# Energy-based models for financial network analysis

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# Outline

1. Overview of energy-based models
2. Financial network reconstruction
    2.1. Data preprocessing and effect of binarization
    2.2. Inference
    2.3. Network clustering structure
    2.4. Evolution of model parameters
3. Conclusions

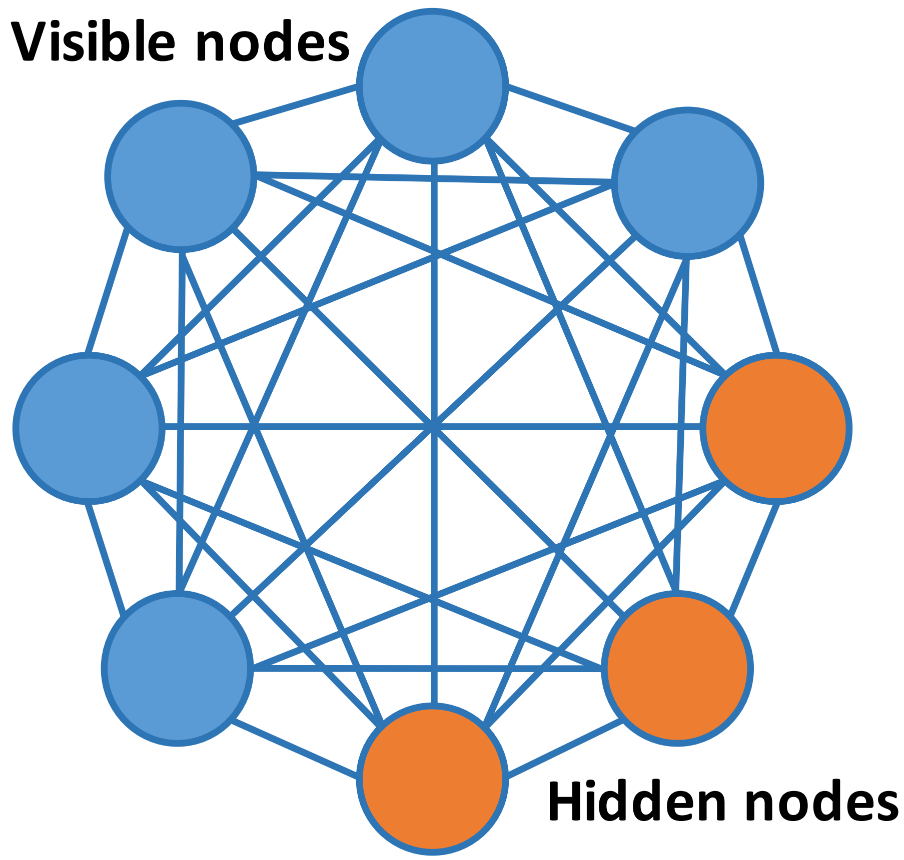
# Outline

# Energy-based models

$$p(\mathbf{x}) = \frac{e^{-\text{Energy}(\mathbf{x})}}{Z}$$

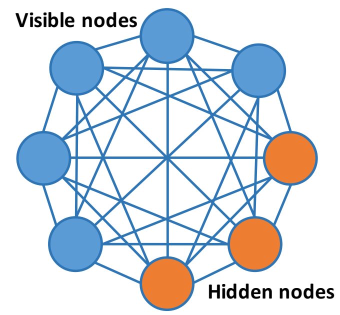
$$Z = \sum_{\mathbf{x}} e^{-\text{Energy}(\mathbf{x})}$$

# Boltzmann machine

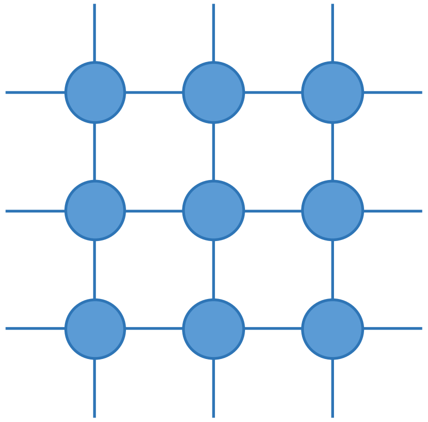



$$\text{Energy}(\mathbf{x}) = -\mathbf{h}_{\text{o}}^{\intercal}\mathbf{x} - \mathbf{x}^{\intercal}\mathbf{J}_{\text{oo}}\mathbf{x} - \mathbf{h}_{\text{u}}^{\intercal}\mathbf{x}_{\text{u}} - \mathbf{x}_{\text{u}}^{\intercal}\mathbf{J}_{\text{uu}}\mathbf{x}_{\text{u}} - \mathbf{x}_{\text{o}}^{\intercal}\mathbf{J}_{\text{ou}}\mathbf{x}_{\text{u}} + O(\mathbf{x}^3)$$

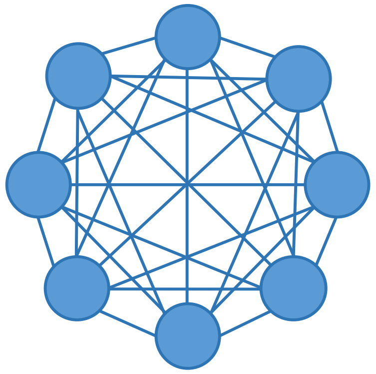
# Boltzmann machine




## 2D square-lattice Ising Model

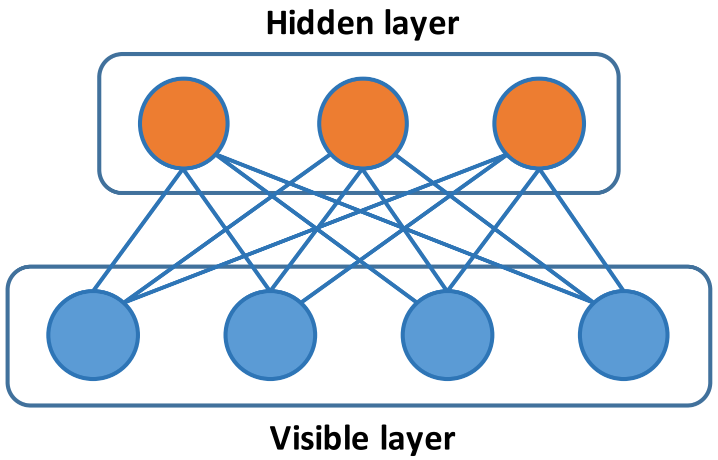


## Sherrington-Kirkpatrick spin glass model

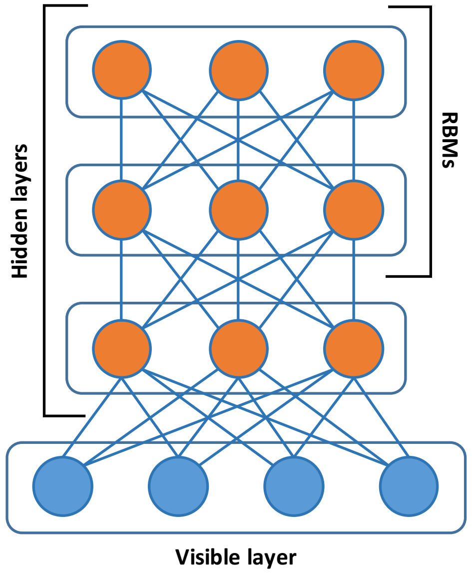


$$\text{Energy}(\mathbf{x}) = -\mathbf{h}^\intercal\mathbf{x} - \mathbf{x}^\intercal\mathbf{J}\mathbf{x}$$

## Restricted Boltzmann machine

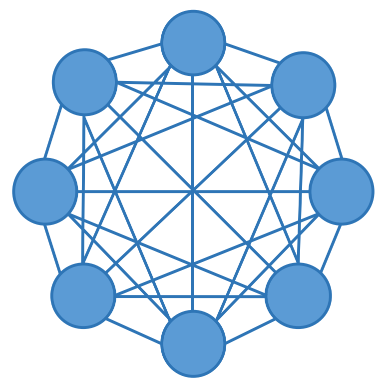



## Deep Boltzmann machine

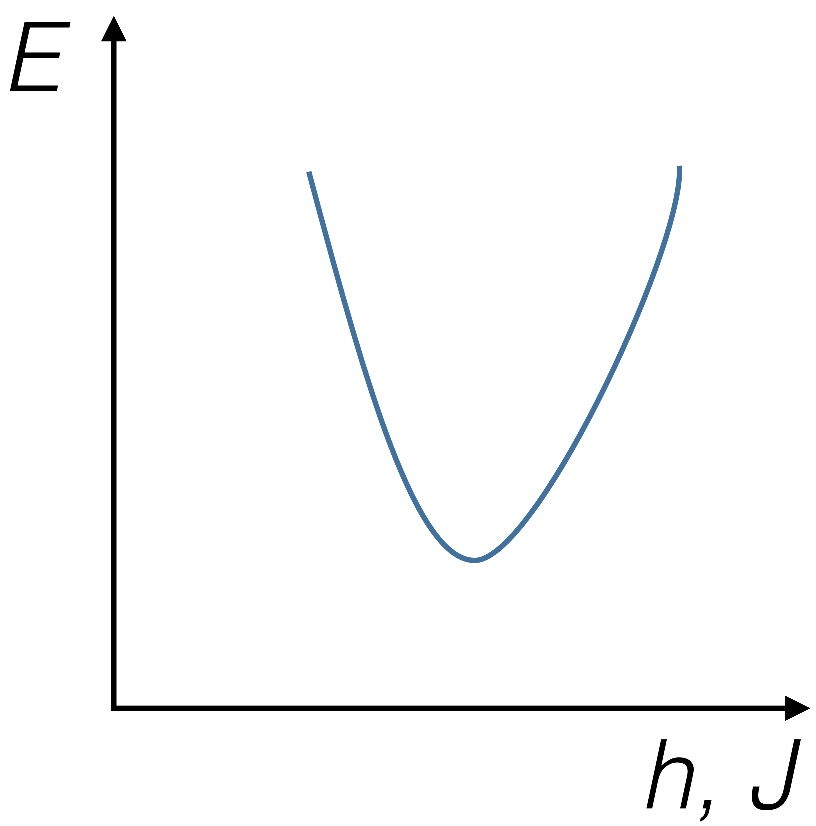


# Inference

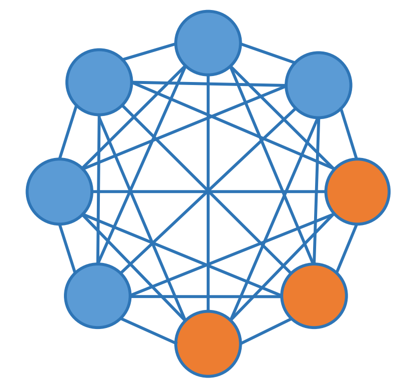
Without hidden nodes



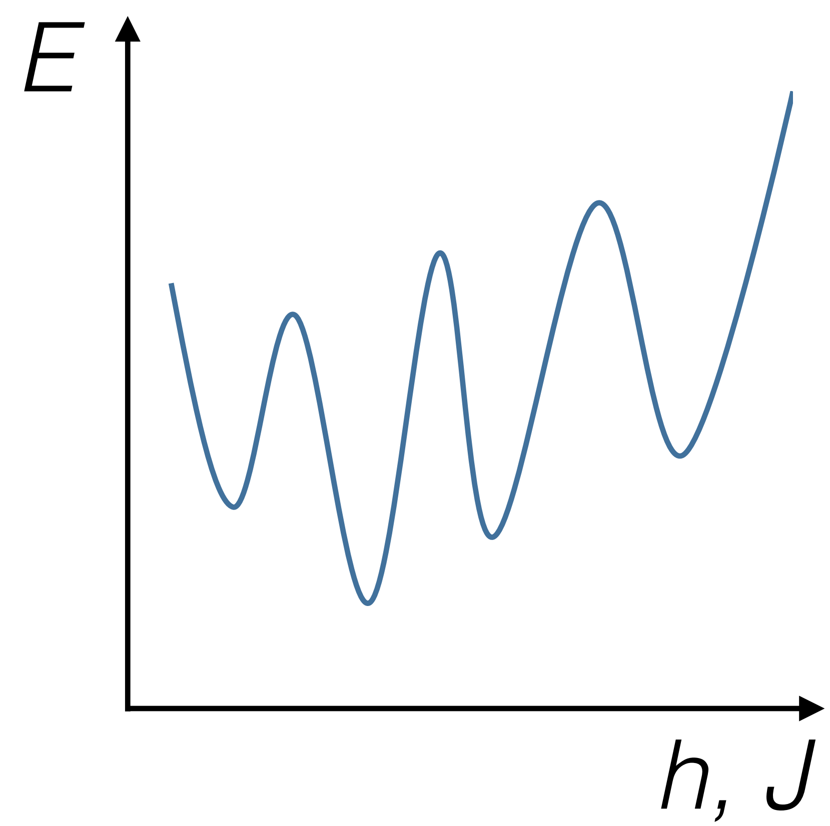
**Convex problem**



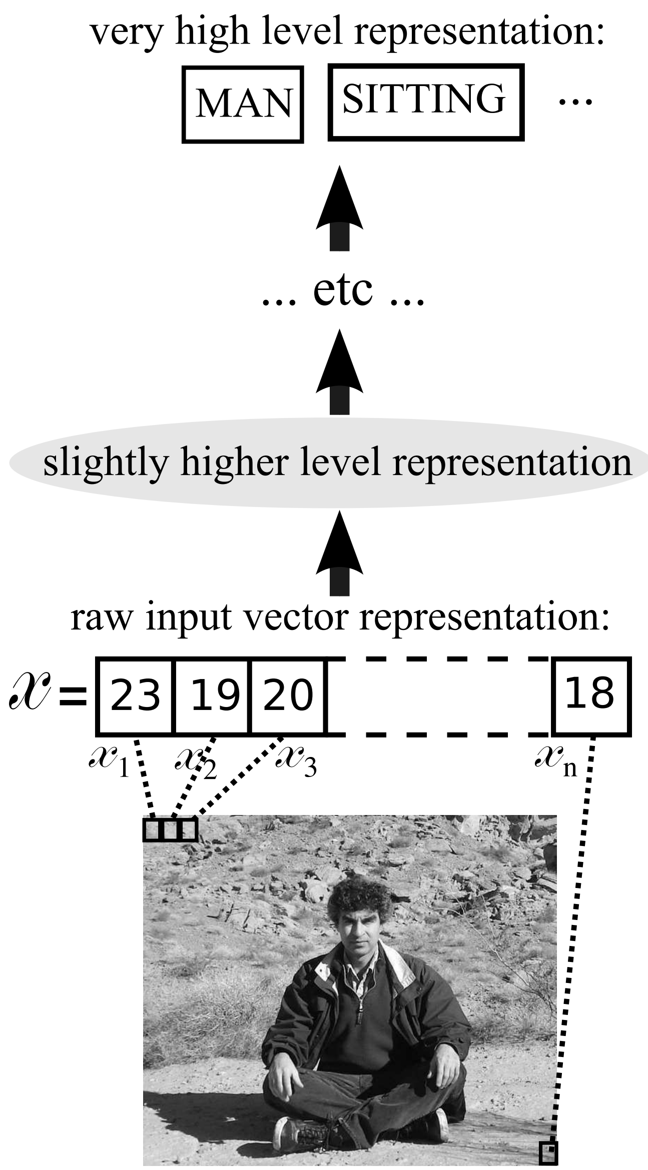

With hidden nodes

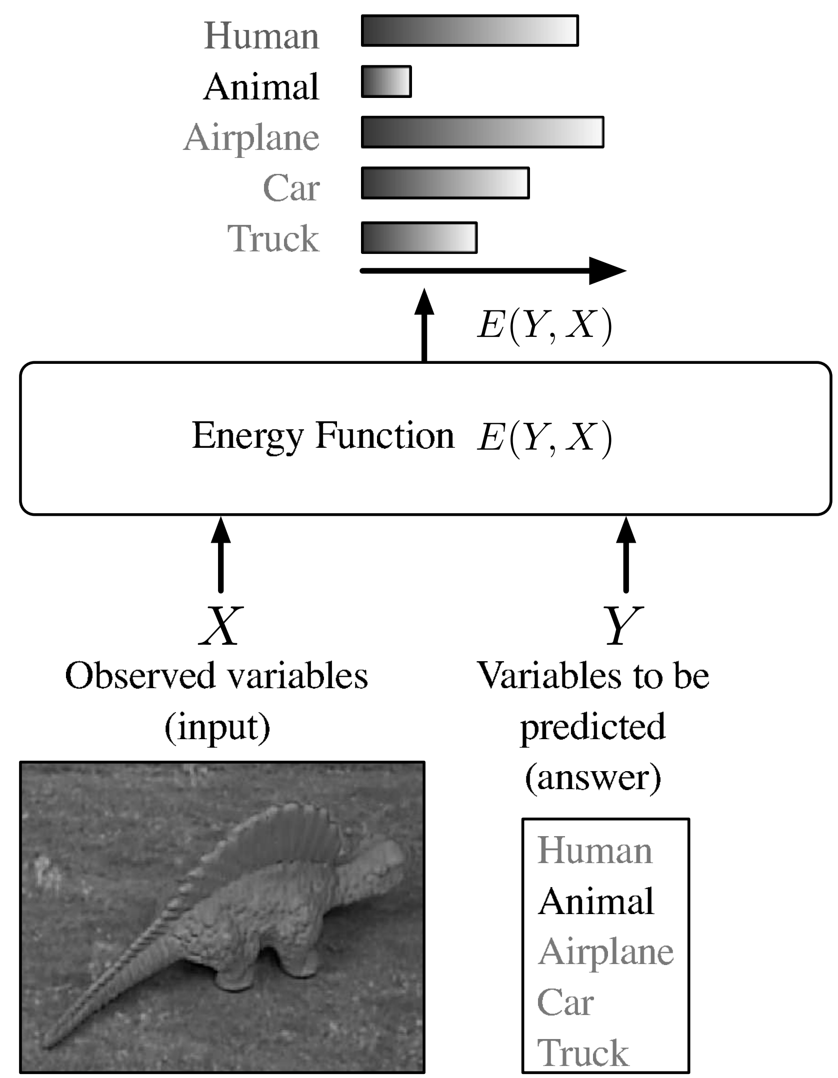


**Non-convex problem**



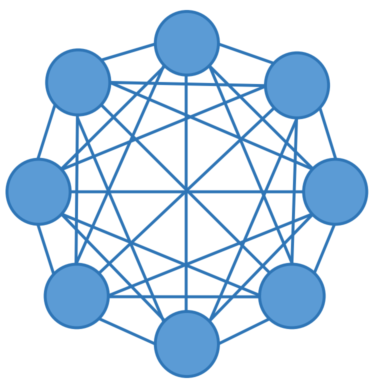
# Deep Boltzmann machine

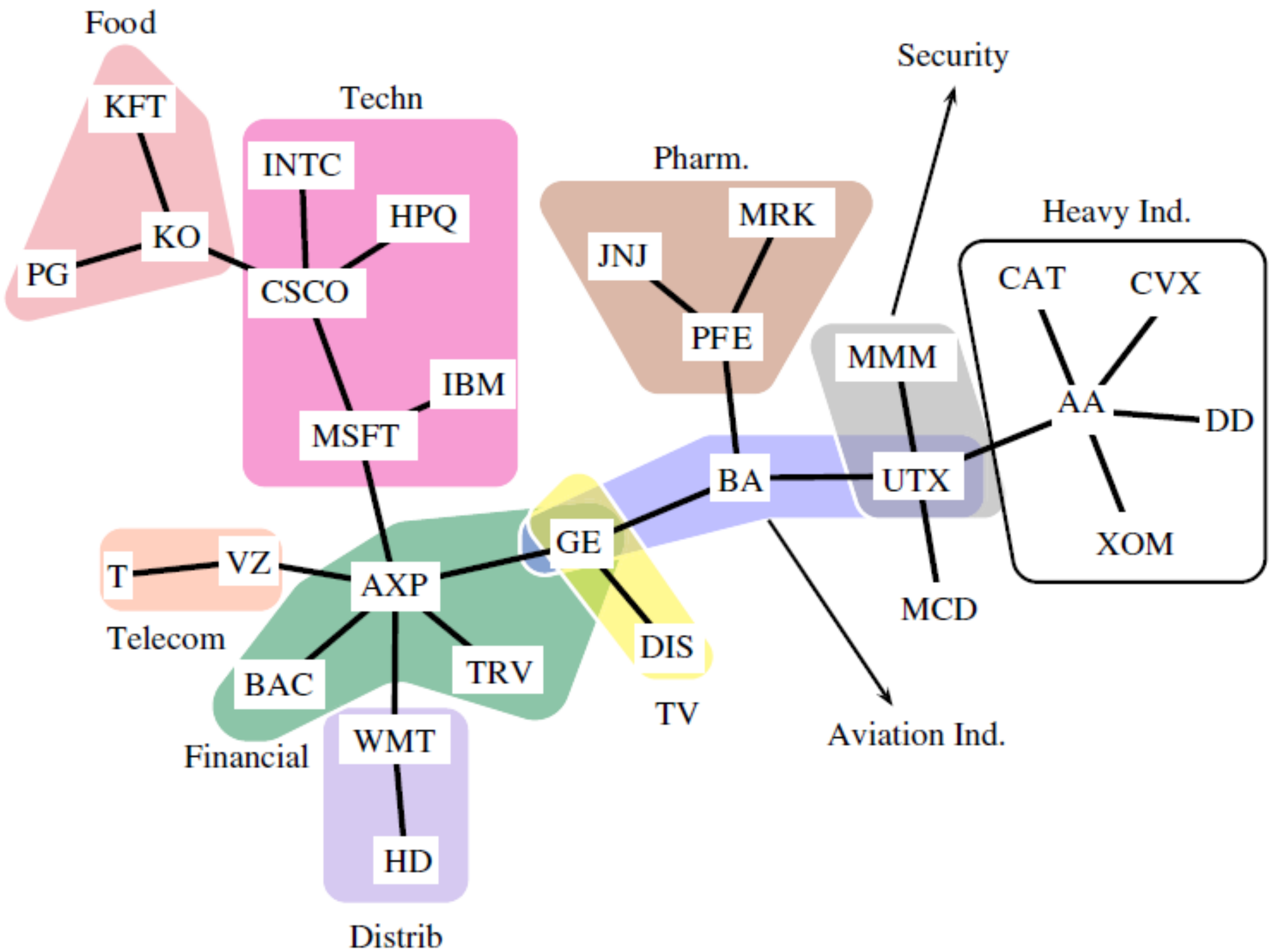


# Stock market correlation clustering structure

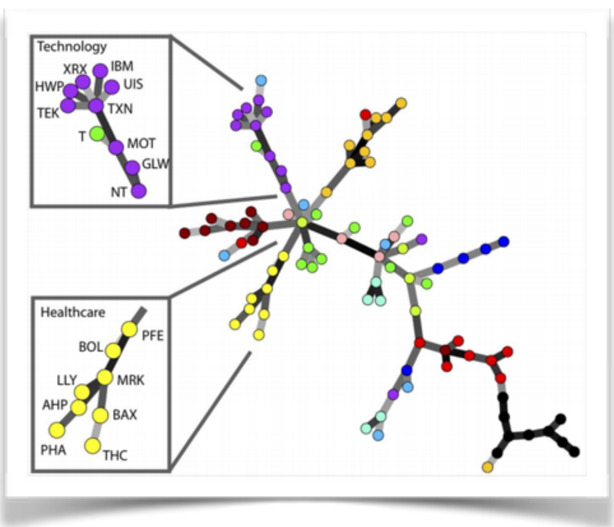
**Spin glass model**

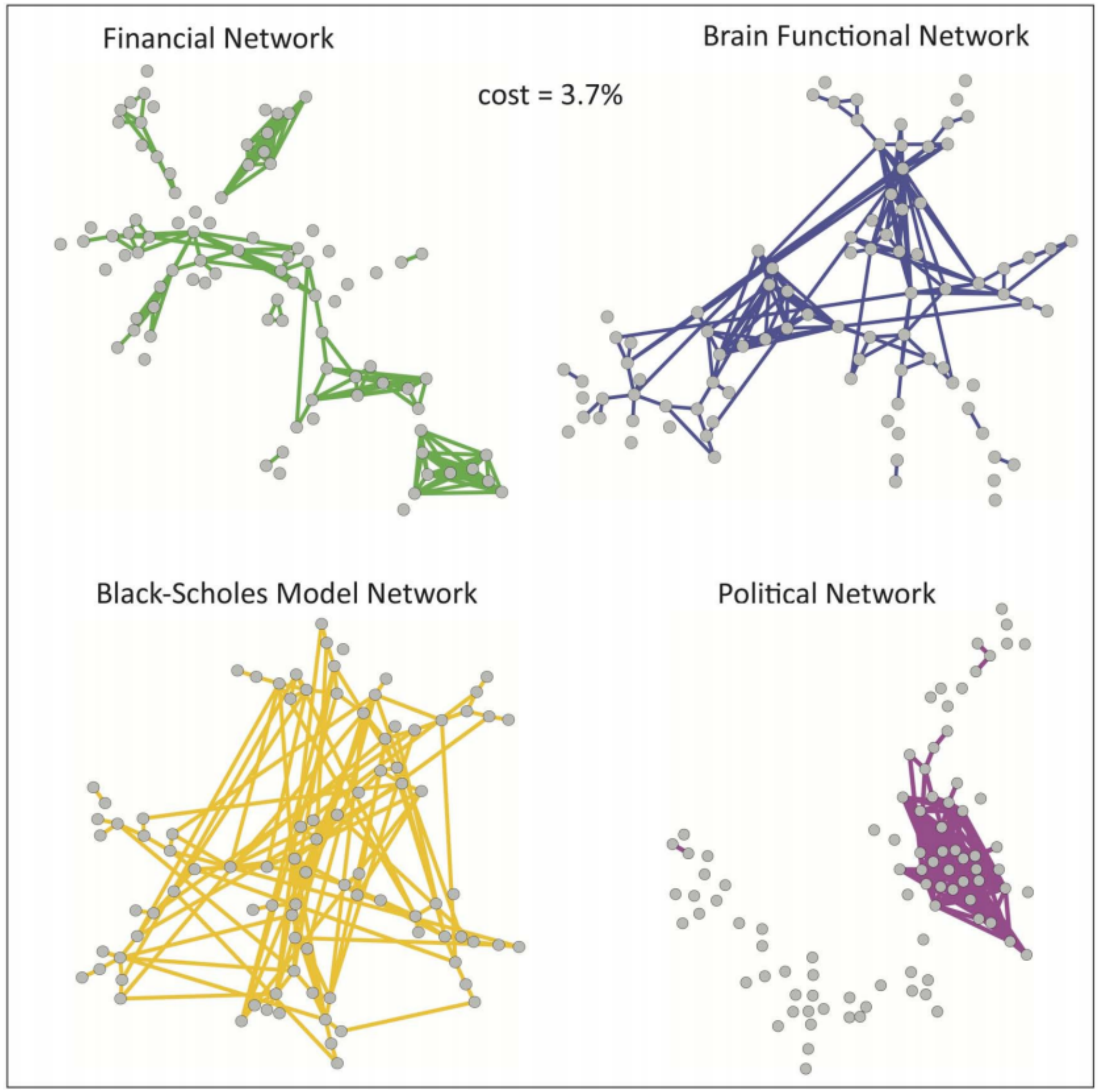


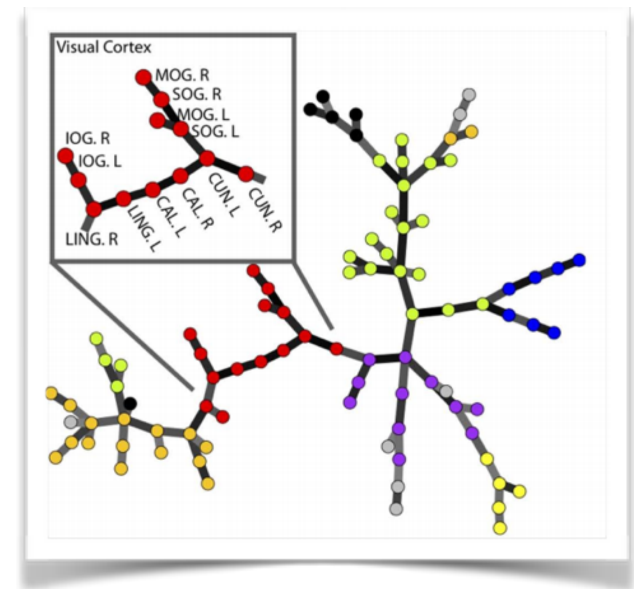



R. Mantegna, Eur. Phys. J. B 11, 193 (1999)

# Financial networks vs neural networks




**Similar:**
**Non-random, small-world, modular, hierarchical, fat-tailed degree distribution**

**However financial networks:**
**Less robust to disintegration (“too big to fail” nodes)**

Vértes PE, et al, Front. Syst. Neurosci. 5:75 (2011)

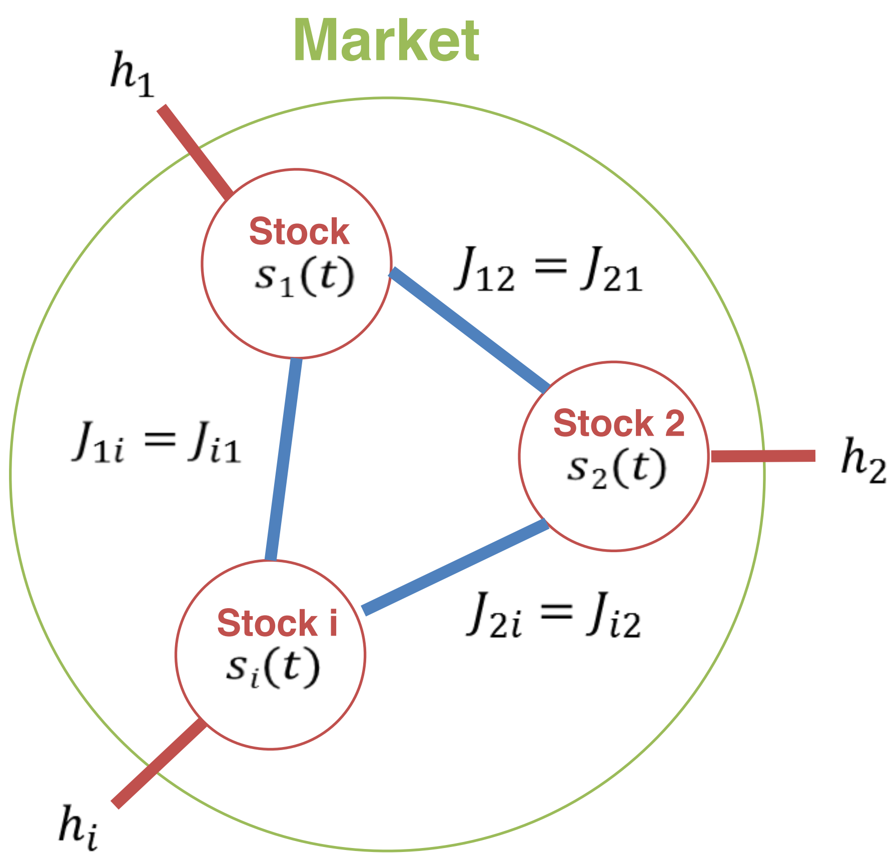
# Joint distribution of stock prices

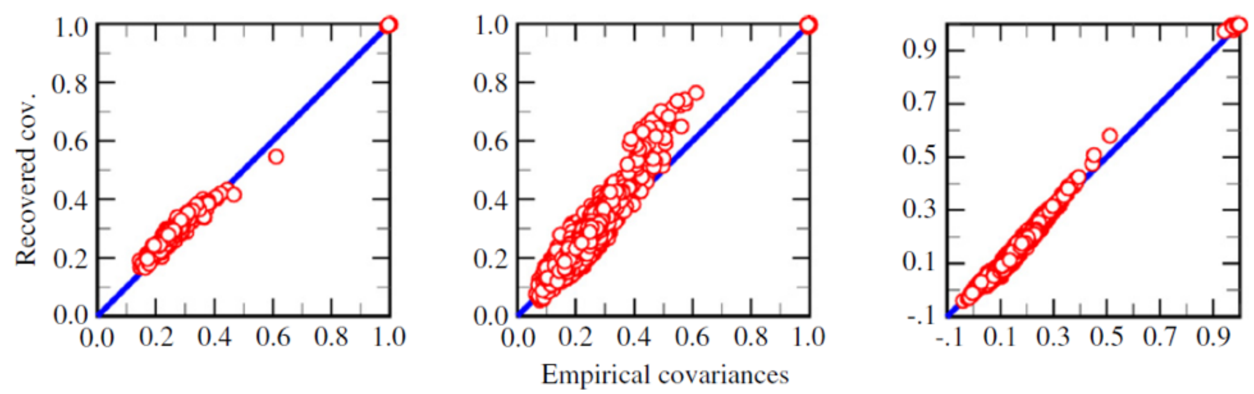
**Equilibrium distribution**

$$p(\mathbf{s}) = \mathcal{Z}^{-1} \exp\{-\mathcal{H}(\mathbf{s})\}$$

**No hidden nodes**

$$\mathcal{H}(\mathbf{s}) = -\mathbf{h}^{\intercal}\mathbf{s} - \mathbf{s}^{\intercal}\mathbf{J}\mathbf{s}$$



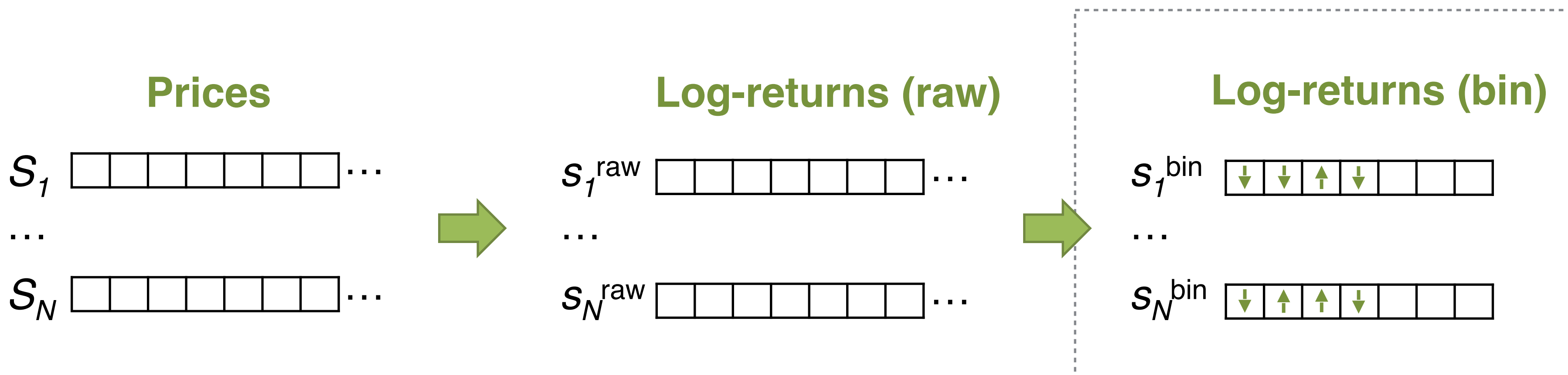




T. Bury, Eur. Phys. J. B 86, 89 (2013); T. Bury, Physica A 392, 1375 (2013); H. Zeng et al, arXiv:1311.3871v1 (2013)

# Outline

# Stock prices time-series

**N=71 stocks from the S&P 500 index**
**Approximately 5000 trading days for 1993-2013**



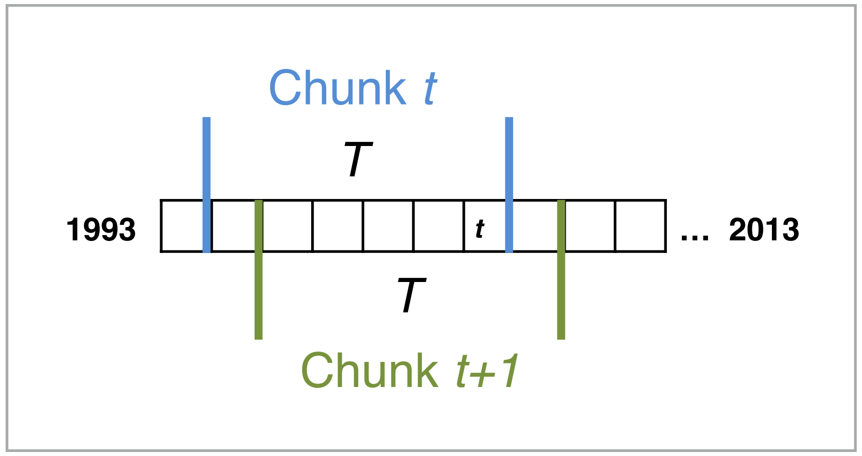

**log-returns**

$$s_i^{\mathrm{raw}}(t) = \ln[S_i(t)/S_i(t-1)]$$

**Binarized log-returns**

$$s_i^{\mathrm{bin}} = \mathrm{sign}(s_i^{\mathrm{raw}})$$

# Moving window approach




Simple moving average (SMA)

$$\langle s_i \rangle := \frac{1}{T} \sum_{t=0}^{T-1} s_i(t)$$

Covariance matrix

$$C_{ij} \equiv \sigma_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

Variance

$$\sigma_i^2 \equiv \sigma_{ii}$$

Correlation matrix

$$Q_{ij} = \sigma_{ij} / \sigma_i \sigma_j$$

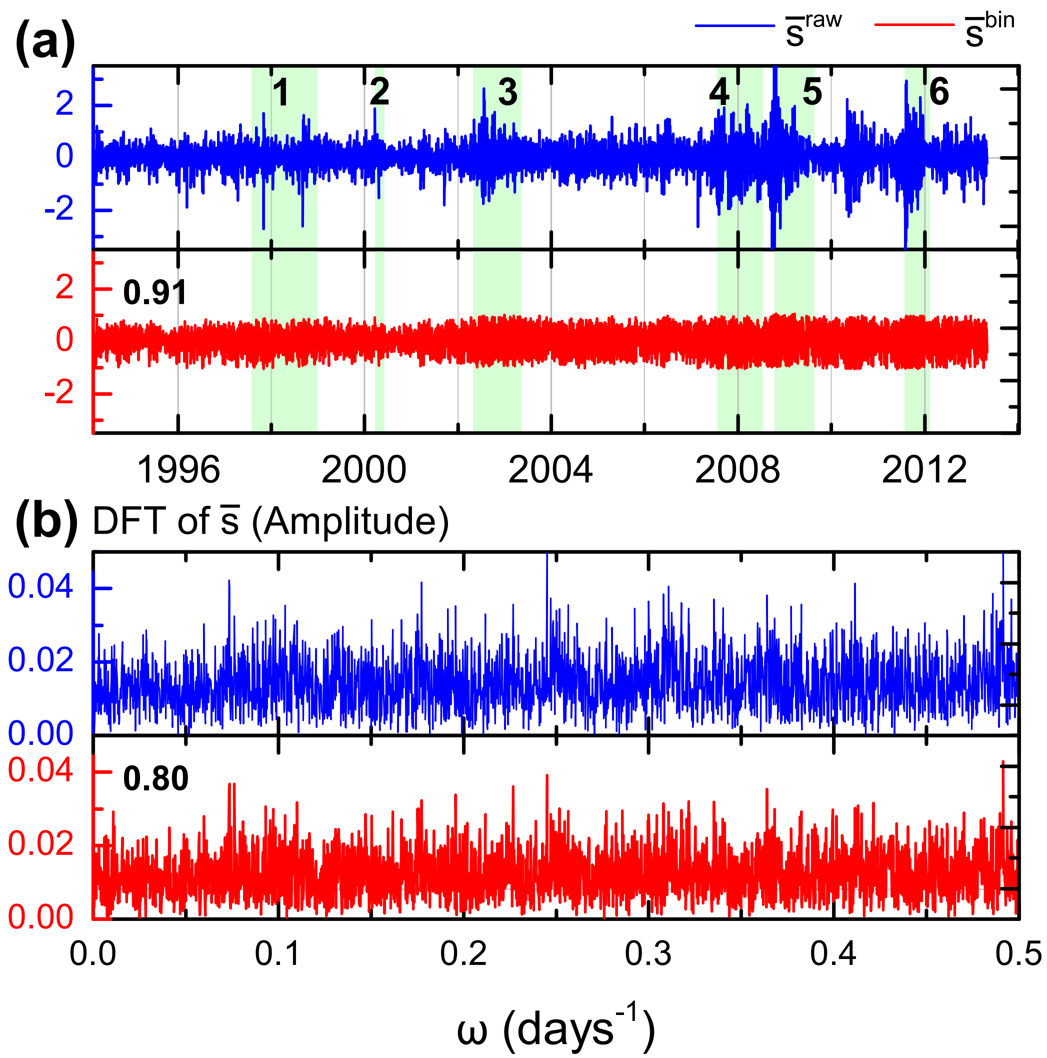
Skewness

$$\gamma_{1i} := \left\langle \left( \frac{s_i - \langle s_i \rangle}{\sigma_i} \right)^3 \right\rangle$$

Kurtosis

$$\gamma_{2i} := \frac{\left\langle (s_i - \langle s_i \rangle)^4 \right\rangle}{\sigma_i^4} - 3$$

# Effect of binarization: Average return

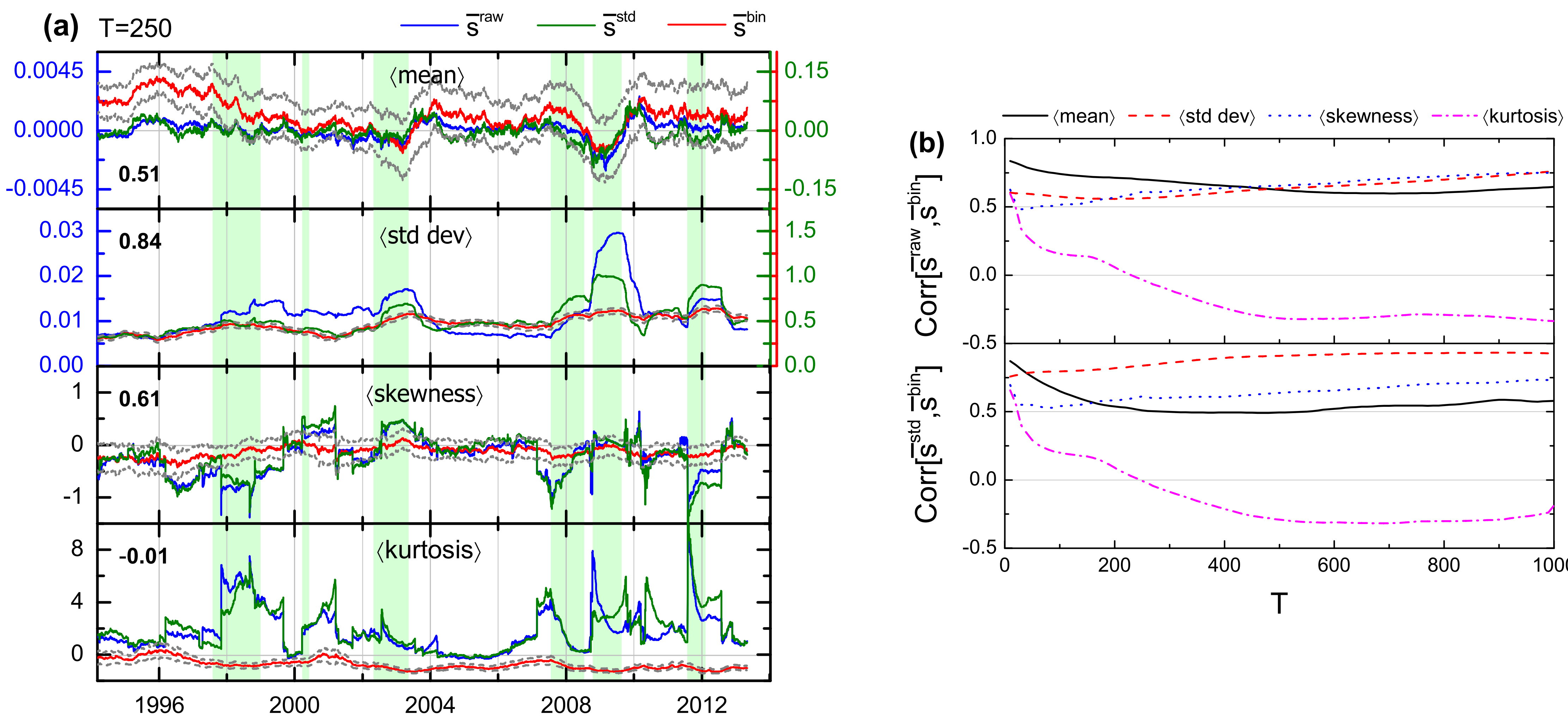



Amplitude is bounded

Cycles are preserved

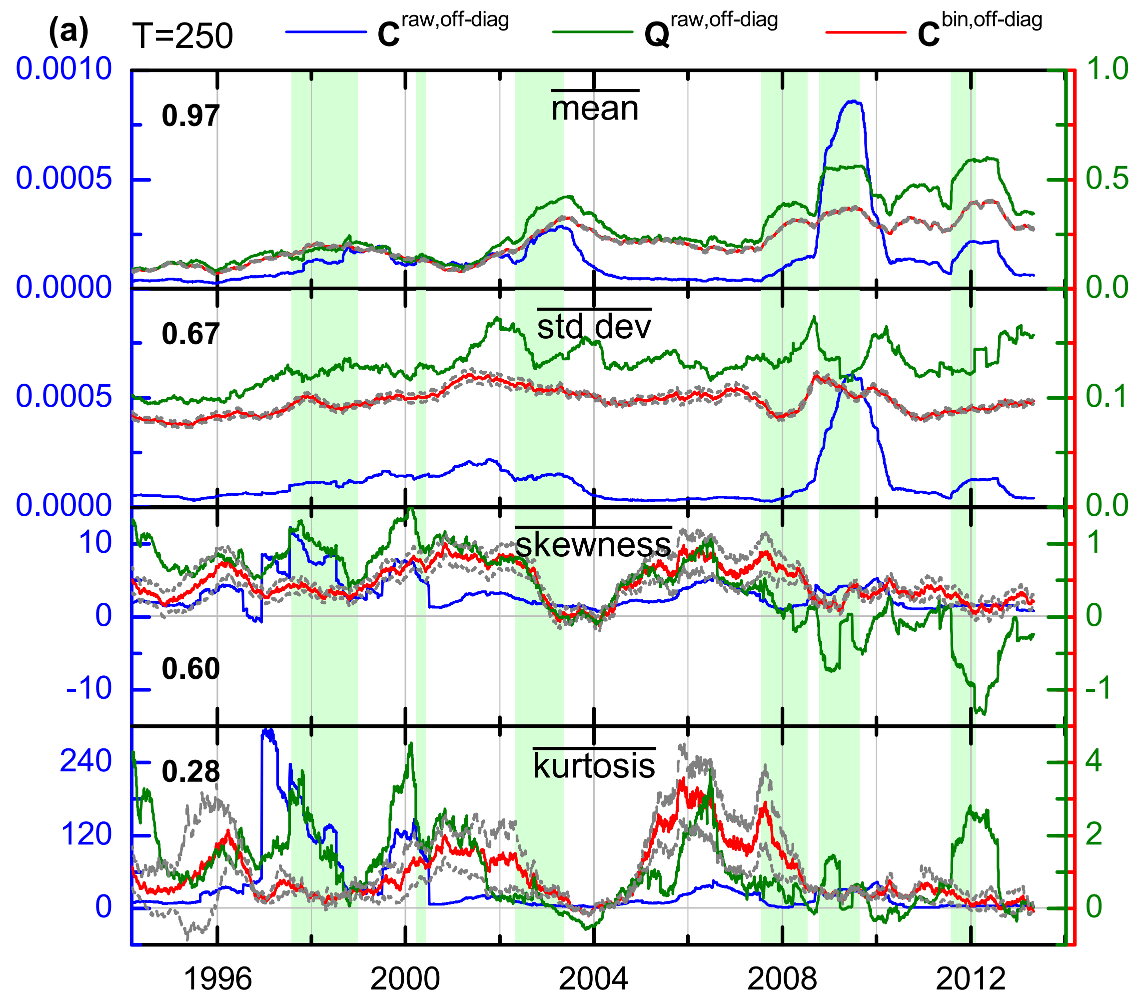
1. Asian and Russian crisis
2. Dot-com bubble
3. US stock market downturn of 2002
4. US housing bubble
5. Global financial crisis
6. European sovereign debt crisis

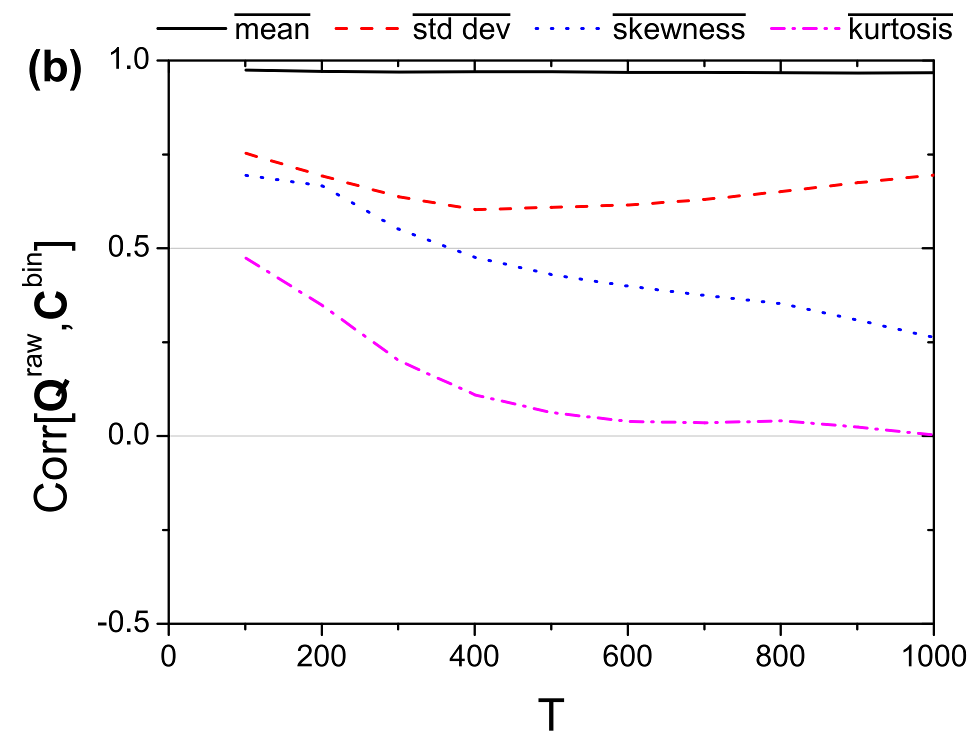
# Effect of binarization: Average return (SMA, T=250)




1. Asian and Russian crisis
2. Dot-com bubble
3. US stock market downturn of 2002
4. US housing bubble
5. Global financial crisis
6. European sovereign debt crisis

# Effect of binarization: Covariance matrix **C** (SMA, T=250)




1. Asian and Russian crisis
2. Dot-com bubble
3. US stock market downturn of 2002
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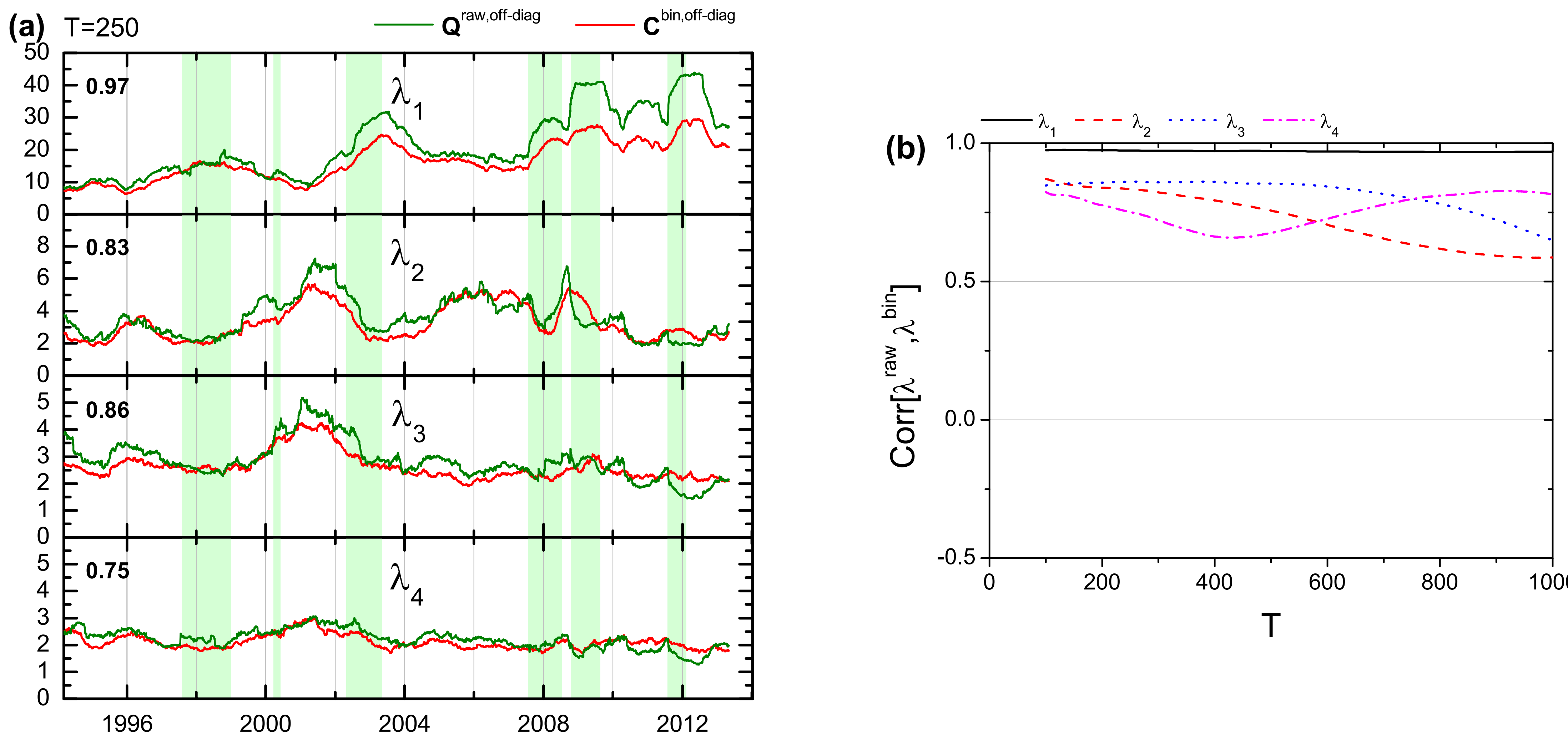
6. European sovereign debt crisis

# Effect of binarization: Eigenvalues of **C** (SMA, T=250)

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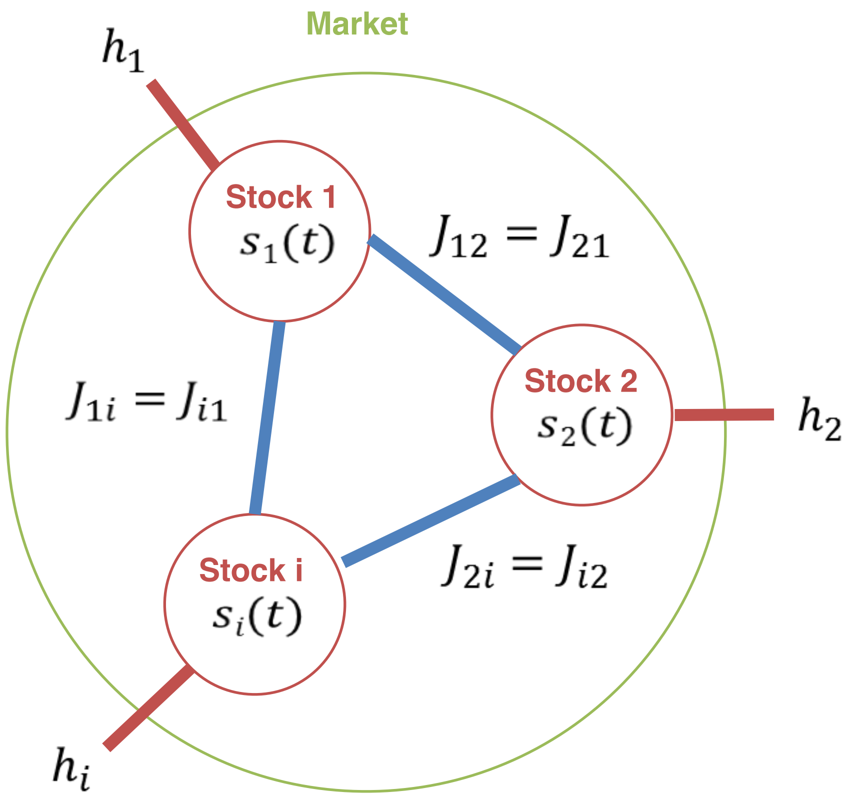
# Statistical inference (learning): Exact

For each moving window
Infer **h** and **J** so that

$$\begin{aligned} \langle s_i \rangle_{\text{data}} &= \langle s_i \rangle_{\text{model}} \\ \langle s_i s_j \rangle_{\text{data}} &= \langle s_i s_j \rangle_{\text{model}} \end{aligned}$$

**Exact inference**

$$\begin{aligned} \delta h_i &= \eta_h \left( \langle s_i \rangle_{\text{data}} - \langle s_i \rangle_{\text{model}} \right) \\ \delta J_{ij} &= \eta_J \left( \langle s_i s_j \rangle_{\text{data}} - \langle s_i s_j \rangle_{\text{model}} \right) \end{aligned}$$



# Statistical inference (learning): Approximations

## Mean field expansion

### Naïve Mean Field (nMF)

$$\mathbf{J}^{\mathrm{nMF}} = \mathbf{A}^{-1} - \mathbf{C}^{-1}$$

$$A_{ij} = (1 - \langle s_i \rangle^2)\delta_{ij}$$

$$h_i^{\mathrm{nMF}} = \tanh^{-1}\langle s_i \rangle - \sum_{j=1}^{N} J_{ij}\langle s_i \rangle$$

### Thouless-Anderson-Palmer (TAP)

$$\left(\mathbf{C}^{-1}\right)_{ij} = -J_{ij}^{\mathrm{TAP}} - 2\left[J_{ij}^{\mathrm{TAP}}\right]^2 \langle s_i \rangle \langle s_j \rangle$$

$$h_i^{\mathrm{TAP}} = h_i^{\mathrm{nMF}} - \langle s_i \rangle \sum_{j=1}^{N} \left[J_{ij}\right]^2 \left\{1 - \langle s_i \rangle^2\right\}$$

## Small correlation expansion

### Independent pair approximation

$$J_{ij}^{\mathrm{pair}} = \frac{1}{4}\ln\left[\frac{\left(1+m_i+m_j+C_{ij}^*\right)\left(1-m_i-m_j+C_{ij}^*\right)}{\left(1-m_i+m_j-C_{ij}^*\right)\left(1+m_i-m_j-C_{ij}^*\right)}\right],$$

$$h_i^{\mathrm{pair}} = \frac{1}{2}\ln\left(\frac{1+m_i}{1-m_i}\right) - \sum_{j}^{N} J_{ij}^{\mathrm{pair}} m_j + O\left(\beta^2\right)$$

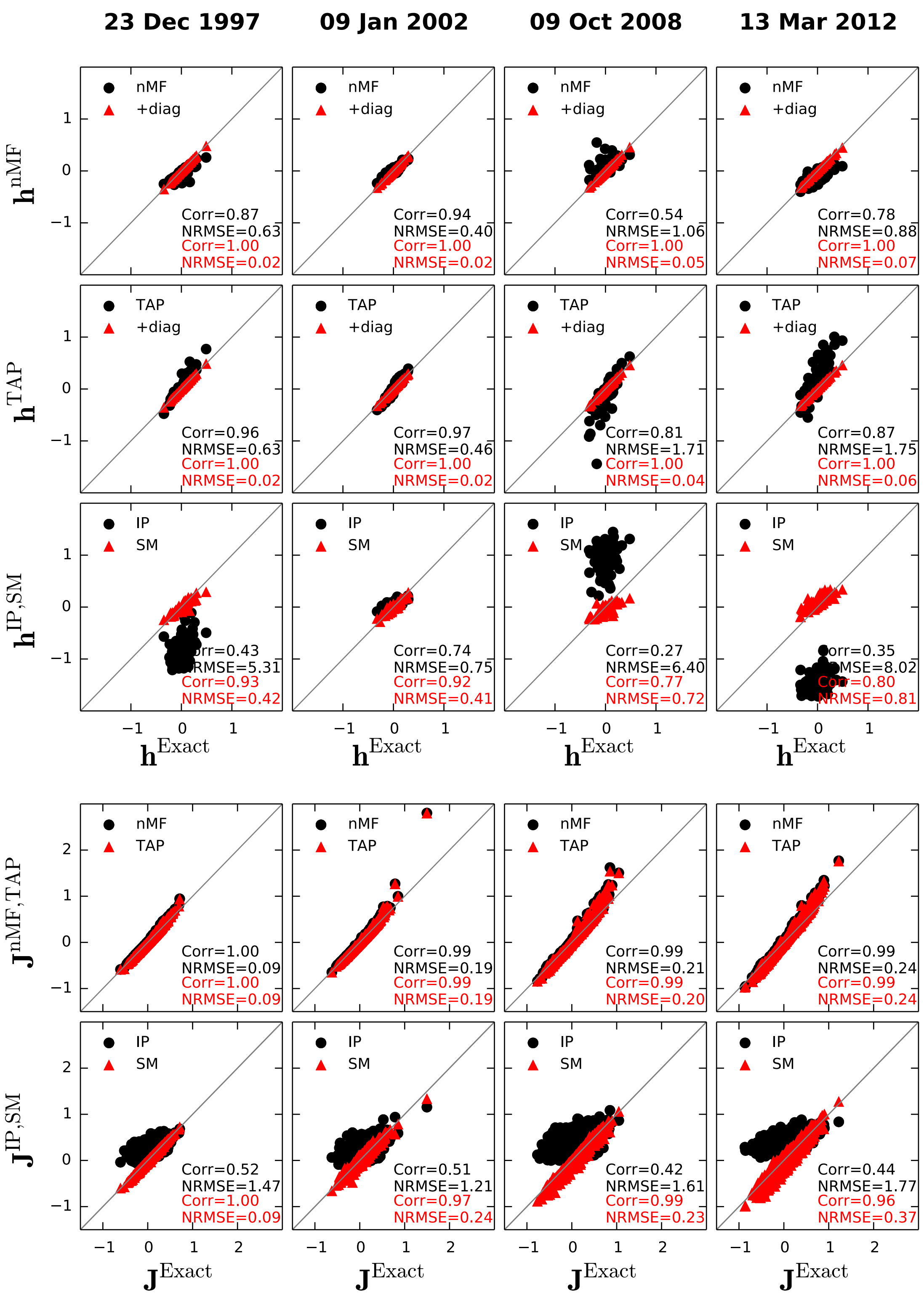
### Sessak-Monasson (SM) correction

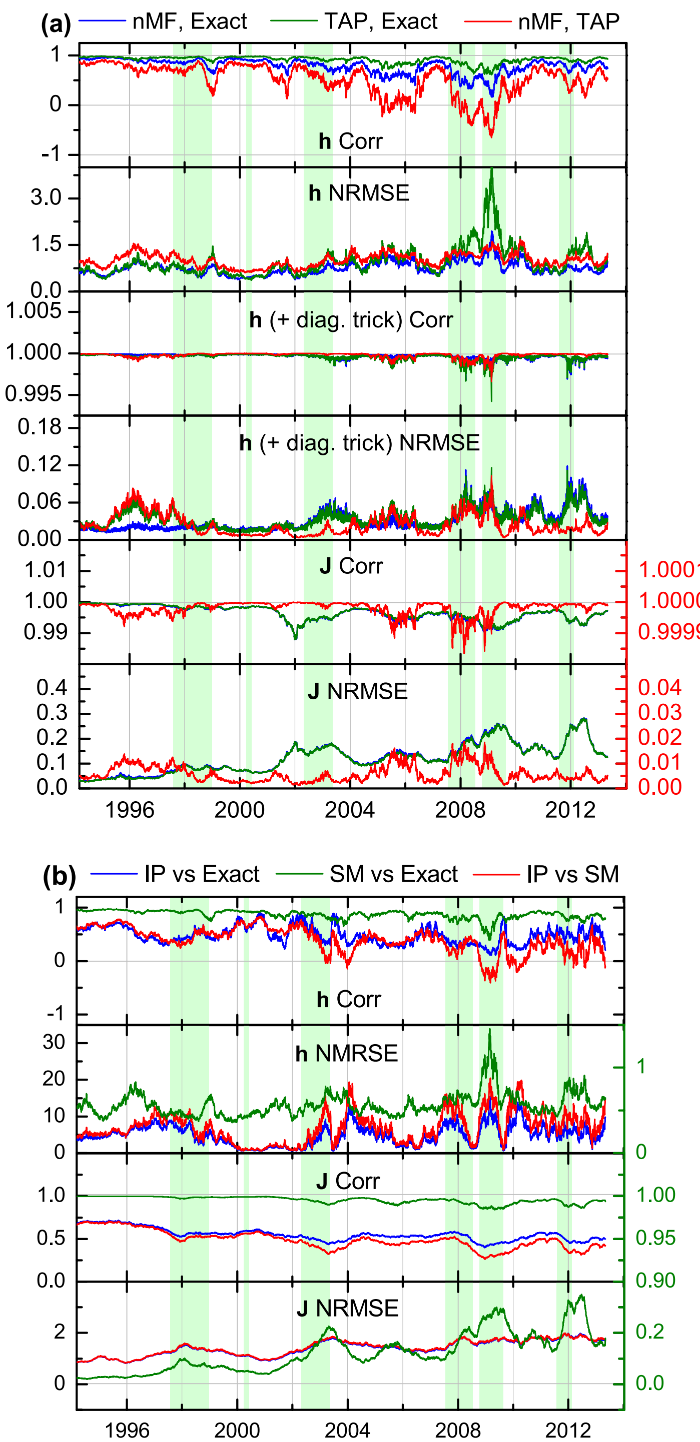
$$J_{ij}^{\mathrm{SM}} = J_{ij}^{\mathrm{nMF}} + J_{ij}^{\mathrm{pair}} - \frac{C_{ij}}{\left(1-m_i^2\right)\left(1-m_j^2\right)-\left(C_{ij}\right)^2},$$

$$h_i^{\mathrm{SM}} = h_i^{\mathrm{pair}}$$

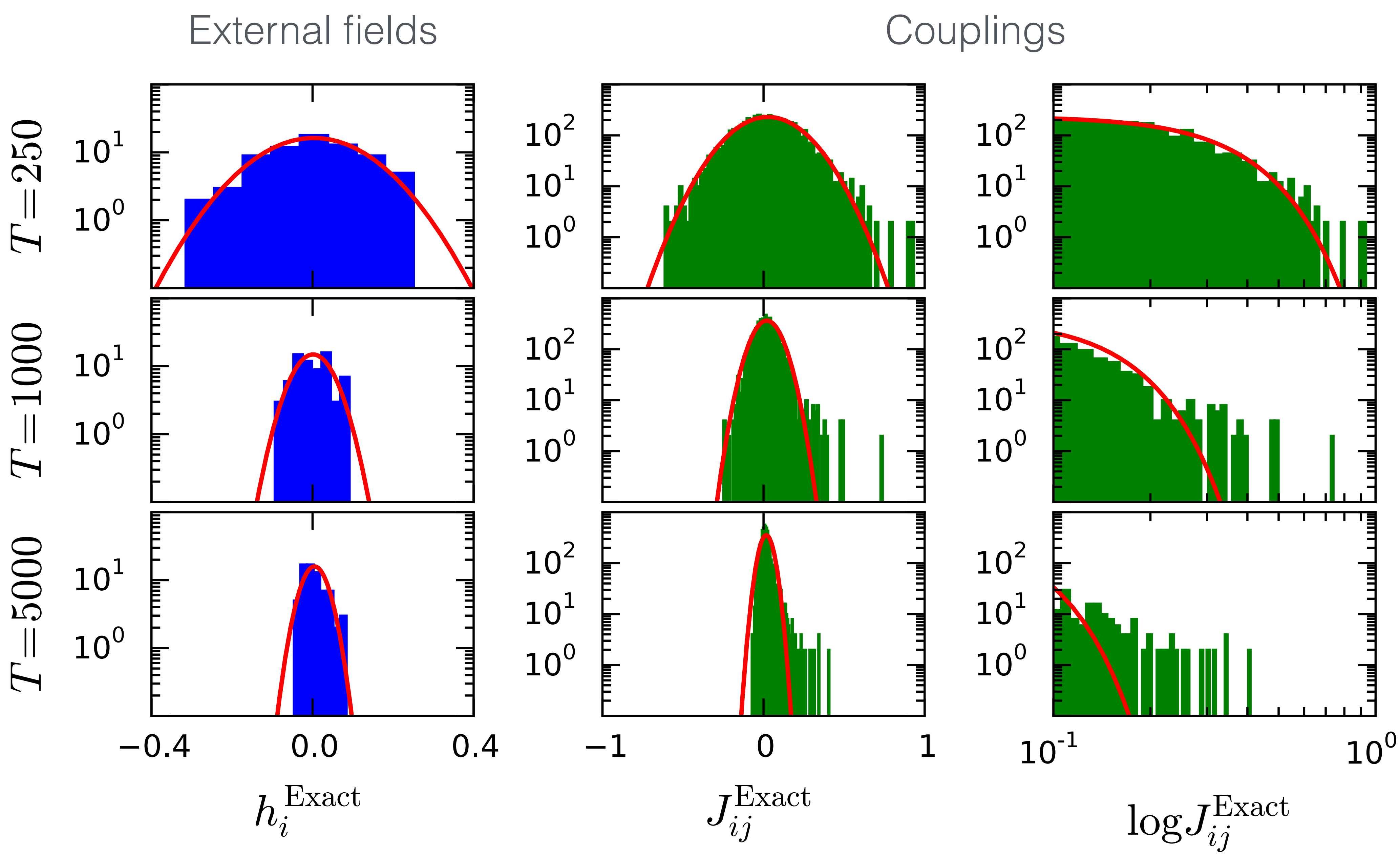

T. Tanaka, Phys. Rev. E 58, 2302 (1998)
Y. Roudi and J. Hertz, PRL 106, 048702 (2011)
V. Sessak and R. Monasson, J. of Phys. A 42, 055001 (2009)

# Accuracy of approximate inference



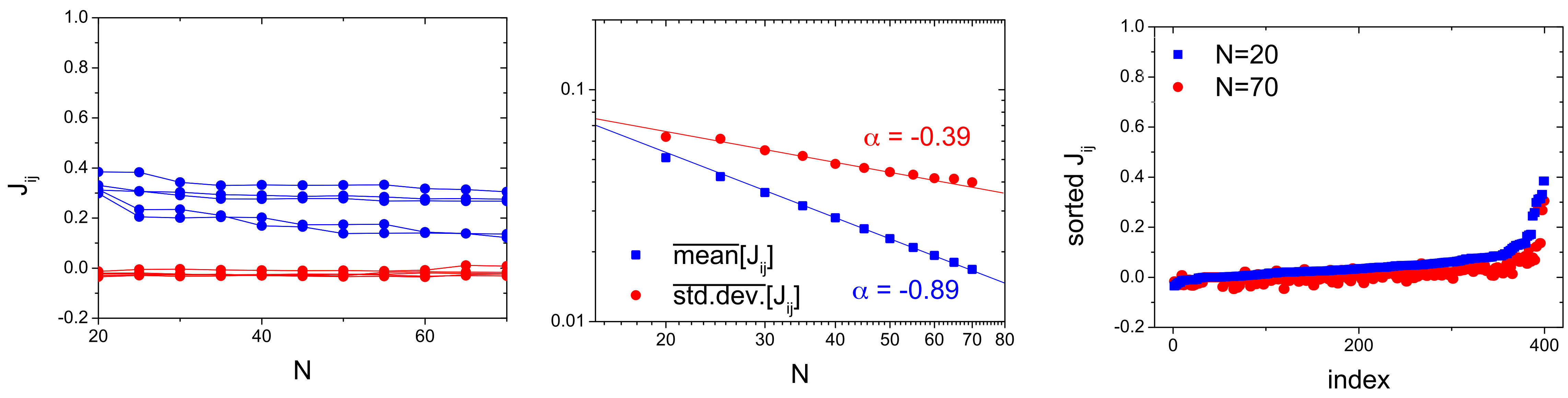



# Distribution of exact external fields and couplings



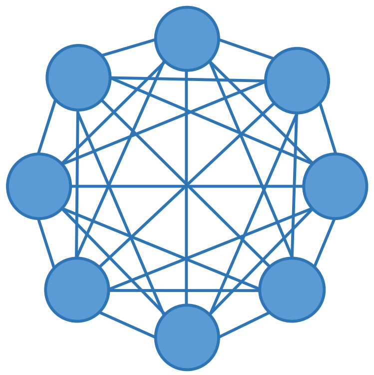

27 Jan 2010

# Couplings structure: Scaling a subset

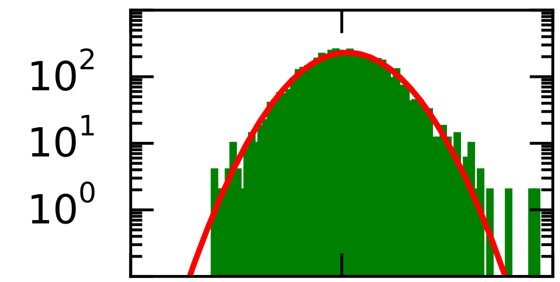


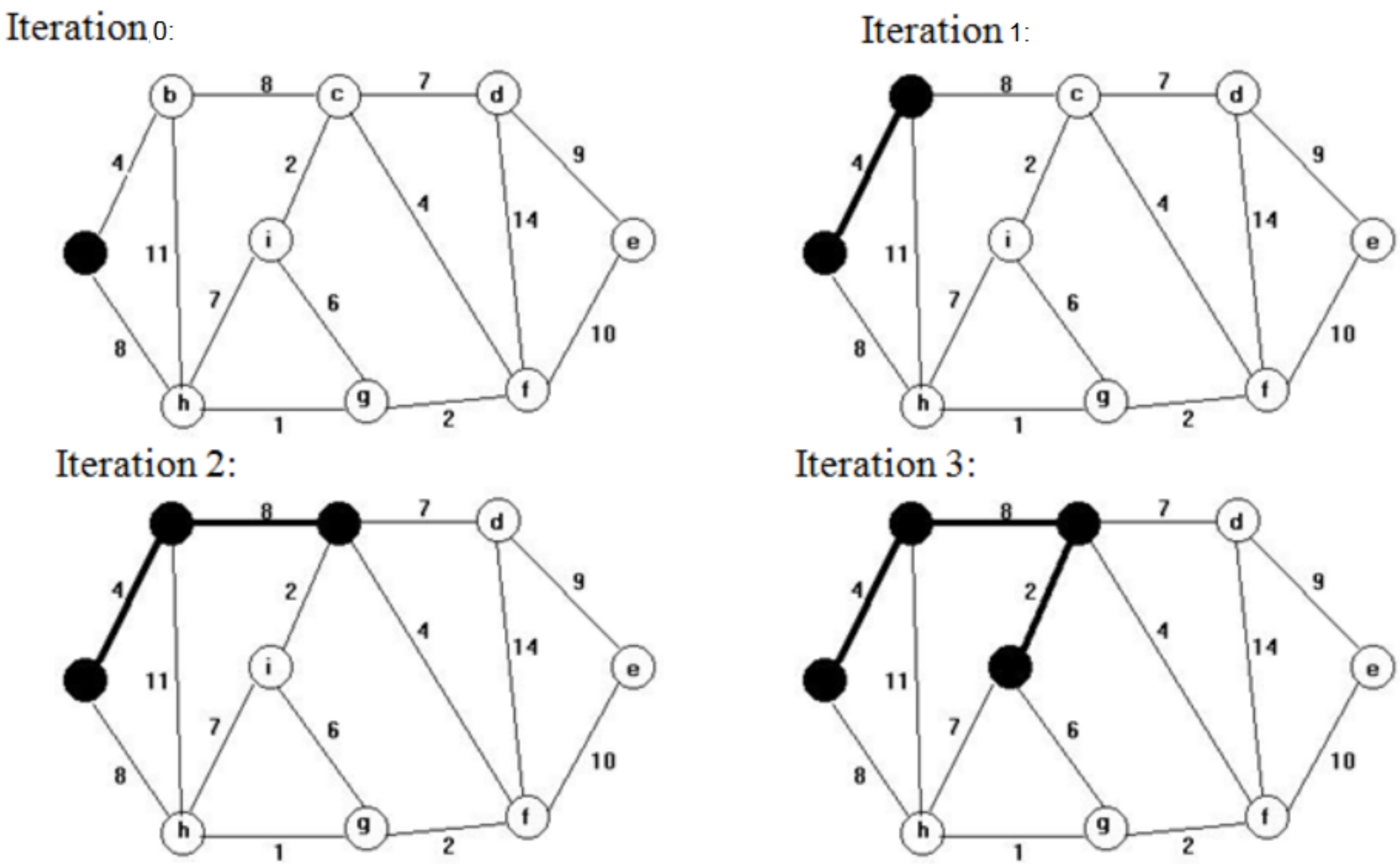

27 Jan 2010, T=5000, a fixed subset of 20 stocks

# Outline



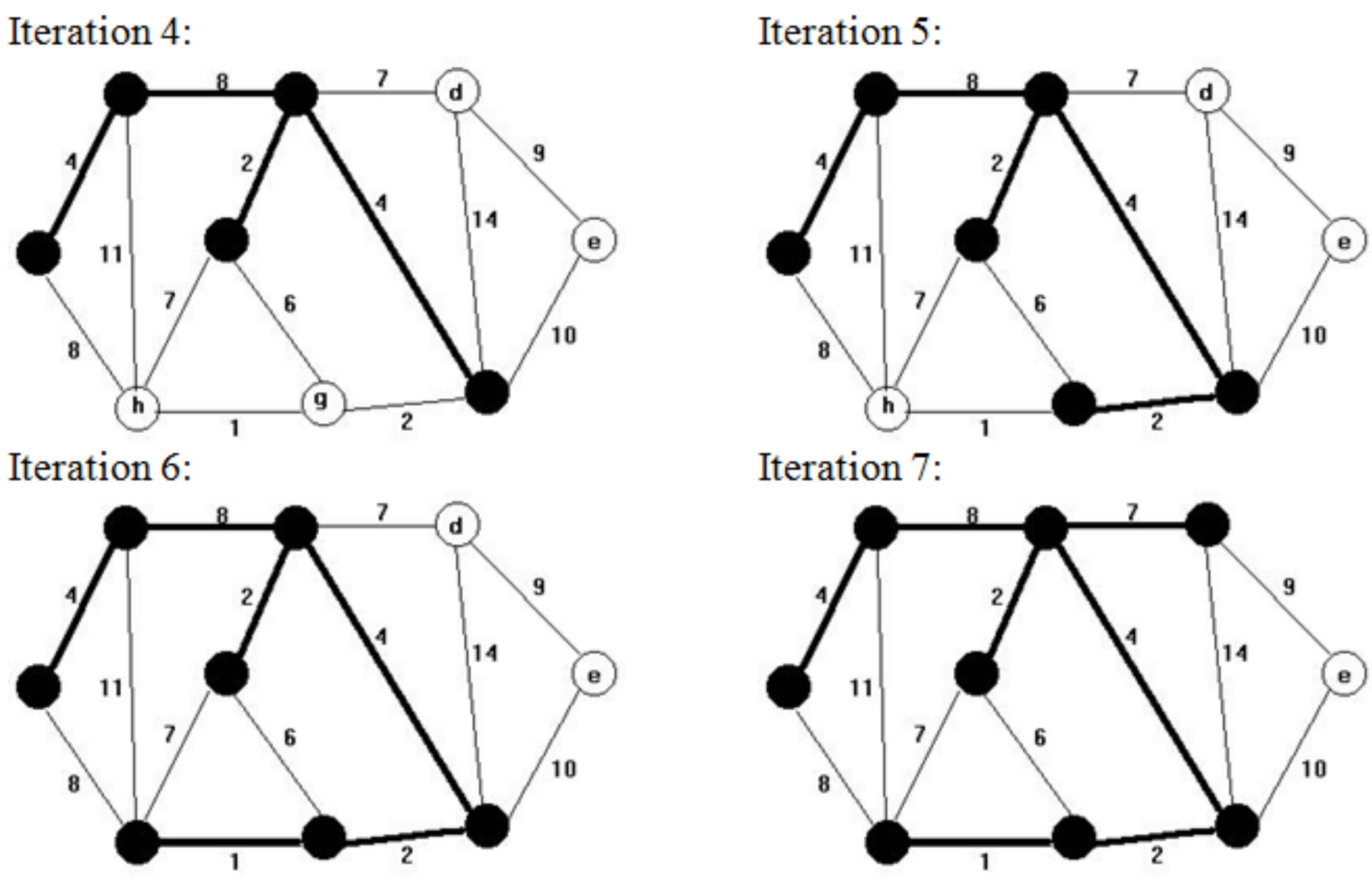
# Stock market clustering structure (minimum spanning tree)




http://www.tutorvista.com/content/math/prims-algorithm/

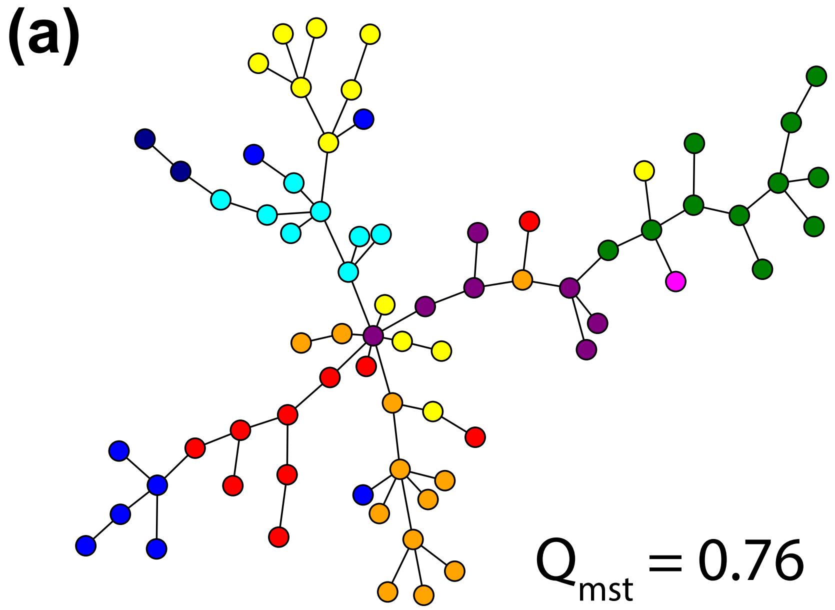
# Stock market clustering structure (minimum spanning tree)



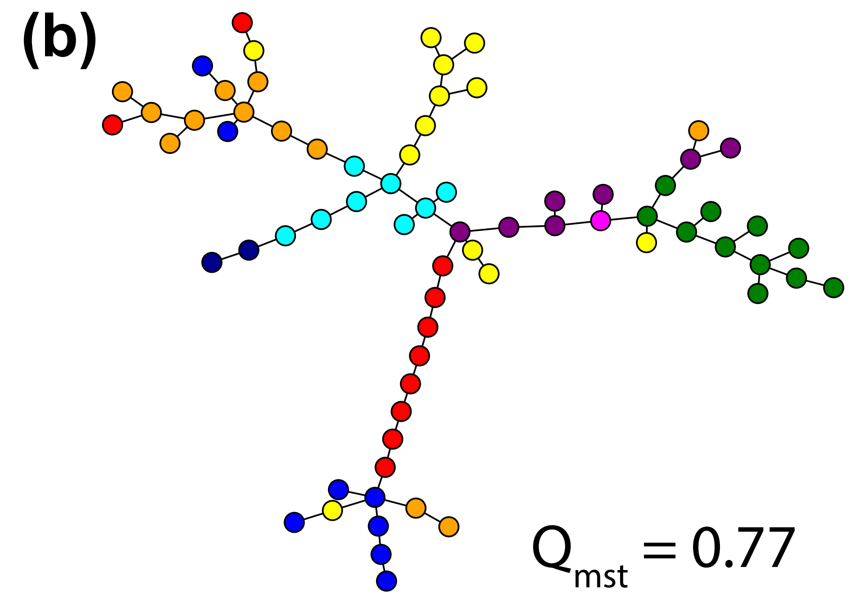

http://www.tutorvista.com/content/math/prims-algorithm/

# Stock market clustering structure (maximum spanning tree)

Correlations




Couplings



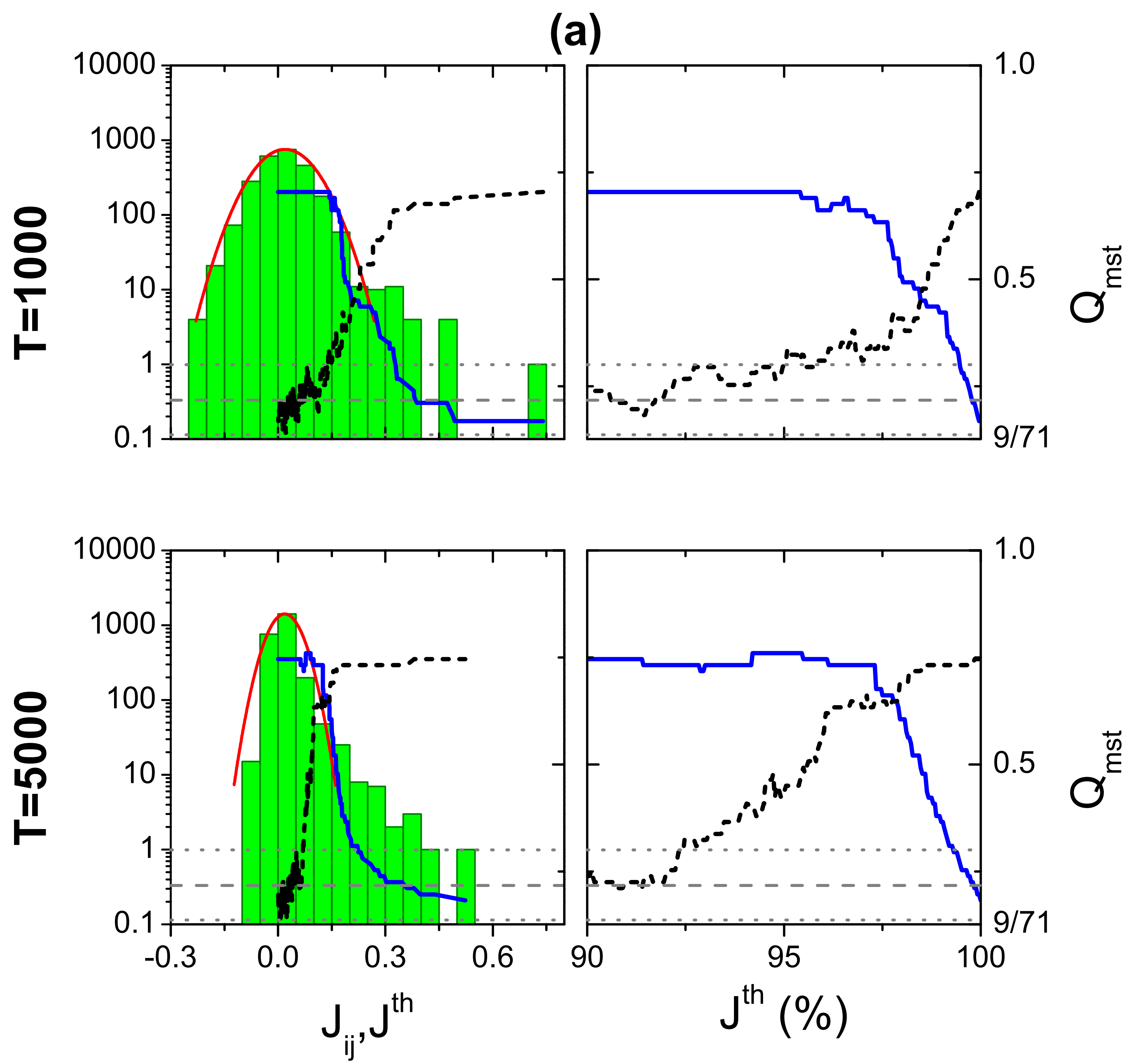

Healthcare (red), Consumer Goods (blue), Basic Materials (green), Financial (cyan), Industrial Goods (purple), Services (yellow), Technology (orange), Conglomerate (magenta) and Utilities (dark blue)

Clustering degree using industry sectors

$$Q_{\rm mst} = \frac{1}{N}\sum_{m=1}^{M} \max_k N_{m,k}$$

S. Borysov et al, Eur. Phys. J. B (2015) 88: 321

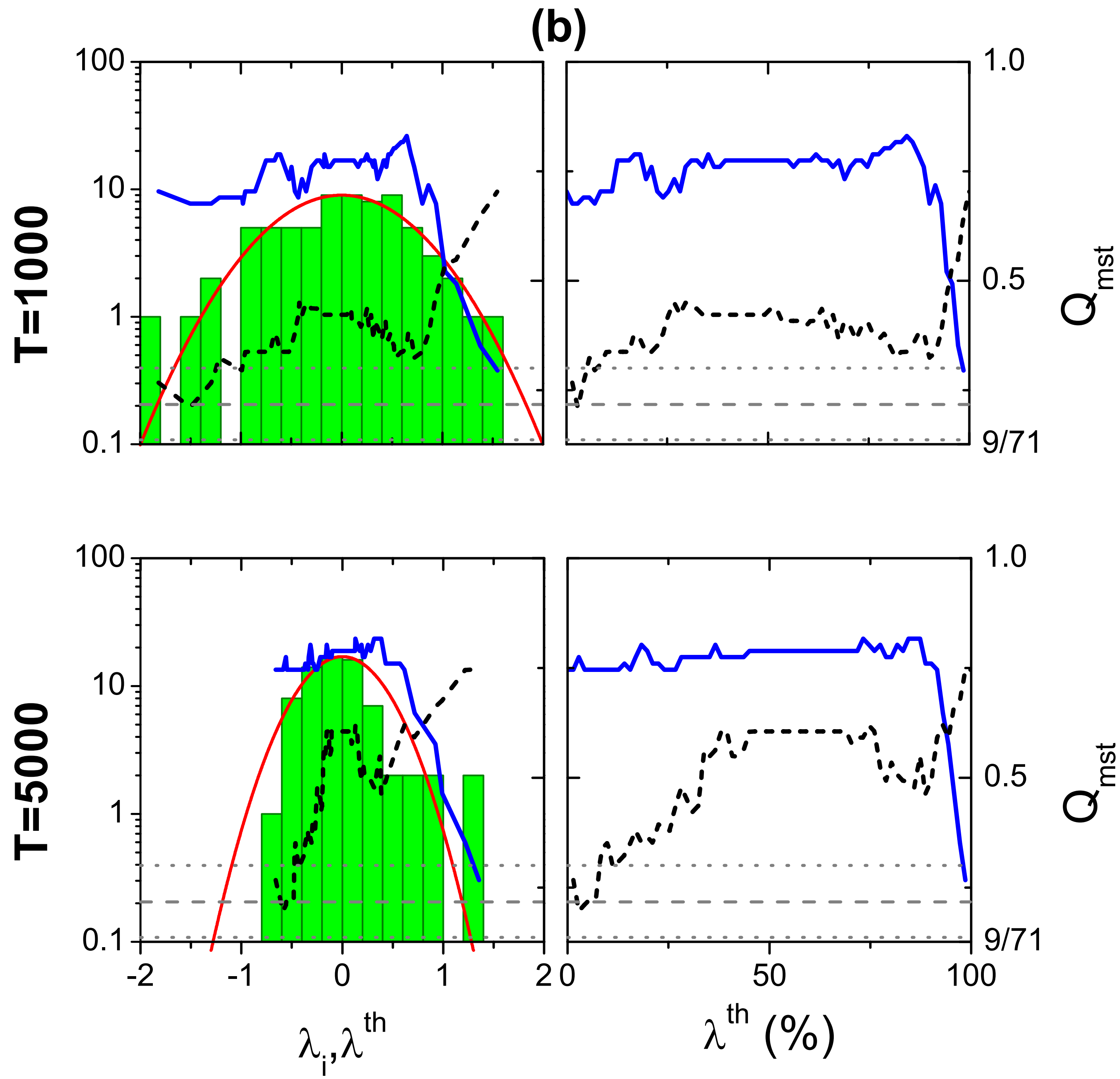
# Clustering degree: Filtering biggest / smallest couplings




27 Jan 2010

S. Borysov et al, Eur. Phys. J. B (2015) 88: 321

# Clustering degree: Filtering biggest / smallest coupling matrix eigenmodes

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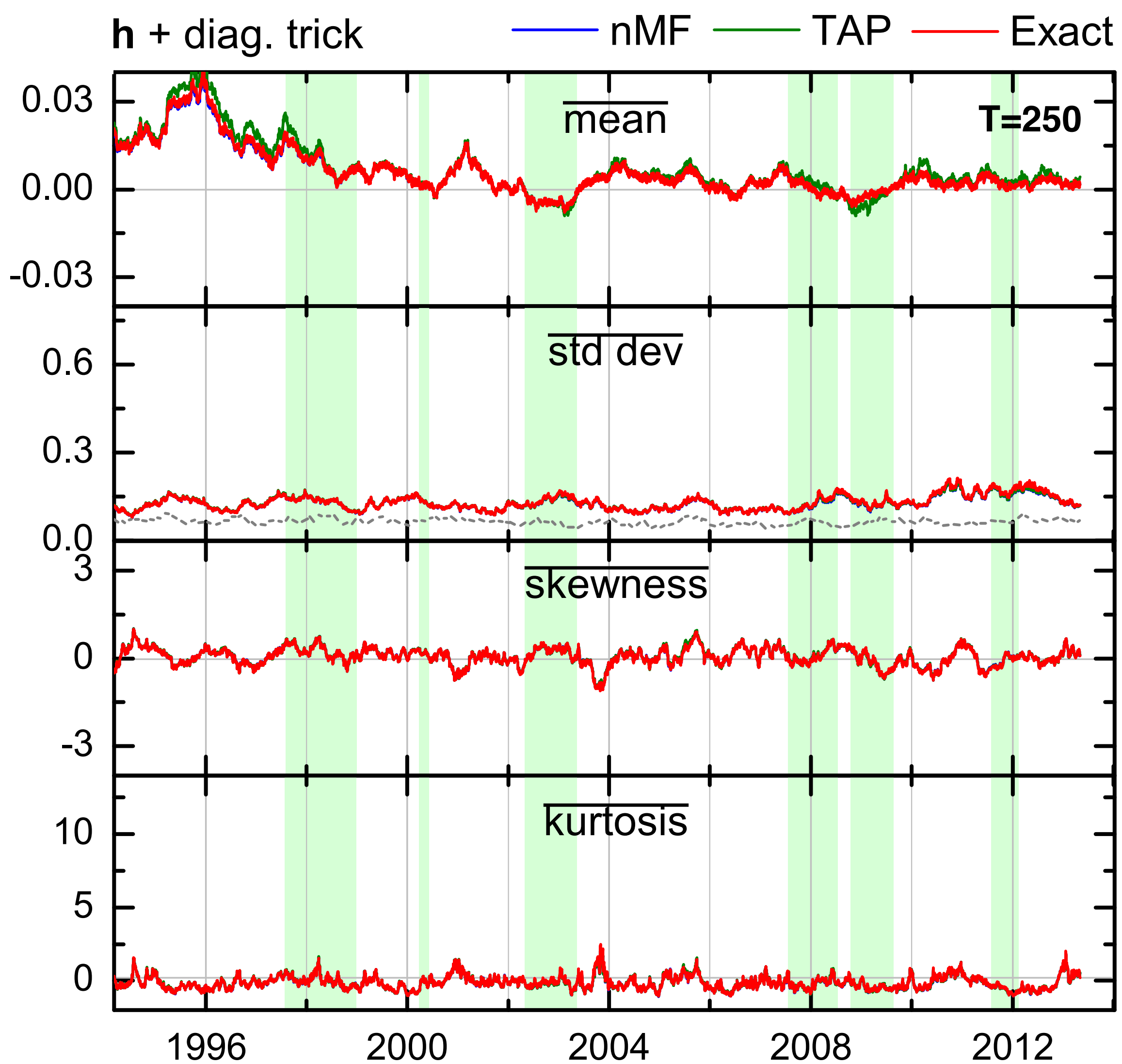
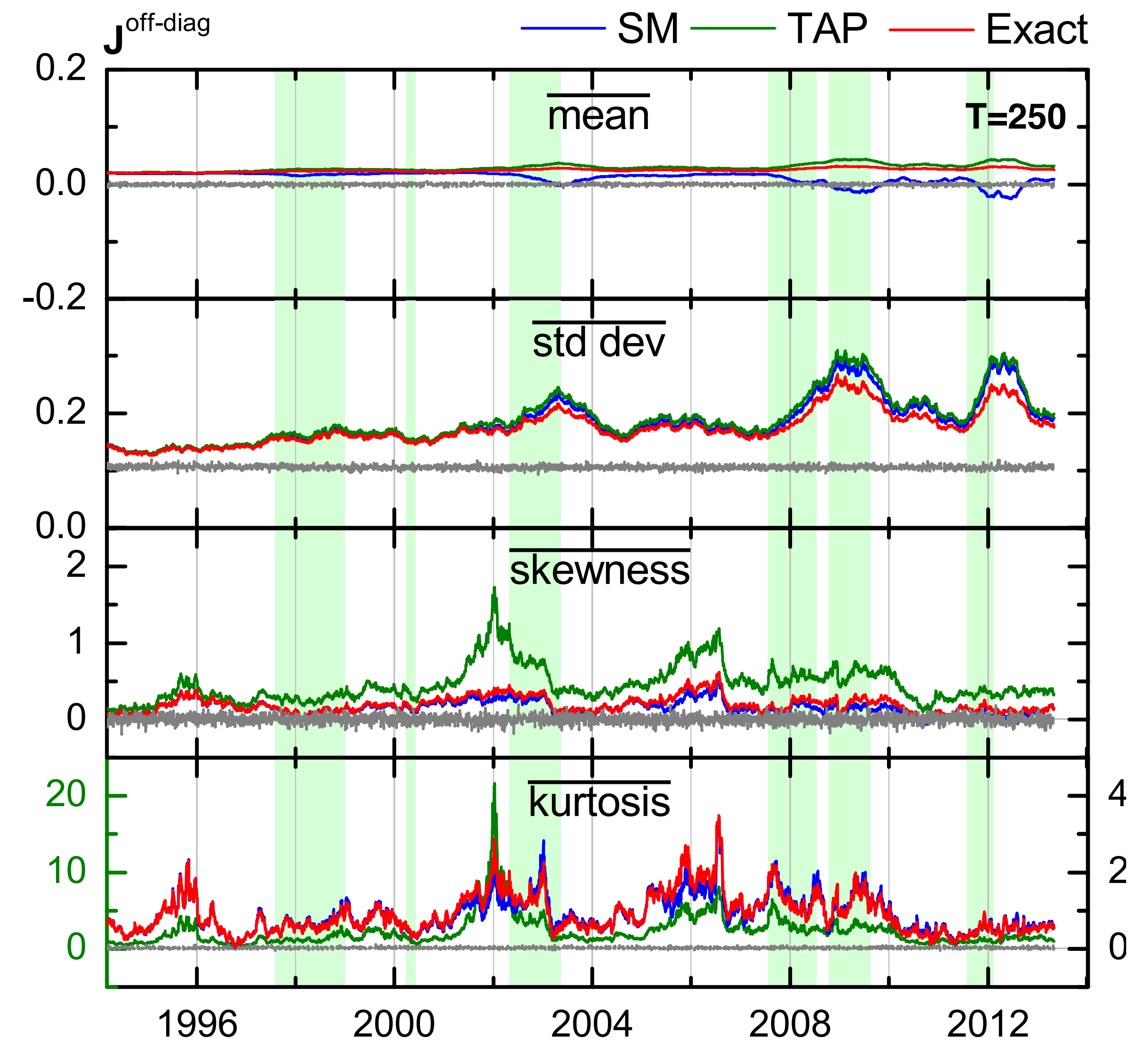
# Outline

# Distribution of external fields and couplings (historical dynamics)

## External fields




## Couplings



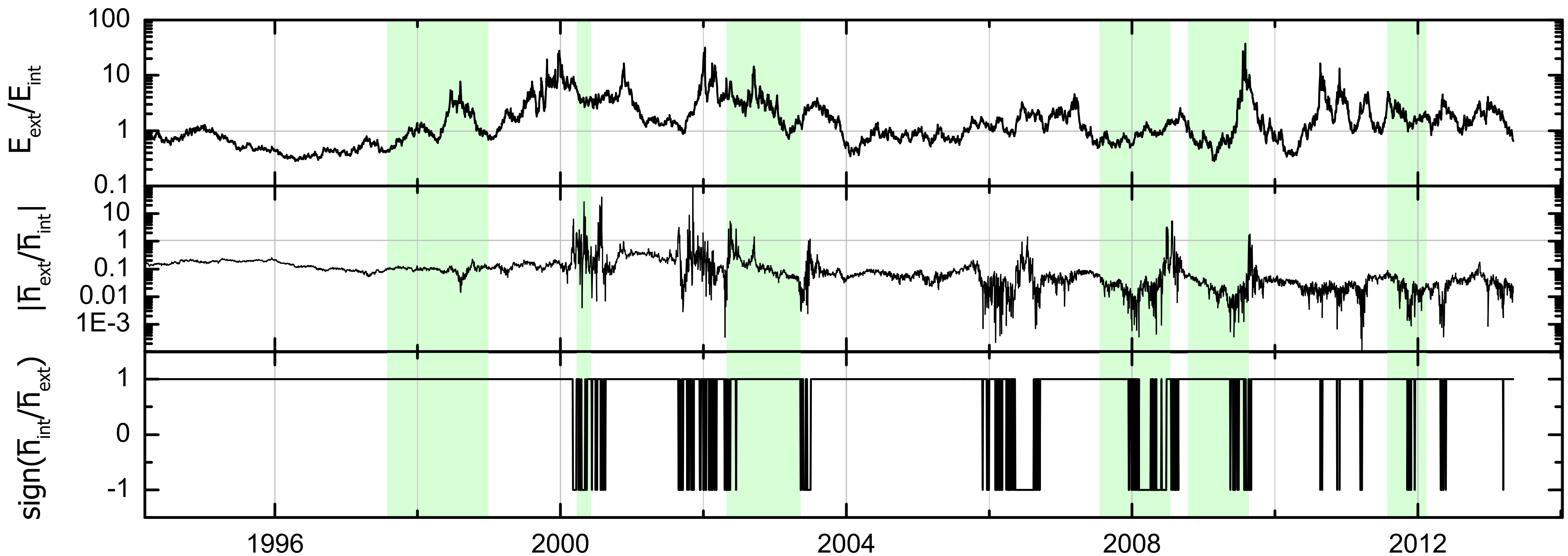

S. Borysov et al, Eur. Phys. J. B (2015) 88: 321

# External and internal biases

$$\mathcal{H}(\mathbf{s}) = -\mathbf{h}^\intercal \mathbf{s} - \mathbf{s}^\intercal \mathbf{J} \mathbf{s}$$

$$\mathbf{h}^{\text{ext}} \equiv \mathbf{h} \qquad \mathcal{H} = E^{\text{ext}} + E^{\text{int}}$$

$$\mathbf{h}^{\text{int}} = \langle \mathbf{s}^\intercal \rangle \mathbf{J} \qquad E^{\text{ext,int}} = -\left(\mathbf{h}^{\text{ext,int}}\right)^\intercal \langle \mathbf{s} \rangle$$




S. Borysov et al, Eur. Phys. J. B (2015) 88: 321

# Conclusions and future work

1. The model statistically captures historical market behaviour.
2. Binarization preserves main market statistical characteristics.
3. Approximation methods in general work well. Both mean field approximations work well for external fields inference and bulk of couplings, while a higher-order small-correlations expansions (SM) can correctly infer the strongest couplings.
4. Distribution of couplings is a mixture of two distributions: Gaussian bulk and heavy tail responsible for the market clustering structure.
5. Changes in external fields and couplings might be used as a leading indicator of financial instabilities
6. Study different models (BM with hidden nodes, deep belief networks), non-equilibrium distributions and factors influencing distribution of parameters, $p(\mathbf{h}, \mathbf{J} \mid x)$

Thank you!

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