It’s the entropy, stupid*

Matteo Marsili
Abdus Salam ICTP, Trieste Italy

*Adapted from the phrase “It’s the economy, stupid” that was key to Clinton’s victory in1992 US presidential elections.

Why does entropy matters?

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Micro state s

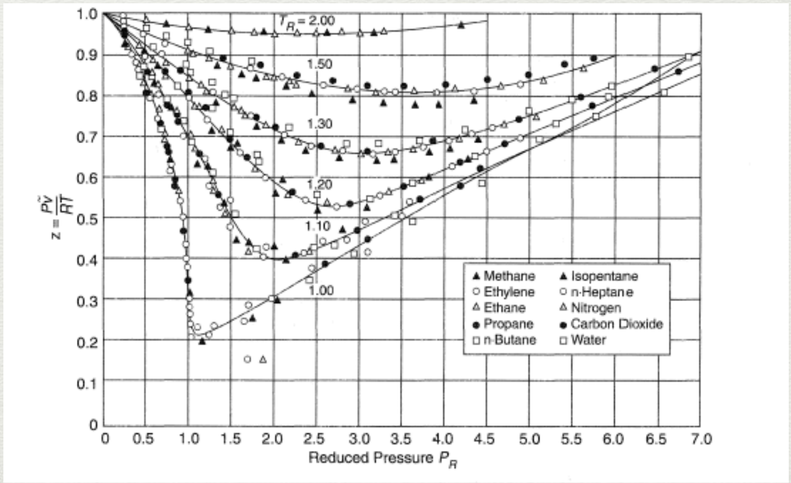
Macro state

Chemistry (NO_3, CO_2, H_2O, …)

Collective behaviour

TABLE 1. *Values of Molecular Weight,* T_c, P_c, Z_c, ω, x, *and Sources of P-V-T Data for Selected Fluids*

Substance	M. Wt.	T_c (°R)	P_c (psia)	Z_c	ω	X	Sources of P-V-T Data
Argon	39.95	271.8	705.4	0.290	0	0	26
Krypton	83.80	377.2	196.9	0.291	0	0	27
Xenon	131.30	521.6	852.4	0.290	0	0	28
Methane	16.04	343.6	673.1	0.290	0.013	0	29, 30, 31, 32
Ethane	30.07	549.9	711.5	0.285	0.099	0	33, 34, 35
Propane	44.09	665.9	617.4	0.277	0.150	0	35, 36, 37, 38, 39, 40
Butane	58.12	765.4	550.6	0.274	0.201	0	34, 35, 39, 41, 42
Pentane	72.15	845.7	489.5	0.269	0.254	0	34, 39
Ethylene	28.05	509.0	739.8	0.270	0.087	0	34, 35, 43, 44
Benzene	78.11	1012.3	714.3	0.274	0.215	0	45, 46
Nitrogen	28.02	227.1	492.6	0.291	0.040	0	34, 47, 48, 49
Carbon monoxide	28.01	239.7	507.6	0.294	0.046	0	43, 50, 51
Carbon dioxide	44.01	547.6	1071.3	0.274	0.420	0	34
Hydrogen sulfide	34.08	672.4	1306.5	0.268	0.100	0	52, 53, 54
Propylene	42.08	657.0	667.5	0.274	0.142	0.002	55
Nitric oxide	30.01	323.9	946.9	0.251	0.577	−0.045	56
Nitrous oxide	44.02	557.4	1051.0	0.273	0.160	−0.003	57, 58
Sulfur dioxide	64.06	775.2	1142.9	0.268	0.252	0.006	59, 60
Methyl chloride	50.49	749.3	986.3	0.276	0.152	0.007	58, 61
Ethylene oxide	44.05	842.0	1043.4	0.255	0.207	0.012	62
Ammonia	17.03	730.2	1641.0	0.242	0.252	0.013	43, 63, 64



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Macro state

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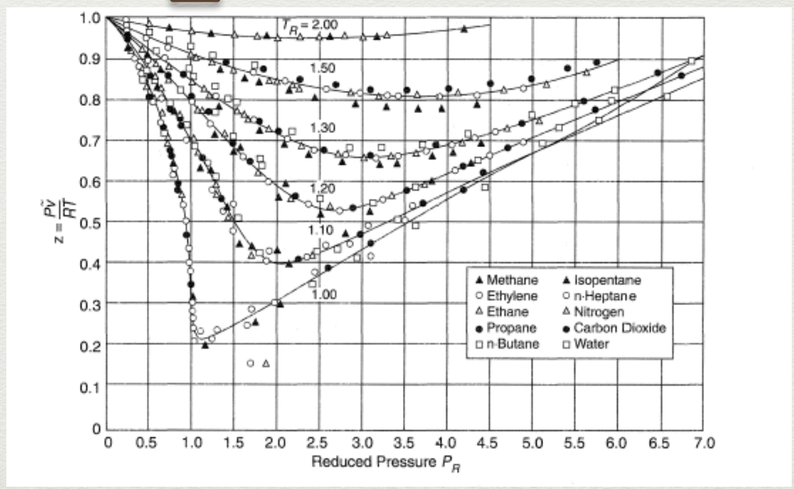
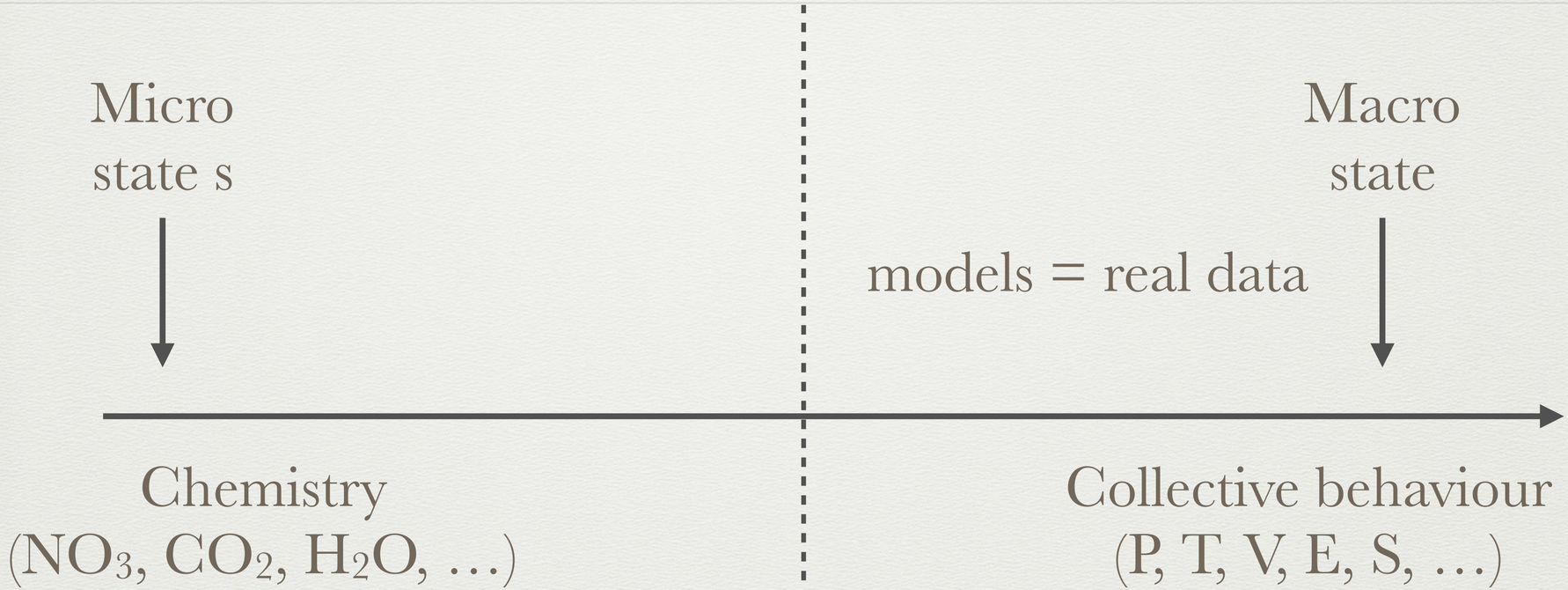


Many micro-motives for the same macro-behaviour



Typical behaviour:
(law of large numbers)

$$P\{s\} = \arg \max_{P:\langle E\rangle = U} H[P]$$

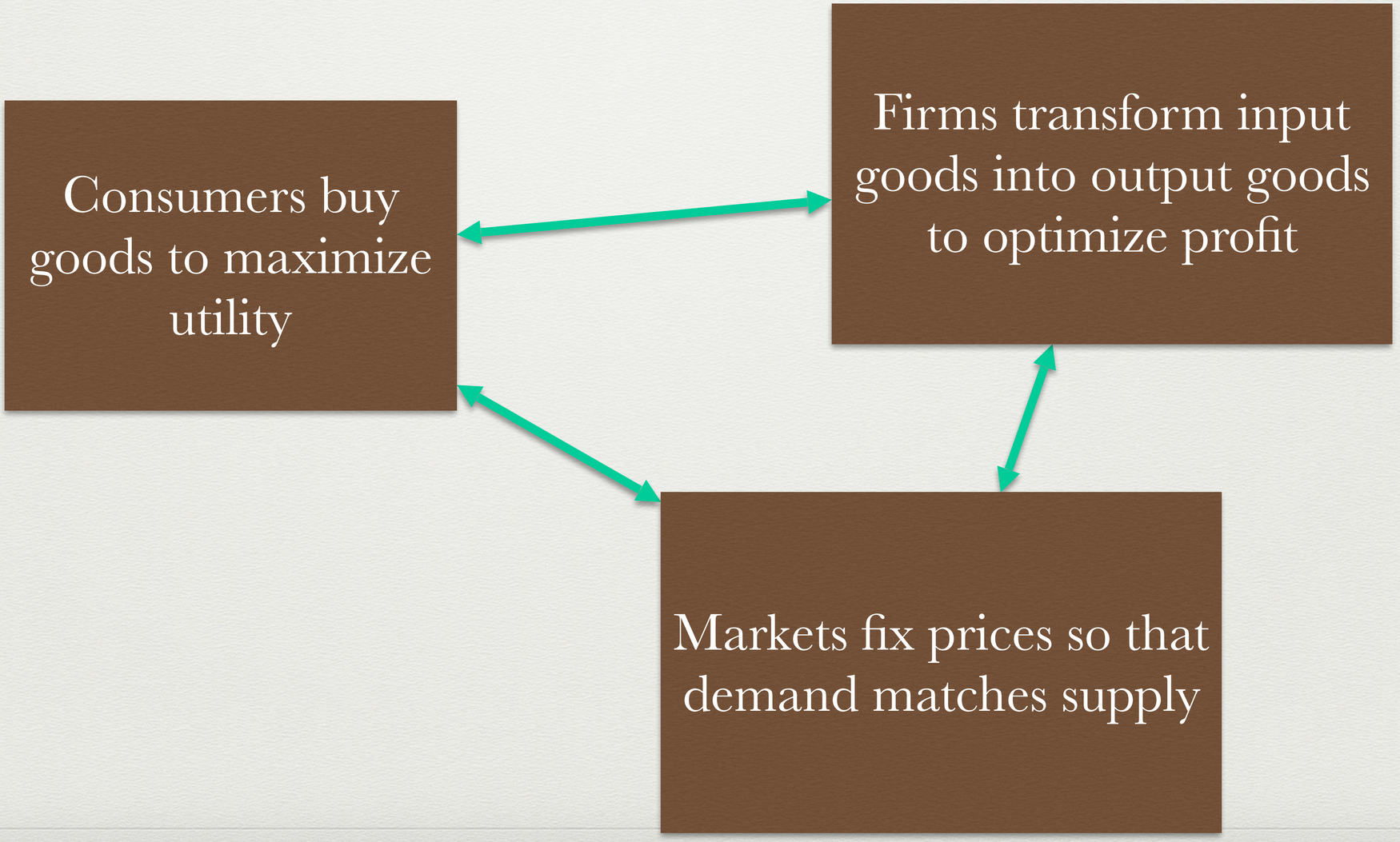
$$= \frac{1}{Z(\beta)} e^{-\beta E_s}$$

Many (large) systems for the same macro-behaviour

Heterogeneity: Typical behaviour is the same for all systems which are large enough

(e.g. Wigner and heavy atom spectra, spin glasses, etc)

General Equilibrium (GE) Theory



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commodity space $\vec{x} \in R^P$

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Consumers $a = 1,\ldots, A$

initial endowments $\vec{y}_a$

utility $U_a(\vec{x})$

budget $B_a(\vec{p}) = \{\vec{x} : (\vec{x} - \vec{y})\vec{p} = 0\}$

$$\Rightarrow \quad \vec{x}_a = \arg \max_{\vec{x} \in B_a(\vec{p})} U_a(\vec{x})$$

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inputs $\vec{z}_i$ outputs $\vec{w}_i = \vec{f}_i(\vec{z}_i)$

profit $\pi_i(\vec{z}_i) = (\vec{w}_i - \vec{z}_i)\vec{p}$

Firm i solves $\max_{\vec{z}_i} \pi_i(\vec{z}_i)$

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Market for commodity $\mu = 1,\ldots, P$

fixes prices $\vec{p}$ such that demand = supply $\forall \mu$

$$\sum_{a=1}^{A} \vec{x}_a + \sum_{i=1}^{N} \vec{w}_i = \sum_{a=1}^{A} \vec{y}_a + \sum_{i=1}^{N} \vec{z}_i$$

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i) Single period economy
ii) Markets are complete
iii) Price taking behavior

Generic results

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- Welfare theorems:
  - at equilibrium everyone is as well off as possible
  - every optimal allocation can be attained

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  - at equilibrium everyone is as well off as possible
  - every optimal allocation can be attained

- Walras' law:
  - every consumer spends all money
  - profit of every firm is zero

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- Get intuition: Few agents
  (e.g. R. Crusoe economies, representative agent …)

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  data (SA matrices) → model → prediction

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- Computational GE approach (calibration!!!)
  data (SA matrices) → model → prediction
- Here:
  Typical behaviour of large random economies
  as a function of A, N, P, distribution of endowments
  and efficiency of production processes

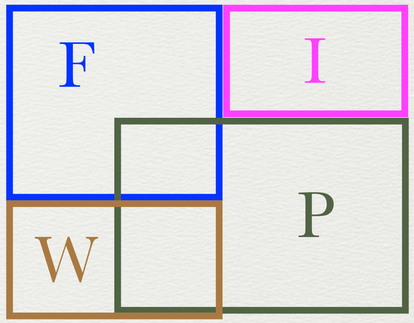
GE of random economies

(KJ Lancaster Mathematical Economics ’87)

- Commodities, consumers and endowments
- Firms and technologies
- Market and prices

The universe of goods and Consumers

- C homogeneous commodities
  - P primary goods: y > 0
  - F final goods: y=0, x>0
  - intermediate goods: y=0, x=0
  - waste x>0



- One consumer (A=1) with separable utility function

$$U(\boldsymbol{x}) = \sum_{c \in \mathcal{F}} u(x^c)$$

(A>1 not difficult)

Firms and technologies

- N linear technologies:

$$\vec{f}_i(\vec{z}_i) = (\vec{z}_i \cdot \vec{u}_i)\,\vec{v}_i, \qquad ||\vec{u}_i|| = 1, \qquad u_i^\mu, v_i^\mu \geq 0$$

- Firms choose the scale s_i at which they operate

$$\max_{\vec{z}_i} \vec{p}\left[\vec{f}_i(\vec{z}_i) - \vec{z}_i\right] \Rightarrow \vec{z}_i^{\,*} = s_i \vec{u}_i, \qquad s_i \geq 0$$

$$w_i^\mu - z_i^\mu = s_i \xi_i^\mu, \qquad \xi_i^\mu = v_i^\mu - u_i^\mu \qquad \xi_i^\mu > 0 \quad \leftrightarrow \quad \mu \text{ output}$$

$$\xi_i^\mu < 0 \quad \leftrightarrow \quad \mu \text{ input}$$

- ξ_i^μ random with no-land-of-Cockaigne constraint

$$\sum_\mu \xi_i^\mu = -\epsilon, \qquad \sum_\mu (\xi_i^\mu)^2 = \Delta$$

(# inputs ~ # outputs finite as P $\rightarrow\infty$)

The solution:

$$\max_{s_i \geq 0} U\left(\vec{y} + \sum_{i=1}^{N} s_i \vec{\xi}_i\right)$$

Parameters: n=N/C (industrial development)

ε (efficiency of technologies)

u(x) (consumer's preferences)

F/C=f, P/C=π (fraction of final/primary goods)

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F/C=f, P/C=π (fraction of final/primary goods)

Note: technologies are drawn i.i.d. at random,
but those which survive (s_i>0) are not

Typical behaviour in the limit N→∞

$$\lim_{N\to\infty}\frac{1}{P}\left\langle \max_{\{s_i\geq 0\}} U\left(\boldsymbol{y}+\sum_{i=1}^{N} s_i\boldsymbol{\xi}_i\right)\right\rangle_{\boldsymbol{\xi}} = \mathrm{extr}_{\boldsymbol{\omega}}\ f(\boldsymbol{\omega})$$

Order parameter:

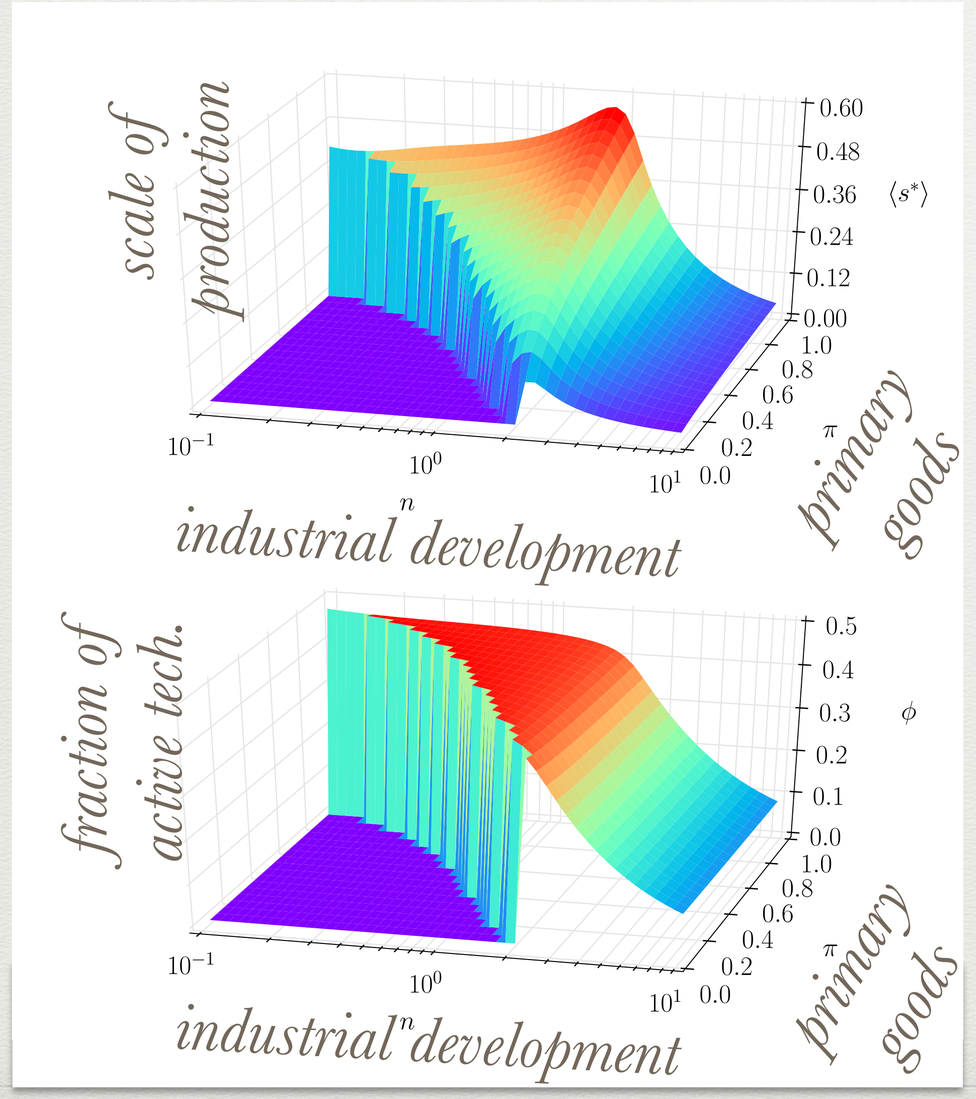
$$q_{a,b}=\frac{\Delta}{N}\sum_{i=1}^{N} s_{i,a}s_{i,b}=q+(Q-q)\delta_{a,b}$$

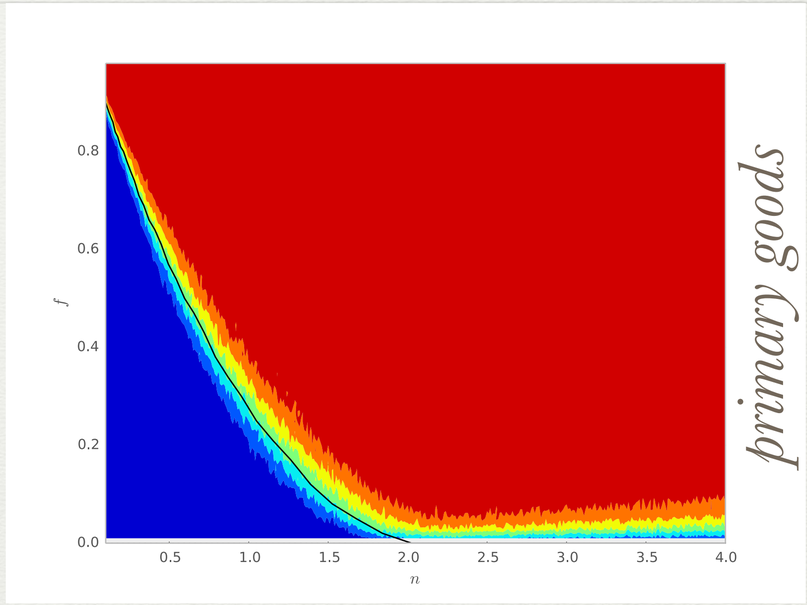
$$f(Q,\gamma,\chi,\widehat{\chi},\kappa,p)=\frac{1}{2}nQ\widehat{\chi}-\frac{1}{2}\gamma\chi+\kappa p$$
$$+\left\langle \max_{x\geq 0}\left[u(x)-\frac{1}{2\chi}\left(x-y+t\sqrt{nQ}+\kappa\right)^2\right]\right\rangle_{t,y}+$$
$$+n\left\langle \max_{s\geq 0}\left[-\frac{1}{2}\Delta\widehat{\chi}s^2+st\sqrt{\Delta(\gamma-p^2)}-s\eta p\sqrt{\Delta}\right]\right\rangle_{t,\Delta}$$

Representative good problem

Representative firm problem

Phase transition



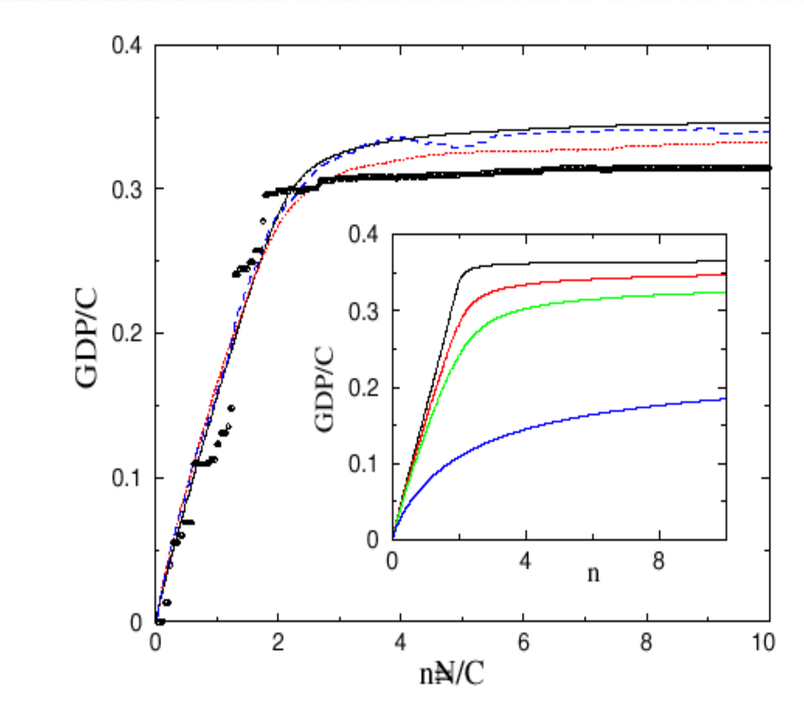


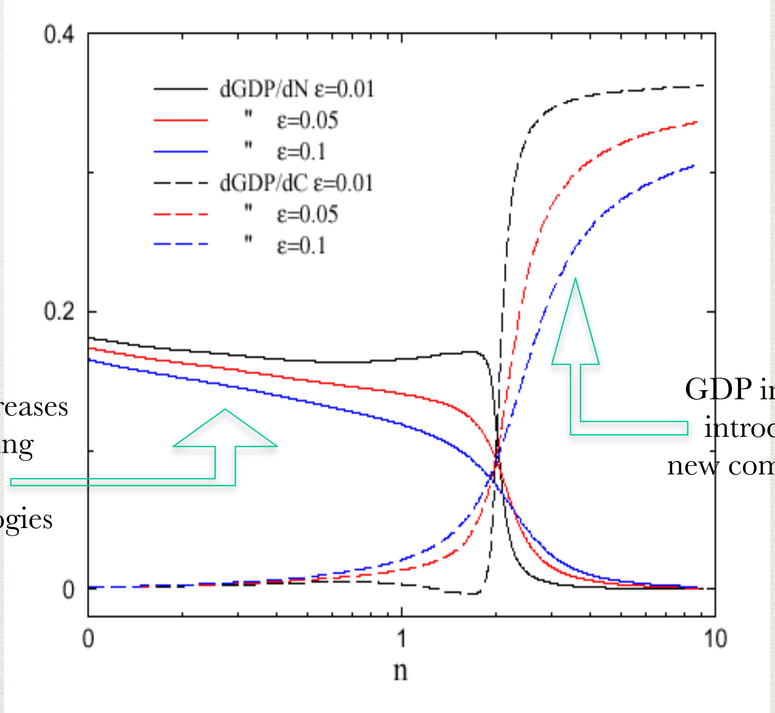
industrial development

Recipes for GDP growth

Modes of technological innovation: $\epsilon \searrow \quad N \nearrow \quad C \nearrow$

GDP=total value of goods produced

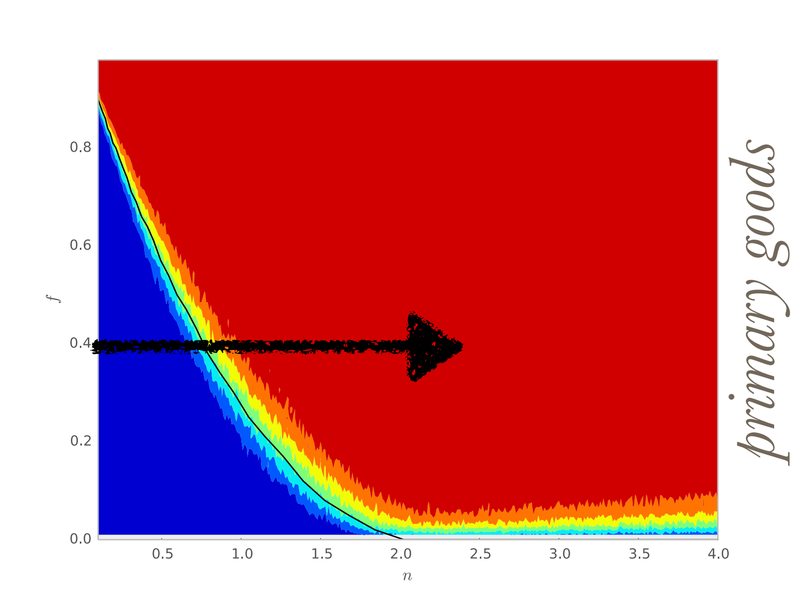




GDP increases inventing new technologies

GDP increases introducing new commodities

Paths of development: N → N+1, C fixed technological innovation



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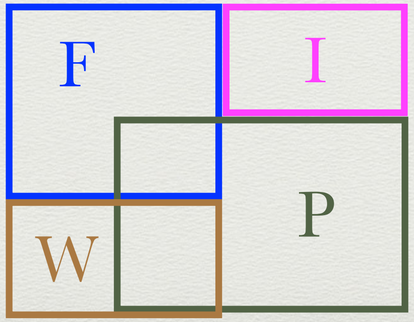
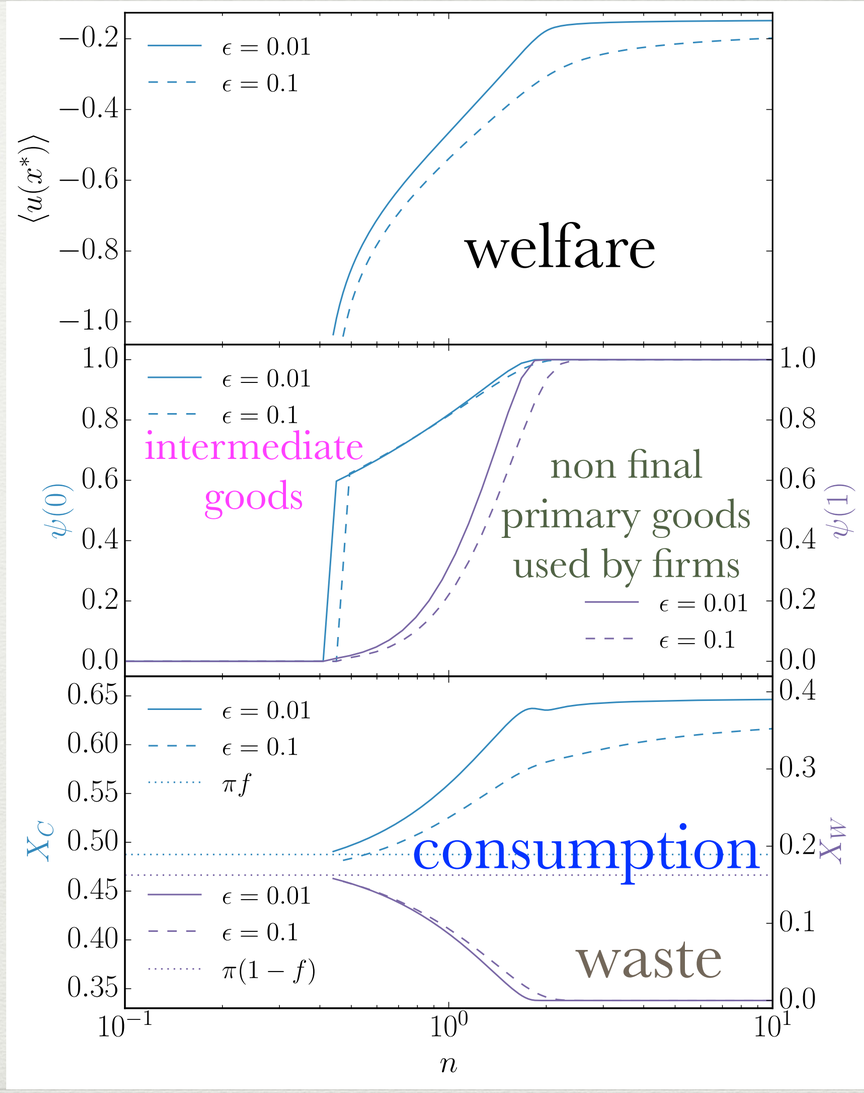
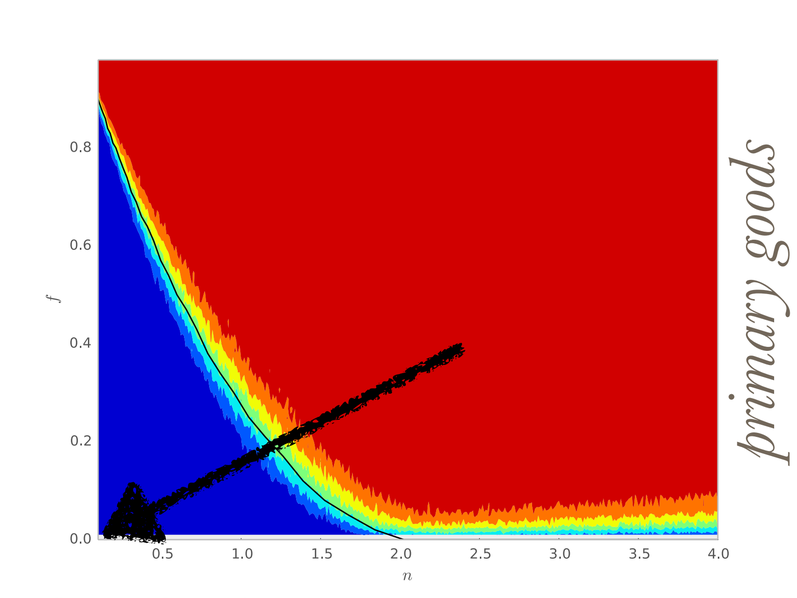




Paths of development: C → C+1, N, F, P fixed outsourcing and the expansion of markets



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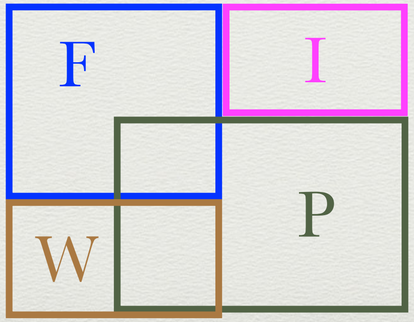
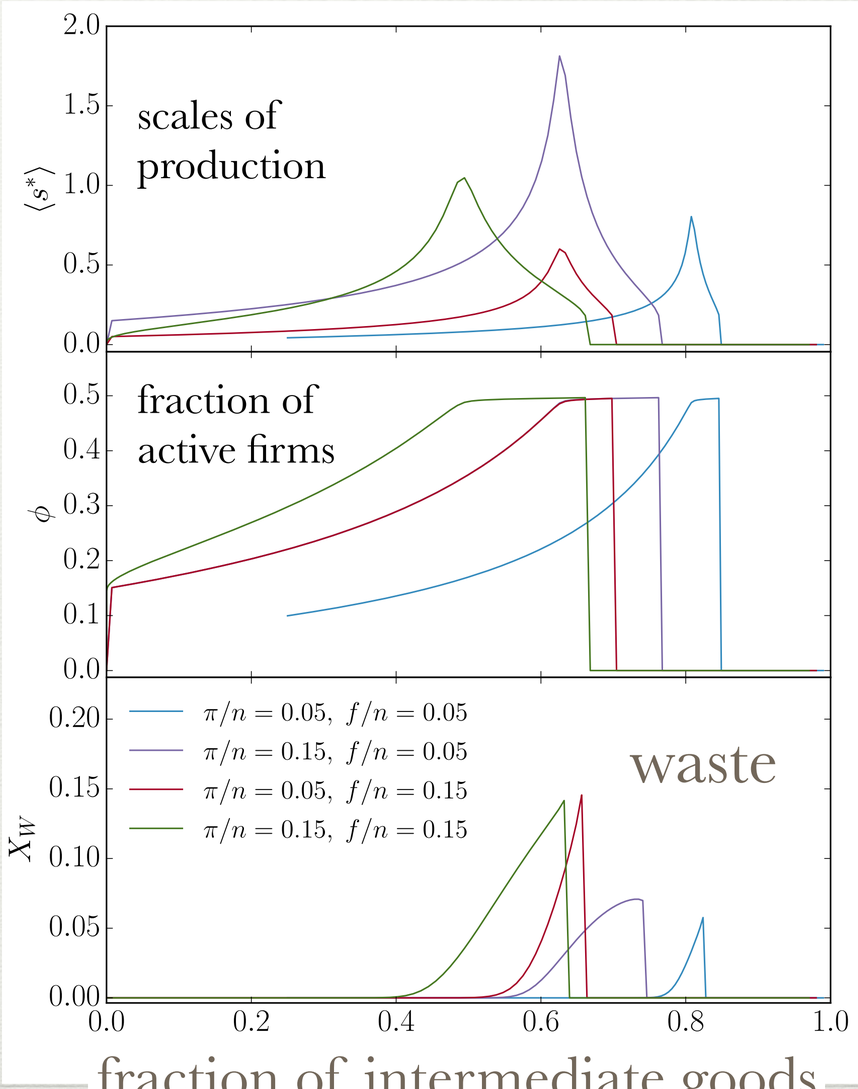
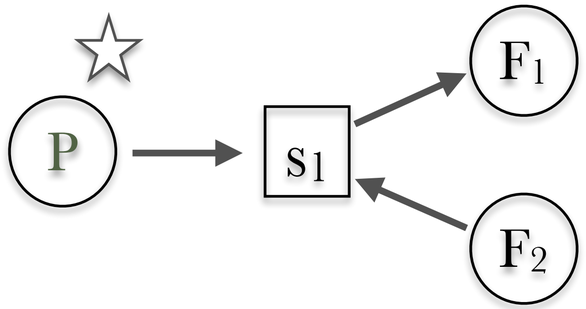
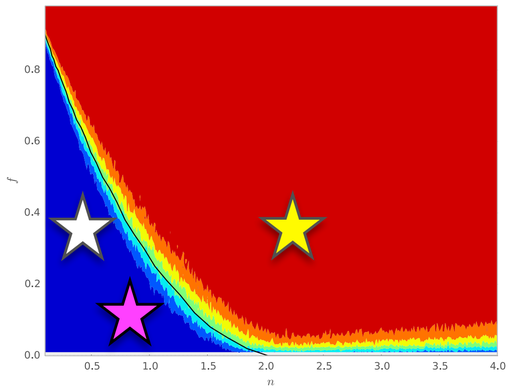


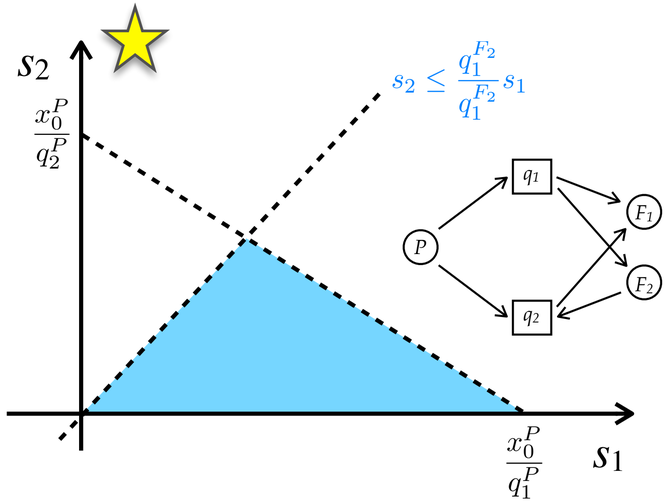


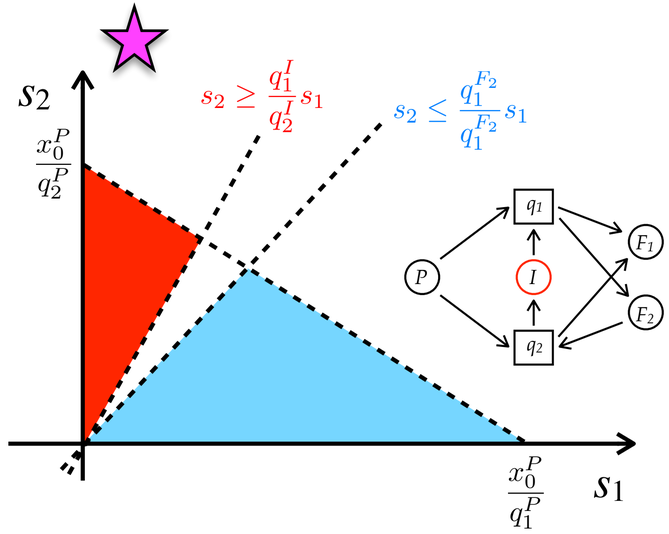
Intuition: a constraint on production for any good

$$y^\mu + \sum_{i=1}^{N} s_i \xi_i^\mu \geq 0$$



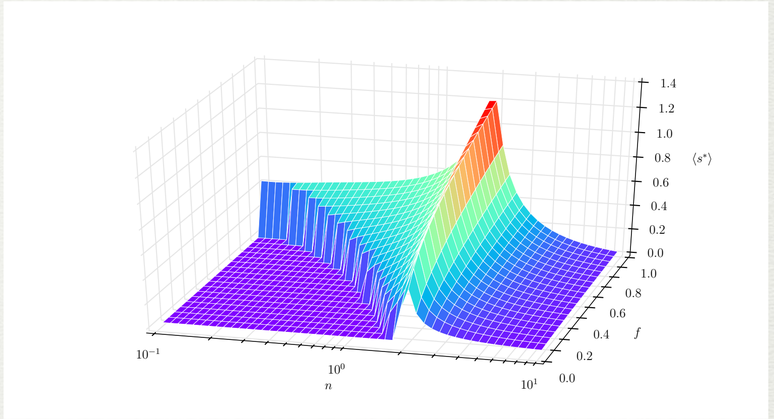






Comments

- Incentive for R&D from private sector only for n<2
- Industrial revolution requires access to primary goods

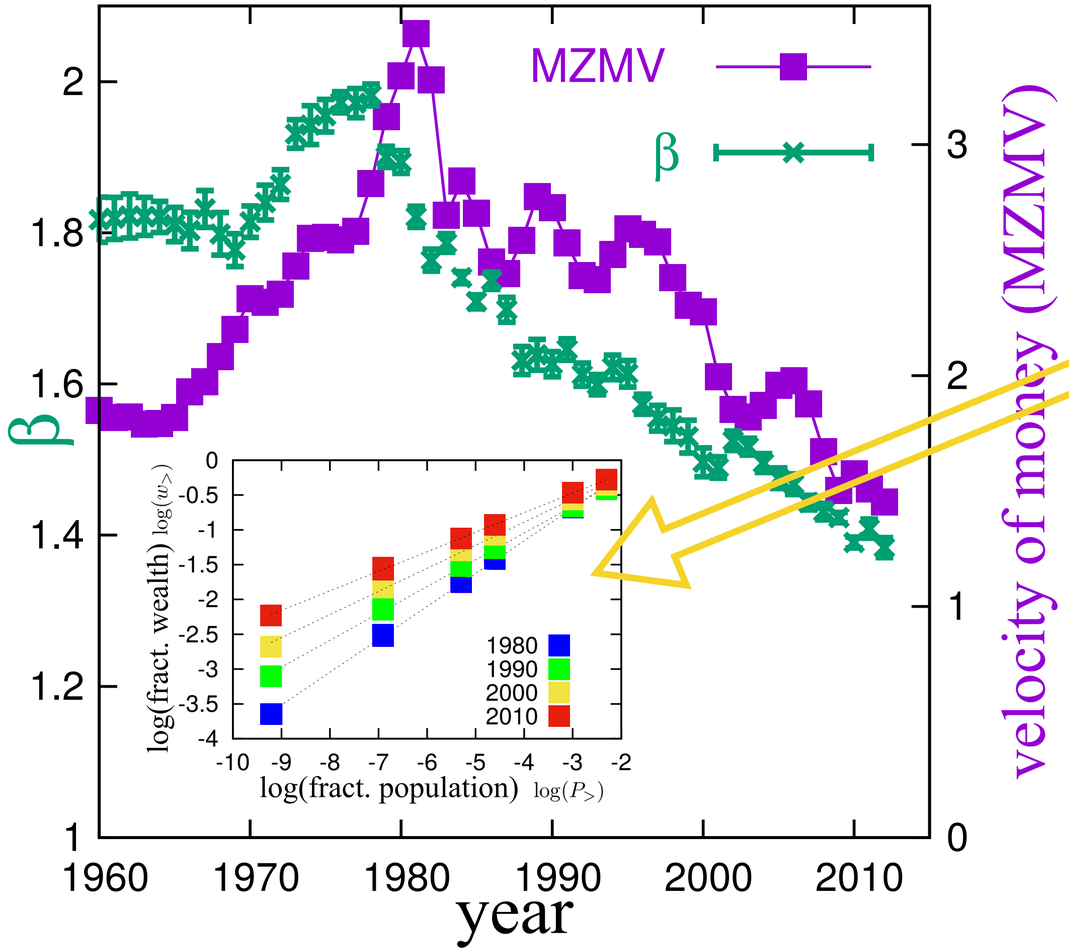


- Industrial dynamics in the last 4 centuries (see e.g. The Vanishing Hand R.N. Langlois 2004): from vertically integrated firms (n<2) to outsourcing (n>2)
- e.g. Carbon emission trading is profitable for n>2 but not for n<2
- The green impact of R&D: Waste decrease with n (and it increases when intermediate goods are introduced)

The debate on inequality

- Inequality is rising and it's back to the pre-WWI levels (Piketty-Saez 2001)
- Return on capital > GDP growth = positive feedback on inequality (Piketty 2014)
- Inequality correlates with many bad things (infant mortality, crime, social (im)mobility… Wilkinson - Pickett 2009)
- Too much inequality with respect to what?
- Inequality and the flow of stuff in an economy (i.e. liquidity)

The data: inequality and liquidity



Data Saez-Zucman (2013)

$$p_>(w) = P\{W > w\} \sim w^{-\beta}$$

$$W_> = \int_x^\infty dp_>(w) w \sim p_>^{1-1/\beta}$$

Fed. Res. Bank St Luis (FRED)
Money with zero maturity
(broadest definition of money)

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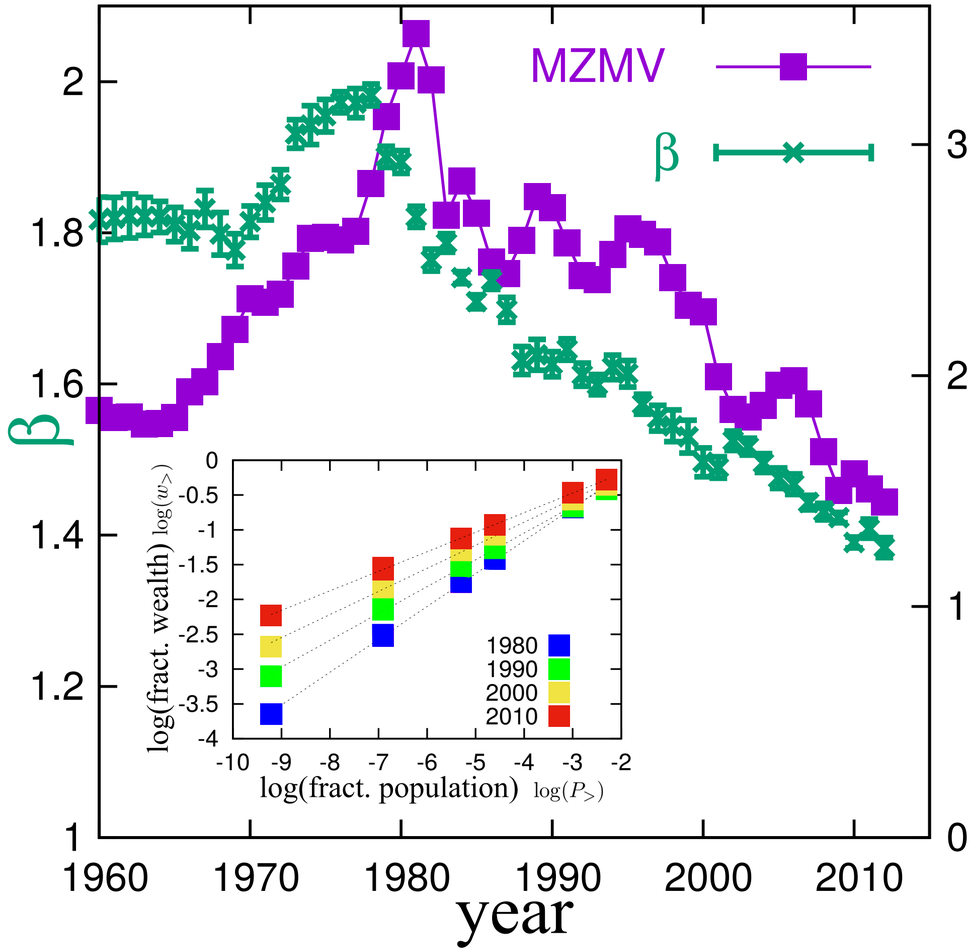

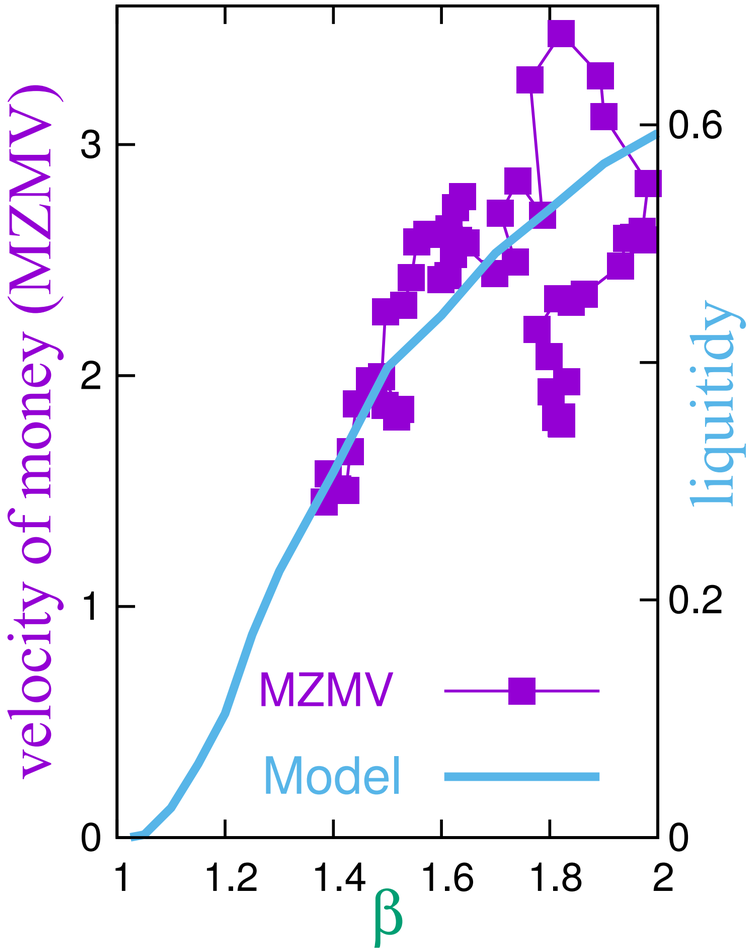

A simple model

- N agents, M goods
  Agent i=1,…,N has wealth w_i drawn i.i.d. from $p(w) \sim w^{-\beta-1}$
  Object o=1,…,M has price π_o

$$\sum_o \pi_o < \sum_i w_i$$

- Feasible assignments A: $\sum_{o \in i} \pi_o < w_i$
- Start from a feasible assignment
  Pick an object o and an agent i at random: i buys o if he has cash>π_o
  Repeat

- Dynamics converges to the maximal entropy state
  P(A)=P(A') for all feasible A, A'

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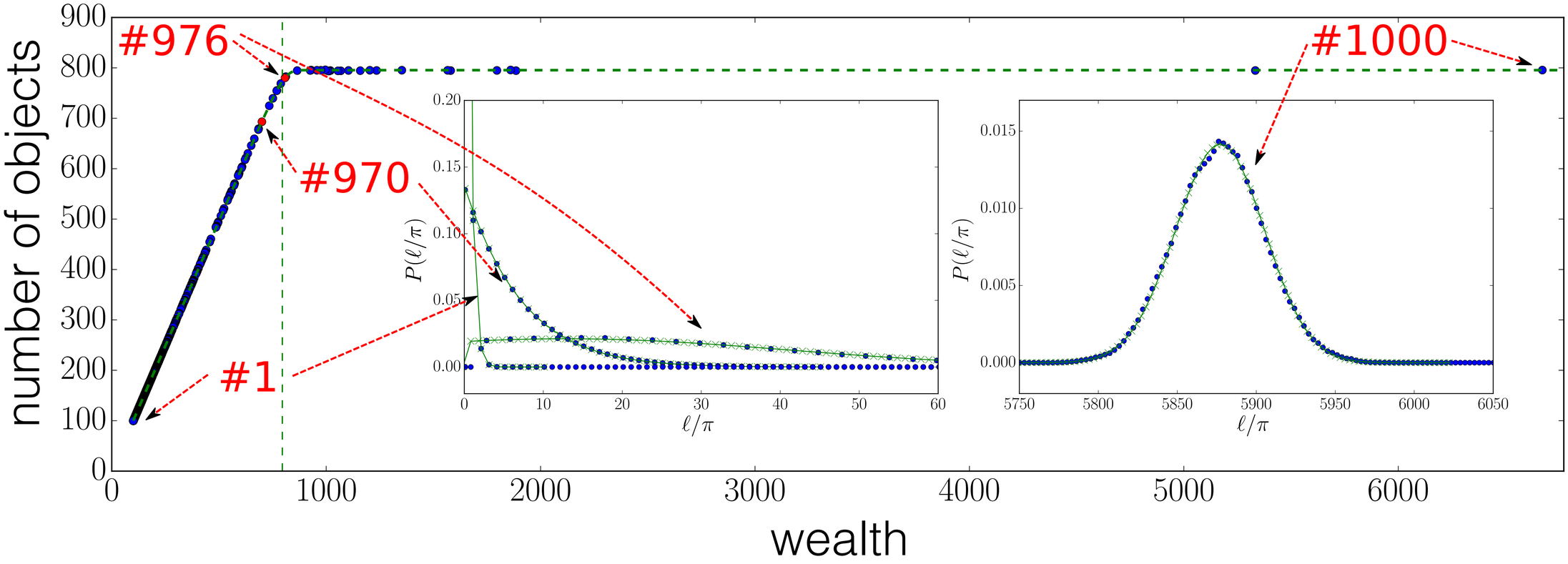
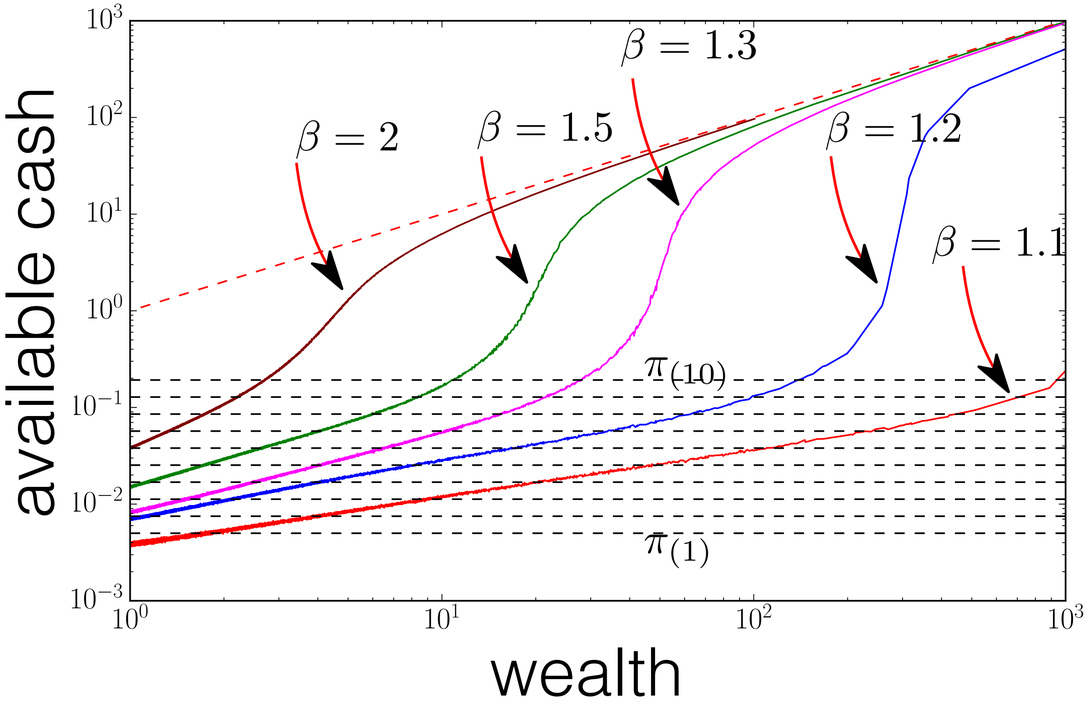
No incentives
infinite T

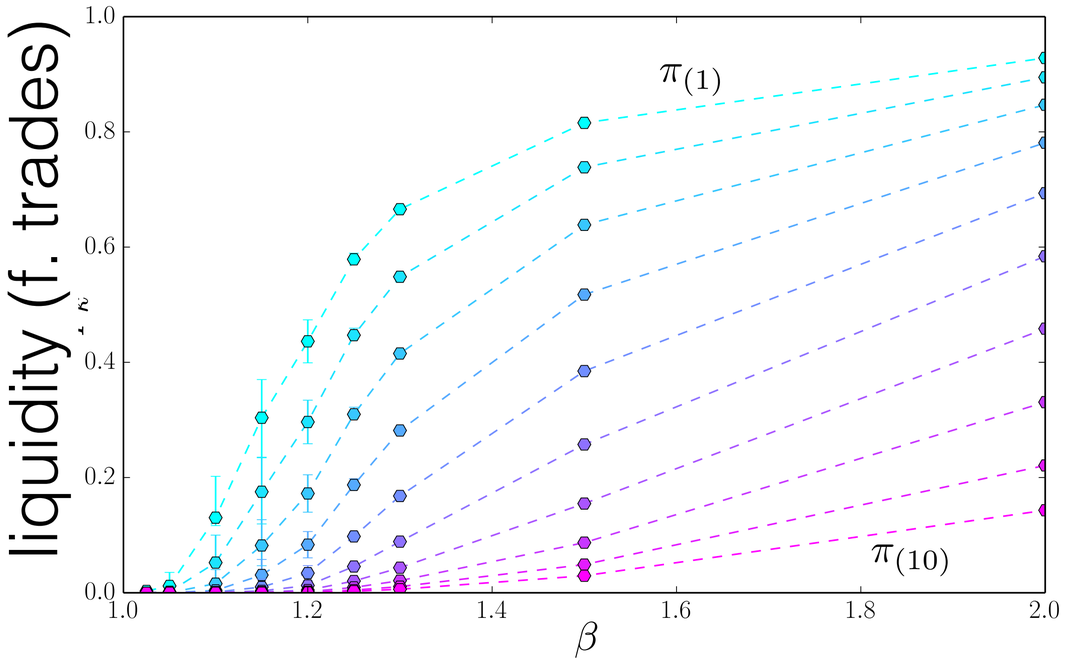
One type of good π_0=1



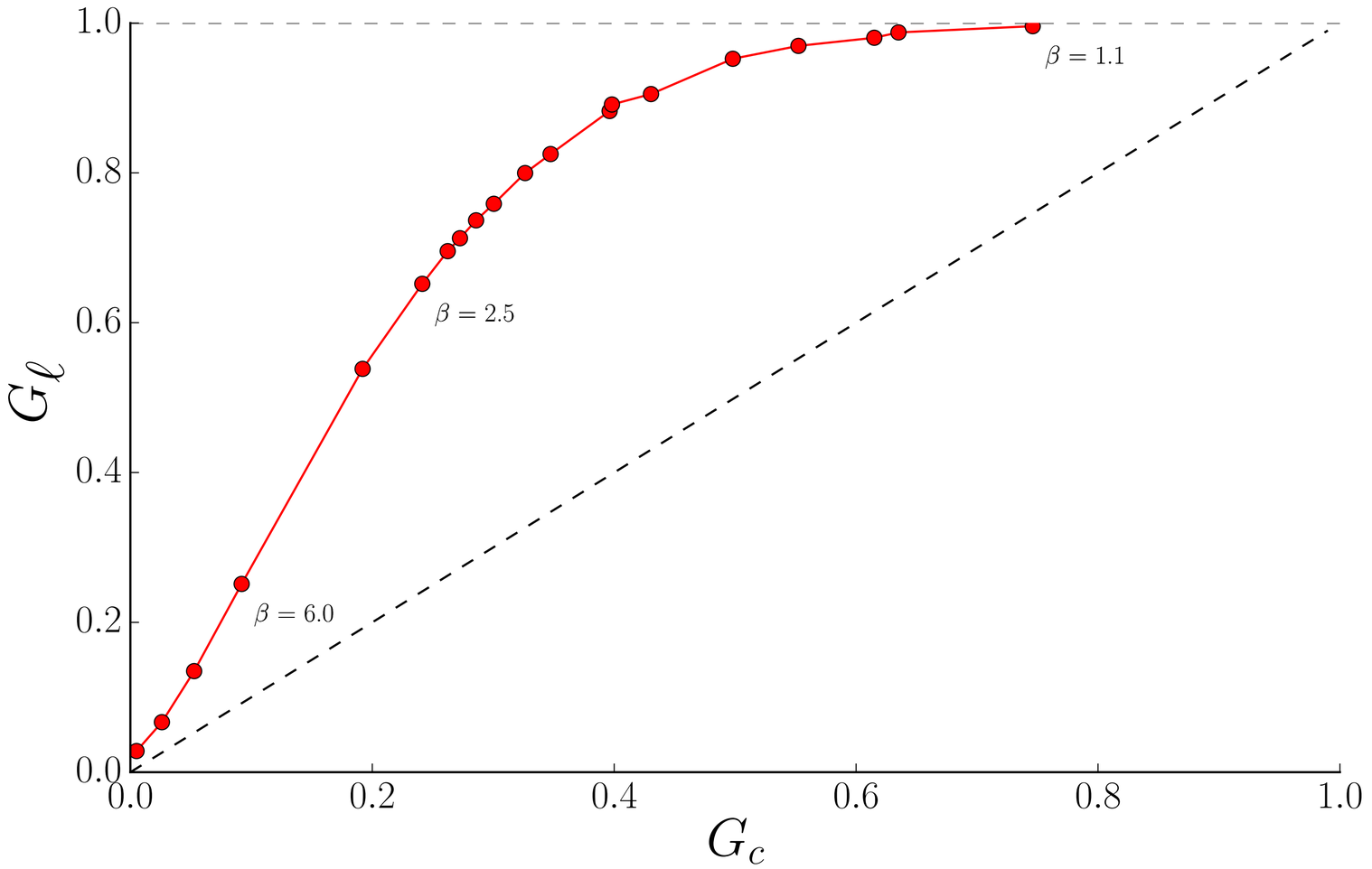
$$N = 10^3, M = 2 \cdot 10^5, \beta = 1.8$$

Ten types of goods





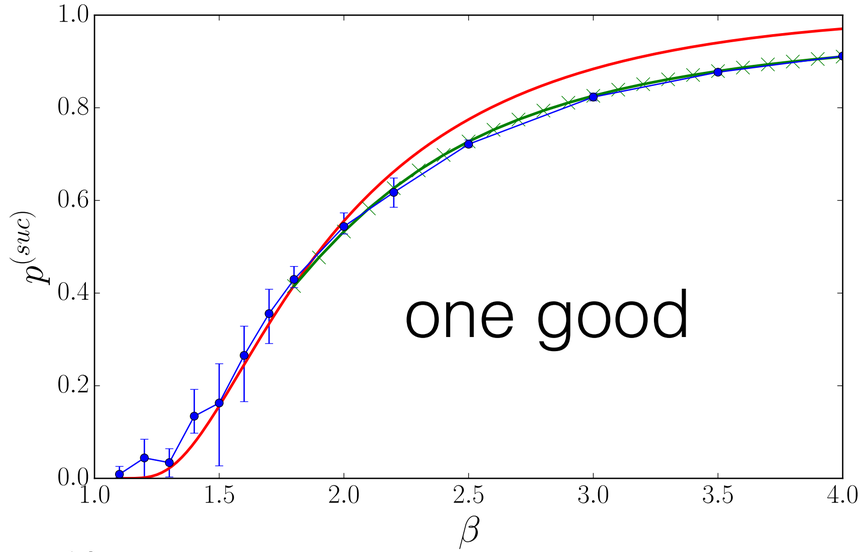
Cash flows to the top

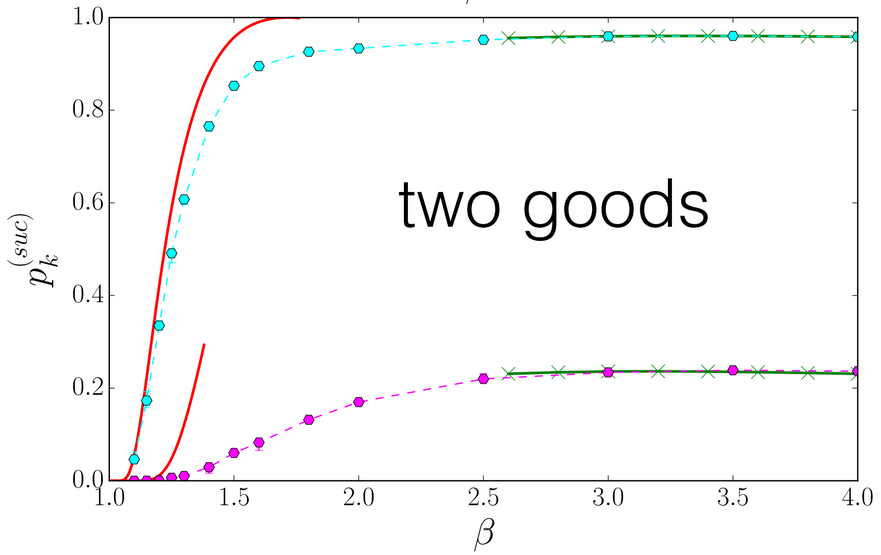


Theory:

$$c^{(k)} \simeq \left[\beta^k - \left(\frac{\beta - \beta^{k+1}}{1-\beta}\right)\frac{\Pi}{KC}\right]^{\frac{1}{1-\beta}}$$

$$p_k^{(\mathrm{suc})} = \frac{M_k}{N\lambda_k} \simeq \frac{\Pi}{KC}\frac{\mathbb{E}\left[c\right]}{c^{(k)}}.$$





Note

- Model: Inequality -> liquidity
- Incentives? Utilities? Preferential trading?
- Endogenous price dynamics?
- Consumption, investment and credit?
- Quantitative Easing for the people?

Few days of blog folly

BloombergView | The Chilling Math of Inequality

The Chilling Math of Inequality

260 MARCH 15, 2016 6:00 AM EST

By Mark Buchanan

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By Mark Buchanan Bloomberg View

First Published Mar 15 2016 04:03PM • Last Updated Mar 16 2016 02:48 pm

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Oh Great, Now This – Part II

Posted on **April 7, 2016**

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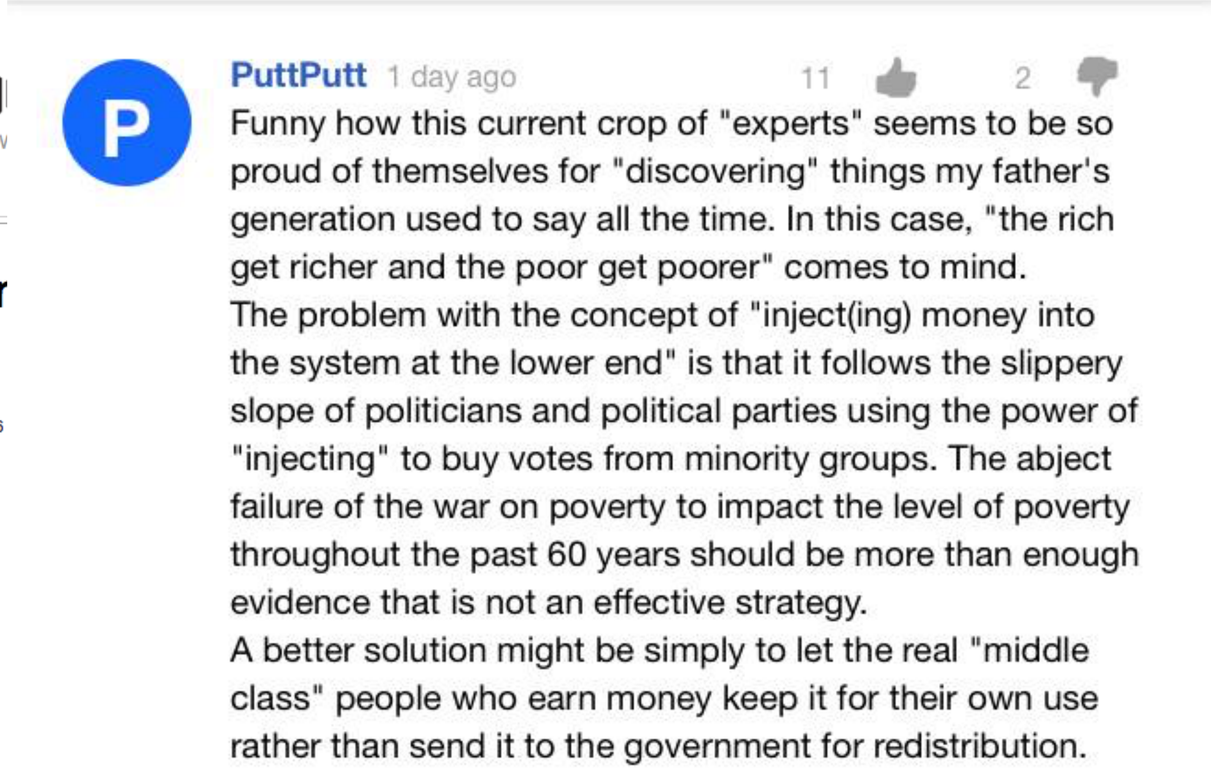

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Thanks

One further reason why entropy matters
Entropy = measure of information
Risk vs transparency