Nordita, April 20-22, 2015

KTH ROYAL INSTITUTE
OF TECHNOLOGY

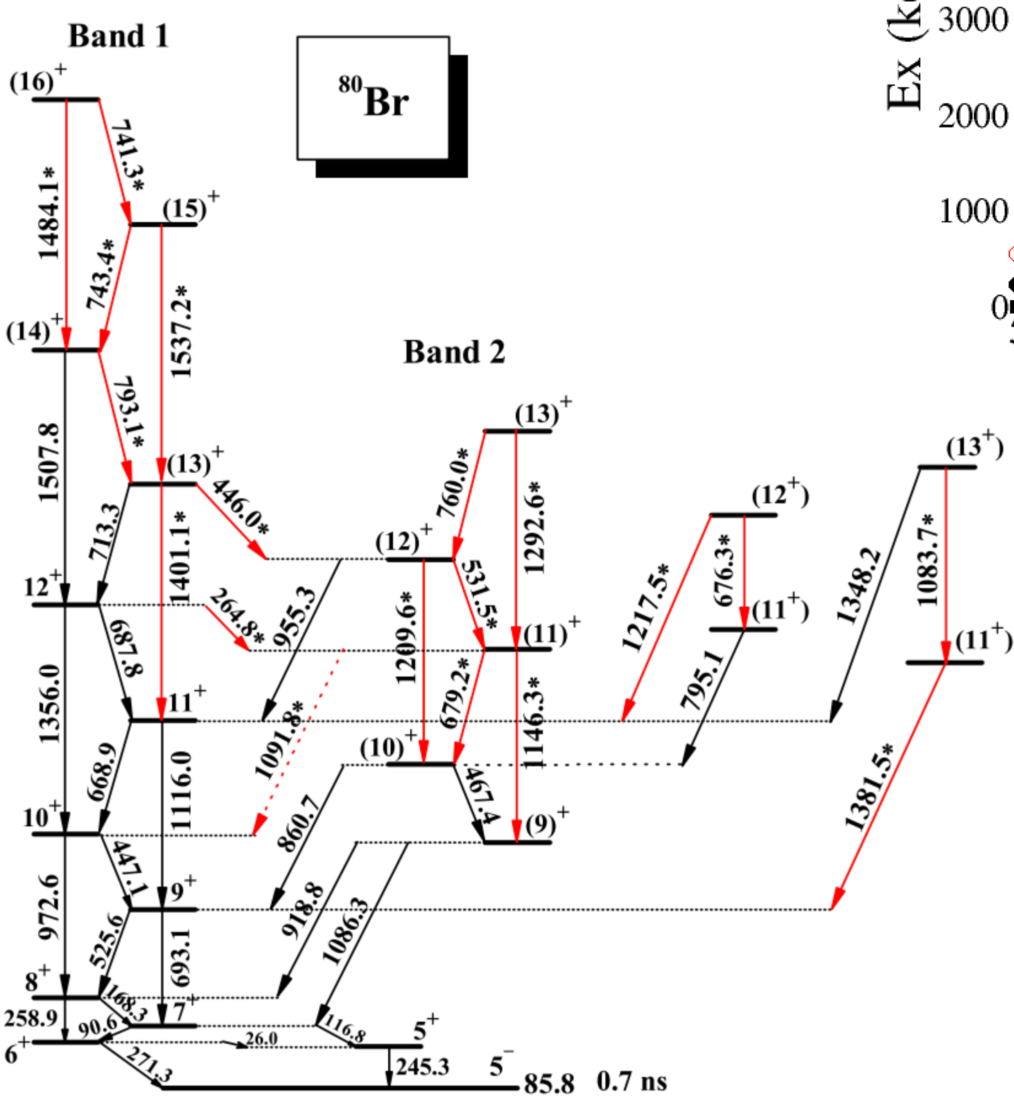
Neutron-proton pair coupling from a shell-model perspective

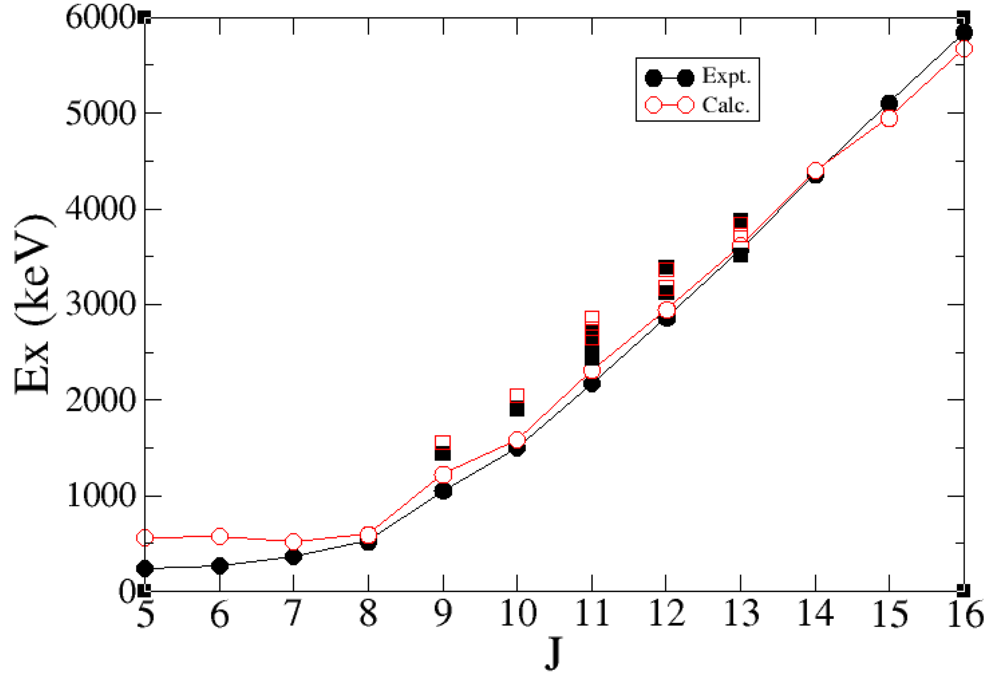
Chong Qi

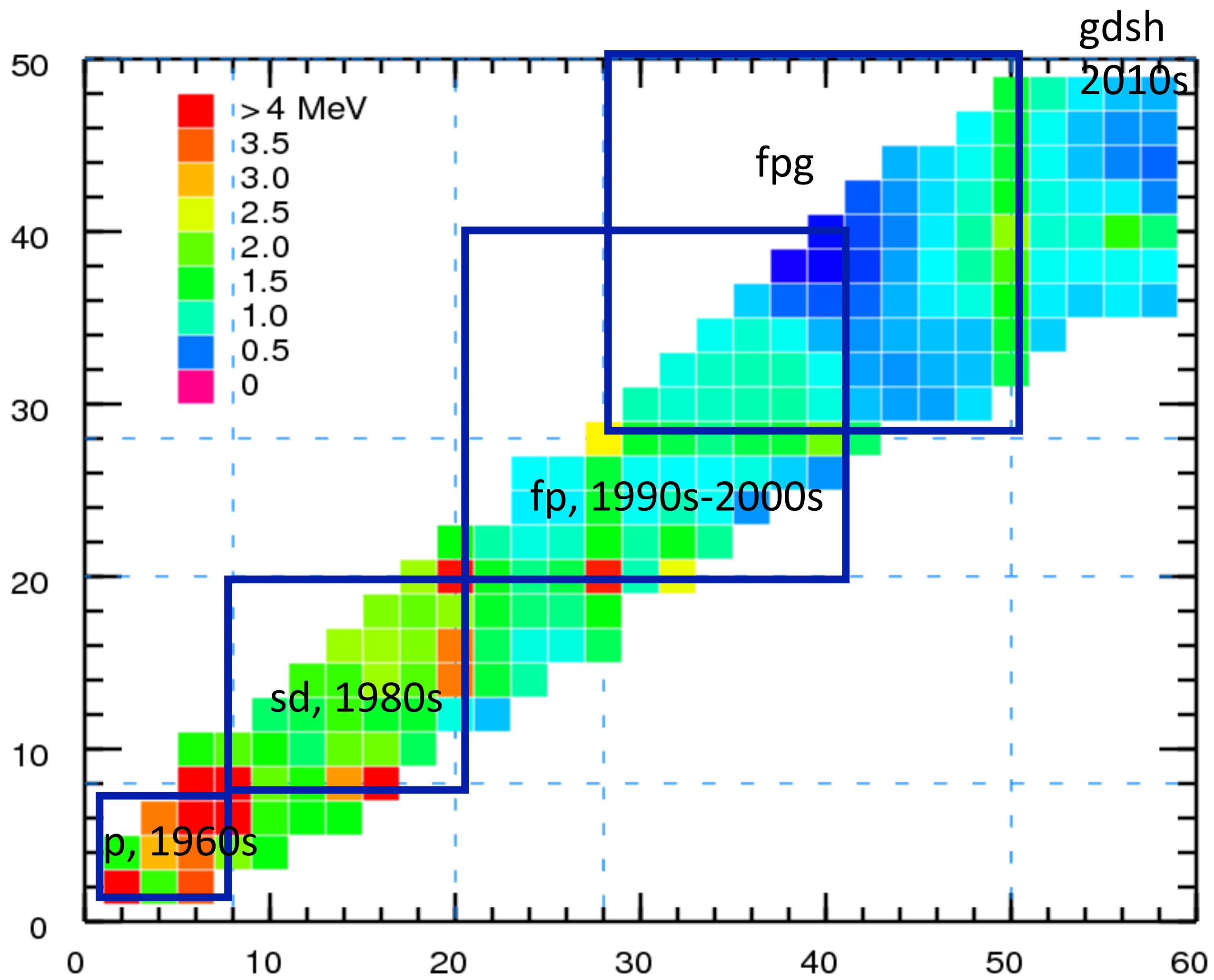
Royal institute of Technology (KTH), Stockholm

Backgroud and motivation
Seniority and systems with identical particles in a single-*j* orbital
Neutron-proton correlation from the binding energy
Neutron-proton spin-aligned pair coupling
>=2 slides can be mistaken.

S.Y. Wang et al. / Physics Letters B 703 (2011) 40–45







gdsh
2010s
fpg
fp, 1990s-2000s
sd, 1980s
p, 1960s
>4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
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0
50
40
30
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10
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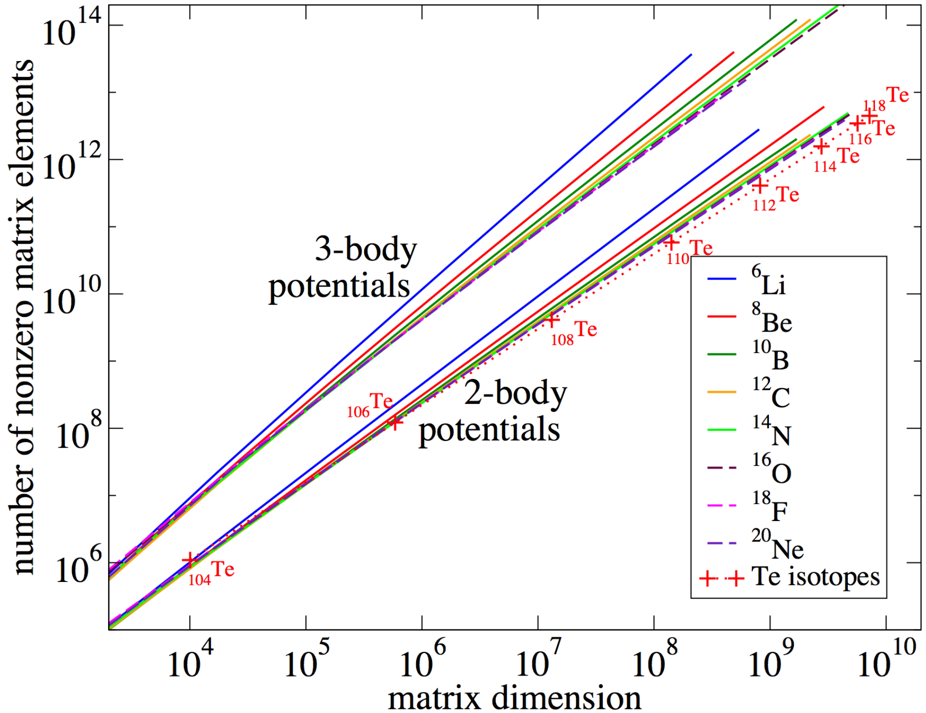
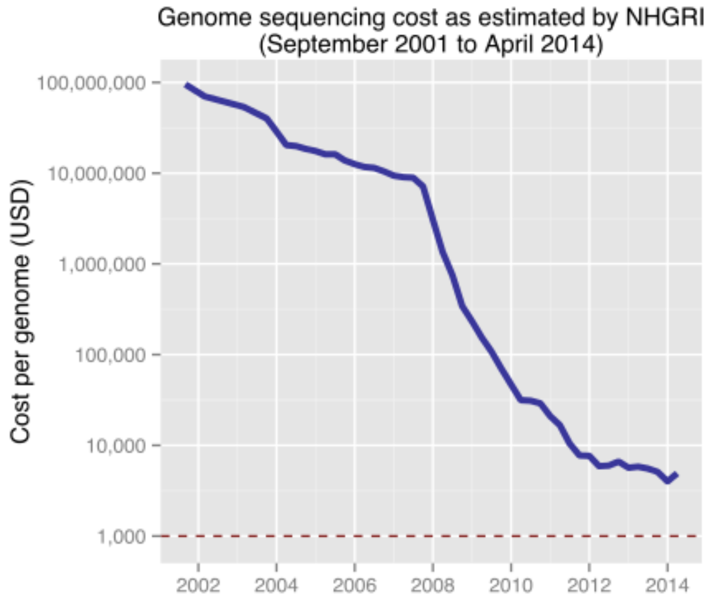
60

No-core shell model vs ^{100}Sn shell model



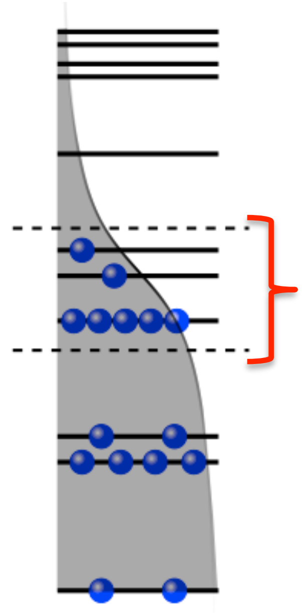


The greatest challenge is to understand the complicated full wave function: How to filter out the relevant components

PDC's supercomputer Beskow

'Physics' in a single-particle orbital?

- The nuclear 'single-particle' state has a complicated nature;
- Strong isovector pairing correlation;
- Strong np quadrupole-quadrupole correlation;



Shell model:
Limited number of orbitals
Full configuration

On single nucleon wave functions in nuclei

Igal Talmi

The Weizamnn Institute of Science, Rehovot 76100 Israel

Abstract. The strong and singular interaction between nucleons, makes the nuclear many body theory very complicated. Still, nuclei exhibit simple and regular features which are simply described by the shell model. Wave functions of individual nucleons may be considered just as model wave functions which bear little resemblance to the real ones. There is, however, experimental evidence for the reality of single nucleon wave functions. There is a simple method of constructing such wave functions for valence nucleons. It is shown that this method can be improved by considering the polarization of the core by the valence nucleon. This gives rise to some rearrangement energy which affects the single valence nucleon energy within the nucleus.

I. Talmi, AIP Conf. Proc. 1355, 121 (2011)

$$E = E_{A+1} - E_A.$$

Non-observability of Spectroscopic Factors

B.K. Jennings*

TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3

(Dated: February 21, 2011)

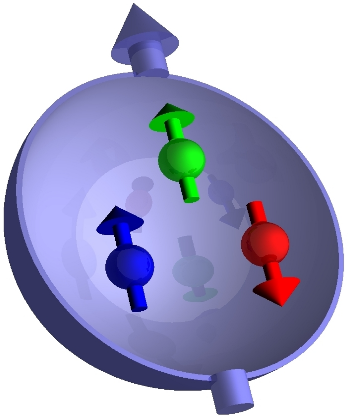
Abstract

The spectroscopic factor has long played a central role in nuclear reaction theory. However, it is not an observable. Consequently it is of minimal use as a meeting point between theory and experiment. In this paper the nature of the problem is explored. At the many-body level, unitary transformations are constructed that vary the spectroscopic factors over the full range of allowed values. At the phenomenological level, field redefinitions play a similar role and the spectroscopic factor extracted from experiment depend more on the assumed energy dependence of the potentials than on the measured cross-sections. The consistency conditions, gauge invariance and Wegmann's theorem play a large role in these considerations.

B.K. Jennings,arXiv:1102.3721

- *Nuclear physics is an emergent phenomenon (Philip Warren Anderson).*
- *Nobody of theoretical physicists could have predicted the existence of a nucleus from first principles. (DJ Rowe & JL Wood)*

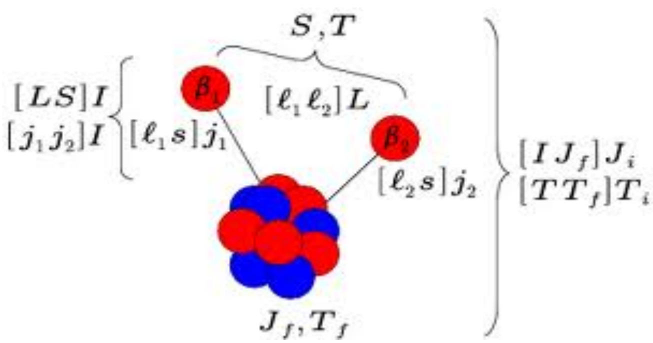
The coupling of few nucleons

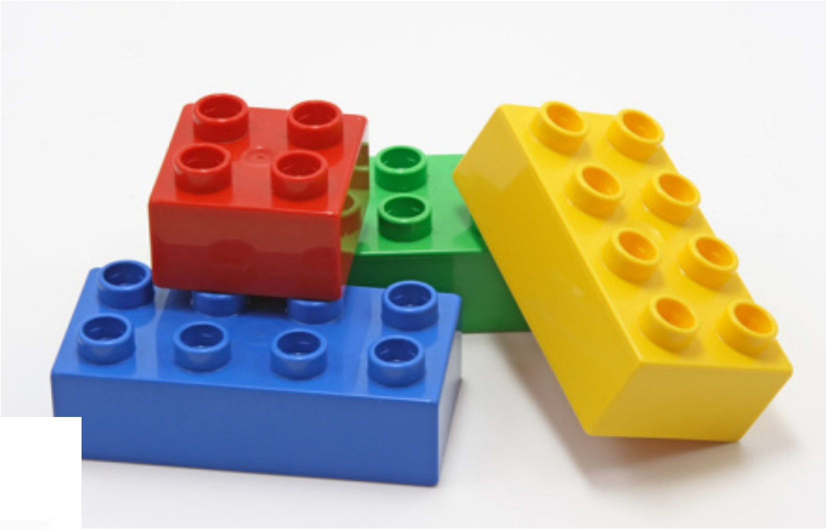


Seniority and systems with identical particles in a single-*j* orbital

Neutron-proton correlation from the binding energy

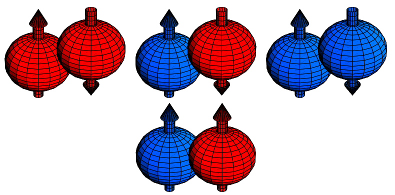
Neutron-proton spin-aligned pair coupling





General properties of the effective interaction

- Isovector (T=1): J=0,2,..,2J-1, J=0 term attractive *(pairing),* others close to zero
- Isoscalar (T=0): J=1,3,..,2j, strongly attractive (mean field)
  - *The J=1 and 2j terms are the most attractive ones.*
  - *L=0,J=1 pairing*
  - *The aligned pair was not much studied*



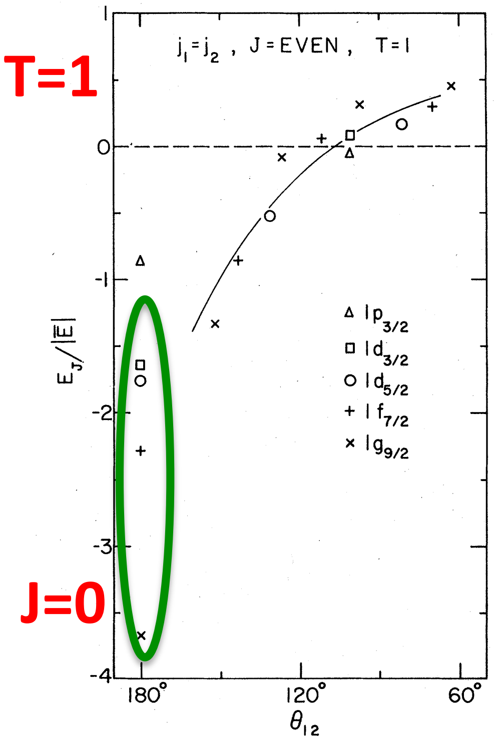


FIG. 3. Comparison of data from various multiplets with $j_1 = j_2$ and $T = 1$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

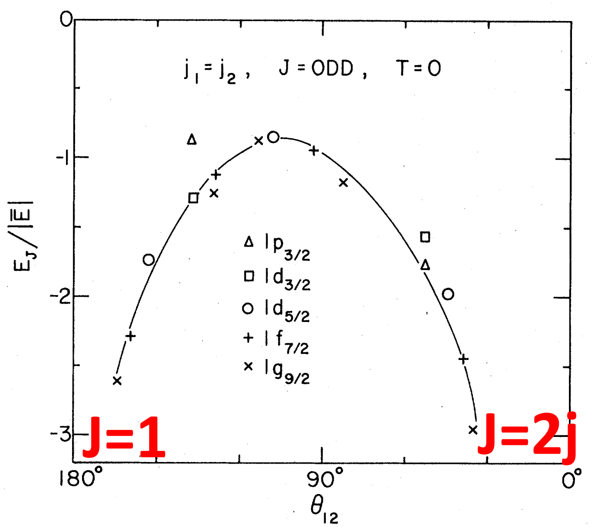


FIG. 2. Comparison of data from various multiplets with $j_1 = j_2$ and $T = 0$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

$$\cos\theta_{12} = \frac{J(J+1)}{2j(j+1)} - 1$$

J.P. Schiffer and W.W. True, Rev.Mod.Phys. 48,191 (1976)

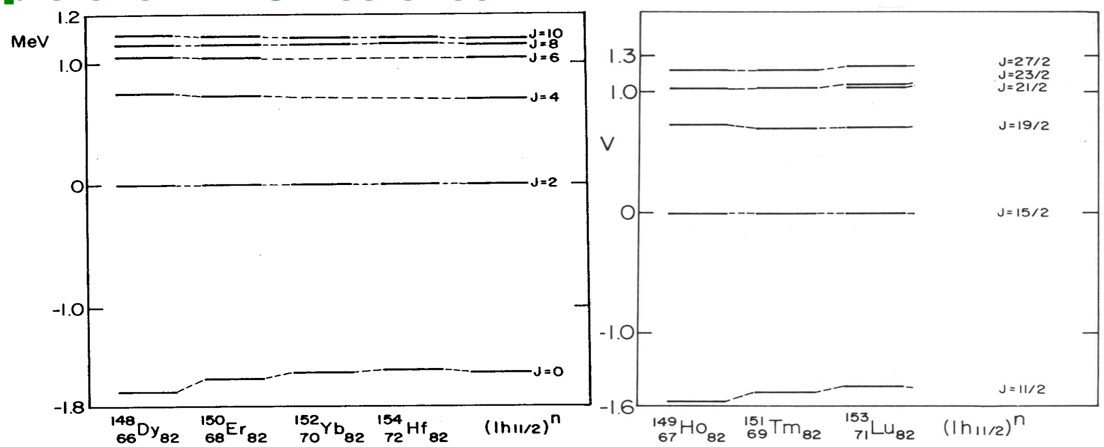
Seniority coupling scheme with realistic interaction

1943 Racah
1949 Goeppert-Mayer
LS coupling should be replaced by the jj coupling
1957 BCS theory

$$|\mathrm{g.s.}\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^{n/2}|\Phi_0\rangle$$

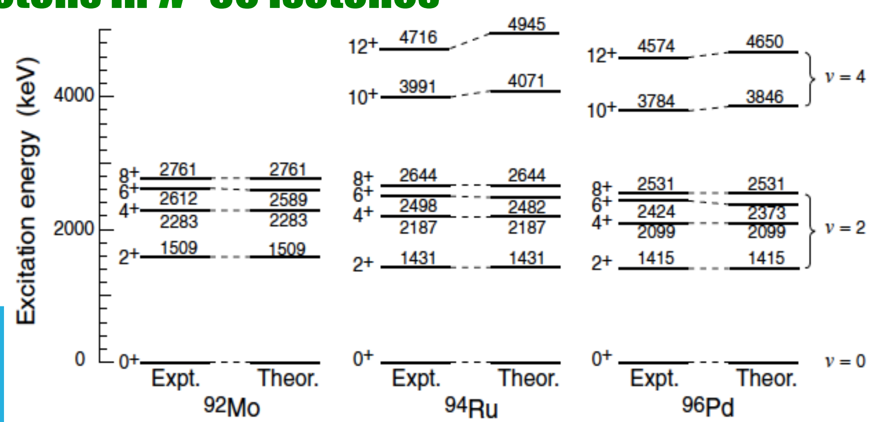
$$|\nu = 2; JM\rangle = (P_j^+)^{(n-2)/2}A^+(j^2JM)|\Phi_0$$

Energy levels of $0h_{11/2}$ protons in N=82 isotones



I. Talmi, Simple models of complex nuclei

Energy levels of $0g_{9/2}$ protons in *N*=50 isotones



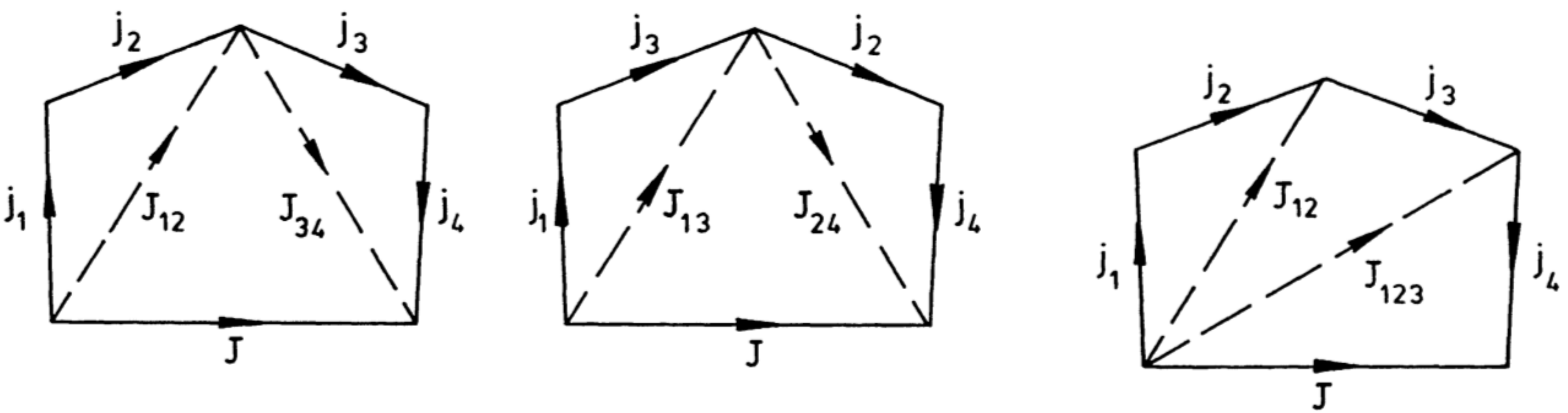
*D.J. Rowe and G. Rosensteel, Phys. Rev. Lett. **87** (2001) 172502*

9j symbol

One can construct a nonorthogonal, unnormalized, and overcomplete two-pair basis

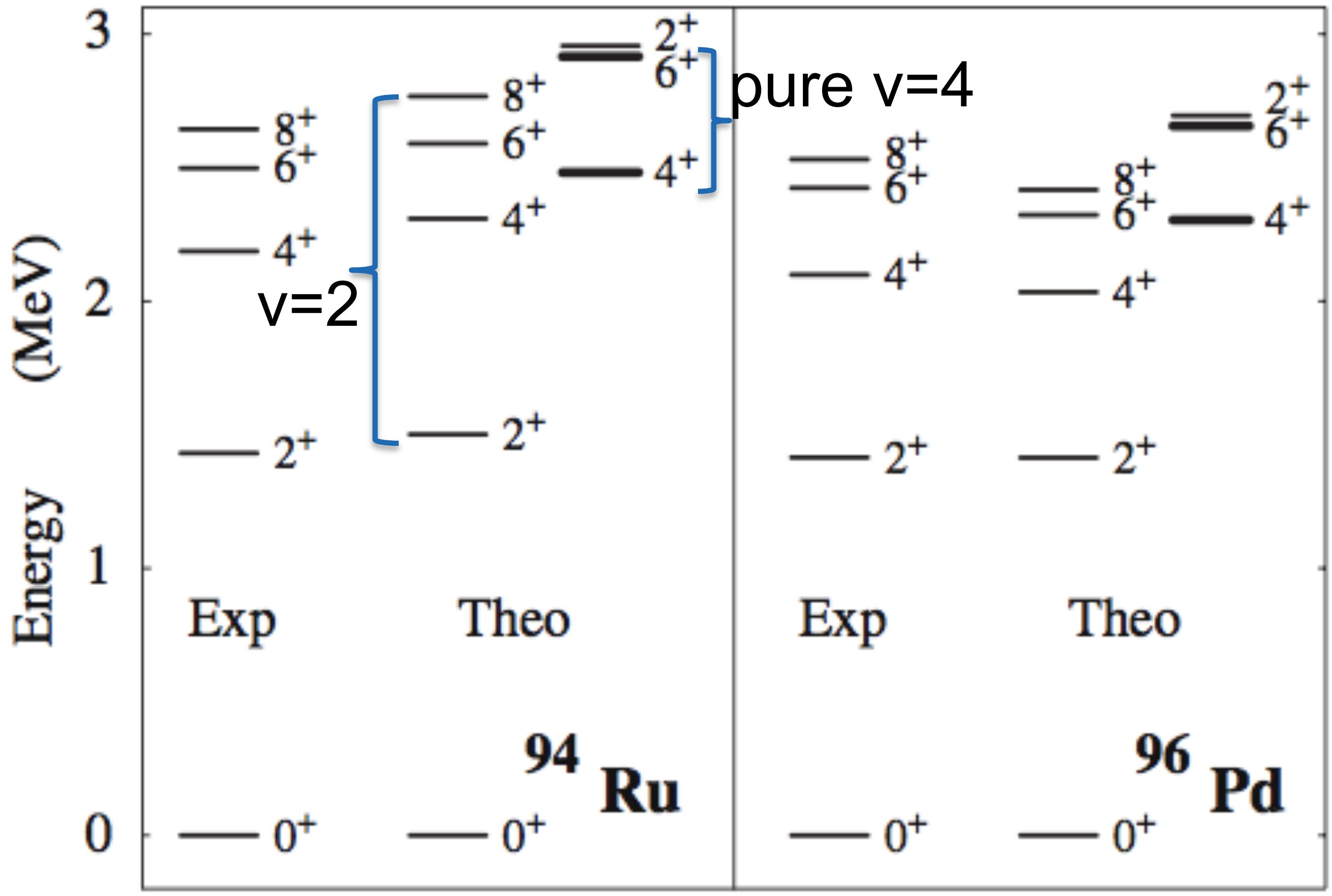
$$|(j_1 j_3) J_{13} (j_2 j_4) J_{24}; JM\rangle = \sum_{J_{12}, J_{34}} \hat{J}_{13} \hat{J}_{24} \hat{J}_{12} \hat{J}_{34}$$

$$\times \begin{Bmatrix} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & J_{24} & J \end{Bmatrix} |(j_1 j_2) J_{12} (j_3 j_4) J_{34}; JM\rangle \,.$$



- For j<9/2, seniority is (automatically) conserved!
- The rotationally invariant interaction has to satisfy **[(2*j* – 3)/6]** linear constraints to conserve seniority.

Partial dynamic conservation of seniority symmetry



Escuderos and L. Zamick, Phys. Rev. C 73, 044302(2006).
L. Zamick, Phys. Rev. C 75, 064305 (2007).
P. Van Isacker and S. Heinze, Phys. Rev. Lett. 100, 052501 (2008).
C. Qi et al, Phys. Rev. C 83 (2011) 014307; Nucl. Phys. A 884–885, 21 (2012).

$$|J_1 J_2 J\rangle = -\sum_{J_1' J_2'} \hat{J}_1 \hat{J}_2 \hat{J}_1' \hat{J}_2' X(jjJ_1; jjJ_2; J_1' J_2' J)|J_1' J_2' J\rangle ,$$

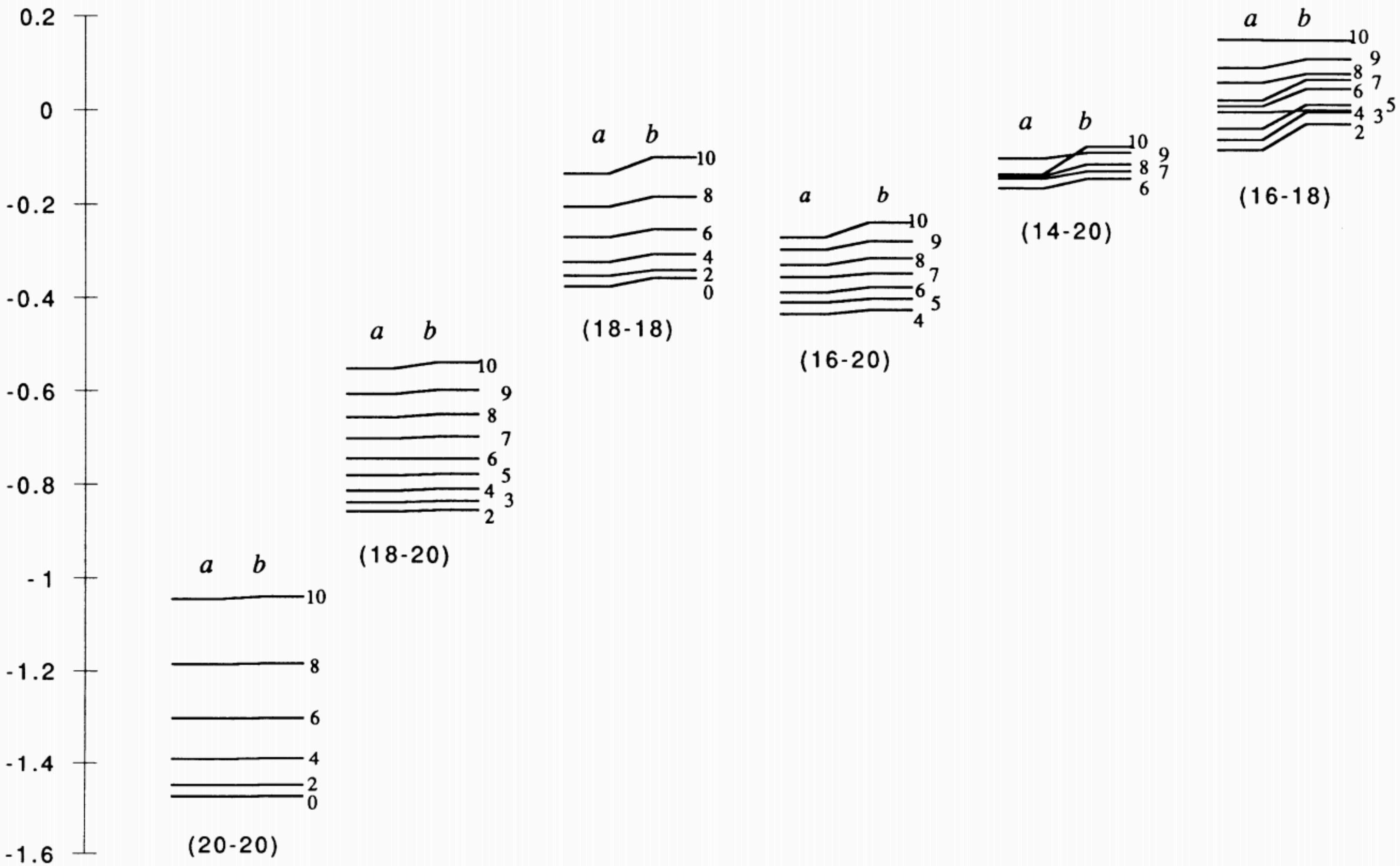


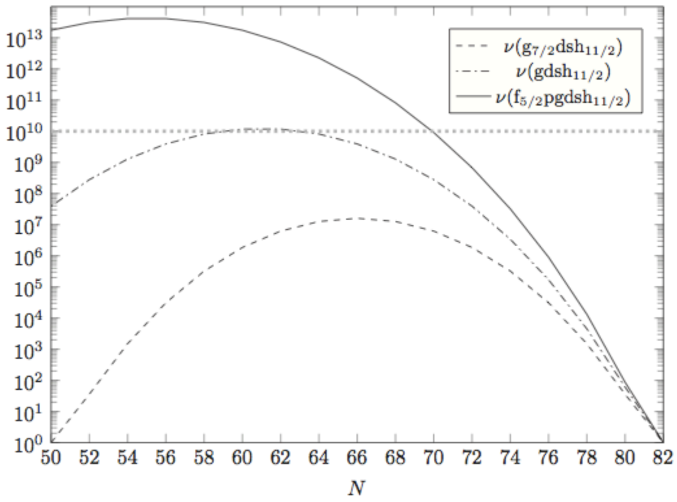
FIG. 1. The spectrum of four particles in a single-j shell ($j=\frac{21}{2}$, $H=-Q\cdot Q$, energies are in arbitrary units). Part a, the shell-model calculation; b, the GPFM calculation.

Hsi-Tseng Chen, Da Hsuan Feng, and Cheng-Li Wu Phys. Rev. Lett. 69, 418 (1992)

Seniority coupling for many shells

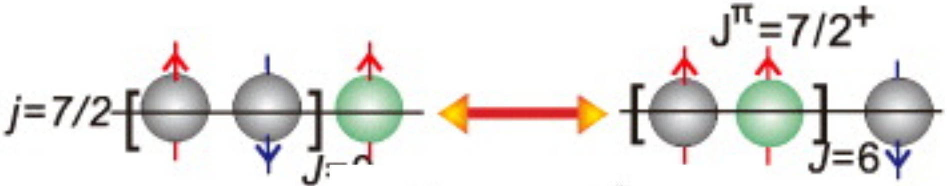
- Shell model calculations with a **pairing Hamiltonian**.
- The physical vector only spans the v=0 subspace
- **There are as many independent solutions as states in the v=0 space.**

$$
\begin{aligned}
&\big\langle \{s_j\}, \ldots N_j + 2, \ldots N_{j'} - 2, \ldots \big| \\
&\quad \times H \big| \{s_j\}, \ldots N_j, \ldots N_{j'}, \ldots \big\rangle \\
&= \frac{G_{j\,j'}}{4} \big[(N_{j'} - s_{j'})(2\Omega_{j'} - s_{j'} - N_{j'} + 2) \\
&\qquad \times (2\Omega_j - s_j - N_j)(N_j - s_j + 2) \big]^{1/2}.
\end{aligned}
$$



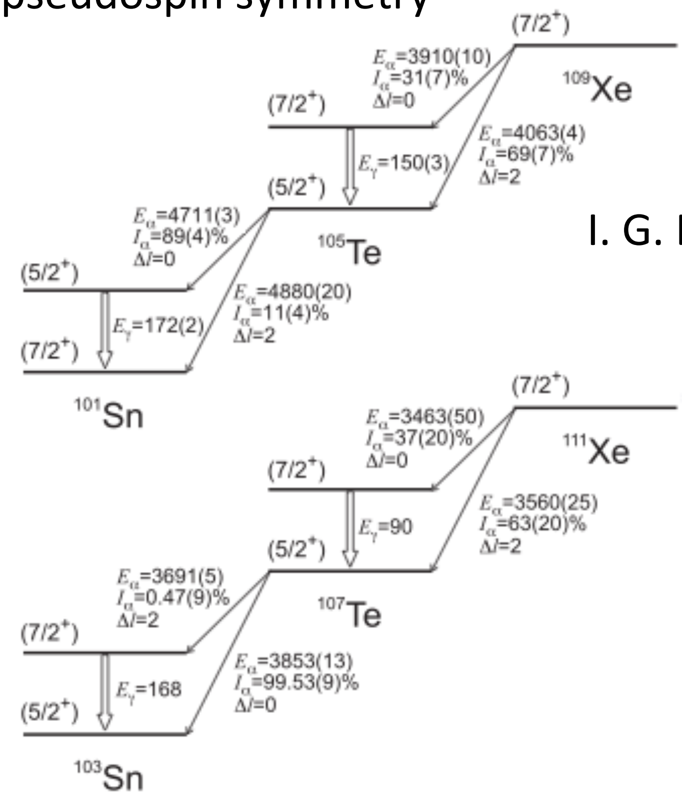
10^2 in seniority v=0 space

Different facets of the nucleus

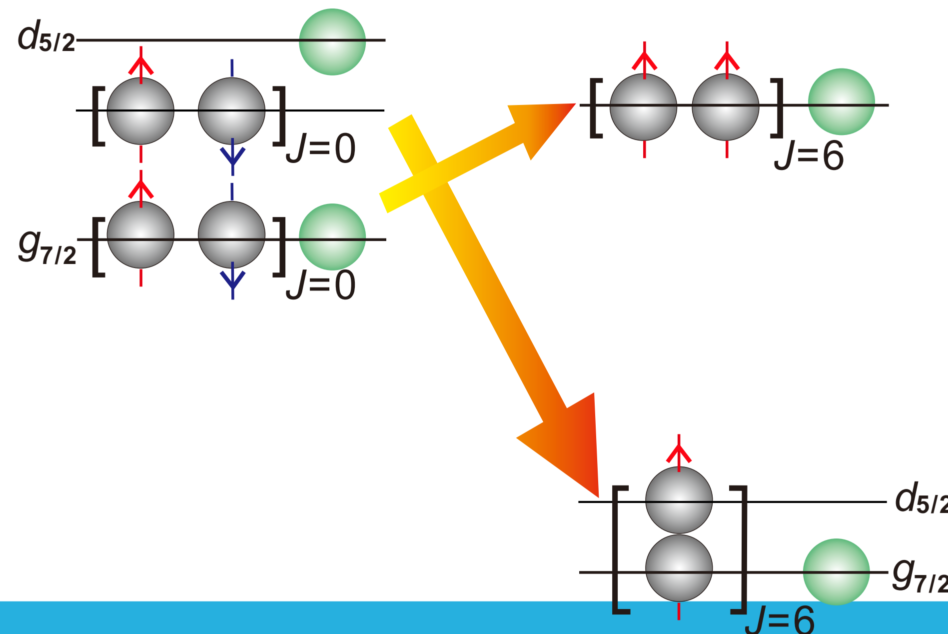


Spin inversion in ^{103}Sn with respect to that of ^{101}Sn

pseudospin symmetry



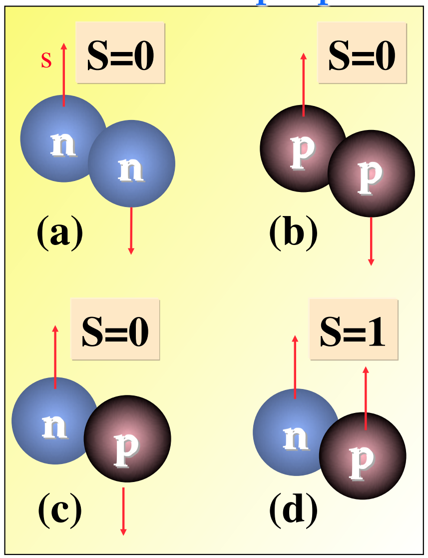
I. G. Darby et al, Phys. Rev. Lett. 105, 162502 (2010)



CQ, Z. Xu, Phys. Rev. C 86, 044323 (2012)

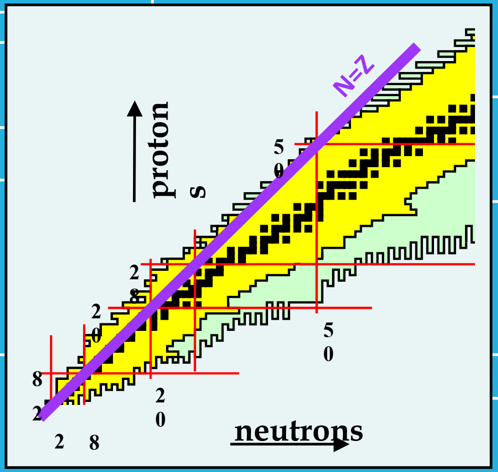
Neutron-proton coupling

nucleonic Cooper pairs



Neutron-proton correlation from the binding energy

Neutron-proton spin-aligned pair coupling



The spin trap isomers:

The 12^+ spin trap in ^{52}Fe

$$E_{12}(^{52}\mathrm{Fe}) = \frac{6}{13}\bar{V}_5 + 3\bar{V}_6 + \frac{33}{13}\bar{V}_7,$$

$$E_{10_1^+}(^{52}\mathrm{Fe}) = 0.310\bar{V}_3 + 1.429\bar{V}_4 + 0.497\bar{V}_5 + 1.571\bar{V}_6 + 2.193\bar{V}_7,$$

CQ, Phys. Rev. C 81, 034318 (2010).

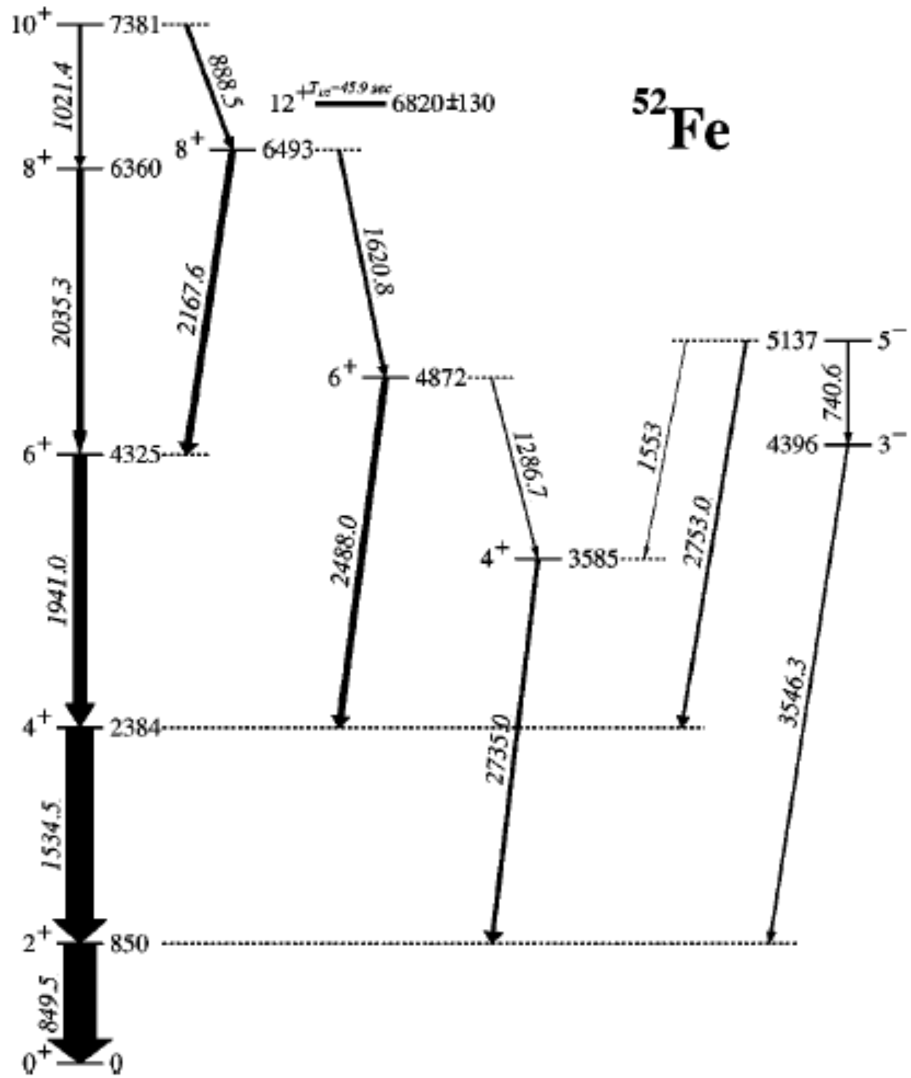
The predicted 16^+ spin trap in ^{96}Cd

$$E_{16}(^{96}\mathrm{Cd}) = \frac{8}{17}\bar{V}_7 + 3\bar{V}_8 + \frac{43}{17}\bar{V}_9.$$

$$E_{14_1^+}(^{96}\mathrm{Cd}) = 0.307\bar{V}_5 + 1.428\bar{V}_6 + 0.493\bar{V}_7 + 1.572\bar{V}_8 + 2.200\bar{V}_9.$$

K. Ogawa, Phys. Rev. C 28, 958 (1983).

CQ, Phys. Rev. C 81, 034318 (2010).



C. A. Ur *et al.*, Phys. Rev. C 58, 3163 (1998).

The relative positions of these spin traps are sensitive to the strength of the interaction $V_{J=2j}$

The energy expression

$$E_I = C_J^I V_J,$$

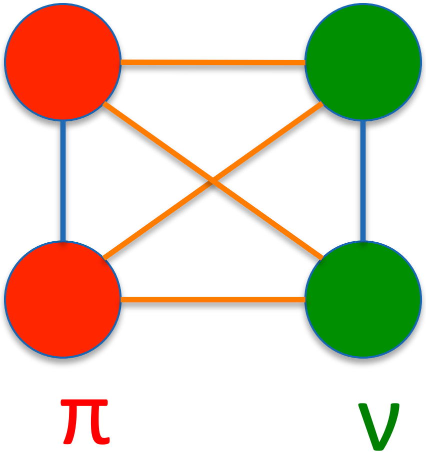
$$C_J^I = \frac{1}{2}\langle \, \left\| \left[\left(a^+ a^+\right)^J \left(a \; a\right)^J \right]^0 \right| \, \rangle$$

The total number of pairs with all spins J is given by

$$\sum_J C_J^I = n(n-1)/2,$$

and

$$\sum_{J,\text{odd}} C_J^I = \frac{1}{2}\left[\frac{n}{2}\left(\frac{n}{2}+1\right) - T(T+1)\right],$$



✧For systems with n=4 and isospin T=0, there are three isoscalar pairs and three isovecto pairs;

✧For those with T=2 (four identical nucleons), we have six isovector pairs.

E. Moya de Guerra, A. A. Raduta, L. Zamick, and P. Sarriguren, Nucl. Phys. A ***727, 3 (2003).***
CQ, Phys. Rev. C ***81, 034318 (2010).***
I. Talmi, Nucl. Phys. A 846 (2010) 31.

PRL **107,** 172502 (2011) PHYSICAL REVIEW LETTERS 21 OCTOBER 2011

16^+ Spin-Gap Isomer in ^{96}Cd

B. S. Nara Singh,[1] Z. Liu,[2] R. Wadsworth,[1] H. Grawe,[3] T. S. Brock,[1] P. Boutachkov,[3] N. Braun,[4] A. Blazhev,[4] M. Górska,[3] S. Pietri,[3] D. Rudolph,[5] C. Domingo-Pardo,[3] S. J. Steer,[6] A. Ataç,[7] L. Bettermann,[4] L. Cáceres,[3] K. Eppinger,[8] T. Engert,[3] T. Faestermann,[8] F. Farinon,[3] F. Finke,[4] K. Geibel,[4] J. Gerl,[3] R. Gernhäuser,[8] N. Goel,[3] A. Gottardo,[2] J. Grębosz,[9] C. Hinke,[8] R. Hoischen,[3,5] G. Ilie,[4] H. Iwasaki,[4] J. Jolie,[4] A. Kaşkaş,[7] I. Kojouharov,[3] R. Krücken,[8] N. Kurz,[3] E. Merchán,[3] C. Nociforo,[3] J. Nyberg,[10] M. Pfützner,[11] A. Prochazka,[3] Zs. Podolyák,[6] P. H. Regan,[6] P. Reiter,[4] S. Rinta-Antila,[12] C. Scholl,[4] H. Schaffner,[3] P.-A. Söderström,[10] N. Warr,[4] H. Weick,[3] H.-J. Wollersheim,[3] P. J. Woods,[2] F. Nowacki,[13] and K. Sieja[13]

'Superallowed' alpha decay around N=Z nuclei

PRL 97, 082501 (2006) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2006

Discovery of ^{109}Xe and ^{105}Te: Superallowed α Decay near Doubly Magic ^{100}Sn

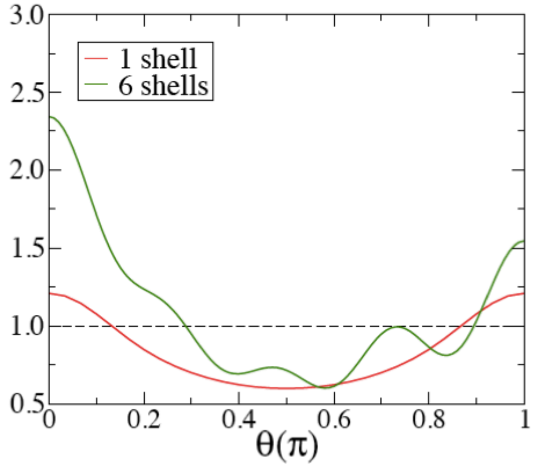
S. N. Liddick,[1] R. Grzywacz,[2,3] C. Mazzocchi,[2] R. D. Page,[4] K. P. Rykaczewski,[3] J. C. Batchelder,[1] C. R. Bingham,[2,3] I. G. Darby,[4] G. Drafta,[2] C. Goodin,[5] C. J. Gross,[3] J. H. Hamilton,[5] A. A. Hecht,[6] J. K. Hwang,[5] S. Ilyushkin,[7] D. T. Joss,[4] A. Korgul,[2,5,8,9] W. Królas,[9,10] K. Lagergren,[9] K. Li,[5] M. N. Tantawy,[2] J. Thomson,[4] and J. A. Winger[1,7,9]

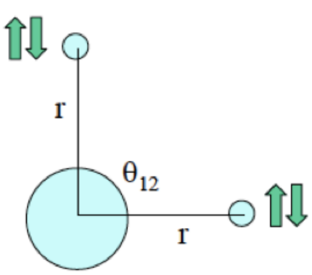
Shell model calculations on the alpha formation amplitude in N=Z nuclei.

The four-body (alpha) wave function can be written as

$$|\gamma_4\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\gamma_4)|\alpha_2 \otimes \beta_2\rangle,$$

where α_2 and β_2 denote proton and neutron wave functions, respectively.





Relative angular distribution of four-particle wave function with (solid lines) and without (dashed line) neutron-proton interactions.

Seniority with isospin

$$E = \varepsilon n + \frac{a}{2}n(n-1) + \frac{b}{2}\left[\mathcal{T}(\mathcal{T}+1) - \frac{3n}{4}\right]$$
$$+G\left[\frac{n-v}{4}(4j+8-n-v) - \mathcal{T}(\mathcal{T}+1) + s(s+1)\right],$$

Odd-even staggering

$$\begin{aligned} E &= \varepsilon n + \frac{2a-G}{4}n(n-1) \\ &\quad + \frac{b-2G}{2}\left[\mathcal{T}(\mathcal{T}+1) - \frac{3n}{4}\right] \\ &\quad + (j+1)G(n-v) + G\left[\frac{v^2}{4} - v + s(s+1)\right], \end{aligned}$$

Pairing energy in mass formula $E_p \propto 2 - v,$

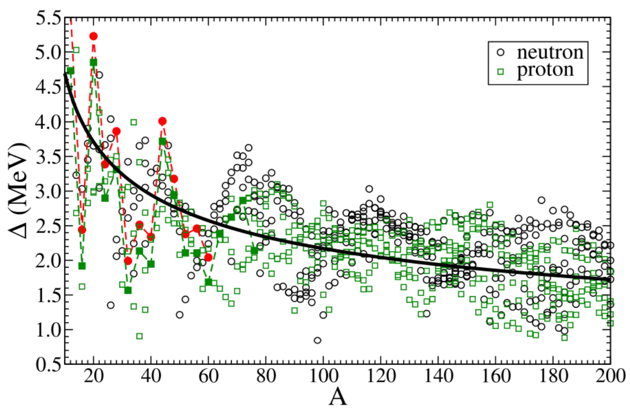
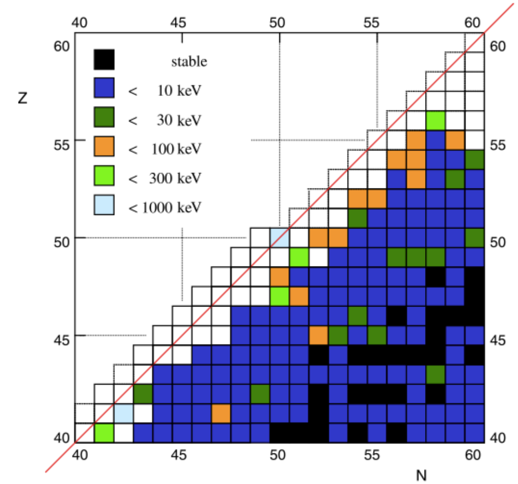


Fig. 2. (Color online.) Empirical proton–proton (squares) and neutron–neutron (circles) interactions in even–even nuclei extracted from experimental nuclear masses as a function of the mass number A [23]. The solid symbols denote those in the $N = Z$ nuclei.



Precision of known masses in the 100Sn region.

PHYSICAL REVIEW C, VOLUME 61, 041303(R)

Is there *np* pairing in $N=Z$ nuclei?

A. O. Macchiavelli, P. Fallon, R. M. Clark, M. Cromaz, M. A. Deleplanque, R. M. Diamond, G. J. Lane, I. Y. Lee, F. S. Stephens, C. E. Svensson, K. Vetter, and D. Ward

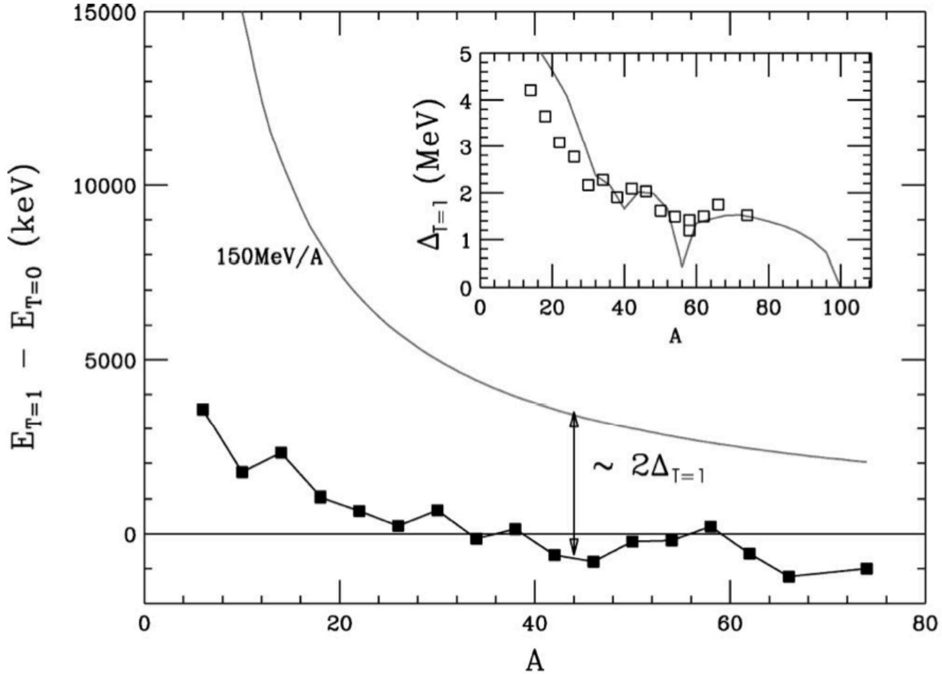


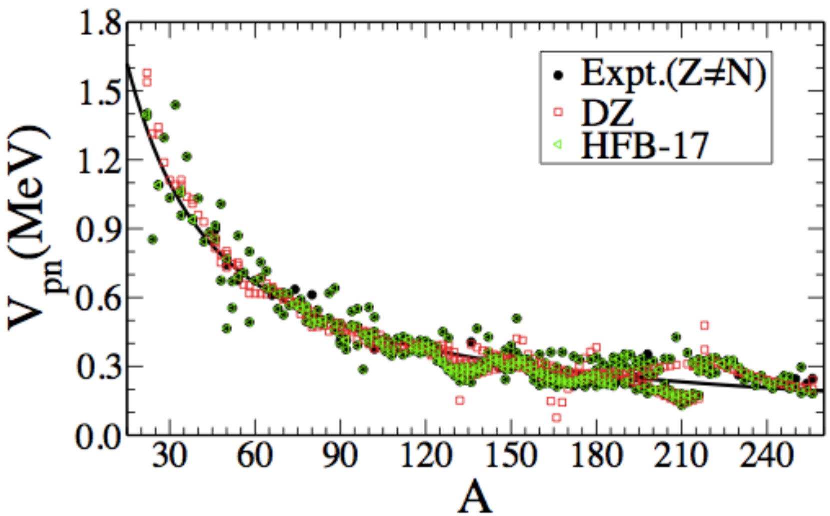
FIG. 4. The difference in level energies between $T=1$ and $T=0$ states in odd-odd $N=Z$ nuclei. For $A<40$ these nuclei have $T=0$ ground states, except ^{34}Cl, above this mass they have $T=1$ ground states, except ^{58}Cu. The solid line represents the isospin correction term $\Delta E_{sym}=150/A$ MeV and corresponds to the

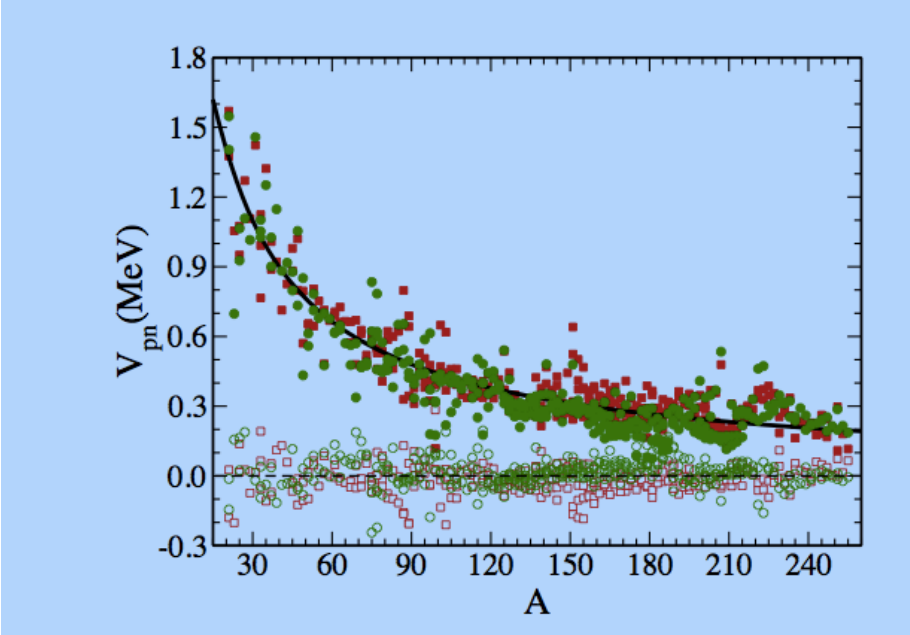
The binding energies of even-even and odd-odd N=Z nuclei are compared. After correcting for the symmetry energy we find that the lowest T=1 state in odd-odd N=Z nuclei is as bound as the ground state in the neighboring even-even nucleus, **thus providing evidence for isovector np pairing.** However, T=0 states in odd-odd N=Z nuclei are several MeV less bound than the even-even ground states. We associate this difference with the T=1 pair gap and conclude from the analysis of binding energy differences and blocking arguments that **there is no evidence for an isoscalar (deuteronlike) pair condensate in N=Z nuclei.**

The average proton-neutron interaction

$$V_{pn}(Z,N) = \frac{1}{4}[B(Z,N) + B(Z-2,N-2) - B(Z-2,N) - B(Z,N-2)],$$

J.-Y. Zhang, R.F. Casten, D.S. Brenner, Phys. Lett. B 227 (1989) 1.





$$V_{pn}(Z,N) = \frac{1}{2}[S_n(Z,N-1) - S_n(Z-2,N-1)] + \frac{1}{4}[\Delta_n(Z,N) - \Delta_n(Z-2,N)].$$

CQ, Physics Letters B 717, 436 (2012)

Additional binding for N=Z nuclei

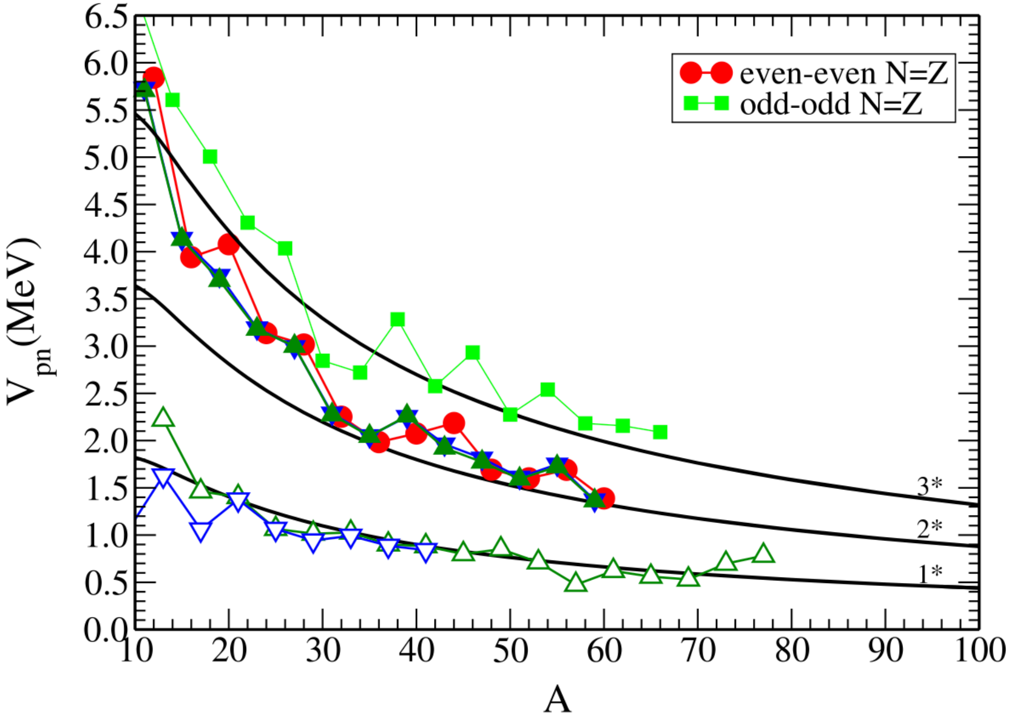


Fig. 4. (Color online.) Experimental V_{pn} values of even–even $N = Z$ nuclei (filled circles) and the adjacent odd–odd (squares) and odd-A nuclei (triangles). The filled and open triangles correspond to systems with one nucleon subtracted from and added to the even–even nuclei, respectively. The solid line labeled 1* describes the average behavior of V_{pn} in even–even $N \neq Z$ nuclei from Fig. 1. 2* and 3* denotes its twice and three time values.

$$\hat{V} = a + b\mathbf{t}_1 \cdot \mathbf{t}_2 + GP_0,$$

For even–even nuclei with $n_\pi \neq n_\nu$,

$$V_{pn} = -\frac{4V_{m;T=1} + 2(V_{m;T=0} - V_{m;T=1})}{4} = \frac{b}{4} - a.$$

in the case of $n_\pi = n_\nu$ (i.e., $N = Z$),

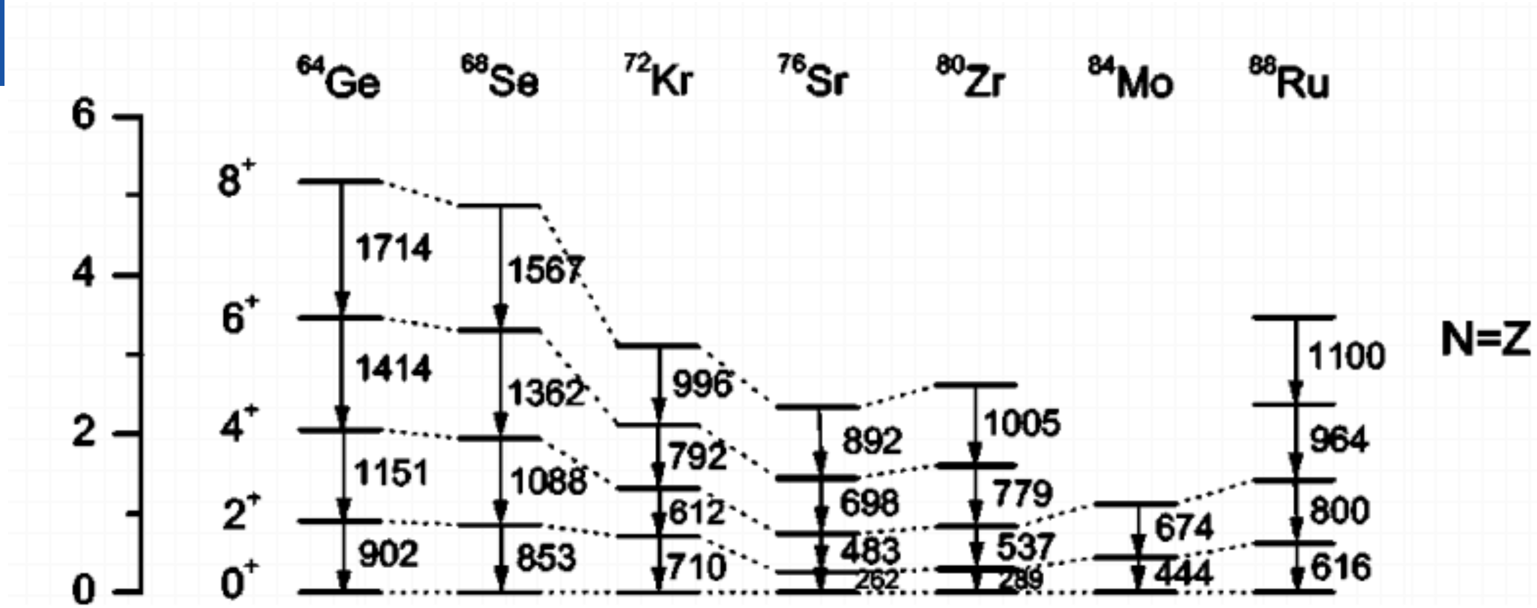
$$V_{pn} = -\frac{4V_{m;T=1} + 3(V_{m;T=0} - V_{m;T=1})}{4} - \frac{G}{2} = \frac{b}{2} - a - \frac{G}{2}.$$

odd–odd $N = Z$

$$\begin{aligned} V_{pn}(Z-1, Z-1) &= B(Z-1, Z-1) + B(Z-2, Z-2) \\ &\quad - B(Z-1, Z-2) - B(Z-2, Z-1) \\ &= \frac{3b}{4} - a. \end{aligned}$$

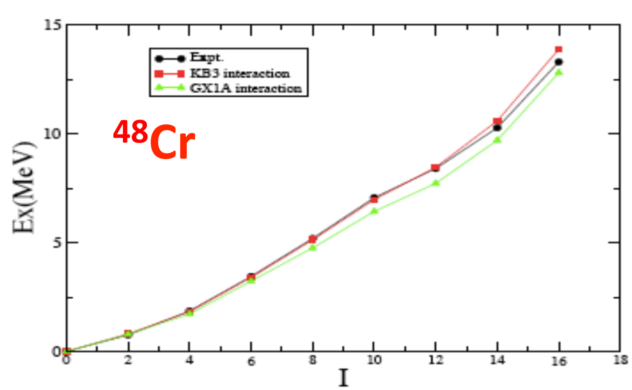
CQ, Physics Letters B 717, 436 (2012)

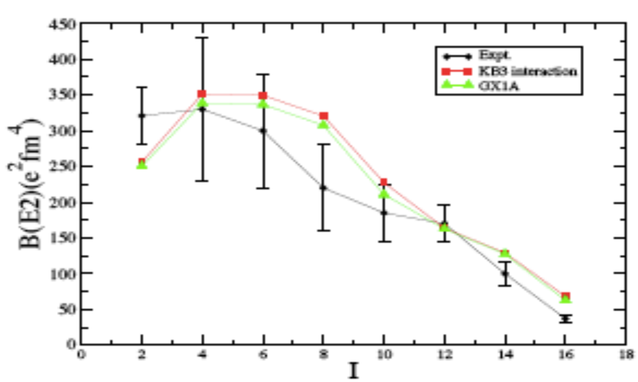
But many N=Z nuclei are deformed



N. Mărginean et al., PRC 63, 031303(R) (2001)

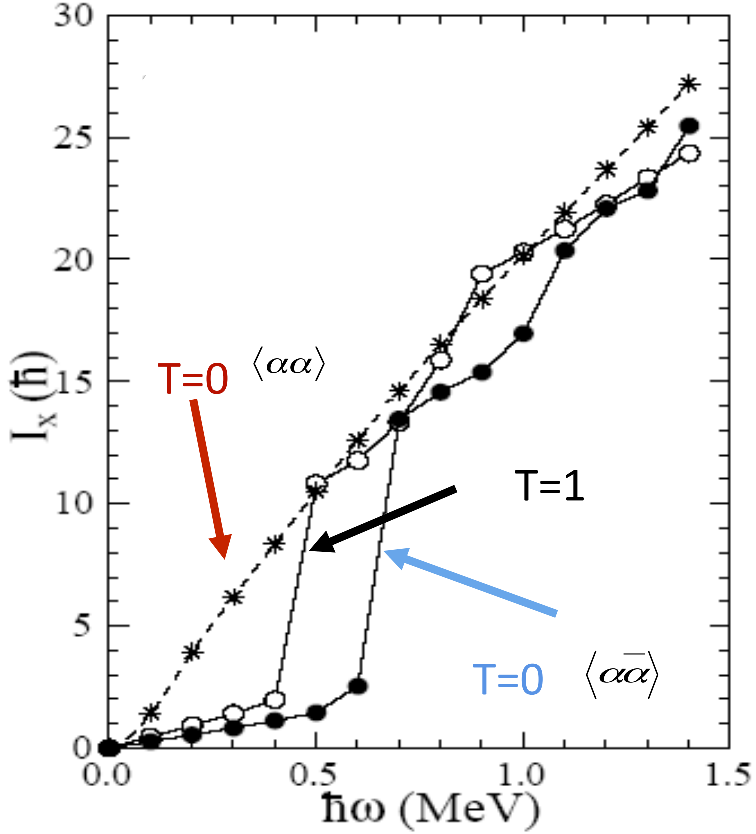
- QQ correlation induces deformation;
- The np interaction also breaks the seniority in a major way





The T=0 $\langle\alpha\alpha\rangle$ pairing exhibits a rotational like behavior as a function of frequency.

The Ix operator may not be the right one for this collective mode



A.L. Goodman, PRC 63, 044325 (2001)

W. Satula and R. Wyss, Phys. Lett. B **393**, 1 (1997).

Nuclei around ^{100}Sn: N=Z=50 shell closures survive

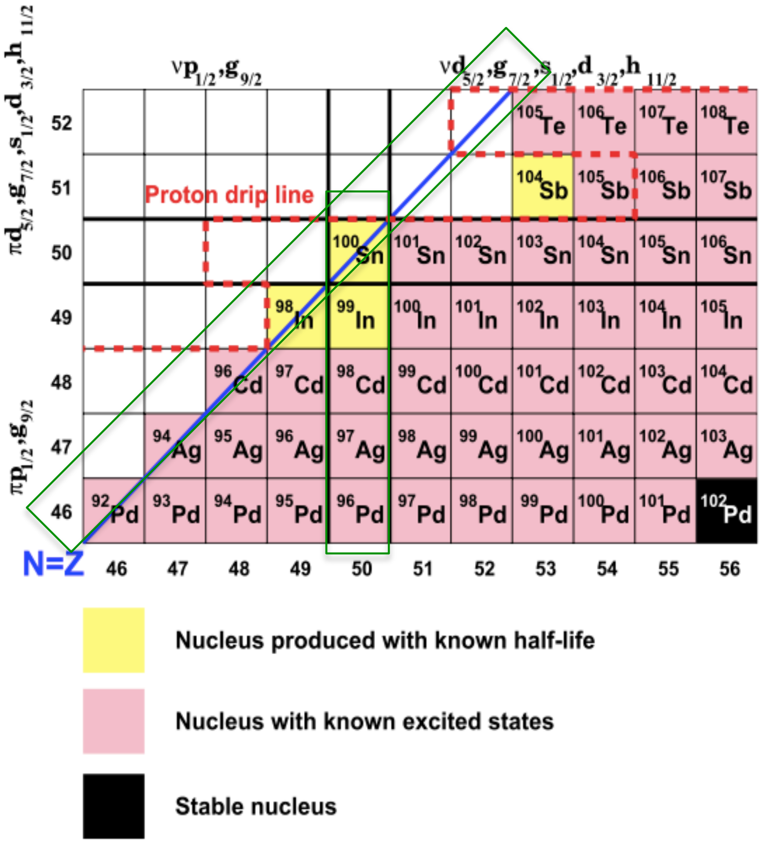


Fig. 1.1. Chart of the ^{100}Sn region showing the status of experimental observation.

T. Faestermann et al. / Prog. Part. Nucl. Phys. 69 (2013) 85–130

Superallowed Gamow–Teller decay of the doubly magic nucleus ^{100}Sn

C. B. Hinke[1], M. Böhmer[1], P. Boutachkov[2], T. Faestermann[1], H. Geissel[2], J. Gerl[2], R. Gernhäuser[1], M. Górska[2], A. Gottardo[3], H. Grawe[2], J. L. Grębosz[4], R. Krücken[1,5], N. Kurz[2], Z. Liu[6], L. Maier[1], F. Nowacki[7], S. Pietri[2], Zs. Podolyák[8], K. Sieja[7], K. Steiger[1], K. Straub[1], H. Weick[2], H.-J. Wollersheim[2], P. J. Woods[6], N. Al-Dahan[8], N. Alkhomashi[8], A. Ataç[9], A. Blazhev[10], N. F. Braun[10], I. T. Čeliković[11], T. Davinson[6], I. Dillmann[2], C. Domingo-Pardo[12], P. C. Doornenbal[13], G. de France[14], G. F. Farrelly[8], F. Farinon[2], N. Goel[2], T. C. Habermann[2], R. Hoischen[2], R. Janik[15], M. Karny[16], A. Kaşkaş[9], I. M. Kojouharov[2], Th. Kröll[17], Y. Litvinov[2], S. Myalski[4], F. Nebel[1], S. Nishimura[13], C. Nociforo[2], J. Nyberg[18], A. R. Parikh[19], A. Procházka[2], P. H. Regan[8], C. Rigollet[20], H. Schaffner[2], C. Scheidenberger[2], S. Schwertel[1], P.-A. Söderström[13], S. J. Steer[8], A. Stolz[21] & P. Strmen[15]

PRL 110, 172501 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Coulomb Excitation of ^{104}Sn and the Strength of the ^{100}Sn Shell Closure

G. Guastalla,[1] D. D. DiJulio,[2] M. Górska,[3] J. Cederkäll,[2] P. Boutachkov,[1,3] P. Golubev,[2] S. Pietri,[3] H. Grawe,[3] F. Nowacki,[4] K. Sieja,[4] A. Algora,[5,6] F. Ameil,[3] T. Arici,[7,3] A. Atac,[8] M. A. Bentley,[9] A. Blazhev,[10] D. Bloor,[9] S. Brambilla,[11] N. Braun,[10] F. Camera,[11] Zs. Dombrádi,[6] C. Domingo Pardo,[5] A. Estrade,[3] F. Farinon,[3] J. Gerl,[3] N. Goel,[3,1] J. Grębosz,[12] T. Habermann,[3,13] R. Hoischen,[2] K. Jansson,[2] J. Jolie,[10] A. Jungclaus,[14] I. Kojouharov,[3] R. Knoebel,[3] R. Kumar,[15] J. Kurcewicz,[16] N. Kurz,[3] N. Lalović,[3] E. Merchan,[1,3] K. Moschner,[10] F. Naqvi,[3,10] B. S. Nara Singh,[9] J. Nyberg,[17] C. Nociforo,[3] A. Obertelli,[18] M. Pfützner,[3,16] N. Pietralla,[1] Z. Podolyák,[19] A. Prochazka,[3] D. Ralet,[1,3] P. Reiter,[10] D. Rudolph,[2] H. Schaffner,[3] F. Schirru,[19] L. Scruton,[9] D. Sohler,[6] T. Swaleh,[2] J. Taprogge,[10,20] Zs. Vajta,[6] R. Wadsworth,[9] N. Warr,[10] H. Weick,[3] A. Wendt,[10] O. Wieland,[11] J. S. Winfield,[3] and H. J. Wollersheim[3]

PHYSICAL REVIEW C 87, 031306(R) (2013)

Transition probabilities near ^{100}Sn and the stability of the $N, Z = 50$ shell closure

T. Bäck,[1,*] C. Qi,[1] B. Cederwall,[1] R. Liotta,[1] F. Ghazi Moradi,[1] A. Johnson,[1] R. Wyss,[1] and R. Wadsworth[2]

PHYSICAL REVIEW C 84, 041306(R) (2011)

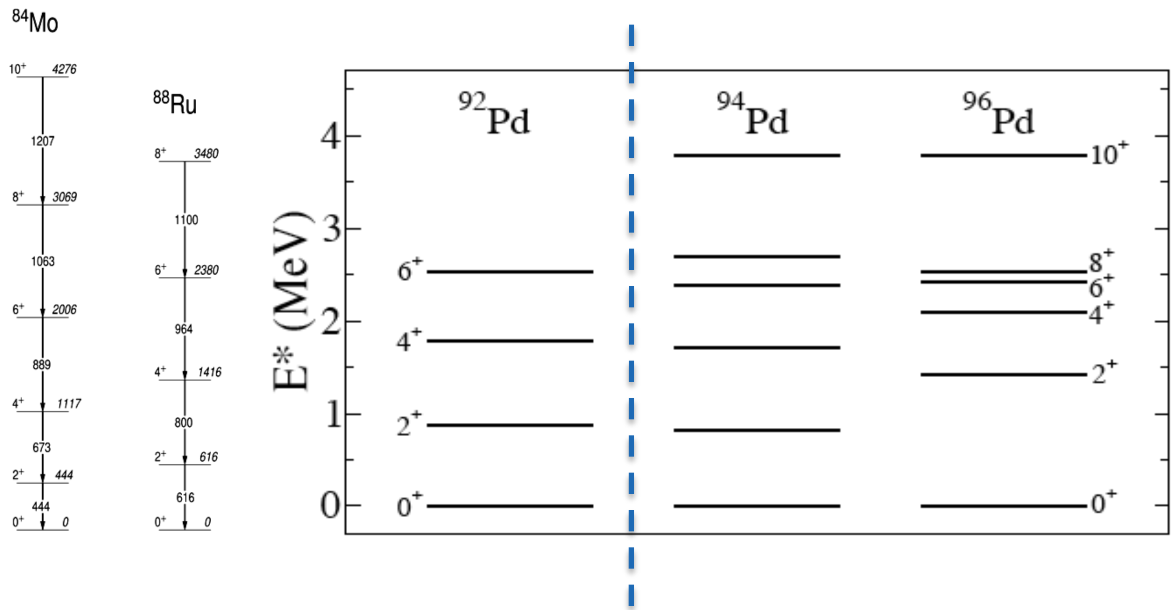
Lifetime measurement of the first excited 2^+ state in ^{108}Te

T. Bäck,[1,*] C. Qi,[1] F. Ghazi Moradi,[1] B. Cederwall,[1] A. Johnson,[1] R. Liotta,[1] R. Wyss,[1] H. Al-Azri,[2] D. Bloor,[2] T. Brock,[2] R. Wadsworth,[2] T. Grahn,[3] P. T. Greenlees,[3] K. Hauschild,[3,†] A. Herzan,[3] U. Jacobsson,[3] P. M. Jones,[3] R. Julin,[3] S. Juutinen,[3] S. Ketelhut,[3] M. Leino,[3] A. Lopez-Martens,[3,†] P. Nieminen,[3] P. Peura,[3] P. Rahkila,[3] S. Rinta-Antila,[3] P. Ruotsalainen,[3] M. Sandzelius,[3] J. Sarén,[3] C. Scholey,[3] J. Sorri,[3] J. Uusitalo,[3] S. Go,[4] E. Ideguchi,[4] D. M. Cullen,[5] M. G. Procter,[5] T. Braunroth,[6] A. Dewald,[6] C. Fransen,[6] M. Hackstein,[6] J. Litzinger,[6] and W. Rother[6]

N=Z nuclei as a probe of np coupling scheme

Experimental status

Spectra of the heaviest even-even N=Z nuclei ^{88}Ru and ^{92}Pd were reported in 2001 and 2011, respectively.
N. Mărginean et al., PRC 63, 031303(R) (2001); B. Cederwall et al., Nature 469, 68 (2011).

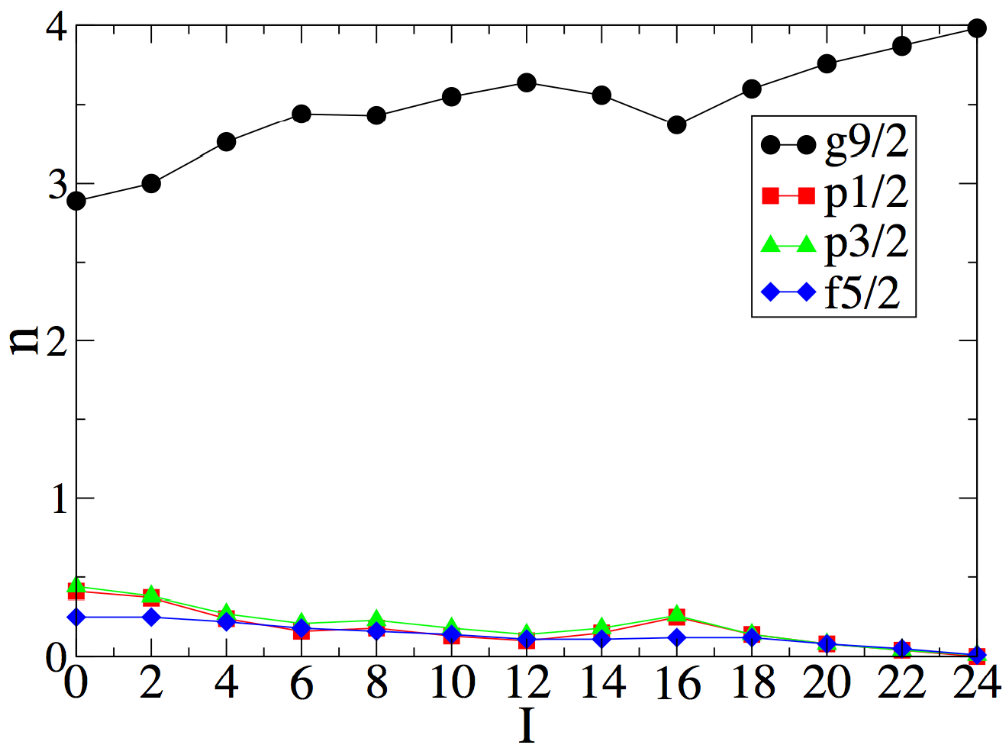


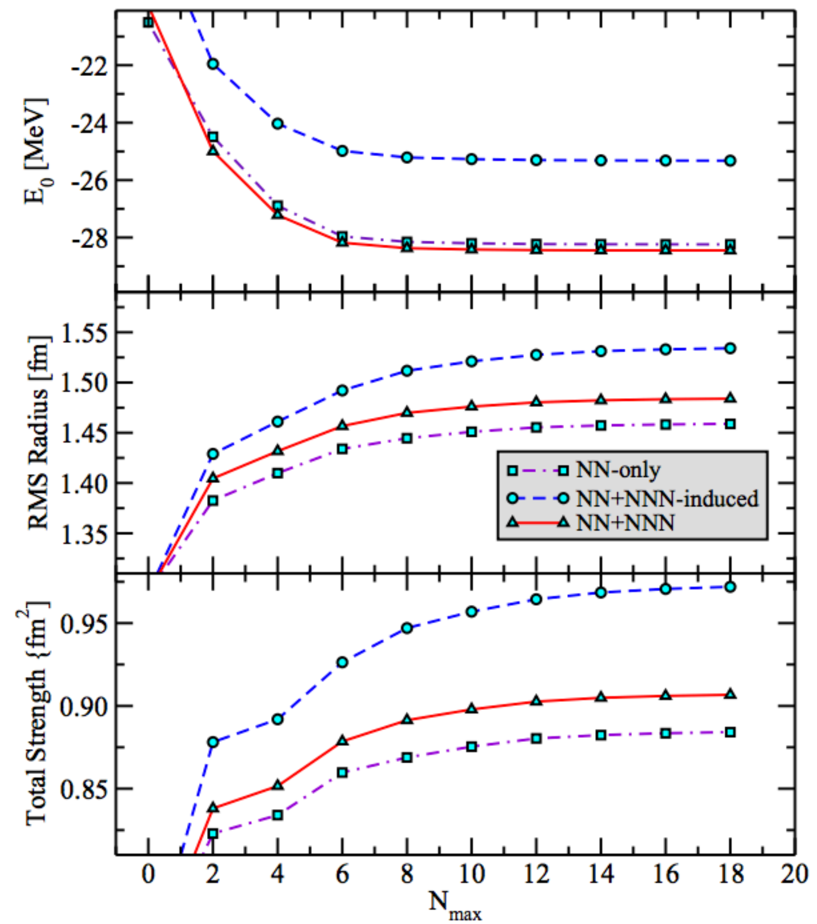
Rotational-like ⟷ Vibrational-like ⟷ Seniority

How to understand the equidistant pattern of the ^{92}Pd spectrum?

Averaged number of particles in different shells for ^{92}Pd

What is the 'minimal' model space one needs?





No-core shell model for ^4He
M.D. Schuster et al., arXiv:1304.5491

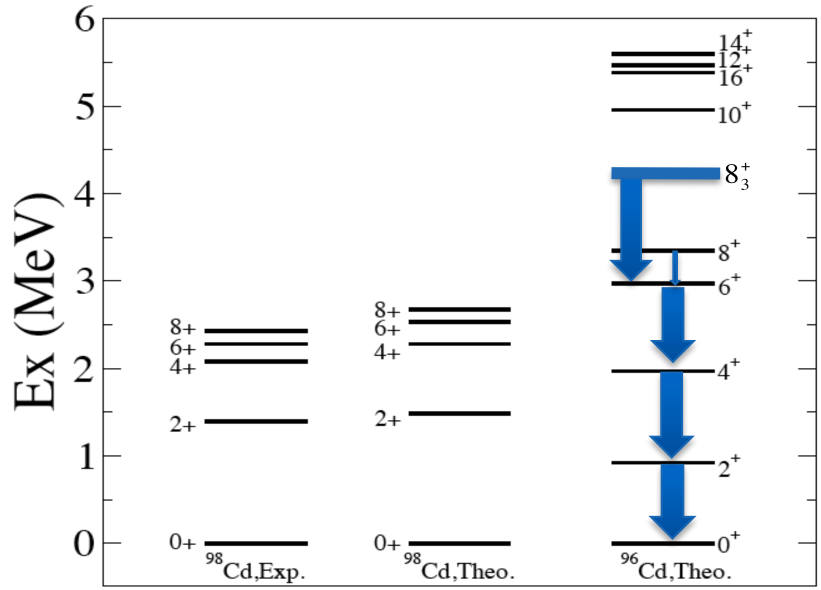
^{96}Cd (2n-2p): A simple example to show the pair content

Usually the wave function can be expanded as

$$|\Psi_I\rangle = \sum_{J_p, J_n} X_I(J_p J_n) | j_\pi^2(J_p) j_\nu^2(J_n); I\rangle,$$

The thus obtained wave function is a mixture of many component as a result of the np interaction

$$\begin{aligned} |\Psi_0(gs)\rangle &= 0.76|[\pi^2(0)\nu^2(0)]_I\rangle + 0.57|[\pi^2(2)\nu^2(2)]_I\rangle \\ &+ 0.24|[\pi^2(4)\nu^2(4)]_I\rangle + 0.13|[\pi^2(6)\nu^2(6)]_I\rangle \\ &+ 0.14|[\pi^2(8)\nu^2(8)]_I\rangle. \end{aligned}$$



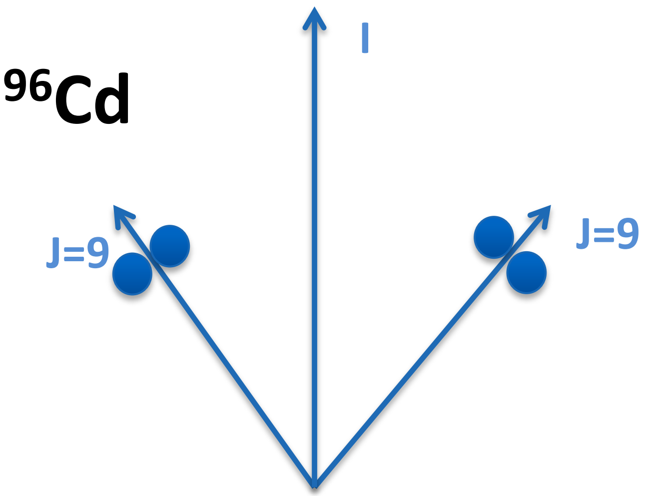
The striking feature is that if we project it on to np coupled terms, the wave function can be represented by a single term $(\nu\pi)_9 \otimes (\nu\pi)_9$

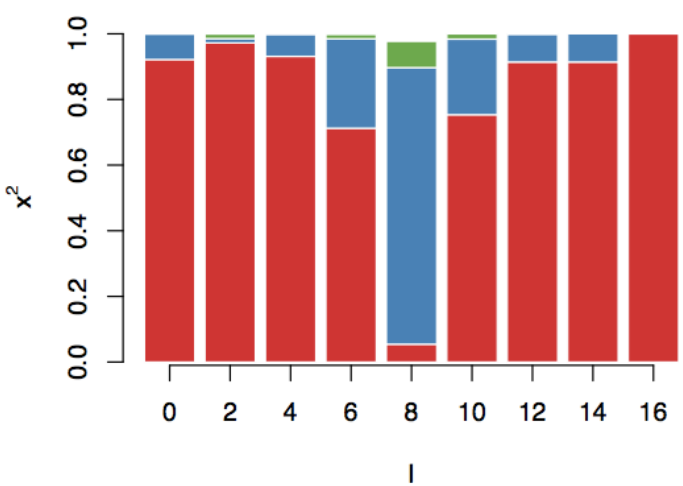
$$\langle [j_p j_n(J_1) j_p j_n(J_2)]_J | [j_p^2(J_p) j_n^2(J_n)]_J \rangle = -2\hat{J}_1 \hat{J}_2 \hat{J}_p \hat{J}_n \left\{ \begin{array}{ccc} j & j & J_p \\ j & j & J_n \\ J_1 & J_2 & J \end{array} \right\}$$

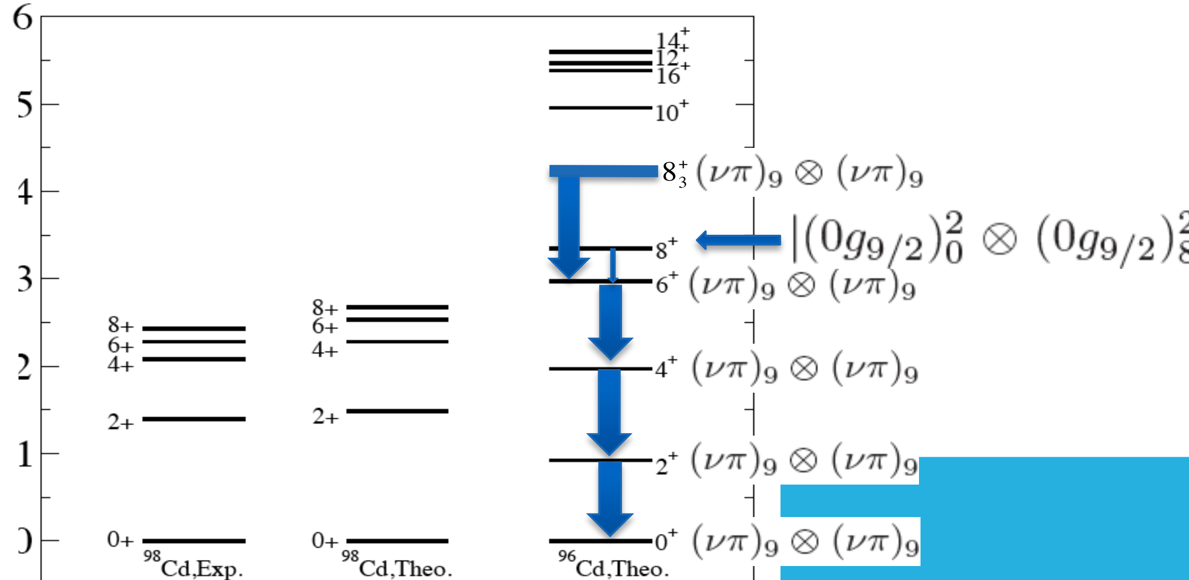
CQ et al, PRC 84, 021301(R) (2011).

^{96}Cd (2n-2p): A simple example to show the pair content

The calculated spectrum show a equidistant pattern along the yrast line up to I=6;
A naive picture is that the angular momenta of the states are generated by the rearrangement of the angular momentum vectors of the aligned np pairs.

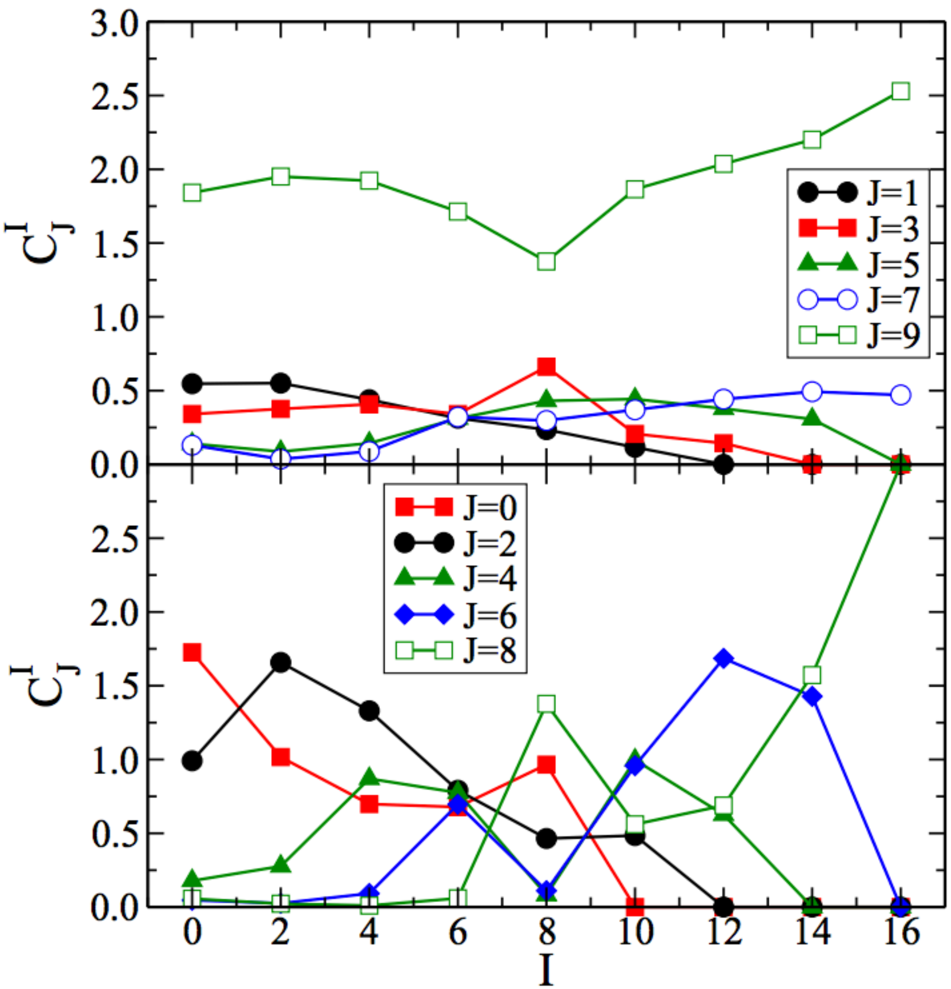






Average number of pairs

$$\langle\Psi_N||((a_i^\dagger a_j^\dagger)_{J^\pi}\times(a_i a_j)_{J^\pi})_0||\Psi_N\rangle \qquad E_I = C_J^I V_J.$$



$$C^I_{J=0}(nn) = C^I_{J=0}(pp) = C^I_{J=0}(np)$$

CQ, PRC 81, 034318 (2010)

PHYSICAL REVIEW VOLUME 161, NUMBER 4 20 SEPTEMBER 1967

Stretch Scheme, a Shell-Model Description of Deformed Nuclei

MICHAEL DANOS AND VINCENT GILLET
Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvett, Seine et Oise, France
and
u of Standards, Washington, D. C.

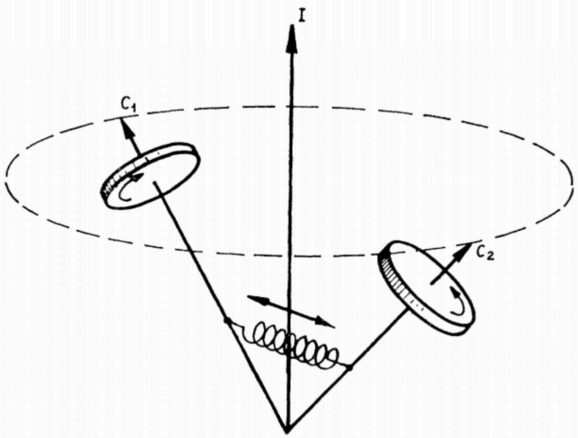
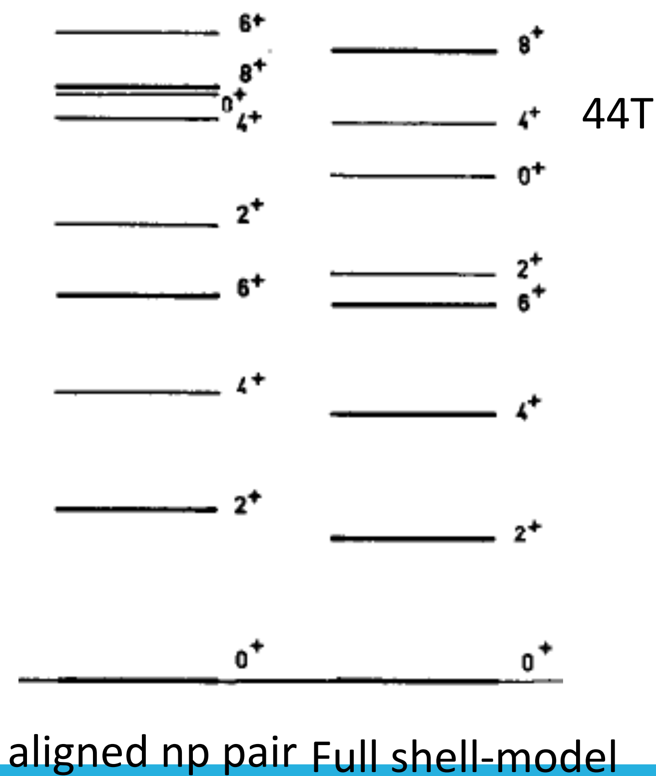


FIG. 3. A classical model for the nuclear rotations. The two



✧Aligned np pair to explain the rotational-like spectra in ^{20}Ne and ^{44}Ti

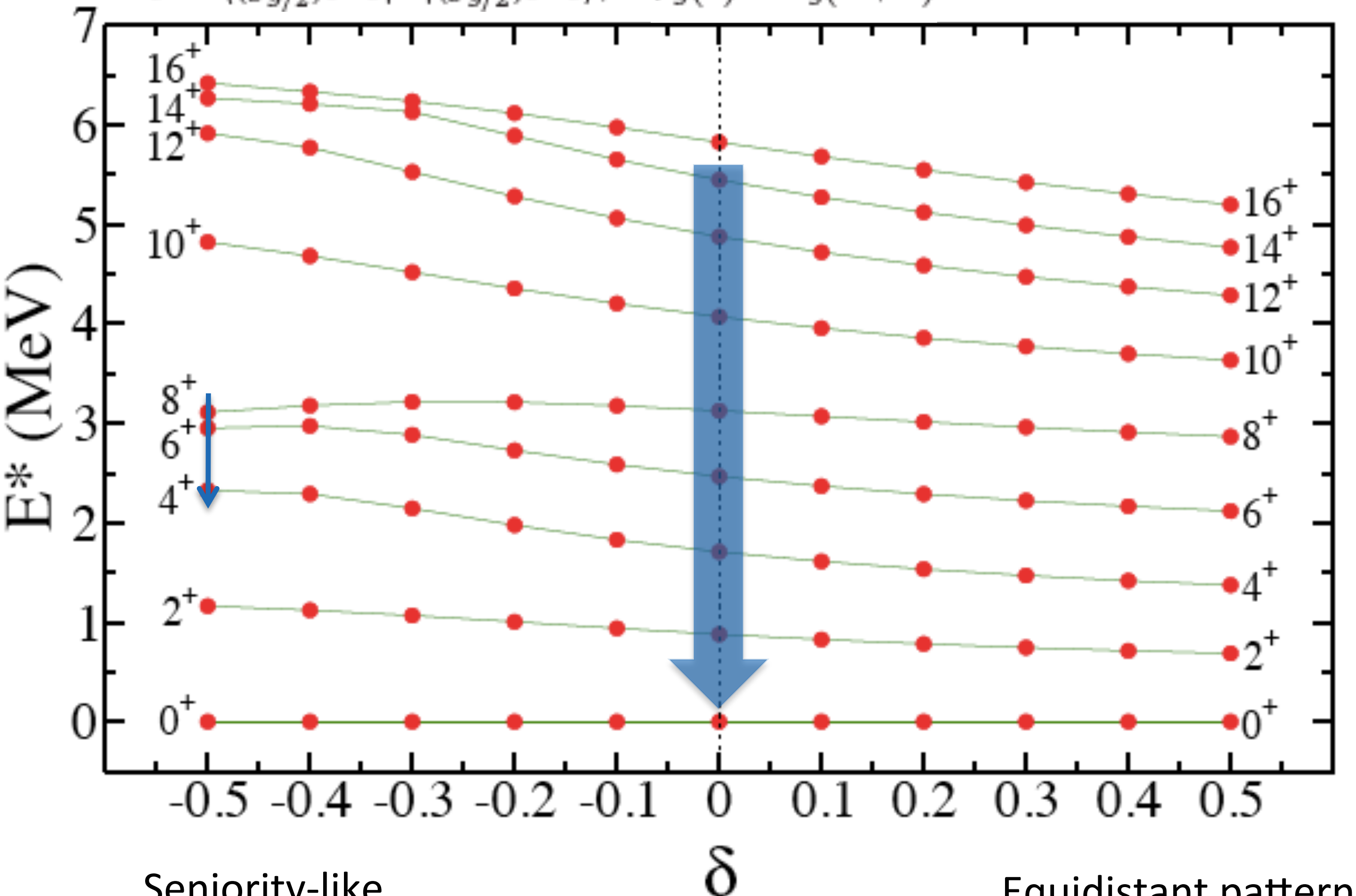
A. Jaffrin, Nucl. Phys. A **196**, 577 (1972).

The spin-aligned pair plays a crucial role

It is strongly attractive since this maximally aligned configuration has maximal overlap between the proton and neutron wave functions

Competition between the np aligned coupling and like nucleon aligned coupling?

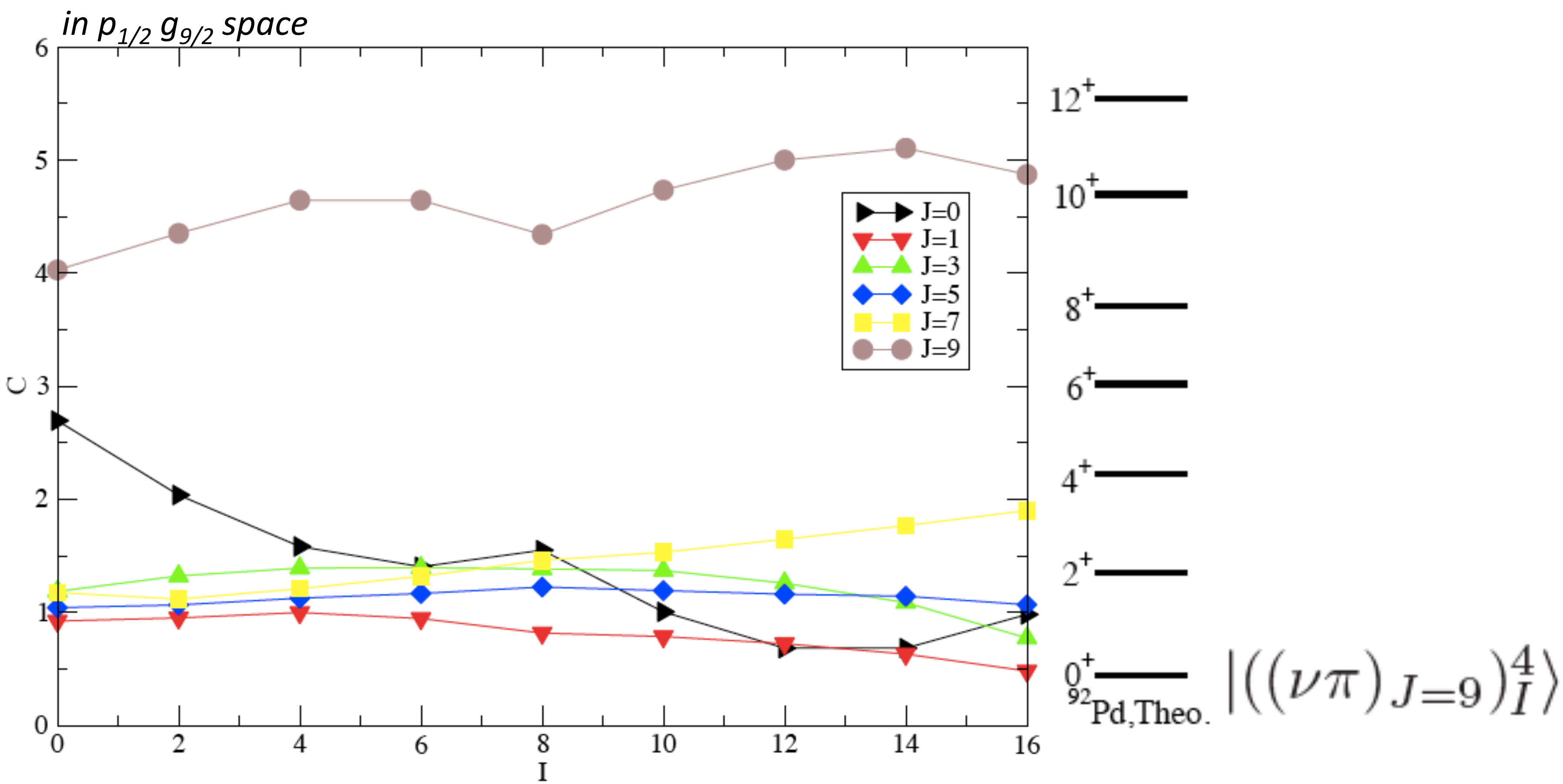
$V_9 = \langle (g^2_{9/2})_{J=9} | V | (g^2_{9/2})_{J=9} \rangle, \quad \mathcal{V}_9(\delta) = V_9(1+\delta)$



Seniority-like

Equidistant pattern

^{92}Pd



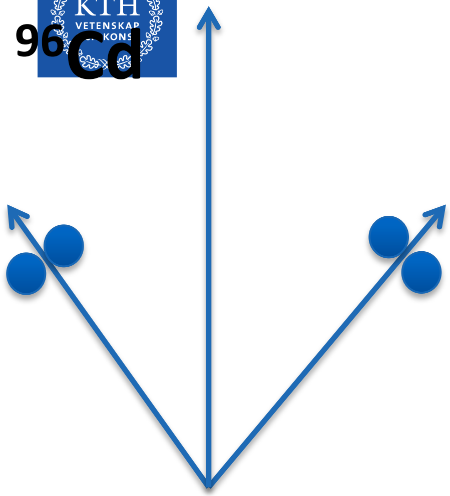
Two particle coefficients of fractional parentage

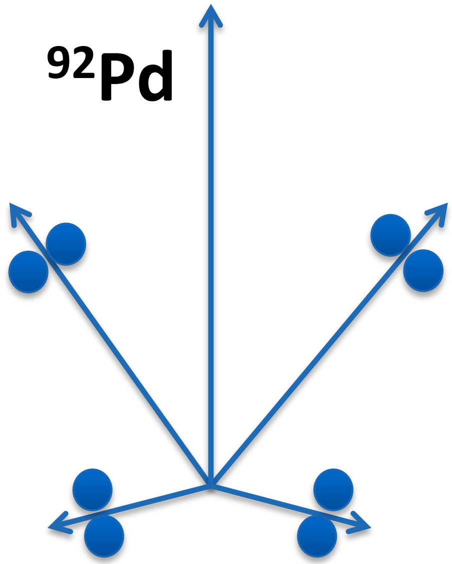
$$\left|\{[(j^2)_{J_1}(j^2)_{J_2}]_{J_3}(j^2)_{J_4}\}_{J_5}(j^2)_{J_6}\right\rangle_I$$

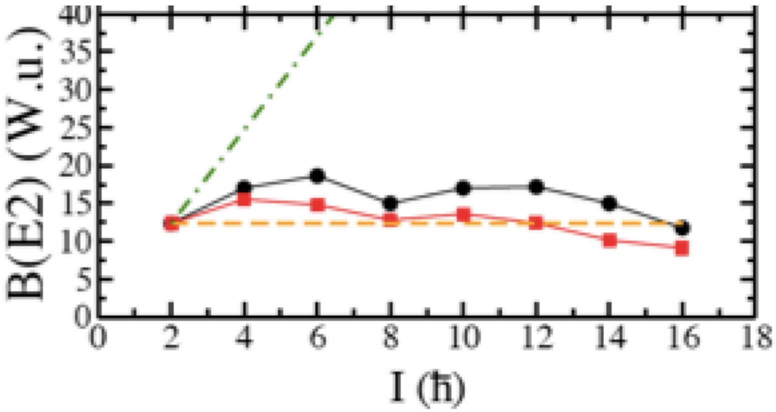
CQ et al, PRC 84, 021301(R) (2011).

Z.X. Xu, C. Qi, J. Blomqvist, R.J. Liotta, R. Wyss Nucl. Phys. A 877, 51 (2012).

The relative motion of the np pairs







The transition strengths remain approximately the same along the yrast line

The B(E2) values connecting yrast states in ^{92}Pd are two times larger than those of ^{96}Cd

Quartet-like coupling

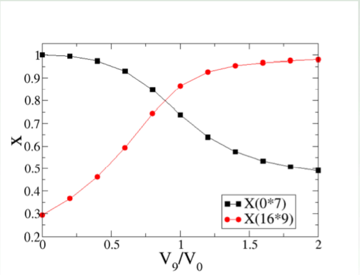
The four $J = 9$ np pairs in ^{92}Pd can couple in various ways. With the help of two-particle cfp one may express the wave function in terms of $((((\nu\pi)_9 \otimes (\nu\pi)_9)_{I'} \otimes (\nu\pi)_9)_{I''} \otimes (\nu\pi)_9)_I$.

e I. Configurations with the largest probabilities for the state $^{92}\mathrm{Pd}(0_1^+)$ corresponding to the ensorial products of different two-particle states (upper) and four-particle states (lower).

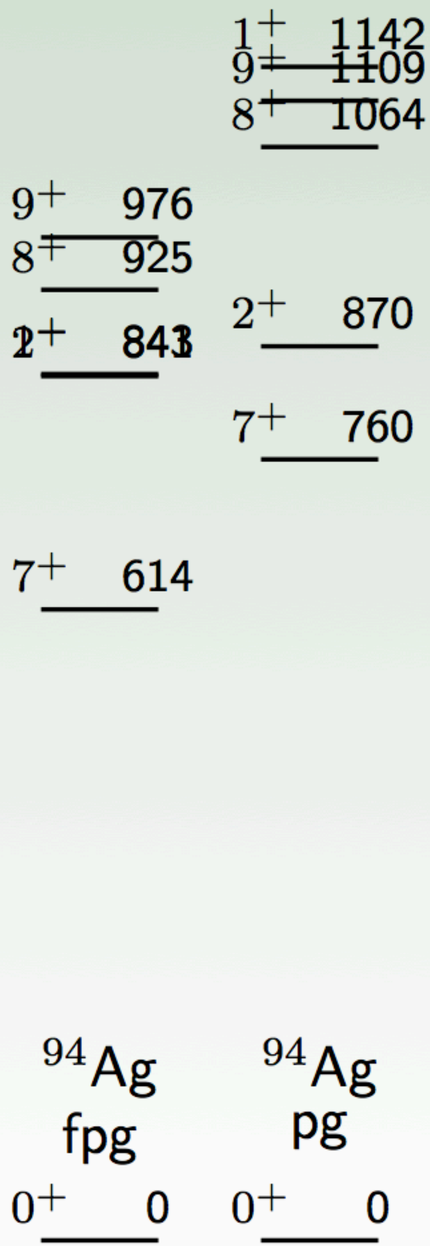
Configuration	x^2
$\lvert\gamma_2 = 9^+ \gamma_2' = 9^+ \gamma_2'' = 9^+ \gamma_2''' = 9^+\rangle$	0.85
$\lvert\gamma_2 = 9^+ \gamma_2' = 9^+ \alpha_2 = 0^+ \beta_2 = 0^+\rangle$	0.76
$\lvert\gamma_2 = 8^+ \gamma_2' = 1^+ \alpha_2 = 0^+ \beta_2 = 8^+\rangle$	0.56
$\lvert\gamma_2 = 8^+ \gamma_2' = 1^+ \alpha_2 = 8^+ \beta_2 = 0^+\rangle$	0.56
$\lvert\gamma_2 = 1^+ \gamma_2' = 1^+ \alpha_2 = 0^+ \beta_2 = 0^+\rangle$	0.52
$\lvert\gamma_4 = 0_1^+ \gamma_4' = 0_1^+\rangle$	0.98
$\lvert\gamma_4 = 8_1^+ \gamma_4' = 8_1^+\rangle$	0.94
$\lvert\gamma_4 = 8_2^+ \gamma_4' = 8_2^+\rangle$	0.92
$\lvert\gamma_4 = 16_1^+ \gamma_4' = 16_1^+\rangle$	0.81

$0g_{9/2}$-shell description of ^{94}Ag (3p-3n): 7_1^+

For simplicity, the Hamiltonian only contain the two matrix elements V_0 and V_9. The wave function is dominated by the configuration of $|[(j^2)_9(j^2)_9]_{16}(j^2)_9\rangle_{I=7}$. Calculation with a realistic Hamiltonian gives a even larger value.



Calculations in different spaces for ^{94}Ag (3p-3n)



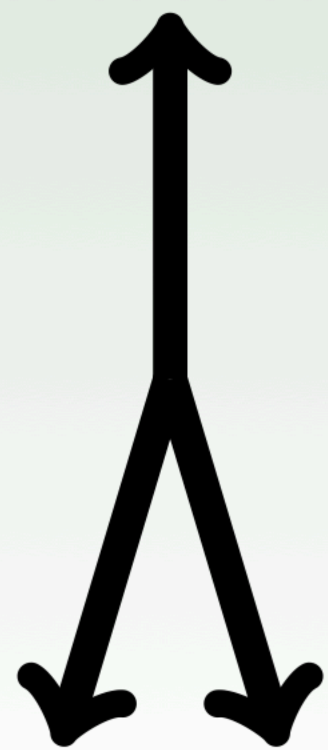
The ground state spin is calculated to be 0^+.

The lowest $T = 0$ state is 7^+.

The wave function is dominated by

$$|[(j^2)_9(j^2)_9]_{16}(j^2)_9\rangle_{I=7}.$$

A stretch isomer?



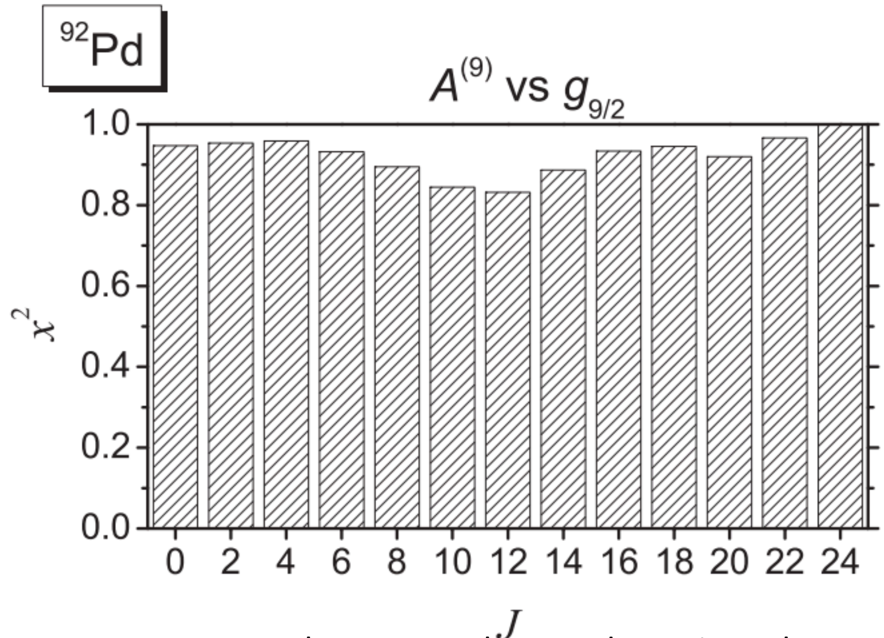
Interacting Boson models with aligned np pair

TABLE IV. Overlaps of the $(1g_{9/2})^4$ yrast eigenstates of the SLGT0 interaction with angular momentum J and isospin $T=0$ with various two-pair states, expressed in percentages.

J	B^2	SP_J	D^2	DG	DI	DK	G^2	I^2	K^2
0	91	80	35				18	7.4	1.9
2	97	85	17	22			1.5	0.0	0.4
4	89	64	42	11	11		0.2	0.2	0.0
6	55	70		43	0.2	4.3	0.0	0.2	0.0
8	5.3	83			7.4	24	1.8	0.2	0.1
10	42					58		6.1	0.5
12	88							57	1.5
14	96								31.4
16	100								100

S. Zerguine and P. Van Isacker, Phys. Rev. C 83, 064314 (2011).

Pair truncation shell model approaches



G. J. Fu, J. J. Shen, Y. M. Zhao, and A. Arima Phys. Rev. C 87, 044312 (2013)

Different facets of the nucleus

The np aligned pair wave functions can be a ‘good’ eigen state of the QQ interaction

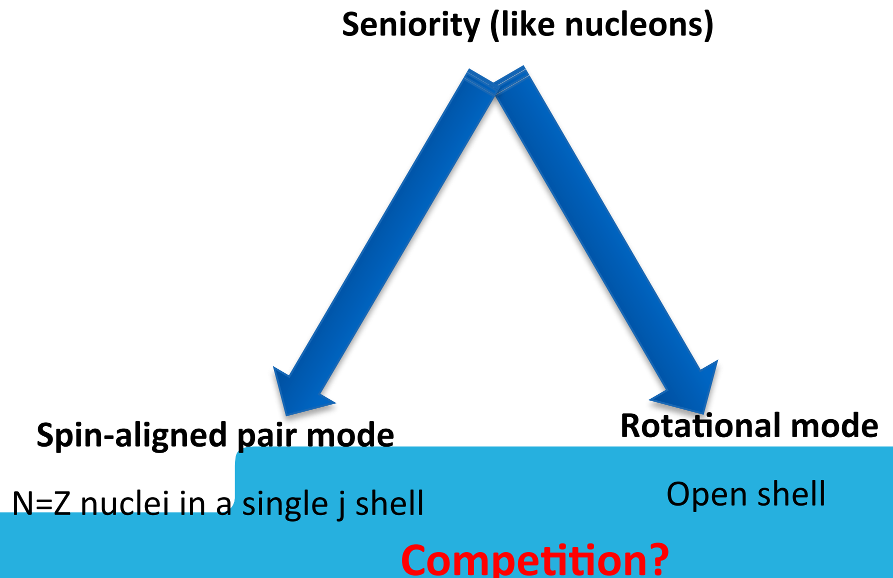
Summary

- Systems with particles in a single-j shell
- The np pair correlation from the binding energy difference
- Neutron-proton spin-aligned pair coupling in N=Z nuclei
- Ongoing:

Extension to systems with many shells

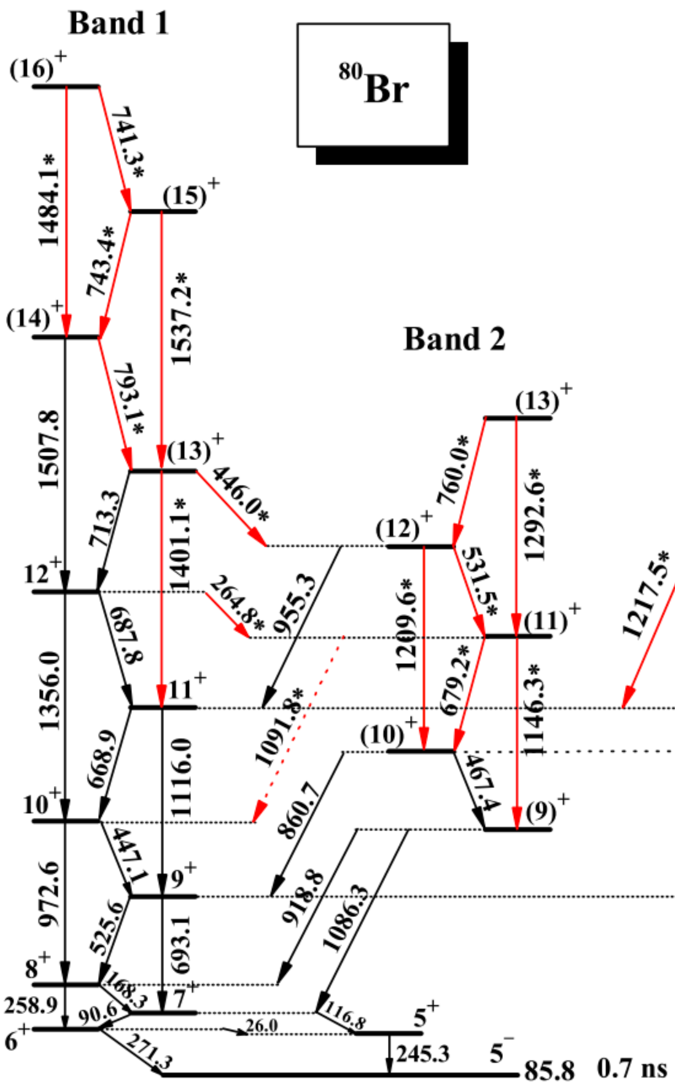
The response to deformation effects

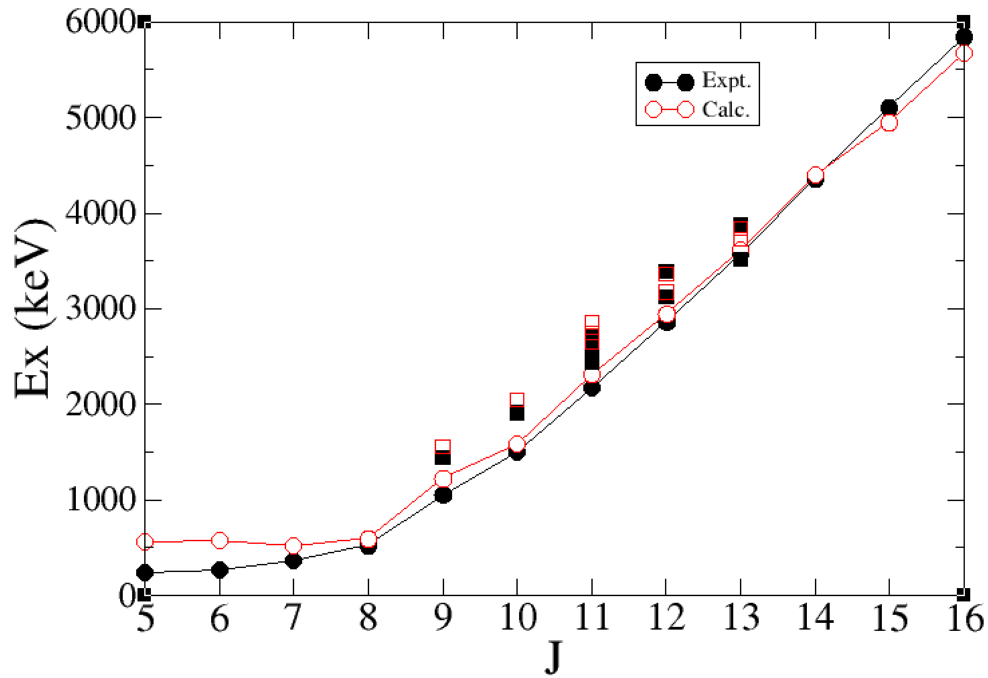
Application in large scale computing: Computer favors uncoupled scheme



Seniority ⟷ Equidistant ⟷ Rotational-like

S.Y. Wang et al. / Physics Letters B 703 (2011) 40–45





Thank you!