Multi-User detection with sparse CDMA

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Neural Computing Research Group
Seminar

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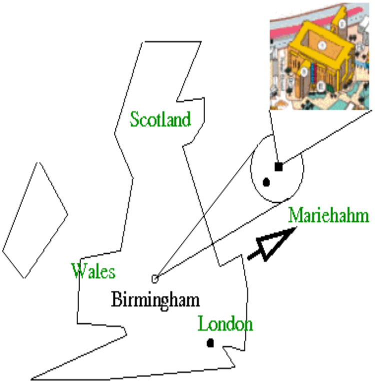
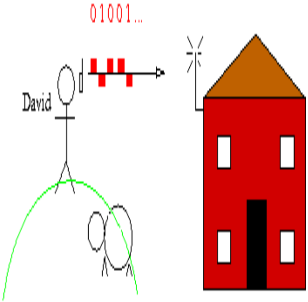
Where am I



Single User Channel

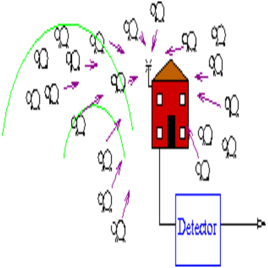


- Capacity limited by channel noise.
- Simple model each bit $\{1, -1\}$ transmitted as a pulse.
- delay $\propto$ bandwidth for single bit.

$$y_\mu = b + \sigma_0 \omega_\mu$$

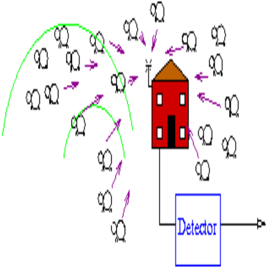
- Channel resources broken up into time-freq blocks called chips (μ)

Multi-User Channel with CDMA



- Limited by multiuser access interference (coordination issues).
- Limited by computational complexity in decoding.

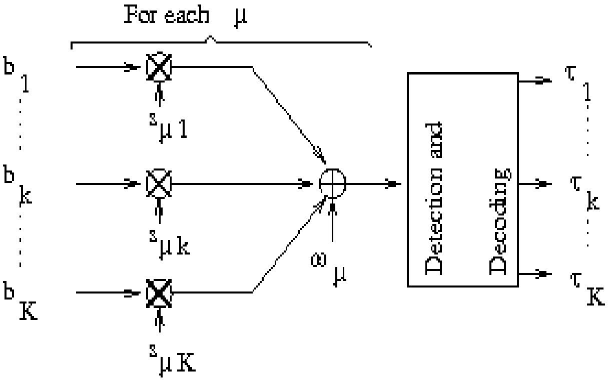
Multi-User Channel with CDMA



- Limited by multiuser access interference (coordination issues).
- Limited by computational complexity in decoding.
- Possible solution CDMA - "multiple access" to each chip
- $$y_\mu = \sum_k s_{\mu k} b_k + \sigma_0 \omega_\mu$$
- Dense Codes: $s_{\mu k} = \{+1, -1\}$ modulation of bits
- Bit interval, $N = \alpha K$ - time/freq blocks for each user transmit 1 bit.

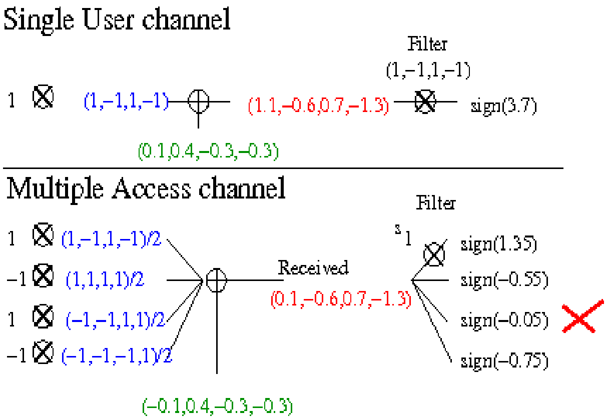
Signal formation

- $$y_\mu = \sum_k s_{\mu k} b_k + \sigma_0 \omega_\mu \quad (1)$$



Example

- Using matched filter for decoding (quick)



CDMA problems

Goals:

- Achieve close to theoretical capacity.
- Robustness against practical implentation problems.
- Robust and quick decoding.
- Versatility.

Methods:

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- TH/FH-CDMA - 1 user/chip
- Orthogonal codes ($\underline{s}_k.\underline{s}_j = \delta_{kj}$)
- Dense CDMA ($s_{\mu k} = \{-1, 1\}$)

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- Orthogonal codes ($\underline{s}_k.\underline{s}_j = \delta_{kj}$)
- Dense CDMA ($s_{\mu k} = \{-1, 1\}$)
- Sparse CDMA ($s_{\mu k} = \{-1, 0, 1\}$), mostly zero $\#(notzero)/N \rightarrow 0$

Optimal performance analysis

- Optimal detectors based upon $P(\underline{\tau}|\underline{y})$
- Can analyse optimal/practical performance with stat. phys (Cavity Method, Replica Method)
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$$H(\underline{\tau}) = \sum_{\mu} \frac{1}{2\sigma_0^2} \left[y_\mu - \sum_k s_{\mu k} \tau_k \right]^2 \quad (2)$$

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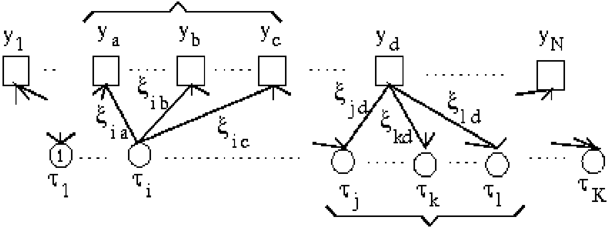
- Free energy found with replica method, assuming simple order parameter structure (RS).

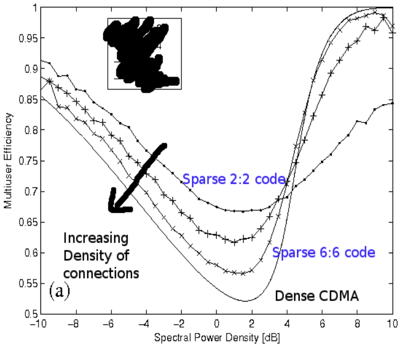
Bipartite graph structure for decoding

- Sparse decoding problem has bipartite graph structure.



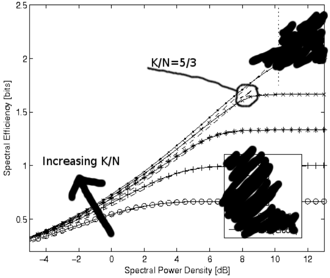
- This is a 3 (chips/user) : 3 (users/chip) regular code. (K/N = 3/3)

How do dense and sparse CDMA compare?



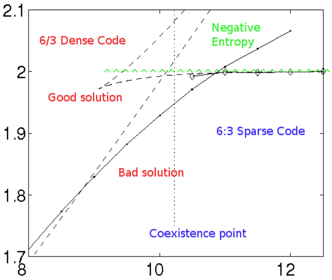
- Both are near optimal.
- With small finite number of connections performance converges quickly.

How does overloading ($K/N > 1$) regime compare?



- Increasing load increases total transmission.
- Phase coexistence appears in dense case.

Phase coexistence also in sparse case



- Phase coexistence in sparse case also - bad performance normally found.
- Appearance of meta-stable states.
- Bad solution shows local linear instability beyond some point.

Conclusions

- Sparse CDMA viable (near optimal) communication method.
- Advantageous power and decoding properties.
- Consider factors within theoretical framework (power control /synchronisation /detectors)....
- Beyond theoretical framework....

Questions

- Overview: Multiuser Detection - Sergio Verdu
- Details: 'Sparsely Spread CDMA - A statistical mechanics based approach' arXiv:0704.0098 (to be updated soon!)

Free energy calculation

- Can calculate free energy through either the cavity method, or replica method.
- All systems converge to similar statistical performance when sufficiently large
-

$$\frac{1}{N} f_N(\beta, s, b, \omega) \doteq f(\beta) \quad (4)$$

$$f(\beta) = \lim_{N\rightarrow\infty} \frac{1}{N} E\left[\log \sum_{\tau} \exp\{\beta H(\underline{\tau})\}\right]_{s,\underline{b},\underline{\omega}} \quad (5)$$

- Deal with ln using replica identity

$$\log Z = \lim_{n\rightarrow 0^+} \frac{\partial}{\partial n} Z^n \quad (6)$$

- Employ Replica Symmetry assumption - assume one pure state can describe system properties.

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- Expressions for free energy cannot be solved exactly, assumptions required and numerical evaluation.
- Solving expressions numerically requires futher approximations which must be tested.

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- Expressions for free energy cannot be solved exactly, assumptions required and numerical evaluation.
- Solving expressions numerically requires futher approximations which must be tested.
- Can we find a solution, is it unique, is it stable?
- Can we valid assumptions necessary in the replica method?