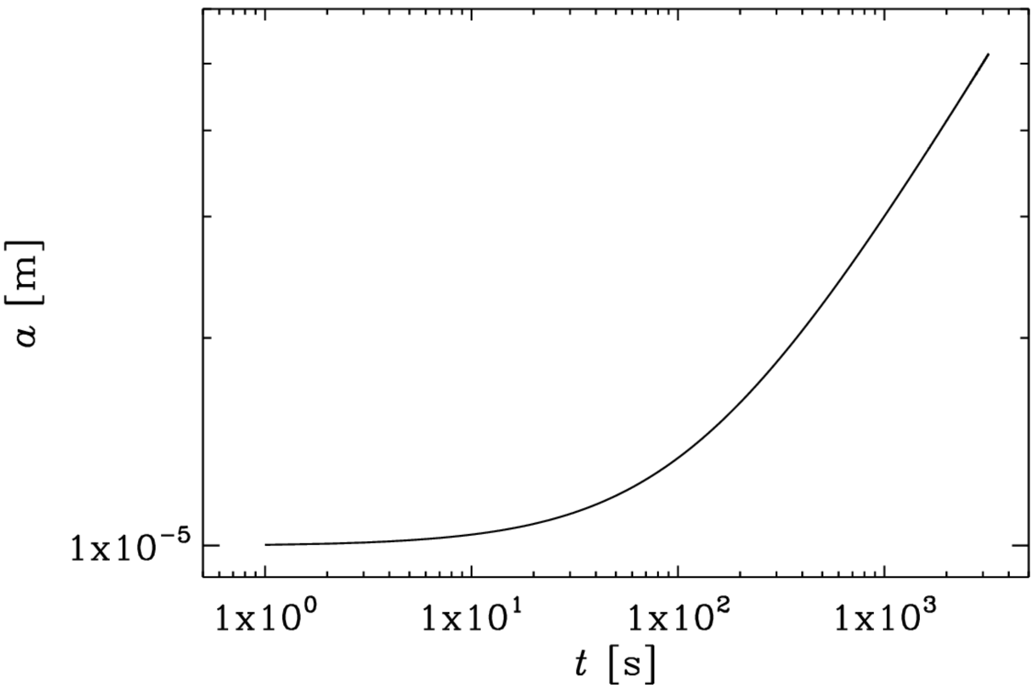
$$s < 0$$

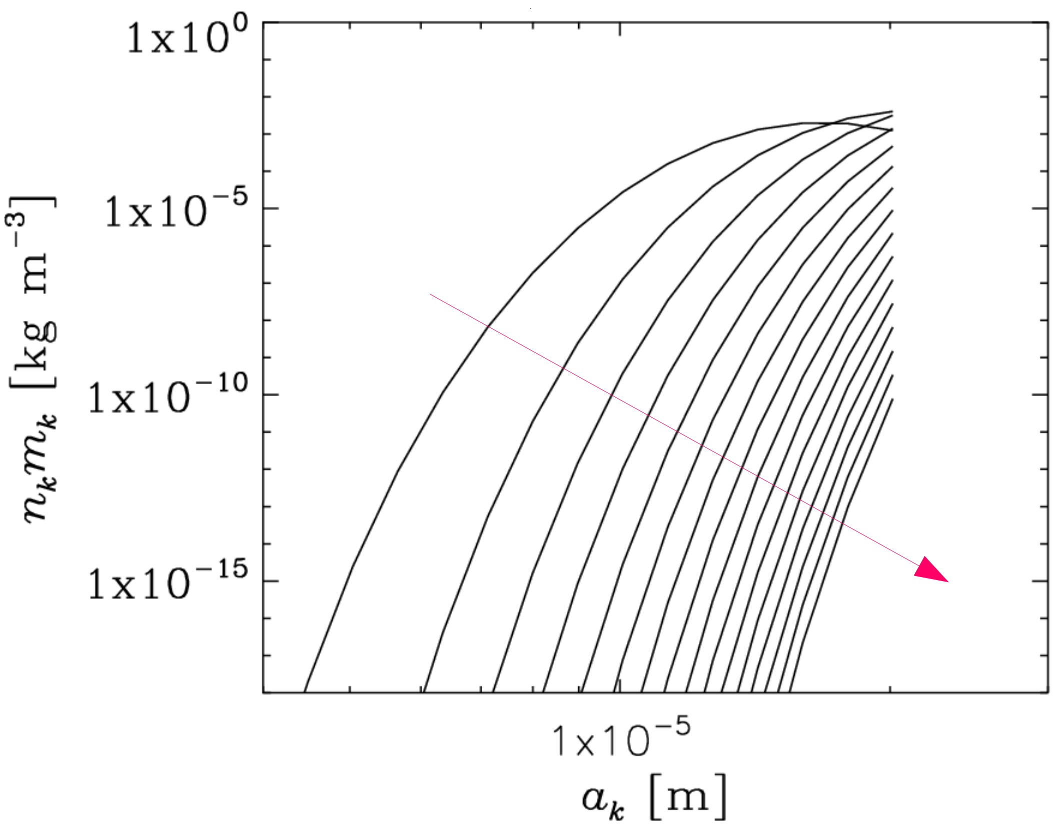
The current condensation model is applicable for both condensation and evaporation.

Cloud droplet growth by condensation (0D)

$$s = constant$$

Model test





Cloud droplet growth by condensation (0D)

Condensation and evaporation

$$s = \cos(\omega t)$$

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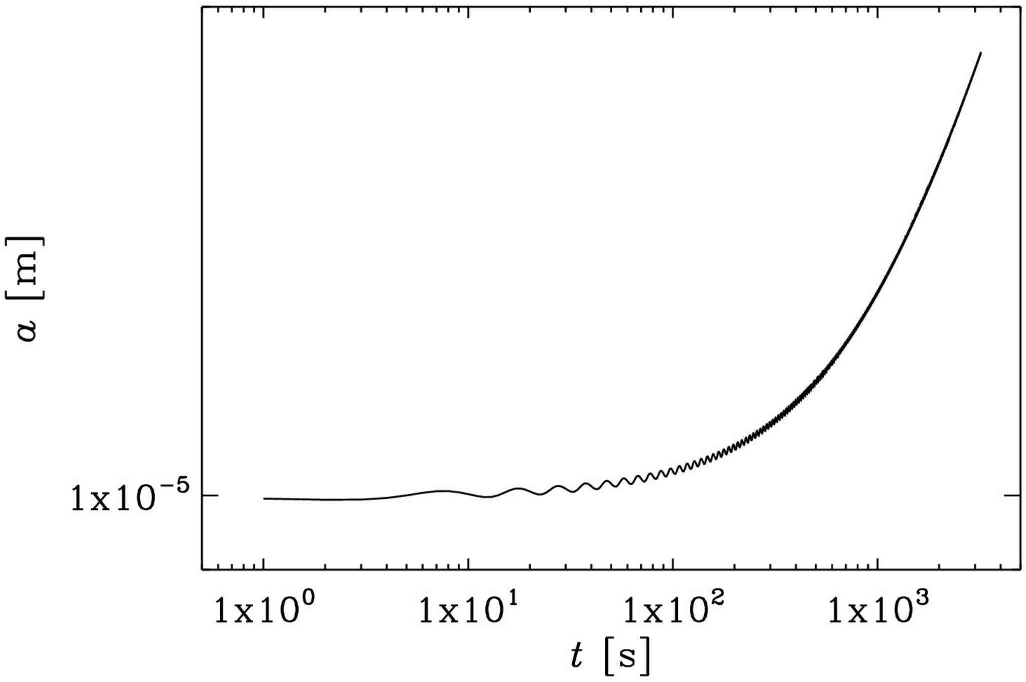
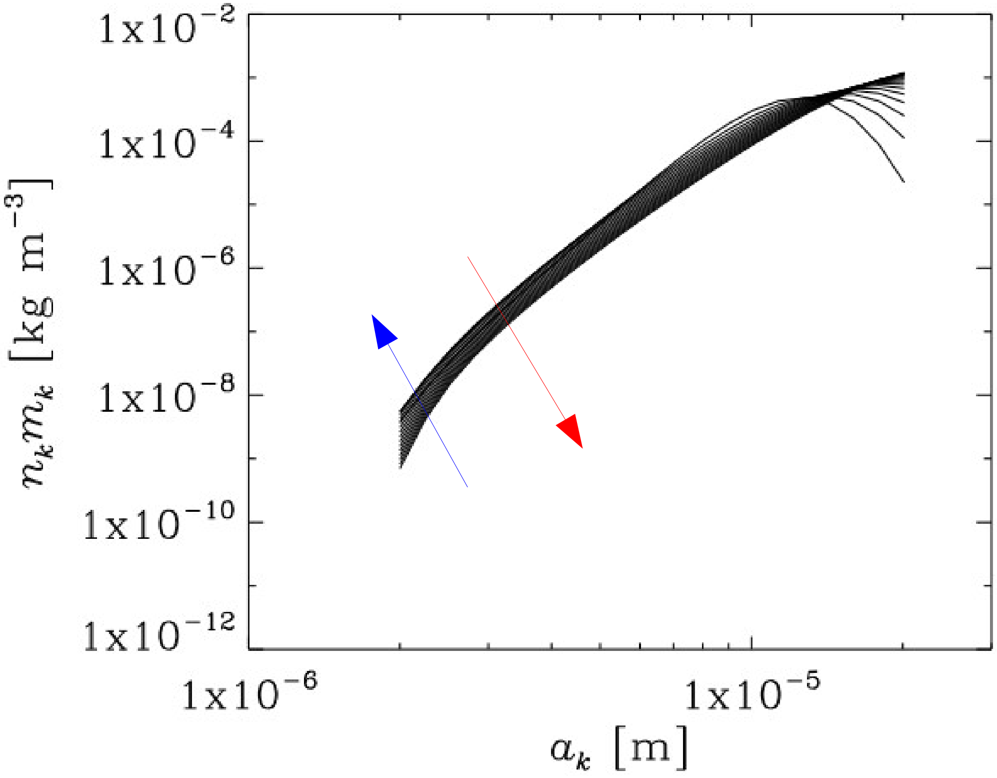




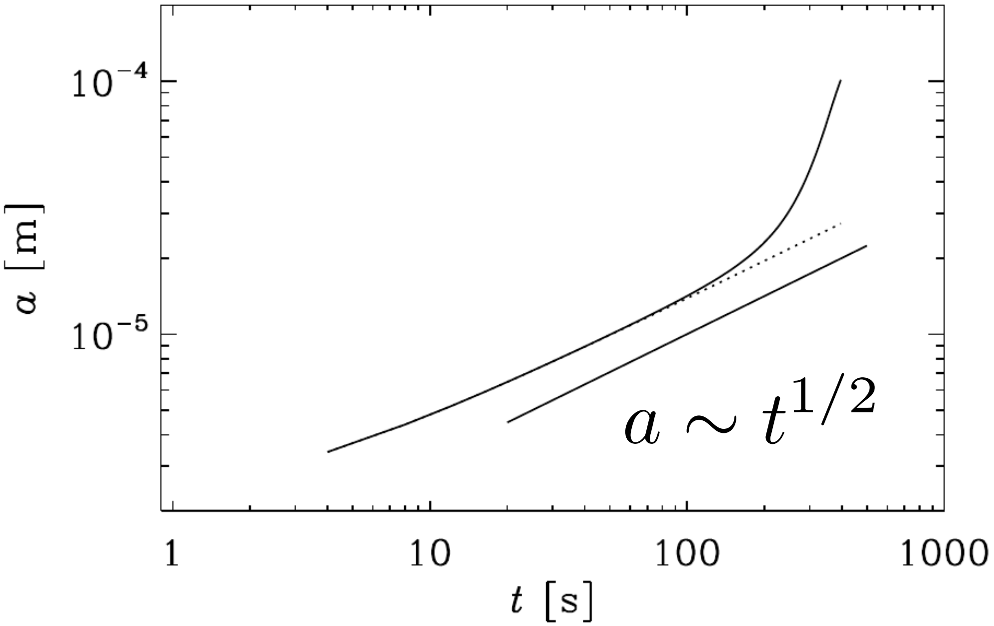
Net growth of cloud droplets with oscillatory supersaturation?

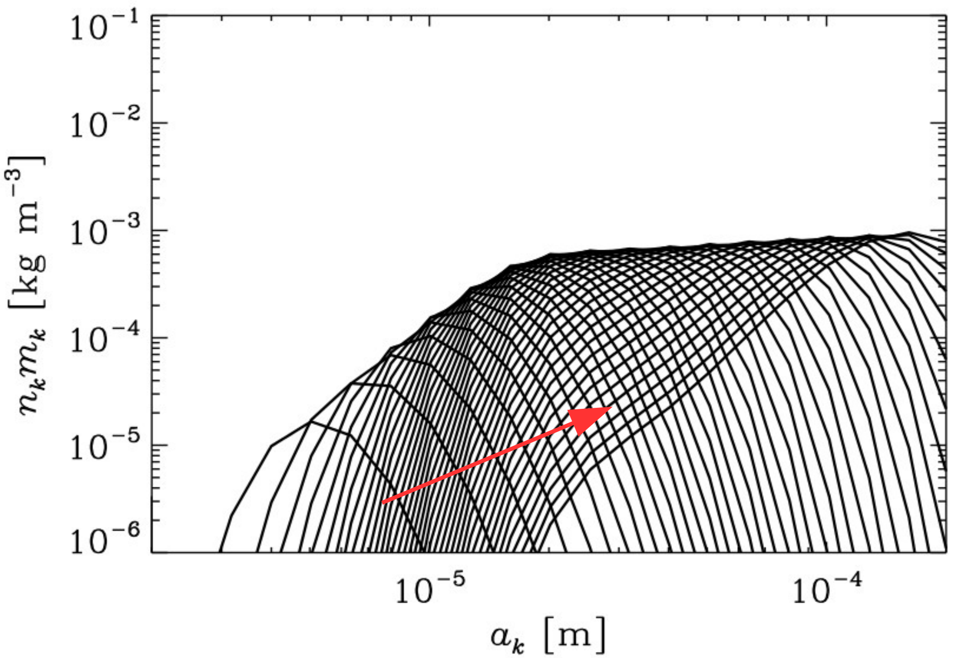
in process

Oscillation of number density distribution function

Cloud droplet growth by condensation and coagulation (0D)

$s = constant$





$s = \cos(\omega t)$

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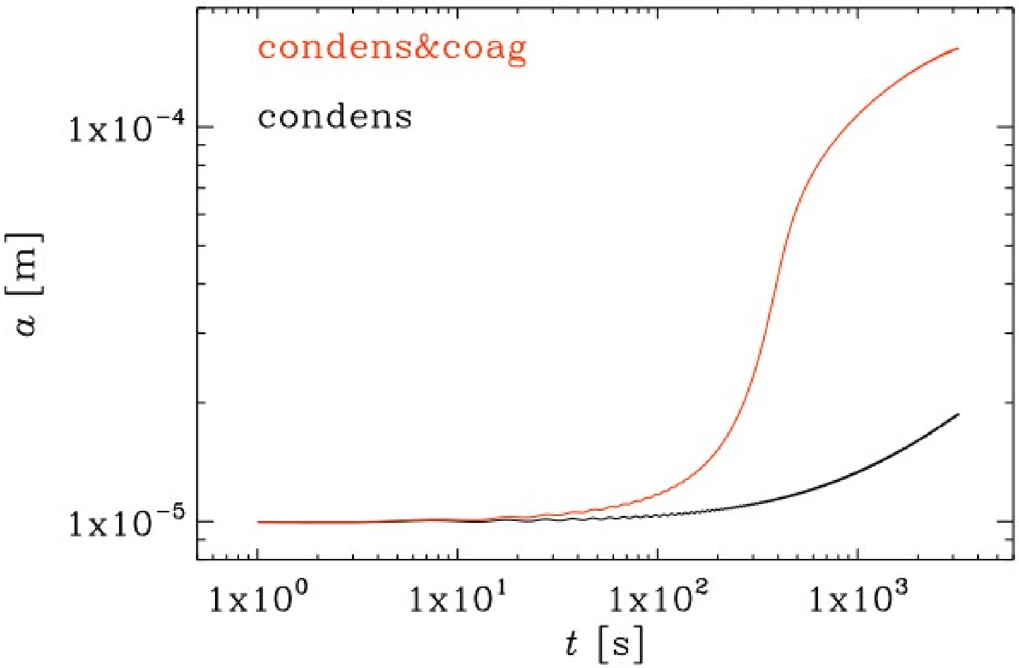

Growth by coagulation is much faster than condensation, but condensation is essential to provide a physical initial particle distribution for coagulation.

Conclusion

- Condensation is added to the coagulation equation, providing a physical initial distribution of number density;
- To do: Simulate cloud droplet growth by condensation and coagulation in 3D case;
- Reynolds number and Stokes number dependency;
- **Bottleneck problem of cloud droplet growth.**

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Thanks for your attention