



Exotic nuclear shapes from multi-dimensionally constrained covariant density functional theories

Shan-Gui Zhou (周善贵)

Institute of Theoretical Physics,
Chinese Academy of Sciences, Beijing

Contents

□ Introduction

□ Multi-dimensionally constrained CDFTs

□ Normal nuclei

- 1-, 2-, & 3-dim PES of ^{240}Pu & fission barriers of actinides
- Non-axial octupole (tetrahedral) shapes in $N = 150$ isotones
- Tetrahedral & cluster structure in ^{16}O & ^{64}Ge
- Superdeformed & hyperdeformed shapes in actinides

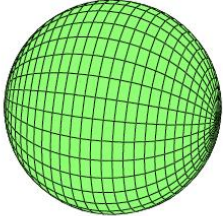
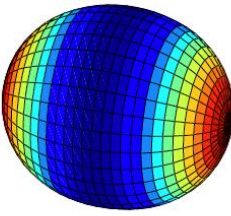
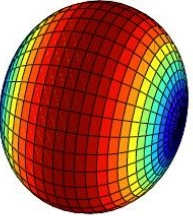
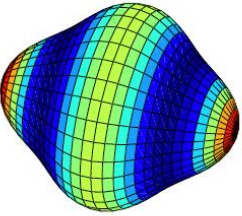
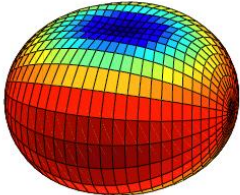
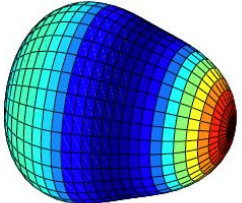
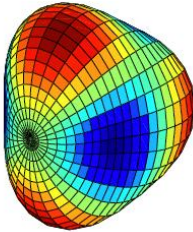
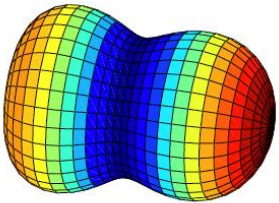
□ Hypernuclei

- Shape polarization effect of Λ
- Superdeformed shapes in Λ hypernuclei

□ Summary & perspectives

Nuclear shapes

$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

(a) $\beta_{\lambda\mu} = 0$	(b) $\beta_{20} > 0$	(c) $\beta_{20} < 0$	(d) $\beta_{40} > 0$
			
(e) $\beta_{22} \neq 0$	(f) $\beta_{30} \neq 0$	(g) $\beta_{32} \neq 0$	(h) $\beta_{20} \gg 0$
			

Courtesy of Bing-Nan Lu (吕炳楠)

Nonaxial quadrupole shape (β_{22} or γ)

J. Phys. G: Nucl. Part. Phys. **37** (2010) 064025

Meng_Zhang 2010_JPG37-064025

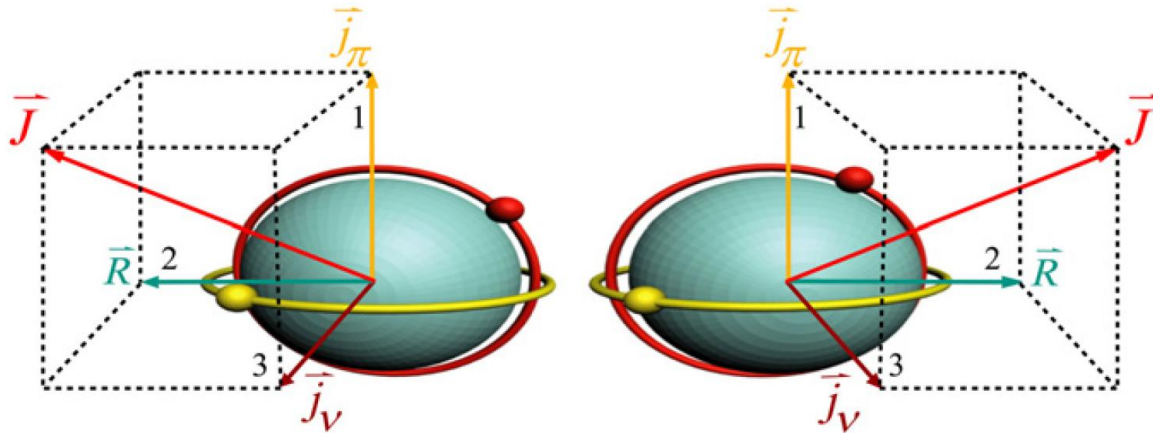


Figure 1. Left- and right-handed chiral systems for a triaxial odd–odd nucleus.

A static triaxial shape in atomic nuclei manifests itself by
the wobbling motion & chiral doublet bands

Bohr & Mottelson 1975
Odegard ... 2001_PRL86-5866

Frauendorf_Meng1997_NPA617-131
Starosta ... 2001_PRL86-971

...

Octupole shape (β_{30})

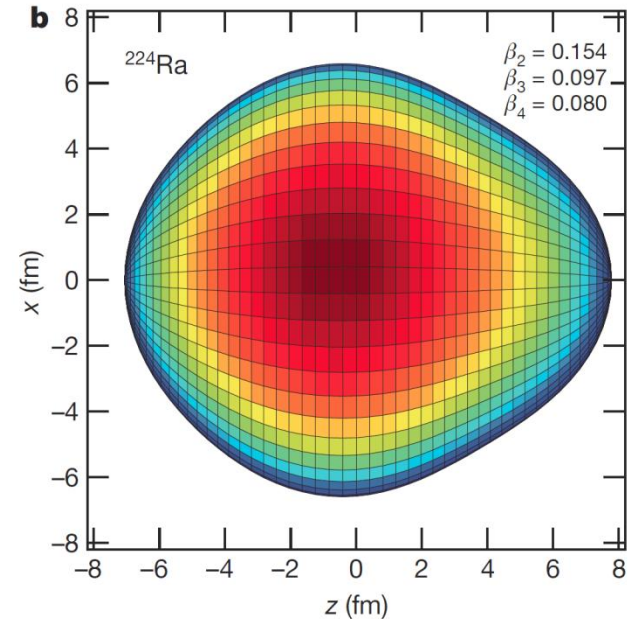
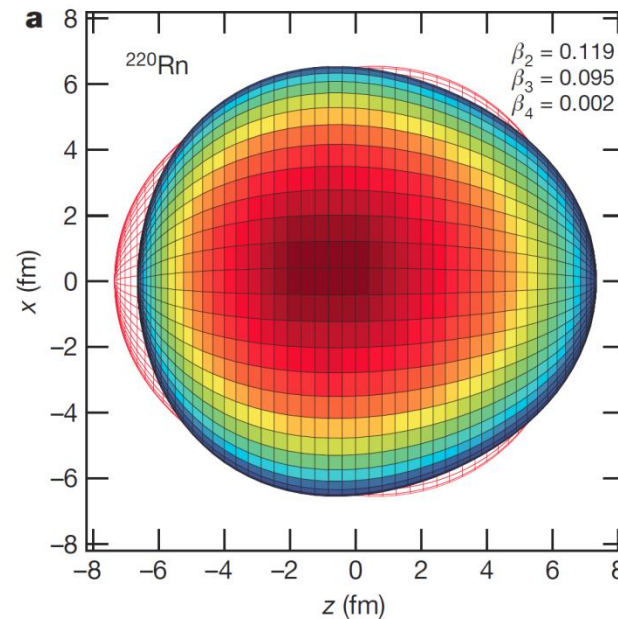
ARTICLE

Gaffney... 2013_Nature497-199

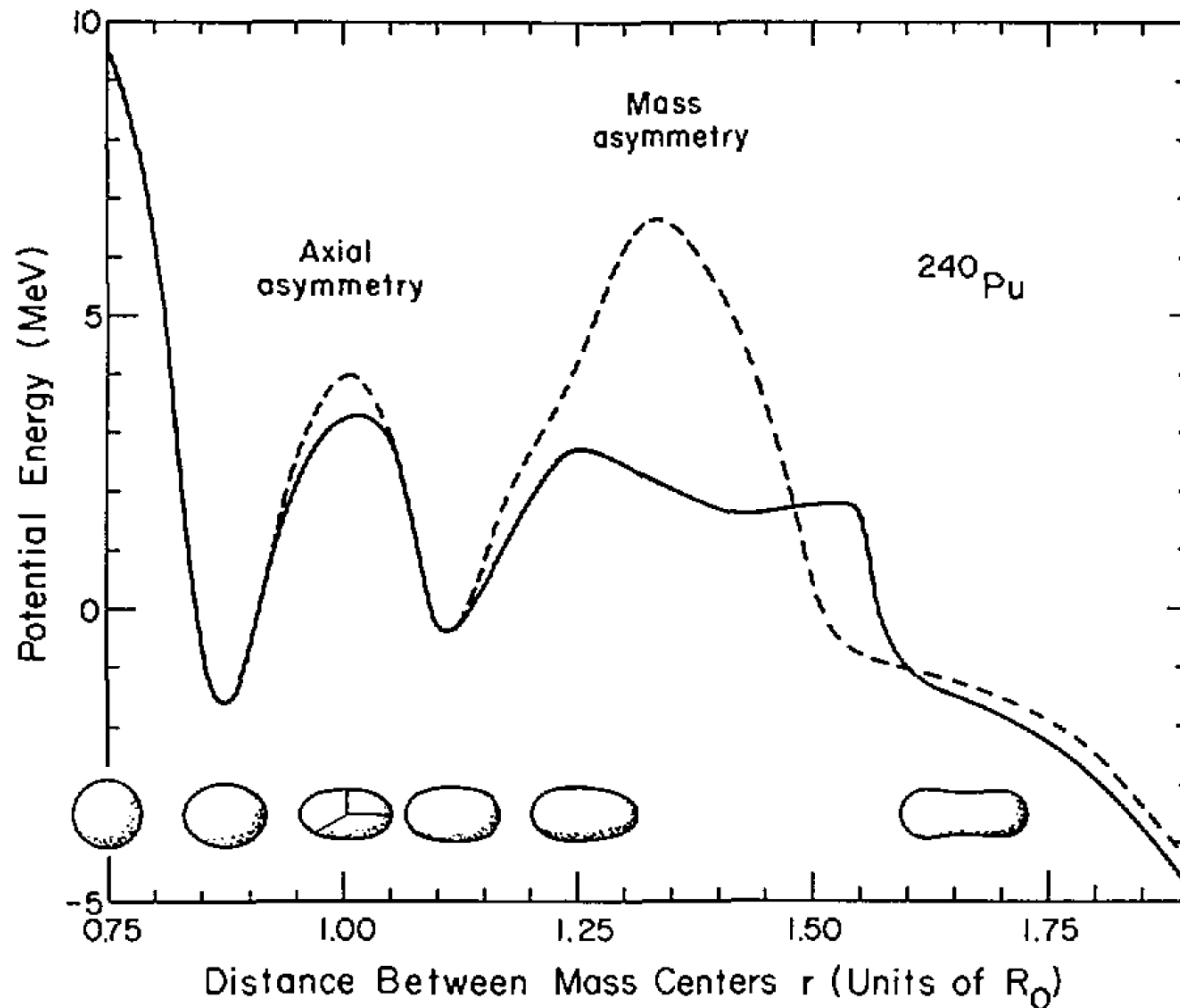
doi:10.1038/nature12073

Studies of pear-shaped nuclei using accelerated radioactive beams

L. P. Gaffney¹, P. A. Butler¹, M. Schec
N. Bree⁷, J. Cederkäll⁸, T. Chupp⁹, D.
M. Huyse⁷, D. G. Jenkins¹³, D. T. Joss¹,
P. Napiorkowski¹⁴, J. Pakarinen^{4,12}, M.
S. Sami⁷, M. Seidlitz⁵, B. Siebeck⁵, T.
K. Wimmer¹⁸, K. Wrzosek-Lipska^{7,14}



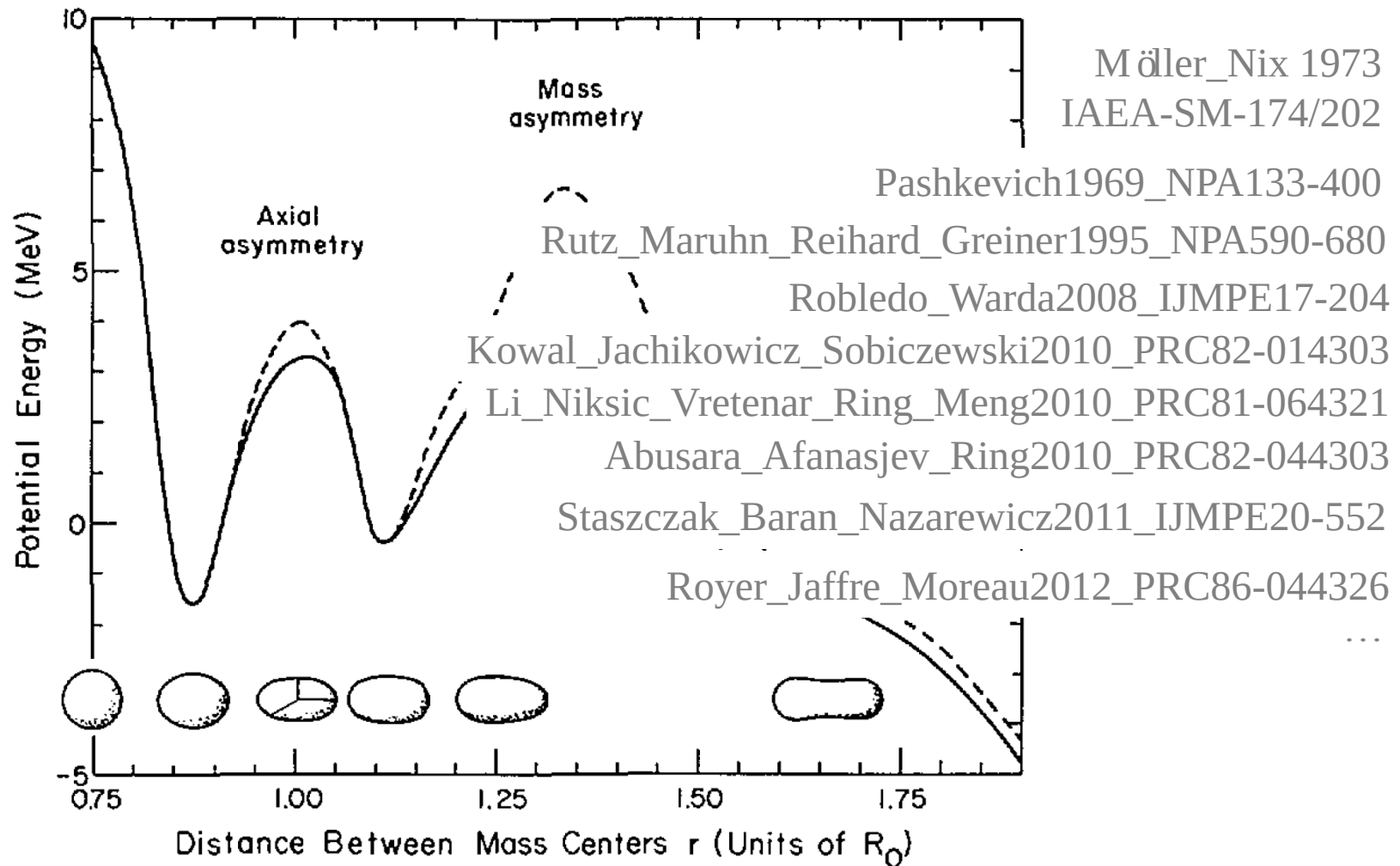
Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Möller_Nix 1973
IAEA-SM-174/202

Axial asymmetry plays important roles around the first barrier
Reflection asymmetry plays important roles around the second barrier

Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Axial asymmetry plays important roles around the first barrier
Reflection asymmetry plays important roles around the second barrier

Covariant Density Functional Theory (CDFT)

$$\begin{aligned}
 \mathcal{L} = & \bar{\psi}_i (i\not{\partial} - M) \psi_i + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - g_\sigma \bar{\psi}_i \sigma \psi_i \\
 & - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - g_\omega \bar{\psi}_i \psi_i \omega \\
 & - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu - g_\rho \bar{\psi}_i \vec{\rho} \vec{\tau} \psi_i \\
 & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_i \frac{1 - \tau_3}{2} A \psi_i,
 \end{aligned}$$

Serot_Walecka1986_ANP16-1

Reinhard1989_RPP52-439

Ring1996_PPNP37-193

Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101

Meng_Toki_SGZ_Zhang_Long_Geng2006_PPNP57-470

$$(\alpha \cdot \mathbf{p} + \beta(M + S(\mathbf{r})) + V(\mathbf{r})) \psi_i = \epsilon_i \psi_i$$

$$(-\nabla^2 + m_\sigma^2) \sigma = -g_\sigma \rho_S - g_2 \sigma^2 - g_3 \sigma^3$$

$$(-\nabla^2 + m_\omega^2) \omega = g_\omega \rho_V - c_3 \omega^3$$

$$(-\nabla^2 + m_\rho^2) \rho = g_\rho \rho_3$$

$$-\nabla^2 A = e \rho_C$$

MDC-CDFT ($\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}, \beta_{40}, \dots$)

□ Axially deformed harmonic oscillator (ADHO) basis

$$\left[-\frac{\hbar^2}{2M} \nabla^2 + V_B(z, \rho) \right] \Phi_\alpha(\mathbf{r}\sigma) = E_\alpha \Phi_\alpha(\mathbf{r}\sigma)$$

Ring_Gambhir_Lalazissis1997_CPC105-77

$$V_B(z, \rho) = \frac{1}{2} M (\omega_\rho^2 \rho^2 + \omega_z^2 z^2)$$

$$\Phi_\alpha(\mathbf{r}\sigma) = C_\alpha \phi_{n_z}(z) R_{n_\rho}^{m_l}(\rho) \frac{1}{\sqrt{2\pi}} e^{im_l \varphi} \chi_{s_z}(\sigma)$$

□ Fourier expansion for densities & potentials

$$f(\rho, \varphi, z) = f_0(\rho, z) \frac{1}{\sqrt{2\pi}} + \sum_{n=1}^{\infty} f_n(\rho, z) \frac{1}{\sqrt{\pi}} \cos(2n\varphi)$$

$f = V \text{ or } \rho$

□ A modified linear constraint method

$$E' = E_{\text{RMF}} + \sum_{\lambda\mu} \frac{1}{2} C_{\lambda\mu} Q_{\lambda\mu}$$

$$C_{\lambda\mu}^{(n+1)} = C_{\lambda\mu}^{(n)} + k_{\lambda\mu} \left(\beta_{\lambda\mu}^{(n)} - \beta_{\lambda\mu} \right)$$

MDC-CDFT ($\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}, \beta_{40}, \dots$)

ph channel	Non-linear	Density-dependent
Meson exchange	NL3, NL3*, PK1, ...	DD-ME1, DD-ME2, ...
Point Coupling	PC-F1, PC-PK1, ...	DD-PC1, ...

MDC-RMF

MDC-RHB

pp channel	BCS	Bogoliubov
Constant gap	√	
Constant strength	√	
Delta force	√	√
Separable force	√	√

Lu_Zhao_SGZ 2011_PRC84-014328

Lu_Zhao_SGZ 2012_PRC85-011301R

Zhao_Lu_Zhao_SGZ 2012_PRC86-057304

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Numerical checks

□ $(\beta_2, \beta_3, \dots)$

Geng_Meng_Toki2007_ChinPhysLett24-1865

□ $(\beta_2, \gamma, \dots)$

Meng_Peng_Zhang_SGZ2006_PRC73-037303

□ $(\beta_2, \dots)$

Ring_Gambhir_Lalazissis1997_CPC105-77

Lu_Zhao_SGZ 2011_PRC84-014328

Lu_Zhao_SGZ 2012_PRC85-011301R

Zhao_Lu_Zhao_SGZ 2012_PRC86-057304

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Numerical checks

□ $(\beta_2, \beta_3, \dots)$

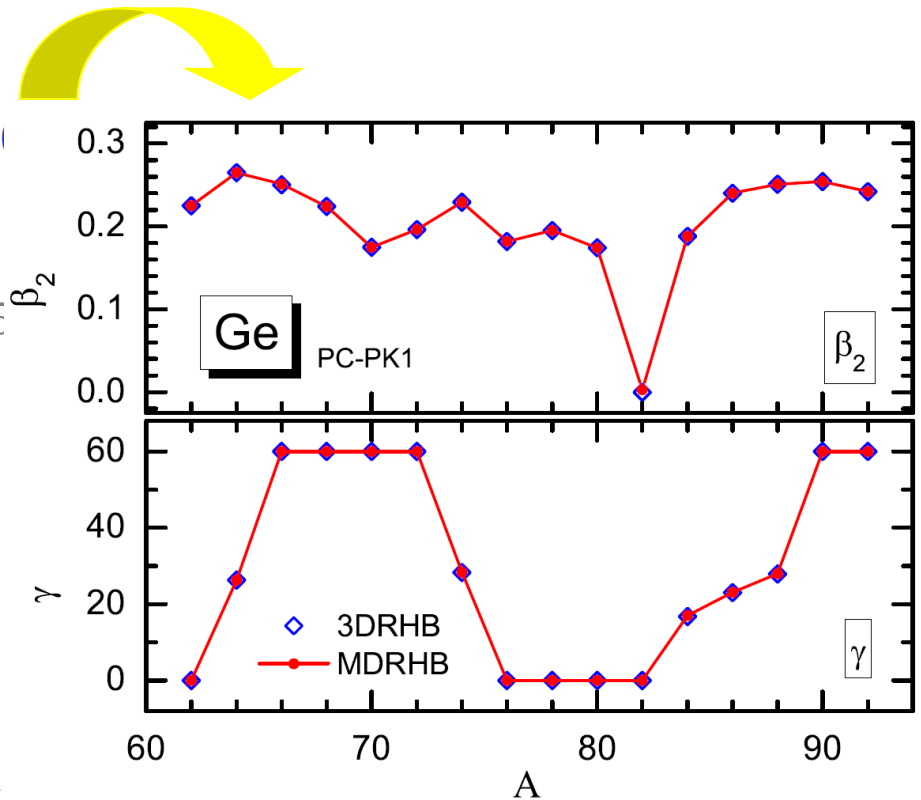
Geng_Meng_Toki2007_ChinPhysLett24-1865

□ $(\beta_2, \gamma, \dots)$

Meng_Peng_Zhang_SGZ2006_PRC73-

□ $(\beta_2, \dots)$

Ring_Gambhir_Lalazissis1997_CPC105



Lu_Zhao_SGZ 2011_PRC84-014328

Lu_Zhao_SGZ 2012_PRC85-011301R

Zhao_Lu_Zhao_SGZ 2012_PRC86-057304

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Numerical checks

□ $(\beta_2, \beta_3, \dots)$

Geng_Meng_Toki2007_ChinPhysLett24-186

□ $(\beta_2, \gamma, \dots)$

Meng_Peng_Zhang_SGZ2006_PRC73-0373

□ $(\beta_2, \dots)$

Ring_Gambhir_Lalazissis1997_CPC105-77

Larger basis size in the elongated direction

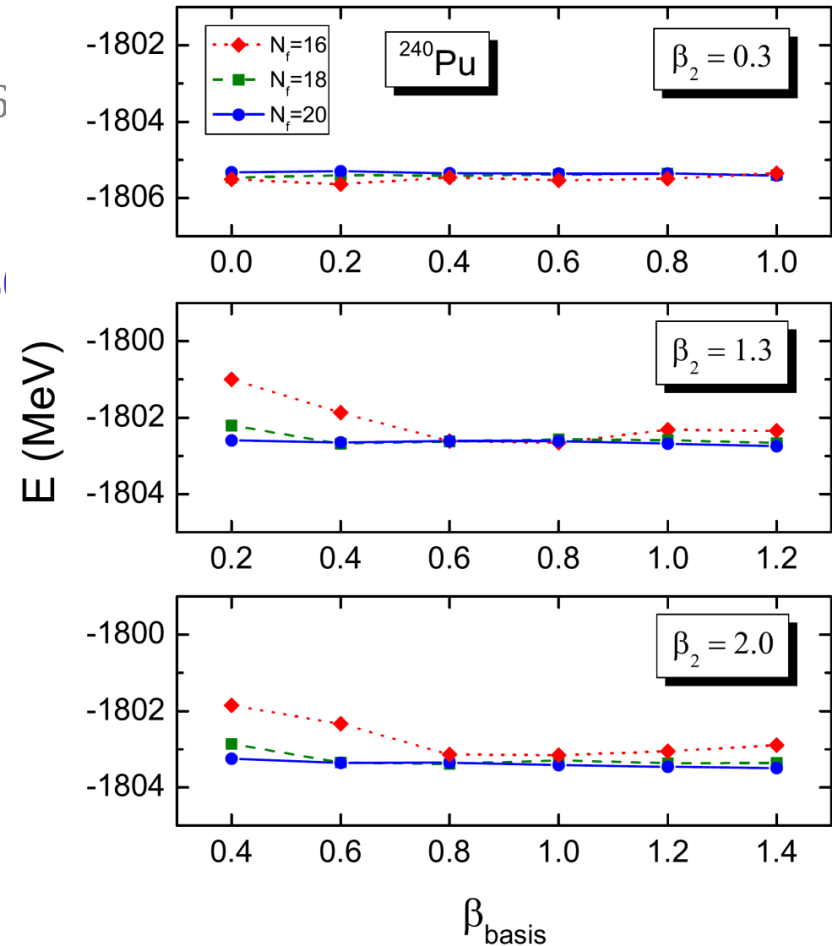
Warda_Egido_Robledo_Pomorski
2002_PRC66-014310

Lu_Zhao_SGZ 2011_PRC84-014328

Lu_Zhao_SGZ 2012_PRC85-011301R

Zhao_Lu_Zhao_SGZ 2012_PRC86-057304

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323



Numerical checks

□ $(\beta_2, \beta_3, \dots)$

Geng_Meng_Toki2007_ChinPhysLett24-18

□ $(\beta_2, \gamma, \dots)$

Meng_Peng_Zhang_SGZ2006_PRC73-037

□ $(\beta_2, \dots)$

Ring_Gambhir_Lalazissis1997_CPC105-77

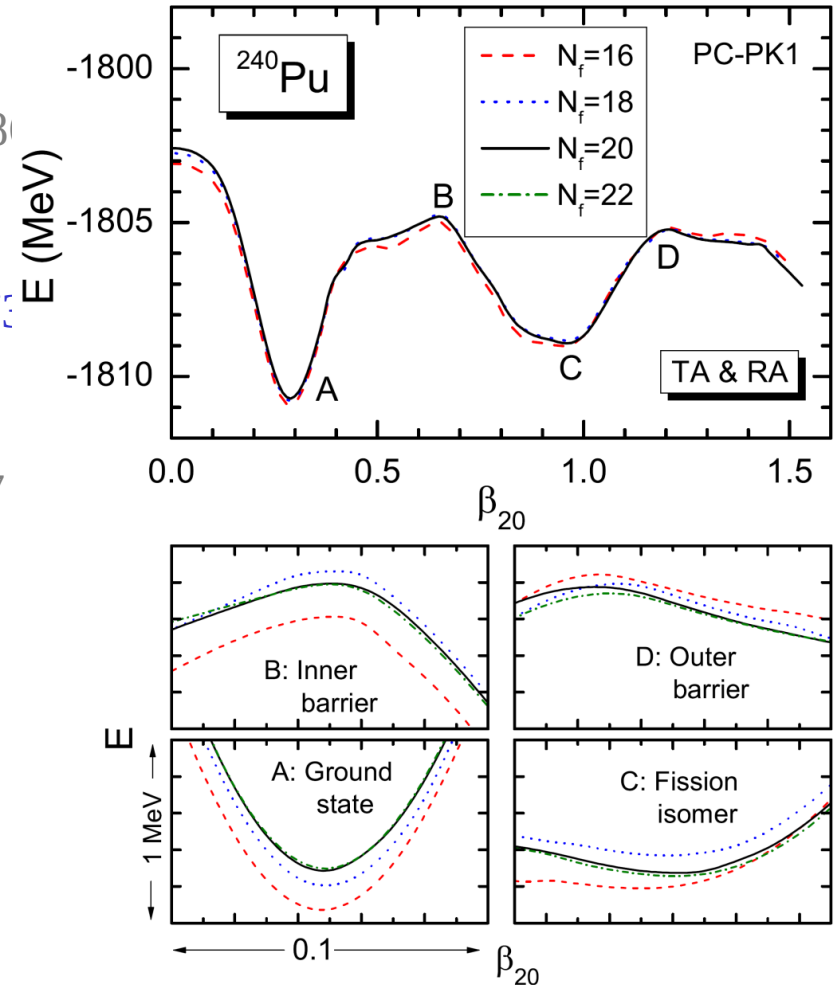
Convergence of the binding energy
w.r.t. the size of the ADHO basis

Lu_Zhao_SGZ 2011_PRC84-014328

Lu_Zhao_SGZ 2012_PRC85-011301R

Zhao_Lu_Zhao_SGZ 2012_PRC86-057304

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323



Numerical checks

□ $(\beta_2, \beta_3, \dots)$

Geng_Meng_Toki2007_ChinPhysLett24-1000

□ $(\beta_2, \gamma, \dots)$

Meng_Peng_Zhang_SGZ2006_PRC73

□ $(\beta_2, \dots)$

Ring_Gambhir_Lalazissis1997_CPC10

The “variational collapse” problem

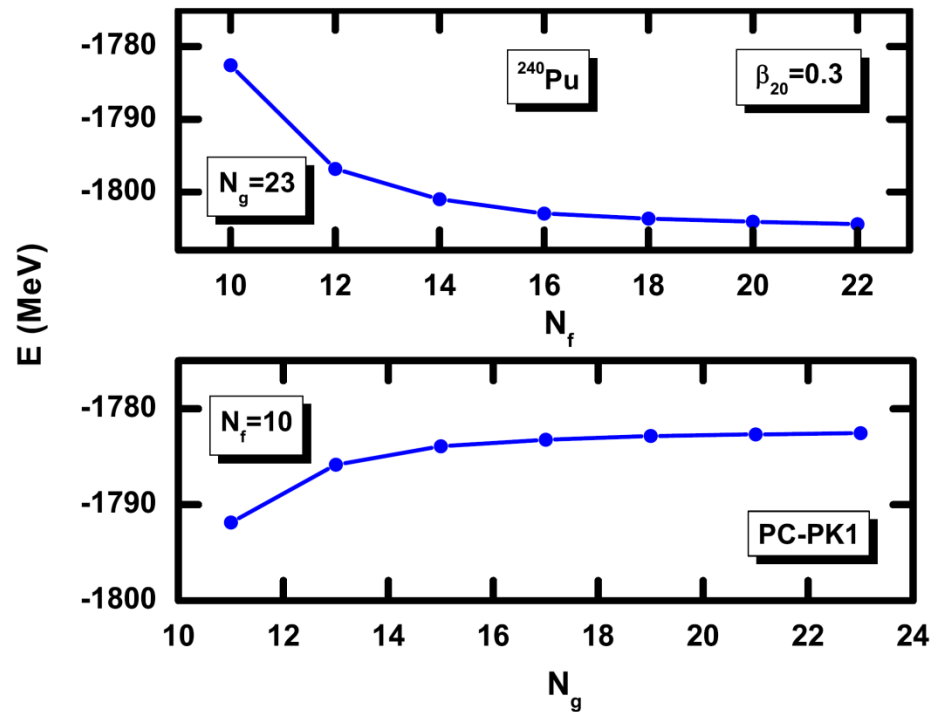
Kutzelnigg1984_IJQuanChem25-107

Lu_Zhao_SGZ 2011_PRC84-014328

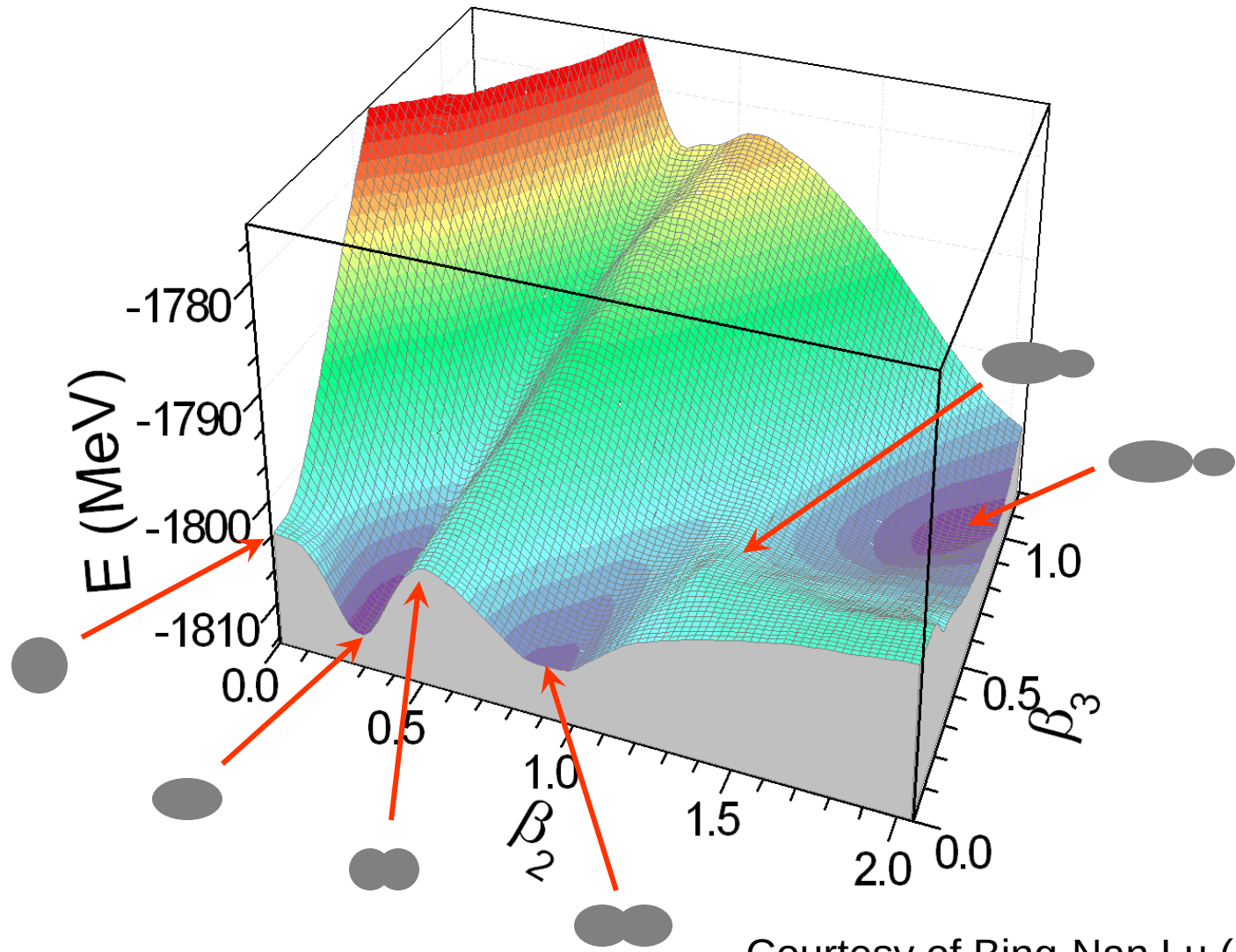
Lu_Zhao_SGZ 2012_PRC85-011301R

Zhao_Lu_Zhao_SGZ 2012_PRC86-057304

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

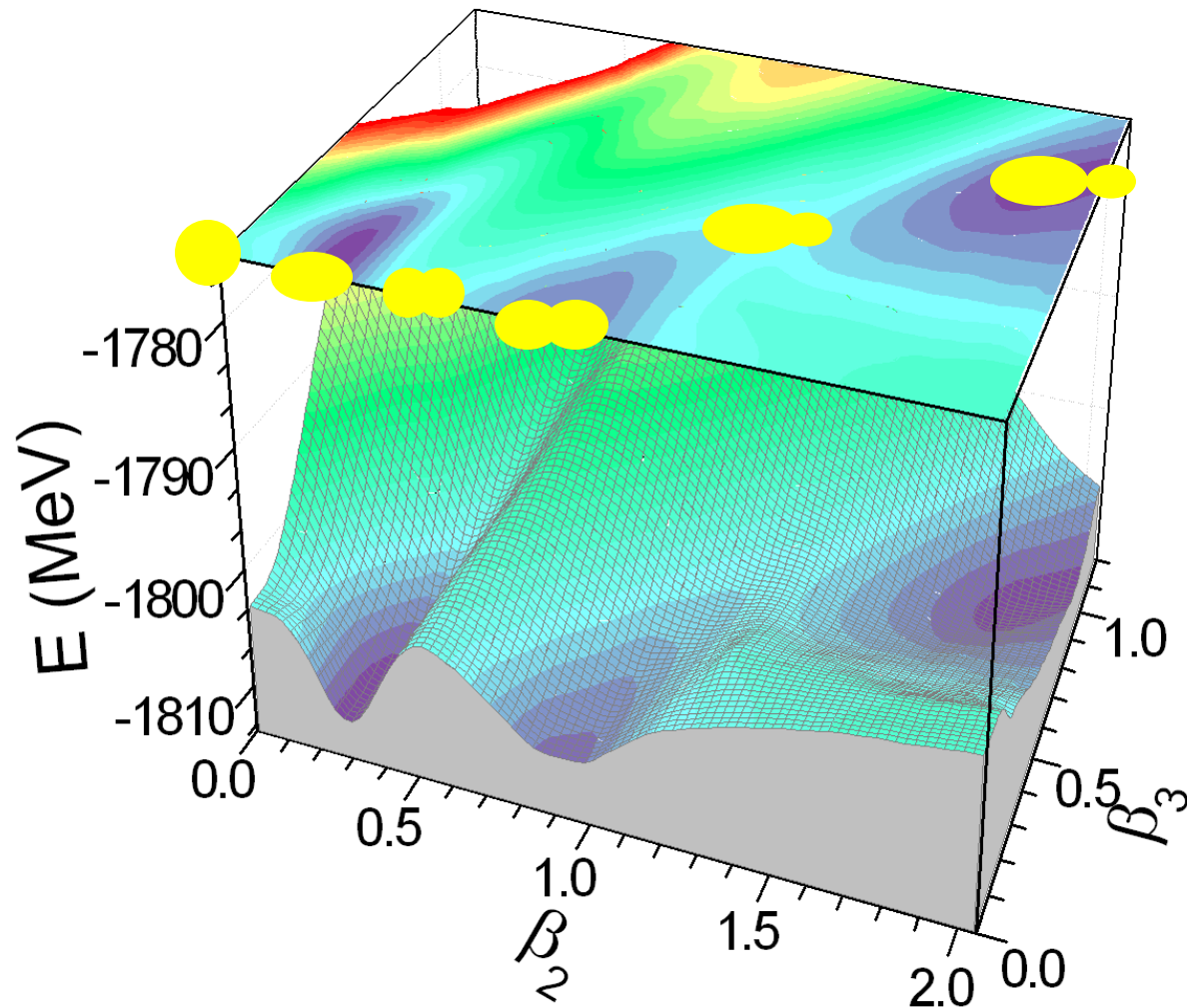


Potential energy surface: an example



Courtesy of Bing-Nan Lu (吕炳楠)

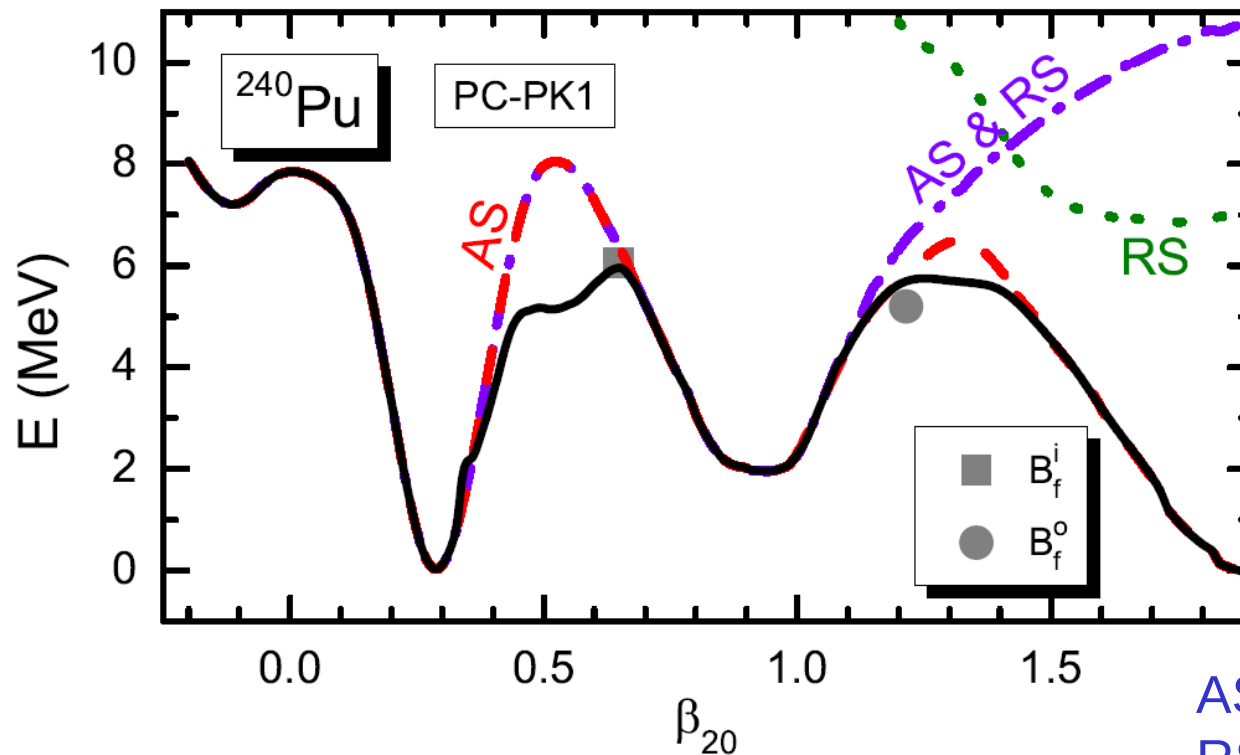
Potential energy surface: an example



Courtesy of Bing-Nan Lu (吕炳楠)

^{240}Pu : 1-dim. potential energy curve (β_{20})

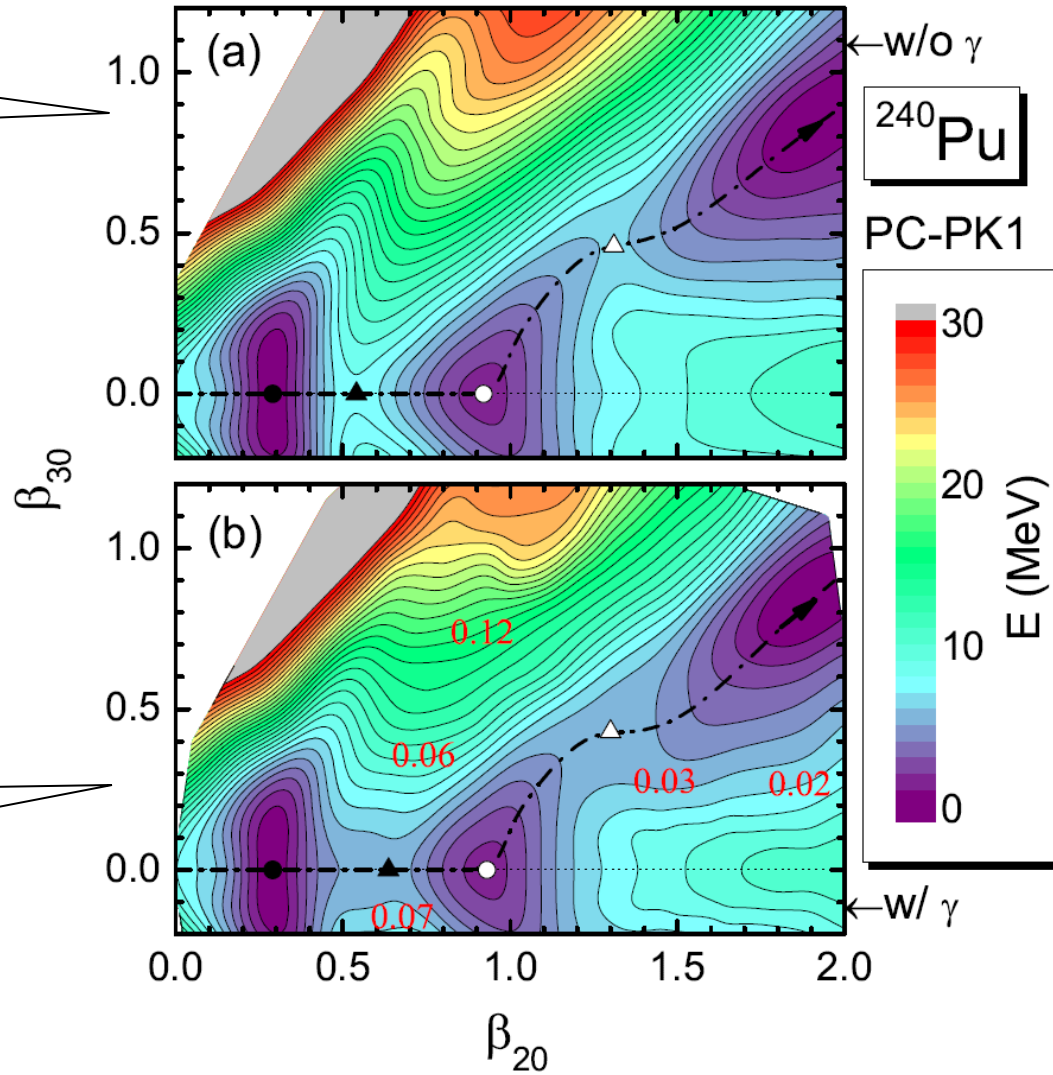
- ❑ Triaxiality lowers inner barrier height by more than 2 MeV
- ❑ Octupole deformation lowers outer barrier dramatically
- ❑ Triaxiality lowers outer barrier height by about 1 MeV



AS: Axially Sym.
RS: Reflection Sym.

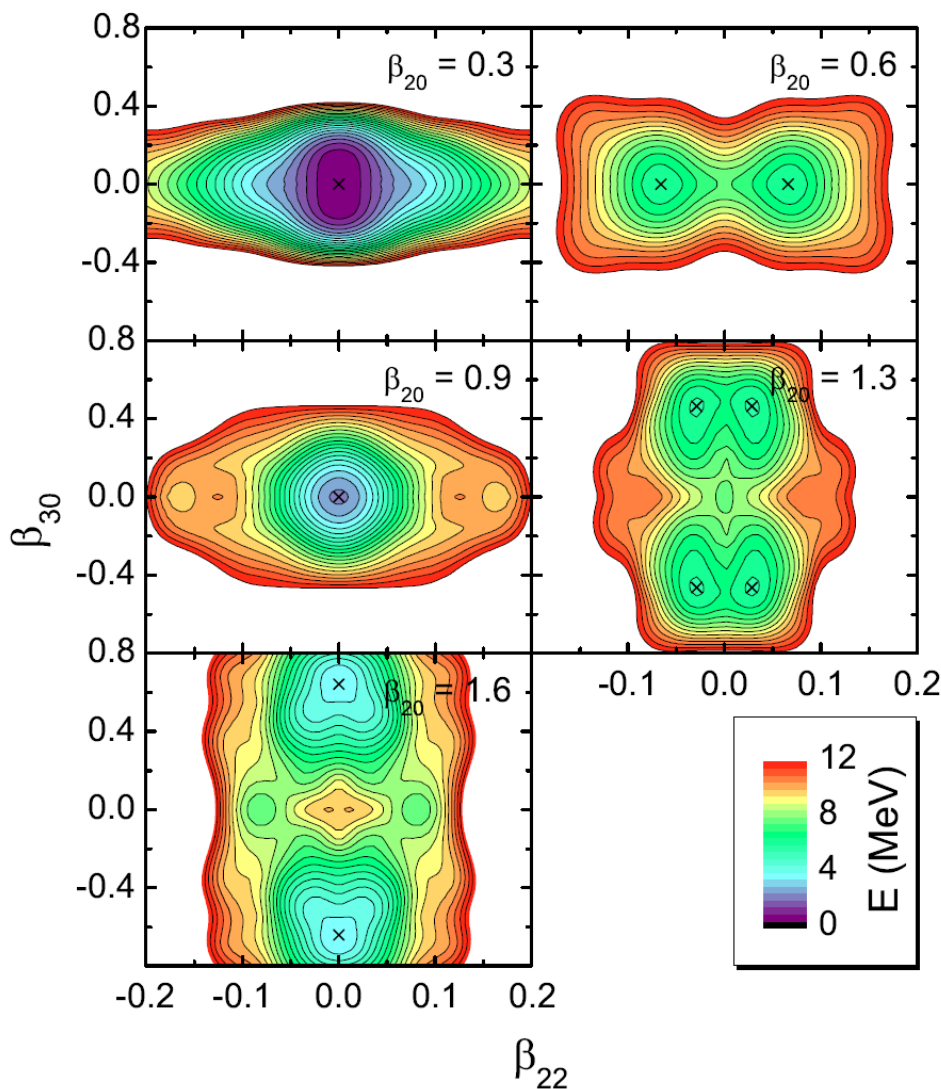
^{240}Pu : 2-dim. PES (β_{20} , β_{30})

Without
triaxility

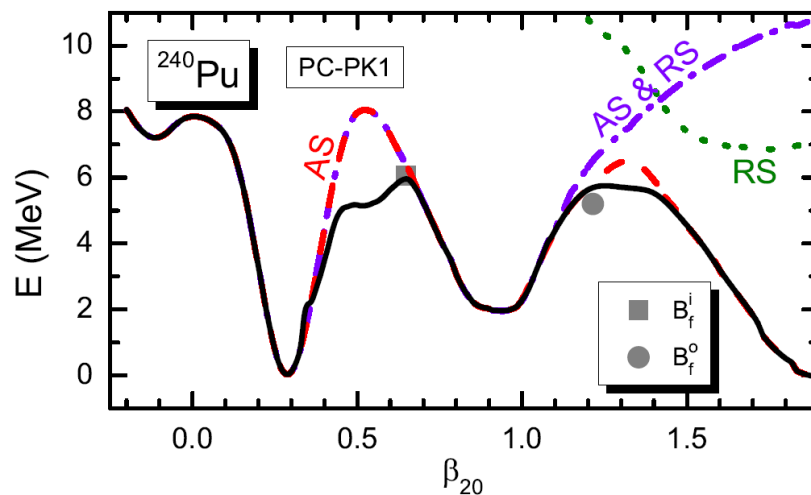


With
triaxility

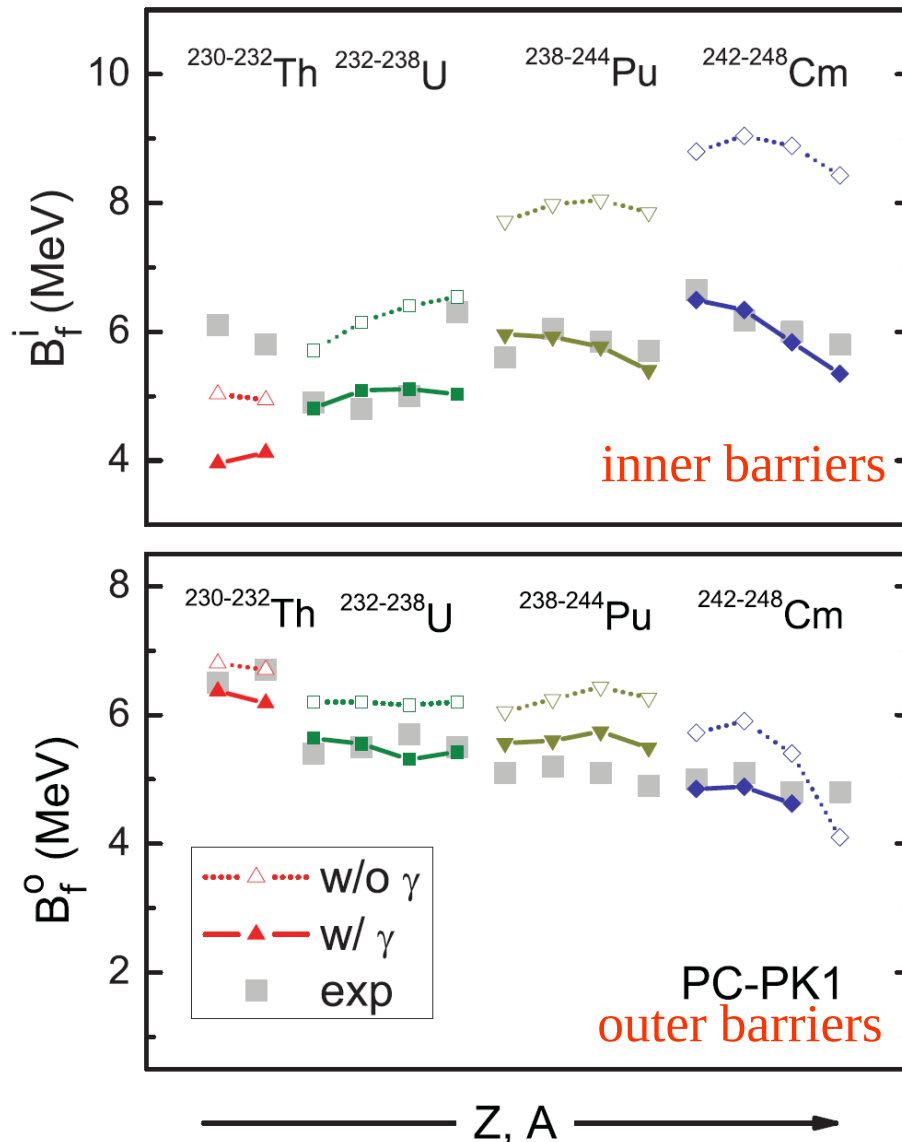
^{240}Pu : 3-dim. PES (β_{20} , β_{22} , β_{30})



- AS & RS for g.s. & isomer, the latter is stiffer
- Triaxial & octupole shape around the outer barrier
- Triaxial deformation crucial around barriers



B_f of actinide nuclei



□ Influence of triaxiality

- Inner fission barriers lowered by 1~2 MeV
- Outer fission barriers lowered by 0.5~1 MeV

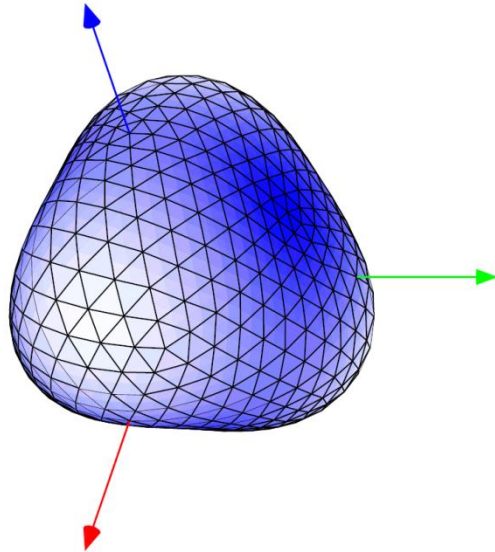
□ Problems

- $^{230-232}\text{Th}$: out barriers primary
- ^{238}U : ?
- ^{248}Cm : two fission paths

Empirical values: RIPL-3 (NDS2010)

Lu_Zhao_SGZ 2012_PRC85-011301R

Non-axial octupole shape in $N=150$ isotones

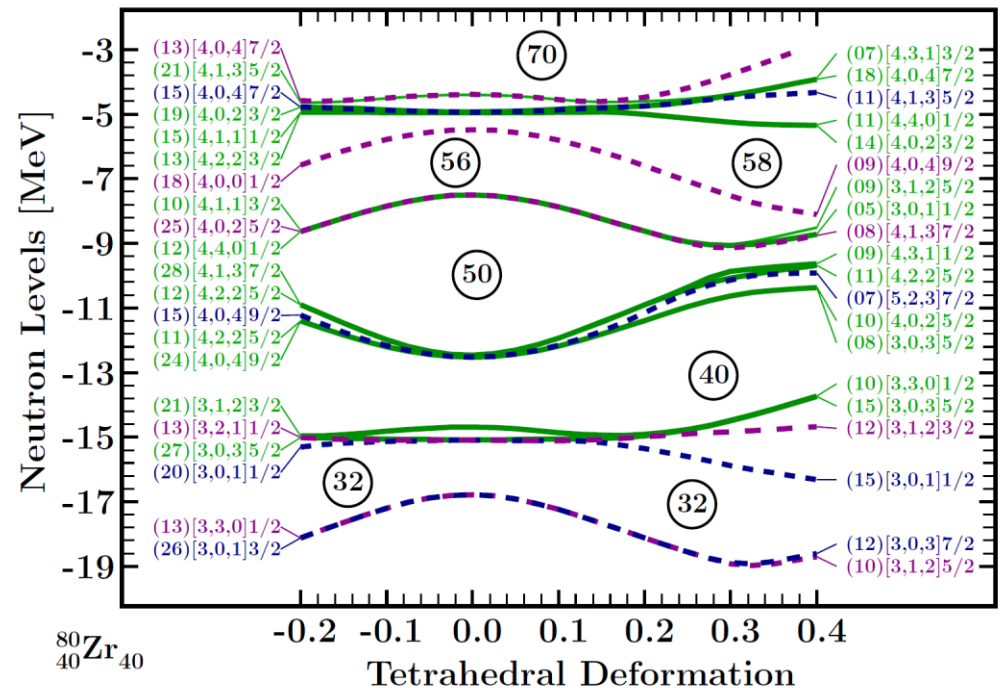


Bark ... 2010_PRL104-022501
Jentschel ... 2010_PRL104-222502

...

Skalski1991_PRC43-140
Hamamoto_Mottelson_Xie_Zhang1991_ZPD21-163
Li_Dudek1994_PRC49-1250R
Takami_Yabana_Matsuo1998_PLB431-242

...



Dudek_Gozdz_Mazur_Molique_Rybak_Fornal
2010_JPG37-064032

Non-axial octupole shape in $N=150$ isotones

PHYSICAL REVIEW C **77**, 061305(R) (2008)

Nonaxial-octupole effect in superheavy nuclei

Y.-S. Chen,¹ Yang Sun,^{2,3} and Zao-Chun Gao^{1,4}

¹*China Institute of Atomic Energy, P. O. Box 275(18), Beijing 102413, People's Republic of China*

²*Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China*

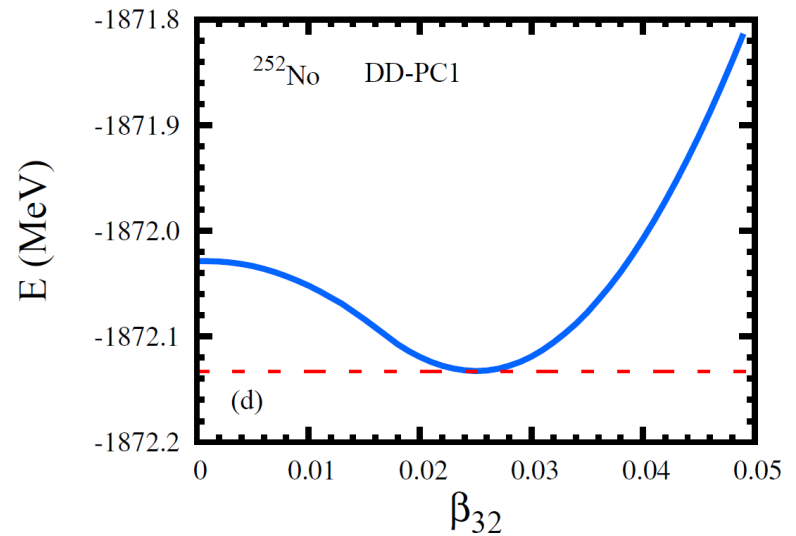
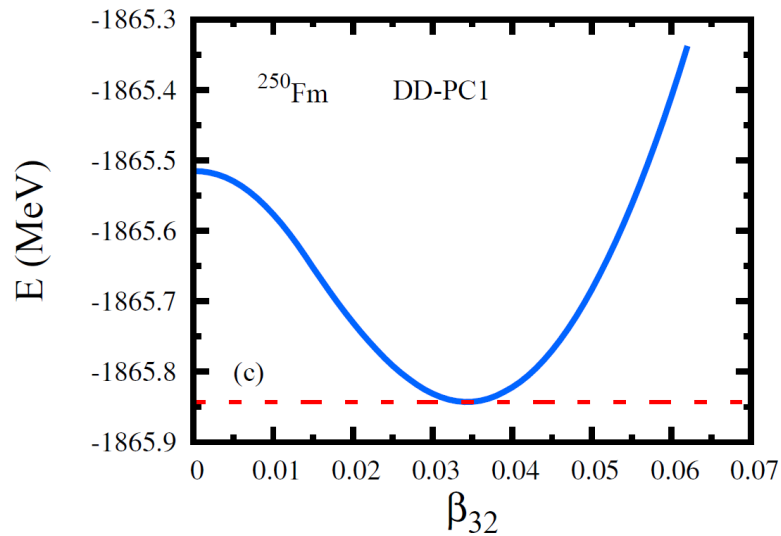
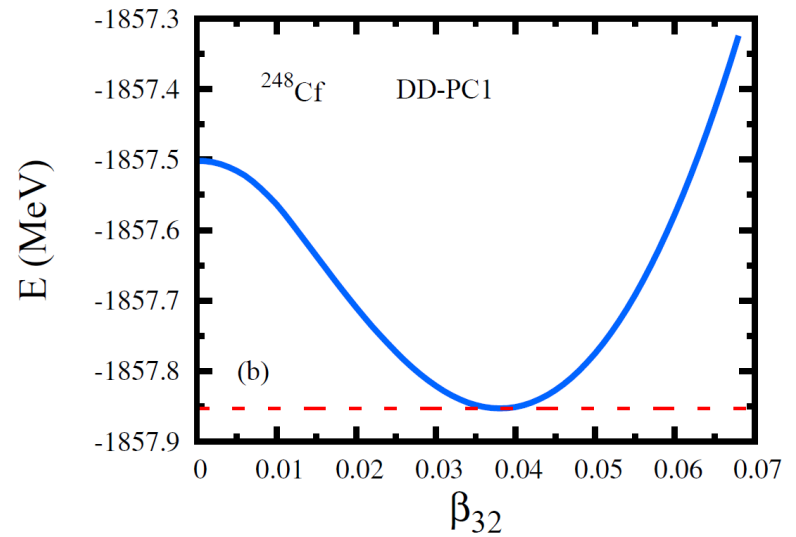
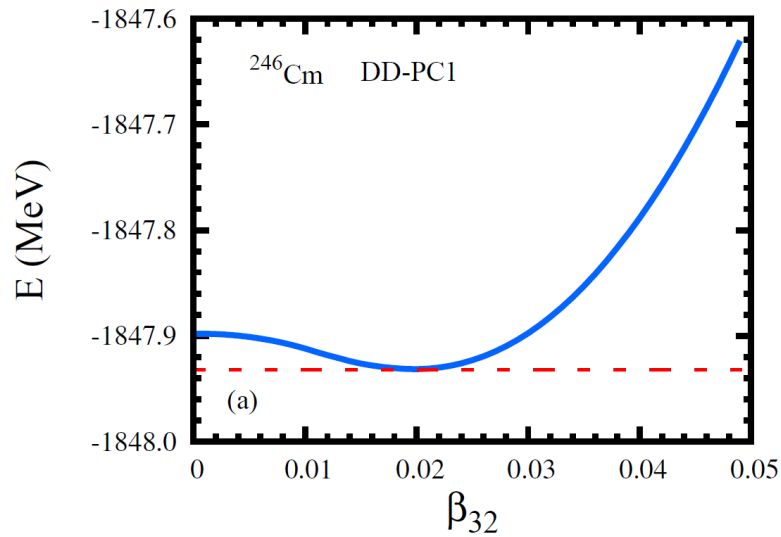
³*Joint Institute for Nuclear Astrophysics, University of Notre Dame, Notre Dame, Indiana 46556, USA*

⁴*Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA*

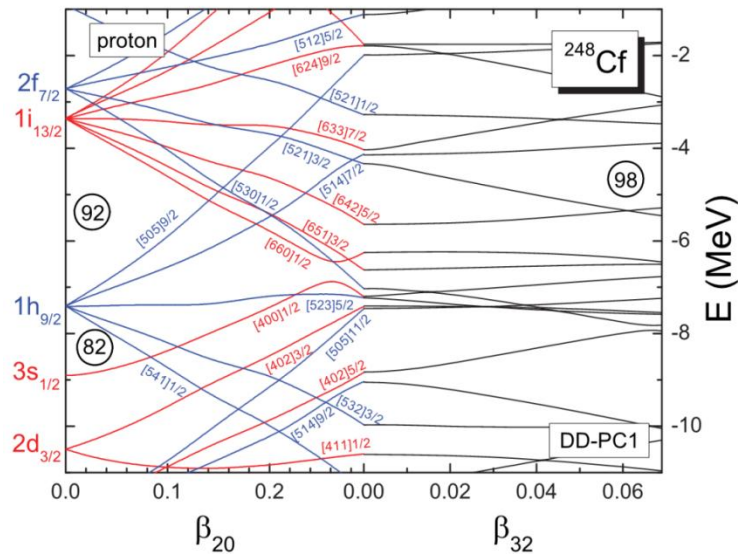
(Received 26 March 2008; published 26 June 2008)

The triaxial-octupole Y_{32} correlation in atomic nuclei has long been expected to exist but experimental evidence has not been clear. We find, in order to explain the very low-lying 2^- bands in the transfermium mass region, that this exotic effect may manifest itself in superheavy elements. Favorable conditions for producing triaxial-octupole correlations are shown to be present in the deformed single-particle spectrum, which is further supported by quantitative Reflection Asymmetric Shell Model calculations. It is predicted that the strong nonaxial-octupole effect may persist up to the element 108. Our result thus represents the first concrete example of spontaneous breaking of both axial and reflection symmetries in the heaviest nuclear systems.

Non-axial octupole shape in $N=150$ isotones



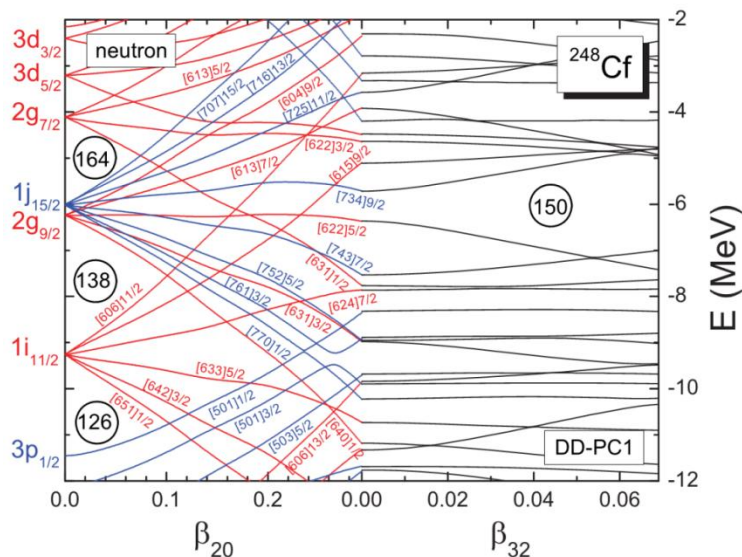
Non-axial octupole shape in $N = 150$ isotones



□ Y_{32} correlations from near degeneracy of pair of orbitals with $\Delta l = \Delta j = 3$ & $\Delta K = 2$

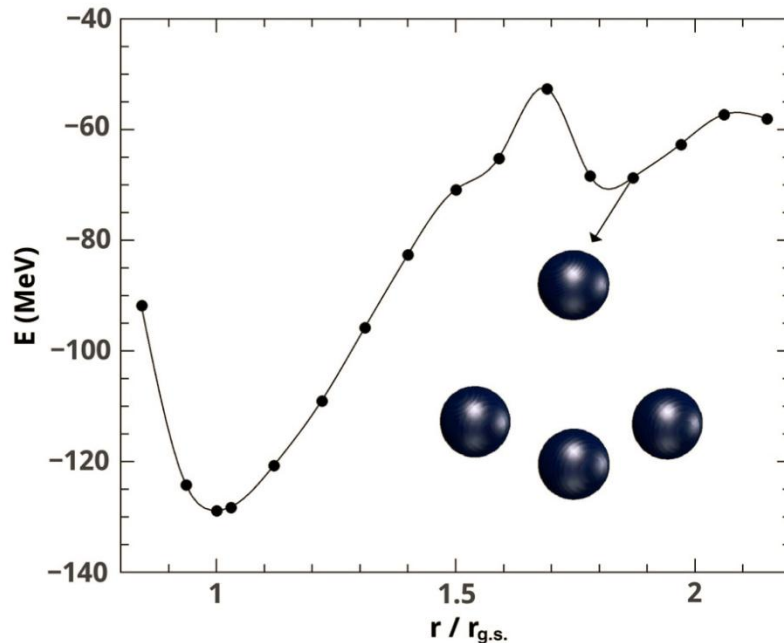
□ For ^{248}Cf

- $\pi 7/2[633] (1i_{13/2})$ & $\pi 3/2[521] (2f_{7/2})$
- $\nu 9/2[734] (1j_{15/2})$ & $\nu 5/2[622] (2g_{9/2})$



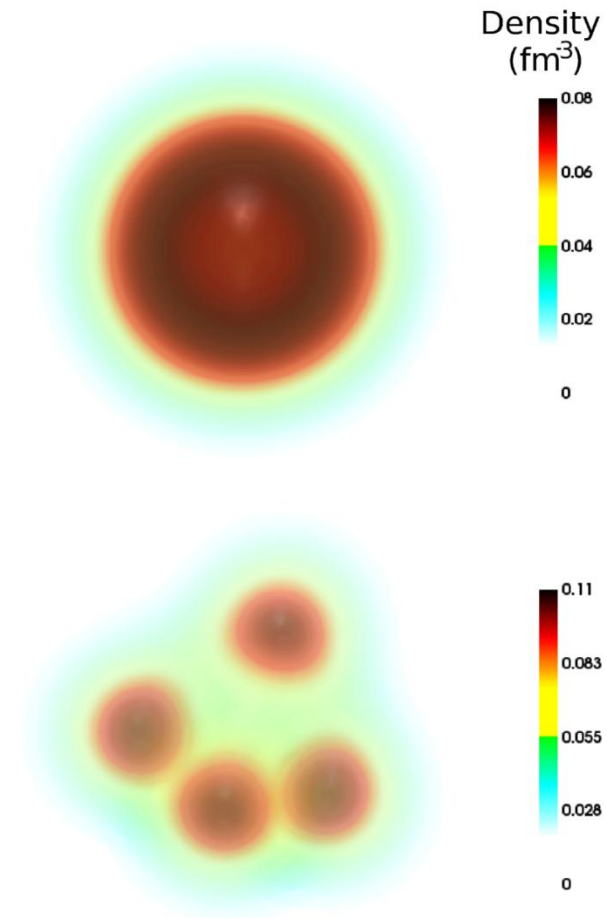
Tetrahedral cluster structure in ^{16}O under radius constraint (expansion)

Hartree-Fock model



Girod_Schuck2013_PRL111-132503

Relativistic mean field model



Ebran_Khan_Niksic_Vretenar
2014_PRC89-031303R

2013 KITPC Program



RELATED LINKS

[Introduction](#)

[Pictures](#)

[Schedule&Talks](#)

[Participants](#)

[Wiki-space](#)

[Associate Conferences/](#)

Clustering Aspects in Nuclei

Date : From 2013-04-01 To 2013-04-26

Advisory committee :

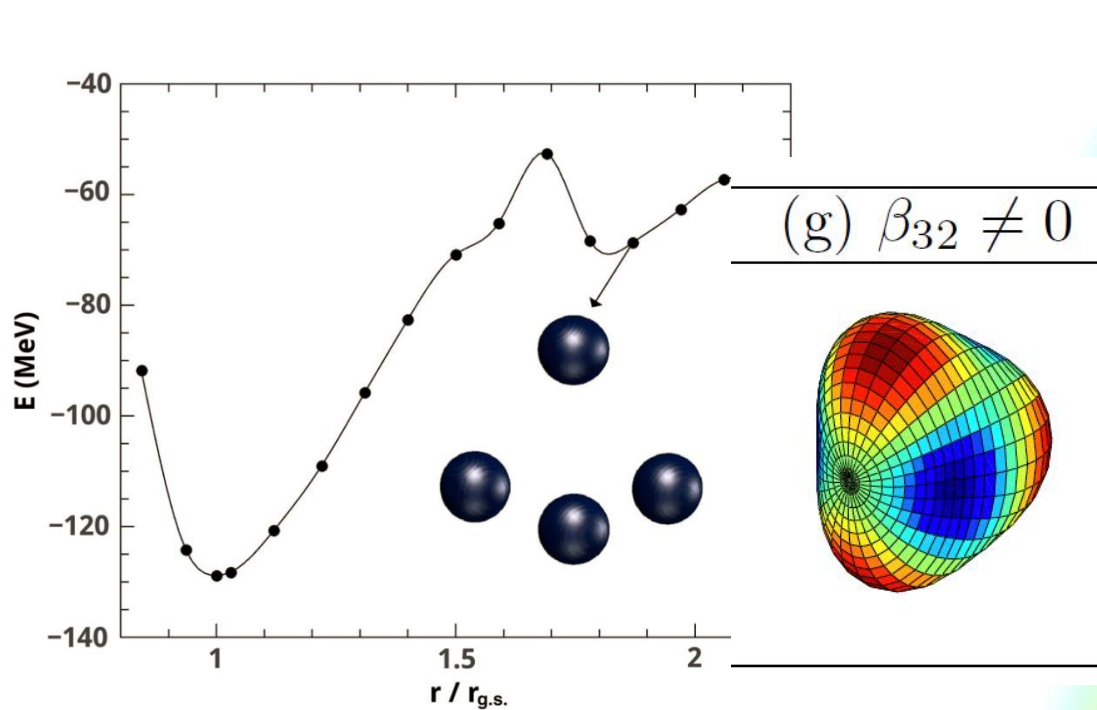
Local coordinators : Zhongzhou Ren (Chair, Nanjing University), Chang Xu (Nanjing University), FuRong Xu (Peking University), Zaiguo Gan (Institute of Modern Physics), Shan-Gui Zhou(KITPC/ITP-CAS)

International coordinators : Zhongzhou Ren (Chair, Nanjing University), Yanlin Ye (Peking University), W. Mittig (Michigan State University), P. Van Isacker (GANIL), G. Audi (Université de Paris Sud)

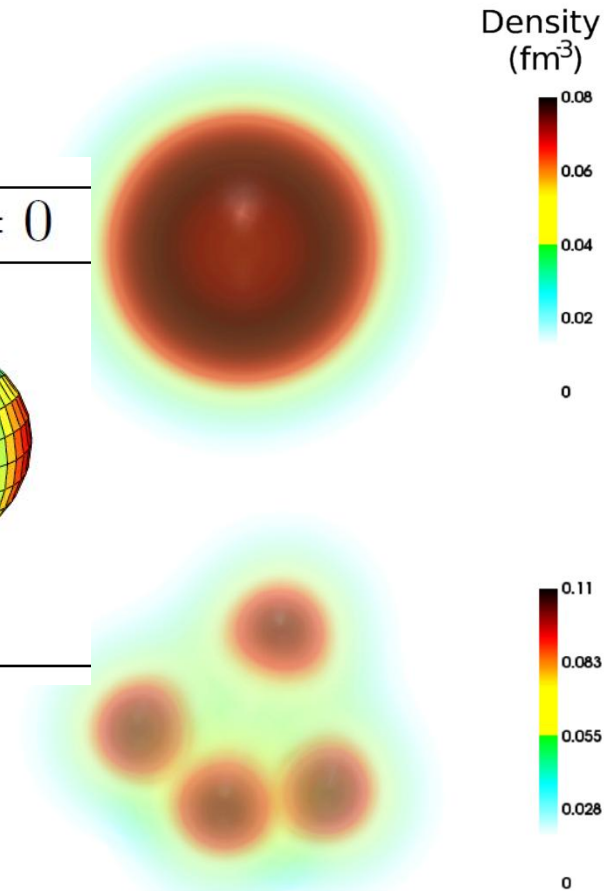
Tetrahedral cluster structure in ^{16}O under tetrahedral constraint ?

Hartree-Fock model

Relativistic mean field model

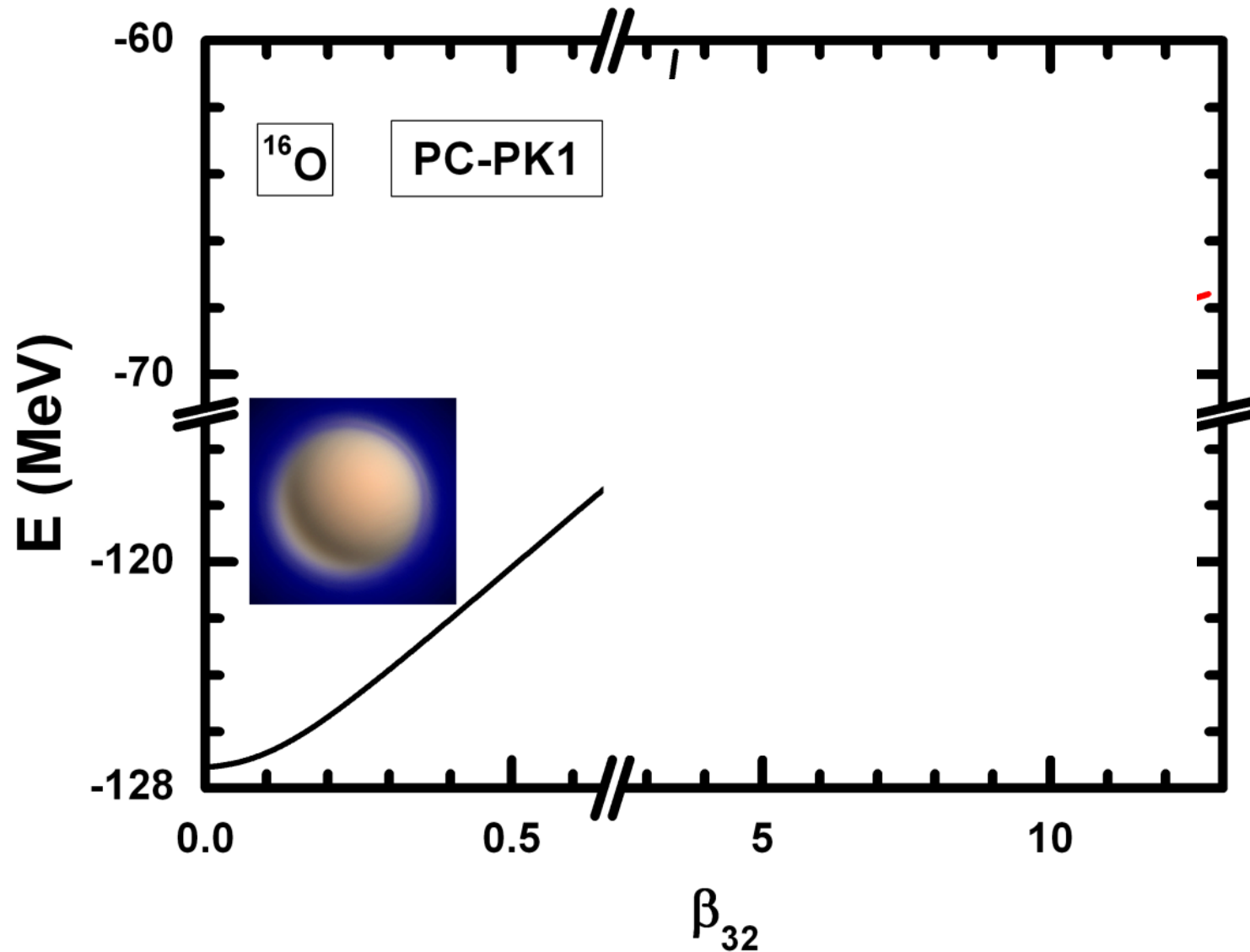


Girod_Schuck2013_PRL111-132503

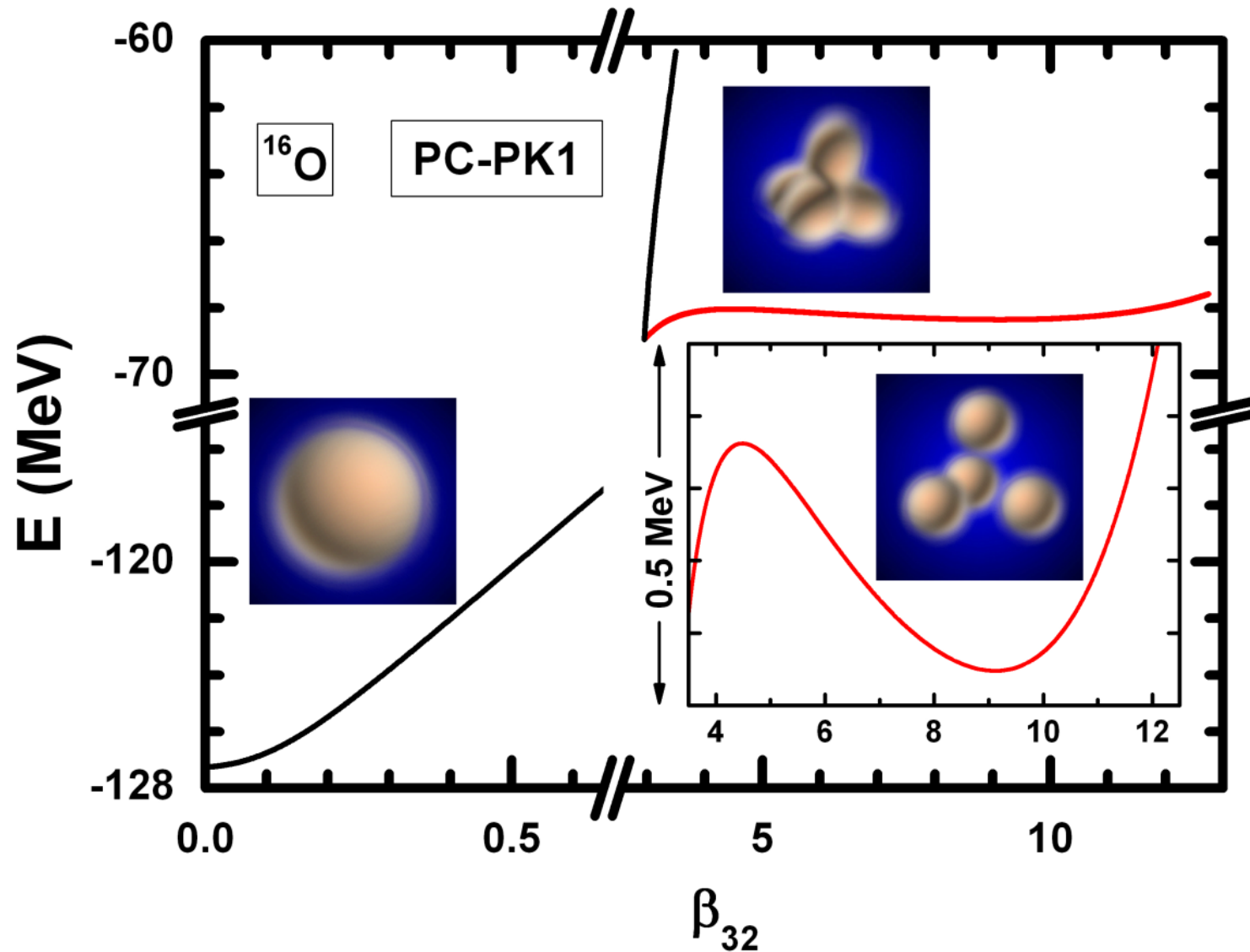


Ebran_Khan_Niksic_Vretenar
2014_PRC89-031303R

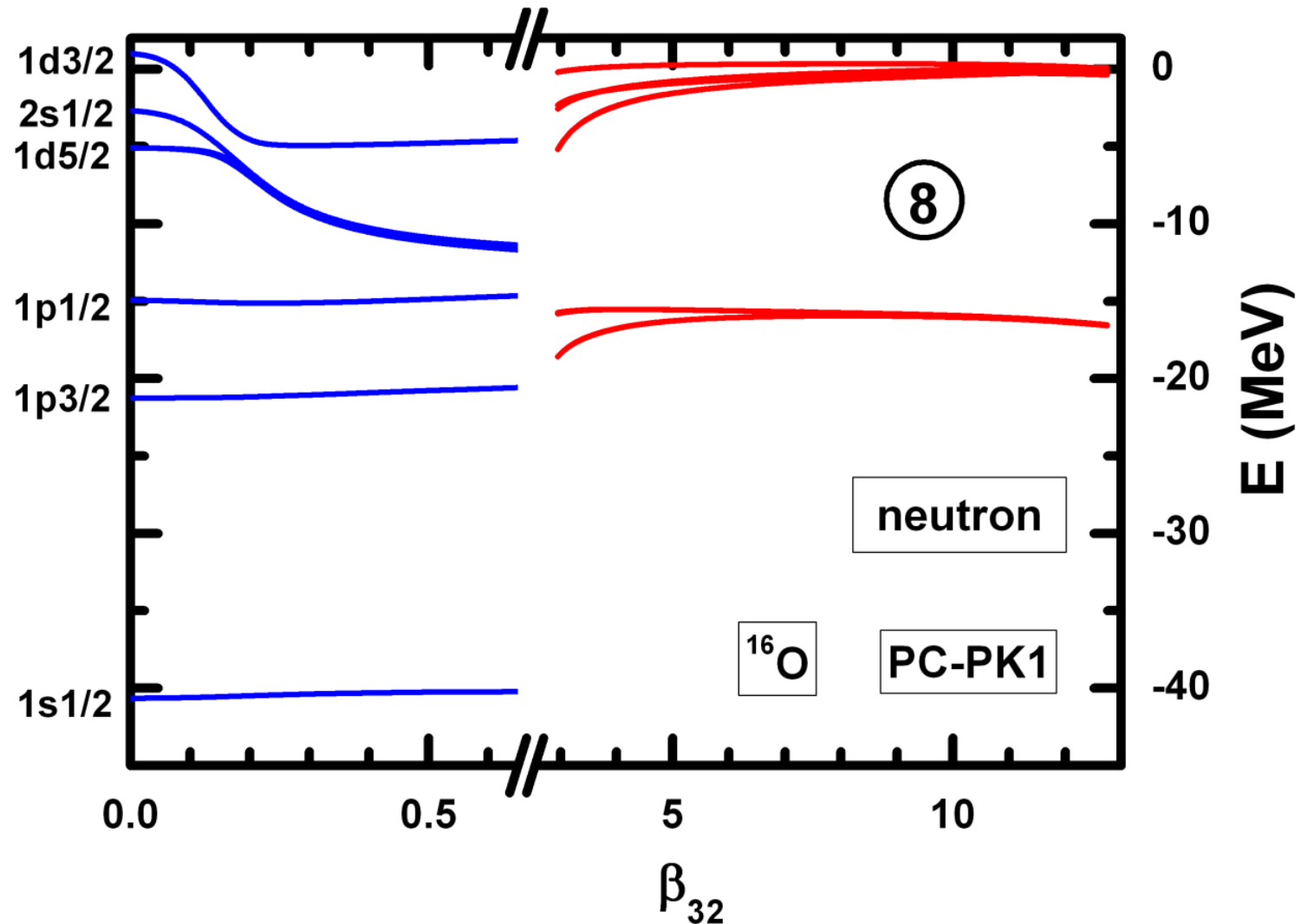
^{16}O under tetrahedral constraint



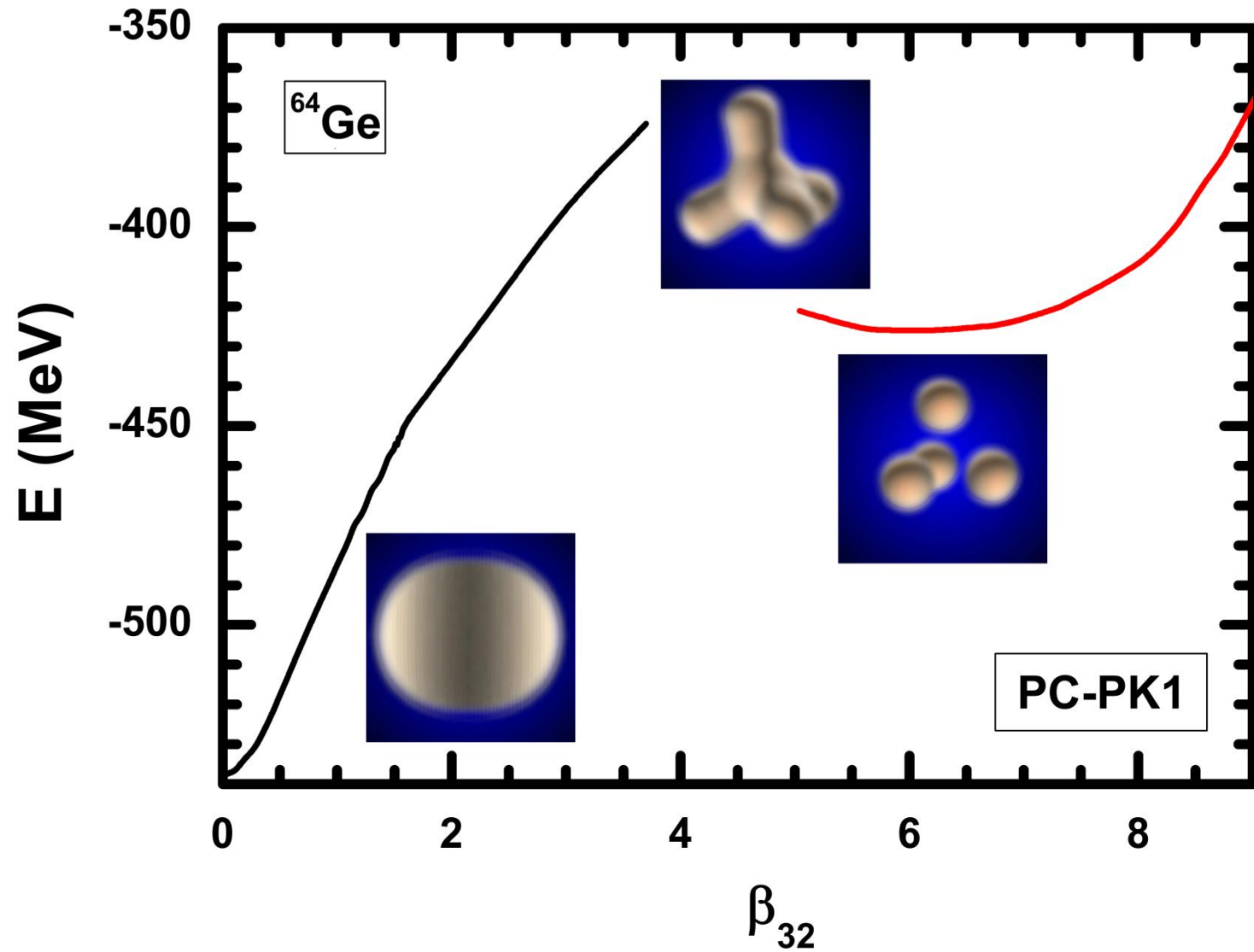
^{16}O under tetrahedral constraint



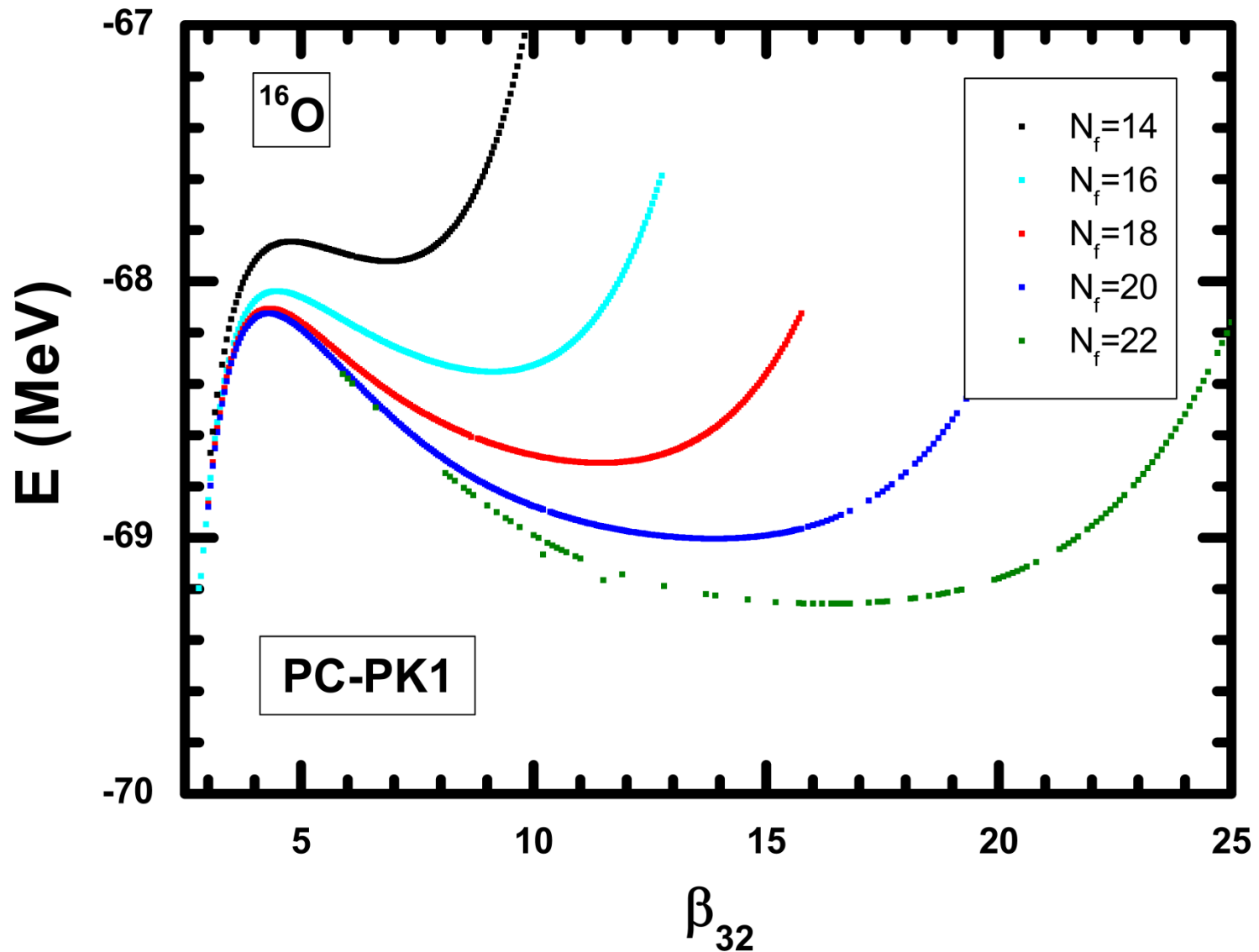
^{16}O under tetrahedral constraint



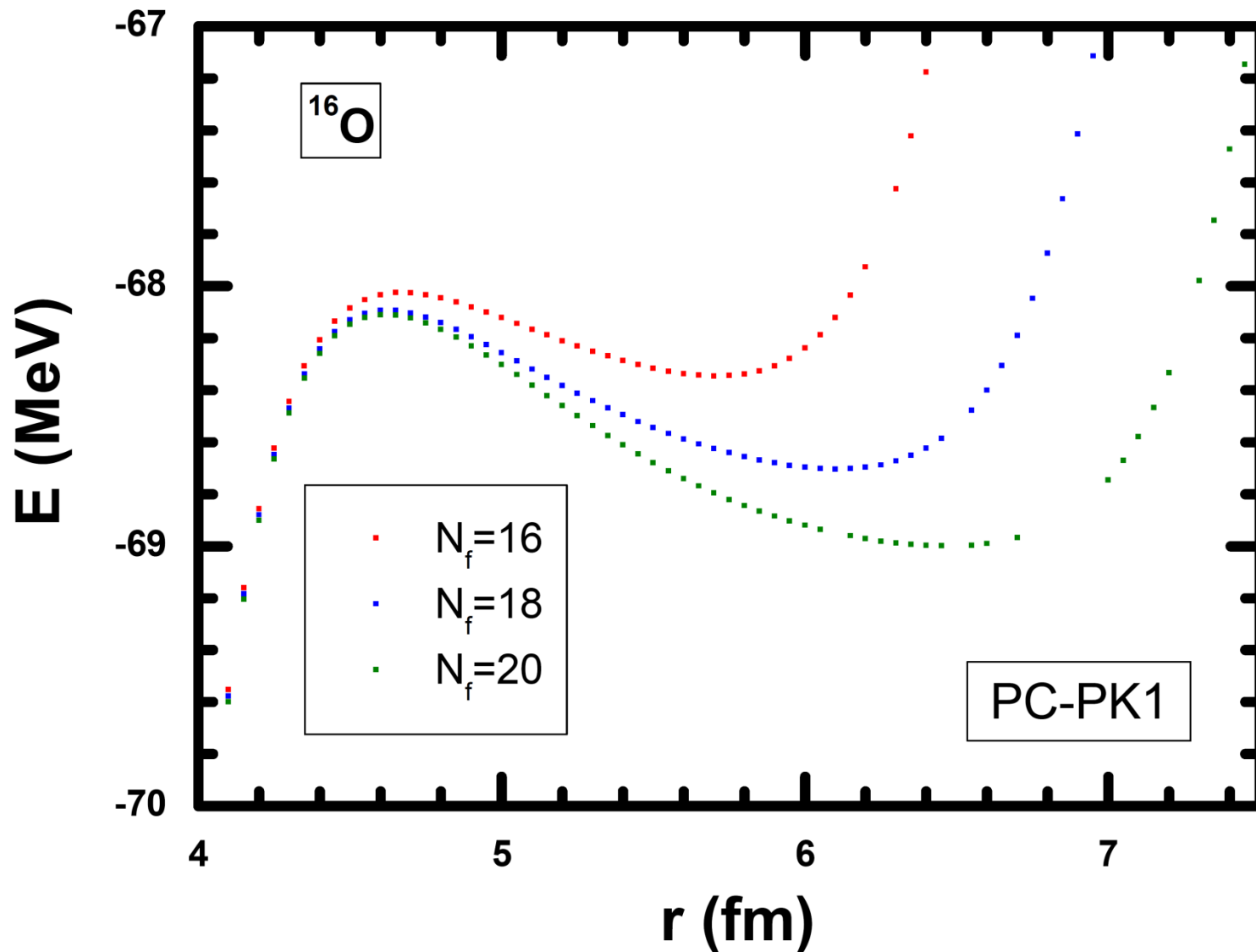
^{64}Ge under tetrahedral constraint



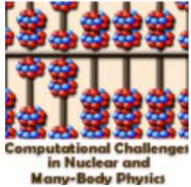
Dependence on basis size



Dependence on basis size



2014 NORDITA Program



Computational Challenges in Nuclear and Many-Body Physics

from 15 September 2014 to 10 October 2014

Nordita, Stockholm



Main Page

➤ List of registrants

♥ Call for Abstracts

- ♦ Submit a new abstract
- ♦ View my abstracts

➤ Daily Program

➤ Author index

➤ Contribution List

♥ Application

- ♦ Application Form

➤ Speaker index

➤ Scientific Programme

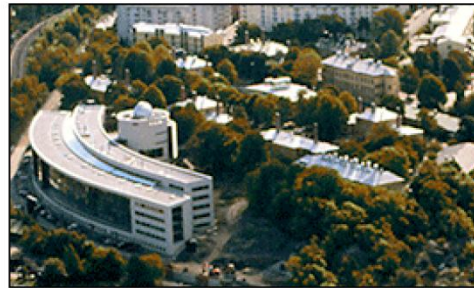
➤ Book of abstracts [PDF]

♥ Practical Information

- ♦ What is Nordita?
- ♦ Directions to Nordita
- ♦ Directions to BizApartment Hotel
- ♦ Accommodation

Home

Venue



Nordita, Stockholm, Sweden

Scope

Advanced theoretical methods play a central role in answering the key questions of many-body physics. We intend in this Program to discuss and compare such methods as are being applied at present in nuclear physics, condensed matter, cold atoms and quantum chemistry. The problems faced in these fields at present are focused in the development of new methods and in the improving of existing techniques to achieve an understanding of existing experimental data and in predicting with high reliability new properties and processes. All this requires computation techniques and methods which at present seems to be dispersed in the various fields that we intend to discuss in this workshop. We propose this workshop as a mean to bring together all these related communities with the goal of creating an enriching dialog across the disciplines.

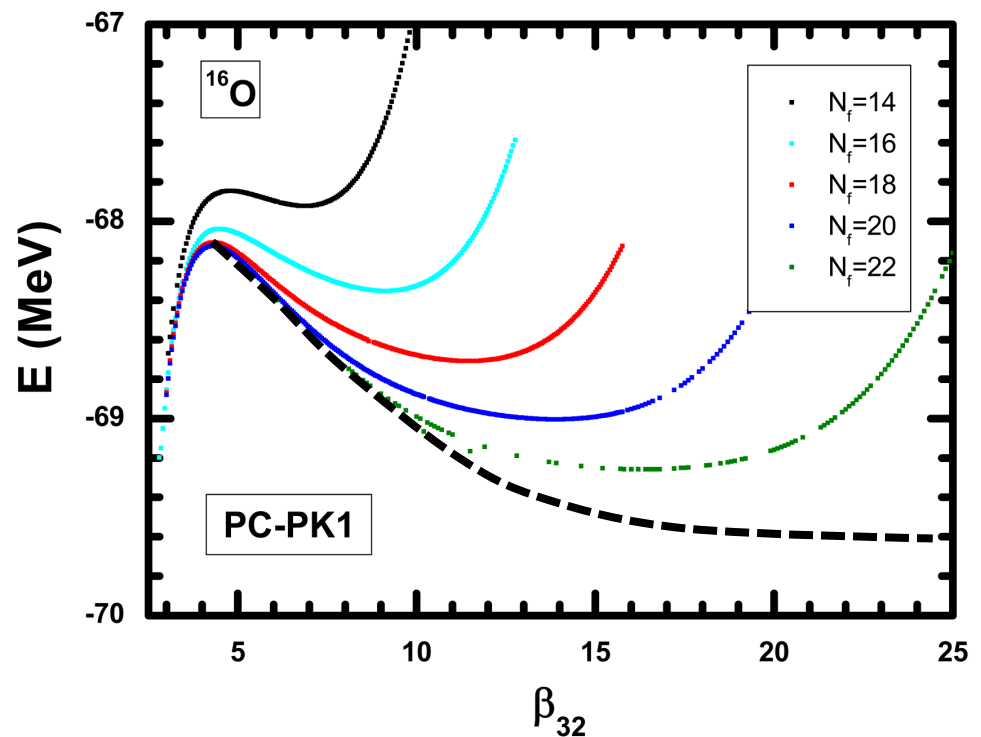
[Timetable - available from start of the program]

Several remarks

- ❑ The minimum outside the barrier is due to the basis truncation !

Several remarks

- ❑ The minimum outside the barrier is due to the basis truncation !
- ❑ What can we learn from such a minimum?
 - **Constraint cluster structure**
 - Clustering at low density



What else ?

Hyperdeformed shapes in actinides

Moller1972_NPA192-529

...

Blons_Mazur_Paya1975_PRL35-1749

...

Cwiok_Nazarewicz_Saladin_Plociennik_Johnson1994_PLB322-304

...

Csige et al. 2013_PRC87-044321

...

Kowal_Skalski2012_PRC85-061302R

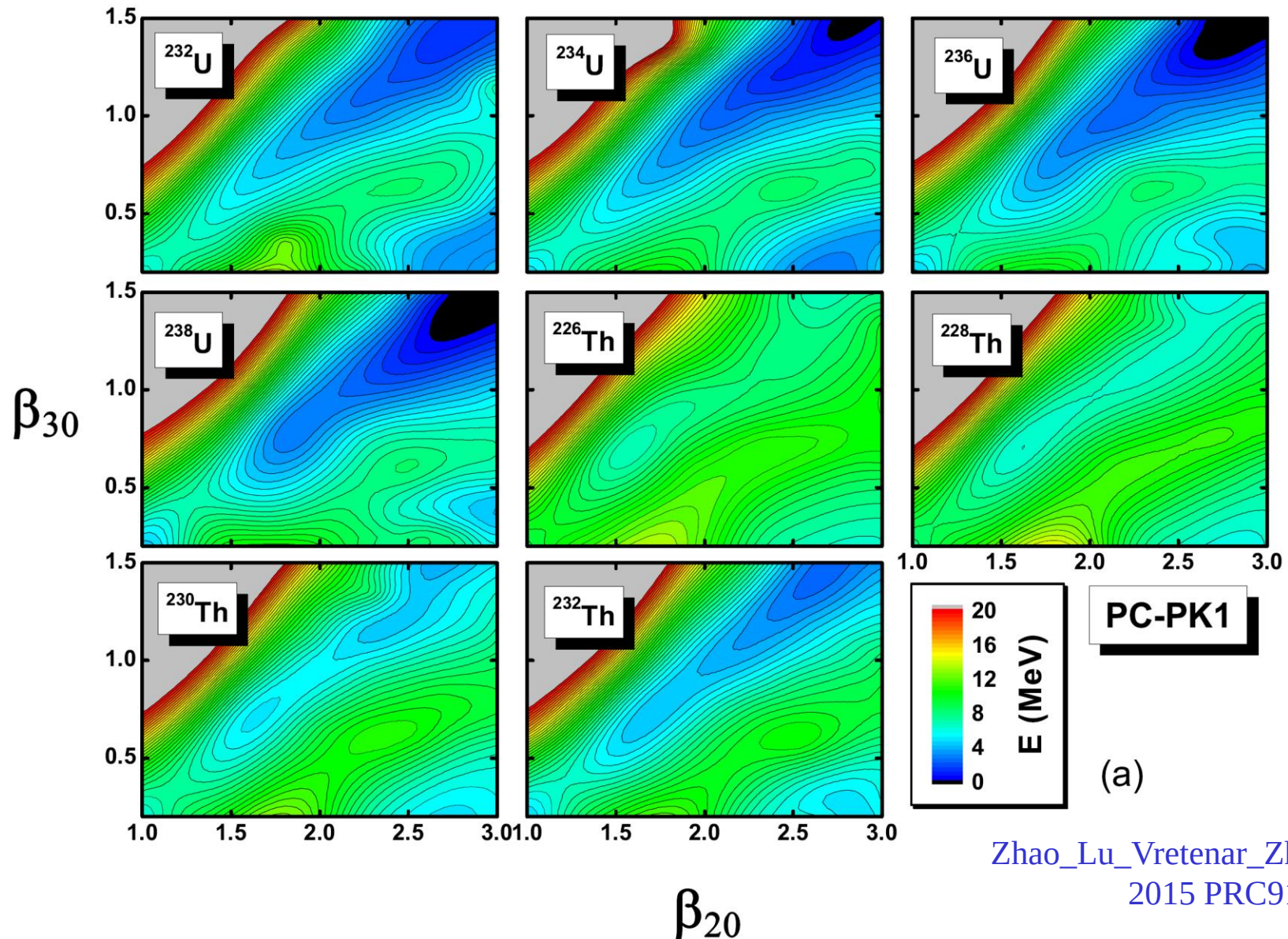
Jochimowicz_Kowal_Skalski2013_PRC87-044308

Ichikawa_Moller_Sierk2013_PRC87-054326

McDonnell_Nazarewicz_Sheikh2013_PRC87-054327

...

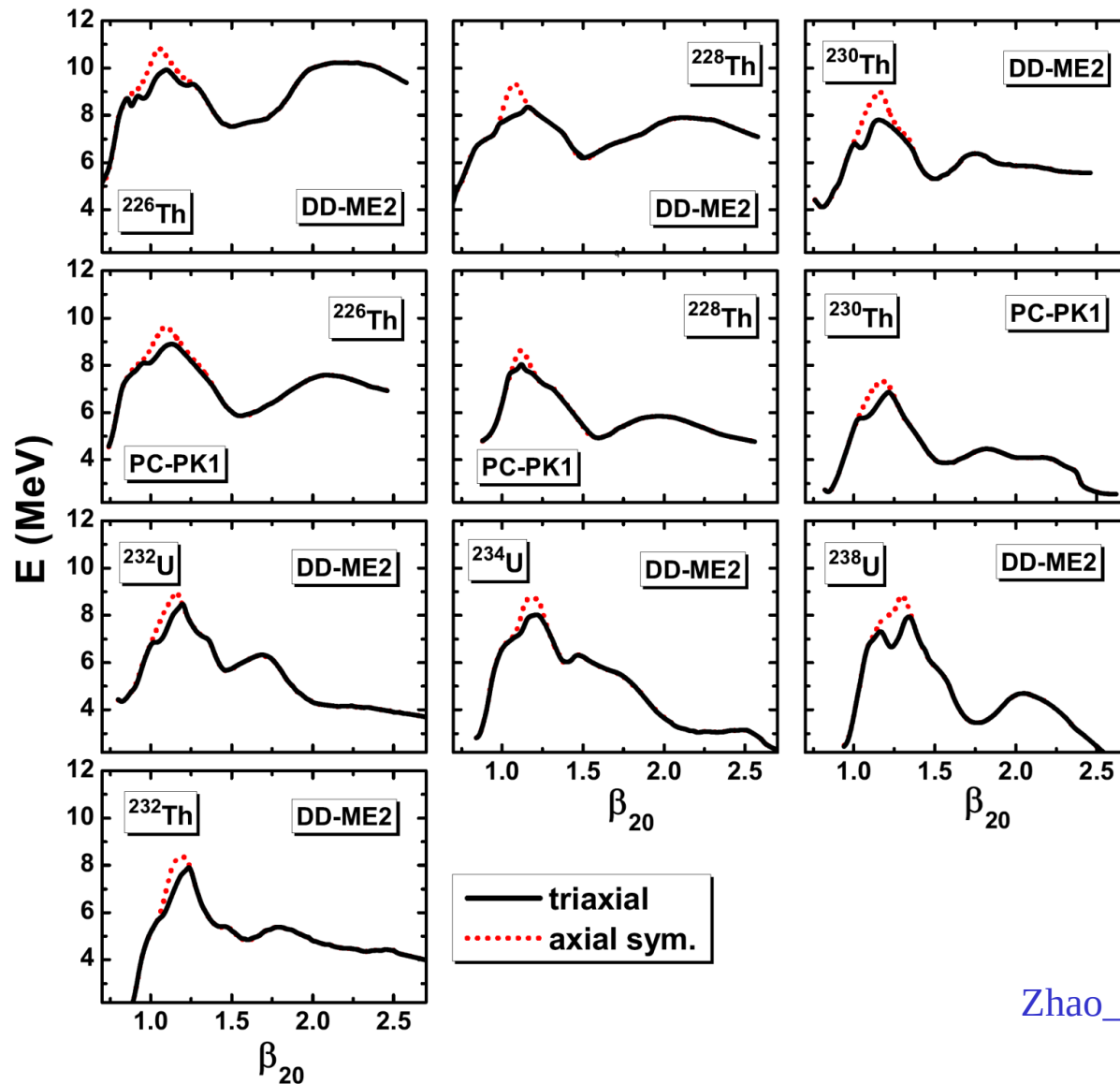
Hyperdeformed shapes in actinides



β_{30} 

Zhao_Lu_Vretenar_Zhao_SGZ
2015 PRC91-014321

Hyperdeformed shapes in actinides



Shapes of Λ hypernuclei w/ MF models

❑ Non-relativistic mean field study of hypernuclei

- An axially deformed Skyrme Hartree-Fock (SHF) model
- Similar shapes of core nuclei & the corresponding hypernuclei

Zhou_Schulze_Sagawa_Wu_Zhao2007_PRC76-034312

❑ However, a RMF study reveals

- In most cases the results are similar to the SHF calculations
- Several exceptions, e.g., $^{13}_{\Lambda}\text{C}$ & $^{29}_{\Lambda}\text{Si}$ whose shapes change dramatically compared to their corresponding core nuclei

Win_Hagino2008_PRC78-054311

❑ Different polarization effect of Λ in SHF & RMF

Schulze_Win_Hagino_Sagawa2010_PTP123-569

Triaxiality in hypernuclei w/ RMF model

	Skyrme HF	RMF
Spherical	Yes	Yes
Axially deformed	Yes	Yes
Triaxially deformed	Yes	???

Triaxially deformed RMF: With an additional Λ , no significant shape change occurs except that the PES becomes softer in the γ direction

Win_Hagino_Koike2011_PRC83-014301

What RMF models predict for triaxiality in hypernuclei?

RMF model for Λ hypernuclei

□ The Lagrangian density: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Lambda$

$$\mathcal{L}_\Lambda = \bar{\psi}_\Lambda (i\gamma^\mu \partial_\mu - m_\Lambda - g_{\sigma\Lambda} \sigma - g_{\omega\Lambda} \gamma^\mu \omega_\mu) \psi_\Lambda \\ + \frac{f_{\omega\Lambda\Lambda}}{4m_\Lambda} \bar{\psi}_\Lambda \sigma^{\mu\nu} \Omega_{\mu\nu} \psi_\Lambda,$$

□ The Dirac equation for Λ

$$[\vec{\alpha} \cdot \vec{p} + \beta (m_\Lambda + S_\Lambda) + V_\Lambda + T_\Lambda] \psi_{\Lambda i} = \epsilon_i \psi_{\Lambda i}$$

$$T_\Lambda = -\frac{f_{\omega\Lambda\Lambda}}{2m_\Lambda} \beta (\vec{\alpha} \cdot \vec{p}) \omega$$

□ Effective interactions

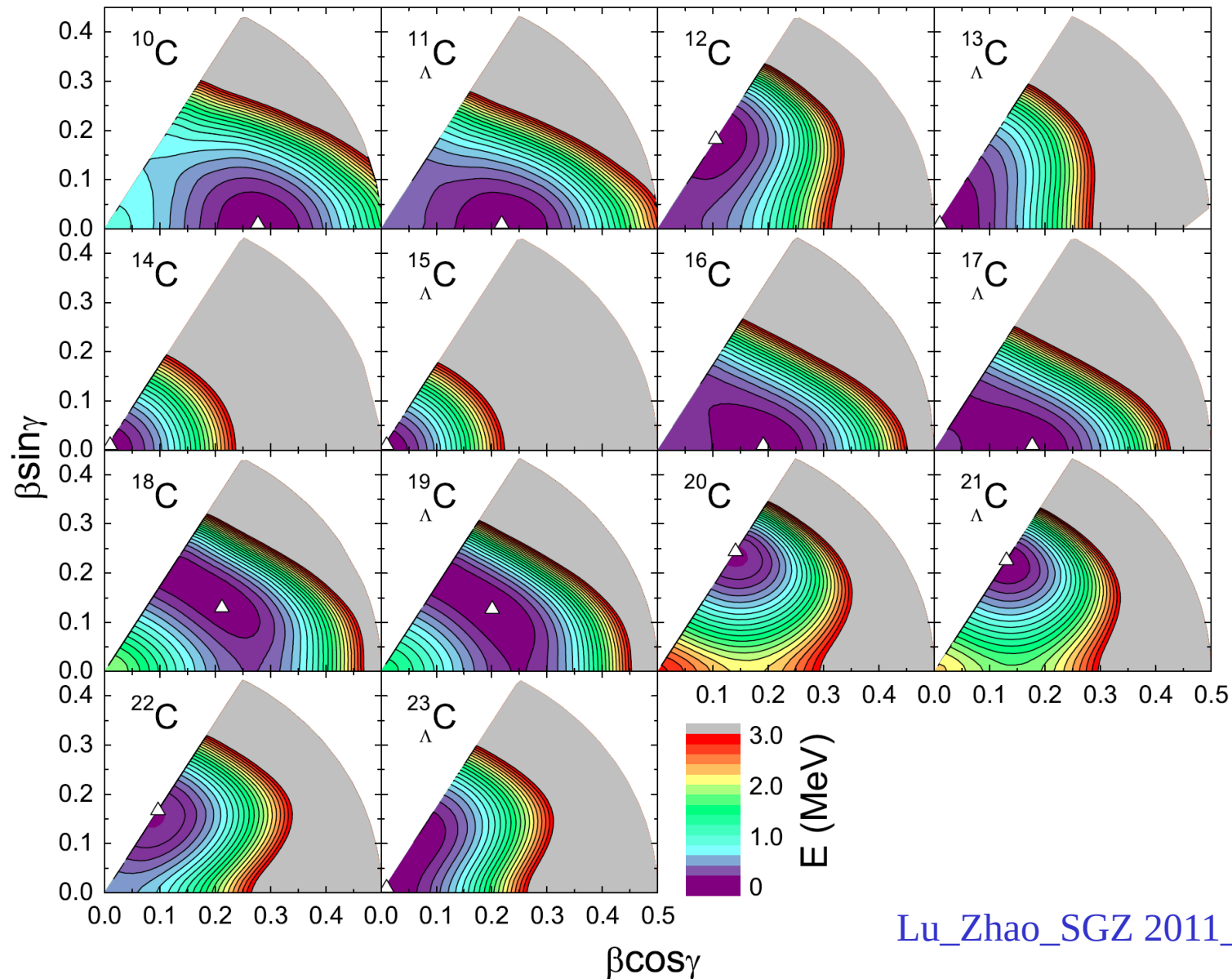
Parameter	m_Λ	$g_{\sigma\Lambda}$	$g_{\omega\Lambda}$	$f_{\omega\Lambda\Lambda}$	N-N interaction
PK1-Y1	1115.6 MeV	$0.580g_\sigma$	$0.620g_\omega$	$-g_{\omega\Lambda}$	PK1
NLSH-A	1115.6 MeV	$0.621g_\sigma$	$0.667g_\omega$	$-g_{\omega\Lambda}$	NLSH

PK1-Y1: Song_Yao_L ü_Meng2010_IJMPE19-2538

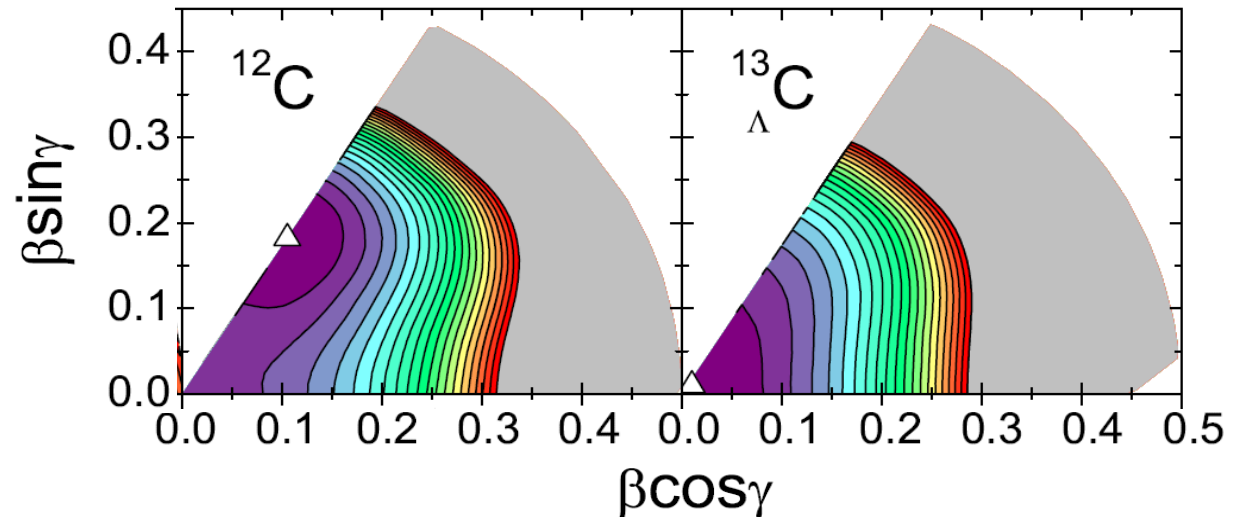
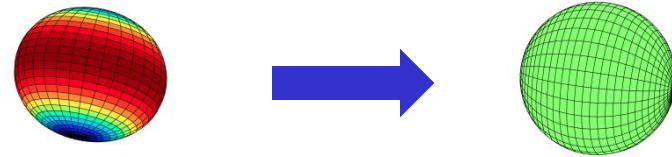
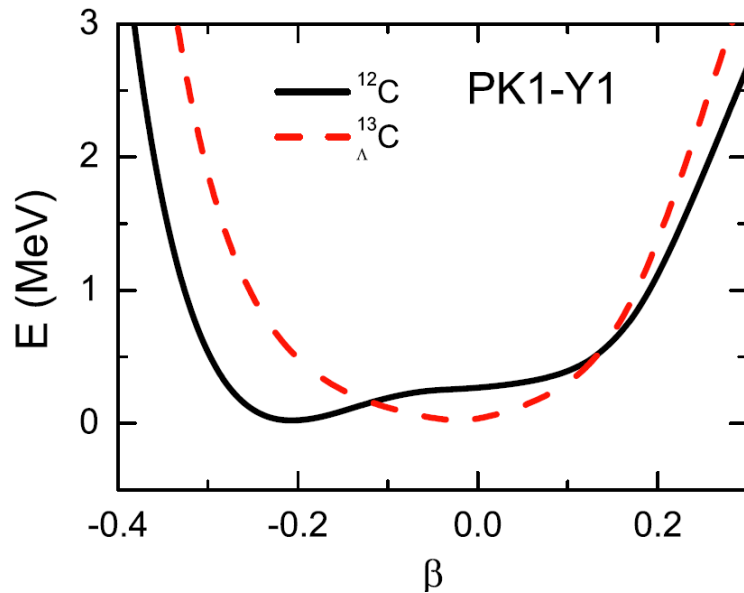
Wang_Sang_Wang_L ü2014_Commu.Theor.Phys.60-479

NLSH-A: Win_Hagino2008_PRC78-054311

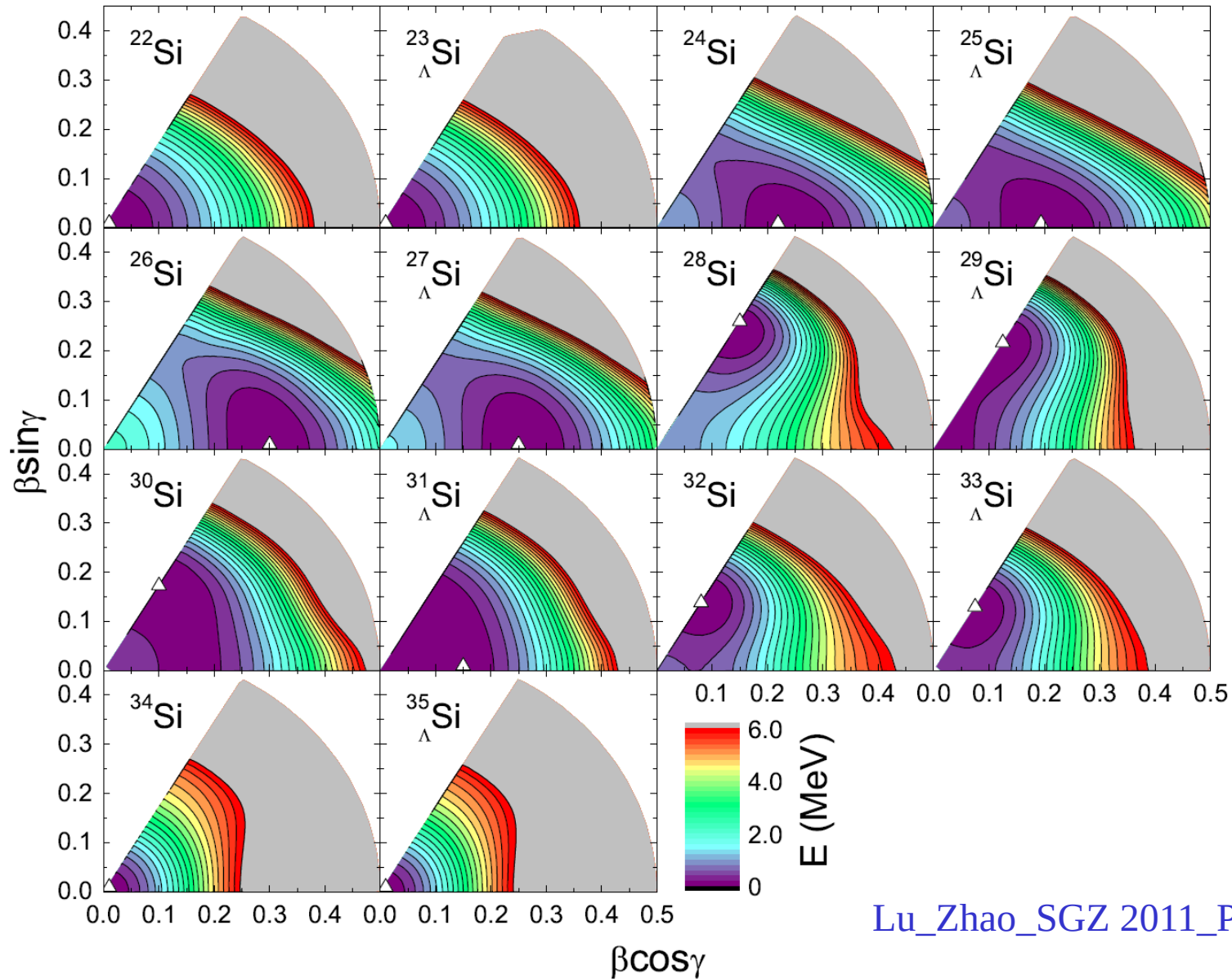
Carbon isotopes: w/ & w/o Λ



Potential energy surfaces of ^{12}C & $^{13}_{\Lambda}\text{C}$

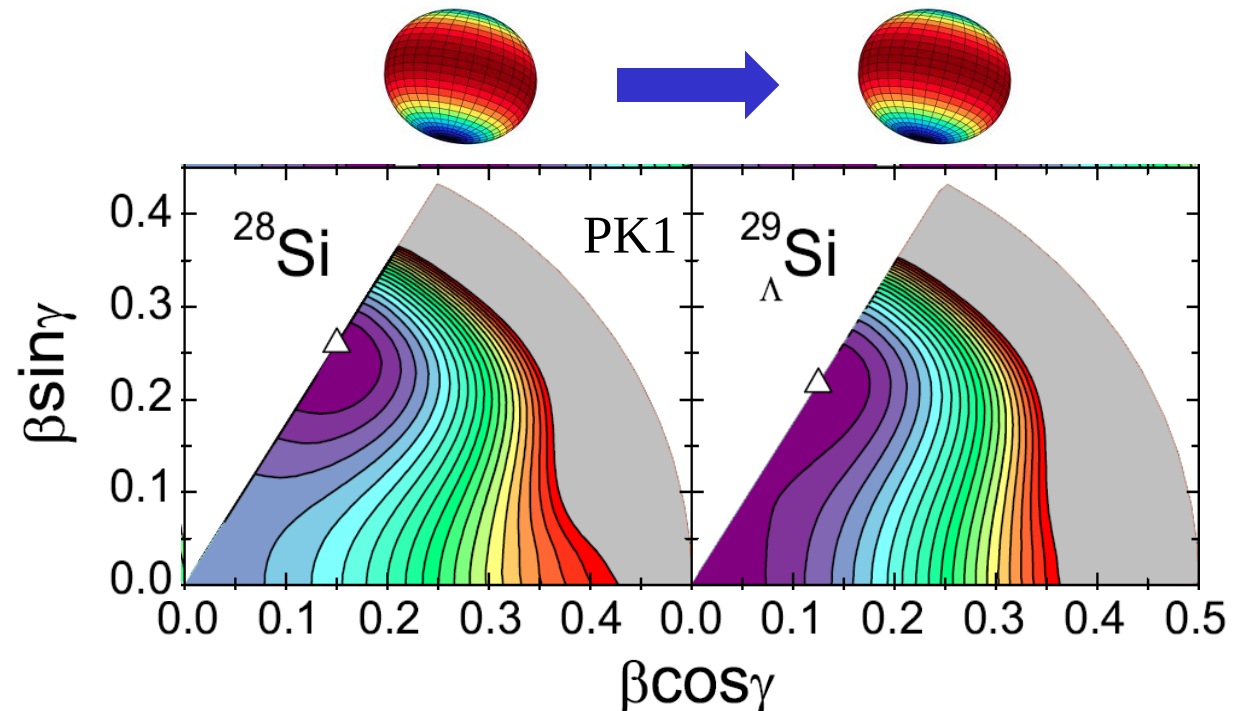


Silicon isotopes: w/ & w/o Λ

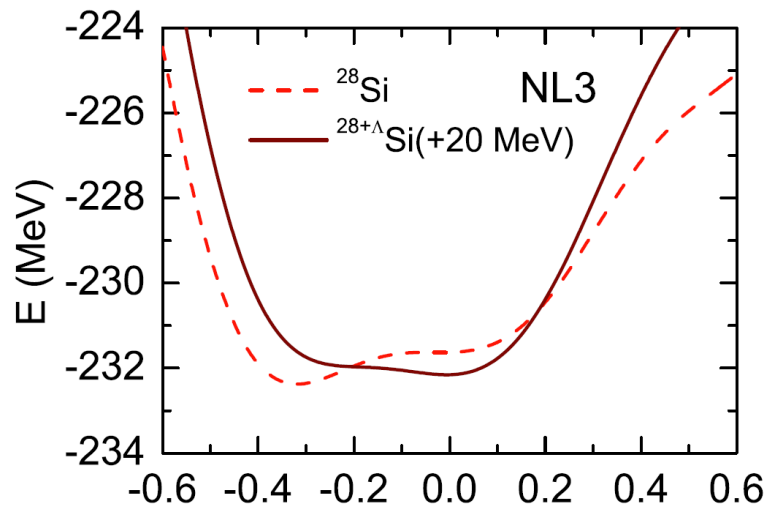


Potential energy surfaces of ^{28}Si & $^{29}_{\Lambda}\text{Si}$

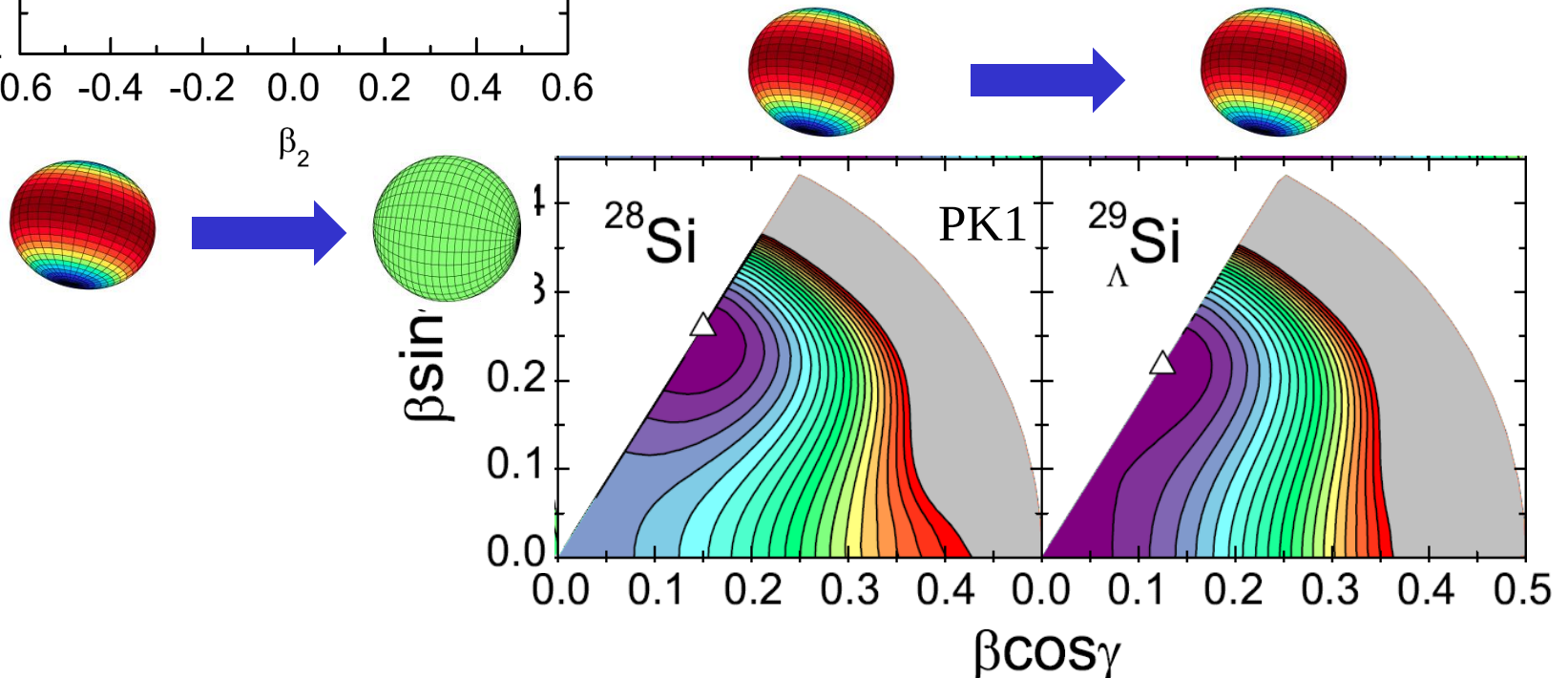
Model dependence
Parameter dependence



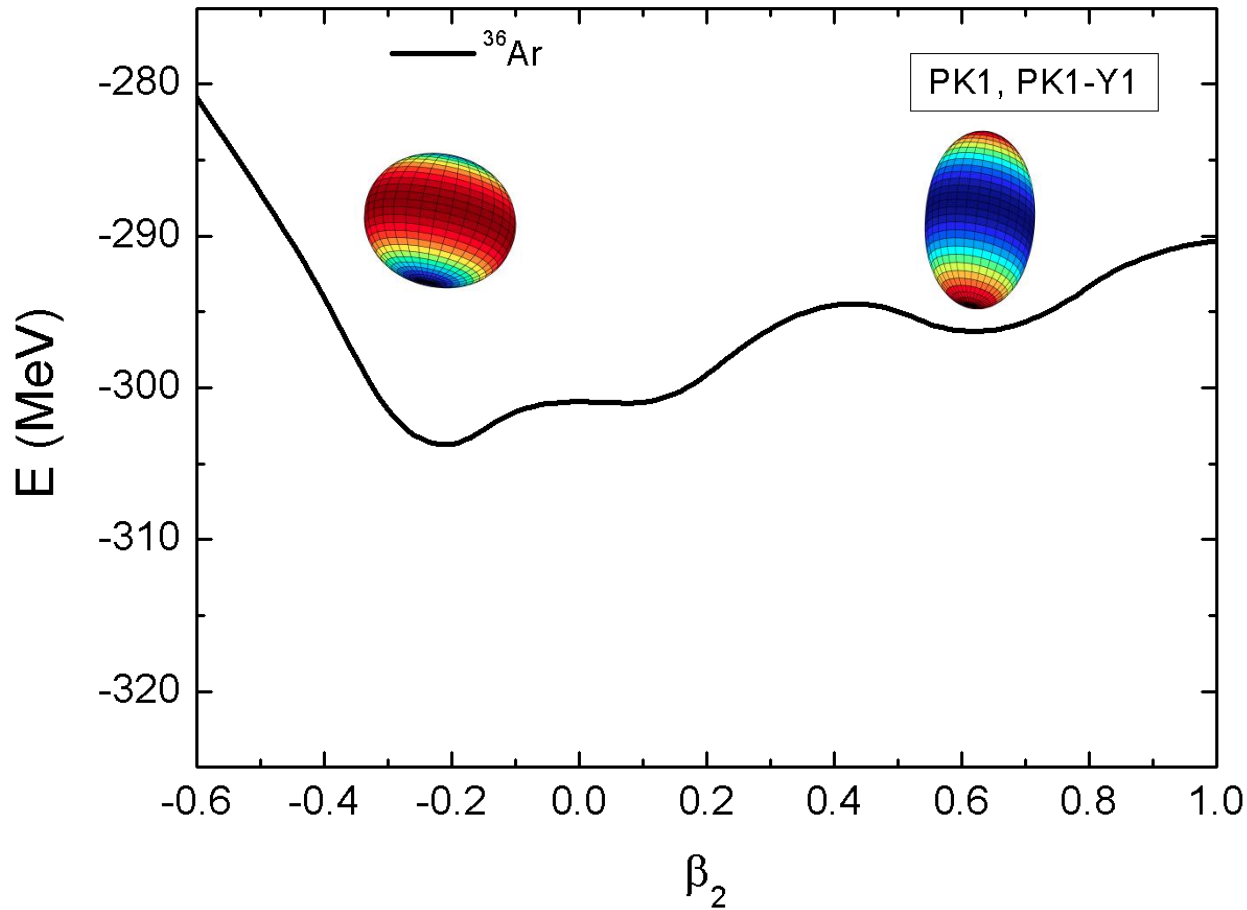
Potential energy surfaces of ^{28}Si & $^{29}_{\Lambda}\text{Si}$



Model dependence
Parameter dependence

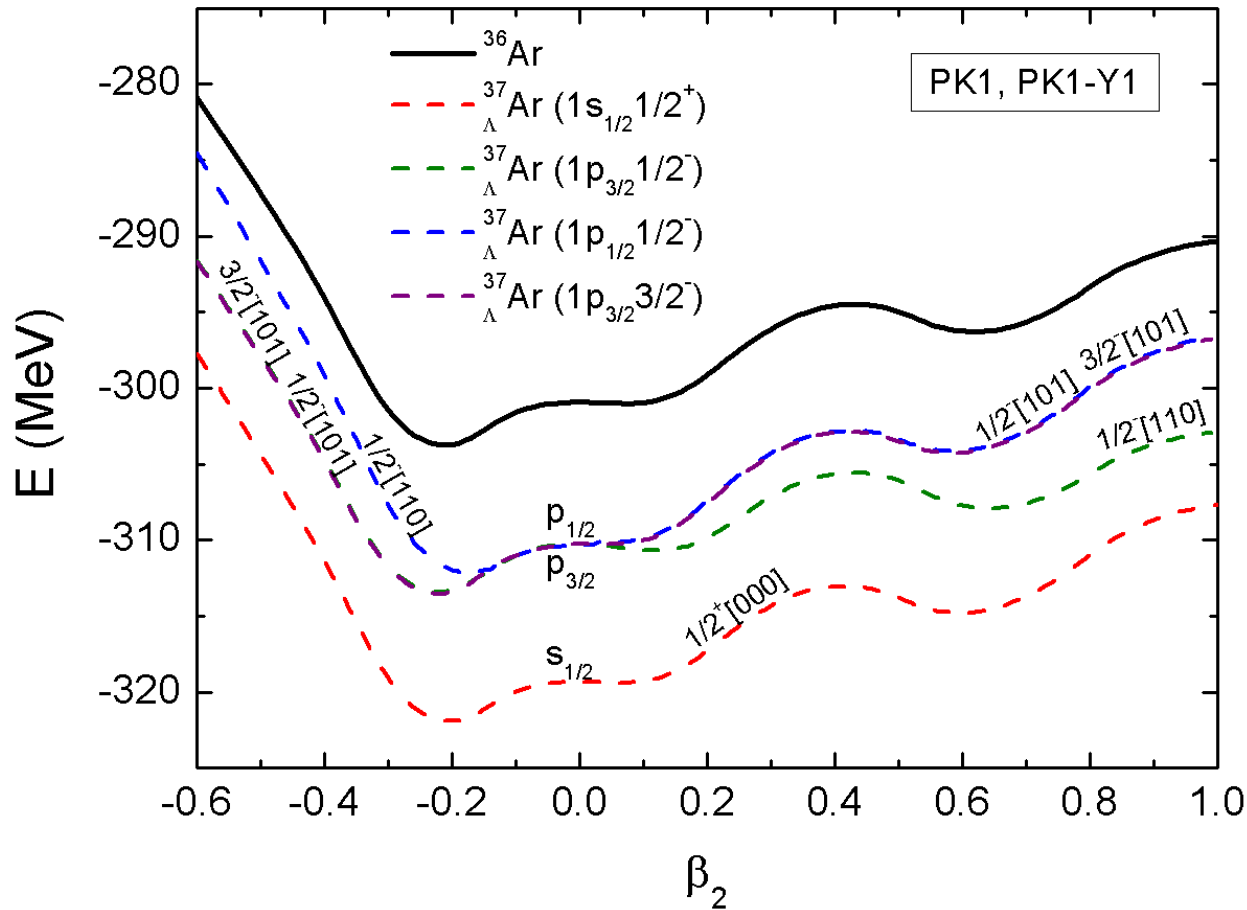


Superdeformed hypernuclei



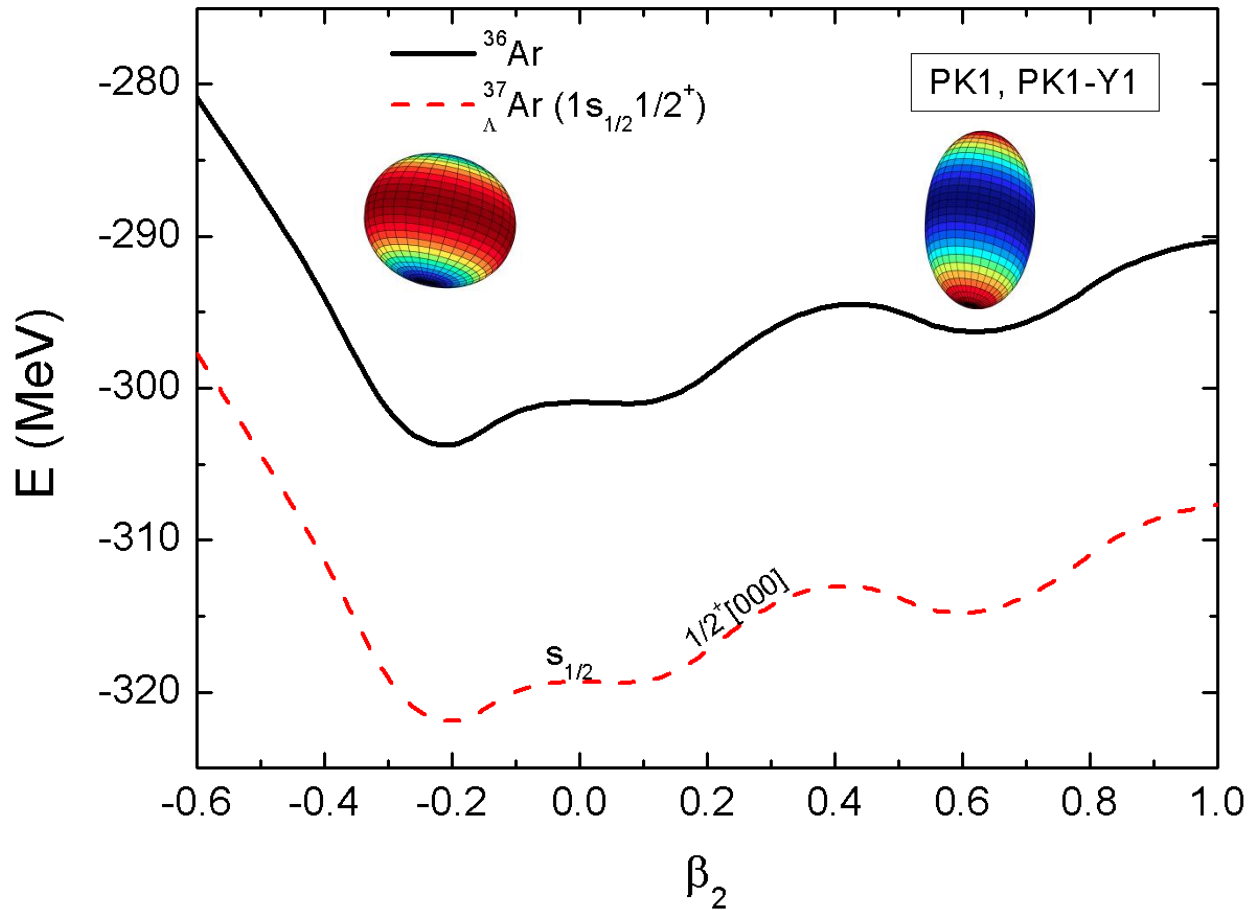
Lu_Hiyama_Sagawa_SGZ
2014_PRC89-044307

Superdeformed hypernuclei



Lu_Hiyama_Sagawa_SGZ
2014_PRC89-044307

Superdeformed hypernuclei



Lu_Hiyama_Sagawa_SGZ
2014_PRC89-044307

Superdeformed hypernuclei

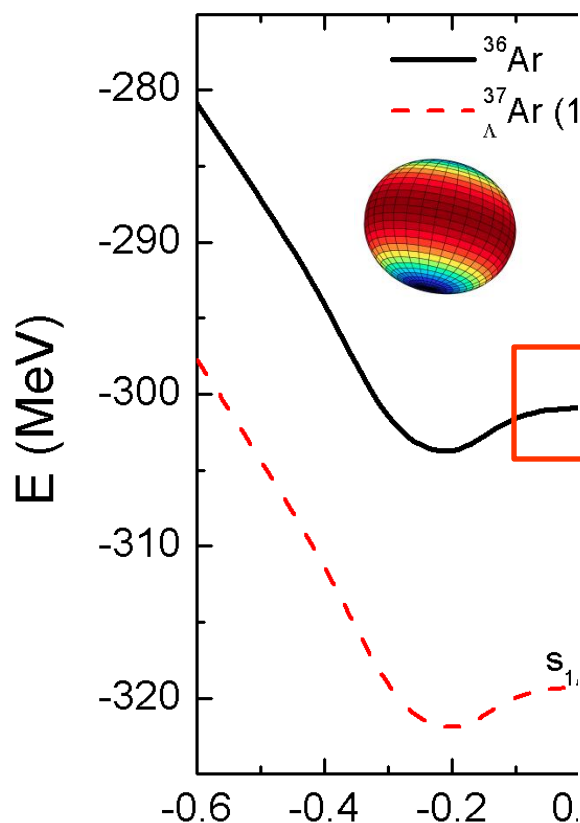
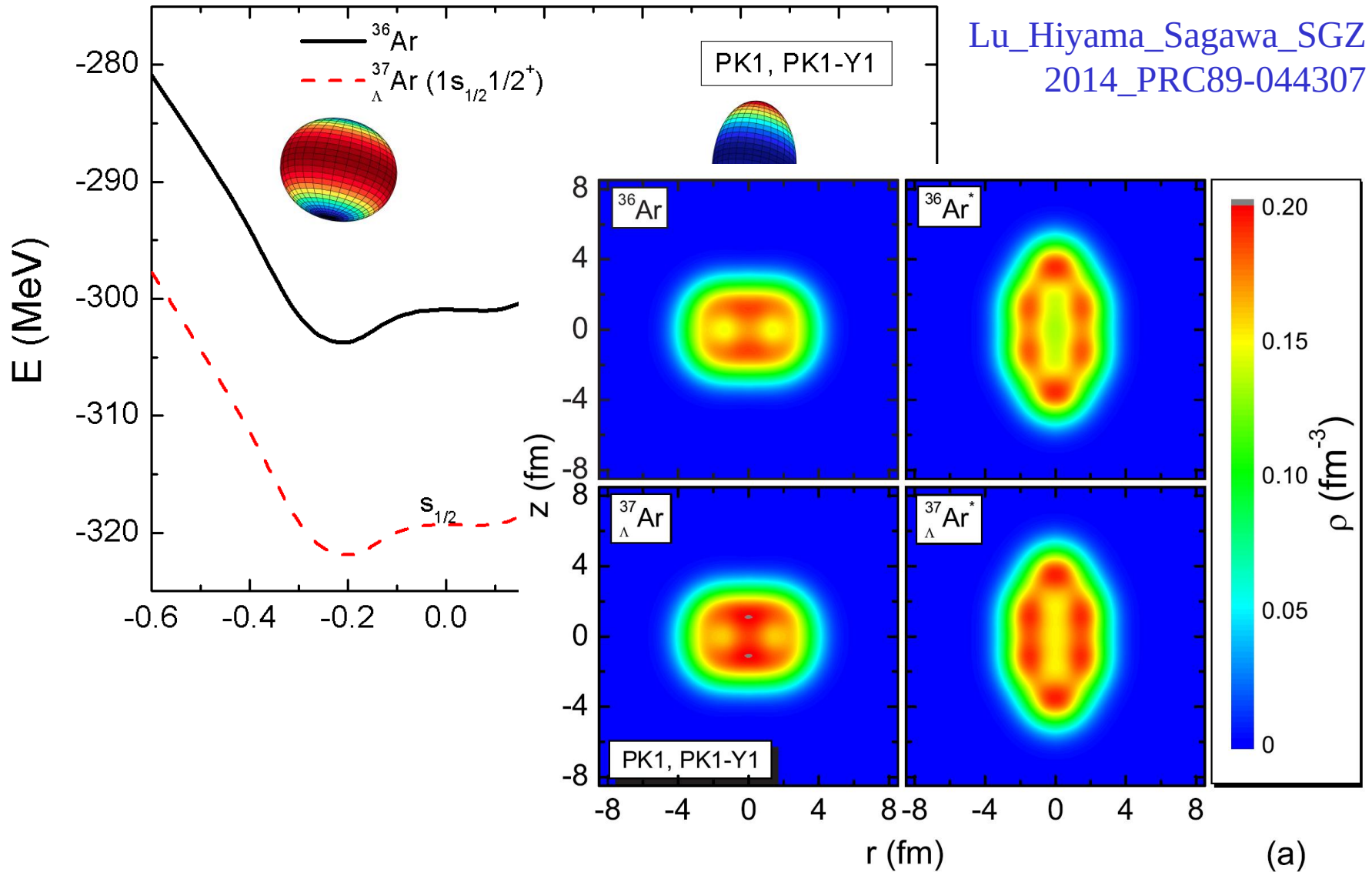


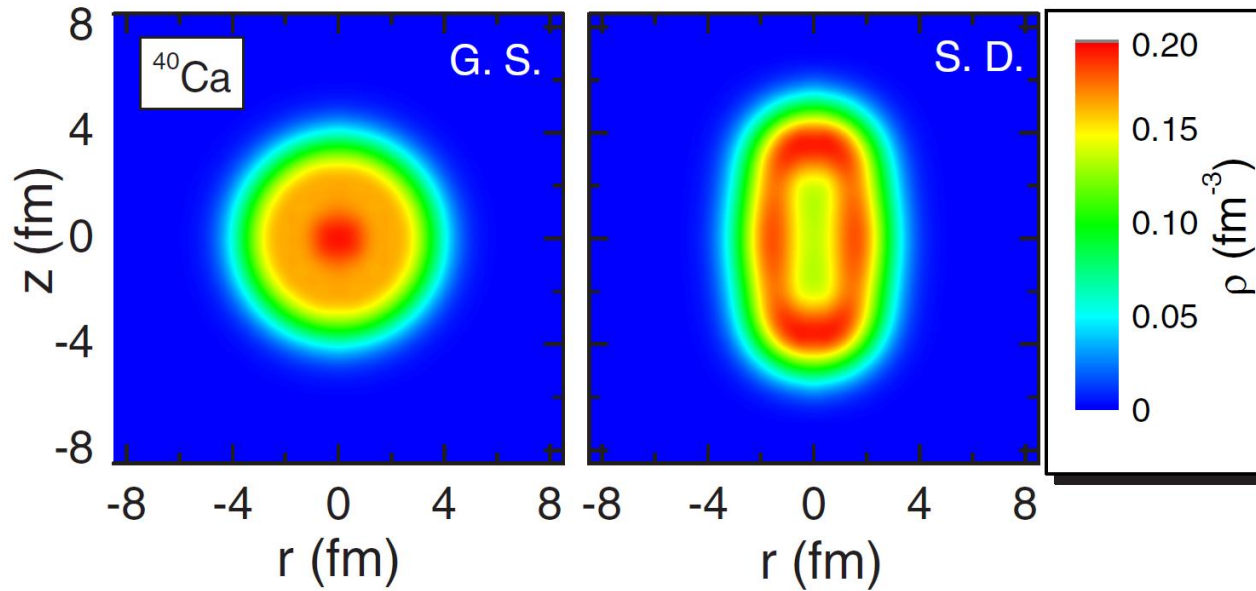
TABLE II. Calculated deformation parameter β_2 , the overlap I_{overlap} defined in Eq. (11), the binding energy E , and Λ separation energy S_{Λ} of some Λ hypernuclei. For comparison, the binding energy of the core nucleus E_{core} is also given. The energies are in MeV.

Nucleus	β_2	I_{overlap}	E_{tot}	E_{core}	S_{Λ}
$^{37}_{\Lambda}\text{Ar}$	-0.204	0.1352	-321.979	-303.802	18.177
$^{37}_{\Lambda}\text{Ar}^*$	0.597	0.1370	-315.194	-296.670	18.524
$^{39}_{\Lambda}\text{Ar}$	0.000	0.1360	-344.896	-326.455	18.441
$^{39}_{\Lambda}\text{Ar}^*$	0.589	0.1378	-336.306	-317.448	18.858
$^{41}_{\Lambda}\text{Ar}$	-0.117	0.1357	-361.398	-342.613	18.785
$^{41}_{\Lambda}\text{Ar}^*$	0.491	0.1378	-355.922	-336.852	19.070
$^{41}_{\Lambda}\text{Ca}$	0.00	0.1361	-361.422	-342.869	18.553
$^{41}_{\Lambda}\text{Ca}^*$	0.70	0.1393	-350.559	-331.317	19.242
$^{33}_{\Lambda}\text{S}$	0.26	0.1376	-285.095	-267.002	18.093
$^{33}_{\Lambda}\text{S}^*$	0.97	0.1243	-274.315	-257.951	16.364
$^{57}_{\Lambda}\text{Ni}$	0.00	0.1461	-506.665	-484.759	21.906
$^{57}_{\Lambda}\text{Ni}^*$	0.40	0.1415	-498.610	-477.892	20.718
$^{61}_{\Lambda}\text{Zn}$	0.22	0.1438	-534.565	-512.924	21.641
$^{61}_{\Lambda}\text{Zn}^*$	0.62	0.1415	-527.168	-506.238	20.930

Superdeformed hypernuclei



Superdeformed hypernuclei



Lu_Hiyama_Sagawa_SGZ
2014_PRC89-044307

Superdeformed hypernuclei

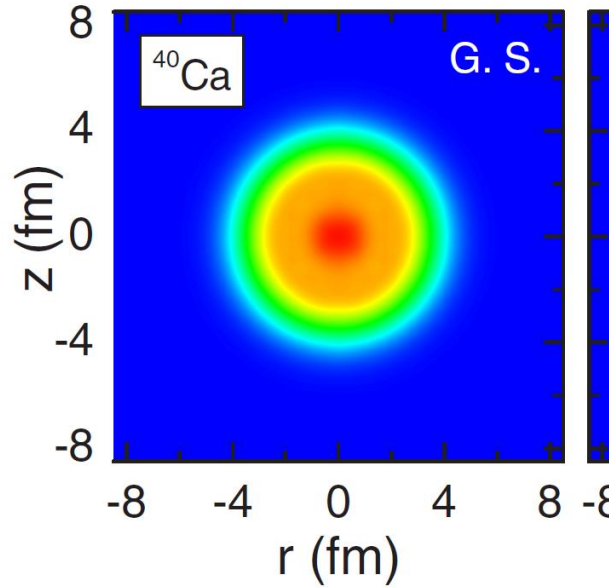


TABLE II. Calculated deformation parameter β_2 , the overlap I_{overlap} defined in Eq. (11), the binding energy E , and Λ separation energy S_Λ of some Λ hypernuclei. For comparison, the binding energy of the core nucleus E_{core} is also given. The energies are in MeV.

Nucleus	β_2	I_{overlap}	E_{tot}	E_{core}	S_Λ
$^{37}_{\Lambda}\text{Ar}$	-0.204	0.1352	-321.979	-303.802	18.177
$^{37}_{\Lambda}\text{Ar}^*$	0.597	0.1370	-315.194	-296.670	18.524
$^{39}_{\Lambda}\text{Ar}$	0.000	0.1360	-344.896	-326.455	18.441
$^{39}_{\Lambda}\text{Ar}^*$	0.589	0.1378	-336.306	-317.448	18.858
$^{41}_{\Lambda}\text{Ar}$	-0.117	0.1357	-361.398	-342.613	18.785
$^{41}_{\Lambda}\text{Ar}^*$	0.491	0.1378	-355.922	-336.852	19.070
$^{41}_{\Lambda}\text{Ca}$	0.00	0.1361	-361.422	-342.869	18.553
$^{41}_{\Lambda}\text{Ca}^*$	0.70	0.1393	-350.559	-331.317	19.242
$^{33}_{\Lambda}\text{S}$	0.26	0.1376	-285.095	-267.002	18.093
$^{33}_{\Lambda}\text{S}^*$	0.97	0.1243	-274.315	-257.951	16.364
$^{57}_{\Lambda}\text{Ni}$	0.00	0.1461	-506.665	-484.759	21.906
$^{57}_{\Lambda}\text{Ni}^*$	0.40	0.1415	-498.610	-477.892	20.718
$^{61}_{\Lambda}\text{Zn}$	0.22	0.1438	-534.565	-512.924	21.641
$^{61}_{\Lambda}\text{Zn}^*$	0.62	0.1415	-527.168	-506.238	20.930

Superdeformed hypernuclei

Isaka_Fukukawa_Kimura
Hiyama_Sagawa_Yamamoto
2014_PRC89-024310

AMD: Opposite predictions !

TABLE I. Calculated excitation energy E_x in MeV, matter quadrupole deformation β and γ (deg), and rms radii (fm). The B_Λ defined by Eq. (14) is also listed in unit of MeV for $^{41}_\Lambda\text{Ca}$.

	J^π	E_x (MeV)	β	γ (deg)	r_{rms} (fm)	B_Λ
$^{41}_\Lambda\text{Ca}$	$1/2_1^+$	0.00	0.10	0	3.38	19.45
	$1/2_2^+$	9.24	0.40	27	3.47	19.15
	$1/2_3^+$	11.41	0.55	13	3.58	18.01
^{40}Ca	0_1^+	0.00	0.12	12	3.39	
	0_2^+	8.94	0.40	28	3.50	
	0_3^+	9.97	0.60	17	3.63	

Superdeformed hypernuclei

LETTER

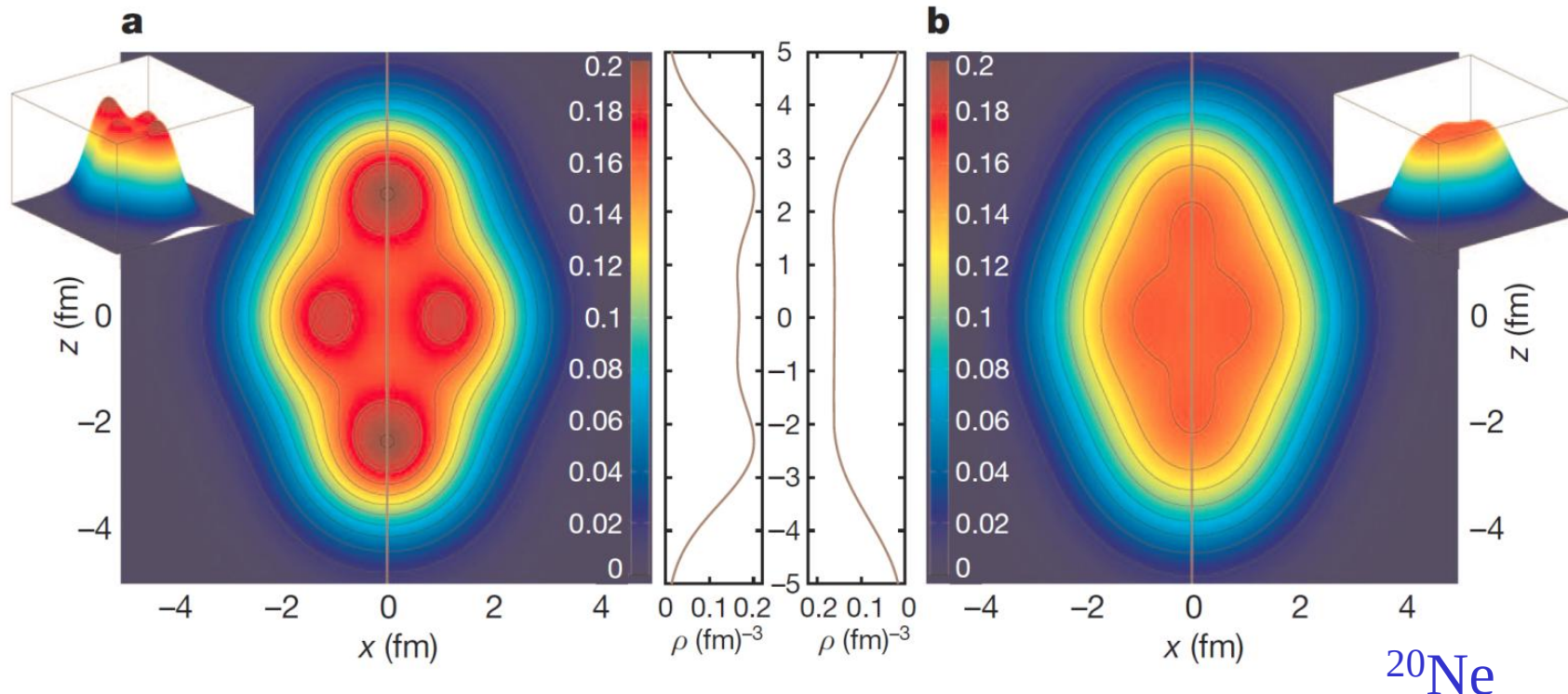
Ebran_Khan_Niksic_Vretenar 2012_Nature487-341

doi:10.1038/nature11246

How atomic nuclei cluster

J.-P. Ebran¹, E. Khan², T. Nikšić³ & D. Vretenar³

What about
Skyrme Hartree-Fock ?



Summary

- ❑ Multidimensionally-constrained covariant density functional theories: (β_{20} , β_{22} , β_{30} , β_{32} , β_{40} , ...)
- ❑ PES & fission barriers of heavy normal nuclei
 - Triaxiality lowering systematically 2nd barrier of actinides
 - Non-axial octupole correlations in $N=150$ isotones
 - Shallow hyperdeformed minima in actinides
- ❑ Shape of hypernuclei
 - Drastic shape changes in ($^{12}\text{C}-^{13}_{\Lambda}\text{C}$), ($^{28}\text{Si}-^{29}_{\Lambda}\text{Si}$), ...
 - Localization effects in superdeformed $^{36,38,40}\text{Ar}$ & ^{40}Ca , leading to larger Λ separation energy than in ground state

Summary & perspectives

- ❑ Multidimensionally-constrained covariant density functional theories: (β_{20} , β_{22} , β_{30} , β_{32} , β_{40} , ...)
- ❑ PES & fission barriers of heavy normal nuclei
 - Triaxiality lowering systematically 2nd barrier of actinides
 - Non-axial octupole correlations in $N=150$ isotones
 - Shallow hyperdeformed minima in actinides
- ❑ Shape of hypernuclei
 - Drastic shape changes in ($^{12}\text{C}-^{13}_{\Lambda}\text{C}$), ($^{28}\text{Si}-^{29}_{\Lambda}\text{Si}$), ...
 - Localization effects in superdeformed $^{36,38,40}\text{Ar}$ & ^{40}Ca , leading to larger Λ separation energy than in ground state
- ❑ Systematic study of PES & B_f of superheavy nuclei
- ❑ Structure of $S=-2$ hypernuclei

Collaborators

□ Emiko Hiyama	RIKEN
□ Bing-Nan Lu (吕炳楠)	ITP/CAS & FZJ
□ Xu Meng (孟旭)	ITP/CAS
□ Hiroyuki Sagawa	RIKEN & Aizu U
□ Dario Vretenar	U Zagreb
□ En-Guang Zhao (赵恩广)	ITP/CAS
□ Jie Zhao (赵杰)	ITP/CAS

Zhou, Shan-Gui

ITP/CAS

周 善 贵

Beijing



Email: sgzhou@itp.ac.cn

URL: www.itp.ac.cn/~sgzhou