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Statistical properties from simulations of the SN-regulated ISM

2015 Pencil-code meeting

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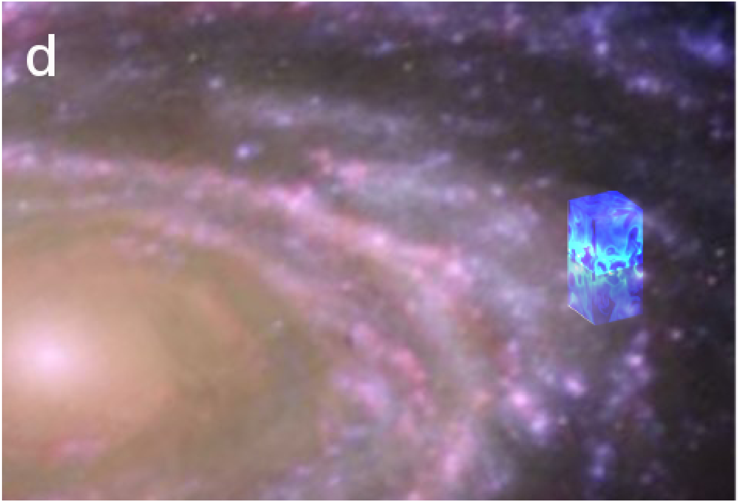
- **SN-regulated ISM simulations**
- Multiphase structure
- Mean and fluctuating fields
- Correlation lengths
- Fieldline-based averages
- Addition of cosmic rays
- Summary

Modelling the interstellar medium in rotating galaxies



Equations

$$\frac{D\rho}{Dt} = -\nabla\cdot(\rho\mathbf{u}) + \dot{\rho}_{\mathrm{SN}}, \tag{1}$$

$$\begin{aligned}\frac{D\mathbf{u}}{Dt} =& -\rho^{-1}\nabla\sigma_{\mathrm{SN}} - c_{\mathrm{s}}^2\nabla\left(s/c_p + \ln\rho\right) - \nabla\Phi - Su_x\hat{\mathbf{y}}\\ &- 2\boldsymbol{\Omega}\times\mathbf{u} + \rho^{-1}\mathbf{j}\times\mathbf{B} + \zeta_\nu\left(\nabla\nabla\cdot\mathbf{u}\right)\\ &+ \nu\left(\nabla^2\mathbf{u} + \tfrac{1}{3}\nabla\nabla\cdot\mathbf{u} + 2\mathbf{W}\cdot\nabla\ln\rho\right),\end{aligned} \tag{2}$$

$$\begin{aligned}\rho T\frac{Ds}{Dt} =& \dot{\sigma}_{\mathrm{SN}} + \rho\Gamma - \rho^2\Lambda + \nabla\cdot\left(c_p\rho\chi\nabla T\right) + \eta\mu_0\mathbf{j}^2\\ &+ 2\rho\nu\left|\mathbf{W}\right|^2 + \zeta_\chi\left(\rho T\nabla^2 s + \nabla\ln\rho T\cdot\nabla s\right) + \rho T\nabla\zeta_\chi\cdot\nabla s,\end{aligned} \tag{3}$$

$$\frac{\partial\mathbf{A}}{\partial t} = (\mathbf{U}+\mathbf{u})\times\mathbf{B} + (\eta+\zeta_\eta)\nabla^2\mathbf{A} + (\nabla\cdot\mathbf{A})\nabla\zeta_\eta, \tag{4}$$

SN-regulated dynamics

Temperature

Gas number density

Composition of the ISM: a multiphase environment
Cox & Smith (1974), McKee & Ostriker (1977)

Differential cooling; compressions

Dynamo action

Non-ideal MHD

- Shear, rotation
- Range of scales
  - $10^{-5} < n < 10^{2}\,\mathrm{cm}^{-3}$
  - $10 < T < 10^{7}\,\mathrm{K}$
  - $0 < \mathcal{M} < 25$
- SN - sources of thermal energy and random velocity
  - shocks
  - density gradient
  - turbulence
- Azimuthal seed **B**
- Magnetic diffusivity

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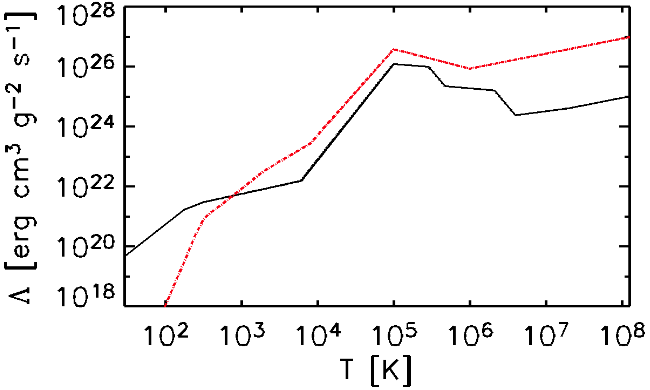
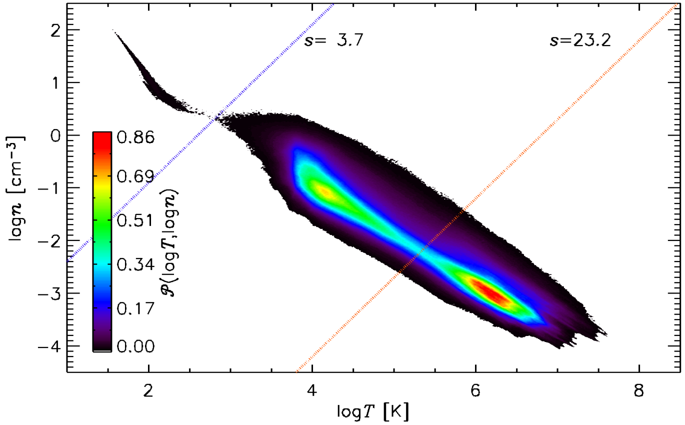
Differential cooling: multiphase structure



WSW cooling (black) derived from Wolfire et al. (1995) $T \leq 10^5\,\mathrm{K}$ and Sarazin & White (1987) $T > 10^5\,\mathrm{K}$

RBN (red,dashed) derived from Rosen, Bregman & Norman (1993)

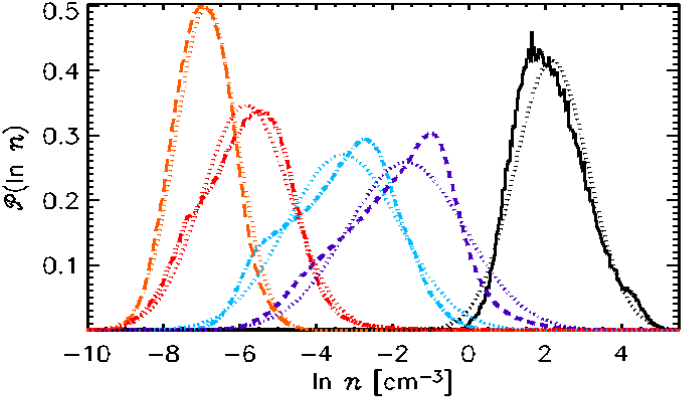
PDF in ρT space



Phases can be separated at boundaries $T = 500\text{K}$, $5 \times 10^5\text{K}$, or $s = 3.7 \times 10^8 \text{erg/g/K}$, $23.2 \times 10^8 \text{erg/g/K}$.

Lognormal fits to gas distributions

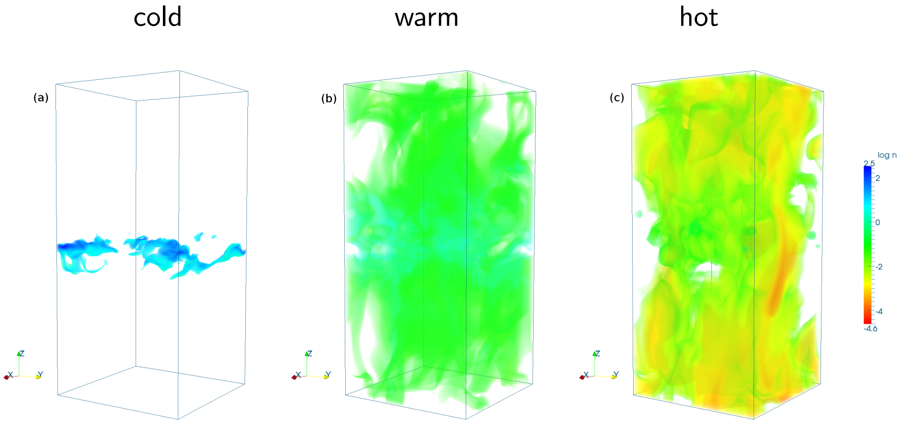
gas number density



hot $|z| \leq 200\,\mathrm{pc}$, hot $|z| > 200\,\mathrm{pc}$, warm $|z| \leq 200\,\mathrm{pc}$, warm $|z| > 200\,\mathrm{pc}$,

cold

Multiphase structure – gas number density



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How do we identify the mean field?

- Observers:

  Gaussian smoothing –

  sliding averages
- Theoreticians:

  Volume averaging
- Horizontal averaging
  Gressel (2008), Dobbs & Price (2008)

  non-galaxy applications

- Applying a low pass filter $G(\mathbf{x}, \ell)$

  Germano (1992) re: turbulent $\mathbf{u}$
- Convolve $\mathbf{B}$ with Gaussian

$$\mathbf{B}_\ell(\mathbf{x}, \ell) = \int_V \mathbf{B}(\mathbf{x}') G(\mathbf{x} - \mathbf{x}', \ell) d\mathbf{x}'$$

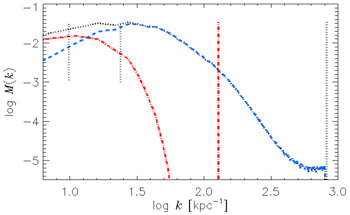
$$G(\mathbf{x} - \mathbf{x}', \ell) = \frac{1}{(\sqrt{2\pi} l_B)^3} \exp\left\{ \frac{-(\mathbf{x} - \mathbf{x}')^2}{2\,\ell^2} \right\}$$

$$\hat{\mathbf{B}}_\ell(\mathbf{k}, \ell) = \hat{\mathbf{B}}(\mathbf{k}) \exp\left\{-2(\ell\pi)^2 \mathbf{k}\cdot\mathbf{k}\right\}$$

$$\hat{\mathbf{B}}_\ell(\mathbf{k}, \ell) = \mathcal{F}\left[\mathbf{B}_\ell(\mathbf{x}, \ell)\right]$$

Scale separation

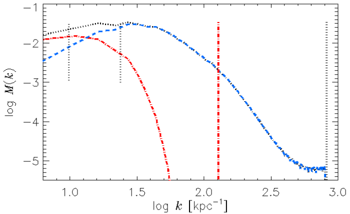
$\ell = 50\,\mathrm{pc}$



- e_{B} black
  $e_{\mathrm{B}\ell}$ red
  e_{b} blue
- vertical red line $k_B = \ell^{-1}$
- vertical black line $k_{\Delta x} = \Delta x^{-1}$

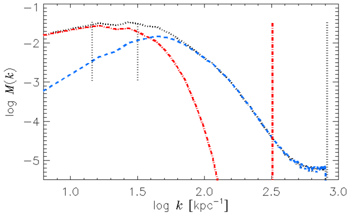
Scale separation

$\ell = 50\,\mathrm{pc}$

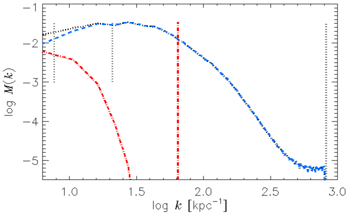


- e_B black
  $e_{B\ell}$ red
  e_b blue
- vertical red line $k_B = \ell^{-1}$
- vertical black line $k_{\Delta x} = \Delta x^{-1}$

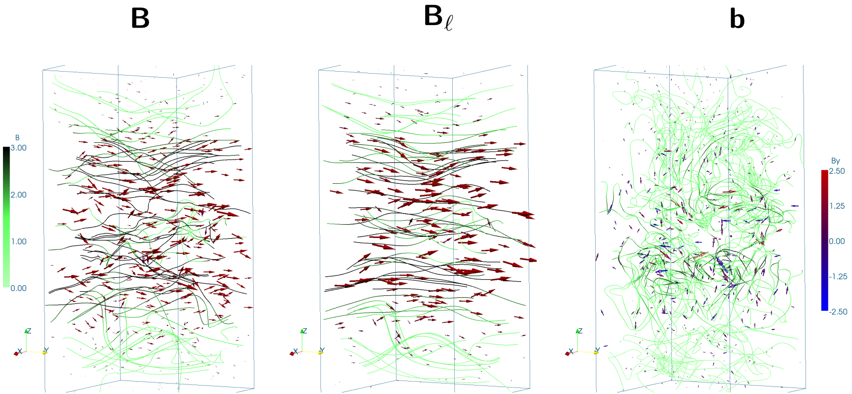
$\ell = 20\,\mathrm{pc}$



$\ell = 100\,\mathrm{pc}$



3D structure of the total, mean and random field



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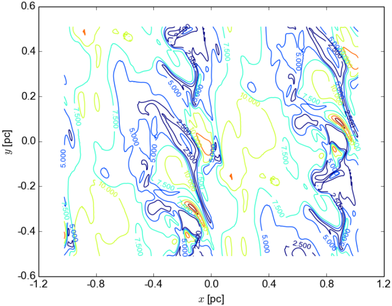
Correlation lengths

We can investigate the correlation lengths in the separate phases.

$$D(l) = \langle [n(\mathbf{x}+\mathbf{l}) - n(\mathbf{x})] \rangle^2$$

$$C(l) = 1 - \frac{D(l)}{2\sigma^2}$$

$$l_0 = \int_0^\infty C(l)\,\mathrm{d}l\,.$$

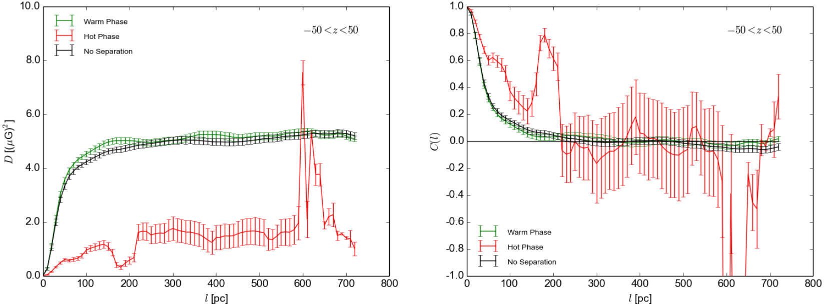


Isotropic horizontal structures investigated to date.

Data from multiple snapshots are stacked, and correlations investigated at varying z-levels.

Correlation lengths

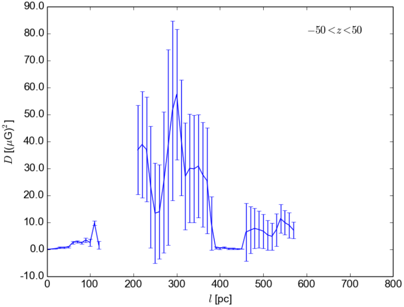
E.g. for the fluctuating field, $b = |\mathbf{b}|$



Fits to the form of $C(l)$ are currently being investigated.

Correlation lengths

Too little cold gas for meaningful statistics.



Integrated correlation lengths l_0 being investigated for different quantities, at differing z-levels.

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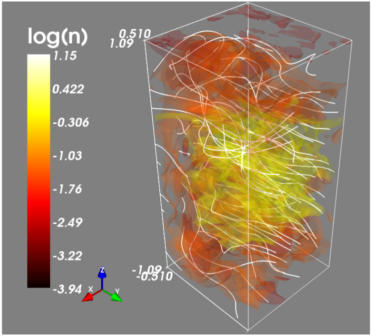
Fieldline-based averages

Statistics obtained along arbitrary (straight) lines correspond to simple volume averages.

If we integrate instead along fieldlines, the statistics associated of the phase carrying the field phase can be isolated.



Fieldline-based averages

Field lines can be simply integrated:

$$\frac{\mathrm{d}x_i}{\mathrm{d}s} = \frac{B_i}{|\mathbf{B}(\mathbf{x})|} \ .$$

The locations of points on a fieldline are then

$$\mathbf{x}_j = \mathbf{x}_0 + \sum_{i=0}^{j-1} \frac{\mathbf{B}(\mathbf{x}_i)}{|\mathbf{B}(\mathbf{x}_i)|} \Delta s \ ,$$

and the maximum displacement (for the $j = k$ which maximises this) is

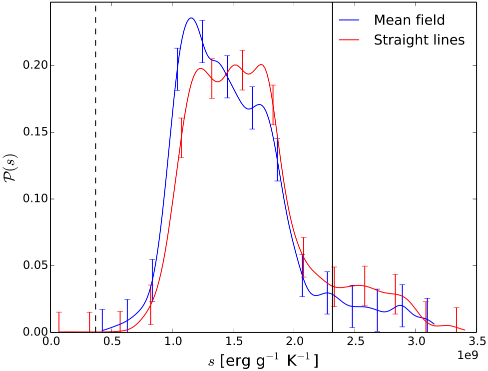
$$r = |\mathbf{x}_k - \mathbf{x}_0| = \left| \sum_{i=0}^{k-1} \frac{\mathbf{B}(\mathbf{x}_i)}{|\mathbf{B}(\mathbf{x}_i)|} \Delta s \right| = \Delta s \left| \sum_{i=0}^{k-1} \frac{\mathbf{B}(\mathbf{x}_i)}{|\mathbf{B}(\mathbf{x}_i)|} \right| \ .$$

A correlation length can then be calculated as

$$\mathcal{C} = \int_0^{r_{\max}} r P(r) \, \mathrm{d}r \ .$$

Fieldline-based averages

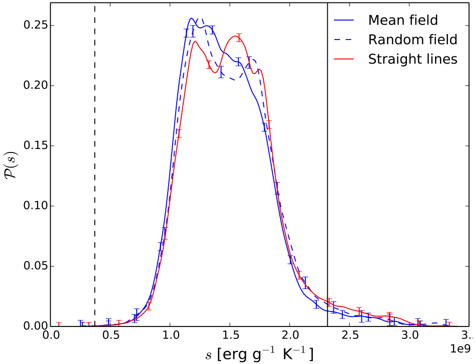
The mean field preferentially samples (the cooler part of) the warm phase, as we would expect.



Analysis for a single snapshot.

Fieldline-based averages

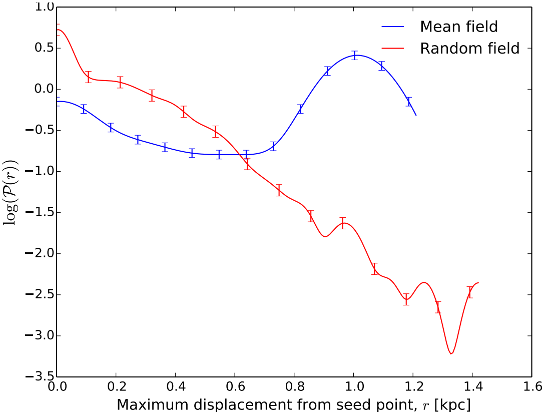
The fluctuating field is somewhat more likely to be found in hotter gas.



Analysis averaged over 12 independent snapshots.

Fieldline-based averages

The morphology of the mean and fluctuating fields is very different, as expected.



Fieldline-based averages

Analysis of this with respect to displacements in different directions confirms the random field is approximately isotropic, the mean field is dominantly azimuthal.

	Correlations [kpc]	
	Mean field	Random field
$\mathcal{C}$	0.735	0.141
$\mathcal{C}_x$	0.045	0.034
$\mathcal{C}_y$	0.382	0.046
$\mathcal{C}_z$	0.085	0.050

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Addition of cosmic rays

Cosmic rays — an important constituent of the ISM — are now being added via the SN explosions.

We model the CR energy density $e_{\rm cr}$ subject to non-Fickian diffusion, so also solve for the cosmicray flux $\boldsymbol{\mathcal{F}}_{\rm cr}$:

$$\frac{\partial e_{\rm cr}}{\partial t} + \nabla\cdot(e_{\rm cr}\mathbf{U}) + p_{\rm cr}\nabla\cdot\mathbf{U} = -\nabla\cdot\boldsymbol{\mathcal{F}}_{\rm cr} + Q_{\rm cr}\,,$$

with $p_{\rm cr} = (\gamma_{\rm cr}-1)e_{\rm cr}$, $\gamma_{\rm cr} = 4/3$, and

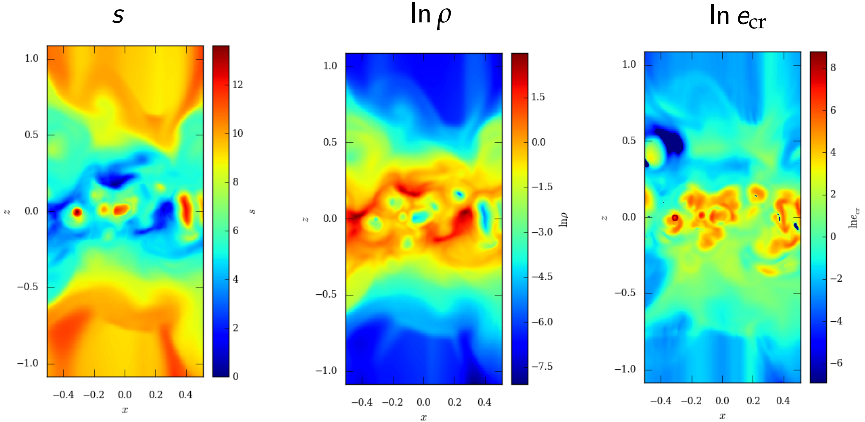
$$\tau\frac{\partial \mathcal{F}_{{\rm cr},i}}{\partial t} = -\kappa_{ij}\nabla_j e_{\rm cr} - \mathcal{F}_{{\rm cr},i}\,,$$

$$\kappa_{ij} = \kappa_\perp\delta_{ij} + (\kappa_\parallel - \kappa_\perp)\hat{B}_i\hat{B}_j\,.$$

Diffusion is strongly anisotropic, with $\kappa_\parallel \gg \kappa_\perp$.

Addition of cosmic rays

Cosmic rays are inserted via the SNe.



See Luiz Felippe's talk for the Parker instability in this context ...

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Summary

- SN-regulated simulations produce realistic ISM structures
- Reasonable distributions of multiphase components, and mean and fluctuating fields
- Simulations still being analysed for the underlying structures
- Addition of cosmic rays now in progress
- Observational diagnostics will be produced