

Dynamics of Networking Social Agents: From Diplomacy to Friendship

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system

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COEVOLUTION OF NETWORKS & OPINIONS: the idea

P. Holme & M. E. J. Newman, Phys. Rev. E **74**, 056108 (2006).

- Opinions spread over social networks.
- People with the same opinion are likely to become acquainted.
- We try to combine these points into a simple model of simultaneous opinion spreading and network evolution.



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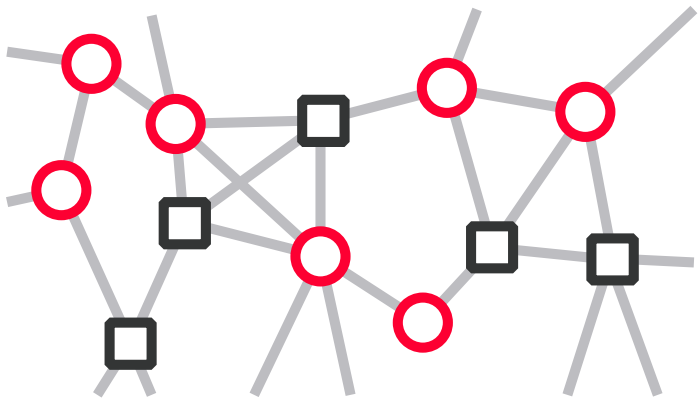
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the voter model

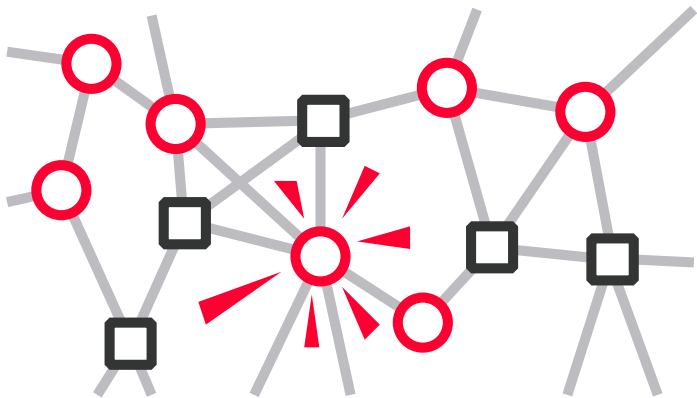


Clifford & Sudbury, *Biometrika* **60**, 581 (1973).

Holley & Liggett, *Ann. Probab.* **3**, 643 (1975).



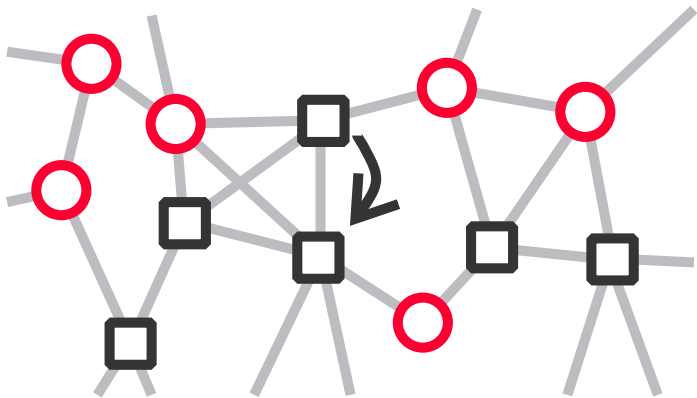
the voter model



choose one vertex randomly



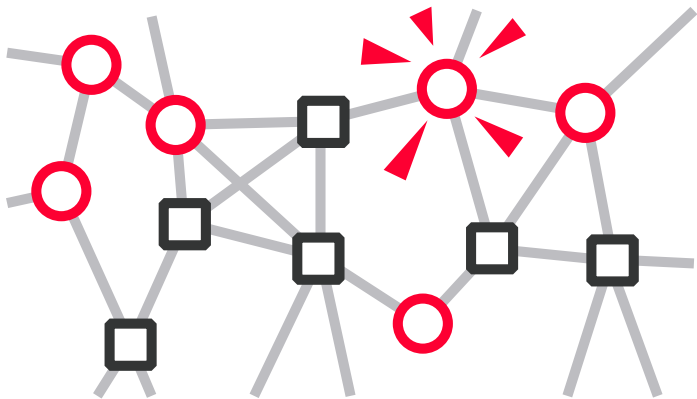
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copy the opinion of a random neighbor



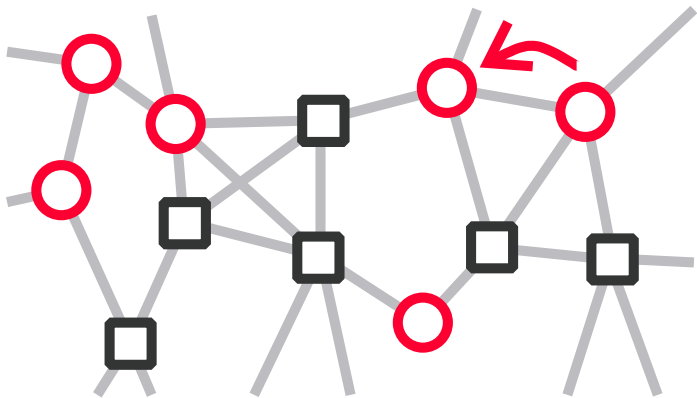
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and so on . . .



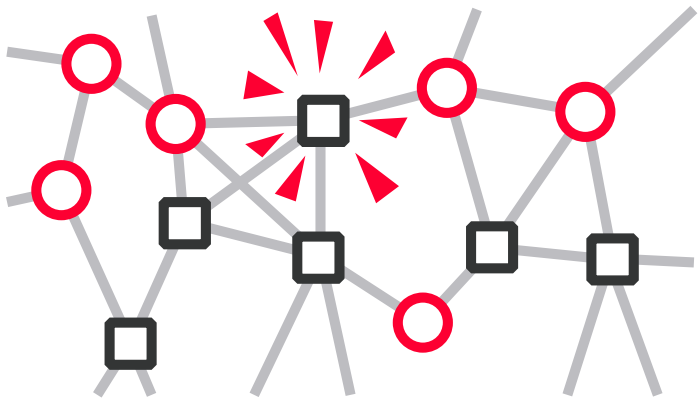
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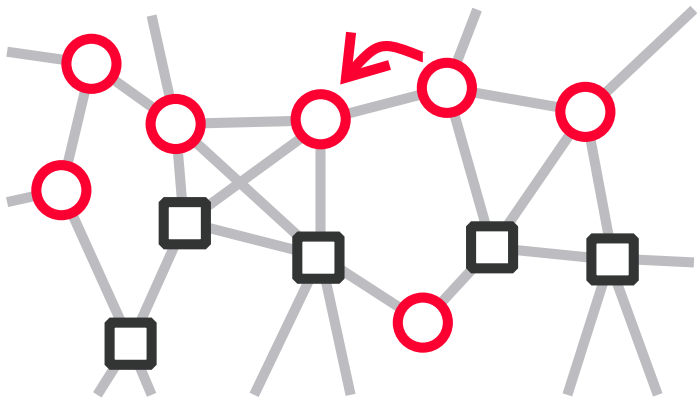
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the voter model

acquaintance dynamics

- People of similar interests are likely to get acquainted. e.g.: McPherson *et al.*, *Ann. Rev. Sociol.* **27**, 415 (2001).
- The number of edges is constant.



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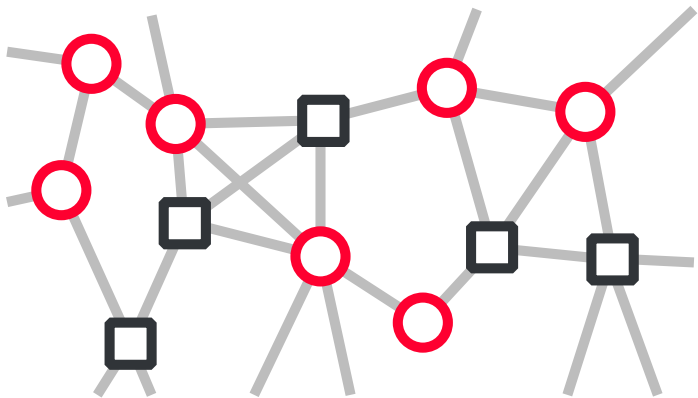
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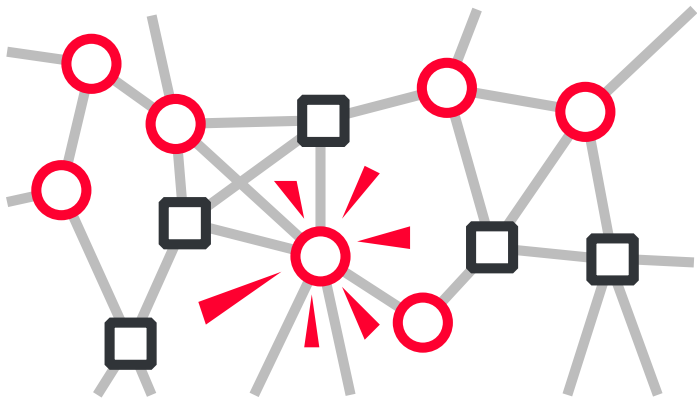
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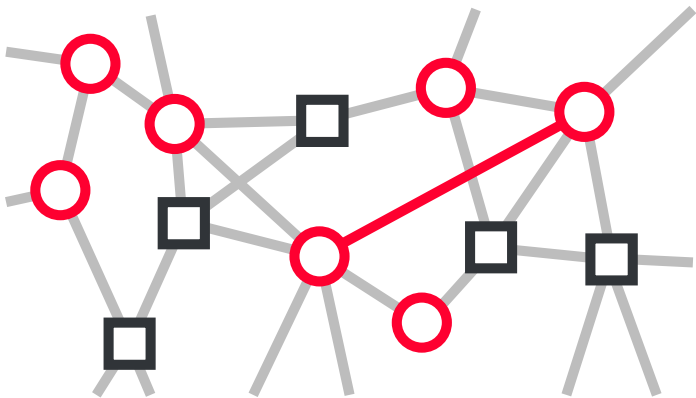
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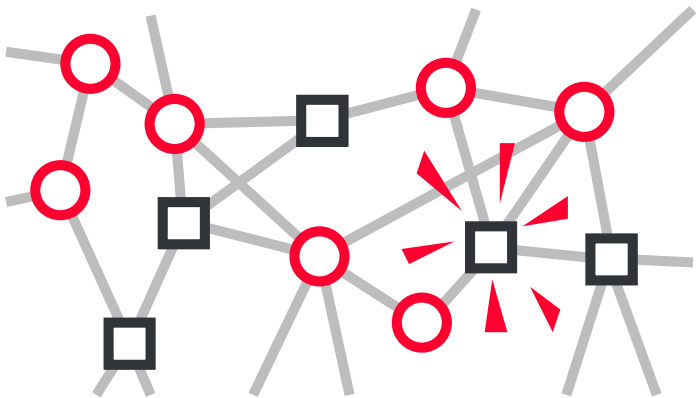
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rewire an edge to a vertex with the same opinion



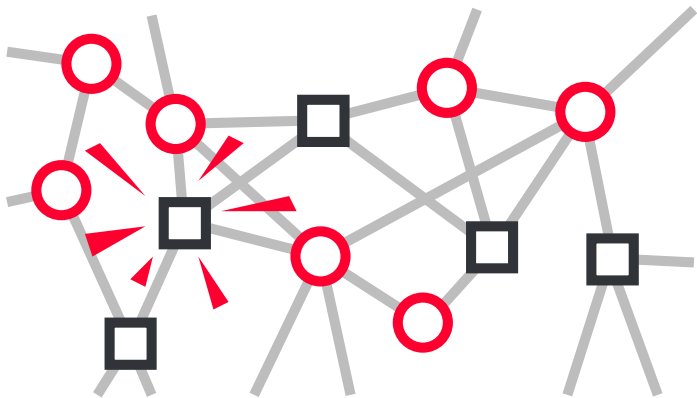
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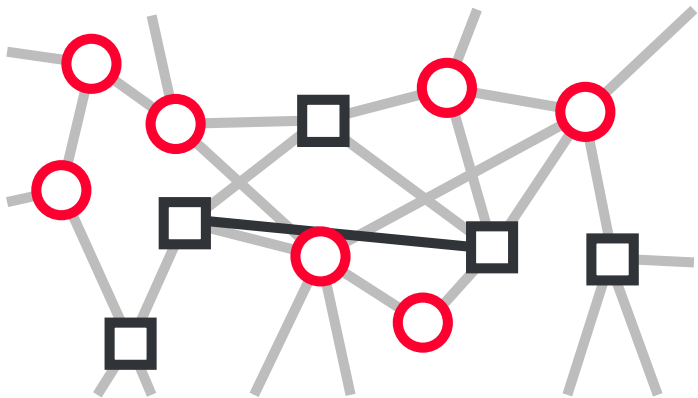
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our model

- 1 Start with a random network of N vertices $M = \bar{k}N/2$ edges and $G = N/\gamma$ randomly assigned opinions.
- 2 Pick a vertex i at random.
- 3 With a probability ϕ make an acquaintance formation step from i .
- 4 . . . otherwise make a voter model step from i .
- 5 If there are edges leading between vertices of different opinions—iterate from step 2.



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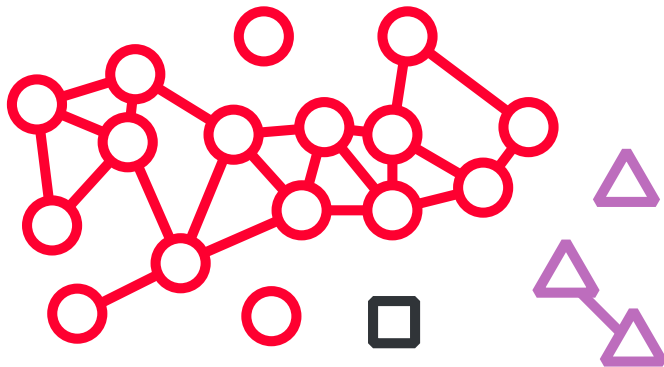


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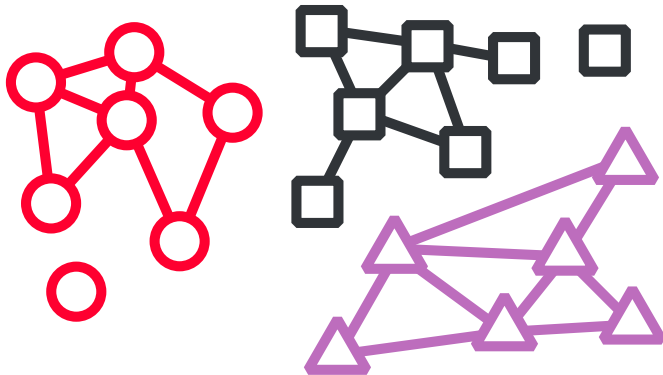
phases



low ϕ —one dominant cluster



phases



high ϕ — clusters of similar sizes



quantities we measure

- The relative largest size S of a cluster (of vertices with the same opinion).
- The average time τ to reach consensus.



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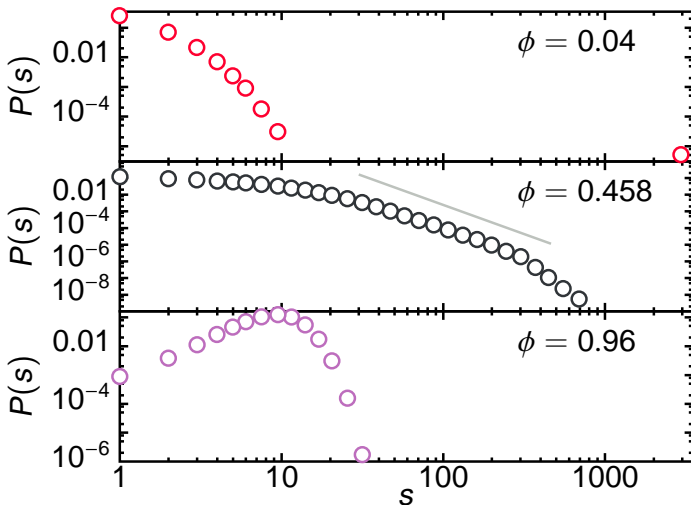


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cluster size distribution



finding the phase transition

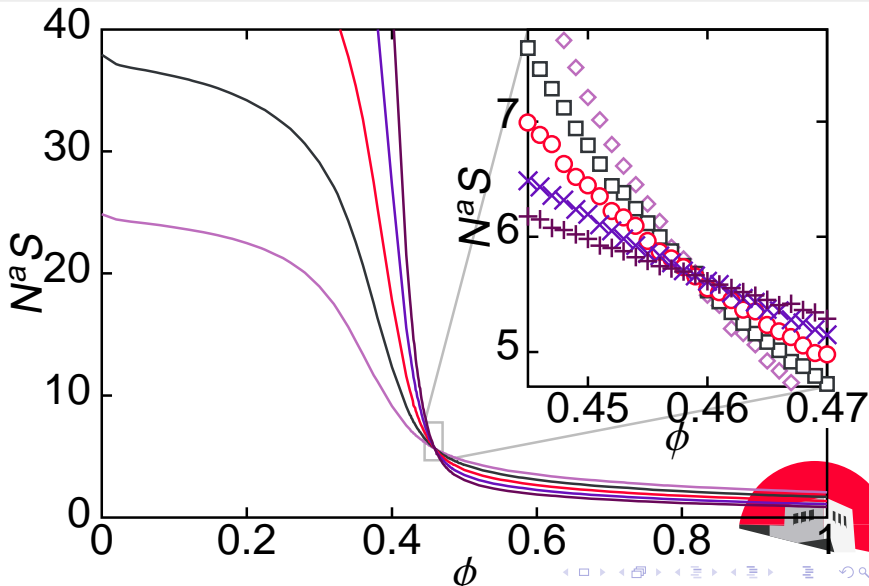
Assume a critical scaling form:

scaling form

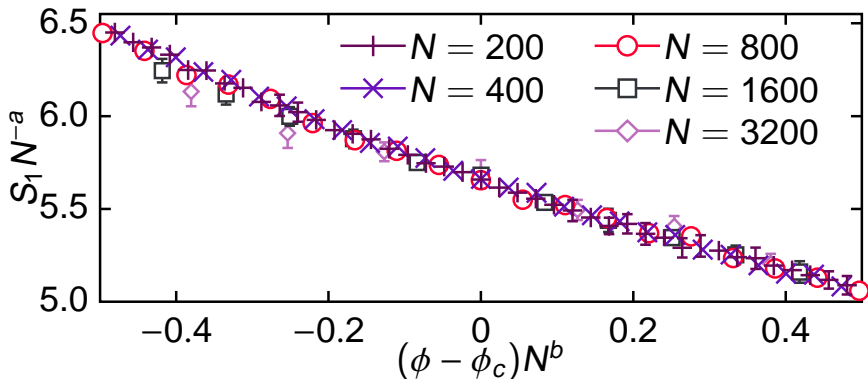
$$S = N^{-a} F\left(N^b(\phi - \phi_c)\right)$$



finding the phase transition



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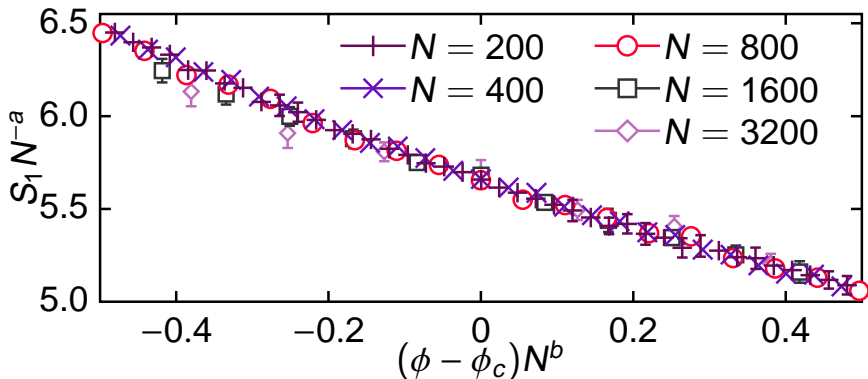


$a = 0.61 \pm 0.05$, $\phi_c = 0.458 \pm 0.008$, $b = 0.7 \pm 0.1$

random graph percolation: $a = b = 1/3$



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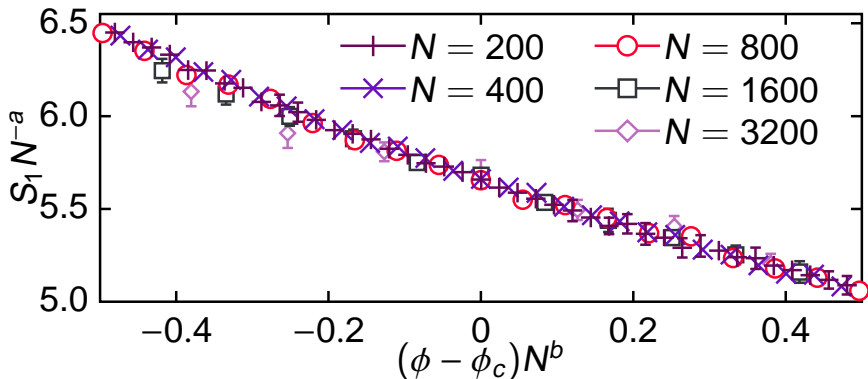


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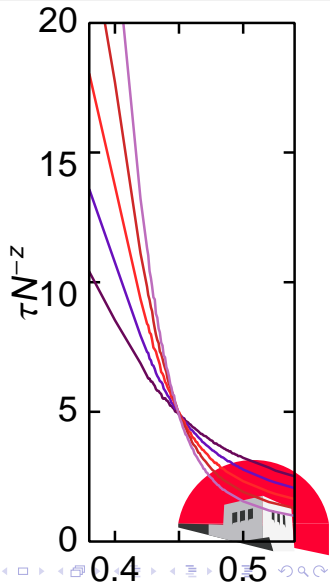
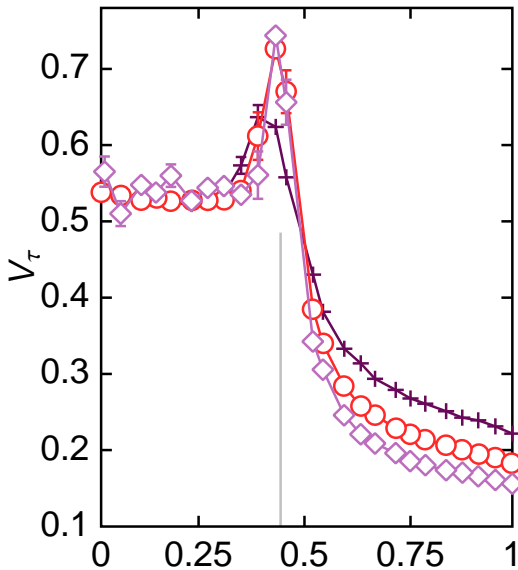


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dynamic critical behavior



conclusions

- We have proposed a simple, non-equilibrium model for the coevolution of networks and opinions.
- The model undergoes a second order phase transition between: One state of clusters of similar sizes. One state with one dominant cluster.
- The universality class is not the same as random graph percolation.
- In society, a tiny change in the social dynamics may cause a large change in the diversity of opinions.



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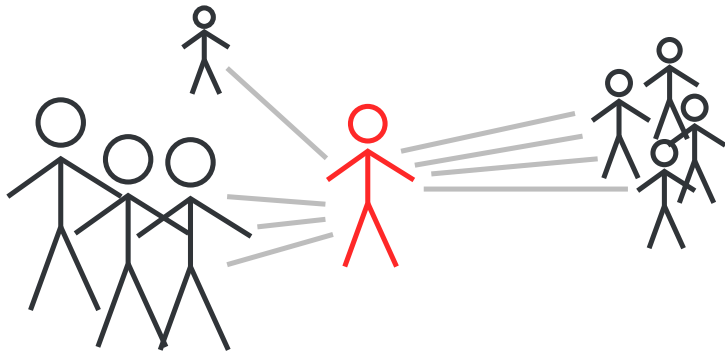
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motivation

In diplomacy, lobbying or other political or corporate networking, it is important to:

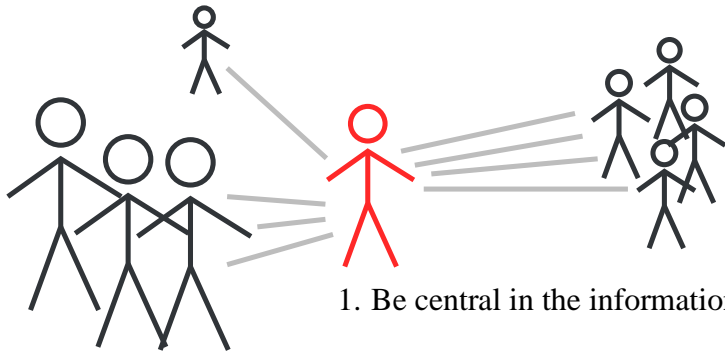


Holme & Ghoshal, Phys. Rev. Lett. **96**, 098701 (2006)



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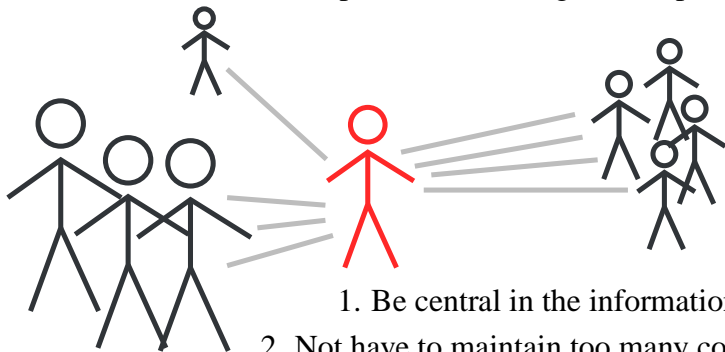
1. Be central in the information flow.

Holme & Ghoshal, Phys. Rev. Lett. **96**, 098701 (2006)



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score function

- Central is good—*closeness centrality*

$$C(i) = (N - 1) / \sum_{j \neq i} d(i, j)$$

- If the network is disconnected, being a part of a large component is good.
- Large degree is bad.



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Component size can be incorporated by modifying the definition of closeness: If we sum the reciprocals (instead of inverting the sum), we get the score function:

Definition

$$s(i) = \begin{cases} (1/k_i) \sum_{H_i} 1/d(i, j) & \text{if } k_i > 0 \\ 0 & \text{if } k_i = 0 \end{cases} \quad (1)$$

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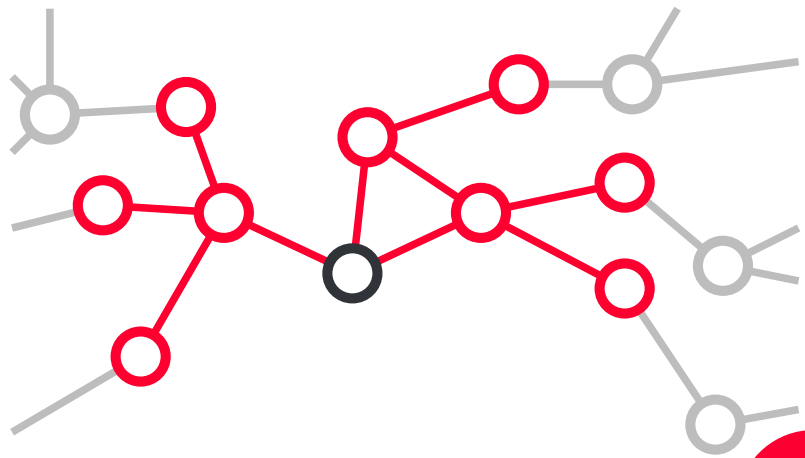
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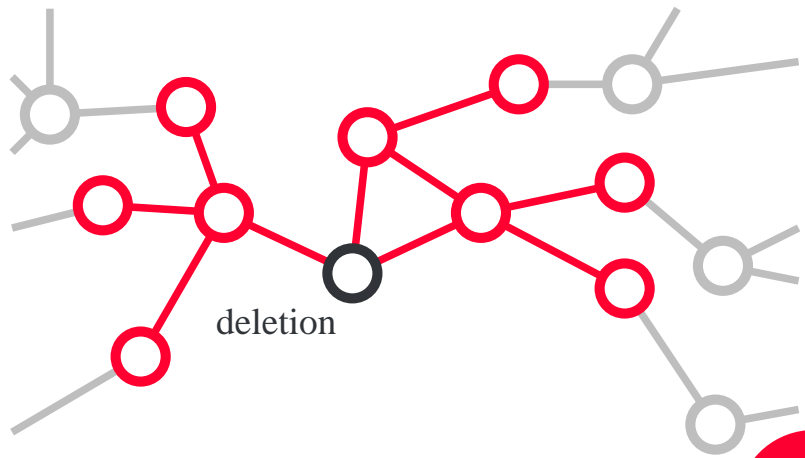
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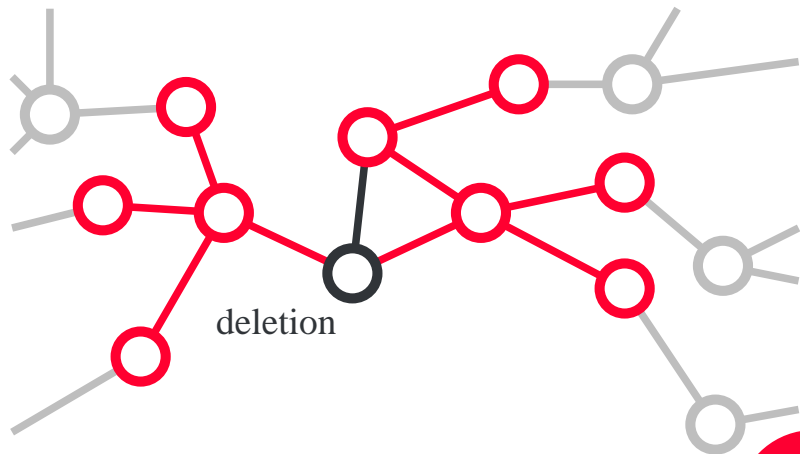
moves



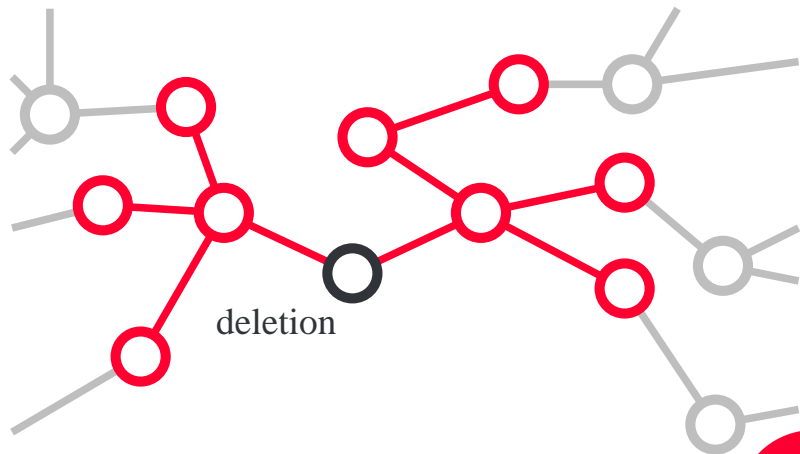
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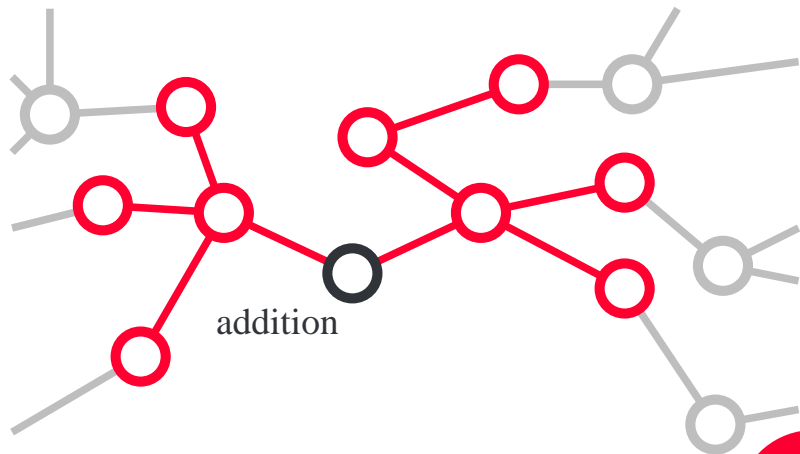
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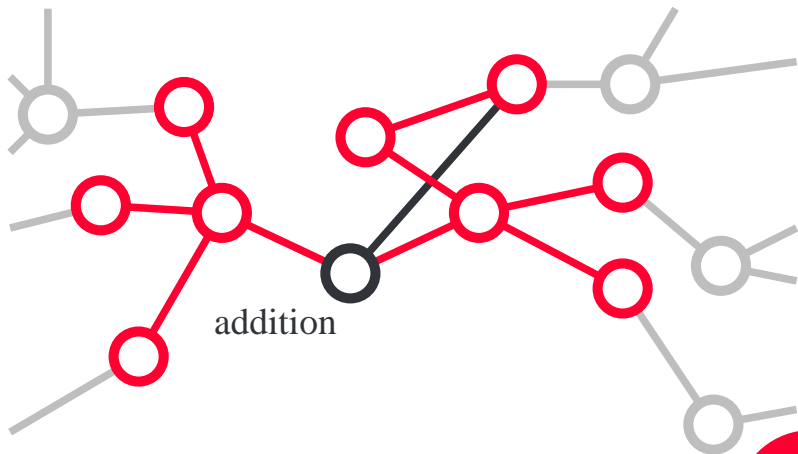
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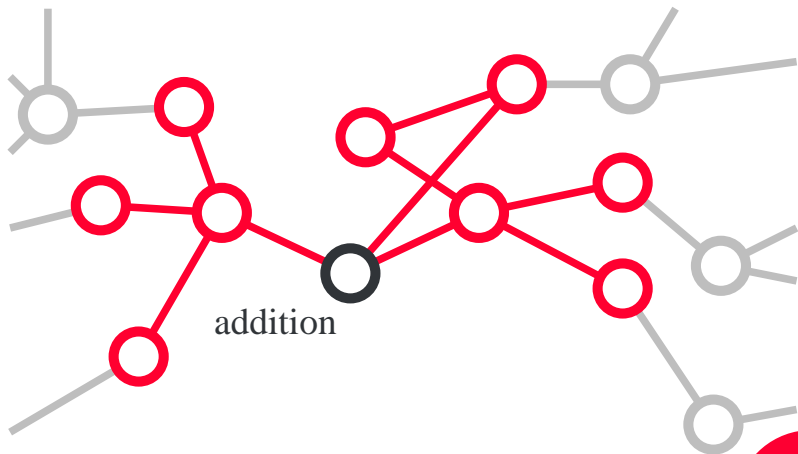
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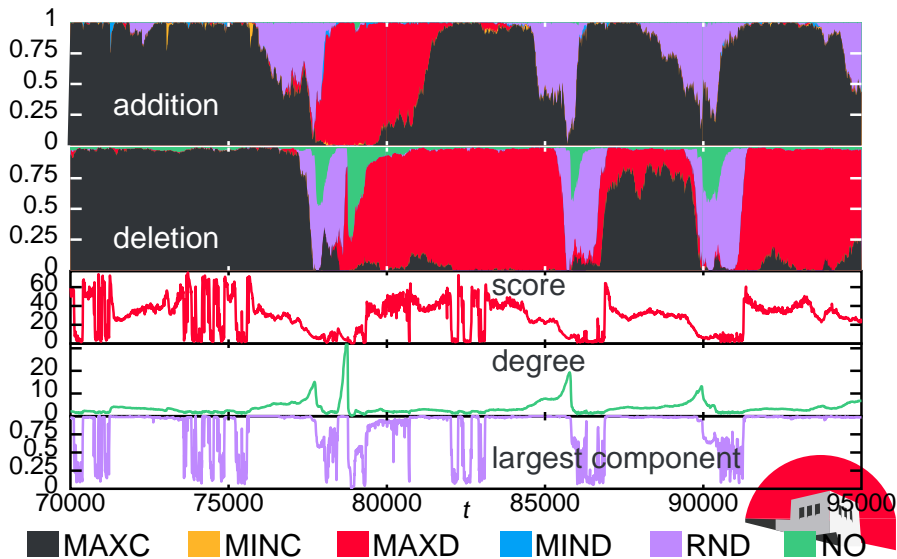
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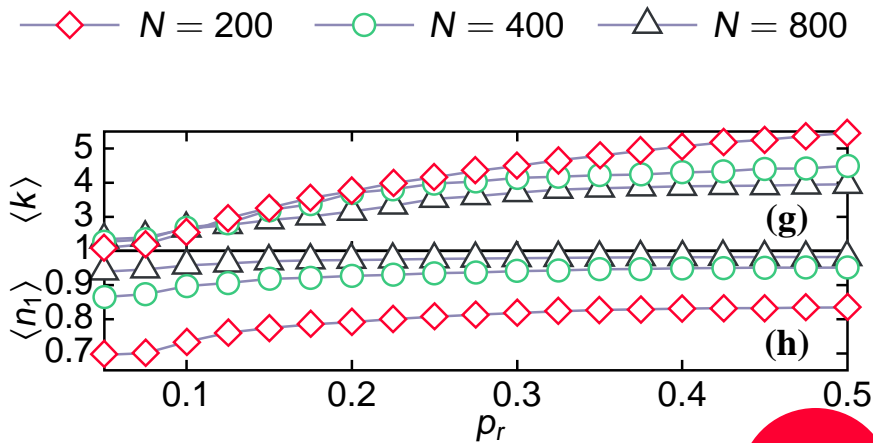
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time evolution



effect of random moves: degree & cluster size



conclusions

- A simple problem that gets quite convoluted when one wants to be general.
- Complex time evolution with spikes, quasi-equilibria and trends.
- Network structure and strategy densities are correlated.
- The most common strategy, over a large range of parameter space, is MAXC.
- MAXC gives a bimodal degree distribution
- The NO/NO strategy is not stable—Red Queen.
- The network gets sparser and more connected with size.



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