

# Google matrix applied (G matrix of the WTN)

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Alexei Chepelianskii and Klaus Frahm

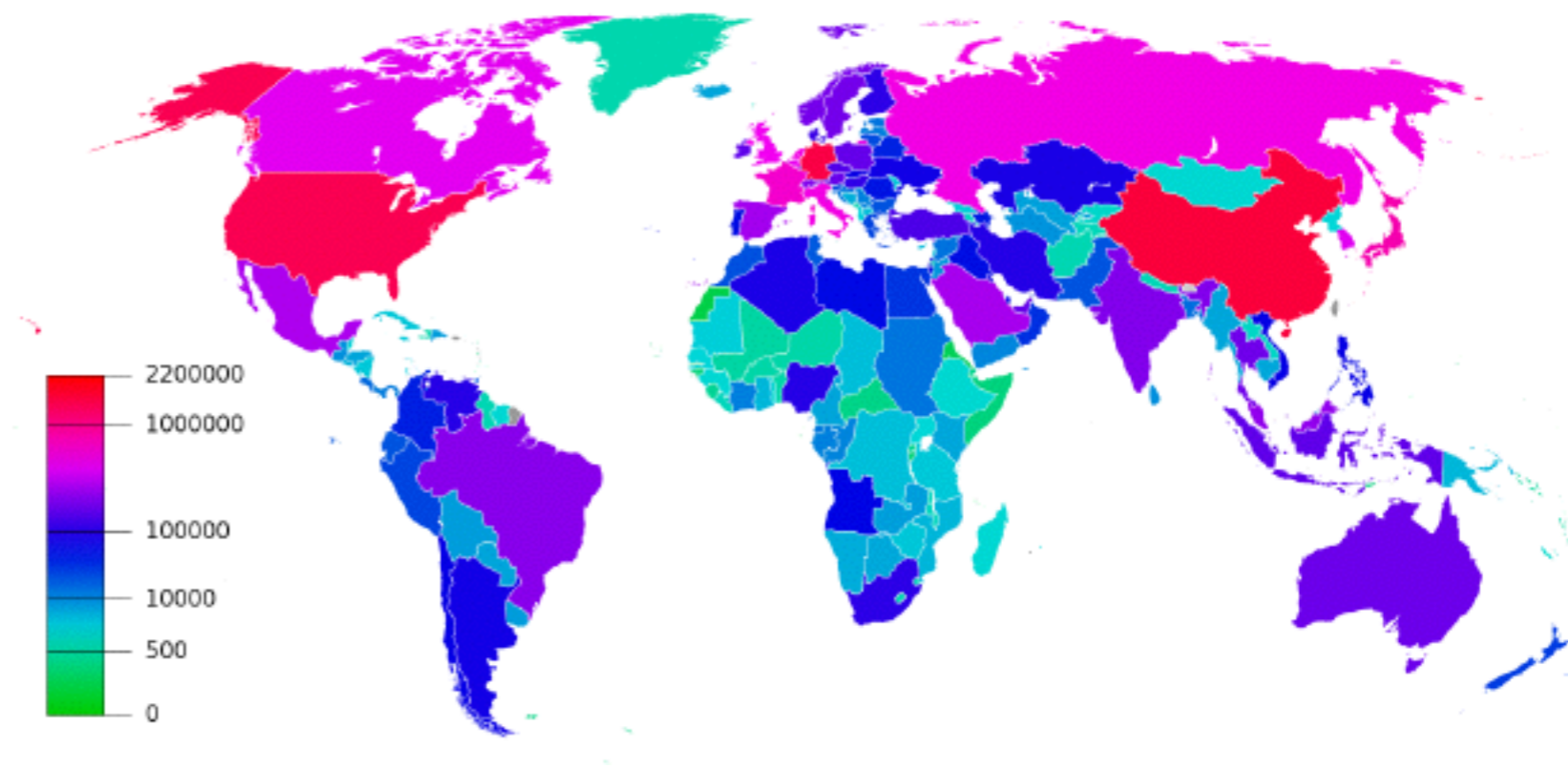
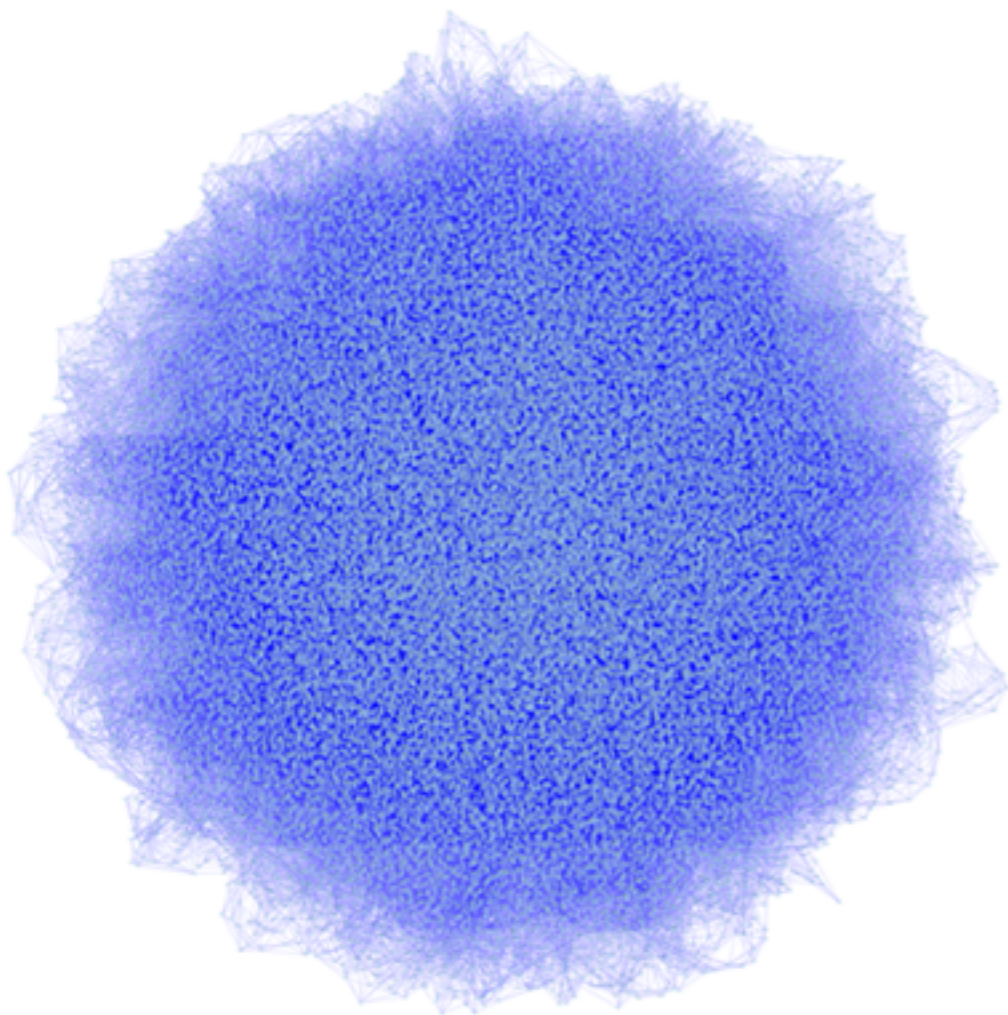
*Physics and Social Network Dynamics of the Markets*

May 31th, 2016

NORDITA, Stockholm, Sweden

# Motivations

Google approach to the World Trade Network



# Google Matrix

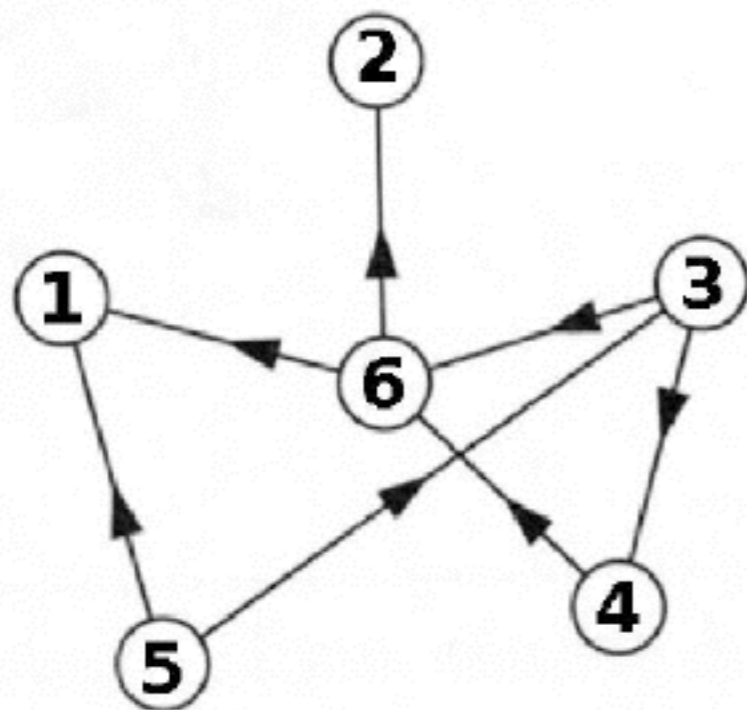
(summary after Dima's talk)

*Brin and Page (1998)*

centrality measure:  
Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network



adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

# Google Matrix

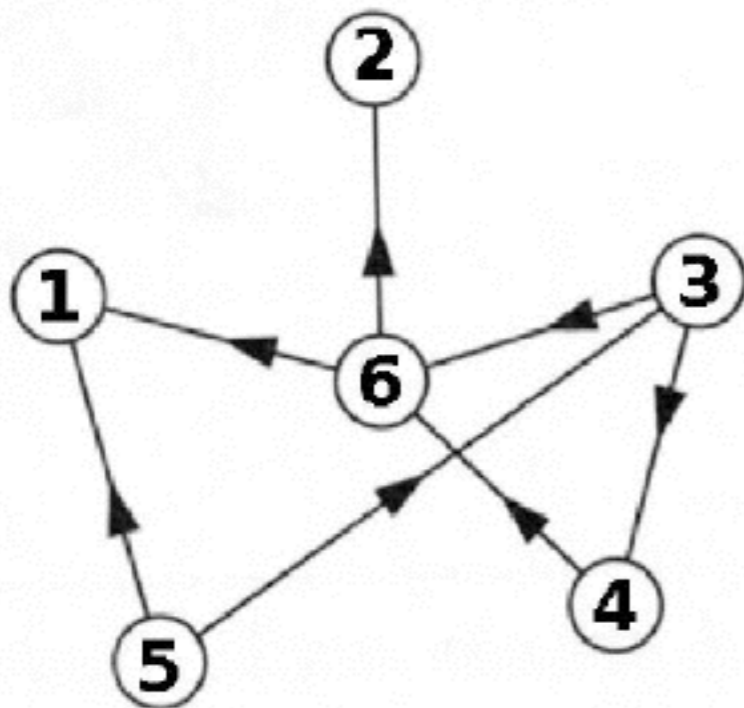
(summary after Dima's talk)

*Brin and Page (1998)*

centrality measure:  
Spectral Indices

- directed networks
- easy to compute
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- non-local properties

directed network



weighted adjacency matrix and dangling nodes

$$S = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 1 & 0 & 0 \end{pmatrix}$$

- $\sum_j S_{ij} = 1$
- Perron-Frobenius (non-negative)
- $\lambda_1 = 1$  (degeneracy)



# Google Matrix

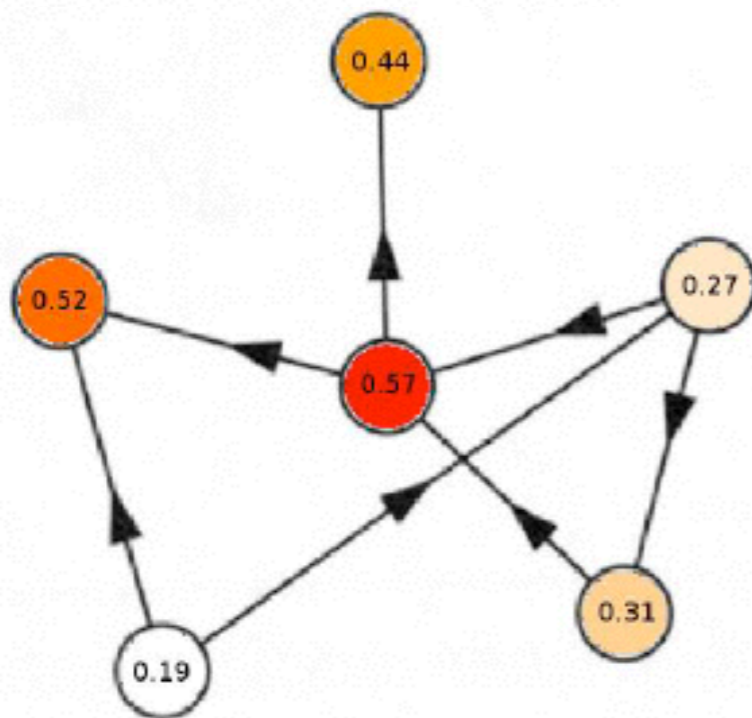
(summary after Dima's talk)

Brin and Page (1998)

PageRank

$$GP = P$$

directed network



centrality measure:  
Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

$$G = \alpha S + (1 - \alpha)E/N \quad (\alpha = 0.85)$$

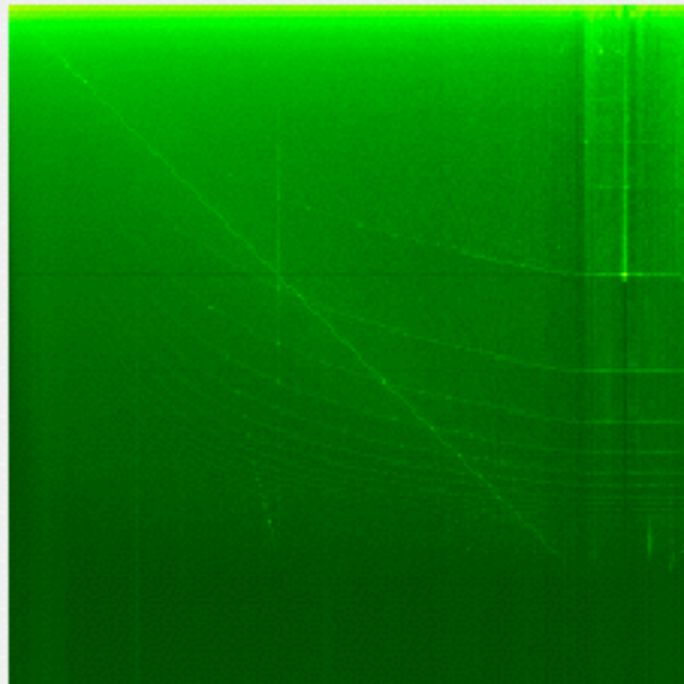
Google Matrix

$$G = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{20}{9} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{20}{9} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{9} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{40}{9} & \frac{40}{7} & \frac{1}{40} & \frac{1}{40} \end{pmatrix}$$

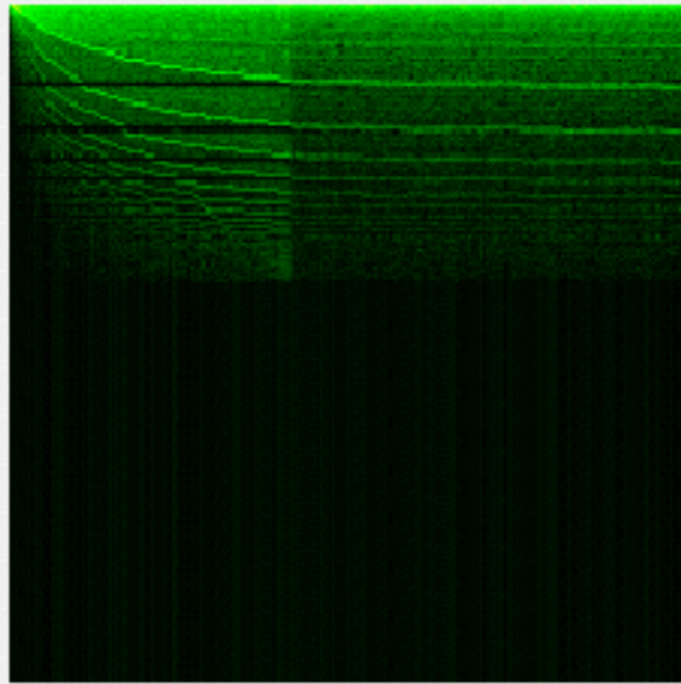
- $\alpha \rightarrow S, (1 - \alpha) \rightarrow$  random node
- Perron-Frobenius (positive)  $\lambda_1 = 1$
- $\Delta \geq (1 - \alpha)$  (global convergence)

# Google Matrix

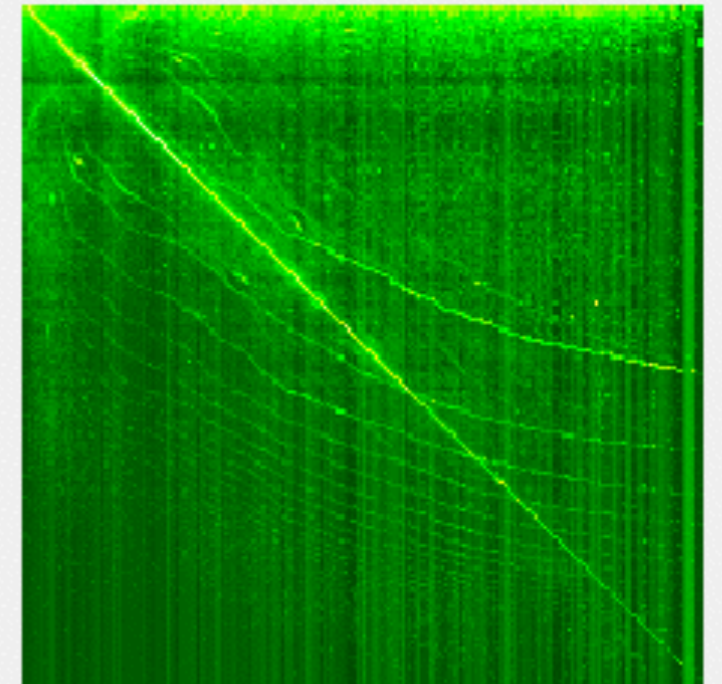
Wikipedia



Kernel Linux V2.6



Cambridge 2006



# Google Matrix

## FWL in Google matrices

Eur. Phys. J. B 75, 299–304 (2010)  
DOI: 10.1140/epjb/e2010-00144-0

THE EUROPEAN  
PHYSICAL JOURNAL B

Regular Article

### Ulam method and fractal Weyl law for Perron-Frobenius operators

L. Ermann and D.L. Shepelyansky<sup>®</sup>

Laboratoire de Physique Théorique du CNRS (IRSAMC), Université de Toulouse, UPS, 118 route de Narbonne, 31062 Toulouse Cedex 4, France

Eur. Phys. J. B 79, 115–120 (2011)  
DOI: 10.1140/epjb/e2010-10774-7

THE EUROPEAN  
PHYSICAL JOURNAL B

Regular Article

### Fractal Weyl law for Linux Kernel architecture

L. Ermann<sup>1</sup>, A.D. Chepelianskii<sup>2</sup>, and D.L. Shepelyansky<sup>1,®</sup>

<sup>1</sup> Laboratoire de Physique Théorique (IRSAMC), Université de Toulouse, UPS-CNRS, 31062 Toulouse, France

<sup>2</sup> LPS, Université Paris-Sud, CNRS, UMR502, 91405 Orsay, France

## 2D ranking of nodes

IOP PUBLISHING

JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. 45 (2012) 275101 (20pp)

doi:10.1088/1751-8113/45/27/275101

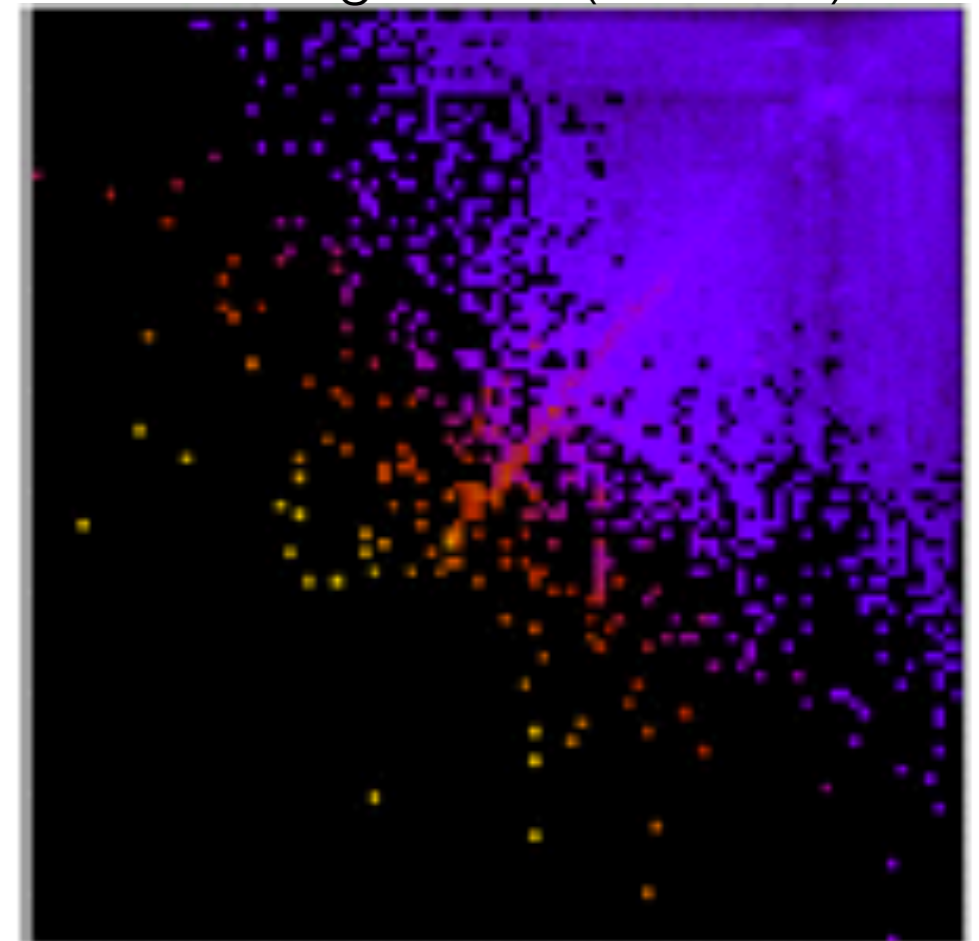
### Toward two-dimensional search engines

L Ermann<sup>1</sup>, A D Chepelianskii<sup>2</sup> and D L Shepelyansky<sup>1</sup>

<sup>1</sup> Laboratoire de Physique Théorique du CNRS, IRSAMC, Université de Toulouse, UPS, 31062 Toulouse, France

<sup>2</sup> Department of Physics, Cavendish Laboratory, University of Cambridge, CB3 0HE, UK

Cambridge 2006 (N~211K)





# World Trade Network

Import-Export trade database:

**United Nation Commodities Trade Network**

**[HTTP://COMTRADE.UN.ORG/DB/](http://COMTRADE.UN.ORG/DB/)**



- \* Each year from 1962 to 2011 (2014)
- \* All UN countries: ~ 220 ( $N_c=227$  in 2008)
- \* Product classification (SITC Rev. 1):  $N_p=61$
- \* Trade volume is given in USD ( $N=13847$  x 50 years)

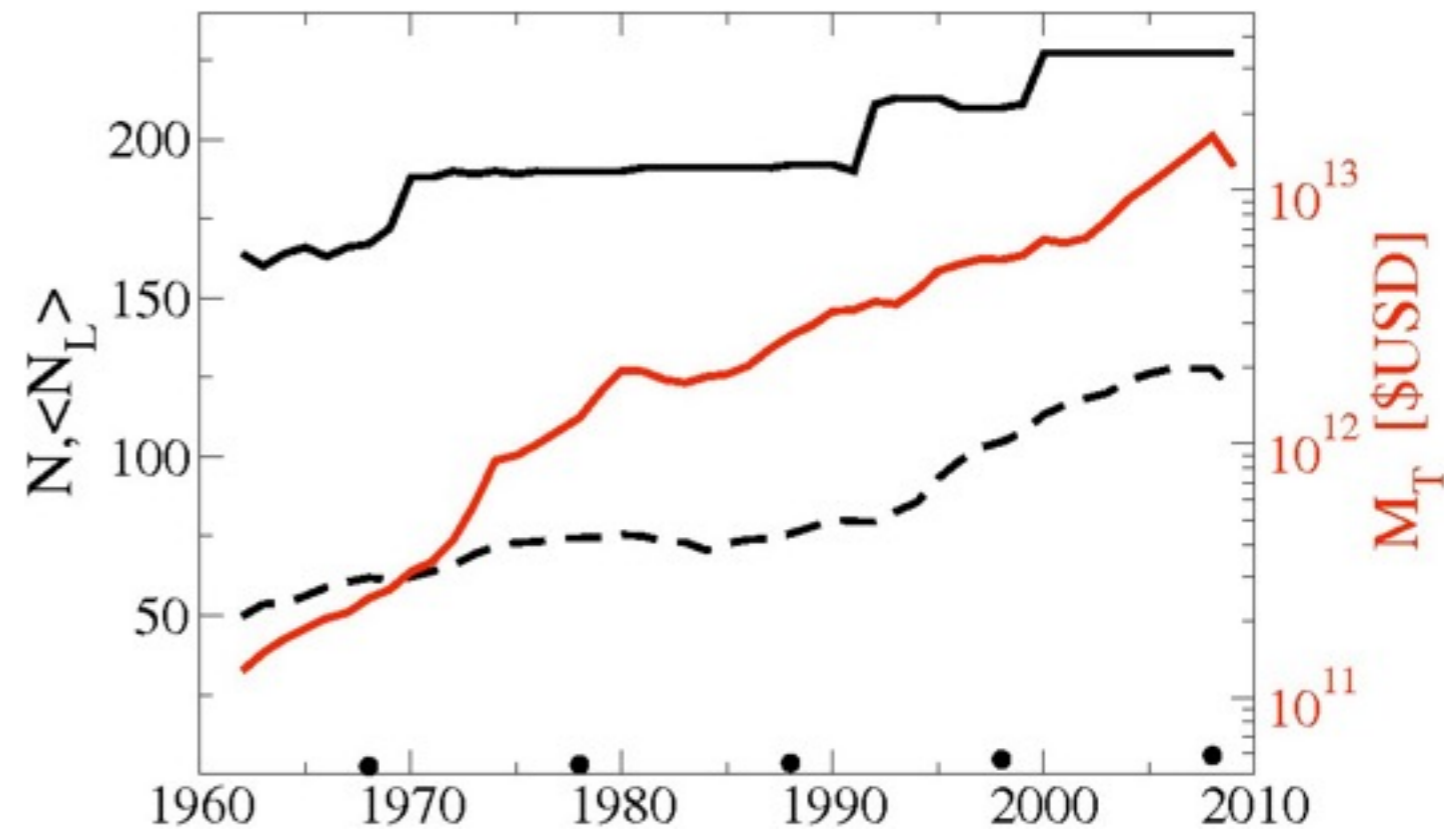
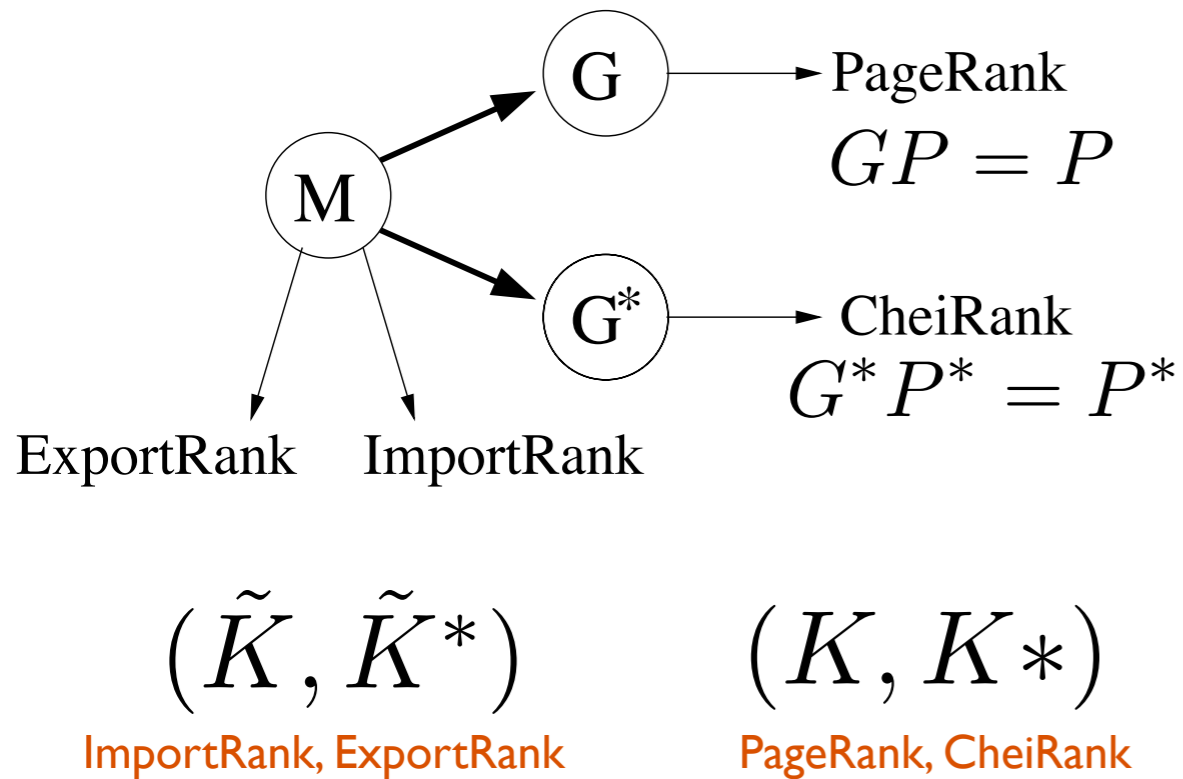
## Money Matrices

$$M_{c,c'} = \$ (c' \rightarrow c)$$

$$M_{c,c'}^p = \$ (c' \rightarrow c)$$



# Google matrix of the WTN



Democracy in countries but not in products

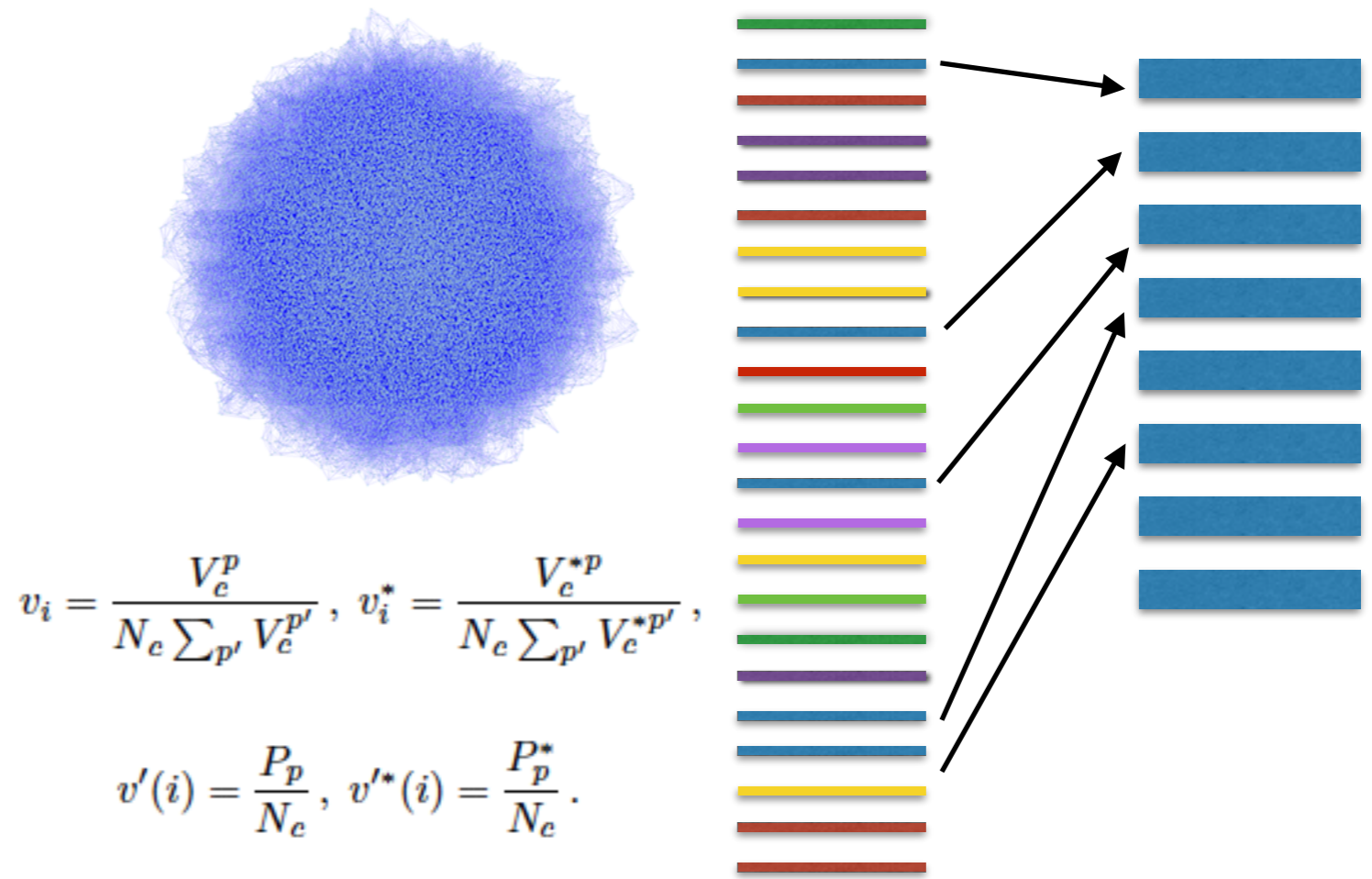
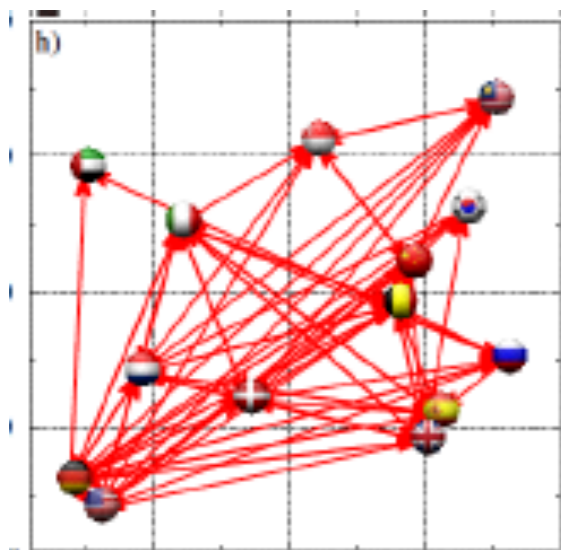
# Google matrix of the WTN

1) WTN (all com. or 1 prod)  
N=227

2) WTN (multiprod)  
N=13847

non-interacting products

personalized vector  $\longrightarrow$  prop to  $V_p$  of each  $c$   
2nd iteration  $\longrightarrow$  reduced  $P_p$  for all  $c$



$$v_i = \frac{V_e^P}{N_e \sum_{p'} V_e^{p'}}, \quad v_i^* = \frac{V_e^{*P}}{N_e \sum_{p'} V_e^{*p'}}$$

$$v'(i) = \frac{P_p}{N_e}, \quad v'^*(i) = \frac{P_p^*}{N_e}$$

L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011),  
<http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank>

L. Ermann and D.L. Shepelyansky, EPJB (2015).

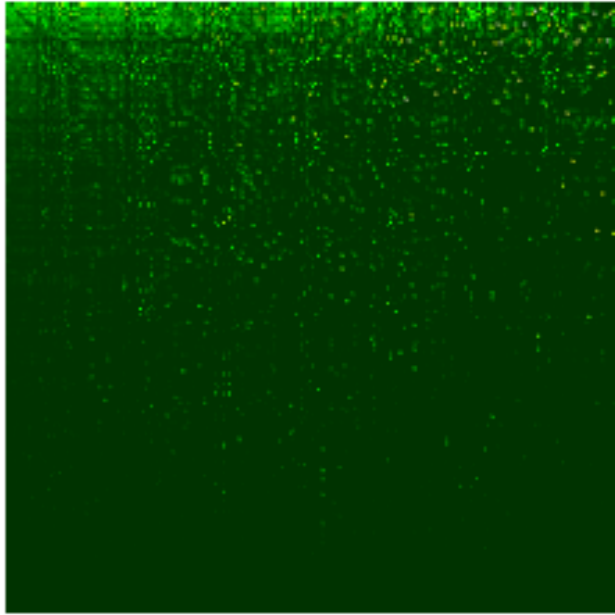
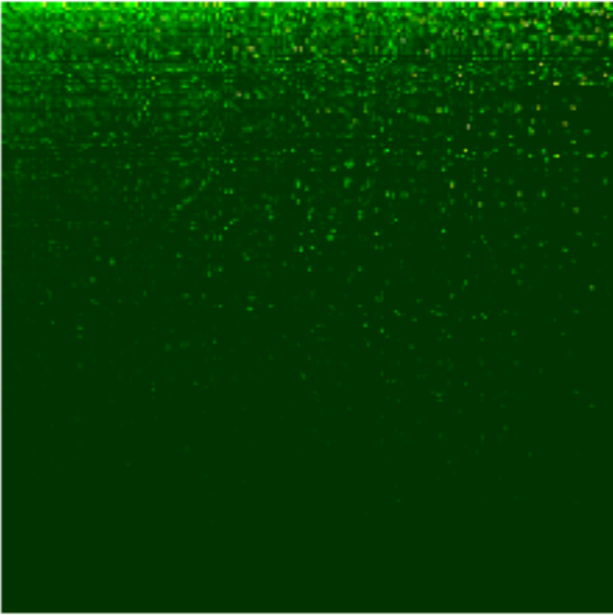
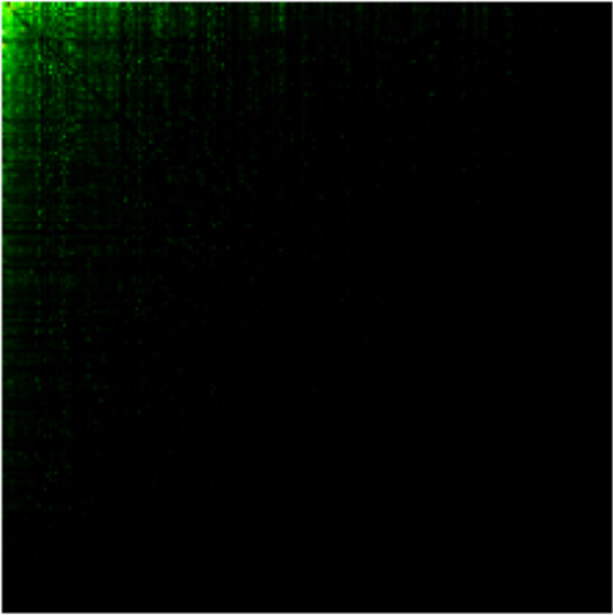
# all commodities and given products (N=227)

M

G

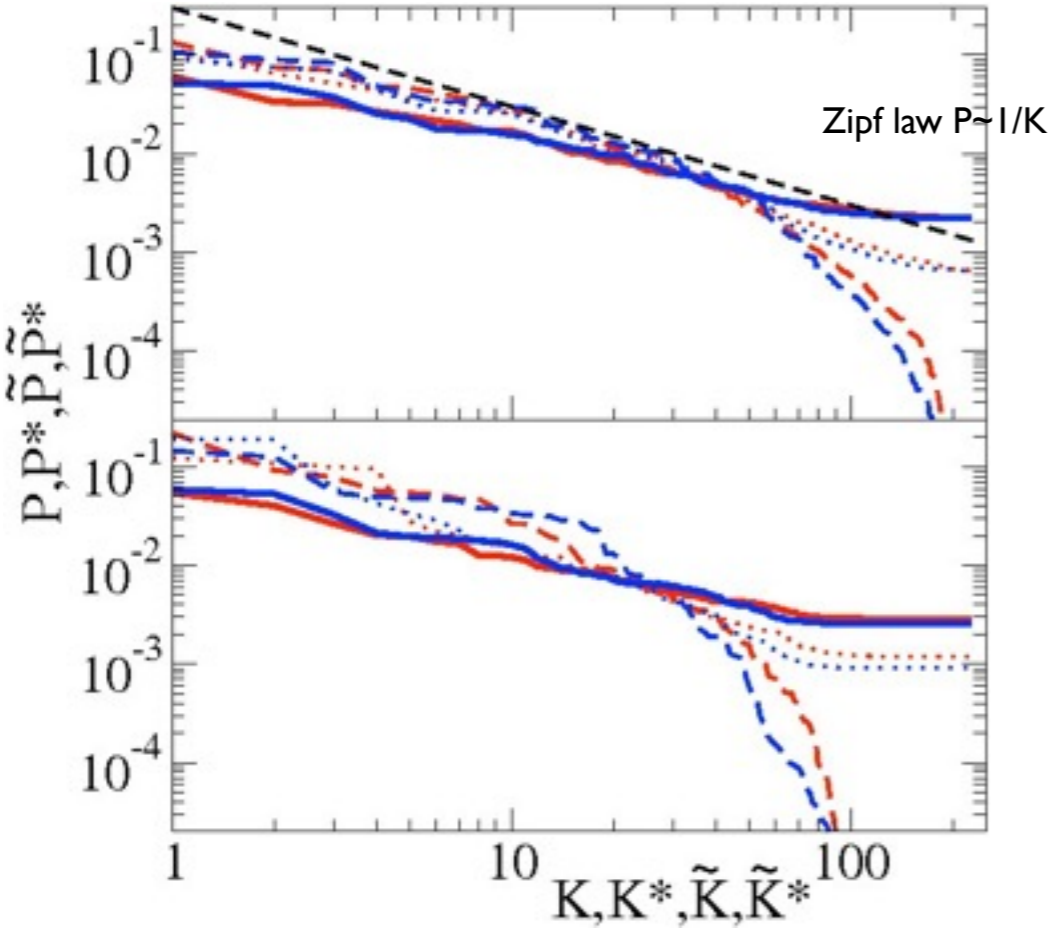
G\*

all commodities

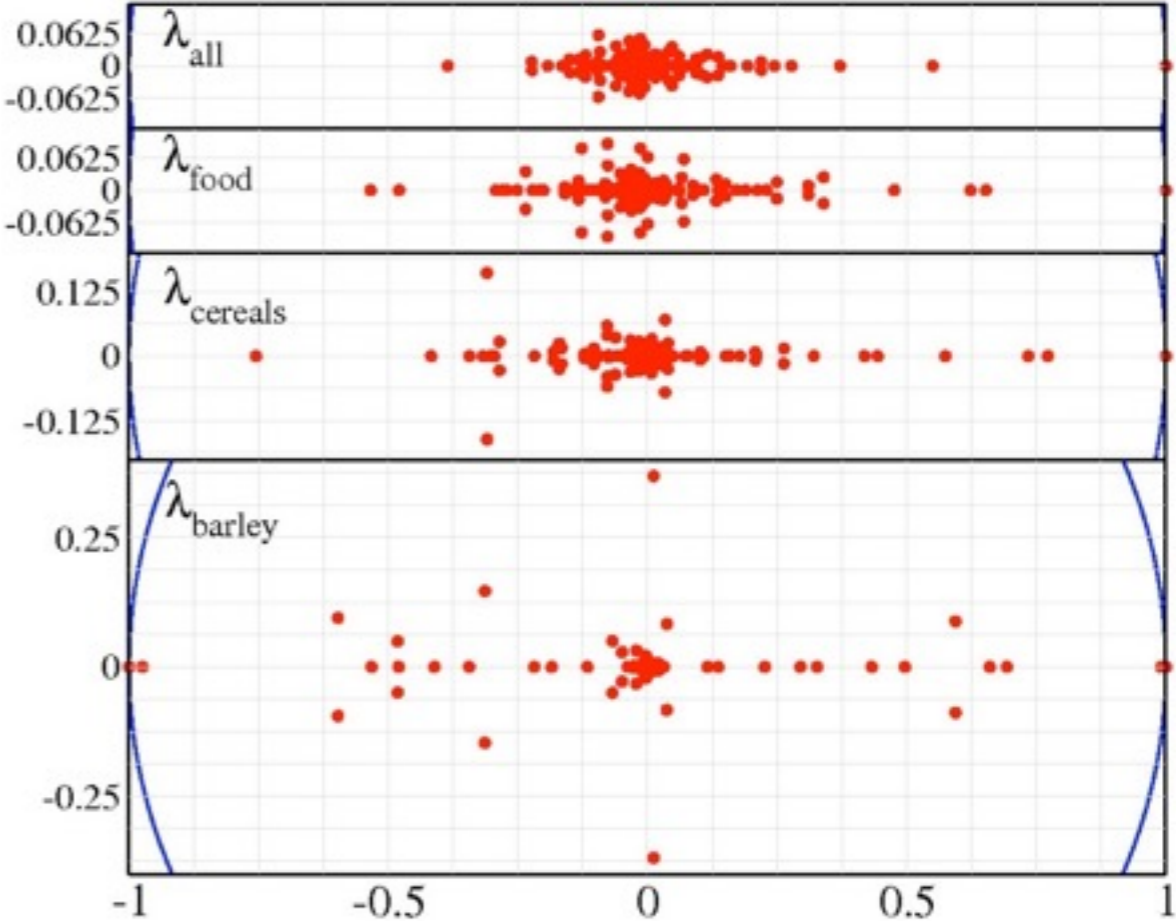


PageRank, CheiRank, ImportRank, ExportRank  $\alpha = 0.5$

Spectra  $\alpha = 1$

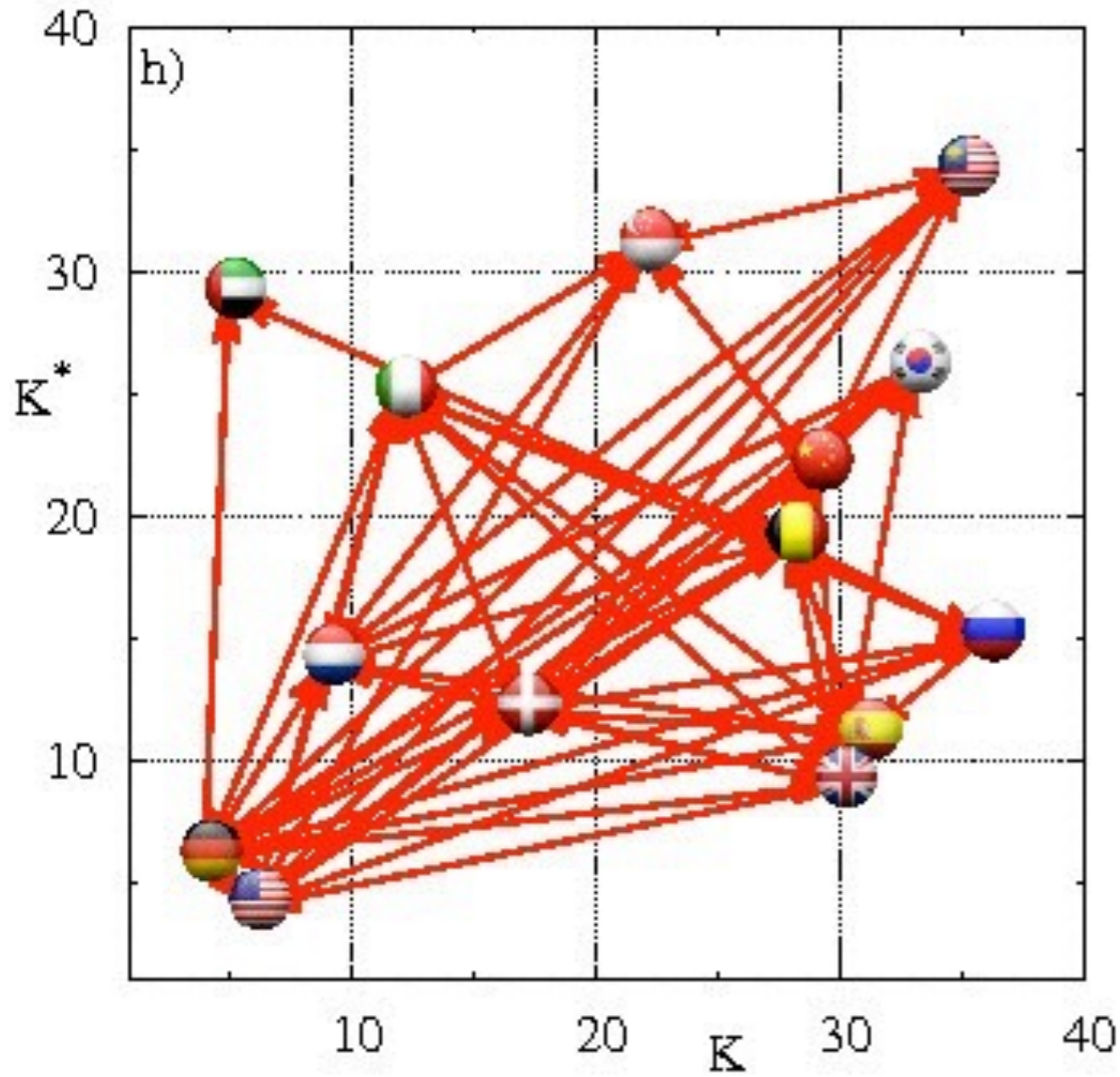


less symmetric



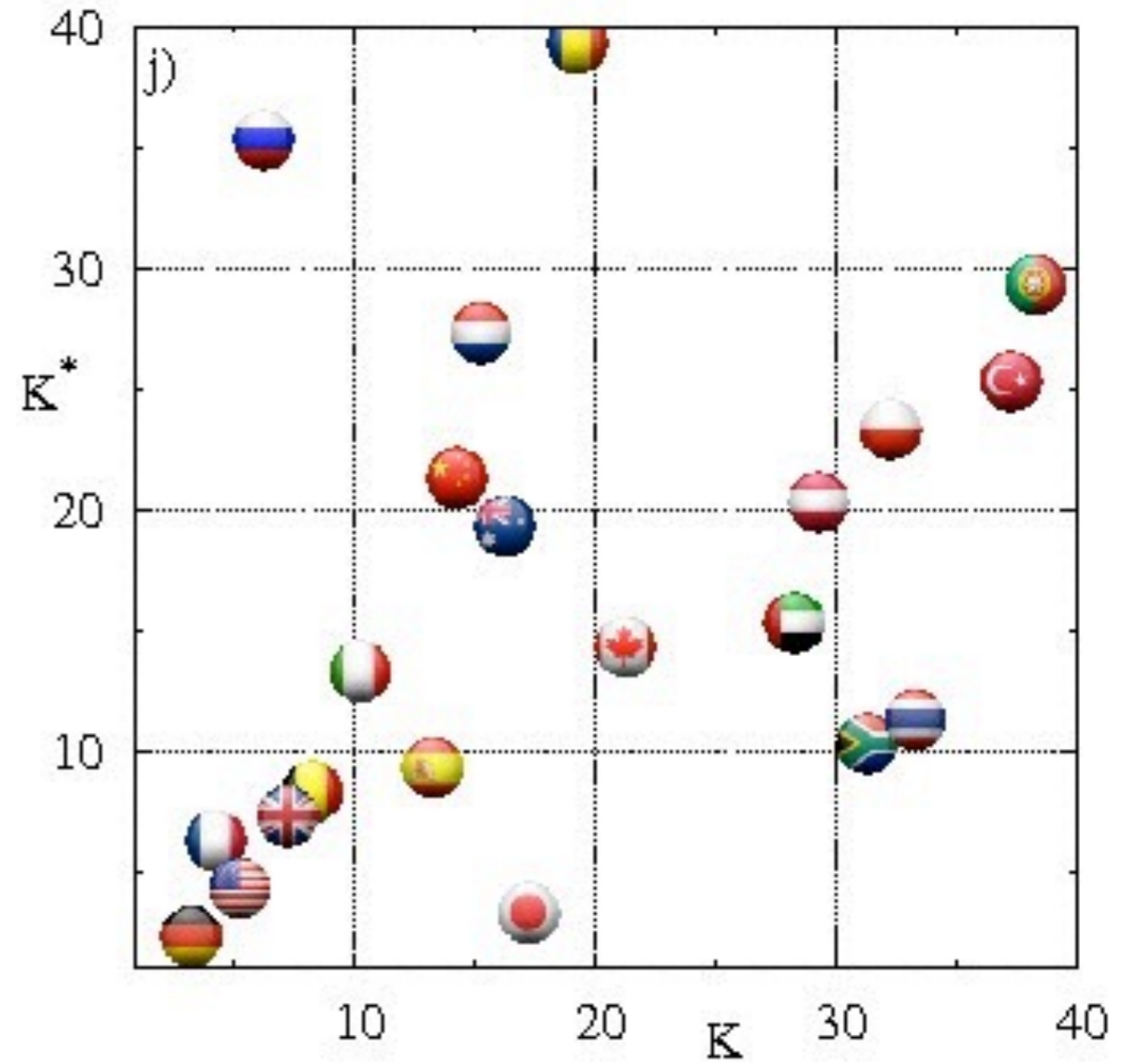


# 2D rank examples: barley and cars



Ukraine ( $K^*$  1st to 6th)

USA ( $K^*$  8th to 3rd)

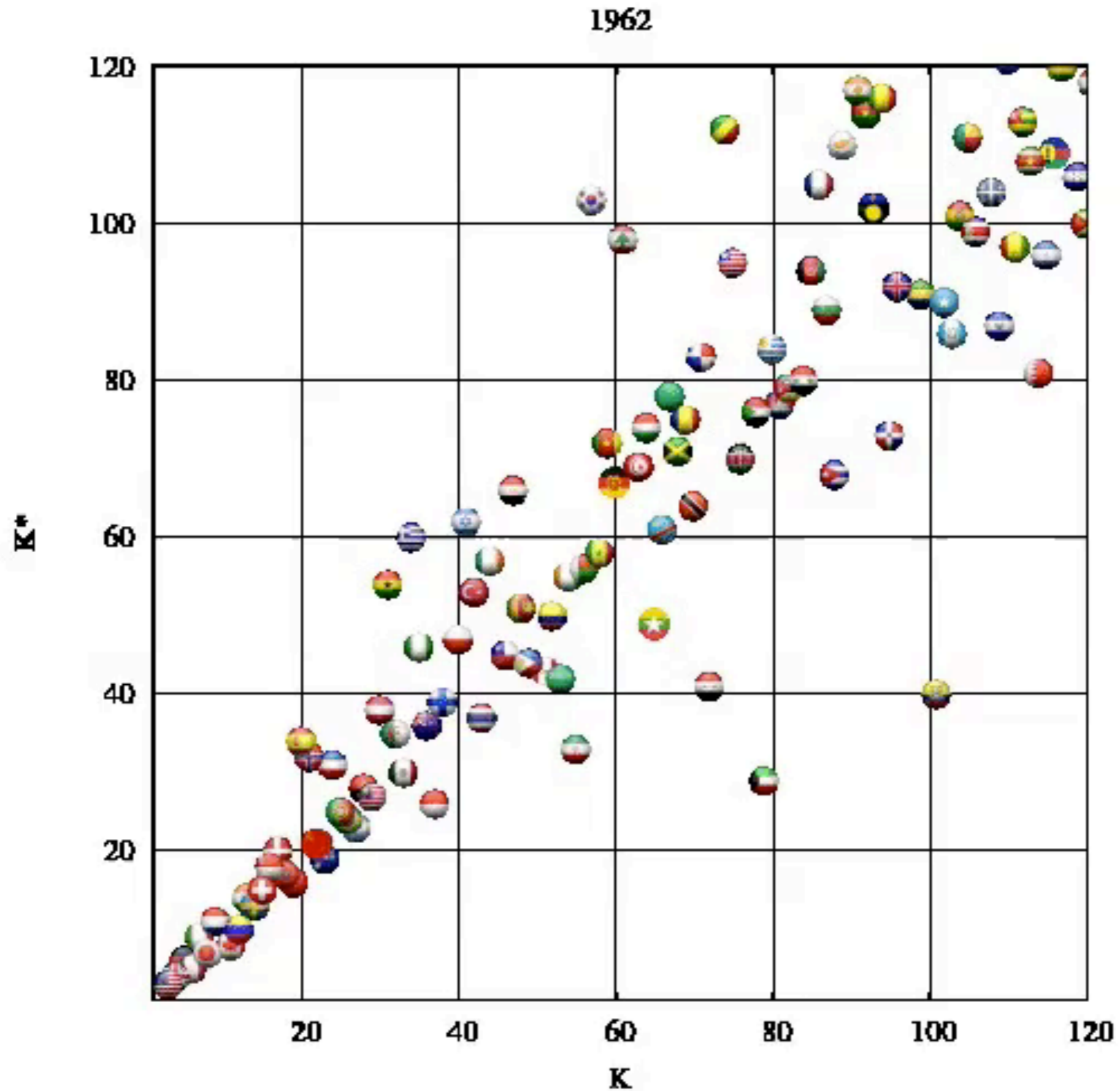


France ( $K^*$  7th to 3rd)

Thailand ( $K^*$  19th to 10th)



# 2d rank evolution (all comm.)



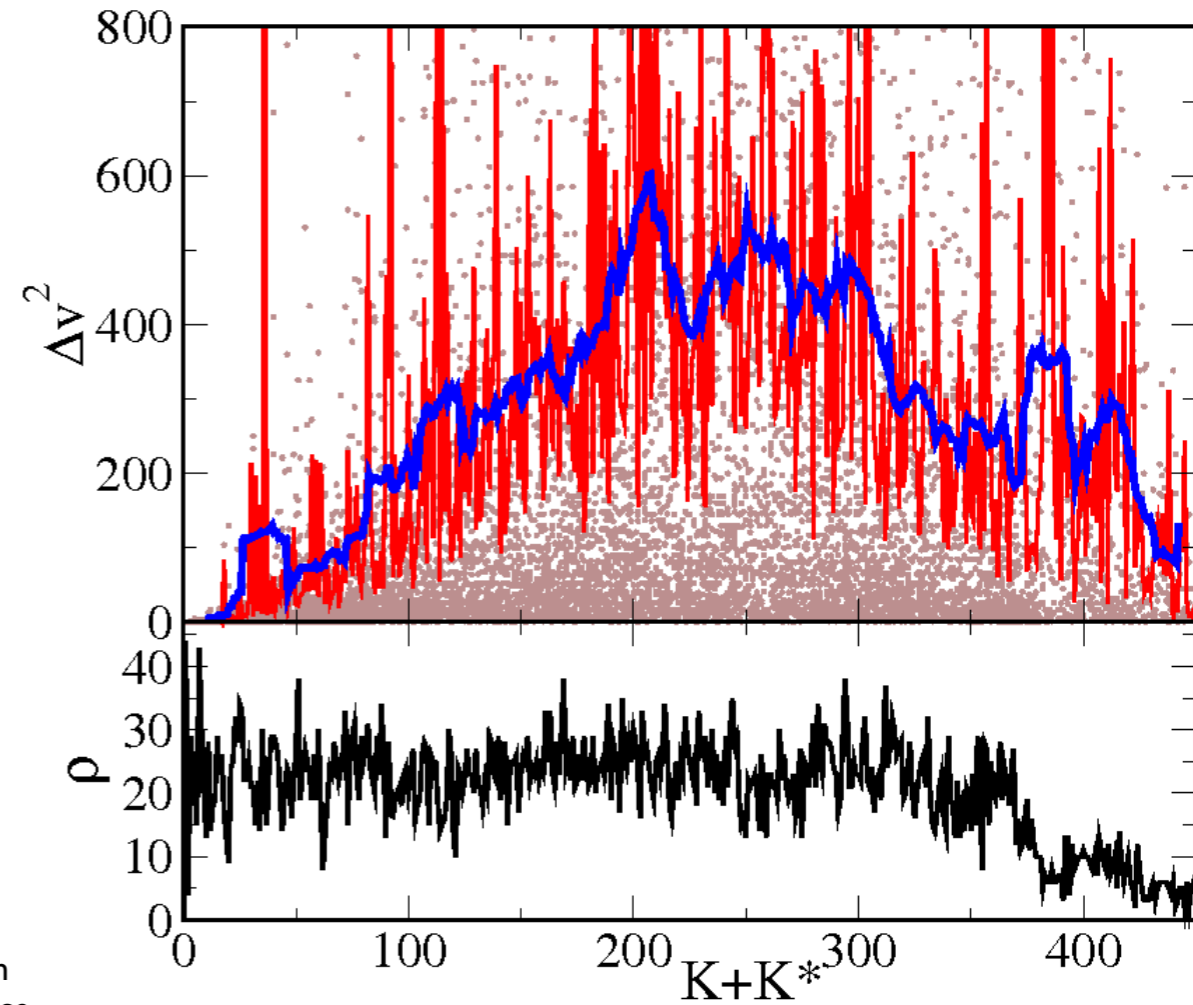
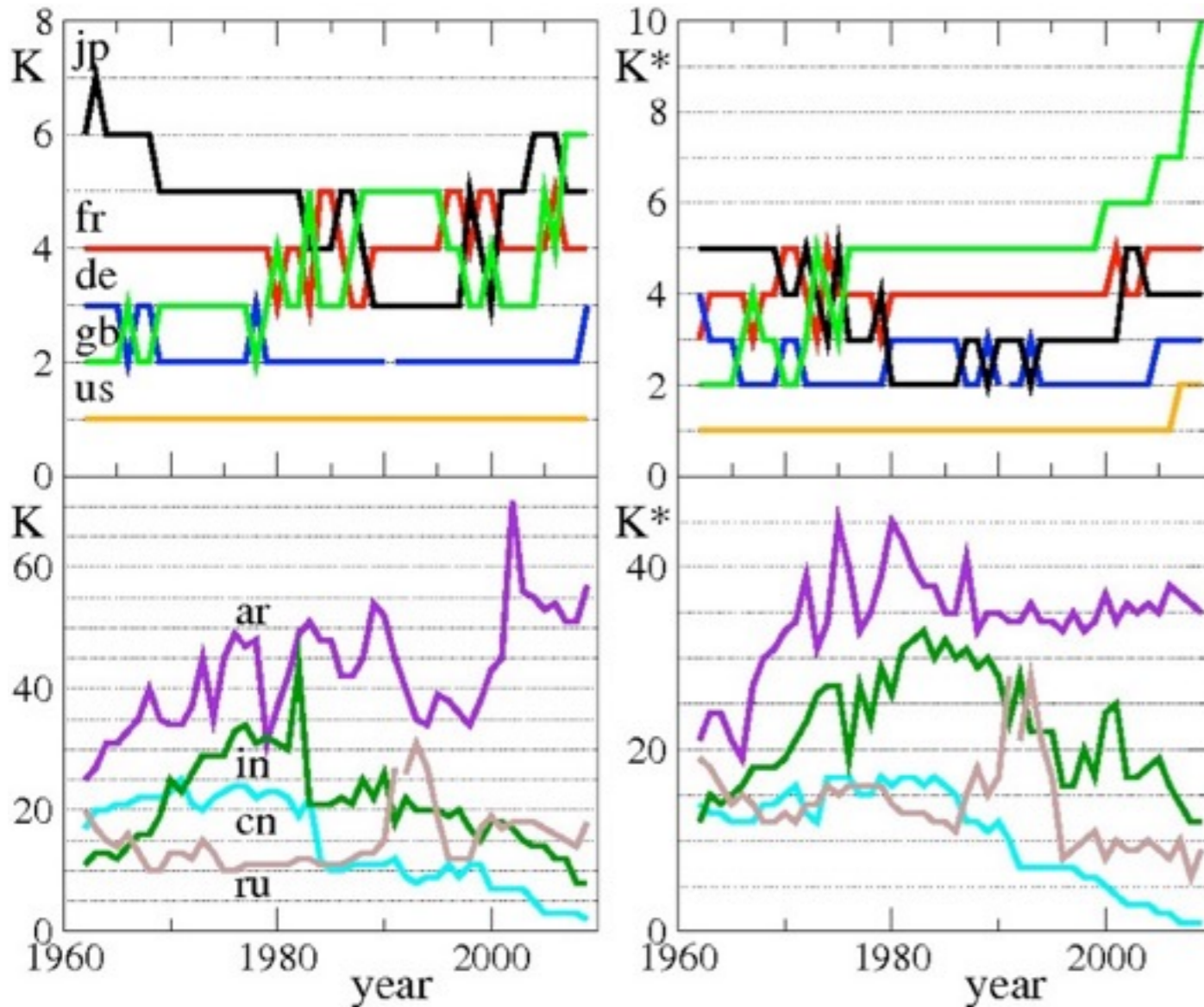
# 2d rank evolution

## Velocity square vs. $K+K^*$

$$\Delta v^2 = [K(t) - K(t-1)]^2 + [K^*(t) - K^*(t-1)]^2$$

average per  $K + K^*$

average in  $[K + K^* - 10, K + K^* + 10]$



Japan  
France  
Fed. Rep. of Germany and Germany  
Great Britain (sublimation?)  
USA

Argentina  
India  
China (deposition)  
USSR and Russian Fed.

crisis detection

# WTN model

- Gravity model of trade:

$$M_{i,j} = gm_i m_j / D_{i,j}$$

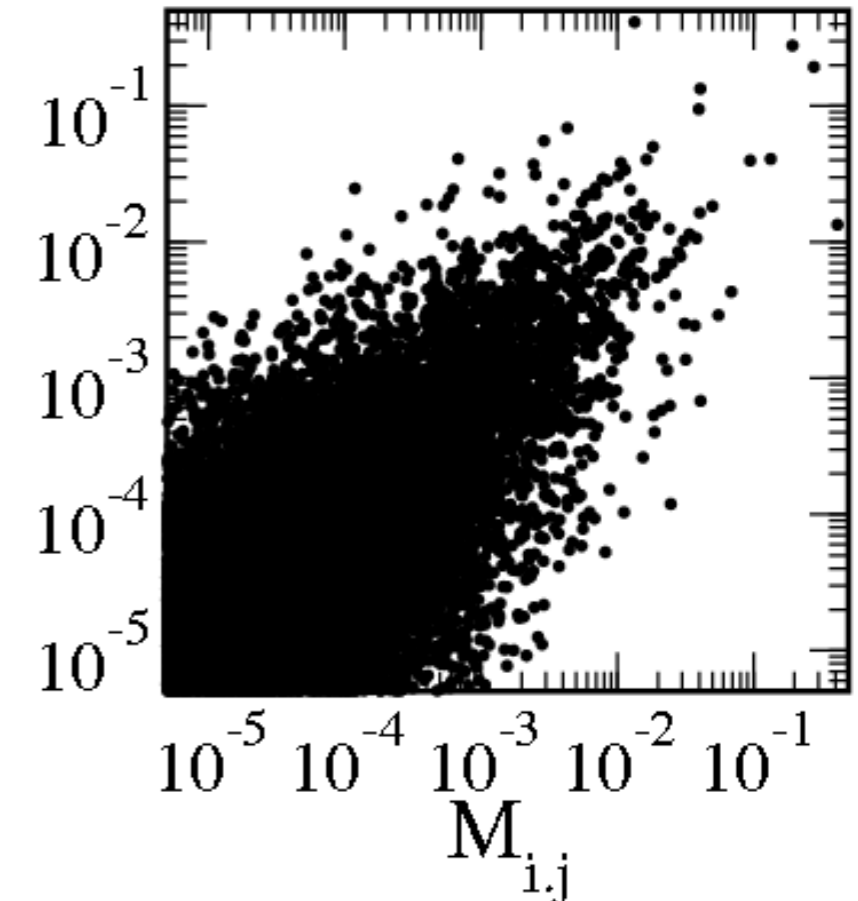
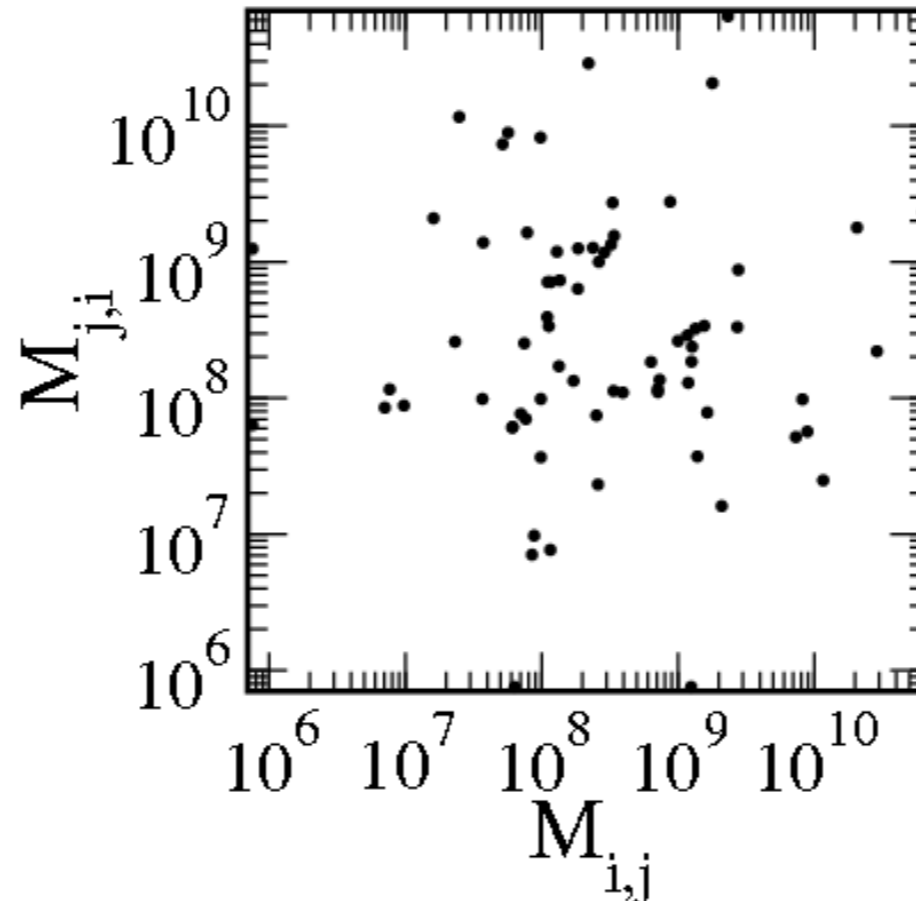
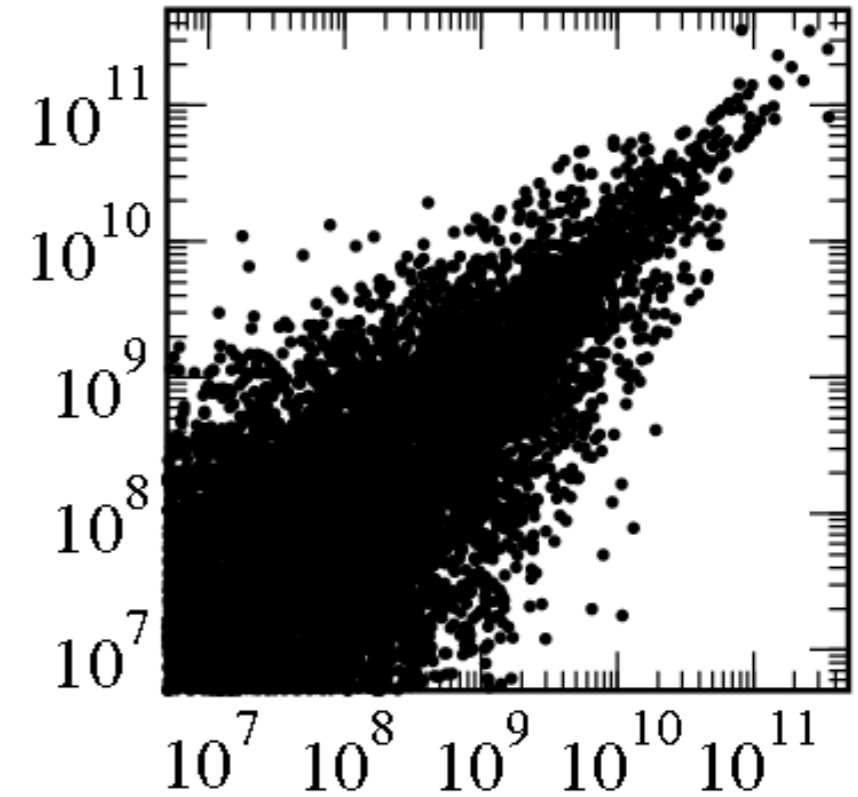
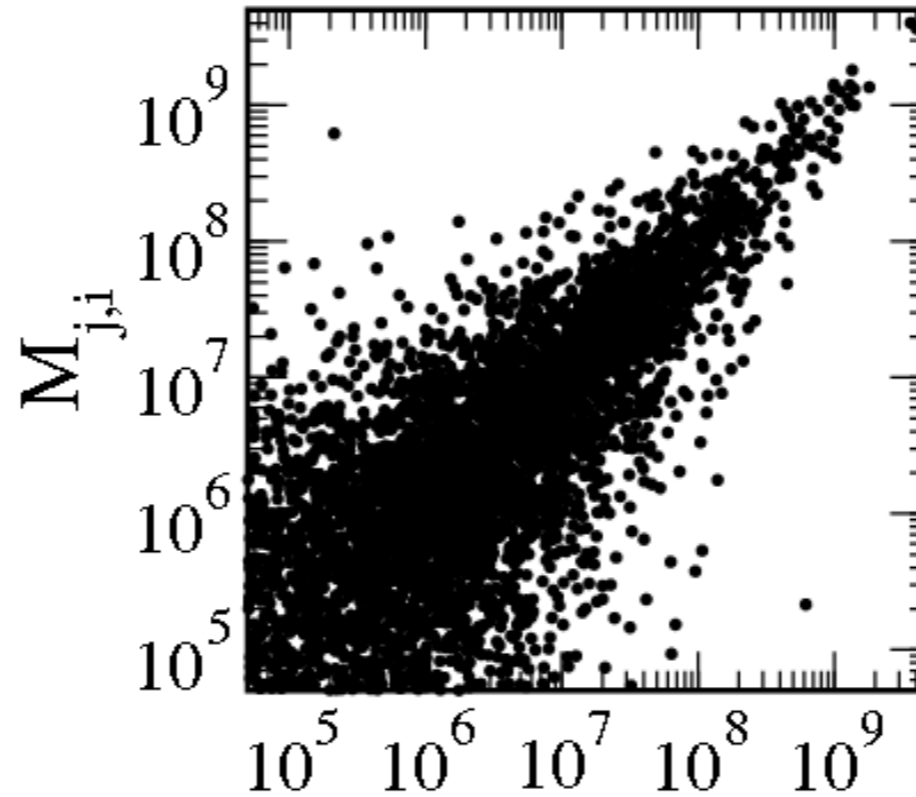
(symmetric)

- Random model

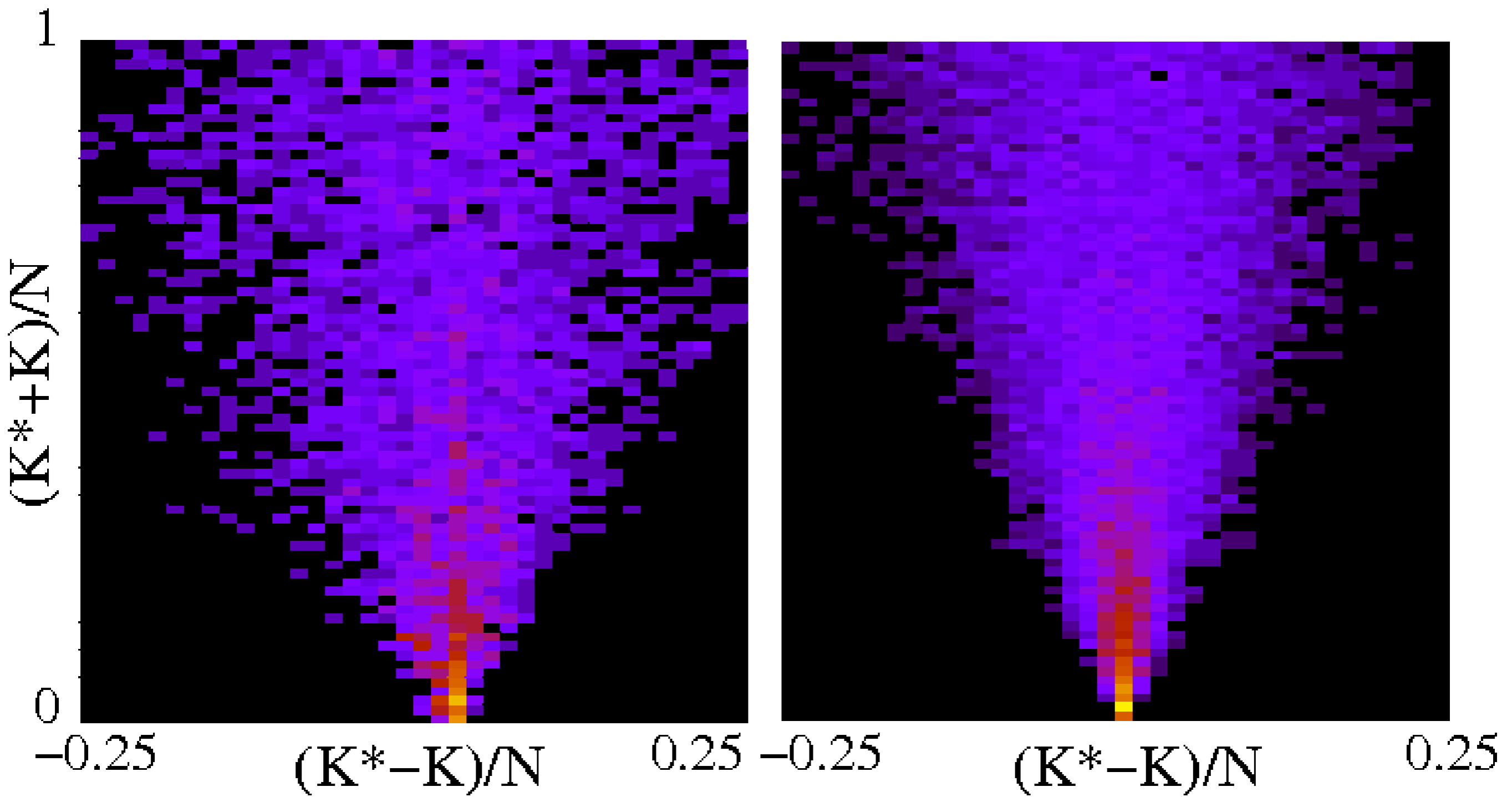
$$M_{i,j} = \epsilon_i \epsilon_j / ij \quad \epsilon_{i,j} \in [0, 1)$$

(preserves Zipf law)

t:: all commodities (1962, 2008);  
b: crude petroleum (2008), random model



# model statistics





# Crisis

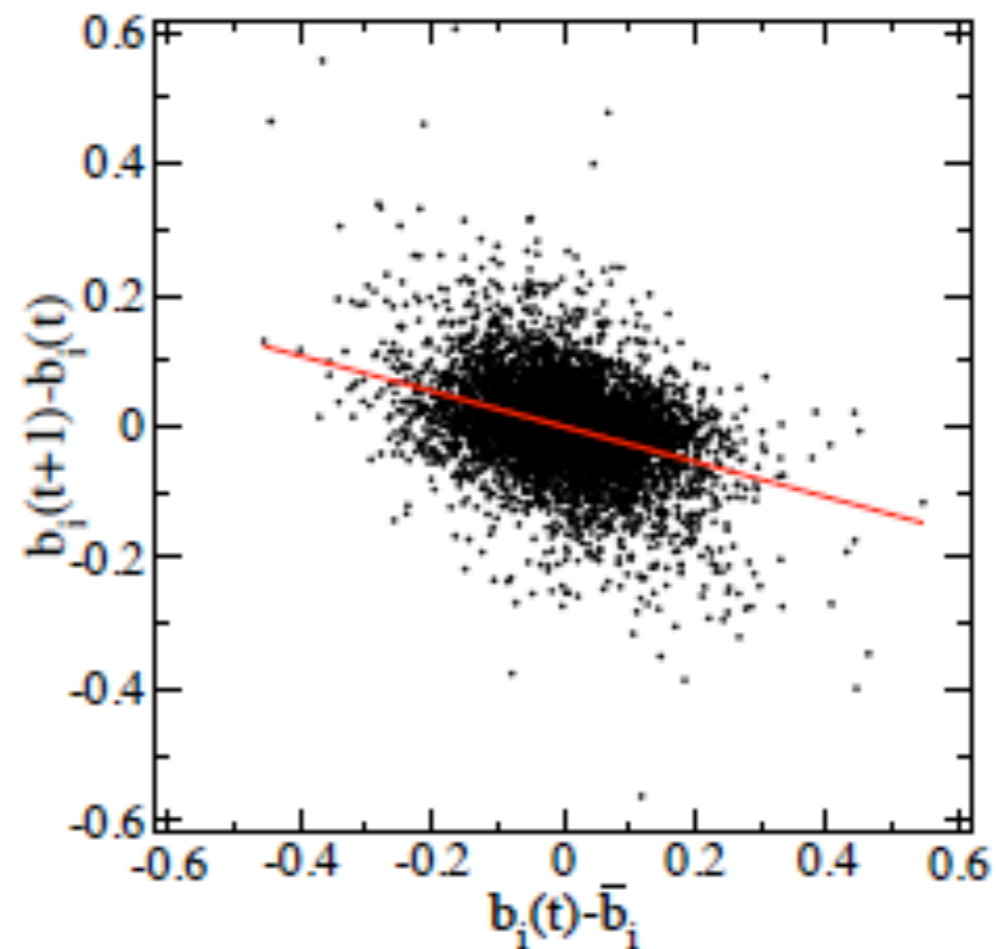
balance  $b_i = \frac{P(i) - P^*(i)}{P(i) + P^*(i)}$

weight  $w_i = P(i) + P^*(i)$

Rank	country	$b_i$ (positive)	global rank	country	$b_i$ (positive)	global rank
1	Greece	0.5131	1	Iran	-0.2846	7
2	Spain	0.2505	15	Malaysia	-0.2676	8
3	Romania	0.2322	19	China	-0.2506	10
4	Portugal	0.2222	23	Saudi Arabia	-0.2470	12
5	Mexico	0.1743	37	Argentina	-0.2388	13
6	Canada	0.1633	40	Russian Fed.	-0.2340	15
7	USA	0.1457	45	Brazil	-0.1939	20
8	UK	0.1397	49	Singapore	-0.1814	22
9	Poland	0.1326	51	Rep. of Korea	-0.1788	23
10	France	0.1086	62	Australia	-0.1693	25

Rank	year	country	$b_i(t+1) - b_i(t)$
1	1973	Lebanon	-0.376
2	2001	Argentina	-0.269
3	1981	Mexico	-0.258
4	1983	Nigeria	-0.253
5	2002	Saudi Arabia	-0.250
6	1982	Venezuela	-0.247
7	1997	Indonesia	-0.244
8	1962	Venezuela	-0.236
9	1973	Nigeria	-0.230
10	1994	Mexico	-0.230
11	1997	Rep. of Korea	-0.219
12	1983	U. Arab Emir.	-0.213
13	2005	Iran	-0.210
14	1978	Iran	-0.210
15	1993	Turkey	-0.204
16	1975	India	-0.202
17	1998	Russian Fed.	-0.202
18	1976	Iraq	-0.200
19	1987	Argentina	-0.196
20	1989	Venezuela	-0.192

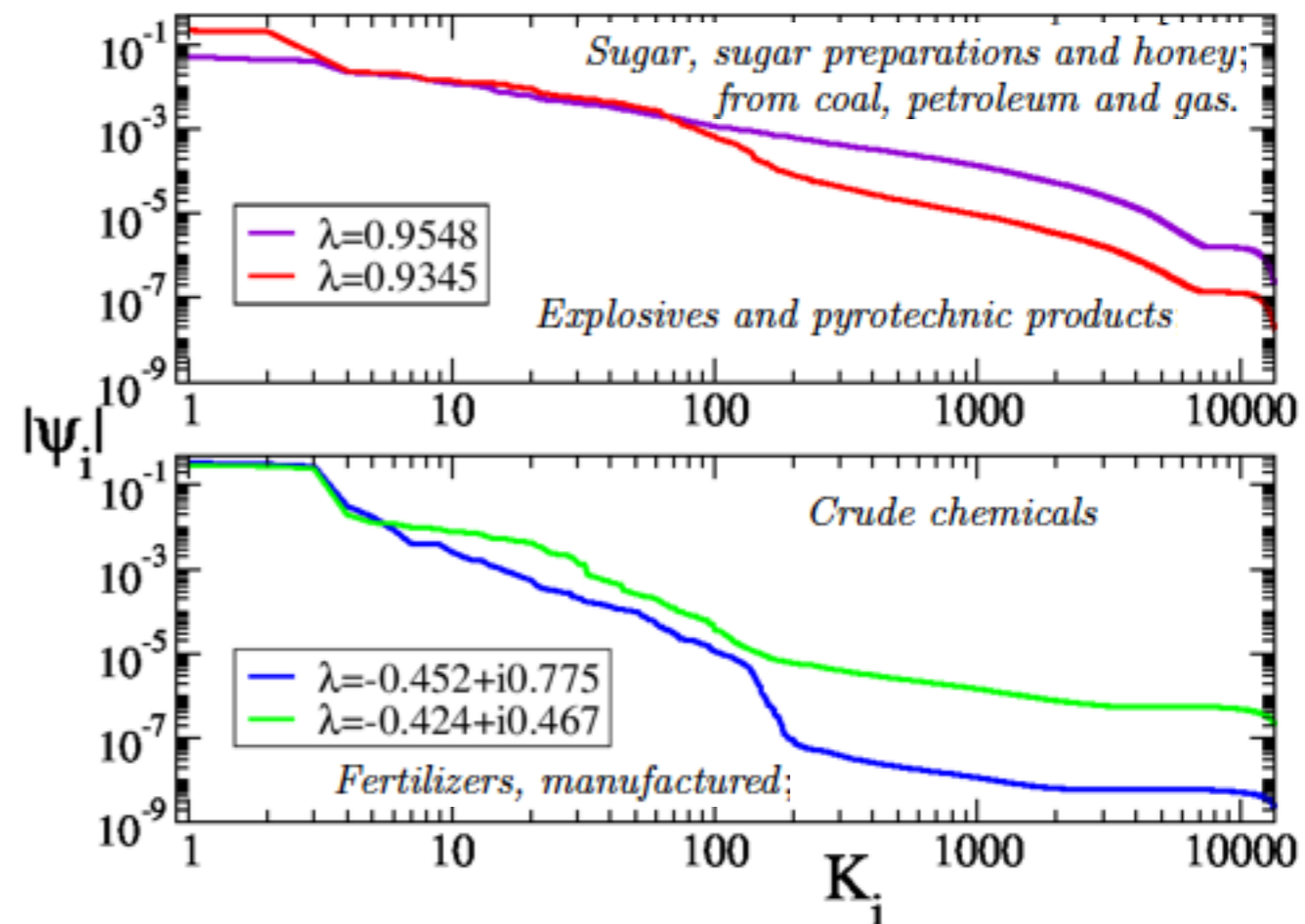
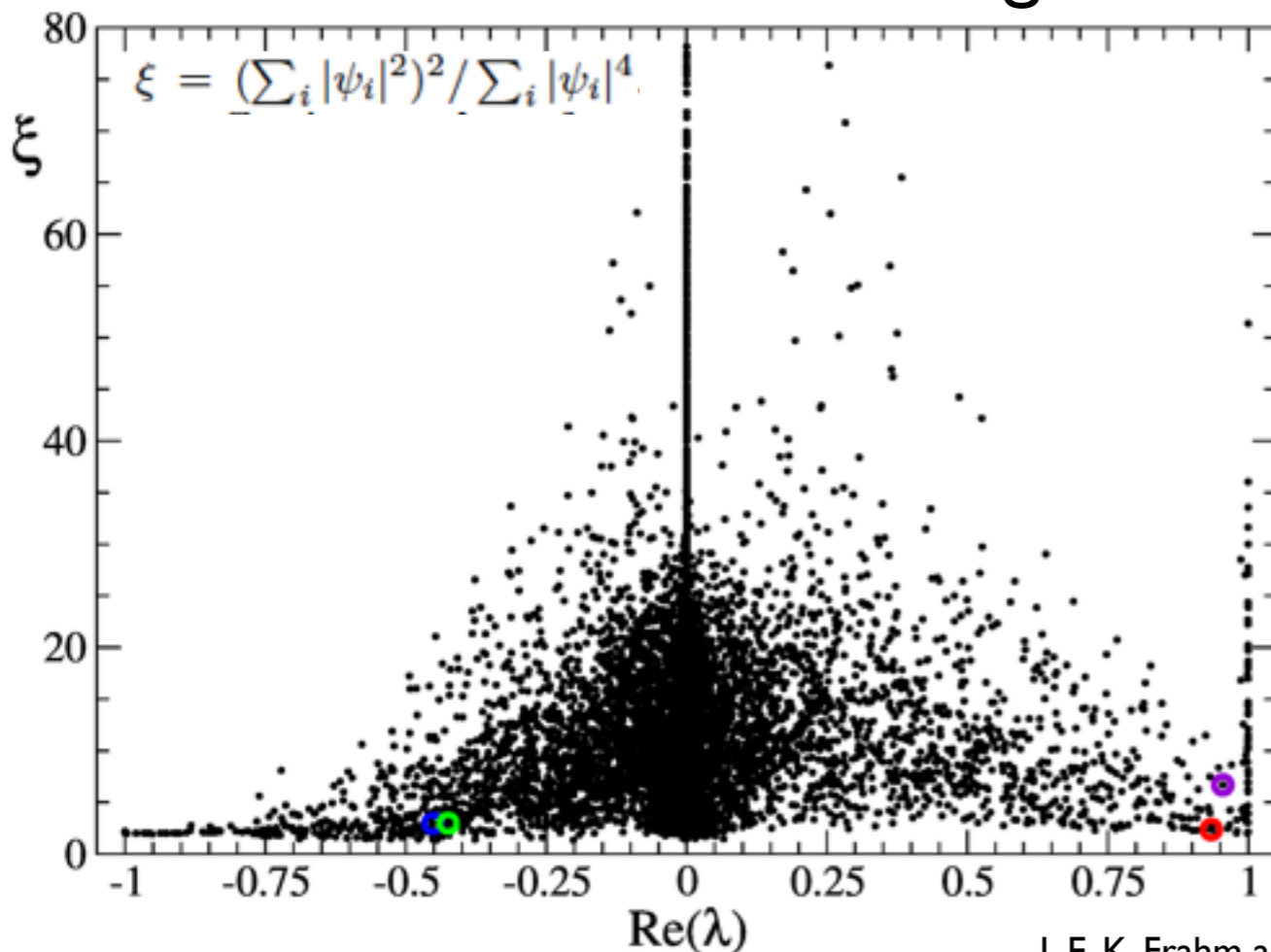
2008;  $w > 0.05$  (~20%)



# multi-prod WTN spectrum

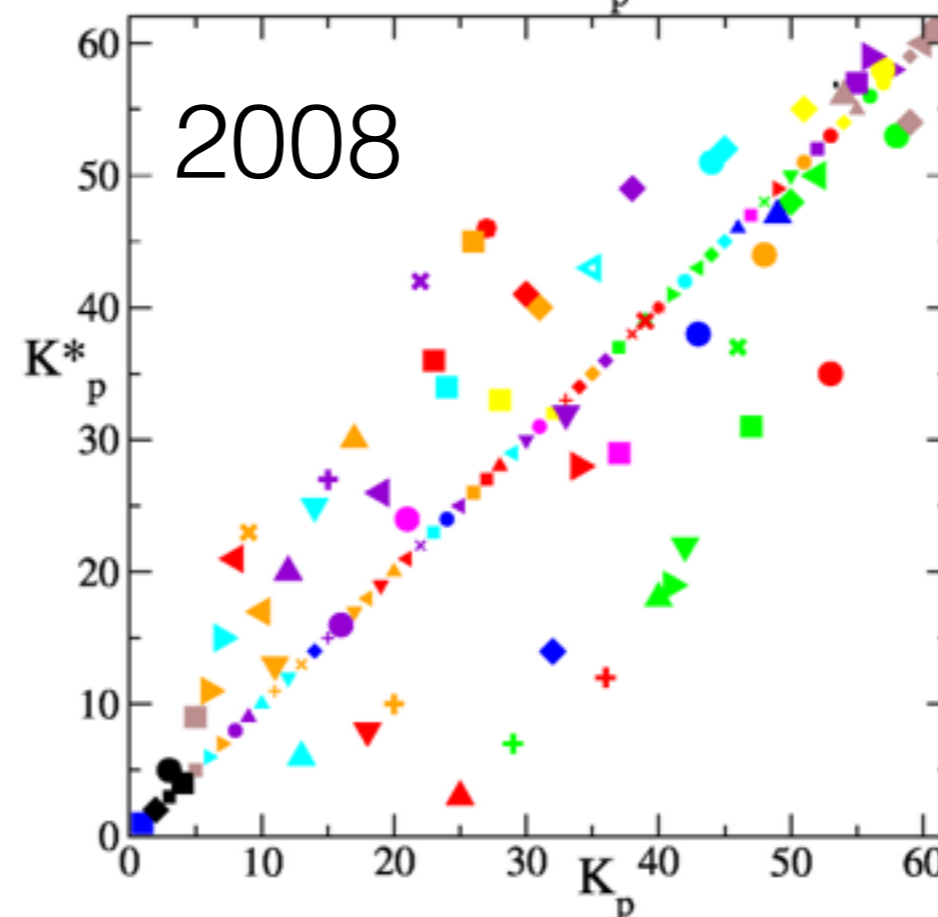
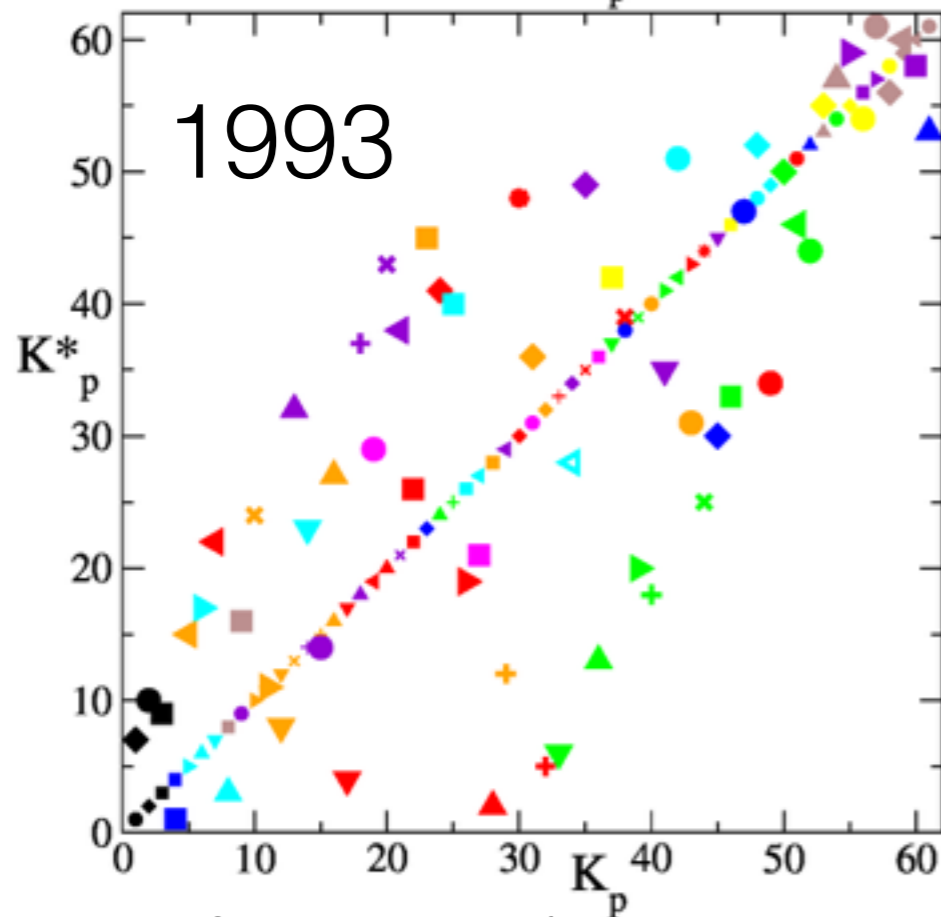
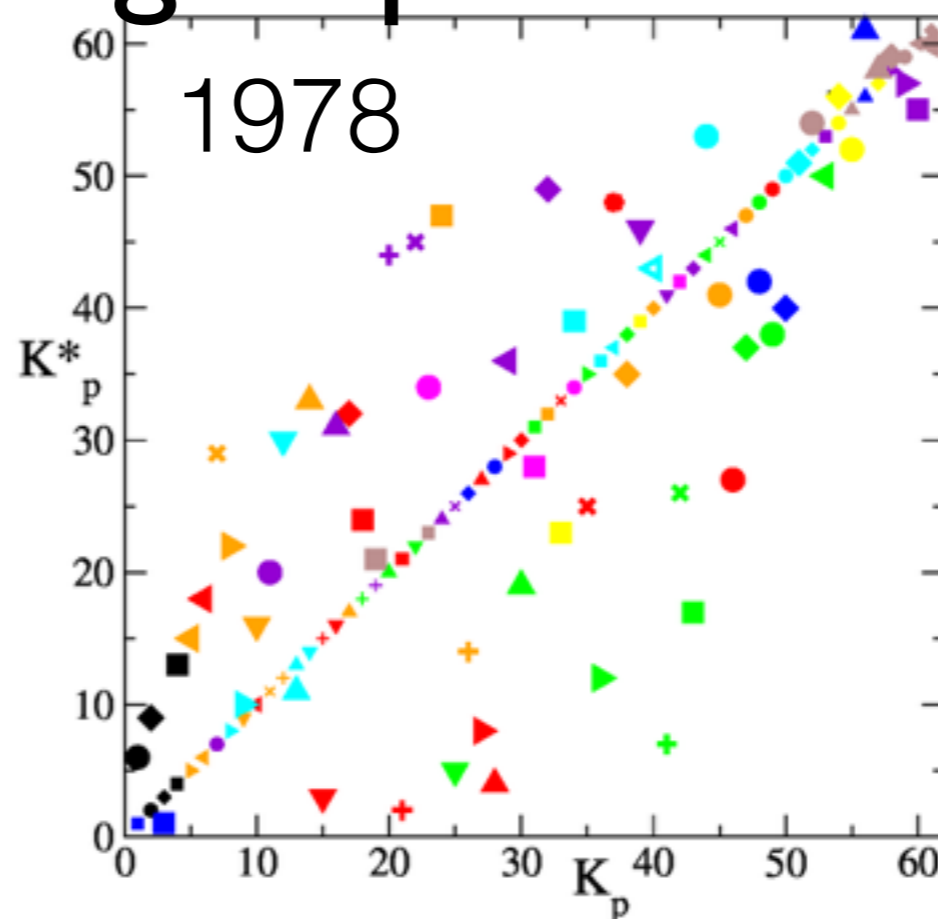
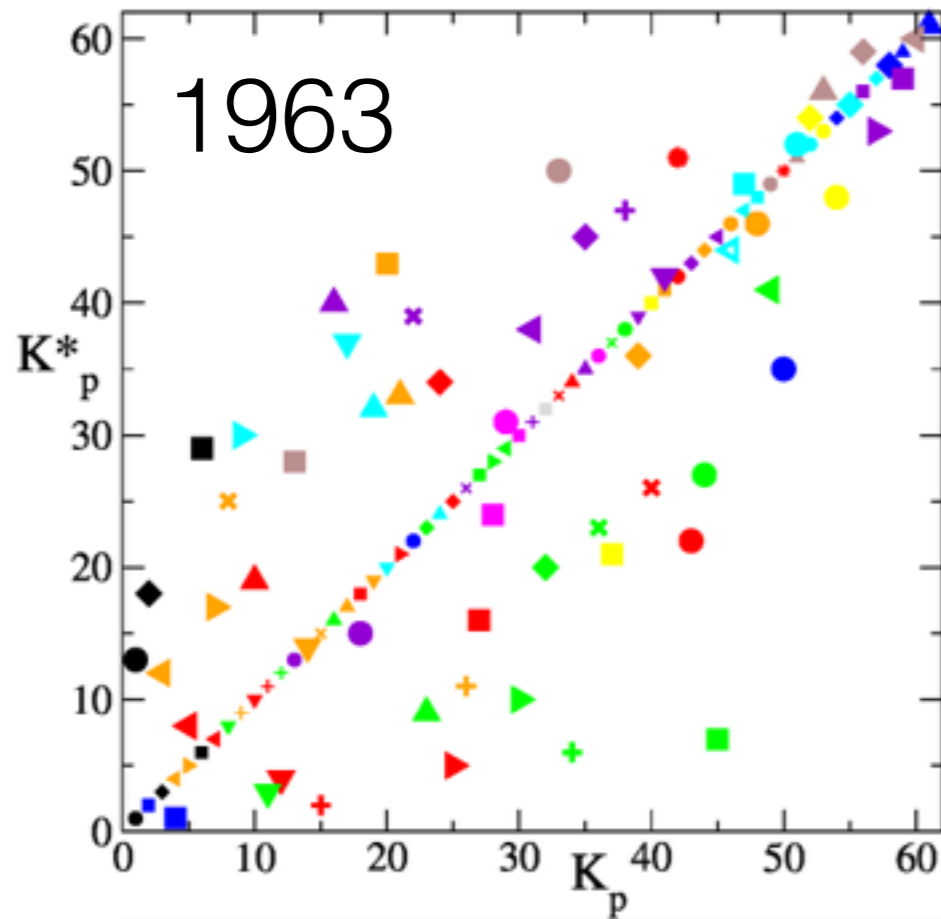
$K_i$	$ \psi_i $	country	$ \psi_i $	country	$ \psi_i $	country	$ \psi_i $	country
		prod: 57		prod:06		prod:56		prod:52
1	0.052	USA	0.216	Mali	0.332	Brazil	0.288	Japan
2	0.044	Tajikistan	0.201	Guinea	0.304	Bolivia	0.279	Rep. of Korea
3	0.042	Kyrgyzstan	0.059	USA	0.274	Paraguay	0.245	China
4	0.022	France	0.023	Germany	0.031	Argentina	0.020	Australia
5	0.021	Mexico	0.021	Mexico	0.017	Uruguay	0.013	USA
6	0.018	Italy	0.021	Canada	0.009	Chile	0.012	U Arab Em
7	0.018	Canada	0.018	UK	0.004	Portugal	0.010	Canada
8	0.015	Germany	0.015	Israel	0.004	Angola	0.010	Singapore
9	0.013	U Arab Em	0.015	C d'Ivoire	0.004	Spain	0.009	Germany
10	0.012	Qatar	0.014	Japan	0.003	France	0.008	New Zealand

eigenstate communities





# 2d ranking of products



- Live animals
- Meat and meat preparations
- Dairy products and eggs
- ▲ Fish and fish preparations
- ▲ Cereals and cereal preparations
- ▼ Fruit and vegetables
- ▶ Sugar, sugar prep. and honey
- + Coffee, tea, cocoa, spices & man
- × Feed. Stuff for animals
- Miscellaneous food preparations
- Beverages
- Tobacco and tobacco manuf.
- Hides, skins and fur skins, undress.
- Oil seeds, oil nuts and oil kernels
- Crude rubber incl. synth & recl.
- ▲ Wood, lumber and cork
- ▼ Pulp and paper
- ▼ Textile fibres, not manuf., & waste
- ▶ Crude fertilizers & crude minerals
- + Metalliferous ores and metal scrap
- × Crude animal & vegetable mat.
- Coal, coke and briquettes
- Petroleum and petroleum products
- Gas, natural and manufactured
- ▲ Electric energy
- Animal oils and fats
- Fixed vegetable oils and fats
- Animal & veg. oils & fats (proc.)
- Chemical elements & compounds
- Crude chem. from coal, petr. & gas
- Dyeing, tanning & colouring mat.
- ▲ Medicinal & pharmaceutical prod.
- ▼ Perfume mat., toilet & clean. prep.
- ▼ Fertilizers, manufactured
- ▶ Explosives and pyrotechnic prod.
- + Plastic materials, etc.
- × Chemical materials and products
- Leather, Manuf. & dressed fur skin
- Rubber manufactures, nes
- Wood & cork manuf. exc. furniture
- ▲ Paper, paperboard and manuf.
- ▼ Textile yarn, fabrics, etc.
- ▼ Non metallic mineral manuf., nes
- ▶ Iron and steel
- + Non ferrous metals
- × Manufactures of metal nes
- Machinery, other than electric
- Electrical machinery, apparatus and appliances
- Transport equipment
- Sanitary, plumbing, heating and lighting fixt.
- Furniture
- Travel goods, handbags and similar articles
- ▲ Clothing
- ▼ Footwear
- ▼ Scientif & control instrum, photogr gds, clocks
- ▶ Miscellaneous manufactured articles, nes
- Postal packages not class. According to kind
- Special transact. Not class. According to kind
- Animals, nes, incl. Zoo animals, dogs and cats
- ▲ Firearms of war and ammunition therefor
- ▼ Coin, other than gold coin, not legal tender

# Sensitivity to price variation

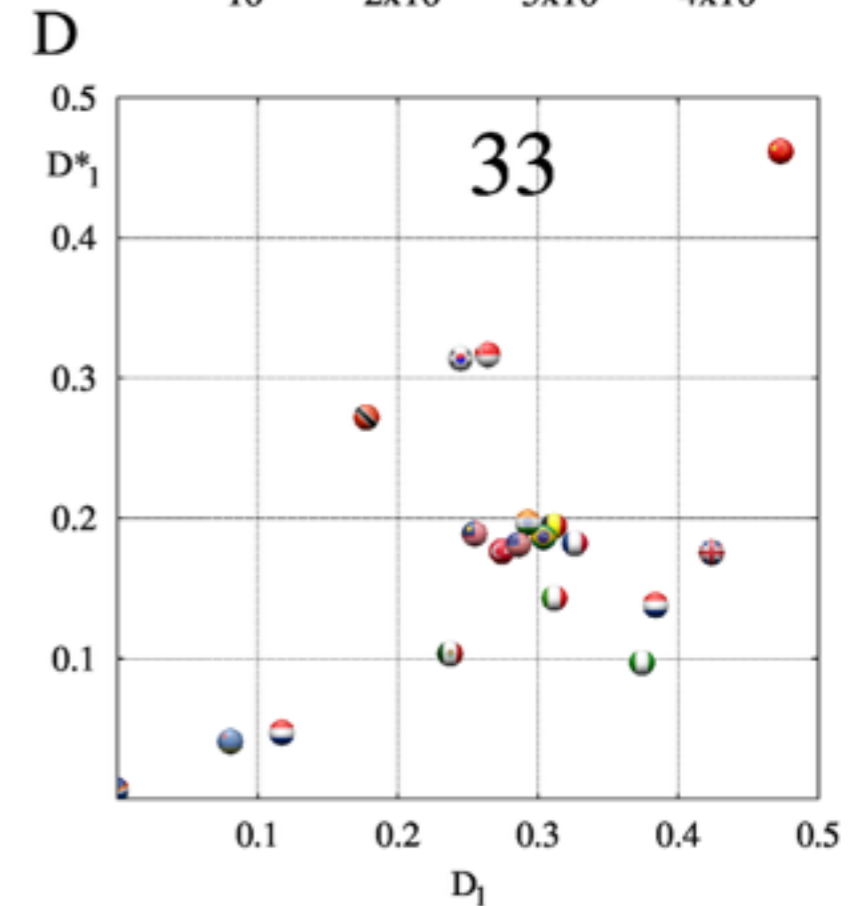
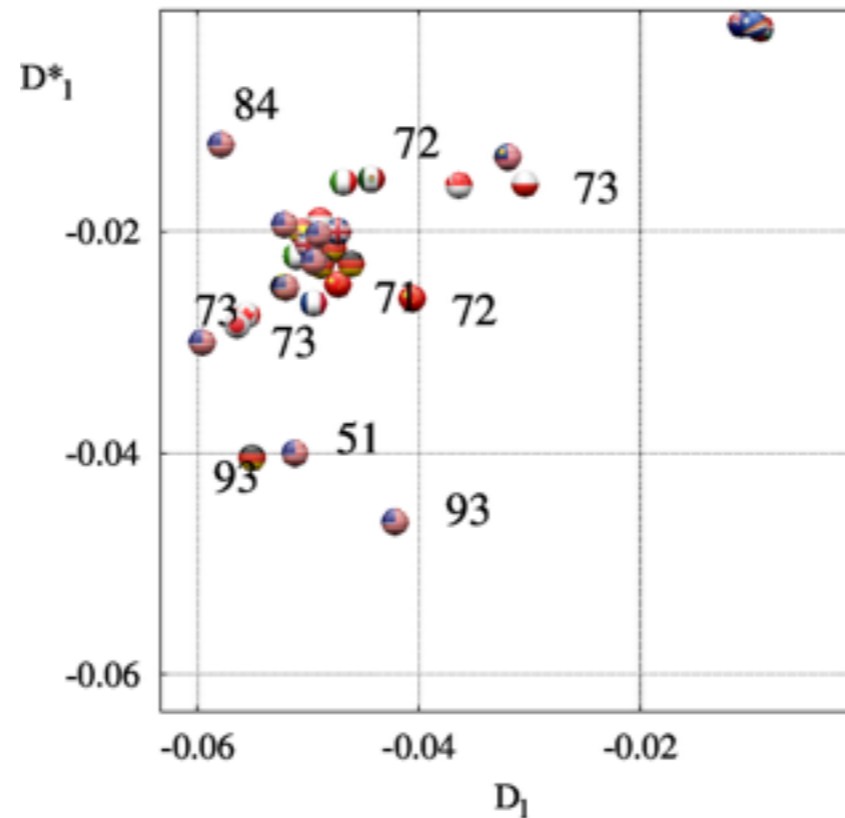
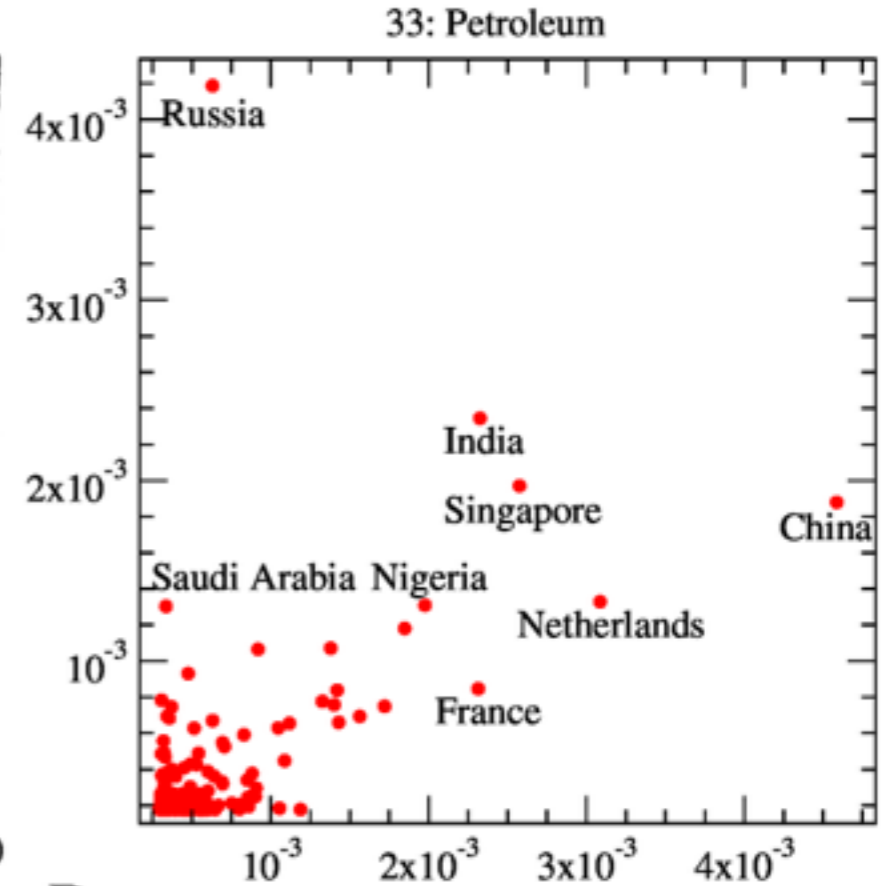
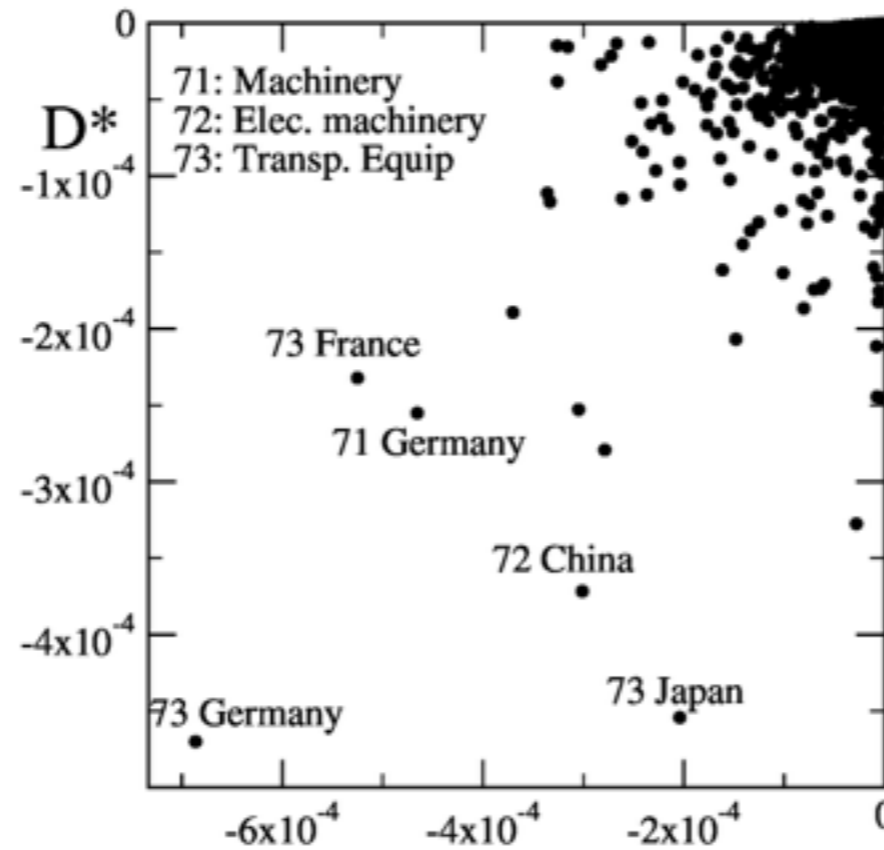
$$D = dP/d\delta = \Delta P/\delta$$

$$D^* = dP^*/d\delta = \Delta P^*/\delta$$

$$\delta' = 0.01, 0.03, 0.05$$

$$D_1 = D/P \text{ and } D_1^* = D^*/P^*$$

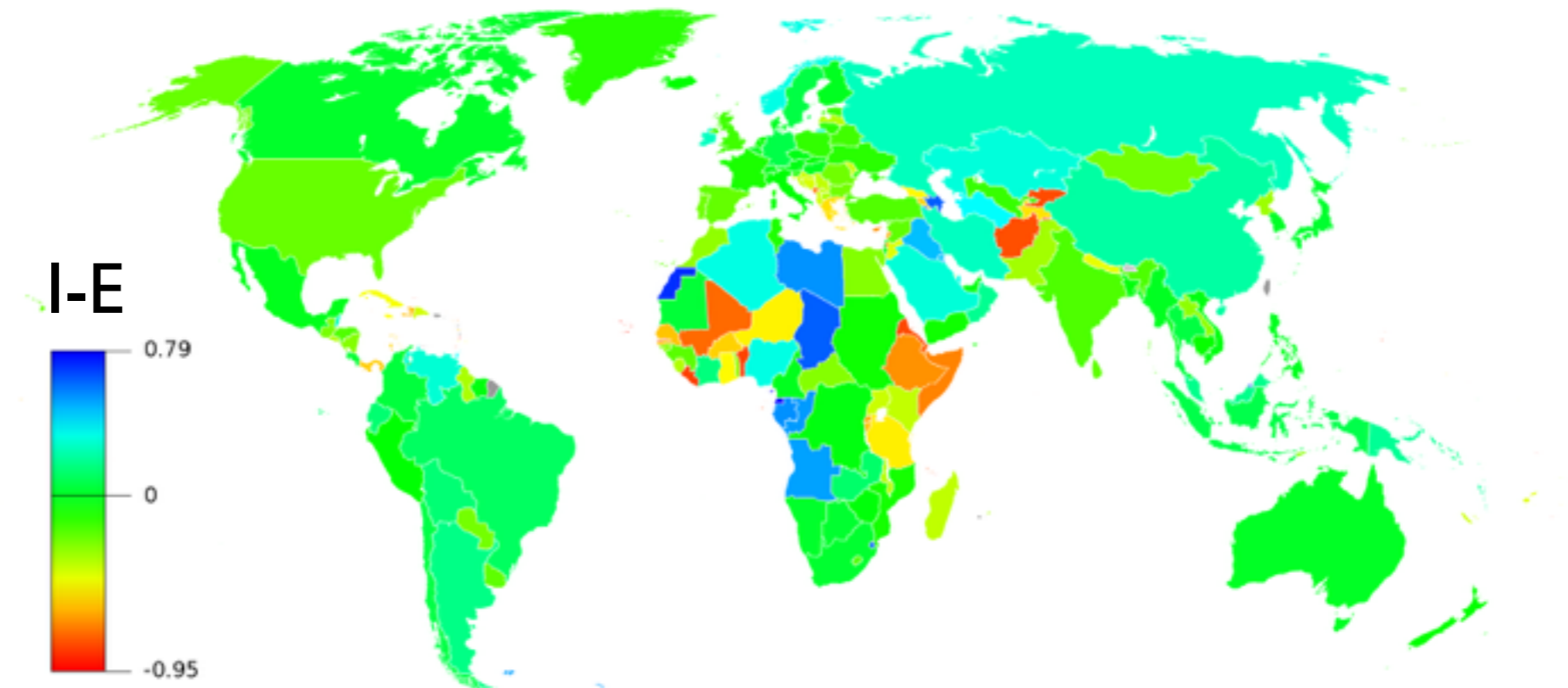
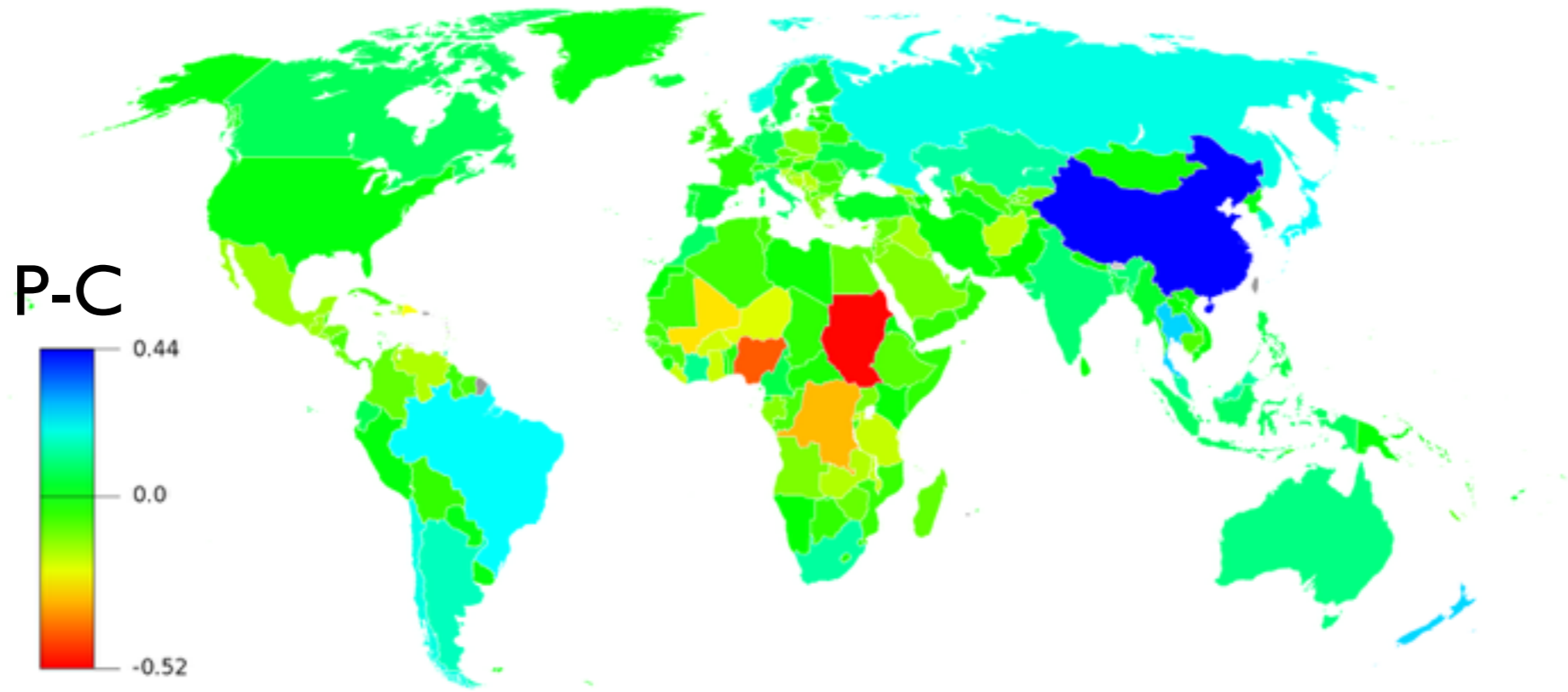
$$K_2 \leq 50,$$





# Country balance

$$B_c = \frac{\sum_p (P_{cp}^* - P_{cp})}{\sum_p (P_{cp}^* + P_{cp})} = \frac{(P_c^* - P_c)}{(P_c^* + P_c)}$$

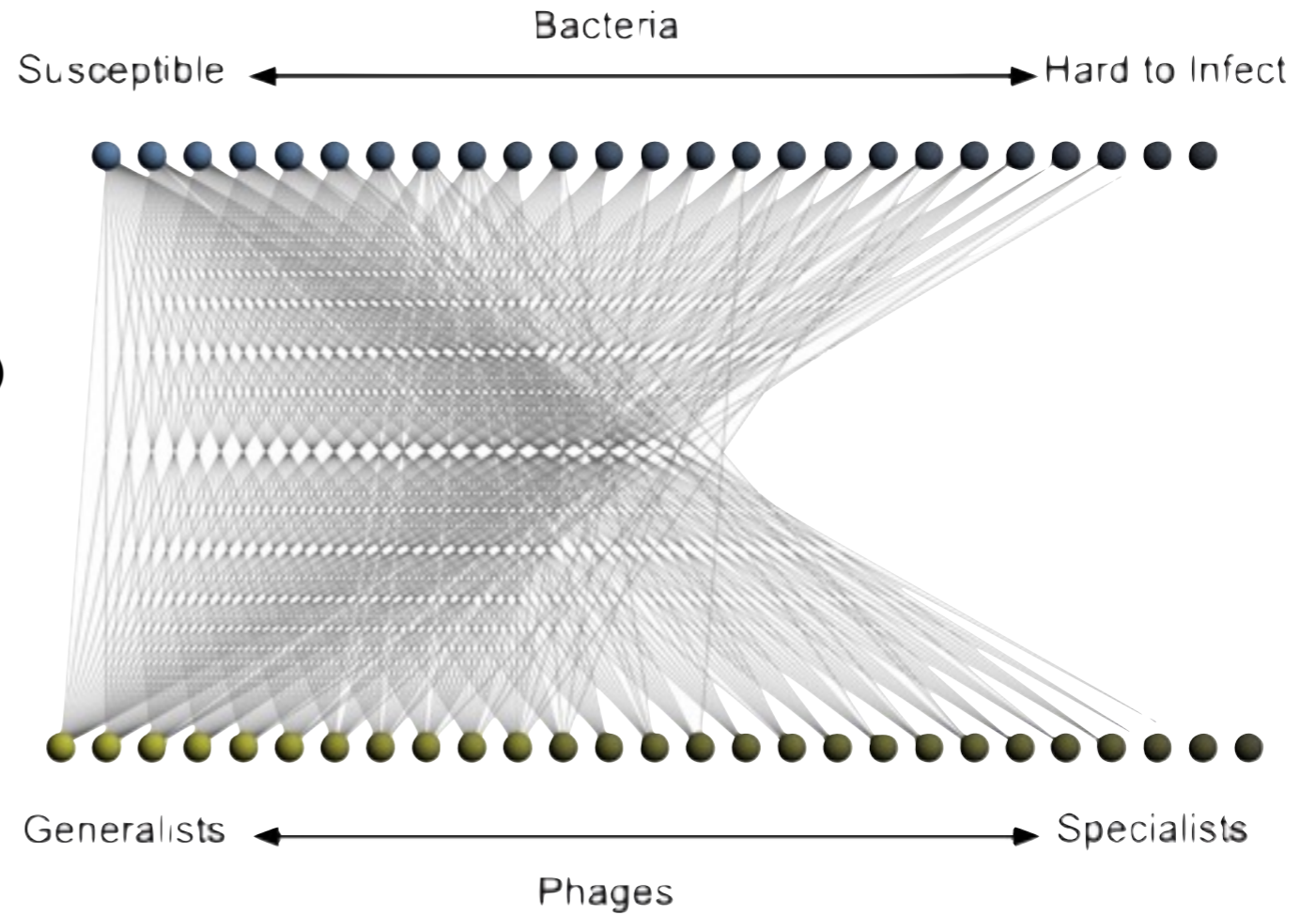


# Nestedness

## biogeography

bipartite networks: species - sites (islands, plants, etc)

- 1937 Hulten
- 1957 Darlington
- 1975 Daubenmire



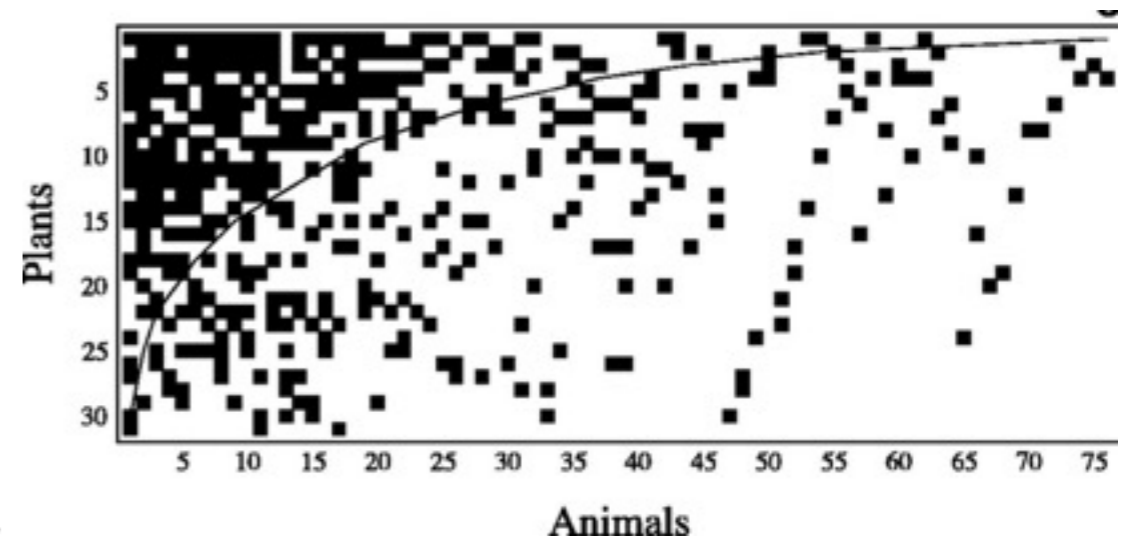
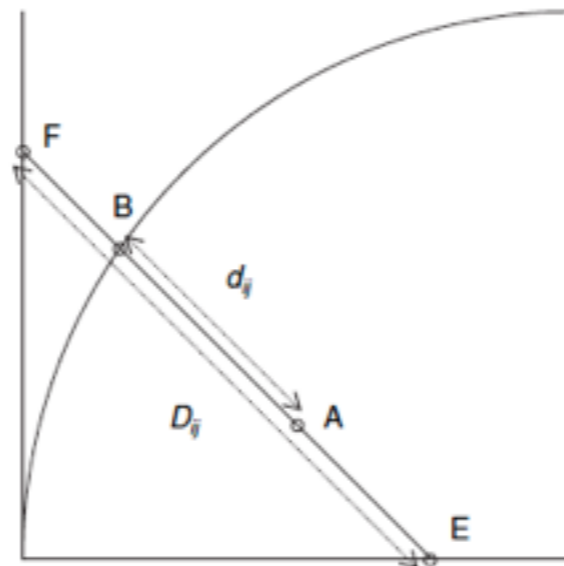
## quantifying nestedness

### BINMATNEST

M.A. Rodriguez-Girones and L. Santamaria,  
Journal of Biogeography 33, 924 (2006)

isocline

$$f(x;p) = \frac{0.5}{n} + \frac{n-1}{n} \cdot \left( 1 - \left( 1 - \frac{m \cdot x - 0.5}{m-1} \right)^p \right)^{1/p}$$



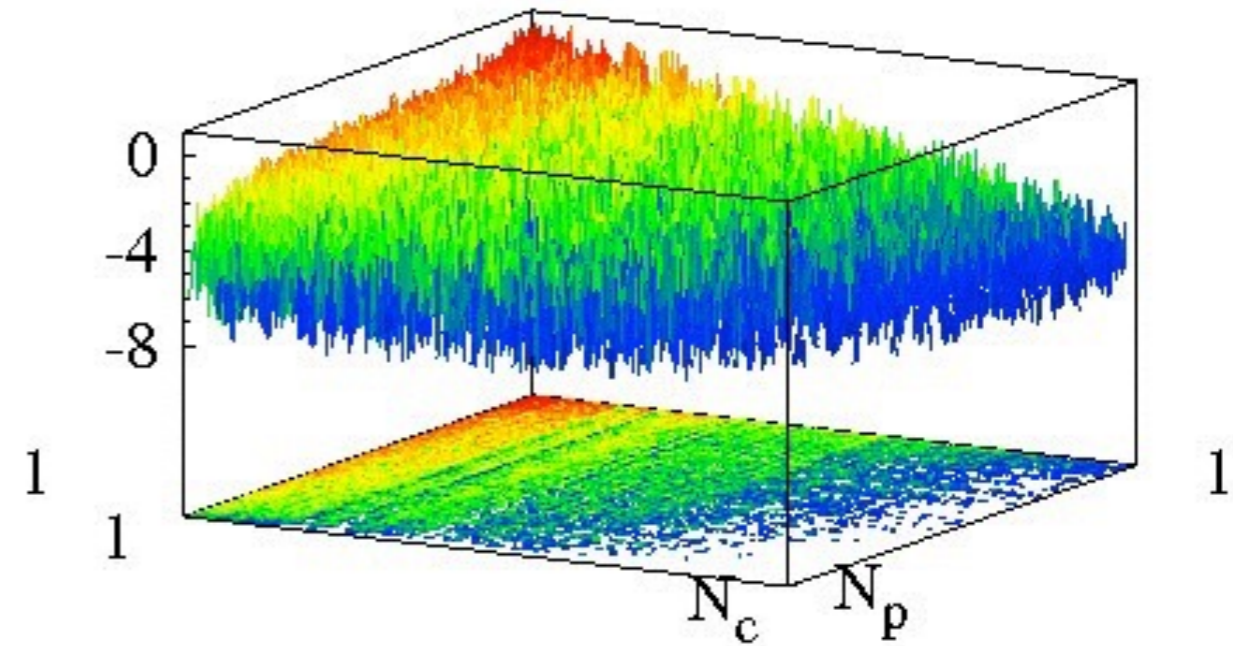
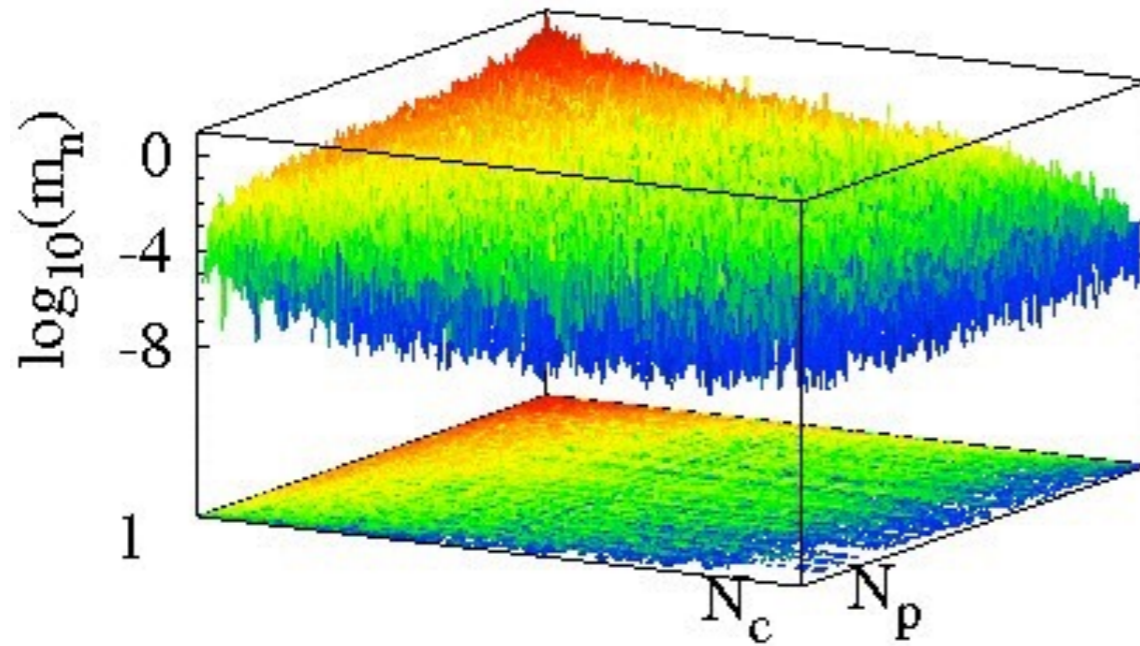


# mutualistic networks (countries-products)

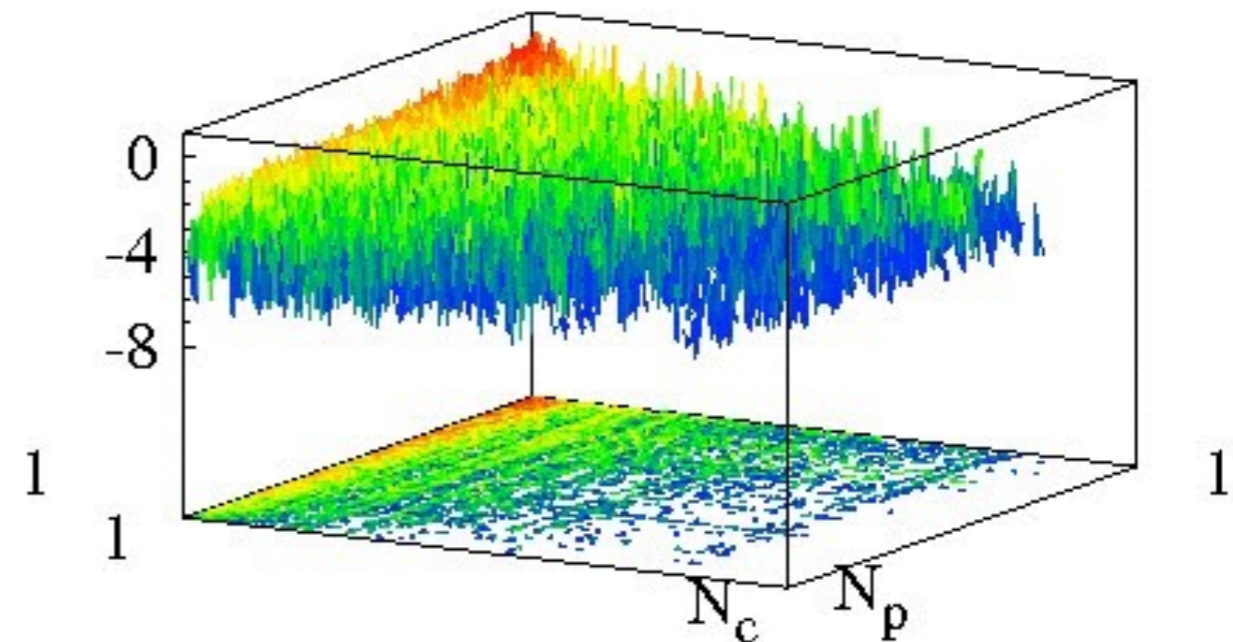
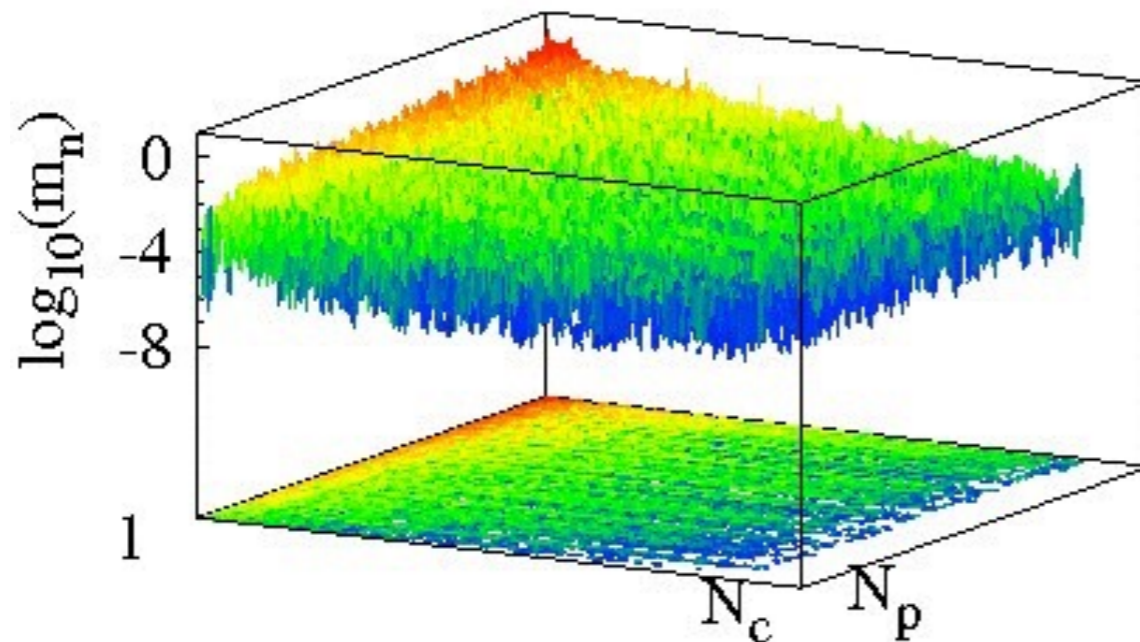
$$m^{(i,e)} = M^{(i,e)} / M_{\max} \quad M_{p,c}^{(i)} = \sum_{c'=1}^{N_c} M_{c,c'}^p$$

$$M_{p,c}^{(e)} = \sum_{c'=1}^{N_c} M_{c',c}^p$$

2008



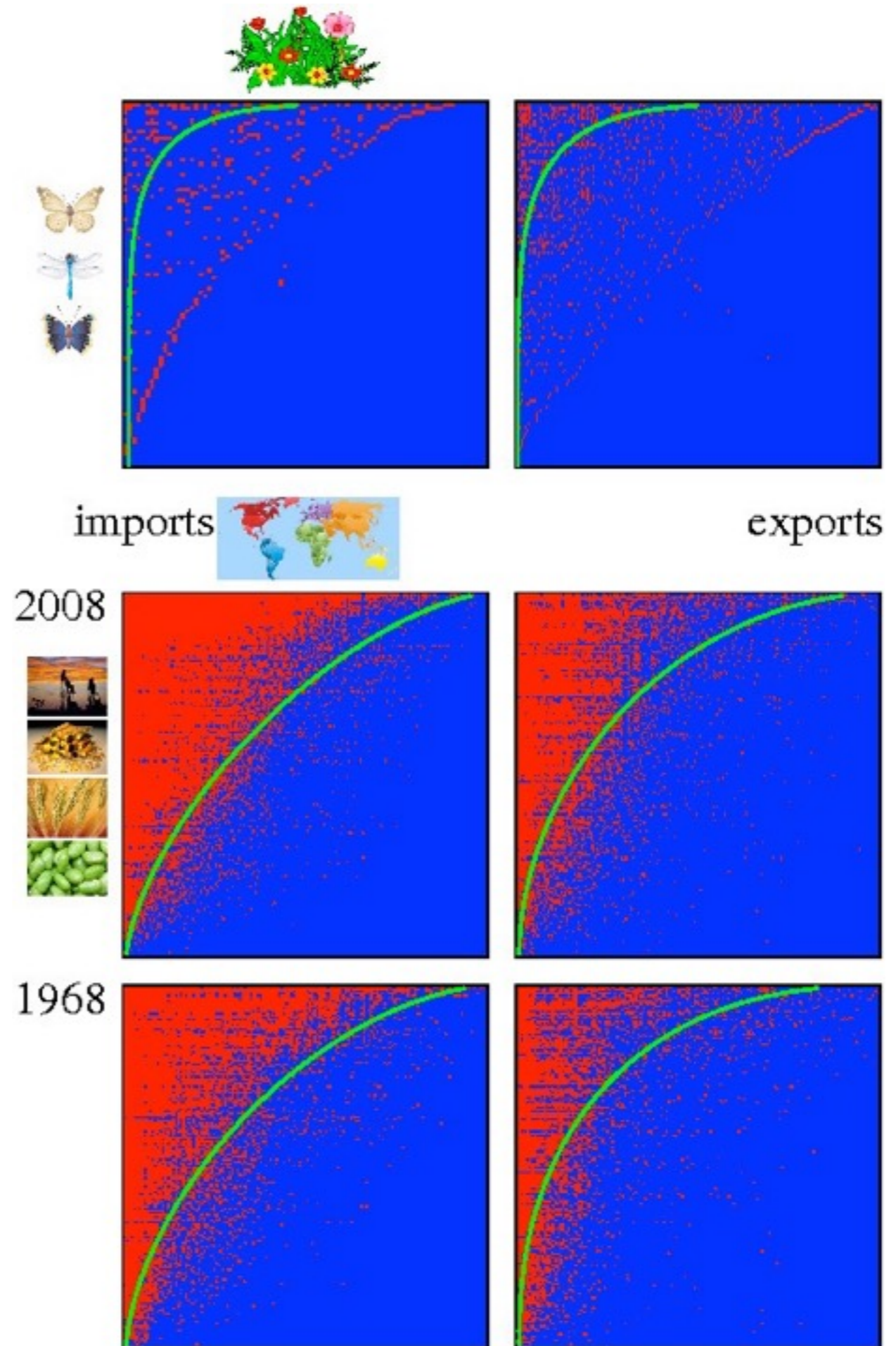
1968





# Nestedness (binary matrices)

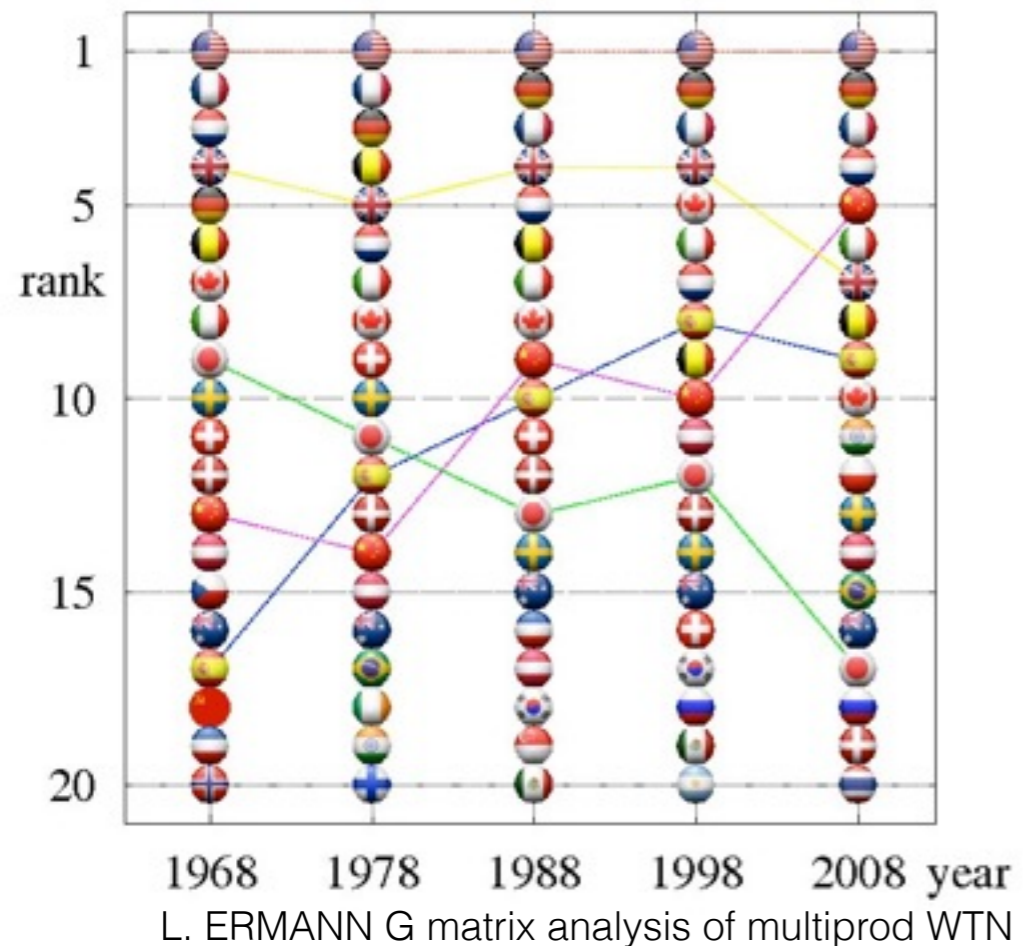
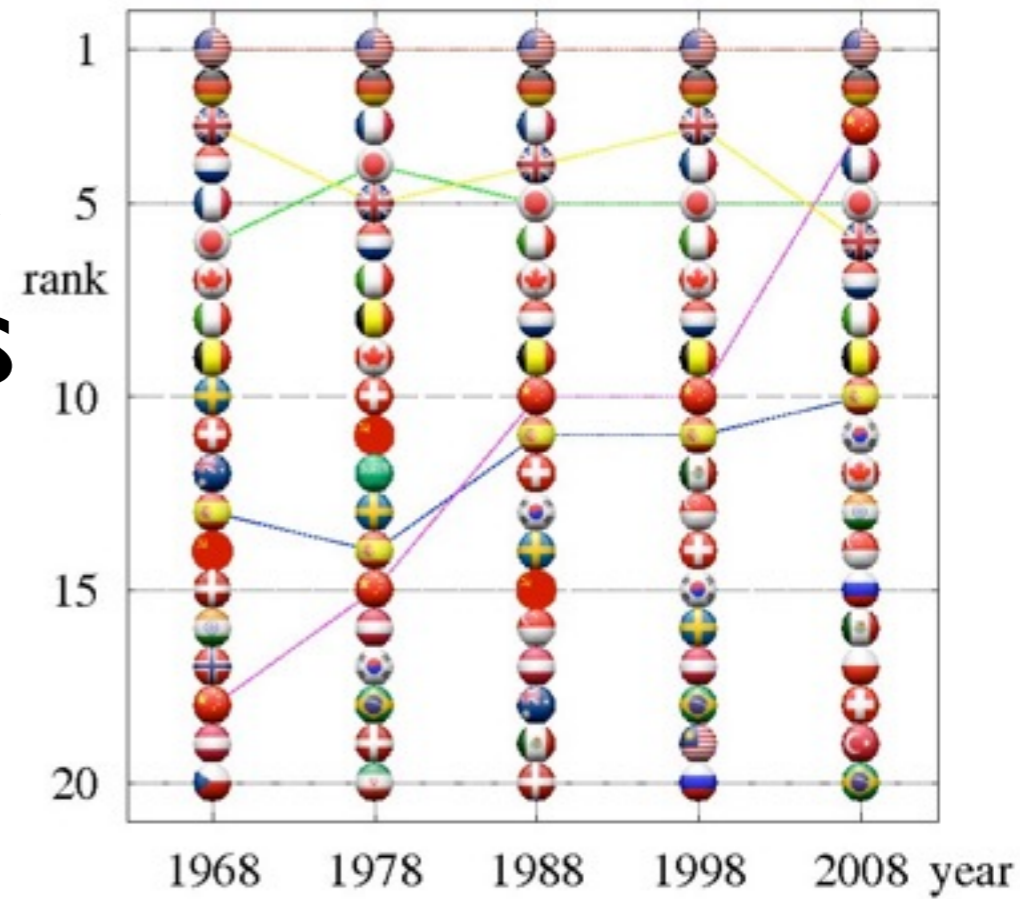
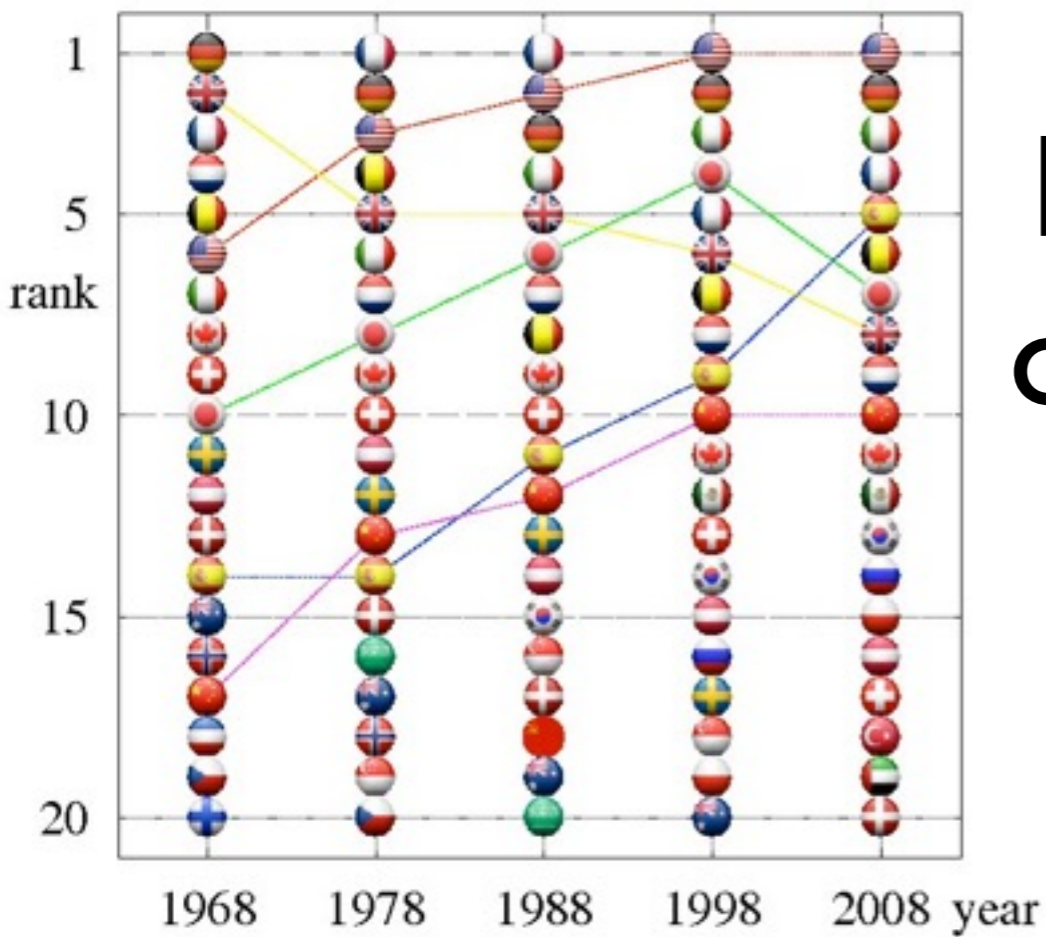
$$Q_{c,p}^{(i,e)} = \begin{cases} 1 & \text{if } m_{c,p}^{(i,e)} \geq \mu \\ 0 & \text{if } m_{c,p}^{(i,e)} < \mu \end{cases}$$



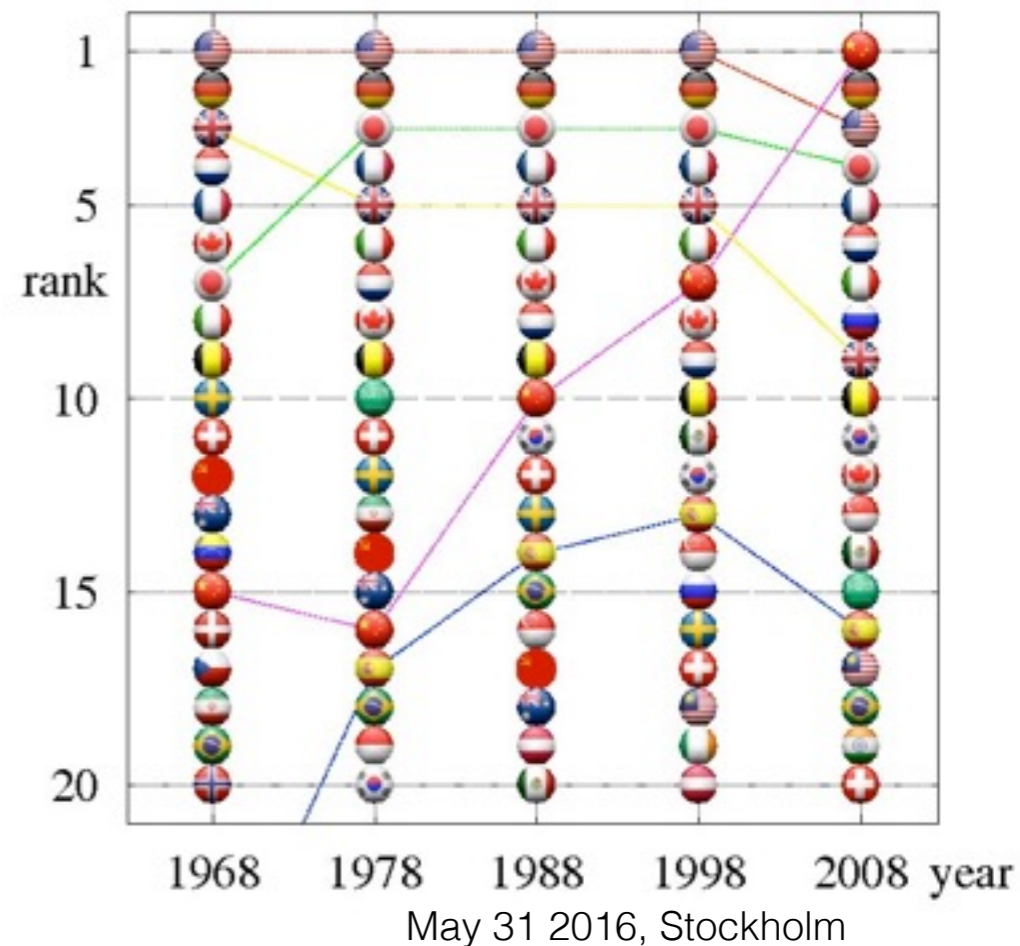


# Ecolo-Rank of countries

Imports Exports



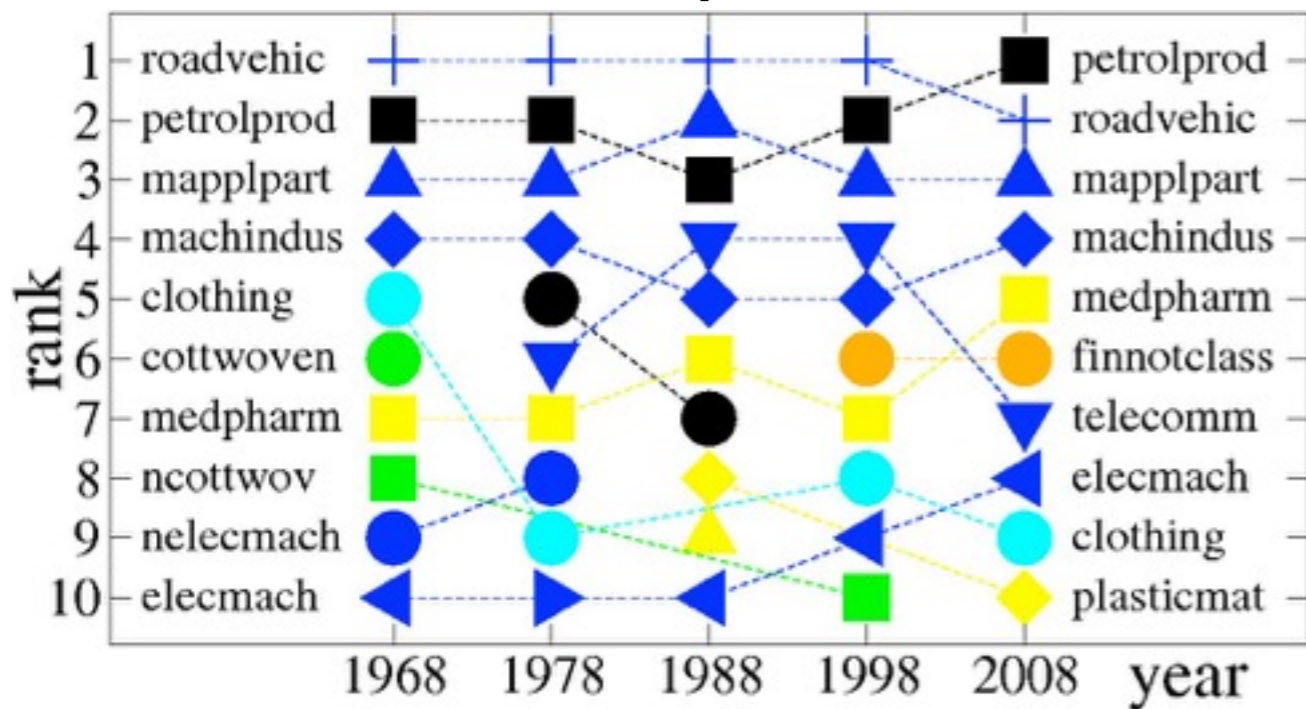
← money ranking →



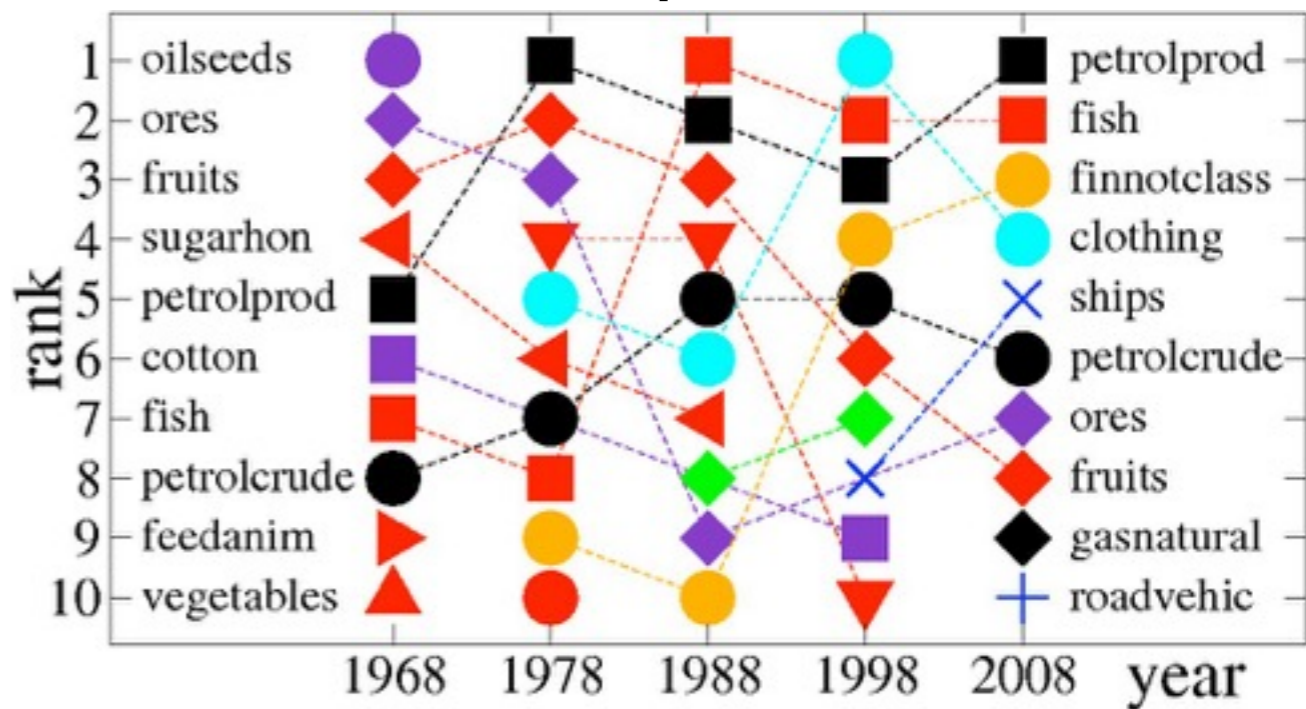


# Ecolo-Rank of products

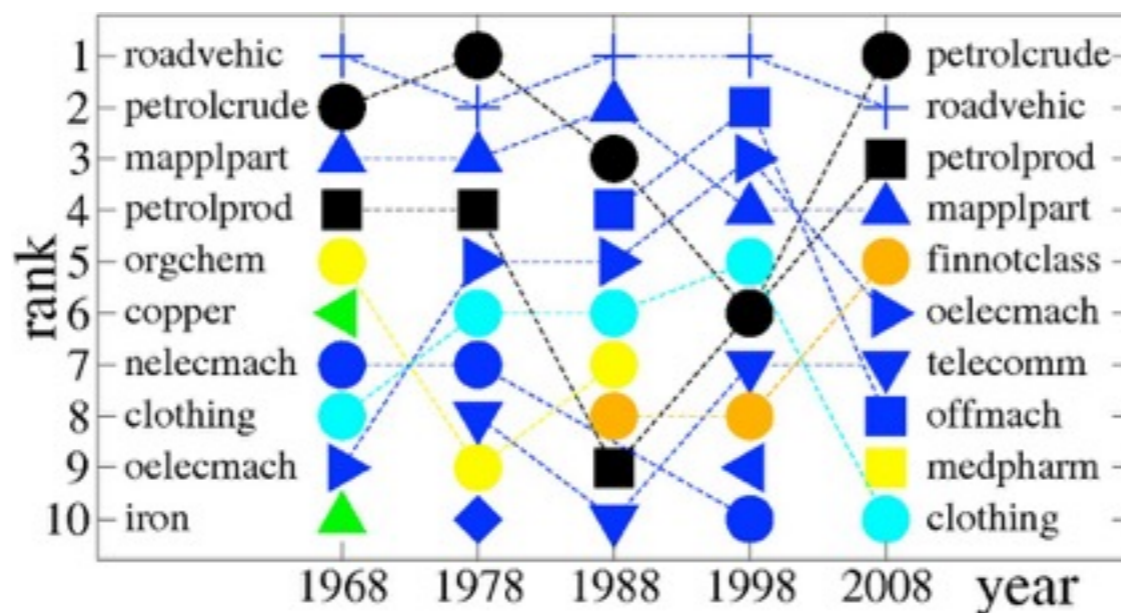
## imports



## exports



## money rank



001	animals	Live animals
031	fish	Fish, fresh & simply preserved
051	fruits	Fruit, fresh, and nuts excl. Oil nuts
054	vegetables	Vegetables, roots & tubers, fresh or dried
061	sugarhon	Sugar and honey
071	coffee	Coffee
081	feedanim	Feed. stuff for animals excl. unshelled cereals
221	oilseeds	Oil seeds, oil nuts and oil kernels
263	cotton	Cotton
283	ores	Ores & concentrates of non ferrous base metals
331	petrolcrude	Petroleum, crude and partly refined
332	petrolprod	Petroleum products
341	gas	Gas, natural and manufactured
512	orgchem	Organic chemicals
541	medpharm	Medicinal & pharmaceutical products
581	plasticmat	Plastic materials, regenerd. cellulose & resins
599	chemmat	Chemical materials and products, nes
652	cottwoven	Cotton fabrics, woven ex. narrow or spec.fabrics
653	ncottwov	Text fabrics woven ex narrow, spec, not cotton
667	pearlaprec	Pearls and precious and semi precious stones
674	iron	Universals, plates and sheets of iron or steel
682	copper	Copper
711	nelecmach	Power generating machinery, other than electric
714	offmach	Office machines
718	machindus	Machines for special industries
719	mapplpart	Machinery and appliances non electrical parts
722	elecmach	Electric power machinery and switchgear
724	telecomm	Telecommunications apparatus
729	oelecmach	Other electrical machinery and apparatus
732	roadvehicles	Road motor vehicles
735	ships	Ships and boats
841	clothing	Clothing except for clothing
931	finnotclass	Special transactions not classd. accord.to kind

# Conclusions

- Google matrix of the WTN (democratic in countries, global network properties):
  - 1) one product of all comm. ( $N_c$ )
  - 2) multiprod ( $N_c \times N_p$ )2d-ranking, spectrum, communities in eigenstates, correlation between P-C, comparison with I-E, new tool for trade analysis
- Dynamics, crisis, sensitivity to price variation
- Bitcoin network
- reduced Google matrix on WTN



# product names

code	name	code	name
00	Live animals	54	Medicinal and pharmaceutical products
01	Meat and meat preparations	55	Perfume materials, toilet & cleansing preparations
02	Dairy products and eggs	56	Fertilizers, manufactured
03	Fish and fish preparations	57	Explosives and pyrotechnic products
04	Cereals and cereal preparations	58	Plastic materials, etc.
05	Fruit and vegetables	59	Chemical materials and products, nes
06	Sugar, sugar preparations and honey	61	Leather, lthr. Manufs., nes & dressed fur skins
07	Coffee, tea, cocoa, spices & manufac. Thereof	62	Rubber manufactures, nes
08	Feed. Stuff for animals excl. Unmilled cereals	63	Wood and cork manufactures excluding furniture
09	Miscellaneous food preparations	64	Paper, paperboard and manufactures thereof
11	Beverages	65	Textile yarn, fabrics, made up articles, etc.
12	Tobacco and tobacco manufactures	66	Non metallic mineral manufactures, nes
21	Hides, skins and fur skins, undressed	67	Iron and steel
22	Oil seeds, oil nuts and oil kernels	68	Non ferrous metals
23	Crude rubber including synthetic and reclaimed	69	Manufactures of metal, nes
24	Wood, lumber and cork	71	Machinery, other than electric
25	Pulp and paper	72	Electrical machinery, apparatus and appliances
26	Textile fibres, not manufactured, and waste	73	Transport equipment
27	Crude fertilizers and crude minerals, nes	81	Sanitary, plumbing, heating and lighting fixt.
28	Metalliferous ores and metal scrap	82	Furniture
29	Crude animal and vegetable materials, nes	83	Travel goods, handbags and similar articles
32	Coal, coke and briquettes	84	Clothing
33	Petroleum and petroleum products	85	Footwear
34	Gas, natural and manufactured	86	Scientif & control instrum, fotogr gds, clocks
35	Electric energy	89	Miscellaneous manufactured articles, nes
41	Animal oils and fats	91	Postal packages not class. According to kind
42	Fixed vegetable oils and fats	93	Special transact. Not class. According to kind
43	Animal and vegetable oils and fats, processed	94	Animals, nes, incl. Zoo animals, dogs and cats
51	Chemical elements and compounds	95	Firearms of war and ammunition therefor
52	Crude chemicals from coal, petroleum and gas	96	Coin, other than gold coin, not legal tender
53	Dyeing, tanning and colouring materials		