Statistical Mechanics of LDPC Codes in MIMO Channels

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Roberto C. Alamino & David Saad
NCRG – Aston University, Birmingham, UK

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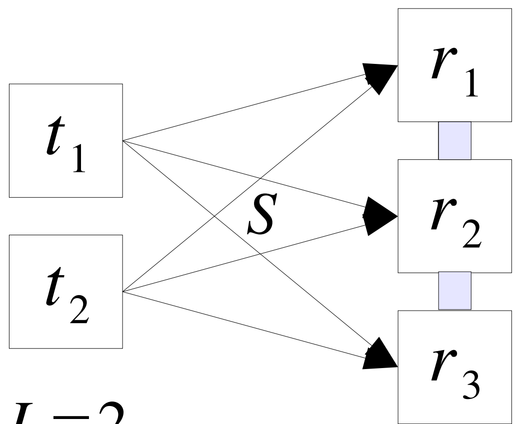
Summary

- Multiple-Input Multiple-Output Channels
- Low-Density Parity-Check Codes
- Decoding: MPM estimator
- Statistical Physics Formulation: Replica Symmetric Ansatz
- Channels:
  - Single Transmitter
  - Multiple Access
  - Interference Channel (Symmetric, Asymmetric)
- Conclusions

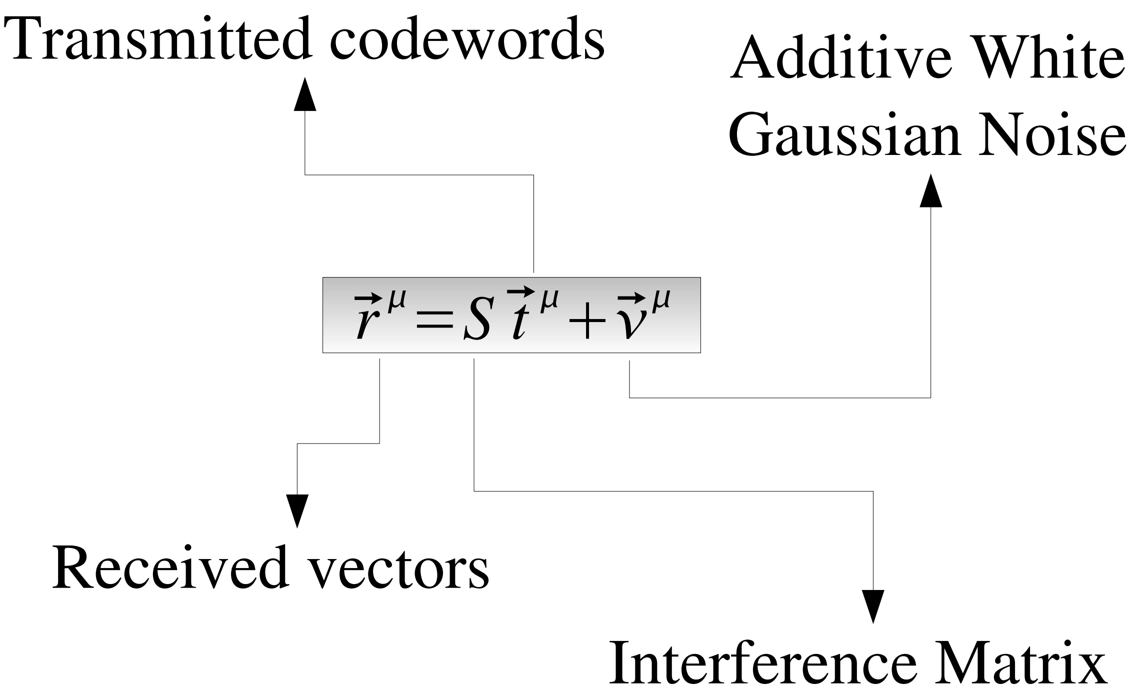
Multiple-Input Multiple-Output (MIMO) Channels

Joint Decoding



$L=2$
(# of senders)

$O=3$
(# of receivers)



Low-Density Parity-Check (LDPC) Codes

Generator Matrix (G): Original Message (w) --> Codeword (t)

$$G=\begin{bmatrix} I \\ B \end{bmatrix} \Rightarrow t=\begin{bmatrix} w \\ Bw \end{bmatrix}$$

$$w \rightarrow t = Gw \,(mod\, 2)$$

Decoding

Parity Check Matrix

$$AG=0 \Rightarrow At=AGw=0$$

Syndrome

$$r=t+v \Rightarrow Ar=Av \equiv z$$

Gallager Codes

$$A=[C_1|C_2]$$

$$G^T=[I|C_2^{-1}C_1]$$

Regular Codes: $R=1-\frac{C}{K}$

C: nonzero entries per column

K: nonzero entries per row

Decoding: Marginal Posterior Maximiser (MPM)

Encoding: i.i.d. Parity-Check Matrices for each sender.

MPM Decoding

$$\hat{t}_i^{\mu} = sgn \langle \tau_i^{\mu} \rangle_{P(\tau|r)}$$

minimizes bit error

Overlap

$$d_i = \frac{1}{M} \sum_{\mu=1}^{M} \langle t^{\mu} \hat{t}_i^{\mu} \rangle_{A_1, \ldots, A_L, r, t}$$

Fast Decoding by Probability (Belief) Propagation

Typical behaviour averaged over all codes, codewords and noise.

Statistical Physics Formulation: Replica Symmetric Ansatz

$$P(\tau|r)=\frac{P(r|\tau)P(\tau)}{P(r)} \rightarrow P(\tau|r)=\frac{e^{-\beta H(\tau|r)}}{Z}$$

inference problem SM problem

$$H(\tau|r)=-\ln[P(r|\tau)P(\tau)]$$

Nishimori's Temperature

$$\beta=1$$

Example: Parity Check Code + Gaussian Channel

$$r=\tau+\nu$$

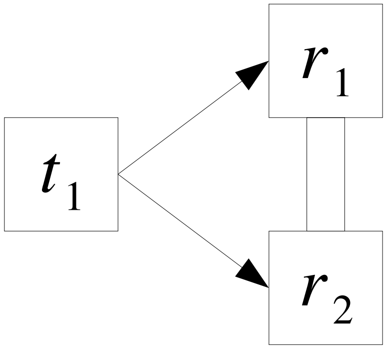
$$P(r|\tau)\propto e^{-\frac{1}{2\sigma^2}(r-\tau)^2}$$

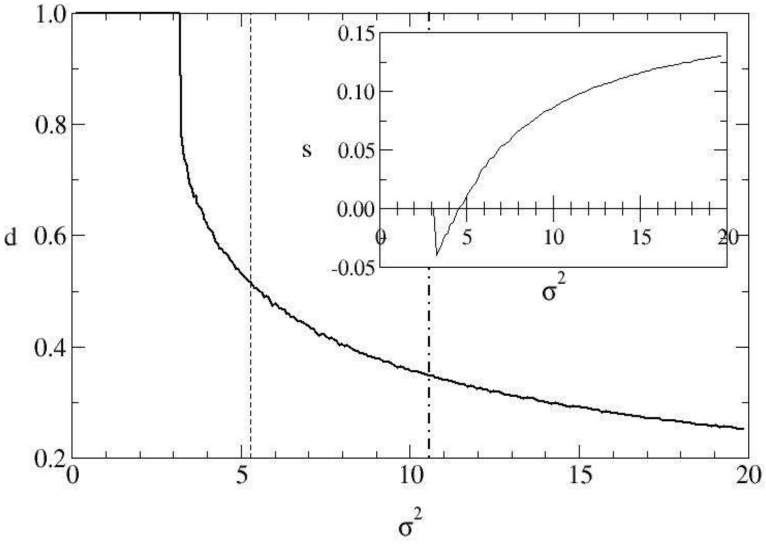
→ **local field**

$$P(\tau)\propto\chi(A\tau=0)$$

→ ***K*-spin interaction**

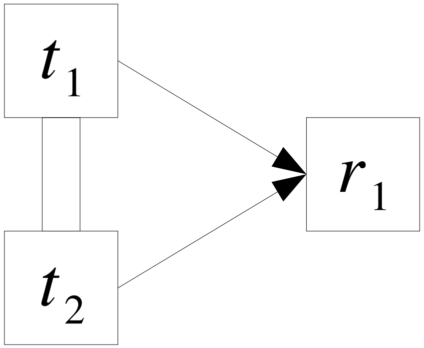
Single Transmitter

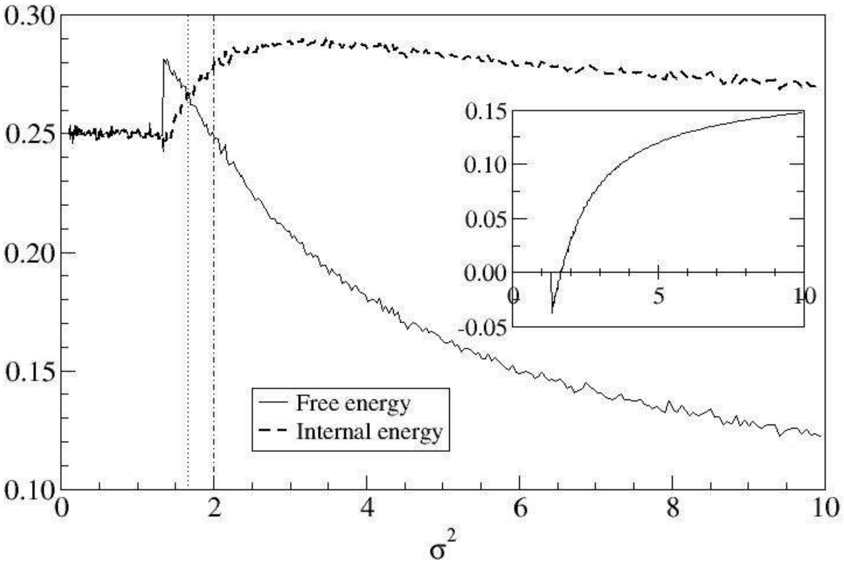




O	Shannon's Limit	Dynamical Transition	Thermodynamical Transition	T.T.-D.T.	S.L. Gaussian Channel O messages
1	2.41	1.59	2.24	0.65	2.41
2	10.57	3.28	4.59	1.31	5.28
3	24.50	4.90	6.68	1.78	8.17

Multiple Access Channel (MAC)

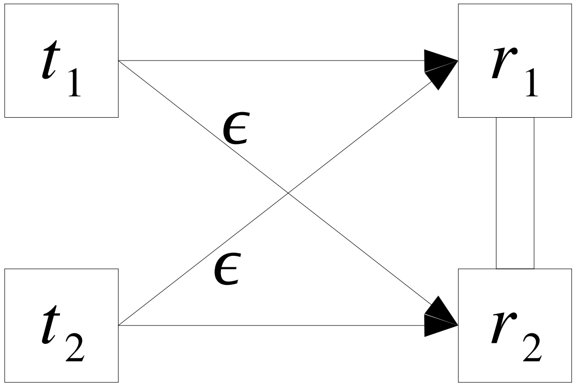


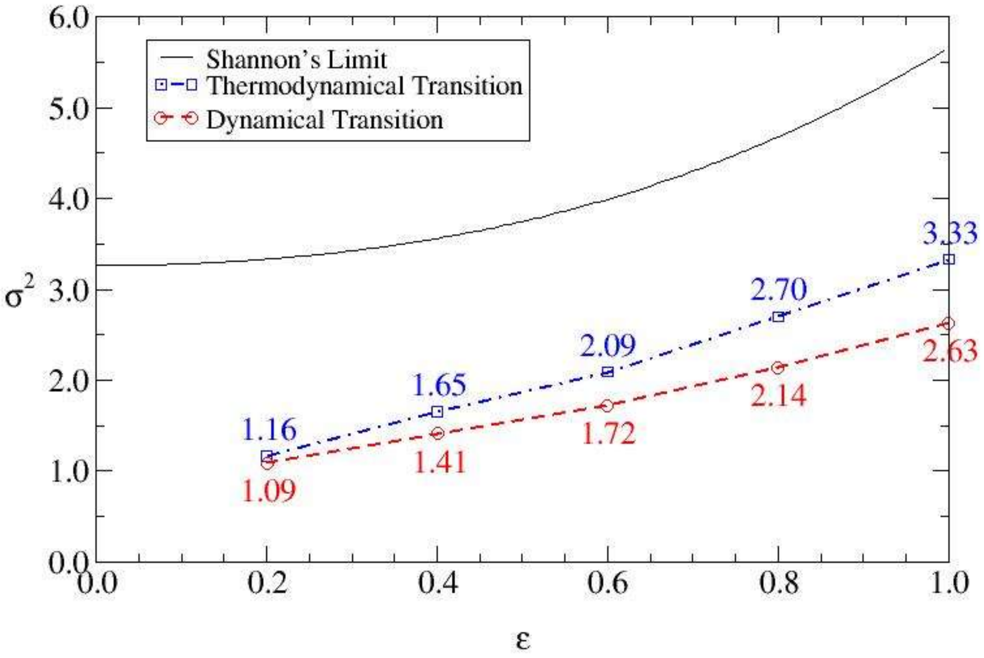


O	Shannon's Limit	Dynamical Transition	Thermodynamical Transition	T.T.-D.T.
1	2.41	1.59	2.24	0.65
2	2.00	1.32	1.66	0.34
3	1.64	1.24	1.45	0.21

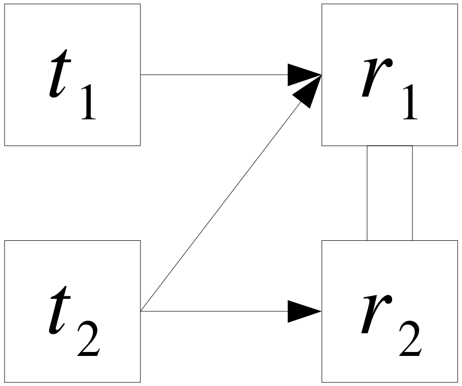
Symmetric Interference Channel

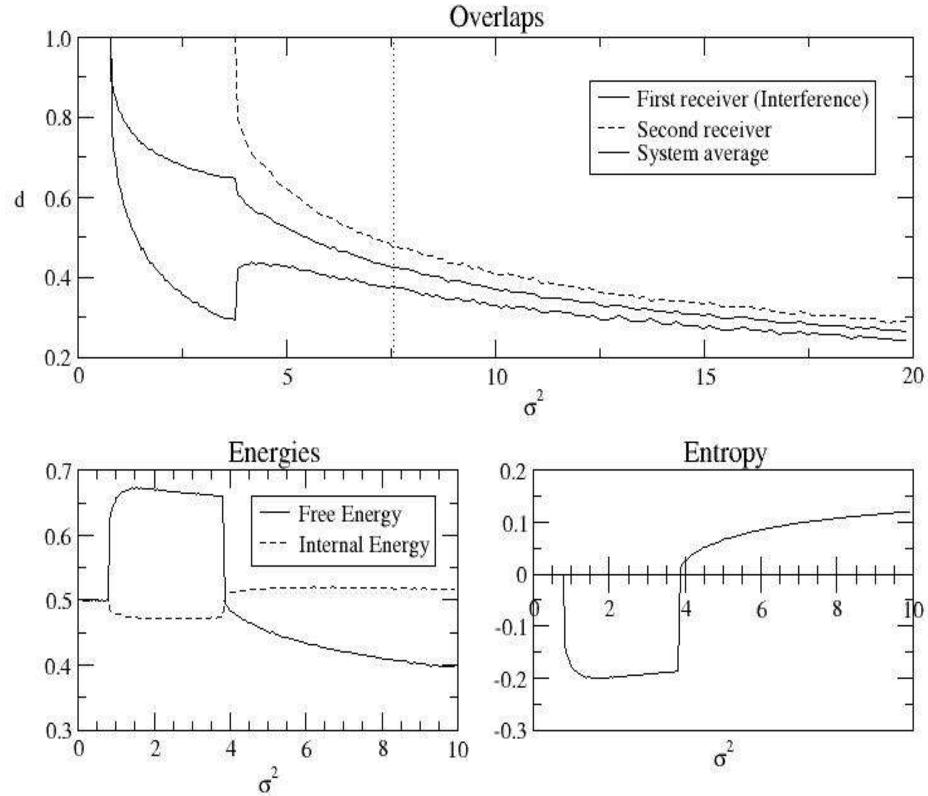
$$0 < \epsilon \leq 1$$





Asymmetric Interference Channel





Conclusions

- Improvement/deterioration in MIMO channels
- Role of metastable states generated by L and O
- Importance of Network Coding
- Increasing threshold with increasing interference in the symmetric interference channel
- Non-trivial information sharing in asymmetric channels