Consequences of γ-softness of the core for some properties of chiral nuclei

Leszek Próchniak

Heavy Ion Laboratory, University of Warsaw

UNIVERSITY OF WARSAW

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Outline

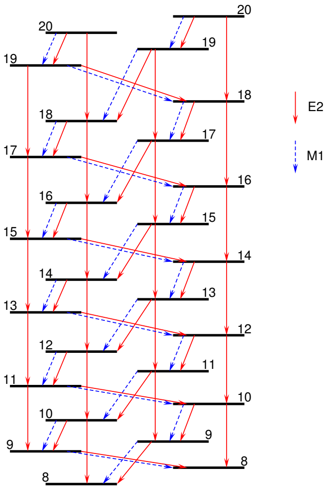
- Introduction
- Core-Particle-Hole-Coupling (CPHC) model
- S symmetry and its consequences
- Nonsymmetric cases
- Microscopic cores

Collaborators: Ch. Droste, S. G. Rohoziński and K. Starosta
EPJ A 47 (2011) 90

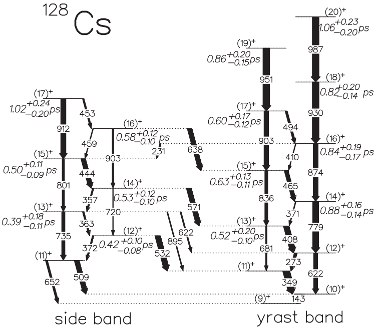
EPJ A 42 (2009) 72 + E. Grodner

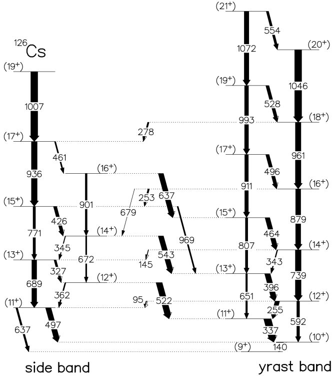
Characteristic features of chiral nuclei
Partner bands:

- ,,almost" degenerate levels $(200 - 300$ keV) with the same parity, $\Delta I = 1$
- similar EM properties inside bands
- staggering of M1 transitions (also E2 with $\Delta I = 1$)



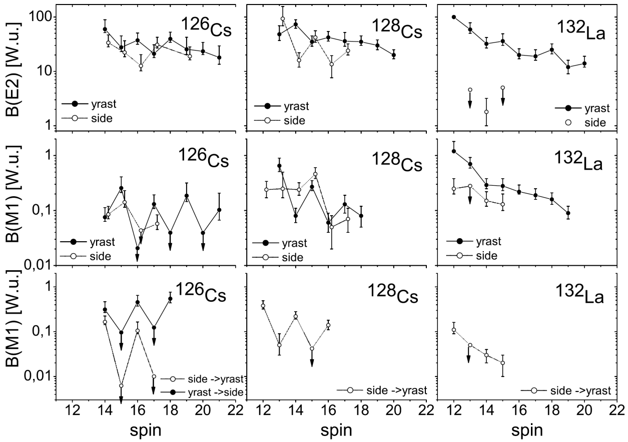
Experimental stimulus: ^{128}Cs, ^{126}Cs, ^{124}Cs

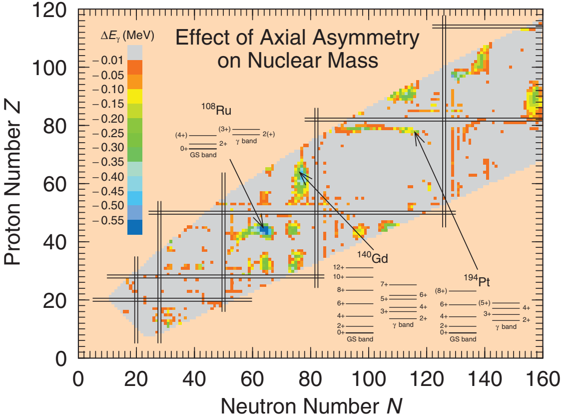




- E. Grodner et al. *128Cs as the Best Example Revealing Chiral Symmetry Breaking*, PRL 97, 172501 (2006)
- E. Grodner et al. *Partner bands of 126Cs - first observation of chiral electromagnetic selection rules*, PLB 703, 46 (2011)

$^{126-128}$Cs nuclei, EM transitions





P. Moller et al., *Global Calculations of Ground-State Axial Shape Asymmetry of Nuclei*, PRL 97, 162502 (2006)

Core-Particle-Hole Coupling model

$$|\mathrm{odd}-\mathrm{odd}, i\rangle = \sum_{r,k,m} U^i_{rkm} |\mathrm{core}, r\rangle |\mathrm{p}, k\rangle |\mathrm{n}, m\rangle$$

$$H_{\mathrm{o-o}} = H_{\mathrm{core}} - \chi Q q_{\mathrm{p}} - \chi Q q_{\mathrm{n}} - \chi' q_{\mathrm{p}} q_{\mathrm{n}} + h_{\mathrm{p}} + h_{\mathrm{n}}$$

$Q, q_{\mathrm{p,n}}$ — quadrupole operators

Cores:

- Bohr Hamiltonian (FQ, full quadrupole); (β, γ, Ω) or (α_μ)
  - given by analytic formulas
  - calculated microscopically
- rigid rotor (DF, Davydov-Filipov); (Ω); $[\tilde{\beta}, \tilde{\gamma}$ — fixed]

Proton-neutron part:

$\mathrm{p}h_{11/2} \otimes \mathrm{n}h^{-1}_{11/2}, \quad A \sim 130$

The Bohr Hamiltonian

$$H_{\mathrm{GBH}}(\beta,\gamma,\Omega) = T_{\mathrm{vib}} + T_{\mathrm{rot}} + V$$

$$\begin{aligned}
T_{\mathrm{vib}}(\beta,\gamma) &= -\frac{1}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\partial_\beta\left(\beta^4\sqrt{\frac{r}{w}}B_{\gamma\gamma}\right)\partial_\beta - \partial_\beta\left(\beta^3\sqrt{\frac{r}{w}}B_{\beta\gamma}\right)\partial_\gamma\right]+\right.\\
&\left.+\frac{1}{\beta\sin 3\gamma}\left[-\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\right)\partial_\beta + \frac{1}{\beta}\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\right)\partial_\gamma\right]\right\}\\
T_{\mathrm{rot}}(\beta,\gamma,\Omega) &= \frac{1}{2}\sum_{k=1}^{3} I_k^2(\Omega)/J_k; \quad J_k(\beta,\gamma) = 4B_k(\beta,\gamma)\beta^2\sin^2(\gamma - 2\pi k/3)
\end{aligned}$$

$$w = B_{\beta\beta}B_{\gamma\gamma} - B_{\beta\gamma}^2; \quad r = B_x B_y B_z$$

Simplest form of the kinetic energy

$$T_{\mathrm{kin}} = \frac{1}{2}B\sum_\mu |\dot{\alpha}_\mu|^2 \quad \leftrightarrow B_{\beta\beta} = B_{\gamma\gamma} = B_k = B, \quad B_{\beta\gamma} = 0$$

$$S = P_5(\text{core})C_{\text{pn}}(\text{proton-neutron})$$

- FQ core

  LAB system

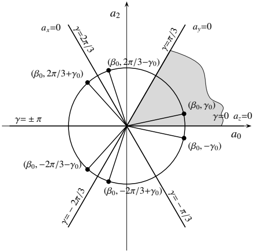
$$P_5\alpha_\mu = -\alpha_\mu \quad \alpha\text{-parity of the core}$$

  INT (principal axes) system

$$\{\alpha_\mu\} \xrightarrow{R(\Omega)} \{\tilde{\alpha}_0, \tilde{\alpha}_1 = \tilde{\alpha}_{-1} = 0, \tilde{\alpha}_2 = \tilde{\alpha}_{-2}\}$$

$$\tilde{\alpha}_0 = \beta\cos\gamma,$$
$$\tilde{\alpha}_2 = \tilde{\alpha}_{-2} = \beta\sin\gamma/\sqrt{2}$$

$$P_5(\beta,\gamma,\Omega) = (\beta,\gamma\pm\pi,\Omega)$$
$$P_5(\beta,\gamma,\Omega) = (\beta,\pi/3-\gamma,R_x(\pi/2)\Omega)$$



- DF core

$$P_5 \to R_x(\pi/2)$$

S symmetry, pn part

Proton-neutron part: C_{pn}

- ”Almost” the same (naturally isomorphic) spaces for proton and neutron

$$C_{\mathrm{pn}}|(\mathrm{p},i)(\mathrm{n},k)\rangle = |(\mathrm{p},k)(\mathrm{n},i)\rangle$$

- Proton and neutron at the same j level

$$C_{\mathrm{pn}}|((\mathrm{p},j)(\mathrm{n},j))^L\rangle = (-1)^{L+1}|((\mathrm{p},j)(\mathrm{n},j))^L\rangle$$

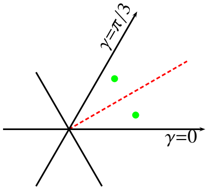
General property

$$P_5^2 = I_{\mathrm{core}}, \quad C_{\mathrm{pn}}^2 = I_{\mathrm{pn}}$$
$$S^2 = (P_5 C_{\mathrm{pn}})^2 = I$$

For the DF core cf. T. Koike, K. Starosta, I. Hamamoto, PRL **93** (2004) 172502

Conditions imposed by the P_5 symmetry

$$f(\beta, \pi/3-\gamma) = f(\beta,\gamma) \quad \text{for } f = B_{\beta\beta}, B_{\gamma\gamma}, B_x$$
$$B_{\beta\gamma}(\beta,\pi/3-\gamma) = -B_{\beta\gamma}(\beta,\gamma)$$
$$B_y(\beta,\pi/3-\gamma) = B_z(\beta,\gamma)$$
$$V(\beta,\pi/3-\gamma) = V(\beta,\gamma)$$



S symmetry S. Quadrupole operators, angular momenta. $H_{\mathrm{o-o}}$ Hamiltonian

Quadrupole operators and angular momenta

$$P_5 Q_\nu P_5^+ = -Q_\nu$$
$$P_5 I_k P_5^+ = I_k$$

$$C_{\mathrm{pn}}\, q_{\nu,\mathrm{p,n}}\, C_{\mathrm{pn}}^+ = -q_{\nu,\mathrm{n,p}}$$
$$C_{\mathrm{pn}}\, j_{k,\mathrm{p,n}}\, C_{\mathrm{pn}}^+ = j_{k,\mathrm{n,p}}$$

Odd-odd nucleus Hamiltonian $\quad H_{\mathrm{o-o}} = H_{\mathrm{core}} - \chi Q q_{\mathrm{p}} - \chi Q q_{\mathrm{n}} - \chi' q_{\mathrm{p}} q_{\mathrm{n}}$

H_{core} is P_5–symmetric $\longrightarrow$ $S H_{\mathrm{o-o}} S^+ = H_{\mathrm{o-o}}$

New additional label $s = \pm 1$ for eigenstates of $H_{\mathrm{o-o}}$

E2

$$\begin{aligned} &T(E2) = \kappa Q + e_\mathrm{p} q_\mathrm{p} + e_\mathrm{n} q_\mathrm{n} \\ &SQS^+ = -Q \qquad\qquad \longrightarrow \langle J, s | T(E2) | J', s\rangle \approx 0 \\ &e_\mathrm{p} q_\mathrm{p}, e_\mathrm{n} q_\mathrm{n} \text{ — small} \end{aligned}$$

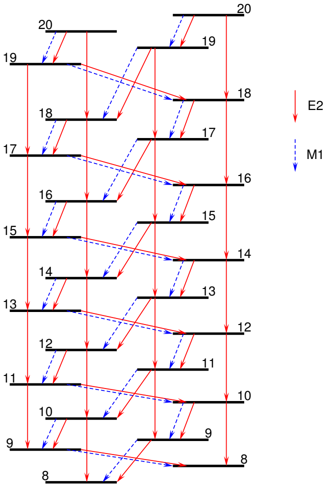
M1

$$\begin{aligned} &T(M1) = \sqrt{\frac{3}{4\pi}} \mu_N (g_I I_\mathrm{core} + g_\mathrm{p} j_\mathrm{p} + g_\mathrm{n} j_\mathrm{n}) \\ &P_5 I_\mathrm{core} P_5^+ = I_\mathrm{core} \\ &g_I - (g_\mathrm{p} + g_\mathrm{n})/2 = 0 \qquad \longrightarrow \langle J, s | T(M1) | J', s\rangle = 0, \quad J \neq J' \end{aligned}$$

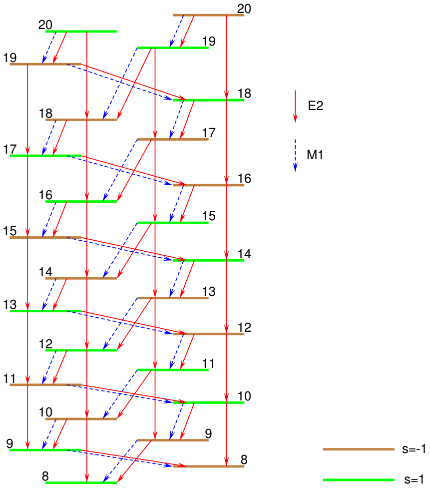
Example (reasonable values): $g_I = 0.44$, $g_\mathrm{p} = 1.22$, $g_\mathrm{n} = -0.21$

$$\begin{aligned} T(M1) \sim (g_I J \quad &+ (g_{l\mathrm{p}} - g_R) l_\mathrm{p} + (g_{s\mathrm{n}} - g_I) s_\mathrm{p}) + \\ &+ (g_{l\mathrm{n}} - g_R) l_\mathrm{n} + (g_{s\mathrm{p}} - g_I) s_\mathrm{n}) \end{aligned}$$

Selection rules



Selection rules



- Simple kinetic energy, B const
- Potential energy

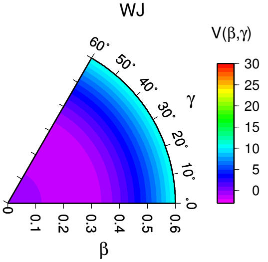
$$V = \frac{1}{2} V_C \beta^2 + [G + h_1 \cos 3\gamma + h_2(\cos^2 3\gamma - 1)^\kappa] \times (\exp(-\beta^2/d^2) - 1)$$

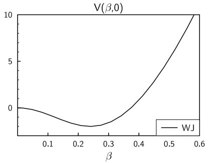
- Properties

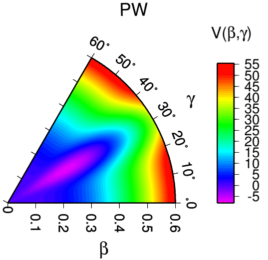
$E(2_1^+) \approx 0.35$ MeV
$\beta_{\mathrm{eq}} \approx 0.25$
$B(E2, 2_1 \to 0_1) \approx 0.28$ eb^2

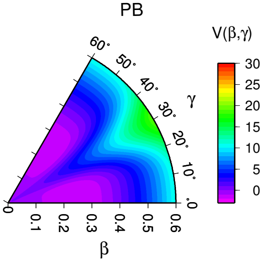
S symmetric Hamiltonians

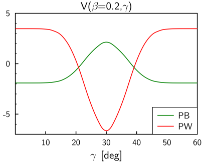
Wilets-Jean, potential well (PW), potential barrier (PB). Simple kinetic energy



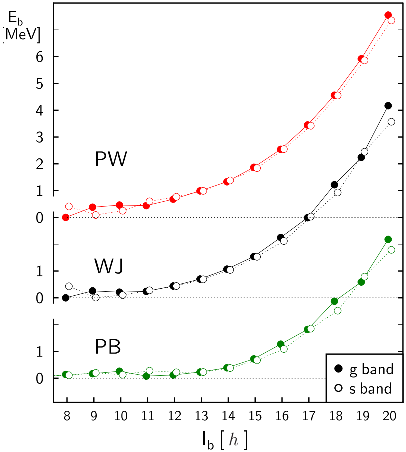




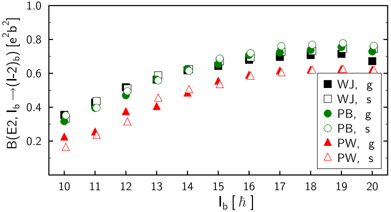




S symmetric cases, energies, E2

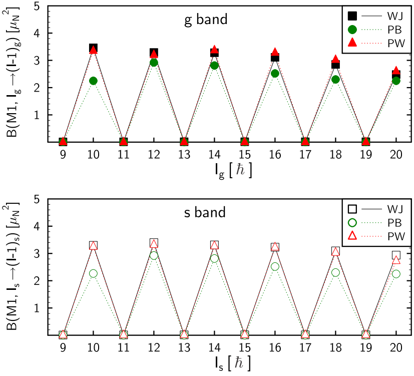


E2 transitions, $\Delta I = 2$

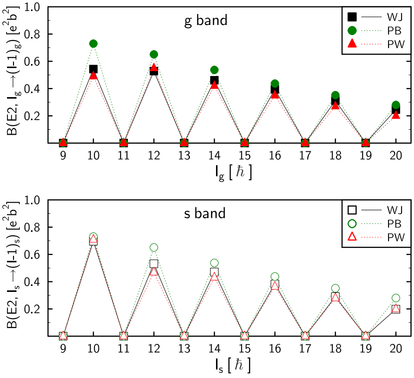


S symmetric cases, intraband transitions

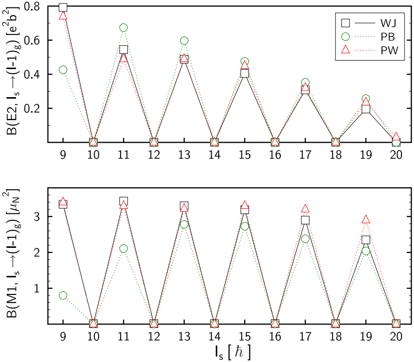
M1 transitions



E2 transitions, $\Delta I = 1$



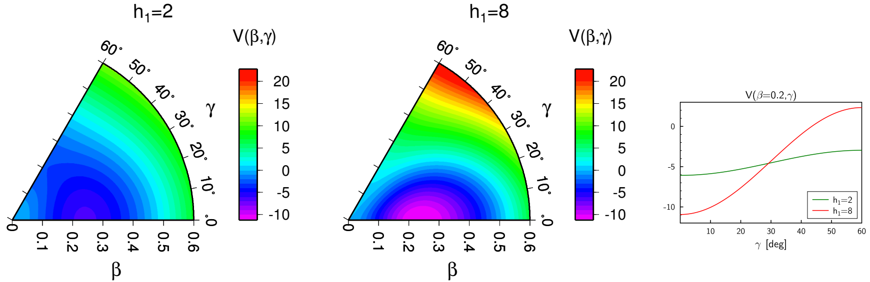
S symmetric cases, s→g transitions



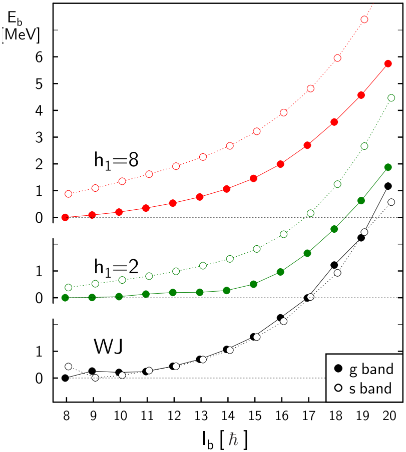
Deviations from the S symmetry. Non-symmetric core

- Non-symmetric core (potential or/and kinetic energy)
- Different one-particle spaces for proton and neutron

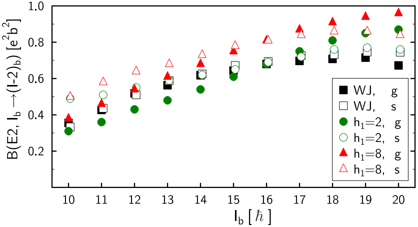
Case 1. Non-symmetric potential energy



Non-symmetric core

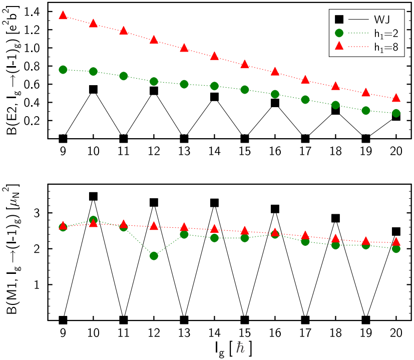


E2 transitions, $\Delta I = 2$

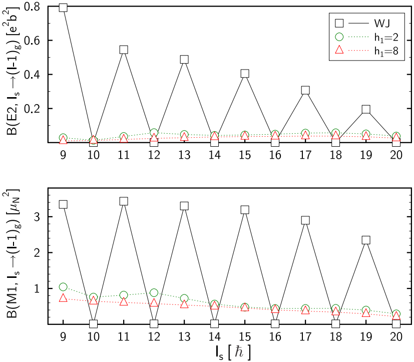


Non-symmetric core, cont.

g→ g transitions



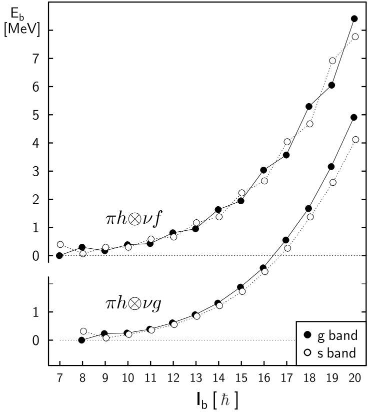
s→ g transitions

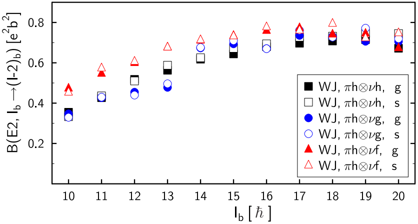


Deviations from the S symmetry. Different orbitals for proton and neutron

Case 2. Proton $h_{11/2}$; neutron $g_{9/2}^{-1}$ or $f_{7/2}^{-1}$

core — WJ potential (P_5 symmetric)

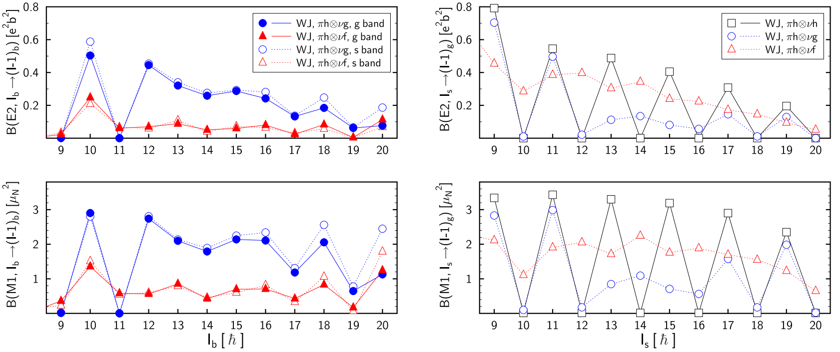




Different orbitals for proton and neutron, cont.

Intraband transitions

s→g band transitions



Microscopic cores

$$
\begin{aligned}
{}^{128}\mathrm{Xe} + p \otimes n^{-1} &\longrightarrow {}^{128}\mathrm{Cs} \\
{}^{128}\mathrm{Ba} + p \otimes n^{-1} &\longrightarrow {}^{128}\mathrm{La}
\end{aligned}
$$

- Mean field:
  Skyrme SIII and SLy4
  RMF NL3
- Pairing: seniority force
- Calculations with constraints to get required deformation
- Kinetic energy — ATDHFB type formulas

- Deformation variables

$$\beta\cos\gamma = cq_0 = c\langle\Psi|Q_0|\Psi\rangle = \langle\Psi|\sum_{i=1}^{A}(3z_i^2 - r_i^2)|\Psi\rangle$$
$$\beta\sin\gamma = cq_2 = c\langle\Psi|Q_2|\Psi\rangle = \langle\Psi|\sum_{i=1}^{A}\sqrt{3}(x_i^2 - y_i^2)|\Psi\rangle; \quad c = \sqrt{\pi/5}/A\overline{r^2}$$

- HFB with constraints

$$\delta\langle\Psi|H_{\mathrm{micr}} - \lambda_0 Q_0 - \lambda_2 Q_2|\Psi\rangle = 0$$
$$\langle\Psi|Q_0|\Psi\rangle = q_0, \quad \langle\Psi|Q_2|\Psi\rangle = q_2 \quad \longrightarrow \Psi(\beta,\gamma)$$

- Potential energy

$$V = \langle\Psi(\beta,\gamma)|H_{\mathrm{micr}}|\Psi(\beta,\gamma)\rangle$$

ATDHFB, some formulas, kinetic energy

HF+BCS case

- Mass parameters (inertial functions)

$$B_{q_i q_j} = \hbar^2 (S_{(1)}^{-1} S_{(3)} S_{(1)}^{-1})_{ij}$$

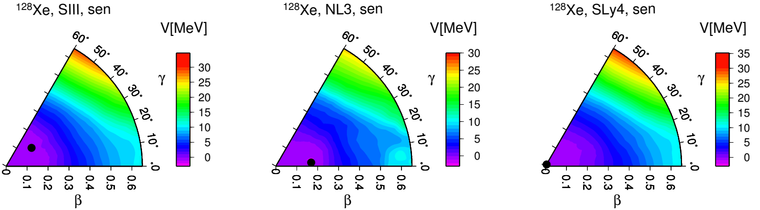
$$(S_{(n)})_{ij} = \sum_{\mu,\nu} \frac{\langle \mu | Q_i | \bar{\nu} \rangle \langle \bar{\nu} | Q_j | \mu \rangle}{(E_\mu + E_\nu)^n} (u_\mu v_\nu + u_\nu v_\mu)^2$$

Then $$B_{q_i q_j} \rightarrow B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma}$$

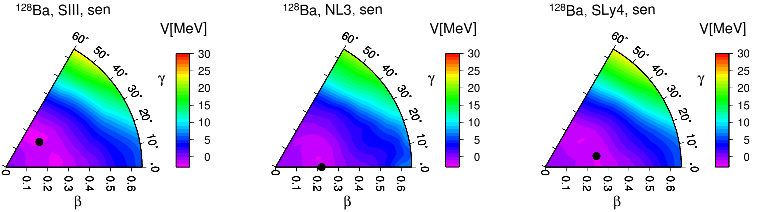
- Moments of inertia

$$J_k = \hbar^2 \sum_{\mu,\nu} \frac{|\langle \nu | j_k | \bar{\mu} \rangle|^2 (u_\mu v_\nu - u_\nu v_\mu)^2}{(E_\mu + E_\nu)}$$

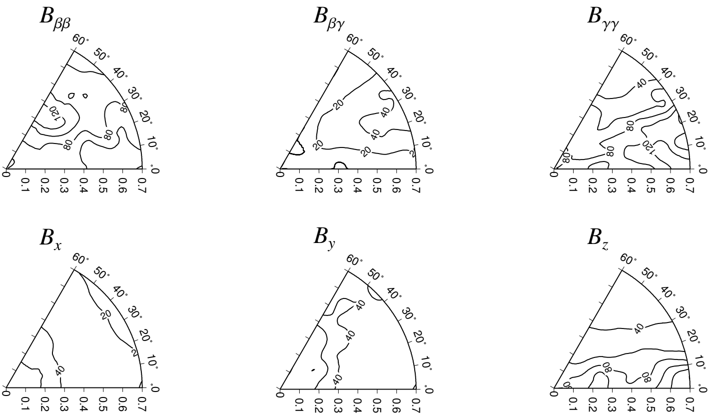
^{128}Xe


^{128}Xe, SIII, sen
^{128}Xe, NL3, sen
^{128}Xe, SLy4, sen
V[MeV]
γ
β

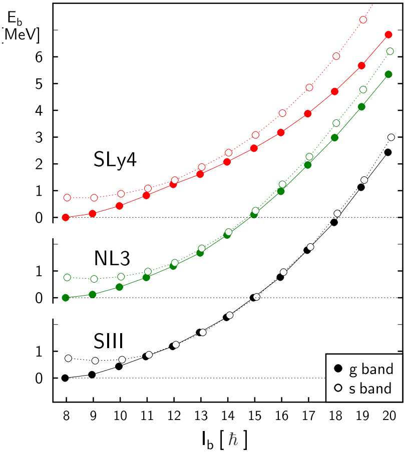
^{128}Ba


^{128}Ba, SIII, sen
^{128}Ba, NL3, sen
^{128}Ba, SLy4, sen
V[MeV]
γ
β

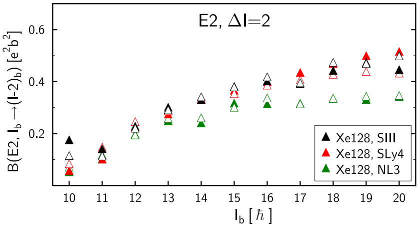
Inertial functions, ^{128}Xe, Skyrme III



Odd-odd nucleus with ^{128}Xe core



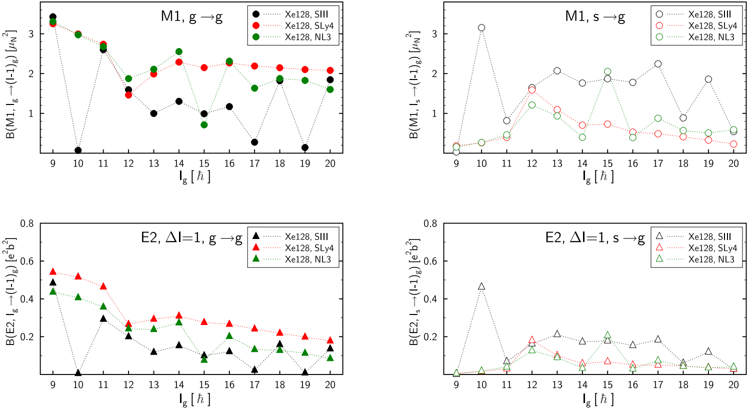
E2 in-band transitions



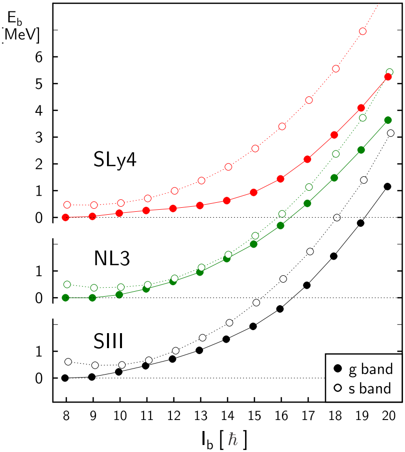
$\Delta I = 1$ M1 and E2 transitions

In-band

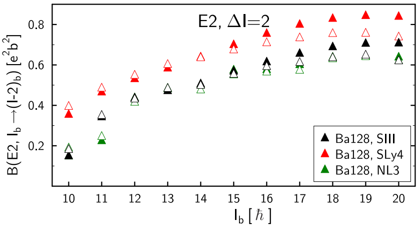
Inter-band



Odd-odd nucleus with ^{128}Ba core

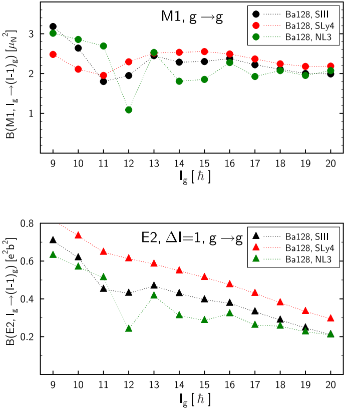


E2 in-band transitions

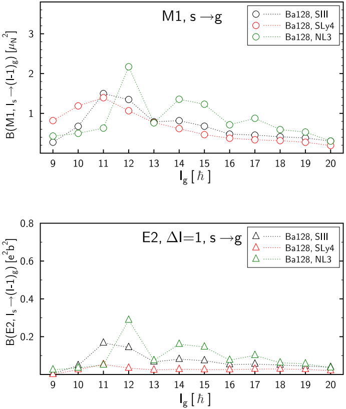


$\Delta I = 1$ M1 and E2 transitions

In-band



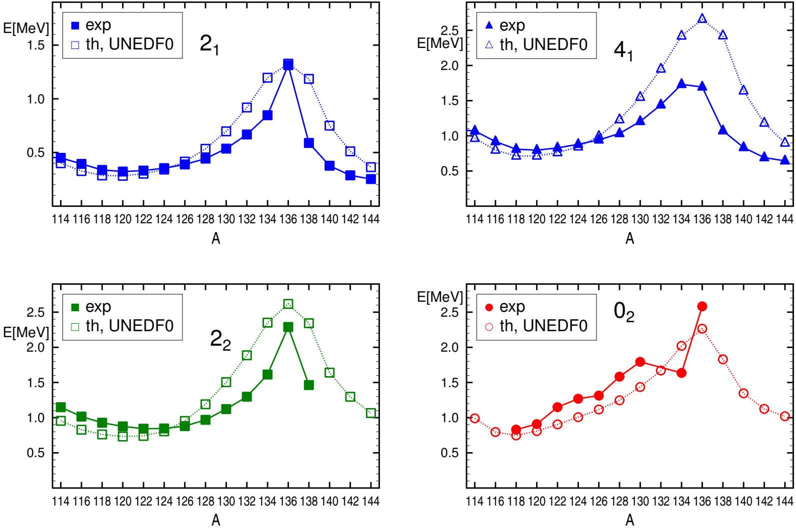
Inter-band



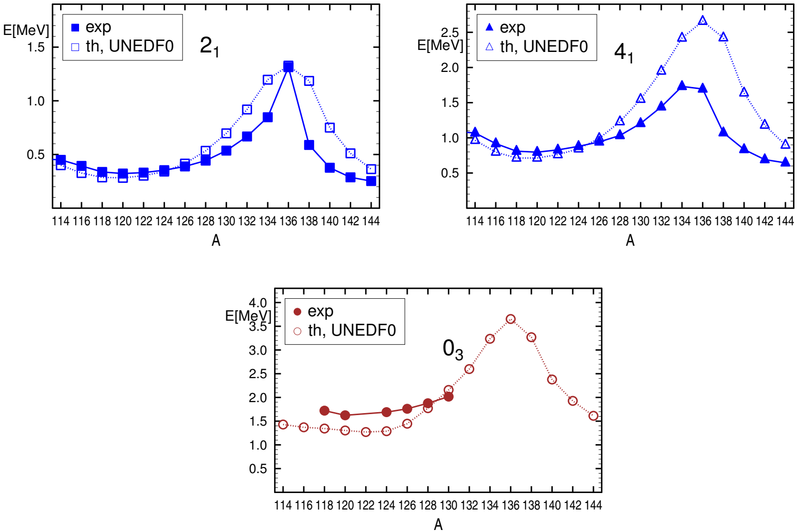
Conclusions

- Existence of the partner bands do not require rigid deformation of the core
- The S symmetry — useful in model studies. Not much probable to exist in real nuclei (?)
- It is possible to obtain partner bands with microscopic cores

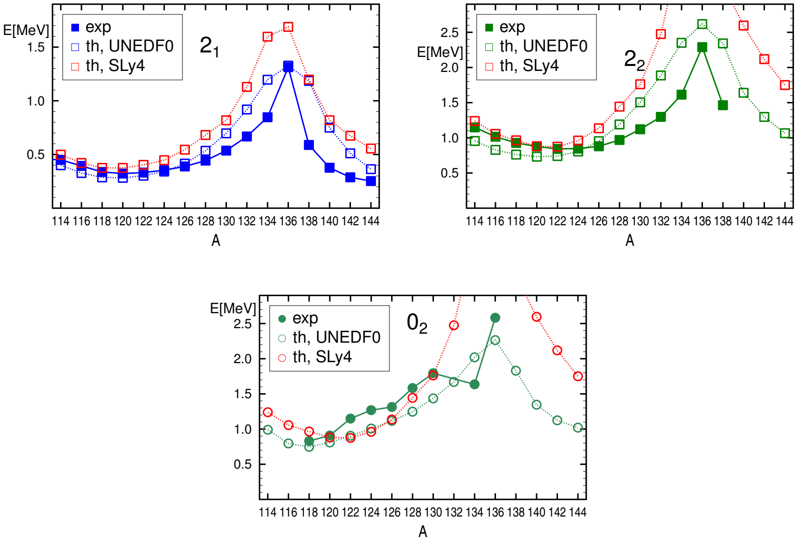
$^{114-144}$Xe. Selected energy levels *cont.*



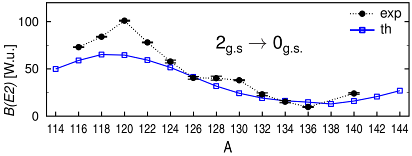
$^{114-144}$Xe. Selected energy levels *cont.*

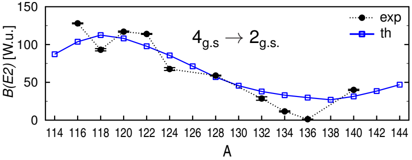


$^{114-144}$Xe. Selected energy levels. Comparison with SLy4 results

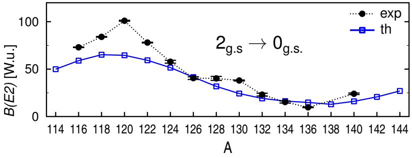


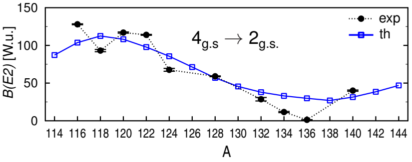
E2 transition probabilities



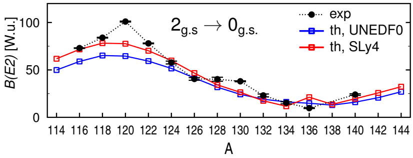


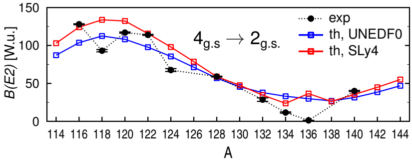
E2 transition probabilities





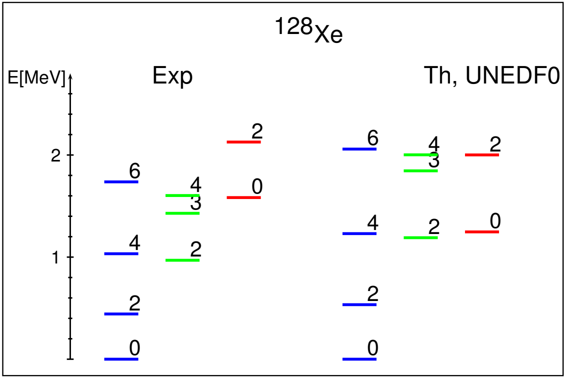
Comparison with SLy4 results





More details on ^{128}Xe. Energy levels

Low energy, positive parity levels in ^{128}Xe



More details on ^{128}Xe. Energy levels

Low energy, positive parity levels in ^{128}Xe