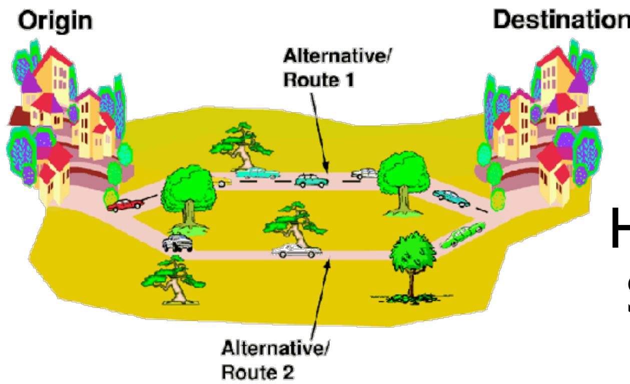
Interacting agents with public and private information (statistical mechanics perspective)

A De Martino (SMC, Roma 1)

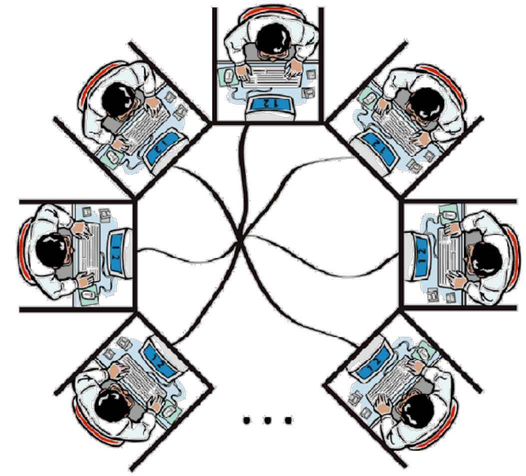
mostly joint work with M Marsili (ICTP Trieste)

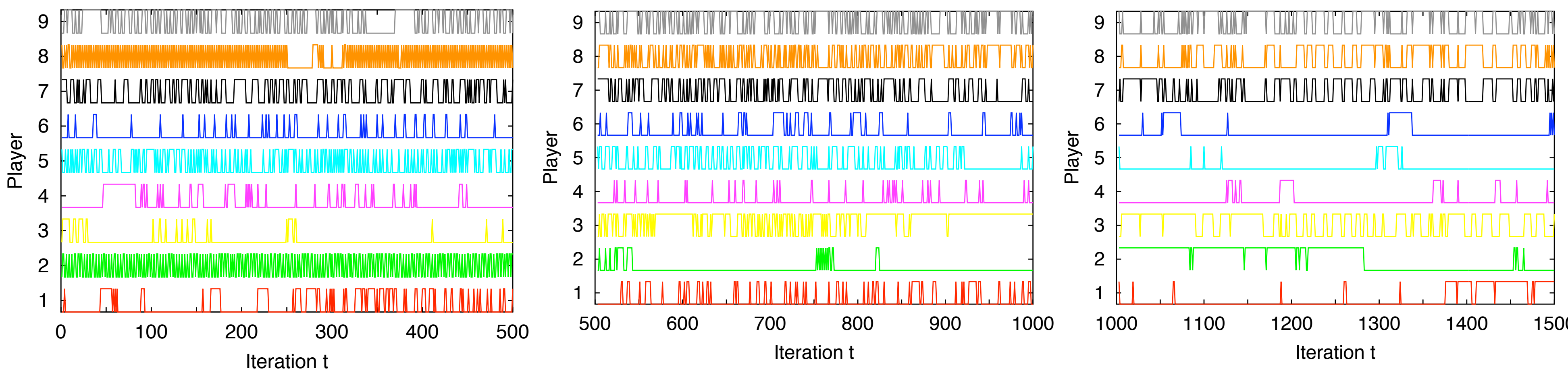
http://chimera.roma1.infn.it/ANDREA

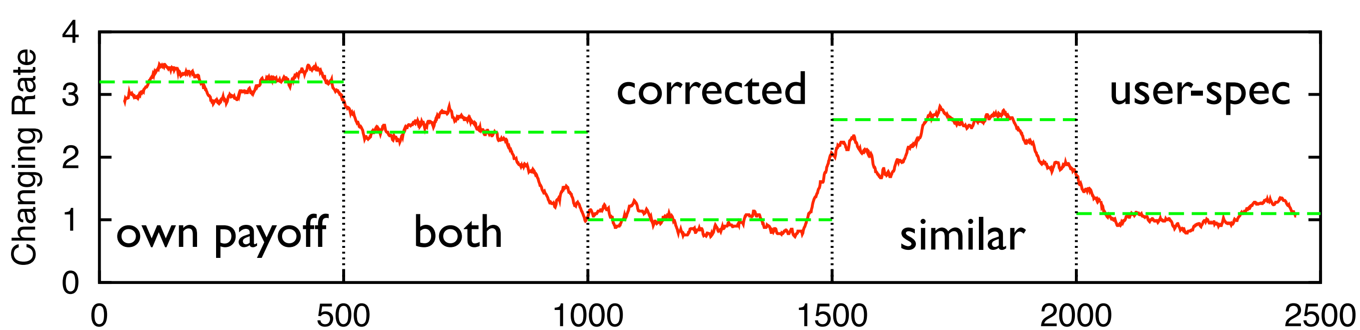
A De Martino and M Marsili, J. Physics A **39** R465 (2006)



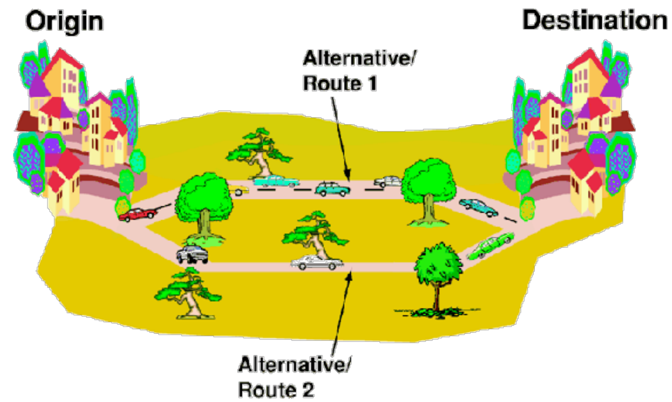
Helbing+2 02
Selten+8 04



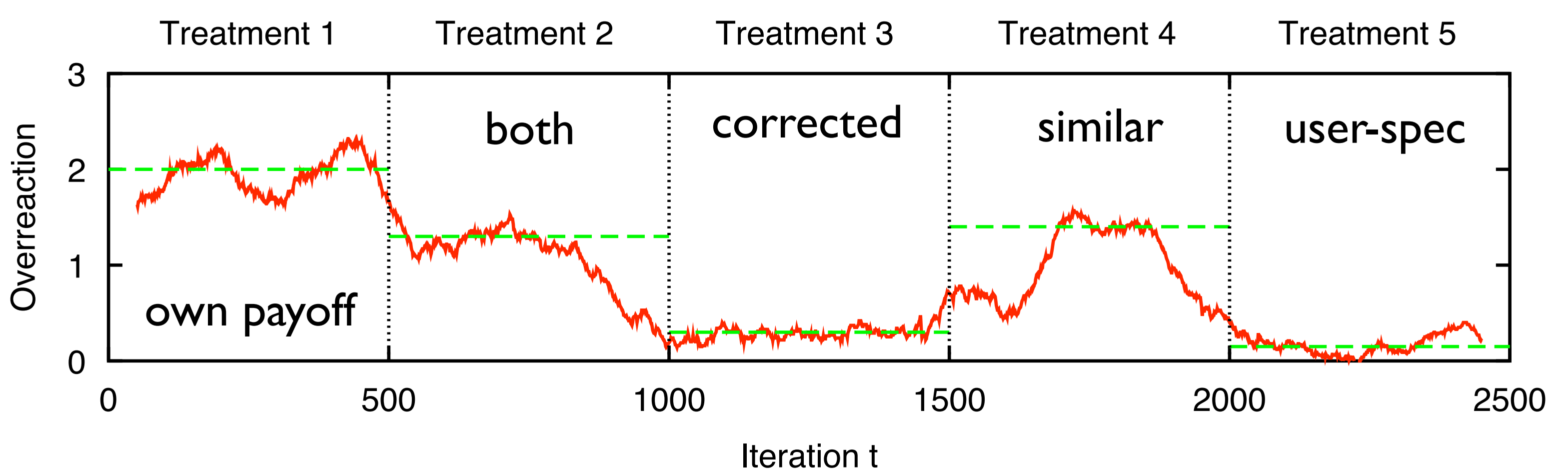




"congestion game"



$$P_1(n_1) = P_1^0 - P_1^1 n_1$$
$$P_2(n_2) = P_2^0 - P_2^1 n_2$$
$$\Rightarrow \quad f_1^{eq} = \frac{P_2^1}{P_1^1 + P_2^1} + \frac{1}{N}\frac{P_1^0 - P_2^0}{P_1^1 + P_2^1}$$

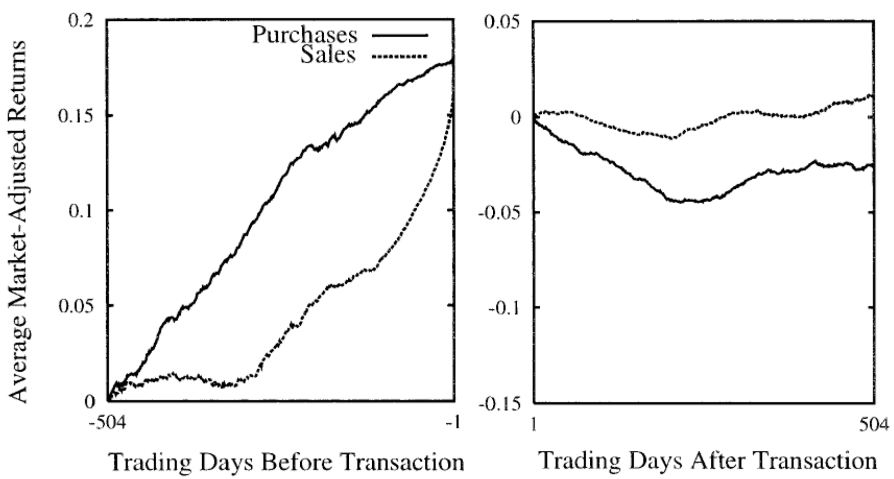


in financial markets (see T Odean's group work)

Do Investors Trade Too Much?

By Terrance Odean*

THE AMERICAN ECONOMIC REVIEW *DECEMBER 1999*



The Courage of Misguided Convictions

Brad M. Barber and Terrance Odean

The field of modern financial economics assumes that people behave with extreme rationality, but they do not. Furthermore, people's deviations from rationality are often systematic. Behavioral finance relaxes the traditional assumptions of financial economics by incorporating these observable, systematic, and very human departures from rationality into standard models of financial markets. We highlight two common mistakes investors make: excessive trading and the tendency to disproportionately hold on to losing investments while selling winners. We argue that these systematic biases have their origins in human psychology. The tendency for human beings to be overconfident causes the first bias in investors, and the human desire to avoid regret prompts the second.

Financial Analysts Journal November/December 1999

"hold losers/sell winners"

people systematically overweight some types of information and underweight others ; overweight their successes and underweight their failures ; overestimate the precision of their information ; etc.

} overconfidence (increases trading volumes, decreases exp. utility, causes underreaction to real information, etc.)

some questions

a. when is convergence to an optimal state possible?
b. when there are many, which one is selected?
c. can one supply information so as to stabilize decision dynamics?
d. how do information schemes affect the dynamics?
e. is there an optimal information scheme?

1 agent , 1resource
N agents , 1 resource
N agents , O(N) resources

learning? (backward-looking induction vs forward-looking deduction)

$\underline{\text{one agent}}$, choices : $s \in \{+, -\}$

payoffs $u_s(t)$ i.r.v. with $Eu_s = v_s$ $(v_+ > v_-)$

decision : $P(s(t) = s) \propto \chi[y_s(t)]$

learning : $y_s(t+1) - y_s(t) = u_s(t)$ (full)

$y_s(t+1) - y_s(t) = u_s(t)\delta_{s(t),s}$ (partial)

learn the best choice?

$$\text{full } + \chi(y) = e^{y^\gamma} \ (\gamma > 0) \ \rightarrow \ \text{yes}$$

$$\text{partial } + \chi(y) = y^\gamma \ (\gamma = 1) \ \rightarrow \ \text{yes}$$

Rustichini 99

one agent , state $a \in \{H, L\}$ (well/badly informed)

forecasts an event θ correctly with prob. a

can he learn a from his past success? $s(t) = \sum_{t' < t} \delta_{\theta(t'), f(t')}$

learning : $P[a = H | s(t) = s] \equiv \phi_\gamma(t|s)$

correct priors, biased updates

$$= \frac{\gamma^s H^s (1-H)^{t-s} \phi_0}{\gamma^s H^s (1-H)^{t-s} \phi_0 + L^s (1-L)^{t-s} (1-\phi_0)}$$

$\gamma = 1 \; \rightarrow$ rational ; $\gamma > 1 \; \rightarrow$ underreact to errors

$\Rightarrow$ if $a = L$ the updated posterior converges a.s. as

$$\phi_\gamma(t|s) \rightarrow \begin{cases} 1 & \text{for } \gamma > \gamma^* \\ \phi_0 & \text{for } \gamma = \gamma^* \\ 0 & \text{for } \gamma < \gamma^* \end{cases}$$

Gervais-Odean 01

1 to N agents

$$\text{choices } a_i \in \{-1, 1\} \ , \ A = \sum_j a_j$$

$$\text{payoffs } u_i(a_i, a_{-i}) = \frac{N - a_i A}{2}$$ (full)

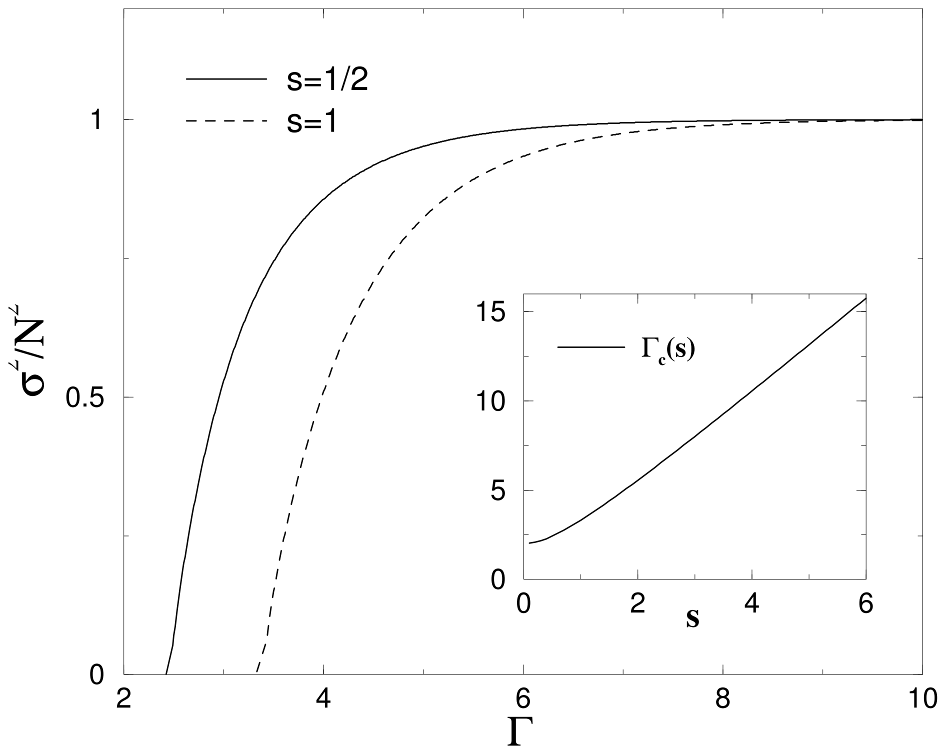
$$\text{decision : } P(a_i(t) = a) \propto e^{y_{ia}(t)}$$ (optimal learning)

$$\text{learning : } y_{ia}(t+1) - y_{ia}(t) = \frac{\Gamma}{N} \frac{N - aA(t)}{2}$$

$$U_i = y_{i+} - y_{i-} \ \rightarrow \ U_i(t+1) - U_i(t) = -\frac{\Gamma}{N} A(t)$$

$y_{ia}(0)$ randomly sampled from $G(0, s^2)$

Fluctuations : $\sigma^2 = \sum_i (\langle a_i^2 \rangle - \langle a_i \rangle^2) = \sum_i (1 - \langle a_i \rangle^2)$



$\Gamma < \Gamma_c \;\Rightarrow\; \sigma^2 \propto N$

$\Gamma > \Gamma_c \;\Rightarrow\; \sigma^2 \propto N^2$

so the larger the spread of i.c.'s (heterogeneity), the smaller the fluctuations but the steady state is not optimal

Optimal state : $\begin{cases} \frac{N-1}{2} & \text{do } a \\ \frac{N+1}{2} & \text{do } -a \end{cases} \;\rightarrow\; \sigma^2 = 1 \qquad \mathcal{N}_{NE} \sim e^{N\Sigma}$

i is in here

Why can't they get to Nash?

$$U_i(t+1) - U_i(t) = -\Gamma A(t)/N \quad , \quad A(t) = \sum_j a_j(t) \ , \ \Gamma > 0$$

remove self-interaction (learn to respond to others, not to yourself)

$$U_i(t+1) - U_i(t) = -\frac{\Gamma}{N}[A(t) - \eta a_i(t)] \qquad \eta \in \{0,1\}$$

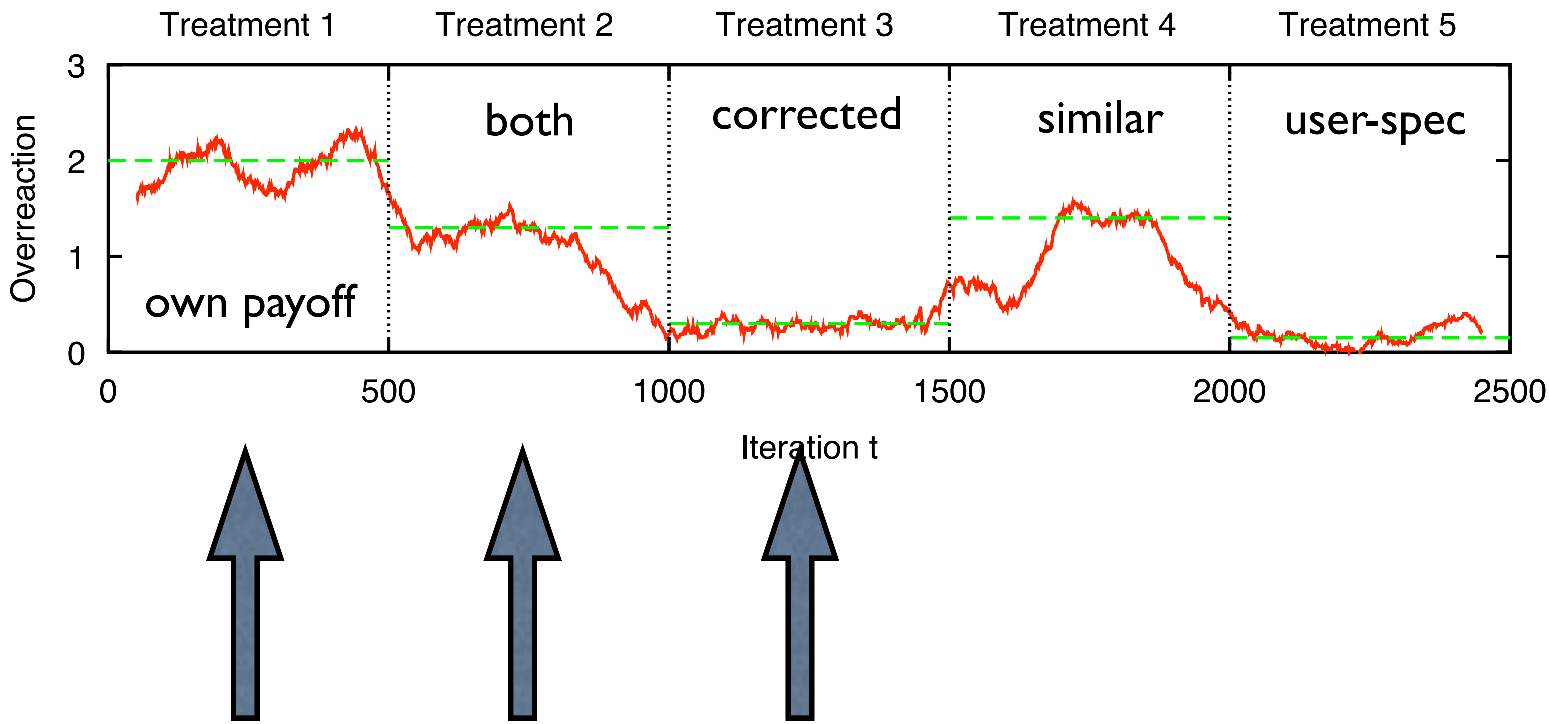
$$\langle U_i(t+1)\rangle - \langle U_i(t)\rangle = -\frac{\Gamma}{N}[\sum_j m_j - \eta m_i] = -\frac{\Gamma}{N}\frac{\partial H}{\partial m_i} \qquad m_i = \langle a_i \rangle$$

$$H = \frac{1}{2}\Big(\sum_i m_i\Big)^2 - \frac{\eta}{2}\sum_i m_i^2$$

minima : $\eta = 0 \ \Rightarrow \ m_i = 0 \ \Rightarrow \sigma^2 = N$

$$\eta = 1 \ : \ H \text{ is harmonic} \ \Rightarrow \ m_i = \pm 1$$

$$\Rightarrow \sigma^2 = 1 \text{ (odd } N)$$

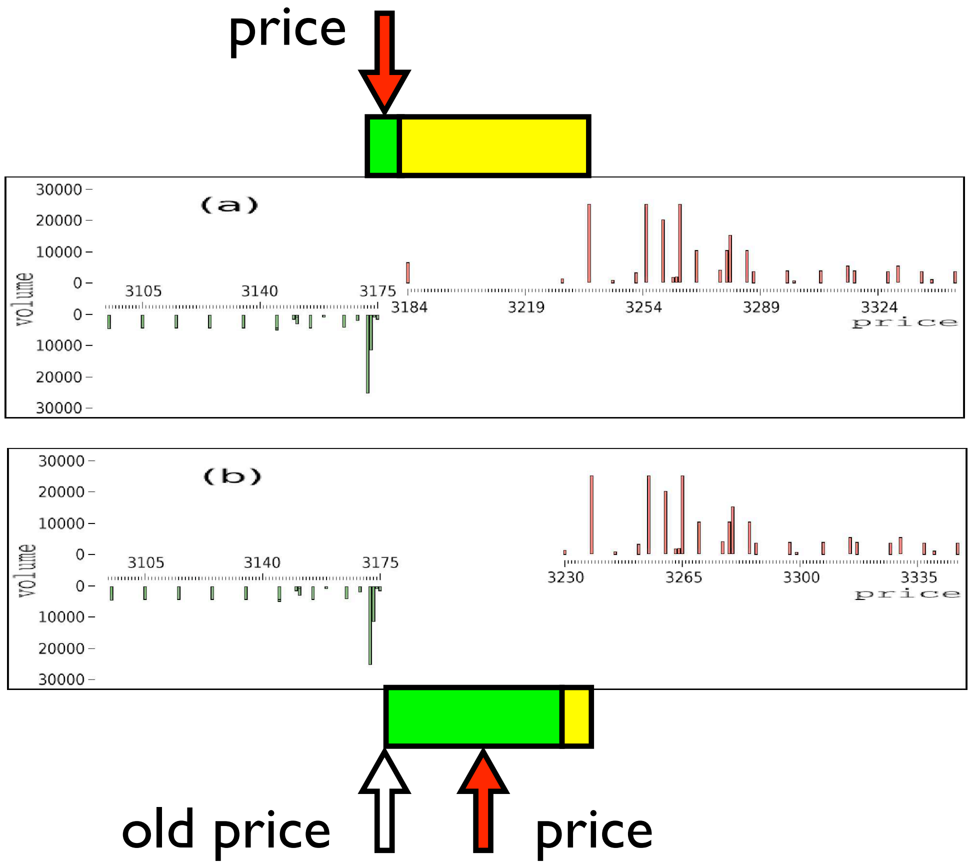


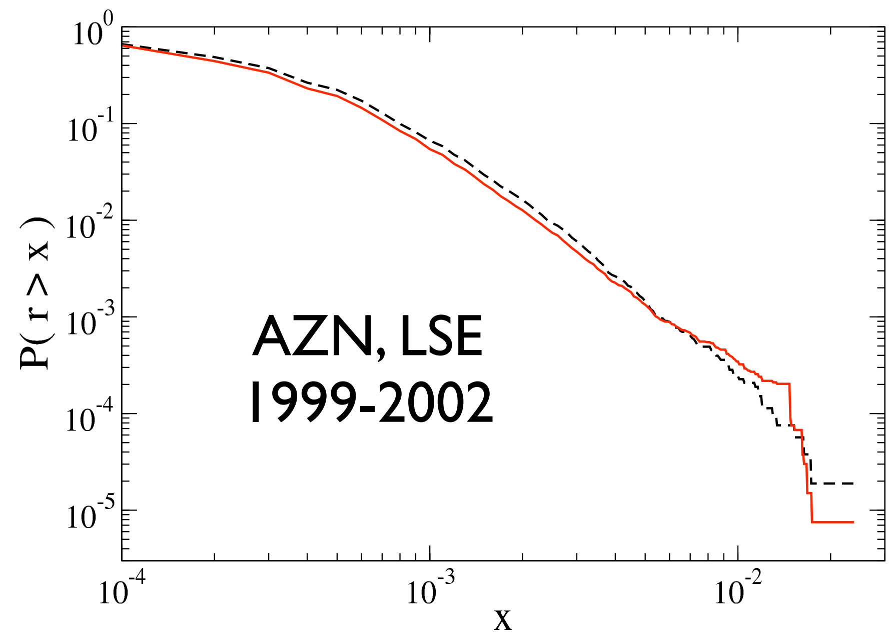
partial full NE

externally supplied, unrealistic
(it is seen as a 1/N effect, "price-taking")

buy/sell difference

first gap





Farmer+2 04

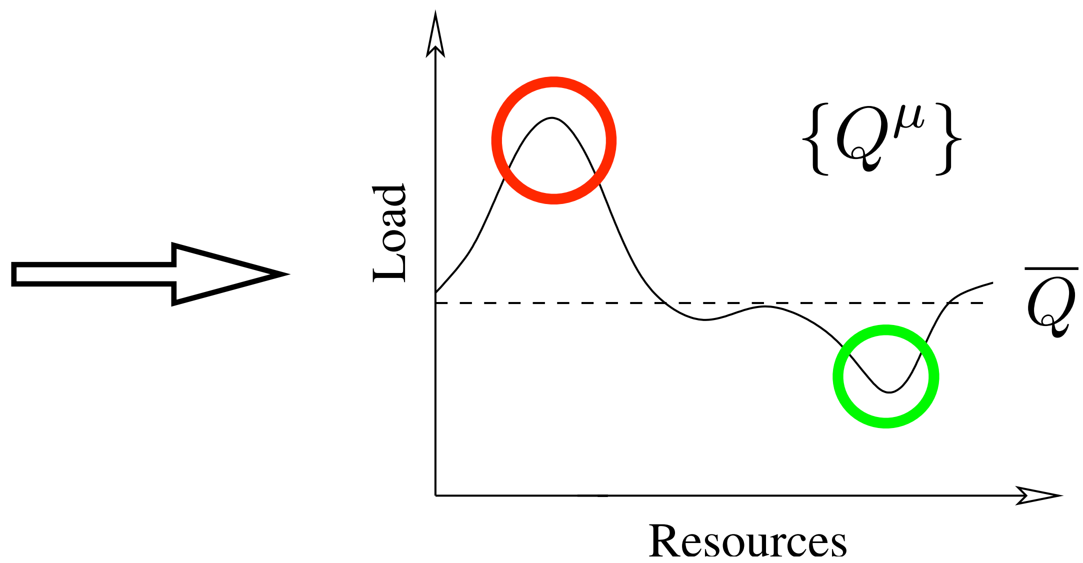
Large price variation =
“closing” of the first gap

N agents, M resources and a matrix of dependencies $\{\xi_i^\mu\}$

Agents are
- heterogeneous
- selfish
- inductive

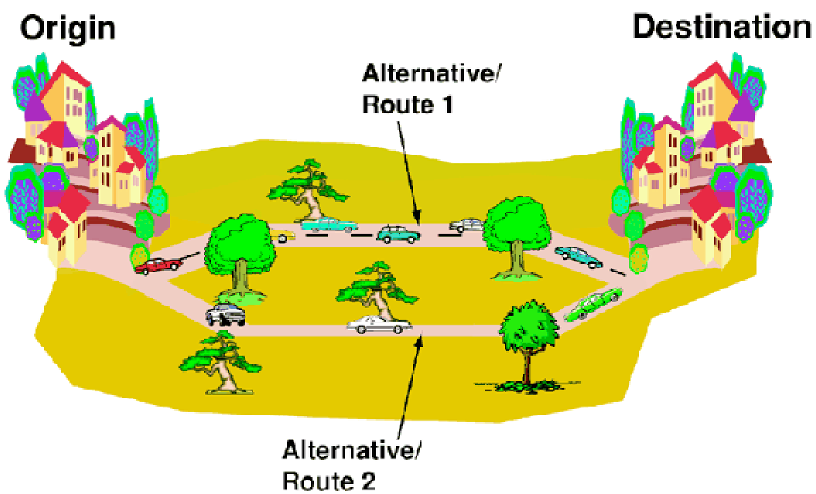
Resources are
- equivalent
- scarce
- (distributed)

No direct interaction

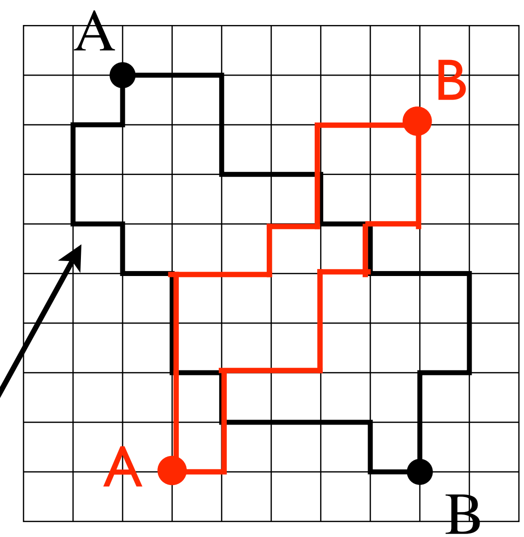


Typical properties : random $\{\xi_i^\mu\}$ + $N \to \infty$, $N = cM$

N drivers on a grid of M streets



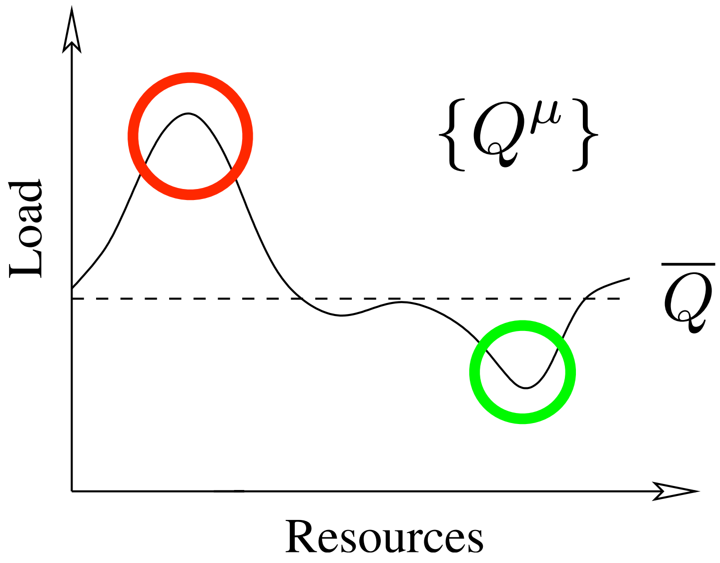
one driver's feasible routes



another driver's feasible routes

$$\text{route for } i : \vec{\xi}_i = \{\xi_i^\mu\}$$

$$c = N/M = \text{density of vehicles}$$



$$A^\mu(t) = Q^\mu(t) - \overline{Q}$$

how evenly are resources used? $H = \frac{1}{M}\sum_\mu \langle A^\mu \rangle^2$

how large are fluctuations? $\sigma^2 = \frac{1}{M}\sum_\mu \langle (Q^\mu)^2 \rangle - \langle Q^\mu \rangle^2$

$$\tau_i(t) \propto \sum_\mu a_i^\mu(t) Q^\mu(t) \quad \rightarrow \quad \sum_i \tau_i(t) \propto \sum_\mu Q^\mu(t)^2$$

two feasible routes per agent : $\vec{\xi}_{is}$, $s \in \{+,-\}$

decision : $P(s_i(t) = s) \propto e^{\Gamma U_{is}(t)}$ $\Gamma \geq 0$

$$\text{learning : } U_{is}(t+1) - U_{is}(t) = -\frac{1}{M}\sum_{\mu} \xi_{is}^{\mu} Q^{\mu}(t) + \frac{1}{2}(1 - \delta_{s,s_i(t)}) z_{is}(t)$$

"congestion game"

$$Q^{\mu}(t) = \sum_i \xi_{is}^{\mu} \delta_{s,s_i(t)}$$

information noise

$\eta = 0$: unbiased information
$\eta > 0$: overestimates unused routes
$\eta < 0$: underestimates unused routes

$$\langle \zeta_{ig}(n) \rangle = \eta$$
$$\langle \zeta_{ig}(n) \zeta_{jh}(m) \rangle = \Delta \delta_{ij} \delta_{gh} \delta_{nm}$$

$\Delta = 0$: same information (biased or not) for all drivers
$\Delta > 0$: user-specific information

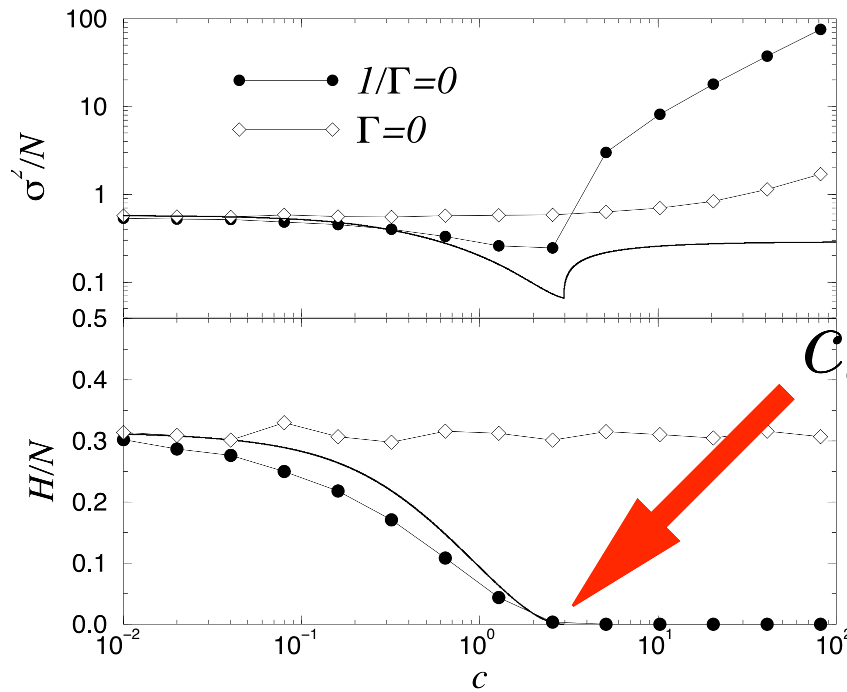
noiseless case $\Delta = \eta = 0$

$\Gamma = 0$

Not optimal but not that bad

$\Gamma = \infty$

Disaster for $c > c_c$ (high density)



$c_c \simeq 3$

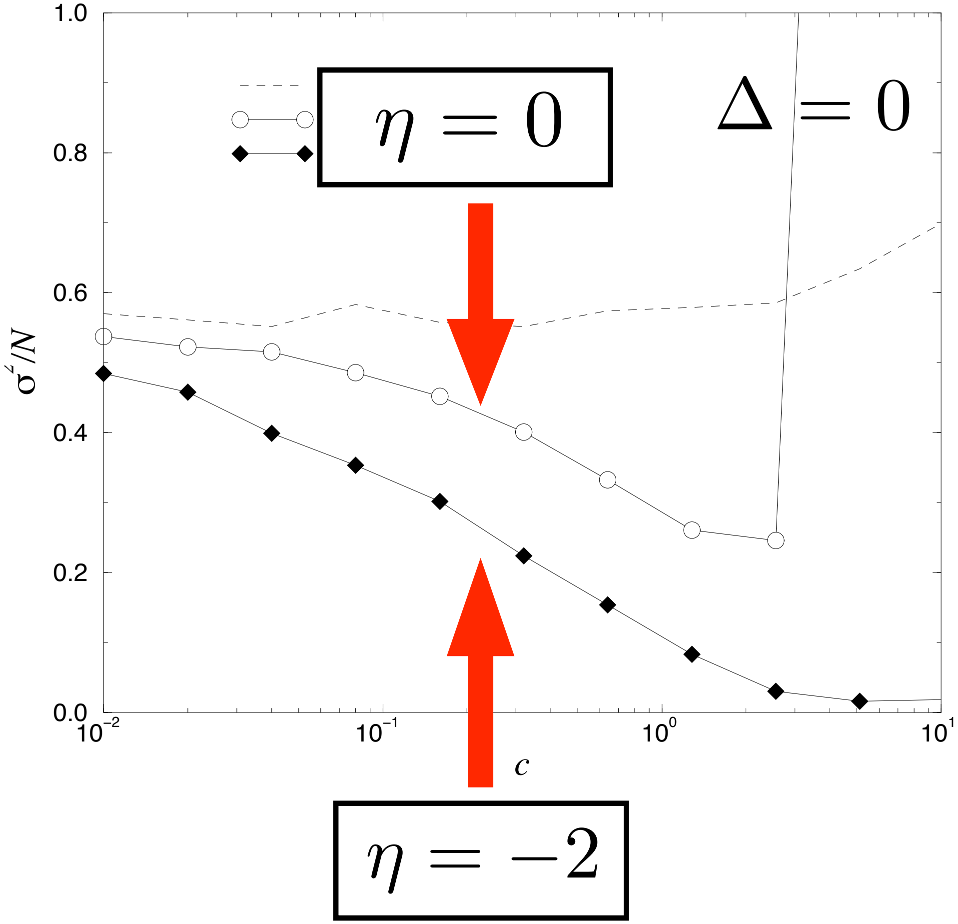
inductive perform worse than random at high densities!

theory (steady state) : the dynamics minimizes H (approx.)

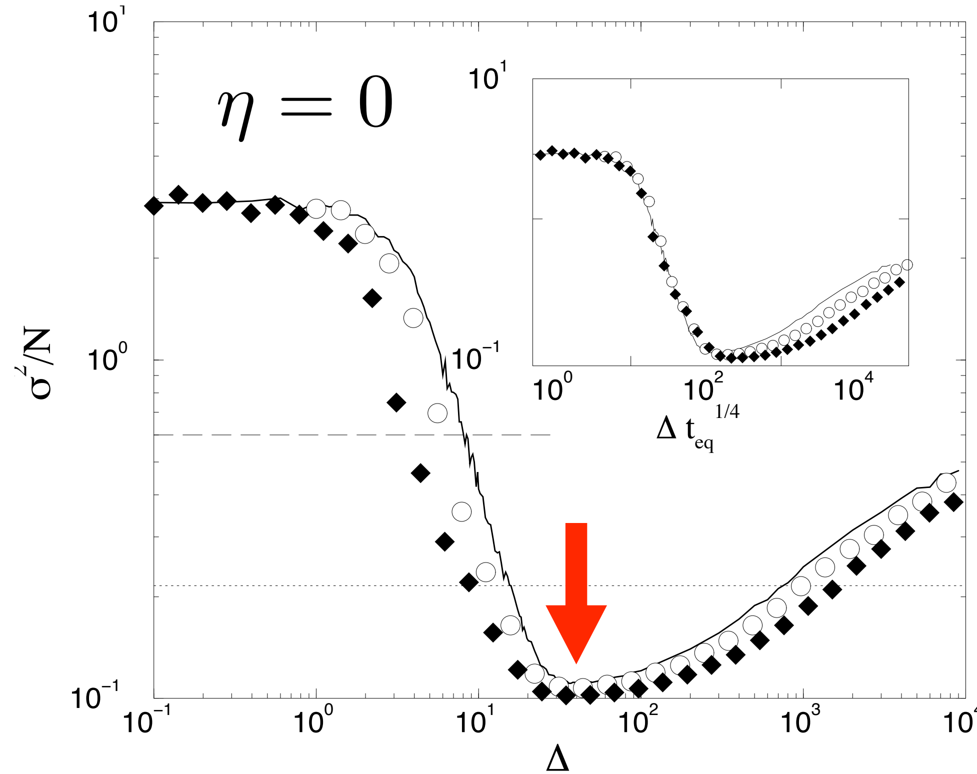
problem : the quenched disorder is spatially correlated ...

Uncorrelated result : $c_c = 2.9638\ldots$

biased case



noisy case

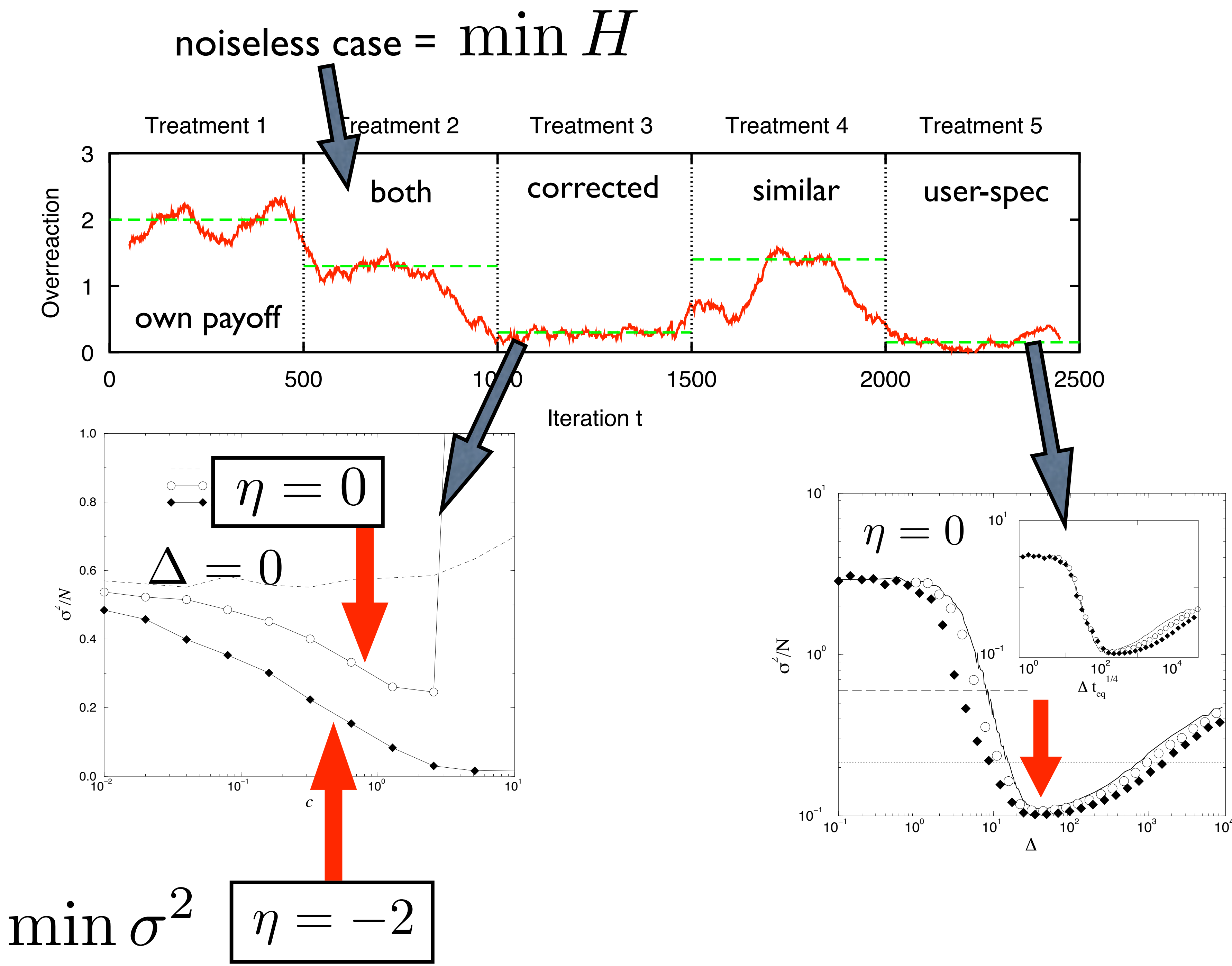


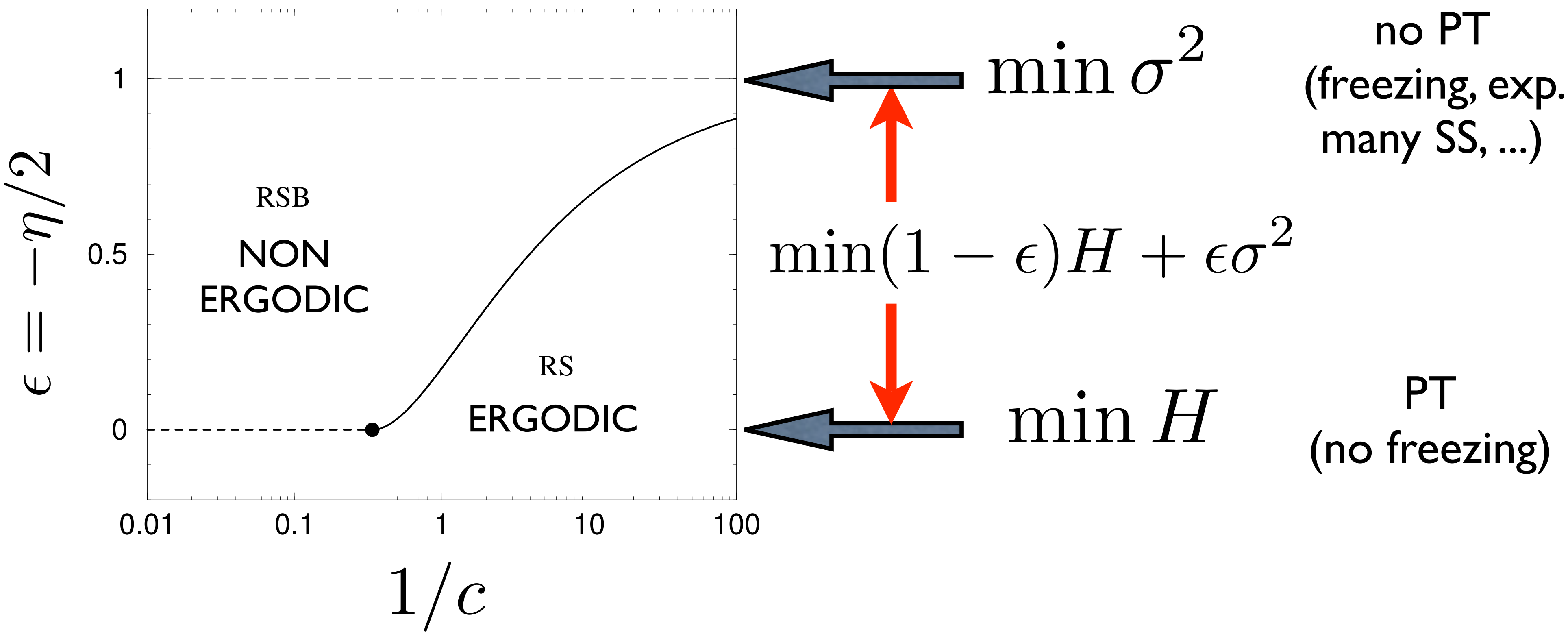
high car density
(hard phase)

\`pessimist' information bias
reduces fluctuations

calibrated user-specific noise
reduces fluctuations

theory (steady state) : the dynamics
minimizes fluctuations


noiseless case = $\min H$
Treatment 1
Treatment 2
Treatment 3
Treatment 4
Treatment 5
own payoff
both
corrected
similar
user-spec
Overreaction
Iteration t
0
500
1000
1500
2000
2500
1
2
3
$\eta = 0$
$\Delta = 0$
σ^2/N
c
$\min \sigma^2$
$\eta = -2$
Δ
$\Delta\, t_{eq}^{1/4}$



$\epsilon = -\eta/2$
1
0.5
0
0.01
0.1
1
10
100
$1/c$
RSB
NON ERGODIC
RS
ERGODIC
$\min \sigma^2$
$\min (1-\epsilon)H + \epsilon\sigma^2$
$\min H$
no PT
(freezing, exp. many SS, ...)
PT
(no freezing)