Nilsson SU3 selconsistency

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Preliminaries

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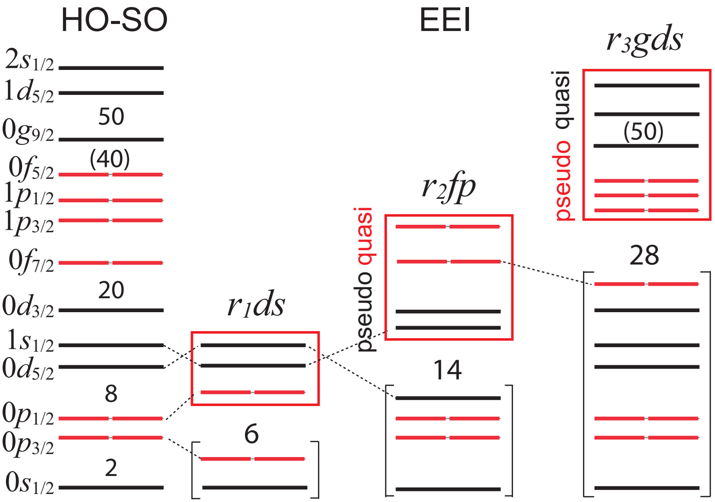
Subjets

- ZBM or EEI spaces
- quasi-pseudo SU3
- Nilsson SU3 selfconsistency
- The heavy N=Z nuclei

HO, EI, ZBM spaces refs. gemo,dz10



SU3 in HO basis ref. Elliott

$$q \equiv q^{2m} = r^2 C^{2m} = r^2 \sqrt{4\pi/(2l+1)} Y^{lm}$$

Quadrupole force $-2q \cdot 2q$

Eigenstates $E(L,i) = E(i) + 3L(L+1)$

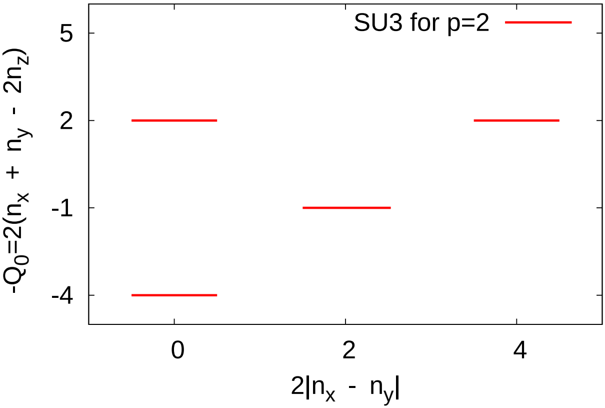
$E(i)$ of intrinsic state maximizes

$$Q_0 = 2q^{20} = (2n_z - n_x - n_y)$$

If $p = n_x + n_y + n_z = 2$ there are six sp states

$[n_z n_x n_y]$=[200],[110],[101],[020],[011],[002]

SU3 platforms



What follows is q_{20} in LS and jj couplings

Please pay attention

q_{20} in LS and jj couplings

$$\langle pl|r^2|pl\rangle = p + 3/2 \langle pl|r^2|pl+2\rangle = -[(p-l)(p+l+3)]^{1/2}$$

$$\langle lm|C_2|lm\rangle = \frac{l(l+1)-3m^2}{(2l+3)(2l-1)}, \quad \langle lm|C_2|l+2m\rangle$$

$$= \frac{3}{2}\left\{\frac{[(l+2)^2-m^2][(l+1)^2-m^2]}{(2l+5)(2l+3)^2(2l+1)}\right\}^{1/2}$$

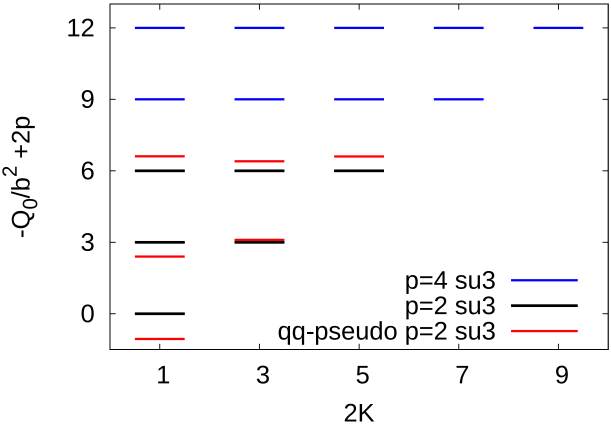
$$\langle jm|C_2|jm\rangle = \frac{j(j+1)-3m^2}{2j(2j+2)}, \quad \langle jm|C_2|j+2m\rangle$$

$$= \frac{3}{2}\left\{\frac{[(j+2)^2-m^2][(j+1)^2-m^2]}{(2j+2)^2(2j+4)^2}\right\}^{1/2}$$

$$\langle jm|C_2|j+1m\rangle = -\frac{3m[(j+1)^2-m^2]^{1/2}}{j(2j+4)(2j+2)}$$

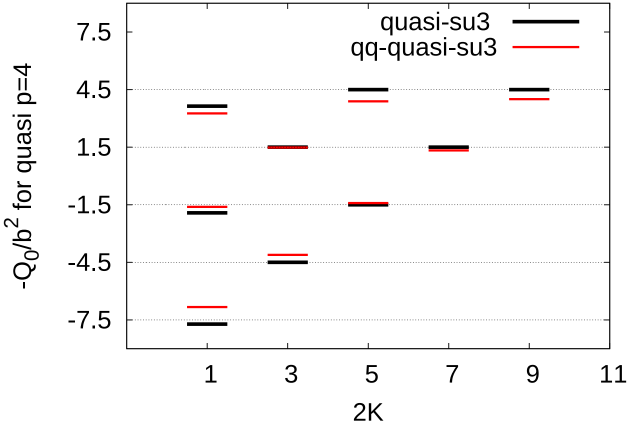
SU3 and pseudo-SU3 in jj coupling ref. pseudo

Diagonalize black+red+blue in *sd* and r_3

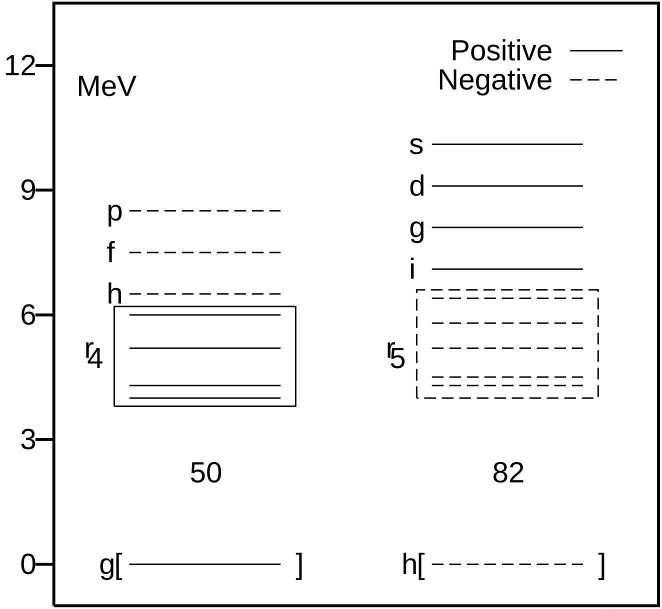


quasiSU3 ref. quasi

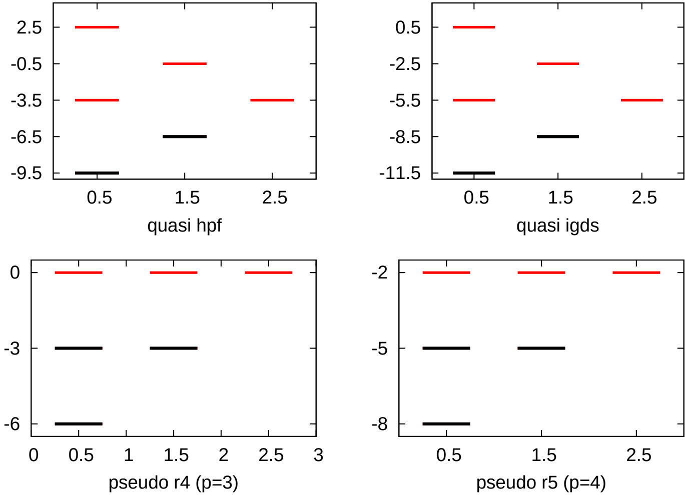
Diagonalize black+magenta in *gds* setting $l \to l + 1/2$ with $q\dot{q}$ and quasi $q\dot{q}$.



Rare earth example figure jj



Rare earth example figure HO



Explain and calculate

Rare earth example table ref. quasi

$$N = 92 \quad 94 \quad 96 \quad 98$$
$$n = \quad 0 \quad 1 \quad 2 \quad 3$$

$$Q_0 = (32+24)e_\pi + (40+36+4n)e_\nu =$$
$$\mathbf{(56)e_\pi + (76+4n)e_\nu}$$
$$B(E2)\downarrow (e^2 f^4) = Q_0^2 b_{osc}^2 / 50.27$$
$$B(E2)\uparrow \approx Q_0^2 A^{2/3} 10^{-5} (e^2 b^2)$$

Table : $B(E2)\uparrow$ in $e^2 b^2$ compared with experiment

N	60Nd	62Sm	64Gd	66 Dy
92	4.47	4.51	4.55	4.58
	2.6(7)	4.36(5)	4.64(5)	4.66(5)
94	4.68	4.72	4.76	4.80
			5.02(5)	5.06(4)
96	4.90	4.95	4.99	5.03
			5.25(6)	5.28(15)
98	5.13	5.18	5.22	5.26
				5.60(5)

Nilsson SU3 selfconsistency intro

$$H_{mq} = \sum \epsilon_i n_i - \frac{\hbar\omega\kappa}{\mathcal{N}_{2q}^2} 2q \cdot 2q$$

$$\mathcal{N}_{2q}^2 = \sum (2q_{rs})^2 = \frac{5}{2} \sum_{k=0}^{p} (k+1)(2p-3k)^2$$

Refresh ideas about pairing and quadrupole ref. mdz

$$H_{\bar{P}} = -0.32\hbar\omega \left(\frac{P_p^+}{\sqrt{\Omega_p}} + \frac{P_{p+1}^+}{\sqrt{\Omega_{p+1}}} \right) \cdot \left(\frac{P_p}{\sqrt{\Omega_p}} + \frac{P_{p+1}}{\sqrt{\Omega_{p+1}}} \right)$$

$$H_{\bar{q}} = -0.216\hbar\omega \left(\frac{q_p}{\mathcal{N}_p} + \frac{q_{p+1}}{\mathcal{N}_{p+1}} \right) \cdot \left(\frac{q_p}{\mathcal{N}_p} + \frac{q_{p+1}}{\mathcal{N}_{p+1}} \right)$$

Nilsson SU3 selfconsistency intro

$$H_{mq} = \sum \epsilon_i n_i - \frac{\hbar\omega\kappa}{\mathcal{N}_{2q}^2} 2q \cdot 2q$$

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Linearize

$$H_{mq0} = \varepsilon H_{sp} - \frac{\beta\hbar\omega\kappa}{\mathcal{N}_{2q_{20}}^2} \langle 2q_{20}^{\nu} \rangle 2q_{20}^{\nu},$$

$$\mathcal{N}_{2q_{20}}^2 = \sum (2q_{20rs})^2 = \frac{2}{5} \mathcal{N}_{2q}^2$$

Nilsson SU3 selfconsistency, linearize correctly ref. a4749

Explain correct linearization $\longrightarrow \beta = 3$

$q_{20}q_{20} = (q^{\nu}_{20} + q^{\pi}_{20})^2 = (q^{\nu}_{20})^2 + 2q^{\nu}_{20}q^{\pi}_{20} + (q^{\pi}_{20})^2$

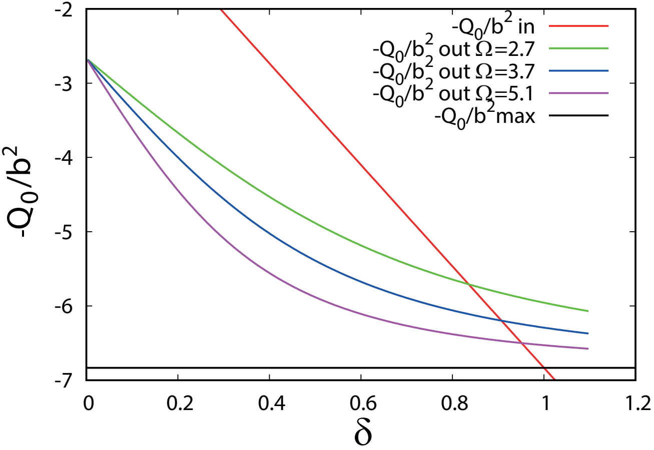
Linearize for ν

$q_{20}\, q_{20} \longrightarrow q^{\nu}_{20}\langle q^{\nu}_{20} + 2q^{\pi}_{20}\rangle \approx 3q^{\nu}_{20}\langle q^{\nu}_{20}\rangle$, if $\langle q^{\nu}_{20}\rangle \approx \langle q^{\pi}_{20}\rangle$. Hence

$$H_{mq0} = \varepsilon H_{sp} - \frac{3\hbar\omega\kappa}{\mathcal{N}^2_{2q_{20}}}\langle 2q^{\nu}_{20}\rangle 2q^{\nu}_{20}$$

Nilsson SU3 selfconsistency enactment

$$H_{mq0} = \varepsilon H_{sp} - \frac{3\hbar\omega\kappa}{\mathcal{N}^2_{2q_{20}}} \langle 2q^{\nu}_{20} \rangle 2q^{\nu}_{20}$$



Nilsson SU3 selfconsistency checks 0

SU3 says $E = -\hbar\omega\kappa 2\lambda(2\lambda + 6)/\mathcal{N}_{2q}^2$ If $H_{sp} = 0$,

$\langle 2q_{20}\rangle = 2\lambda$. Assume form of E holds

$$BE2 = B(E2 : 2^+ \rightarrow 0^+) = ((\langle 2q_{20}\rangle + 3)b^2)^2/50.3$$

$$Q_s = Q2s = ((\langle 2q_{20}\rangle + 3)b^2/3.5$$

$$E = \langle \varepsilon H_{sp}\rangle - \frac{\hbar\omega\kappa}{\mathcal{N}_{2q}^2}\langle 2q_{20}\rangle(\langle 2q_{20}\rangle + \zeta) \qquad (1)$$

Here the norms are those of the full quadrupole interaction *i.e.*, $\mathcal{N}_{2q} = \sqrt{2.5}\mathcal{N}_{2q20}$.

Nilsson SU3 selfconsistency checks I

Table : $q\dot{q}$ calculations in ^{28}Si(sd) (upper panels) and ^{68}Se(r_3) (lower panels). To the left $q\dot{q}$ is made monopole free. Int stands for intrisic values defined in Eq. (1), with $\zeta = 3$ in the left panel and 0 in the rigt one. Absolute energies given for the ground state, excitation energies for the other states.

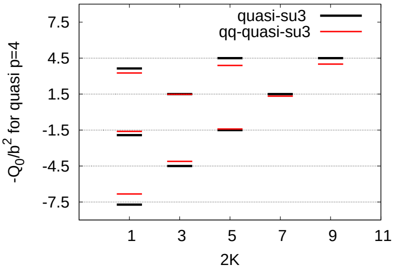
2J	E	Q_s	$BE2$	2J	E	Q_s	$BE2$
0	-27.26329			0	-22.044		
0	0.00192			4	0.958	-26.330	166.98
4	0.91714	-0.7785	167.299	0	1.646		
4	0.91730	0.7785	167.301	4	2.494	26.323	166.90
Int	-26.20	-26.40	169.74	int	-26.20	-23.23	169.74
0	-14.97822			0	-12.176		
0	0.00042			4	0.533	-42.087	426.80
4	0.50677	-2.8707	427.591	0	0.996		
4	0.50683	2.8708	427.590	4	1.467	42.117	426.92
Int	-13.99	41.21	413.64	int	-12.73	41.21	413.64

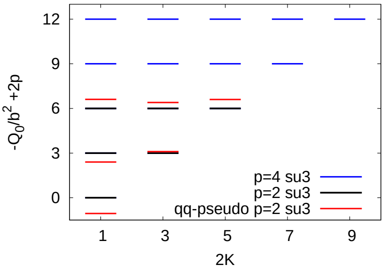
Nilsson SU3 selfconsistency checks II

Table : Monopole-free $q\dot{q}$ calculations in $(gds)^{8-12}$. Single particle energies in MeV: ε_i=[0.0, 0.0 and 0.0](e0) and [0.0, 1.0, and 2.0](e1) for $i = g, d, s$ respectively. Int stands for intrisic values defined in Eq. (1)

gds8 e0				gds8 e1			
2J	E	Q_s	$BE2$	J	E	Q_s	$BE2$
0	-12.977			0	-8.976		
4	0.113	-59.735	857.96	4	0.103	-57.313	795.57
Int	-12.34	-59.98	876.05	int	-8.28	-57.50	805.07
gds12 e0				gds12 e1			
0	-17.894			0	-12.641		
4	0.125	-65.296	1161.57	4	0.136	-65.61	1065.72
Int	-15.84	-69.16	1165.04	int	-8.63	-67.49	1109.34

Heavy N=Z nuclei intro





Heavy N=Z nuclei formulae 1

$$H_{mq} = \sum \epsilon_i n_i - \hbar\omega\kappa \left(\frac{2q_3}{\mathcal{N}_{2q3}} + \frac{2q_4}{\mathcal{N}_{2q4}} \right)^2 \qquad (2)$$

Now $\mathcal{N}_i = \mathcal{N}_{2q_{20}i}$

$$\begin{aligned} H_{sp} - 4\hbar\omega\kappa \frac{q_4^\nu}{\mathcal{N}_4^2} &\left(\langle q_4^\nu \rangle + \langle 2q_4^\pi \rangle + \langle 2q_3^\nu \rangle \frac{\mathcal{N}_4}{\mathcal{N}_3} + \langle 2q_3^\pi \rangle \frac{\mathcal{N}_4}{\mathcal{N}_3} \right) \\ &\approx H_{sp} - 4\hbar\omega\kappa \frac{q_4^\nu}{\mathcal{N}_4^2} \left(3\langle q_4^\nu \rangle + 6\langle q_3^\nu \rangle \right) \\ &= H_{sp} - \beta\hbar\omega\kappa \frac{2q_4^\nu}{\mathcal{N}_4^2} \langle 2q_4^\nu \rangle \qquad (3) \end{aligned}$$

Heavy N=Z nuclei formulae 2

the generalization of Eq (1) for the energy for a $r_3^k(gds)^l$ configuration takes the form

$$E = \sum \epsilon_i \langle n_i \rangle + l(\varepsilon_g - \varepsilon_{r_3})$$
$$-\hbar\omega\kappa \left(\frac{Q_3}{15} + \frac{Q_4}{23}\right)\left(\frac{Q_3+\zeta}{15} + \frac{Q_4+\zeta}{23}\right)$$
$$= hsp + l\varepsilon_{gr} + E_q \qquad (4)$$

$\mathcal{N}_{2q3} = \sqrt{90 \times 2.5} \approx 15,$
$\mathcal{N}_{2q4} = \sqrt{210 \times 2.5} \approx 23.$

Table : Properties of $r_3^k(gds)^l$ configurations. Total (E), quadrupole (E_q) and single particle (hsp) energies from Eq. (4) with $\zeta = 0$ in MeV; quadrupole moment $Q_4 = \langle 2q_{20(4)} \rangle$; $be2 = B(E2: 2^+ \to 0^+)$ in $\mathrm{e}^2\mathrm{fm}^4$. $\beta = 8$. $\epsilon_i =$0,3,4 (0,4,5 in last 2 lines)

k	l	a	E	$-E_q$	hsp	Q_4	$be2$
12	4	72	-12.29	25.53	3.24	23.00	1225.15
16	0	72	-8.37	8.37	0.0		323.54
12	4	72	-10.63	20.63	0.0		939.00
12	8	76	-12.29	40.05	7.76	41.17	2211.89
16	4	76	-2.46	15.62	3.15	22.85	867.33
14	6	76	-4.90	19.90	0.0		987.90
16	4	76	-6.23	16.23	0.0		805.57
12	12	80	-**0.30**	47.01	16.71	49.15	2792.44
16	8	80	1.76	27.45	7.69	41.17	1733.60
18	6	80	-0.04	15.04	0.0		822.71
12	16	84	5.91	51.92	17.82	54.71	3271.36
16	12	84	**13.47**	33.20	16.67	49.01	2239.57
20	8	84	6.05	13.95	0.0		840.95
12	16	84	3.02	51.25	22.26	54.05	322.97
20	8	84	2.05	13.95	0.0		840.95

^{72}Kr

Table : Properties of the yrast bands of ^{72}Kr calculated in the r_3gd space with the R3GD interaction (see text) (E's in MeV, Q in efm^2 and $B(E2) \equiv B(E2: J_i \to J_{fx})$ in e^{2}fm^4). Bottom, first line: measured values from **kr72** (error bars subject to caution as explained in text), second line: $t \leq 4$ results boosted as explained in the text.

$t = 4$			$B(E2)$		$t \leq 4$		$B(E2)$	
J_i	Ex	Q_s	J_{f1}	J_{f2}	Ex	Q_s	J_{f1}	J_{f2}
0_1	0.0				0.00			
0_2	0.24				0.30			
2_1	0.28	-65	1089	6	0.46	-54	586	372
2_2	0.56	58	3	897	0.66	45	103	536
4_1	0.83	-77	1509	1	1.05	-75	1387	75
4_2	1.23	69	0	1286	1.43	64	36	1093
$B(E2: 2_1 \to 0_1)$=810(150), $B(E2: 4_1 \to 2_1)$=2720(550)								
$t \leq 4 \times 1.4$; $B(E2: 2_1 \to 0_1)$=740, $B(E2: 4_1 \to 2_1)$=1750								

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pd92

^{88}Ru

In ^{88}Ru we come at last to a genuine r_3g nucleus. The table corresponds to an yrast oblate band exhibiting 50% $r_3^{-4}g^{12}$ oblate dominance.

J	E(exp)	E(th)	-Q_s	$B(E2\downarrow)$(th)
0_1^+	0.0	0.0		
2_1^+	0.62	0.56	37	492
4_1^+	1.42	1.31	44	766
6_1^+	2.38	2.12	47	888
8_1^+	3.48	2.88	52	980

Not obvious: g^{12} is now beyond midshell and the largest $\langle 2q_{20}\rangle$ is prolate. However the oblate $\langle 2q_{20}\rangle$ in r_3^{-4} is sufficiently strong to dominate but the prolate admixtures reduce and distort the original $\langle 2q_{20}\rangle$ =-(18.66+20.24) and $Q_0 \approx -182$ to $Q_{0s} \approx -125$ and $Q_{0t} \approx -160$ in the Table. It is seen that in this nucleus the prolate-oblate competition within the r_3g space is played up. ^{92}Pd will bring further news.

^{92}Pd and $J = 9$ condensation. ref. pd92

Examine 1) condensation; 2) g^{12} dominance

1	2	3	4	5	6	7	8	9	10
J	Ex	Et	con	qq	Ω	$BE2$	$BE2_{r_3g}$	Q_s	Q_{s,r_3g}
0	0.0	0.0	.000	.00	.99	—	—	—	—
2	0.874	0.84	.026	.22	.99	225	304	-28	-3.63
4	1.786	1.72	.058	.62	.99	316	382	-34	-8.20
6	2.563	2.52	.085	1.20	.98	340	364	-31	-2.77

1. J value 2. Experimental spectrum, in very good agreement with
3. JUN45 spectrum.
4. Spectrum of the condensate defined by $-H_{\rm con} = 0.1\mathrm{P}_0 + 0.9\mathrm{P}_9$, where P_0 and P_9 are the pairing Hamiltonians for $J = 0$ and 9.
5. Spectrum of the quadrupole force scaled so as to have unit $J = 9$ matrix element.
6. Overlap, $\Omega = \langle qq|\mathrm{con}\rangle^2$, of the wavefuntions indicating structural identity. **state leitmotiv** and look at issue 2).
7, 8. A Hamiltonian $-H \approx .6qq + .4P_0$ yields g^{12} energies close to exact and $B(E2)$ close to pure qq in column 7 and not far from exact in column 8. So g^{12} dominance in spite of 30% of wf? No. look at 9 and 10.
9. Spectroscopic Q_s for qq. 10. Spectroscopic Q_s for JUN45