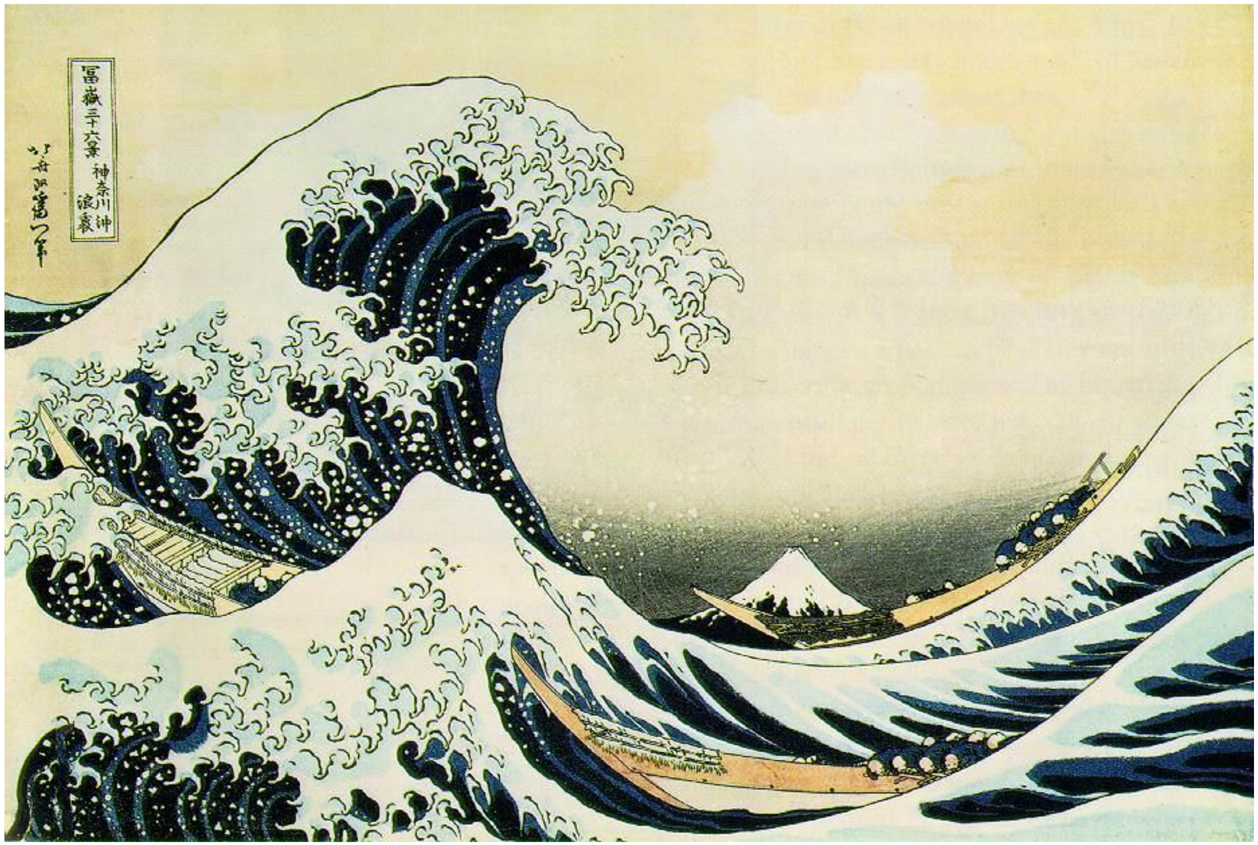
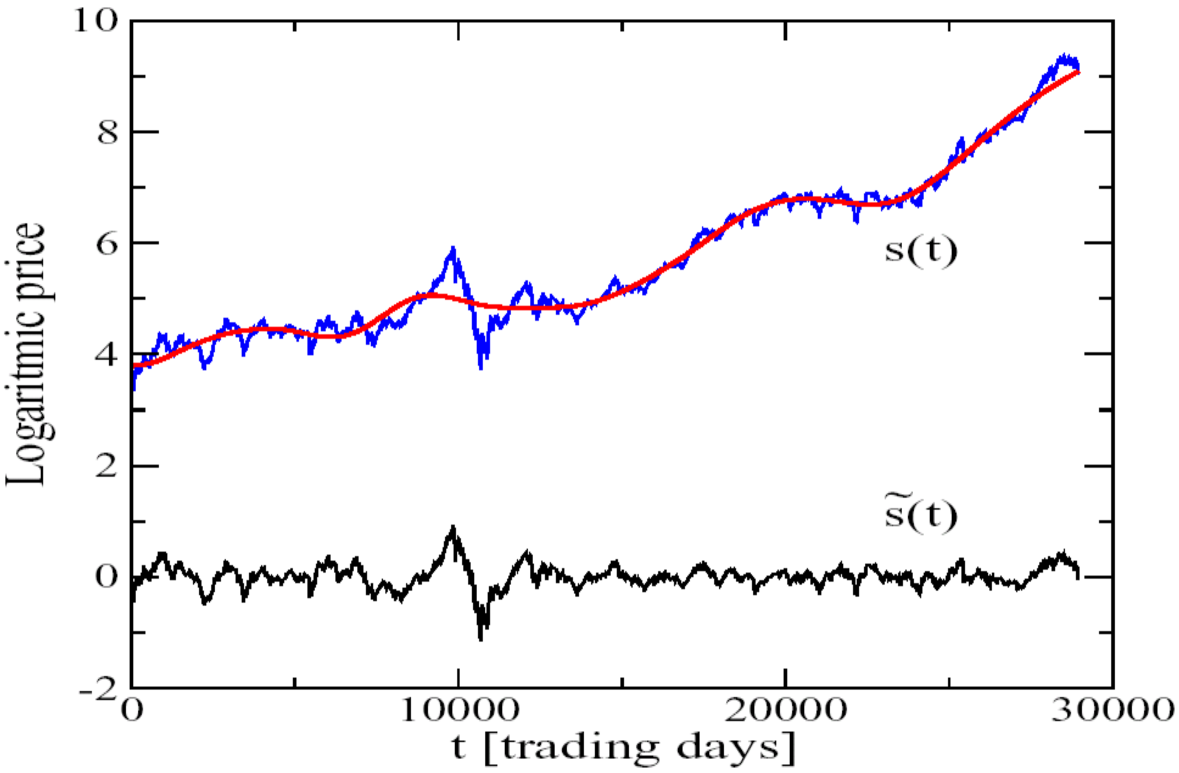
Asymmetry and Synchronization in Stock Markets



Mogens H. Jensen, Niels Bohr Institute

Dynamics of the Markets, Nordita 30 May-3 June 2016

Dow Jones Industrial Average: Some extreme events during 120 years!



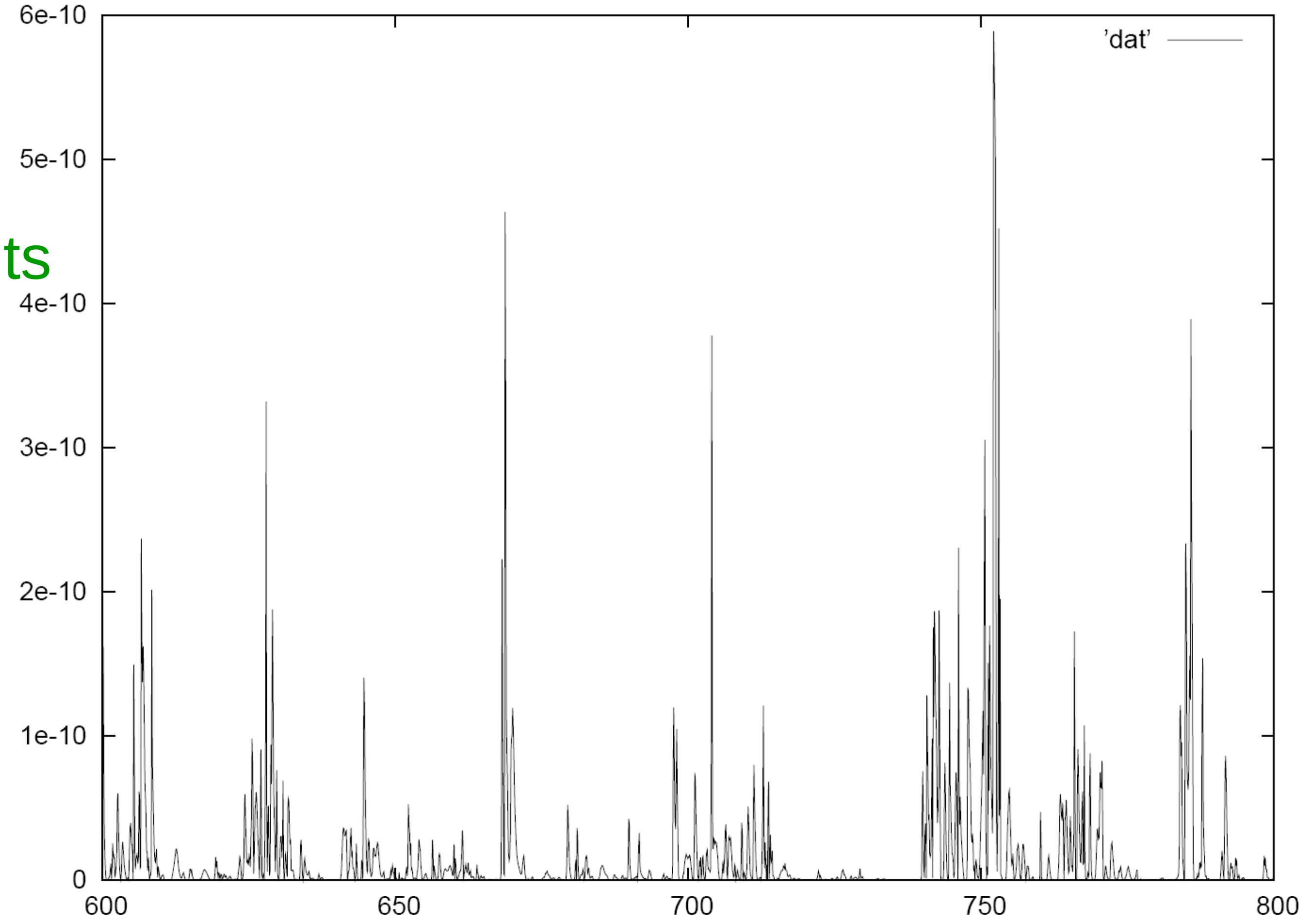
(c) DJIA 1896–2001 (incl. detrended data)

With inflation

Detrended
(over 1000 days)

Fluctuations in finance and turbulence are similar: Non-Gaussian distributions !

Energy dissipation: Extreme events



Time

1. Turbulence: Strong velocities vs. quiet, laminar periods: Intermittency – Extreme events !

2. Inverse statistics: pre-described return level: When reached for the first time: Laminar periods important.

3. Turbulence statistics = Market statistics !!

4. Dow Jones Index: Distribution of investment times: well defined maximum, optimal investment horizon.

5. Gain/loss levels of same magnitude: maximum for gain is twice the size of maximum of loss: Asymmetry! Absent for single stocks.

6. Model: Fear-Factor-Model (FFM), correlations between stocks

7. Growth of scientific paradigms: Another example of extreme events

8. Extreme events: Positive feed-back -> amplification -> Financial crises, climate changes, cell dynamics, ….

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Publikationer:

MHJ, ”Multiscaling and Structure Functions in Turbulence: An Alternative Approach, Phys.Rev.Lett. 83, 76 (1999).

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R. Donangelo, MHJ, I. Simonsen and K. Sneppen, “Synchronization and Asymmetry in Stock Markets: The Consequences of Fear”, J. Stat. Mech. 11, L11001 (2006).

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K. Sneppen, A. Trusina, MHJ, and S. Bornholdt, “A Minimal Model for Multiple Epidemics and Immunity Spreading, PLoSone 5, e13326 (2010).

The atmosphere behaves chaotic and turbulent

Intermittency!

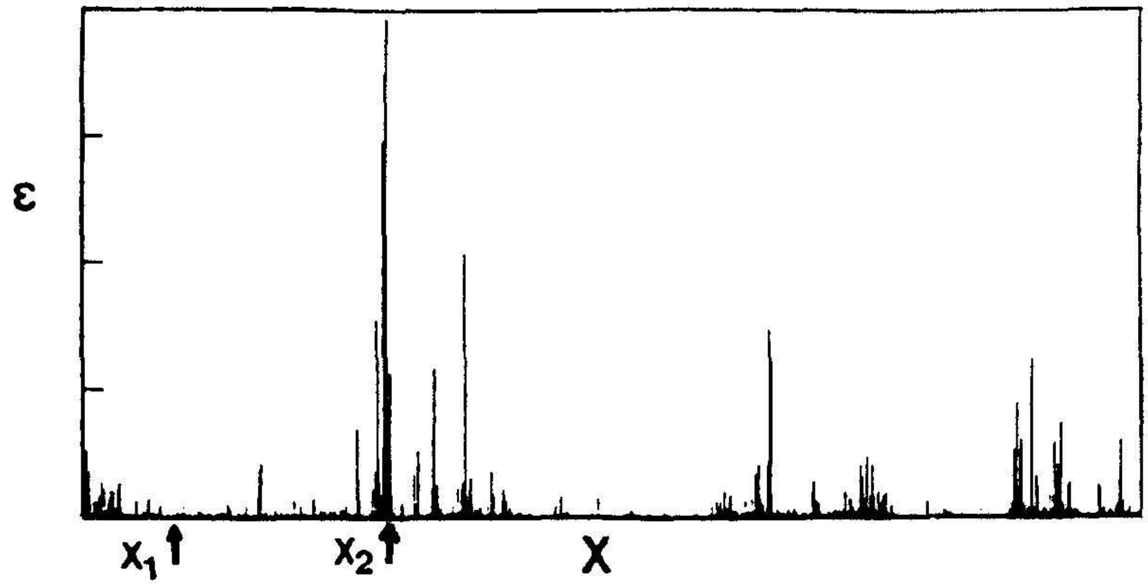
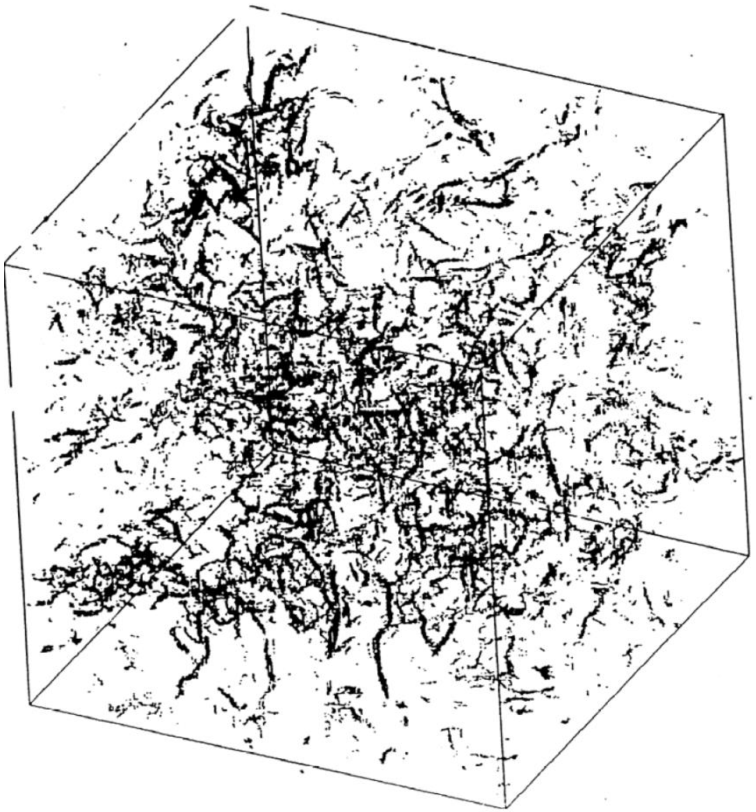


Figure 1a. Typical time trace of $(\partial u_1/\partial t)^2$, representative of the rate of dissipation of turbulent kinetic energy.

How to monitor (and use!) the laminar periods !

Direct Numerical Simulations (DNS):



$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}\Big)}^{\text{Inertia}} = \underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}} + \underbrace{\mathbf{f}}_{\text{Other forces}}$$

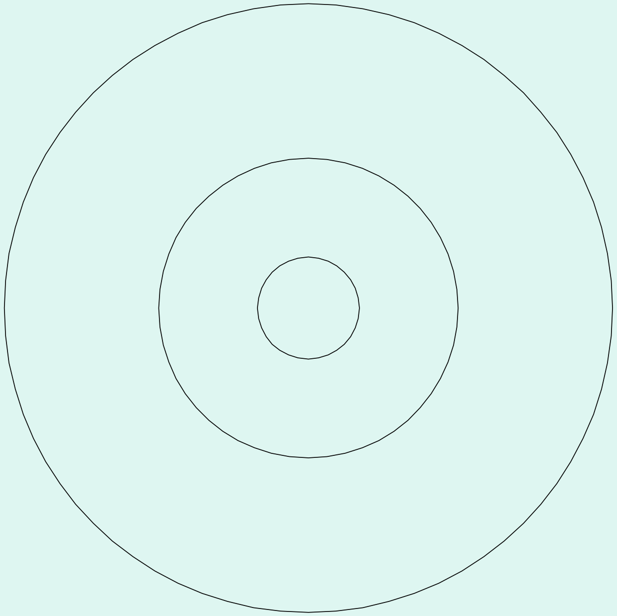
In simulations $Re \sim 1000$

Not very large !

Vincent-Meneguzzi

Shell Models: Discretized approximations to Navier Stokes

Shells in k-space:



Wave vectors (1d): $k_n = r^n \qquad (r = 2)$

Exponentially separated !

Gledzer-Ohkitani-Yamada (GOY) model:

We have N shells, complex Fourier amplitude u_n at shell n:

A dynamical equation for each Fourier amplitude

$\longrightarrow$ **N coupled non-linear ordinary differential equations:**

$$\left(\frac{d}{dt} + \nu k_n^2\right) u_n =$$

$$i\,(a_n\, k_n\, u_{n+1}^* u_{n+2}^* + b_n\, k_{n-1}\, u_{n-1}^* u_{n+1}^* + c_n\, k_{n-2}\, u_{n-1}^* u_{n-2}^*) + f\delta_{n,4},$$

with $n = 1, \cdots N,\ k_n = r^n\, k_0\ (r = 2),$

Deterministic model !

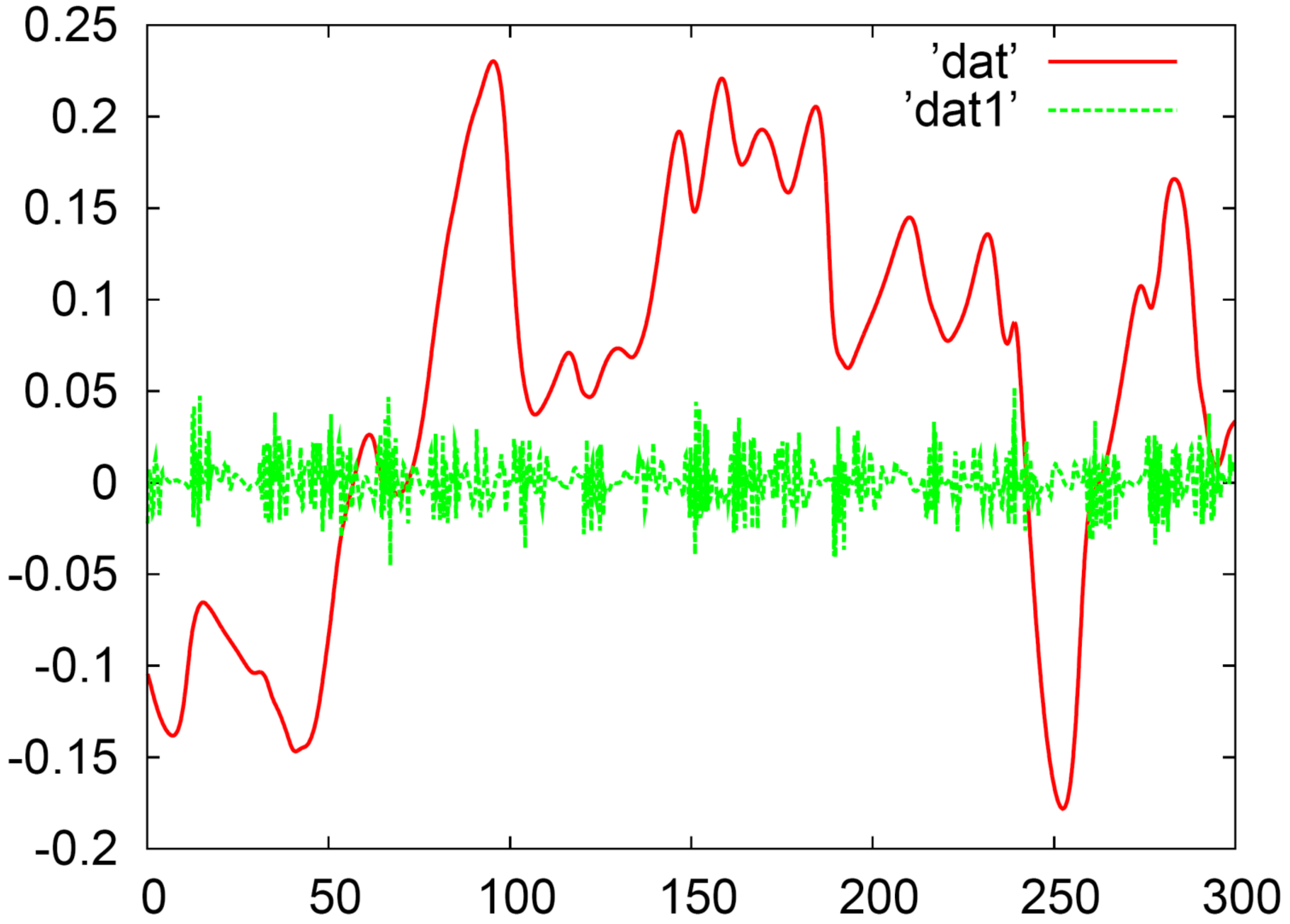
Boundary conditions

$$b_1 = b_N = c_1 = c_2 = a_{N-1} = a_N = 0.$$

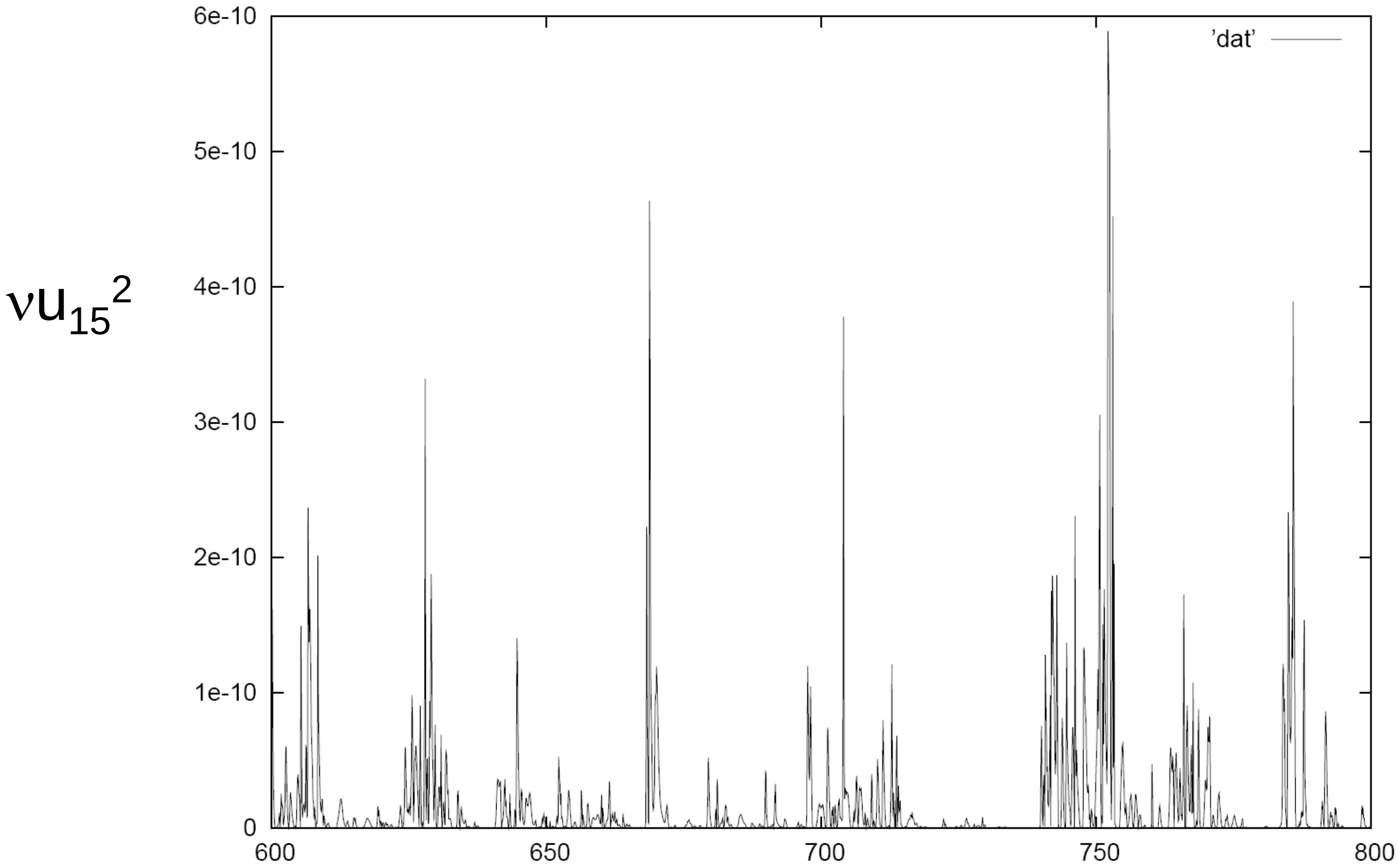
f is an external forcing.

Re u_5

Re u_{14}



Completely deterministic model !



Strong spikes (extreme events) in between laminar periods !

Probability distribution function for small gradients NOT normal (or Gaussian)!

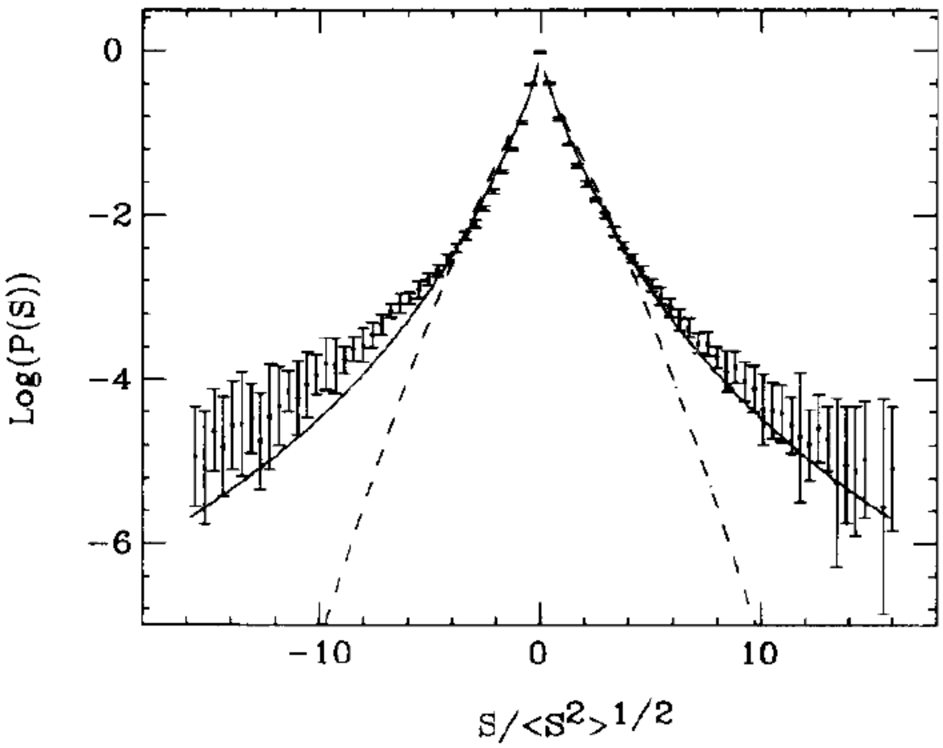
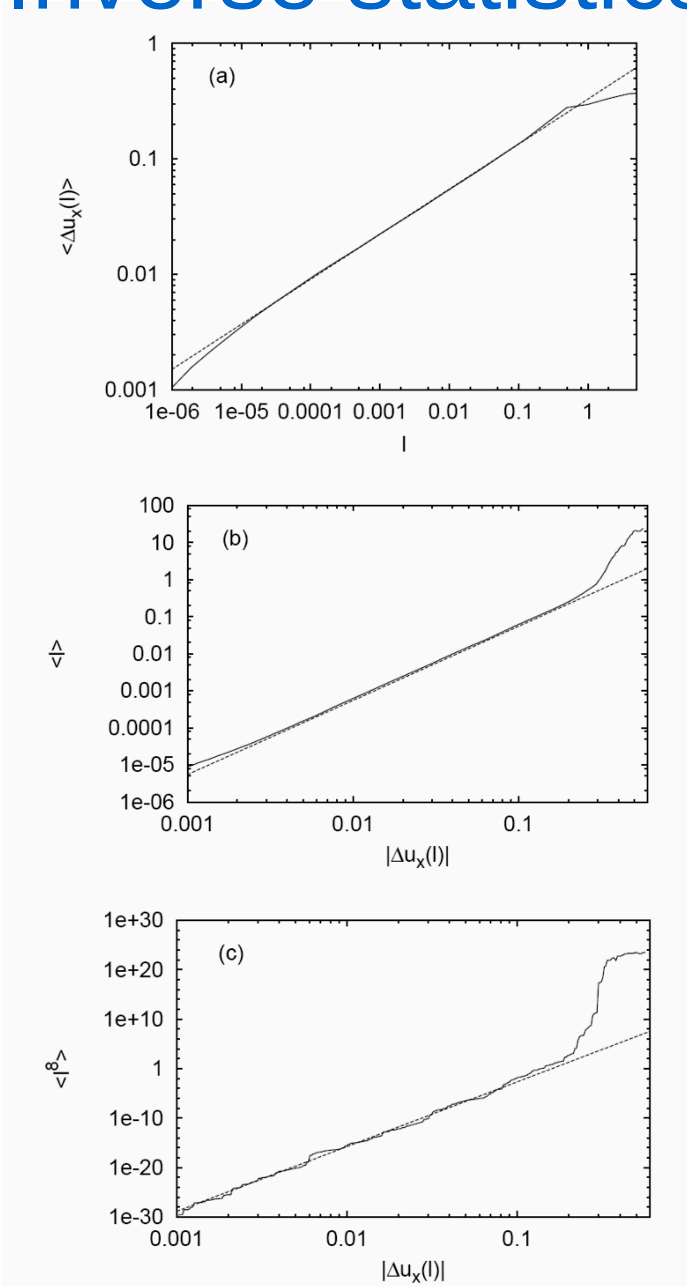


Fig. 2. Log–linear plot of the PDF of the gradients $P(s)$ versus s/σ, where $\sigma^2 = \langle s^2 \rangle$. $\langle v_0^2 \rangle = 10^{-2}$, $\nu = 10^{-6}$ and $N = 19$. The full line is the multifractal prediction given by eq. (15). $D(h)$ is given by fig. 1 through eqs. (4) and (5). Dashed line is the K41 prediction, dots are the numerical data.

Inverse statistics: The laminar periods



Normal Kolmogorov statistics:

Predescribe **lengthscale**
Calculate **velocity difference**

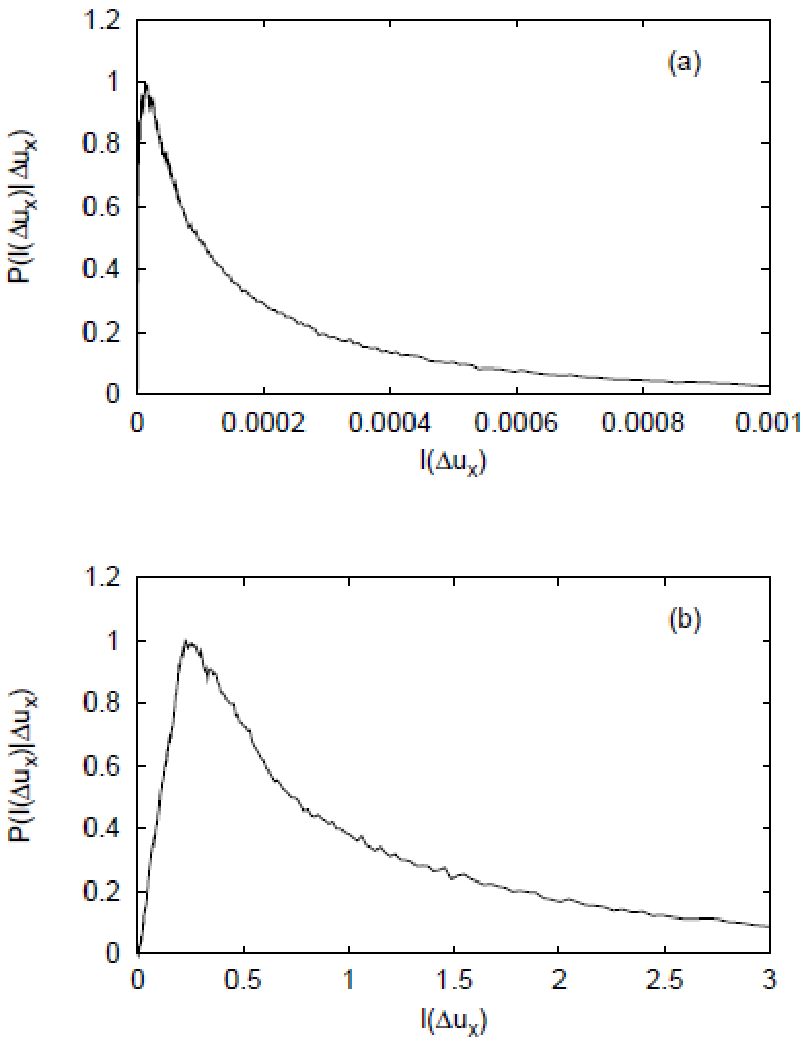
Inverse statistics:

Predescribe **velocity difference**
Calculate **lengthscale**

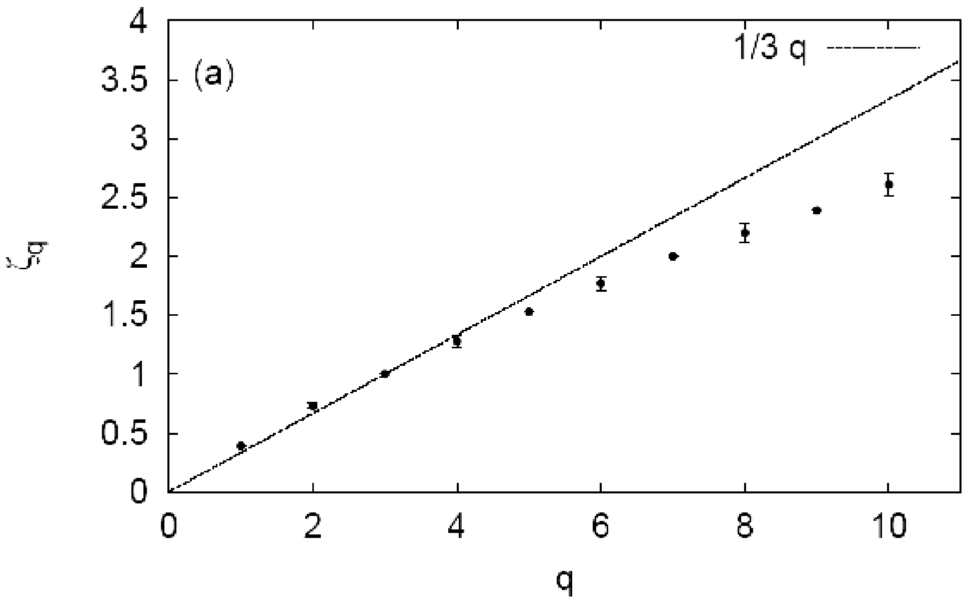
→ : Like first passage time

(M.H. Jensen, Phys.Rev.Lett. 1999)

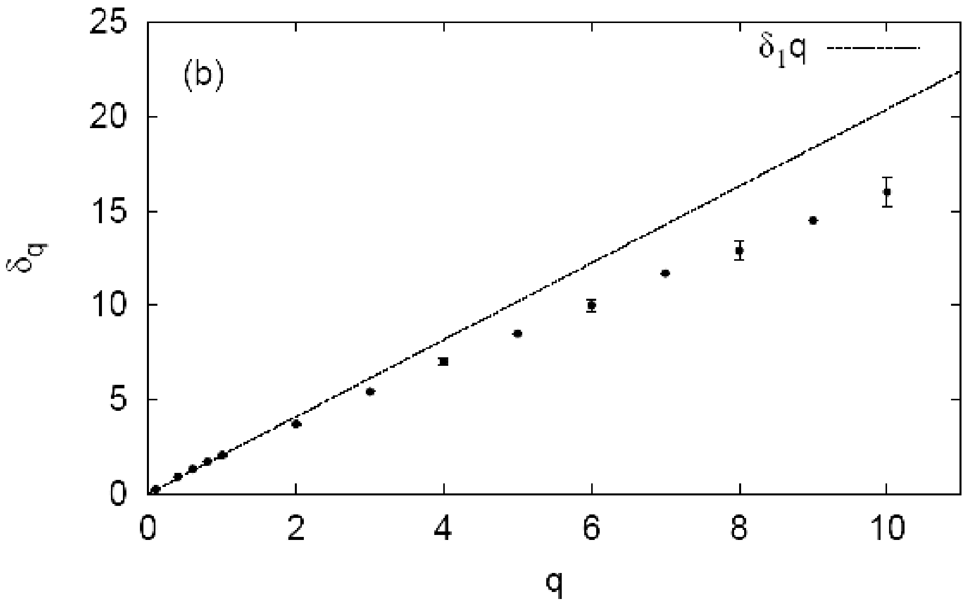
Invers statistics distributions



Maximum and a long tail !

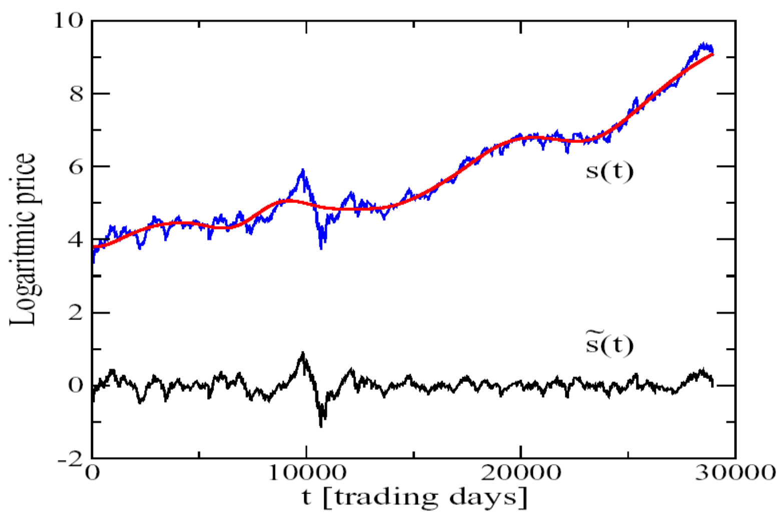


Normal multiscaling statistics



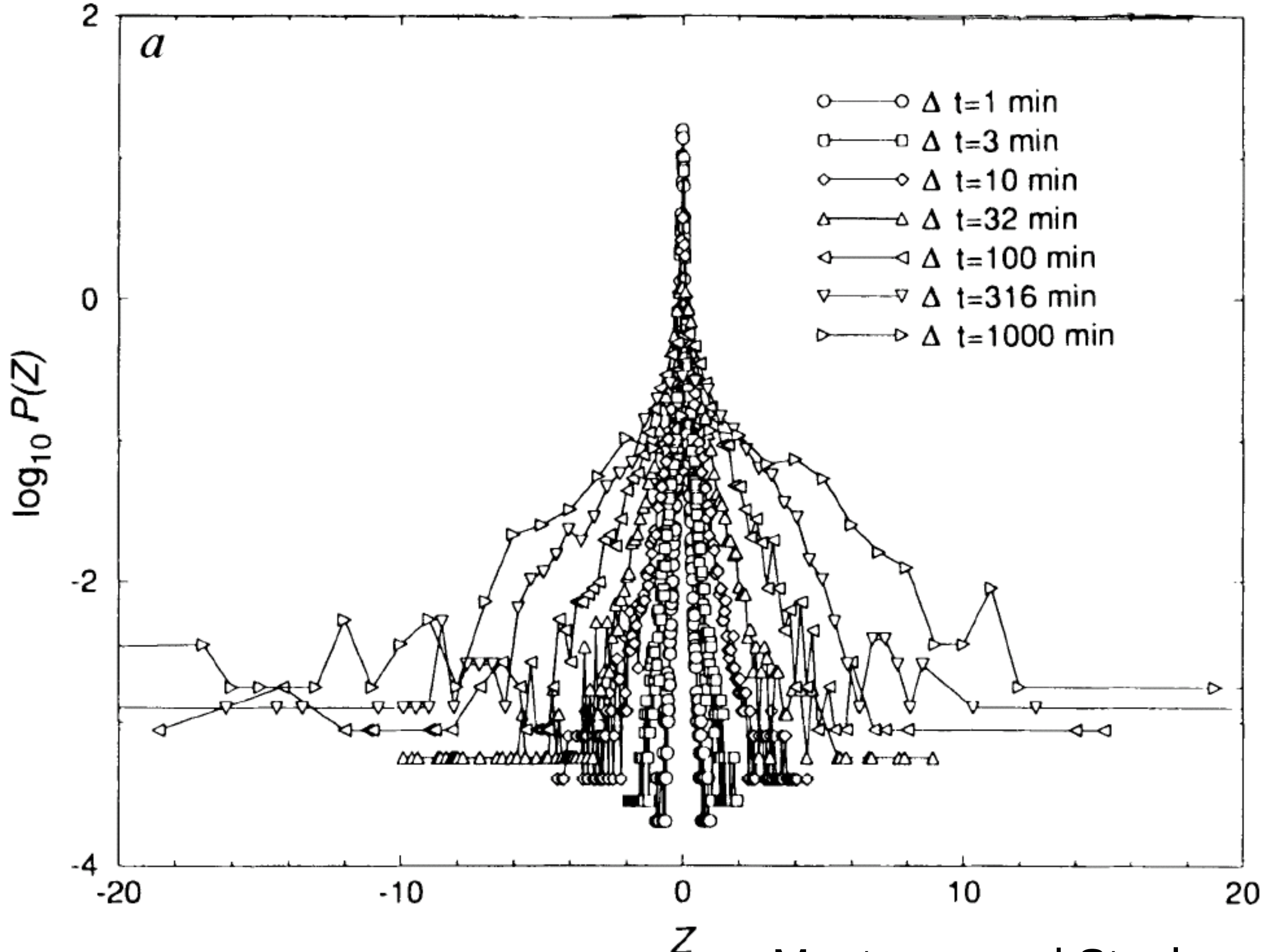
Inverse multiscaling statistics

Fluctuations in Turbulence is like in Finance !



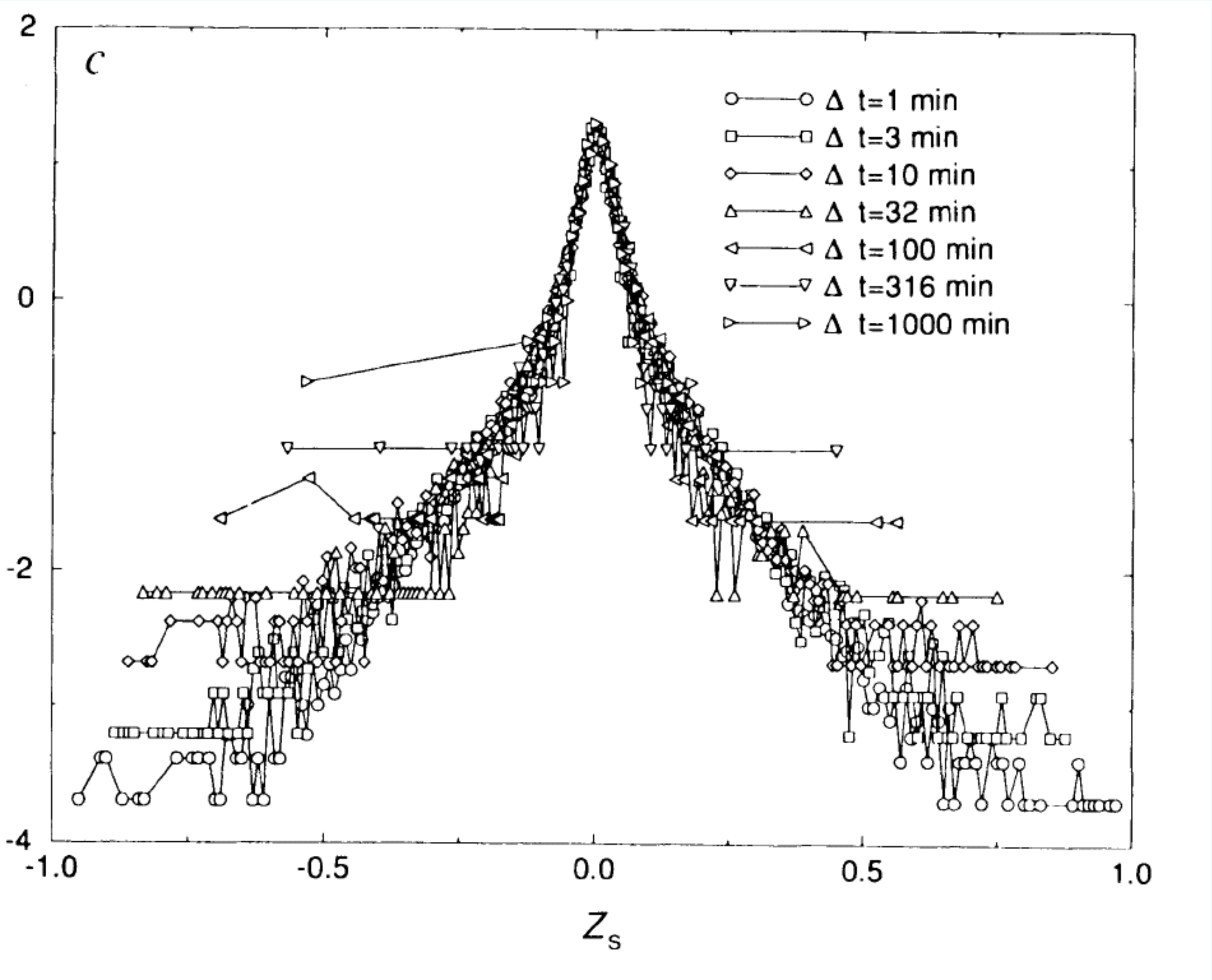
(c) DJIA 1896–2001 (incl. detrended data)

"Forward" statistics

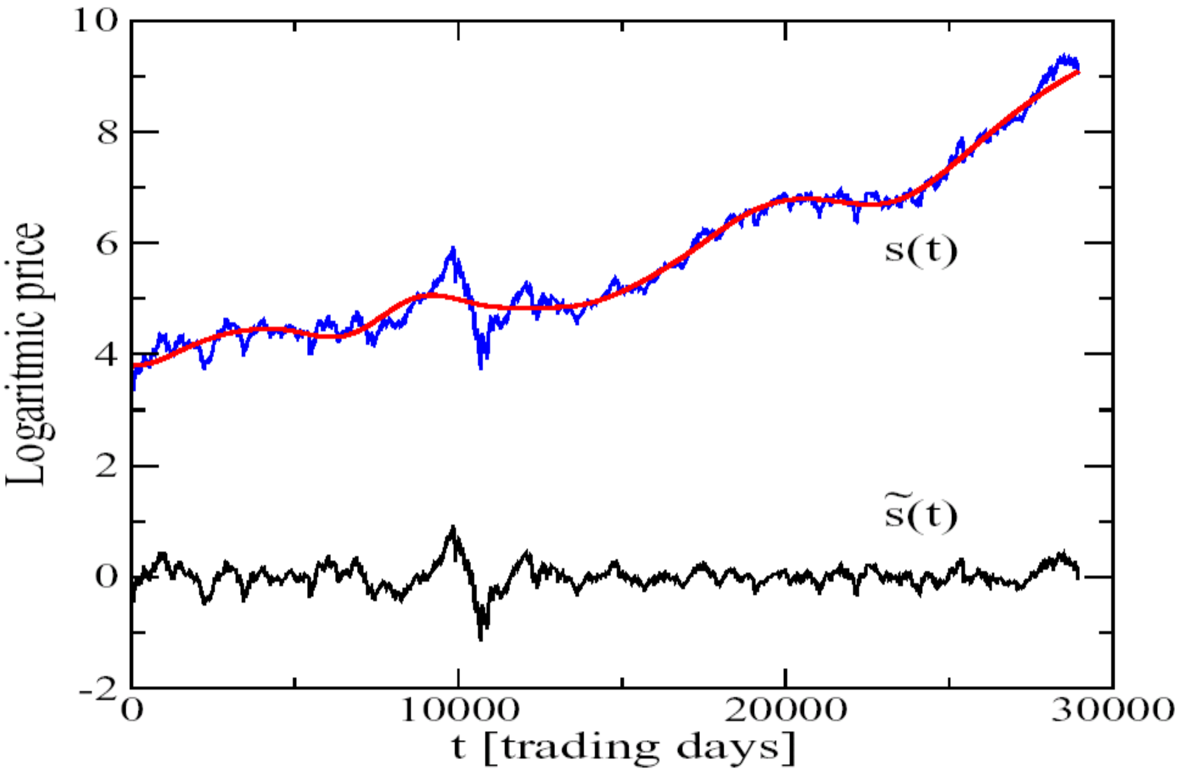


Mantegna and Stanley

Not normal statistics ! Stretched exponential: Extreme events!



Dow Jones Industrial Average



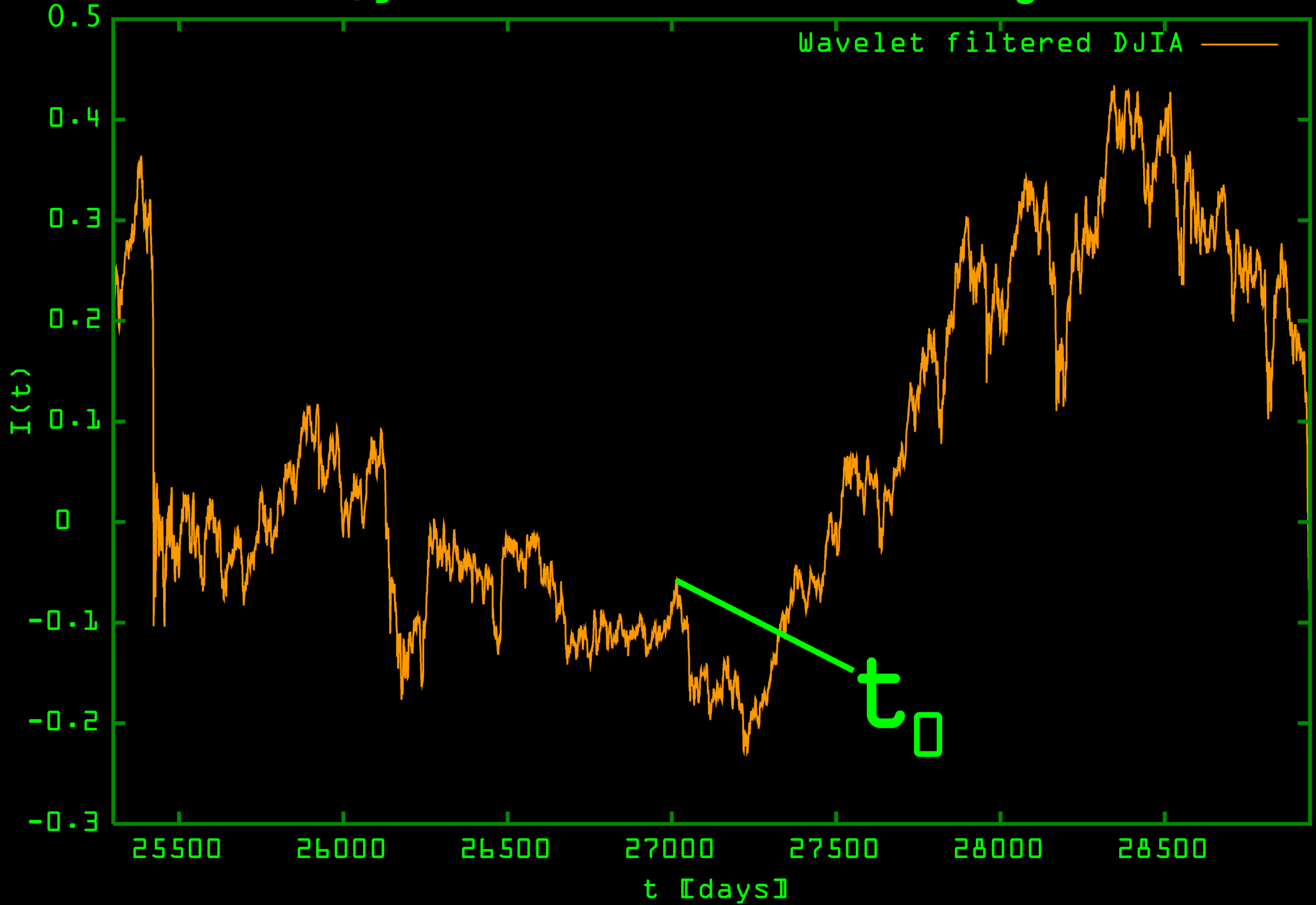
(c) DJIA 1896–2001 (incl. detrended data)

With inflation

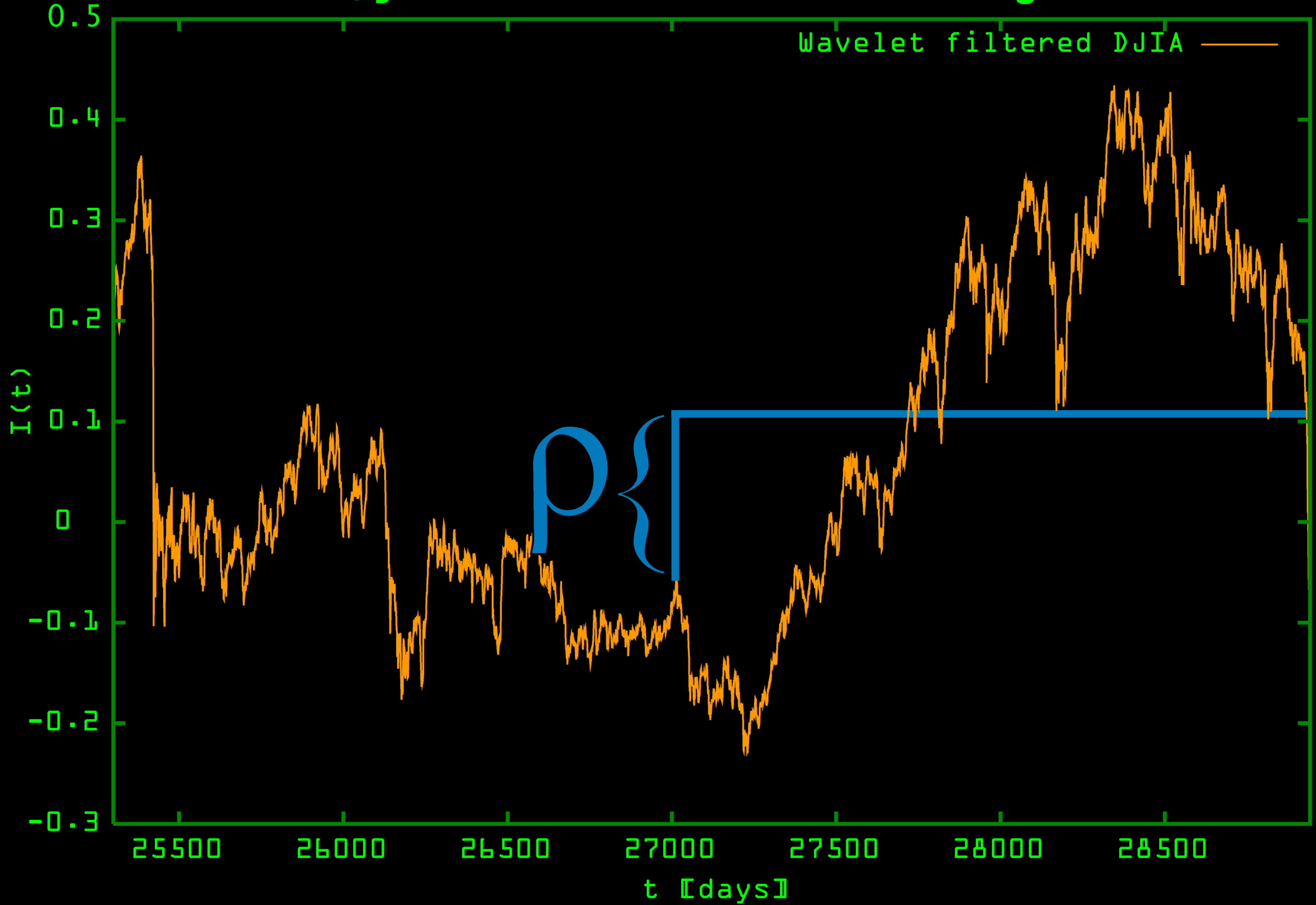
Detrended
(over 1000 days)

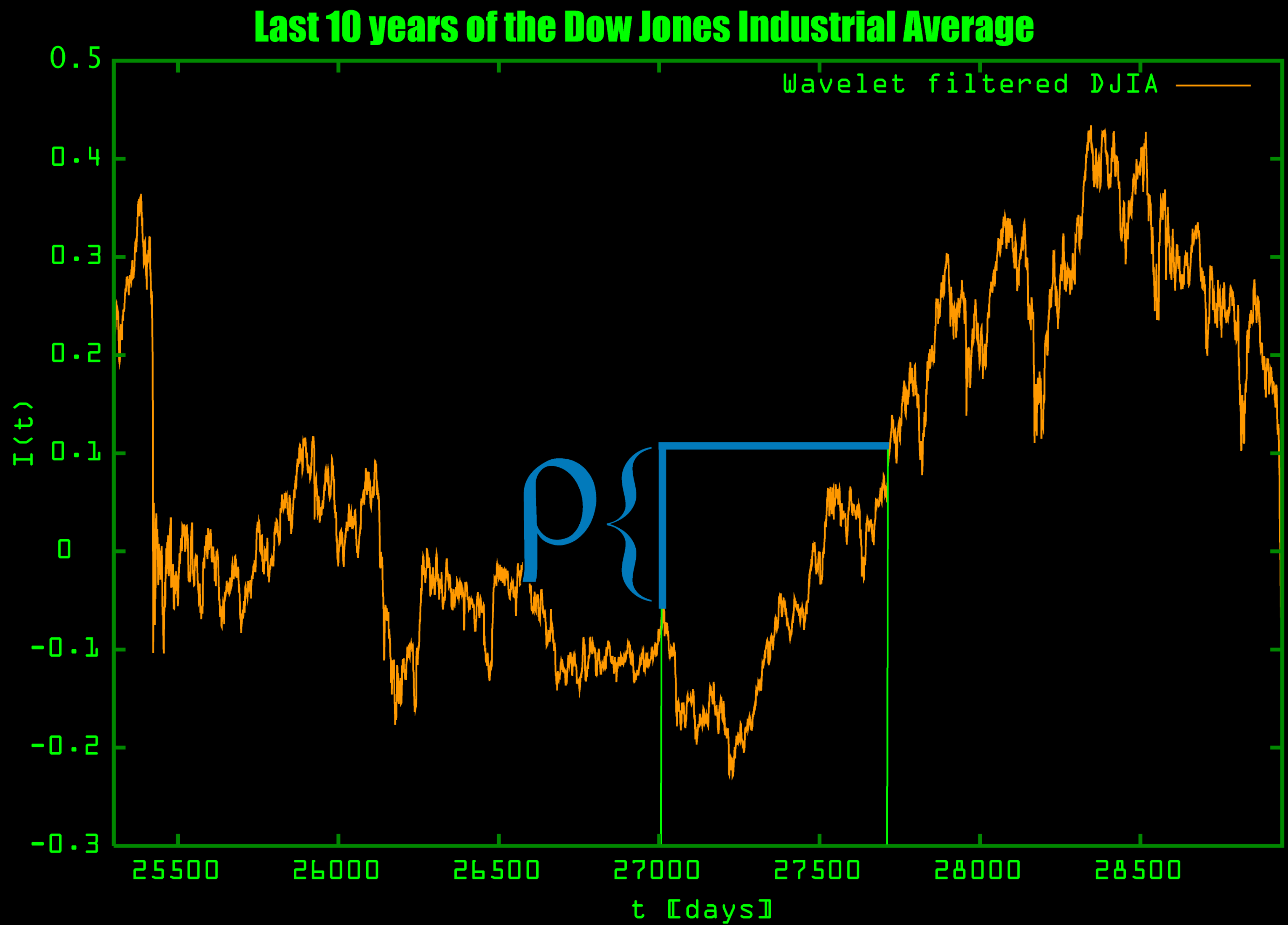
If I invest in asset A at time t, how many time units do I have to wait to get a return of $x\%$?

Last 10 years of the Dow Jones Industrial Average



Last 10 years of the Dow Jones Industrial Average

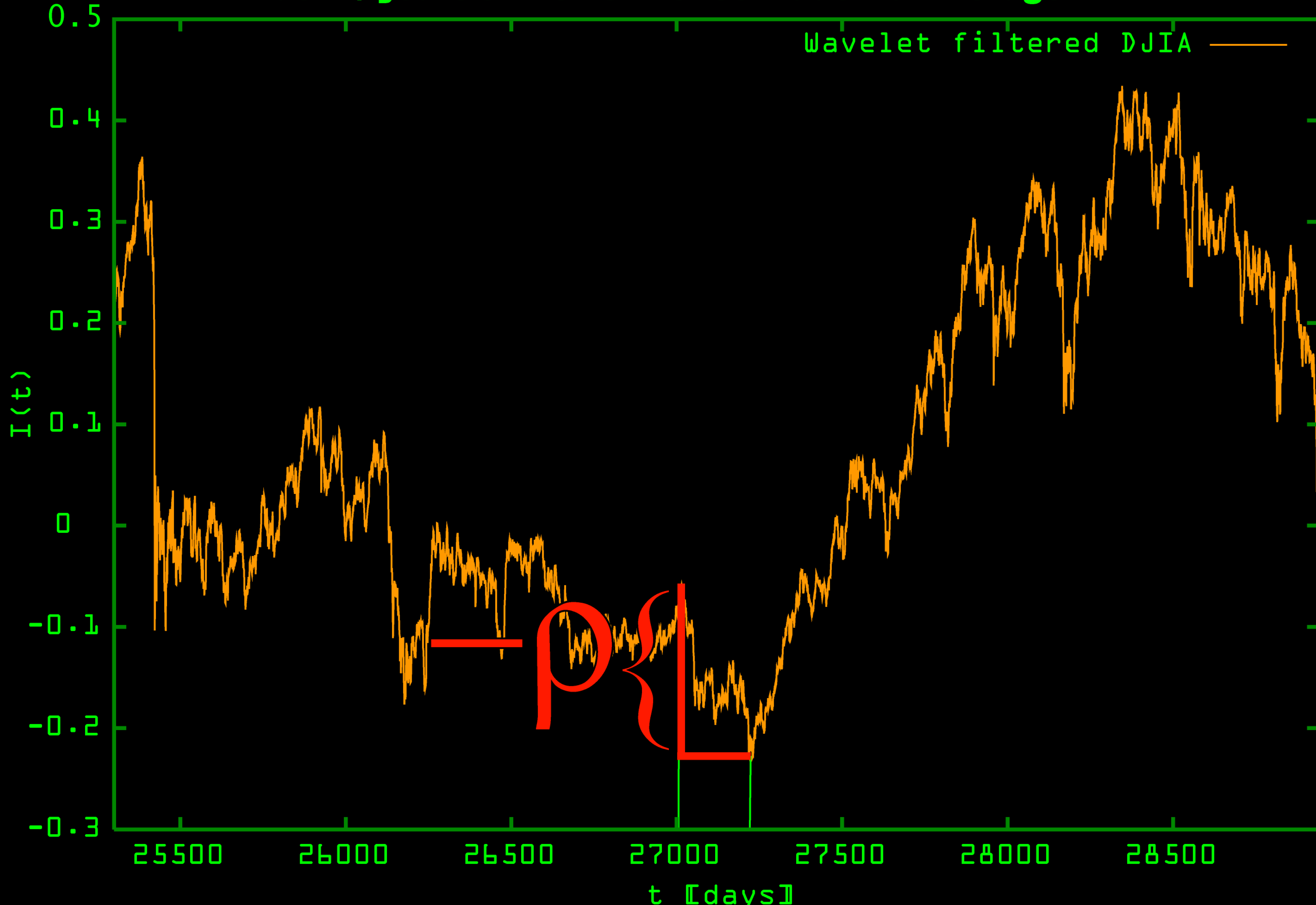


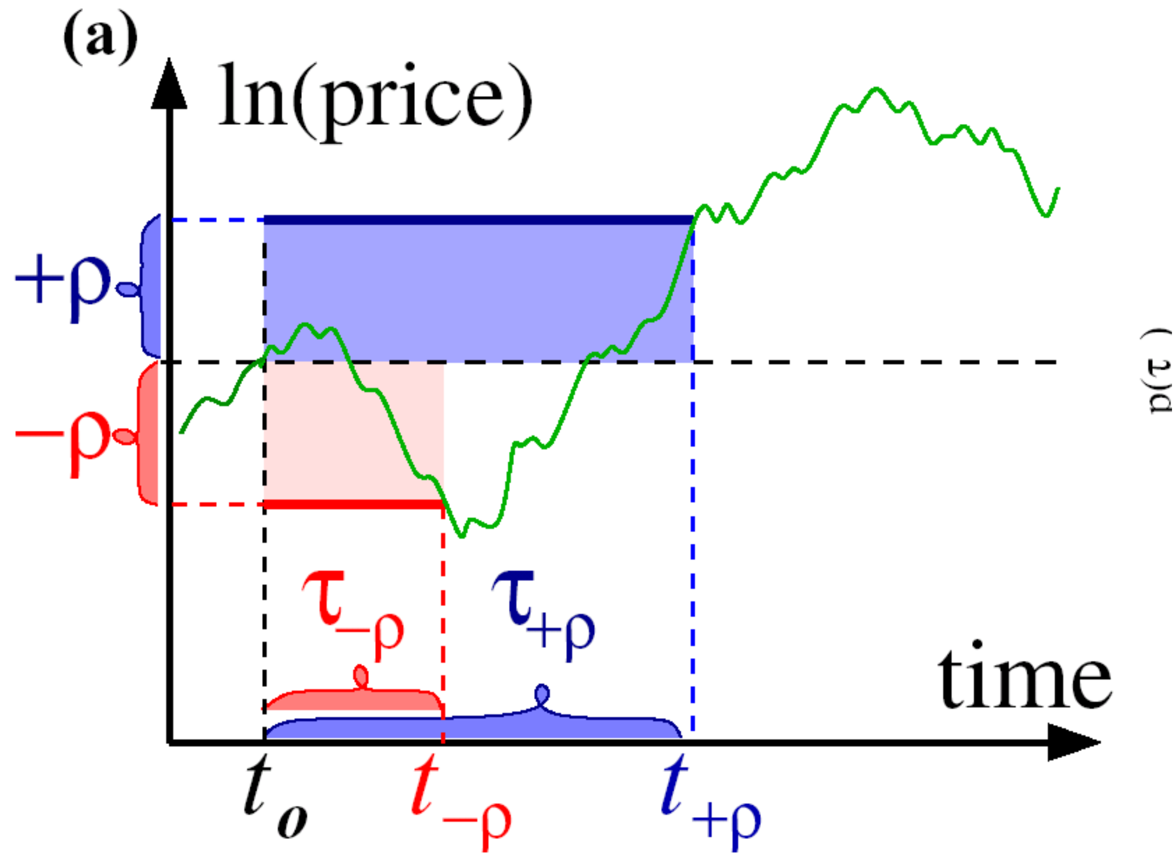


Last 10 years of the Dow Jones Industrial Average
Wavelet filtered DJIA
I(t)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
25500
26000
26500
27000
27500
28000
28500
t [days]
ρ

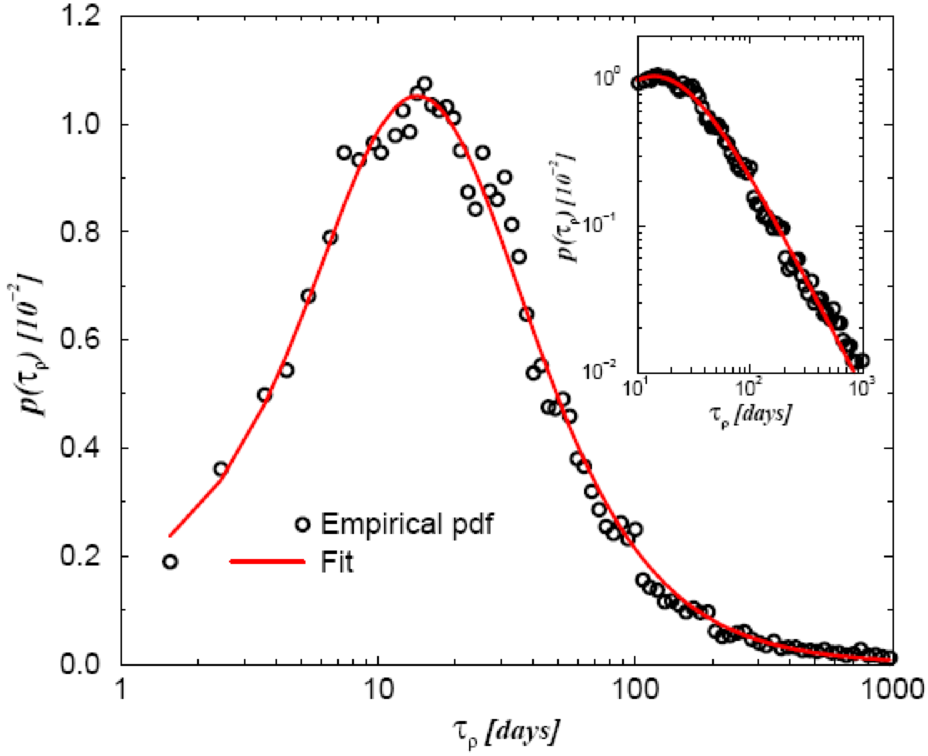

Last 10 years of the Dow Jones Industrial Average
Wavelet filtered DJIA
I(t)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
25500
26000
26500
27000
27500
28000
28500
t [days]
$-\rho\{$

Last 10 years of the Dow Jones Industrial Average




(a)
ln(price)
+ρ
−ρ
τ−ρ
τ+ρ
time
to
t−ρ
t+ρ

Inverse statisctis for ρ=0.05



Maximum: Optimal Investment horizon

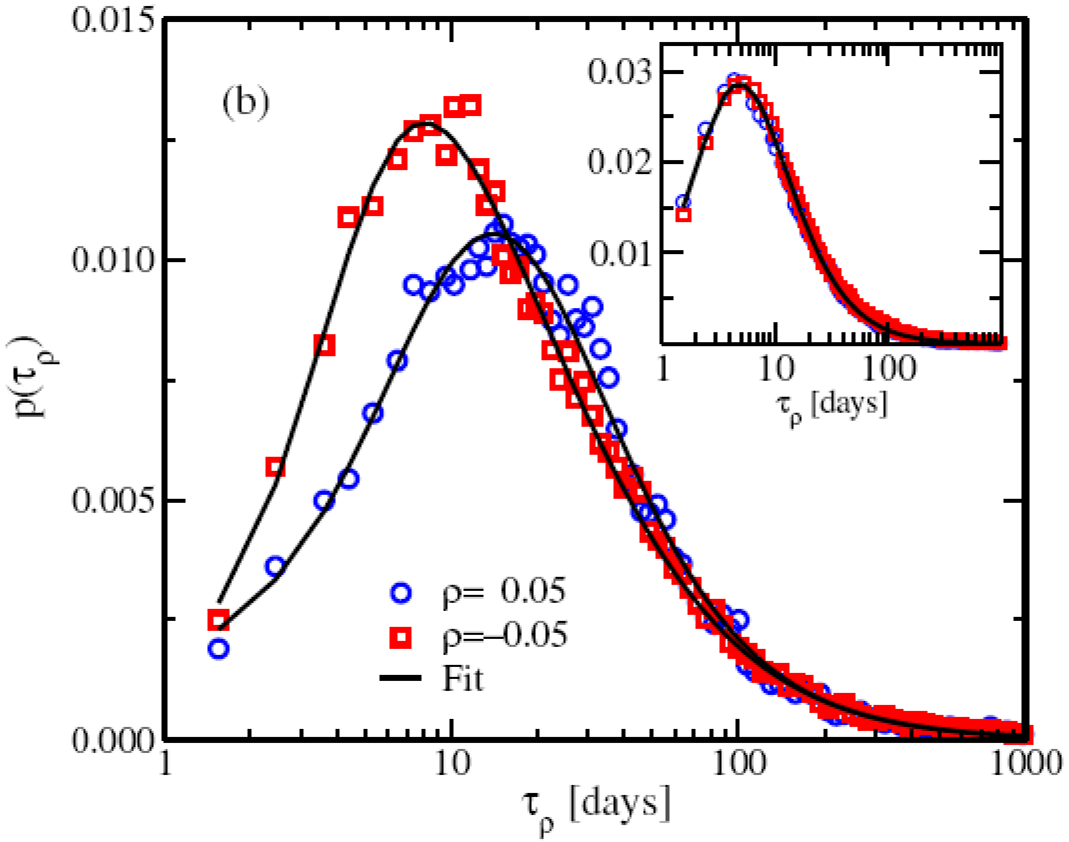
Power law tail

Fit: generalized Gamma function:

$$p(t) = \frac{\nu}{\Gamma(\alpha/\nu)} \frac{|\beta|^{2\alpha}}{(t+t_0)^{\alpha+1}} \exp\left\{-\left(\frac{\beta^2}{t+t_0}\right)^{\nu}\right\}$$

Positive ρ : Gains

Negative ρ : Losses

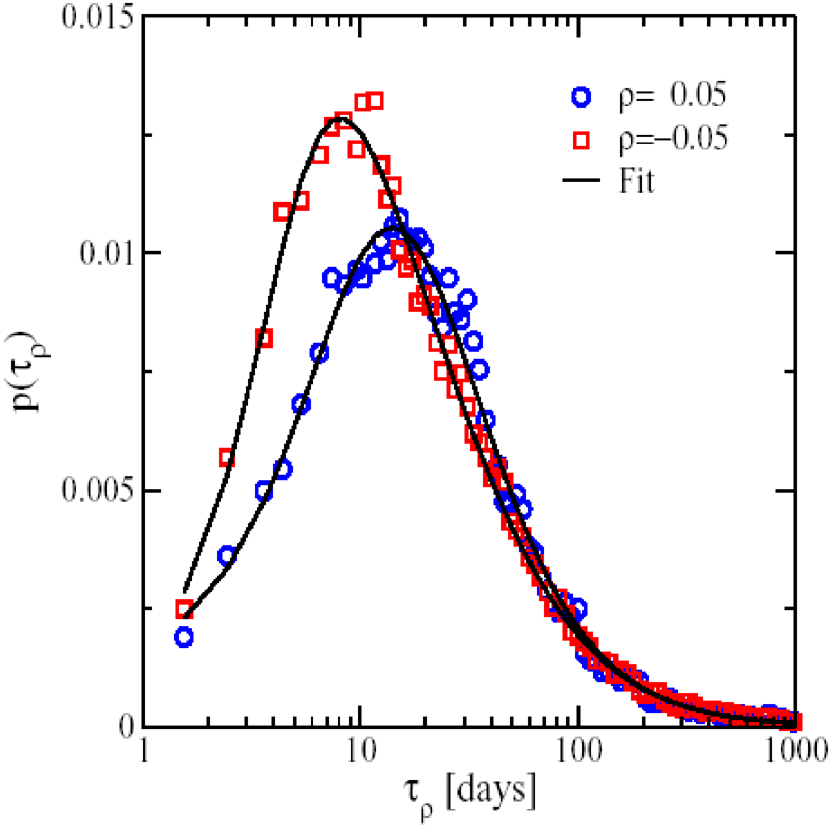


DJIA

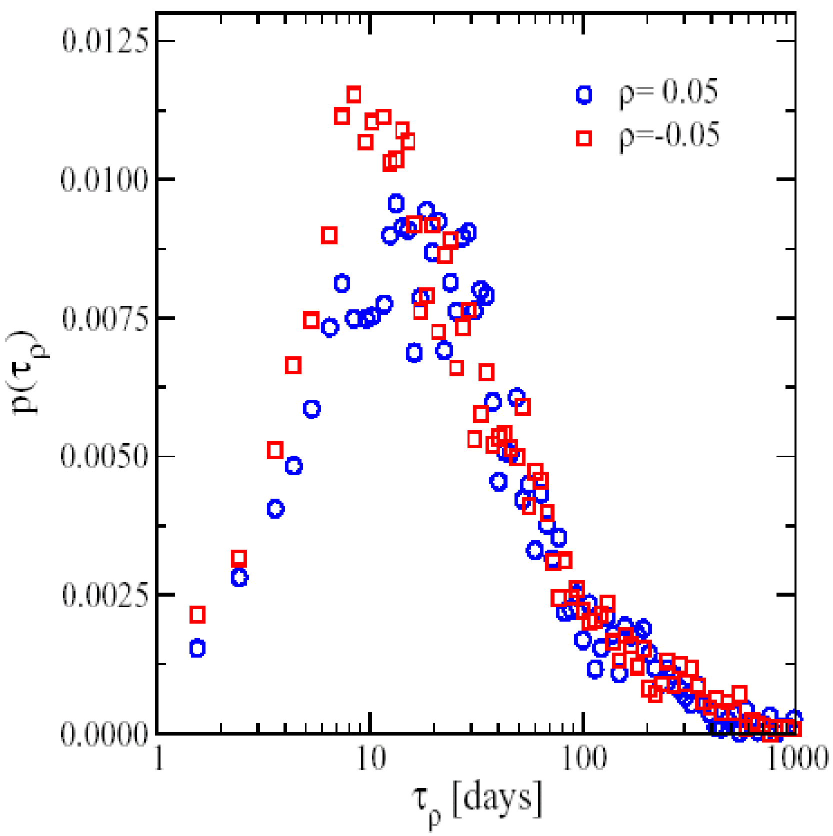
Note: Asymmetry between gains and losses

(maybe related to leverage effect.)

DJIA

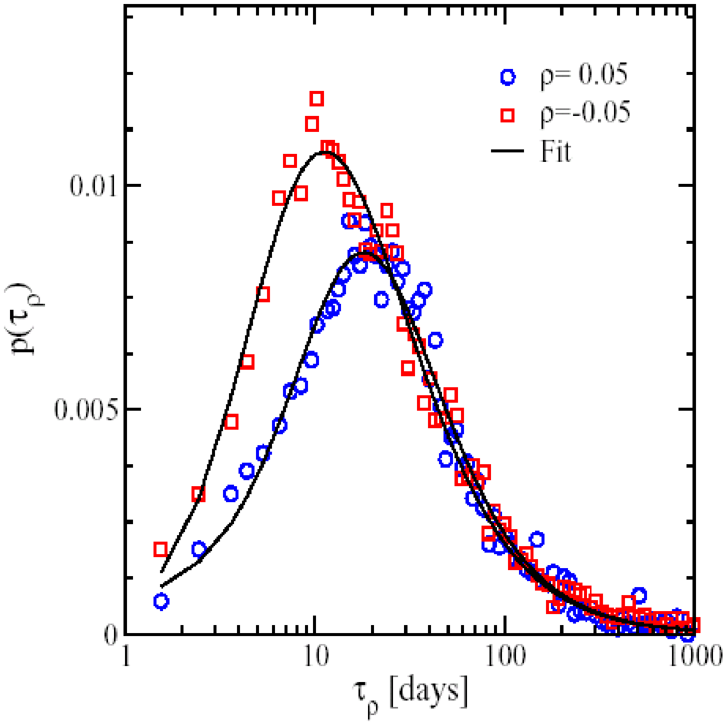


(a) DJIA (1896.5–2001.7)

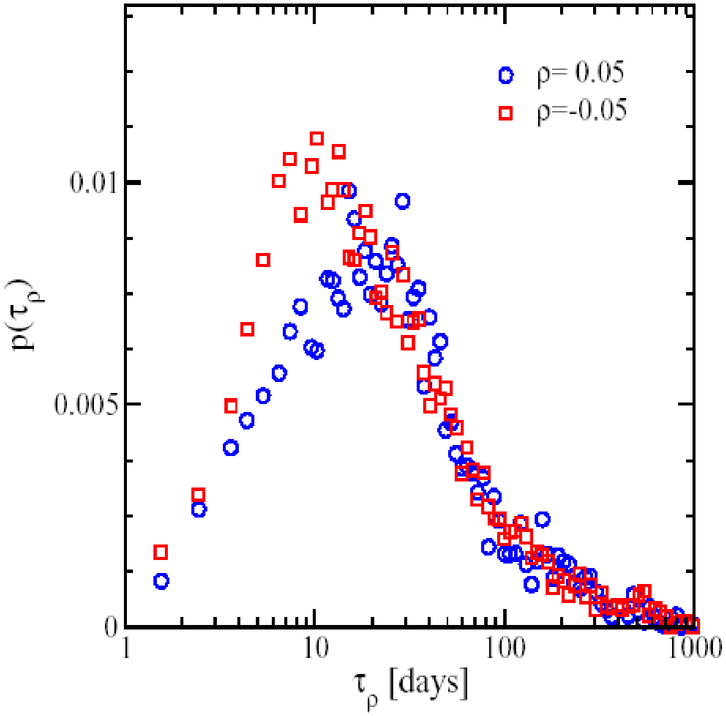


(b) DJIA (1960.8–1999.8)

SP500



(a) SP500 (1940.9–2000.3)



(b) SP500 (1960.8–2000.3)

NASDAQ

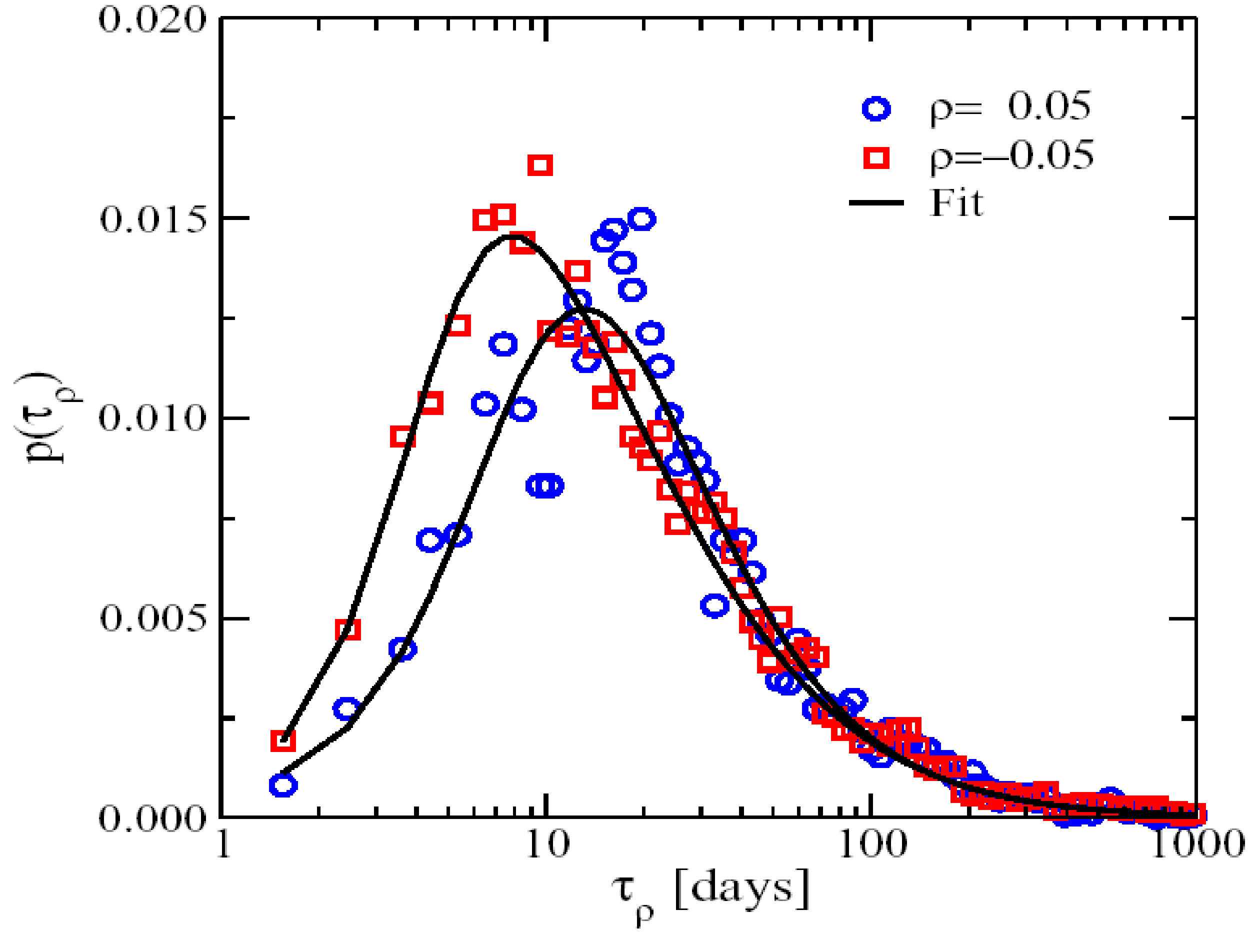
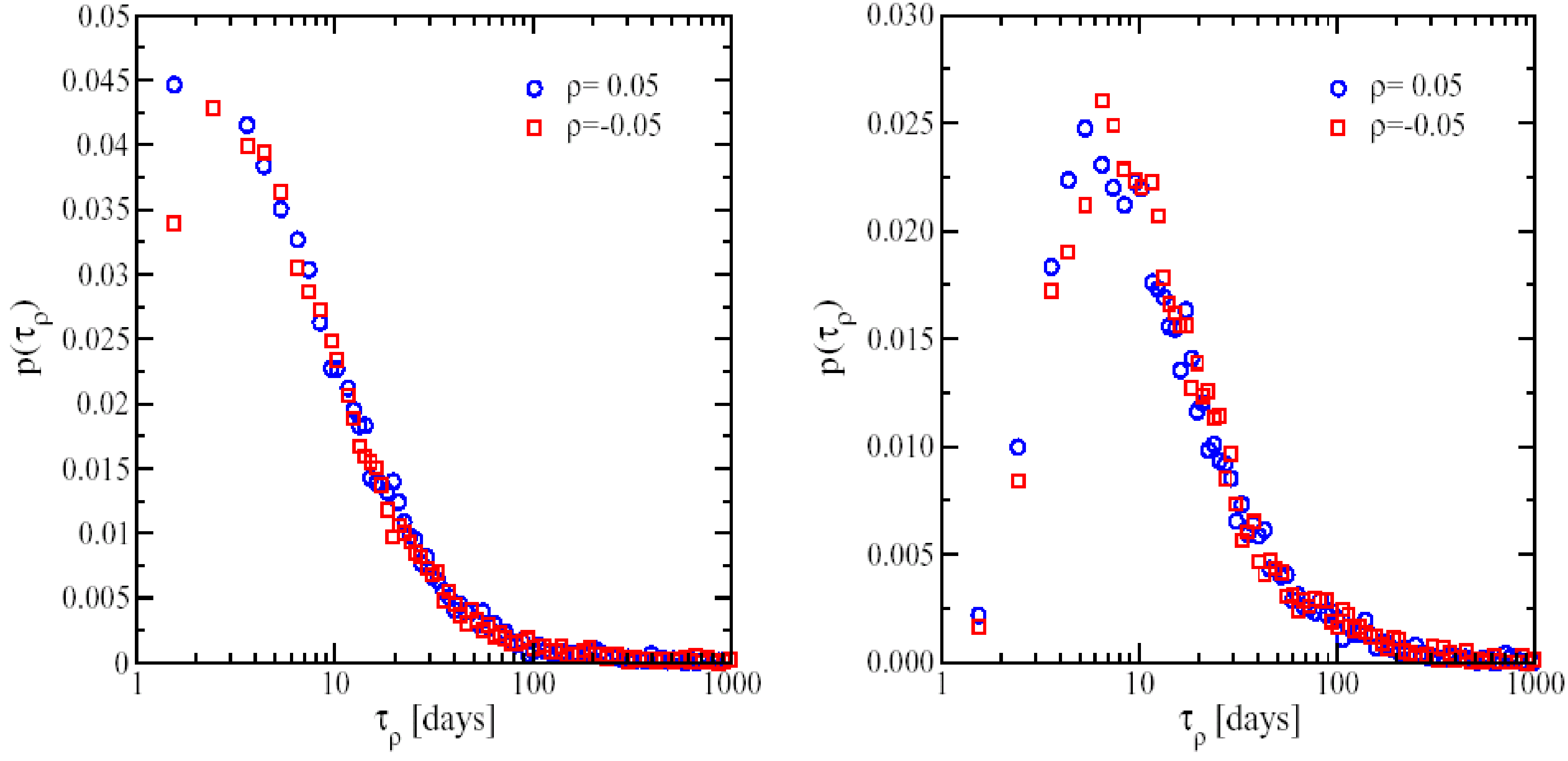


FIG. 5: Same as Fig. 3(a), but for the Nasdaq. The historical time period considered is 1971.2 to 2004.6. Again he solid lines represent fit of the empirical data against Eq. (2) with parameters: $\alpha \approx 0.51$, $\beta \approx 4.72$, $\nu \approx 0.73$ and $t_0 \approx 7.92$ (loss distribution); $\alpha \approx 0.51$, $\beta \approx 4.16$, $\nu \approx 2.41$ and $t_0 \approx 0.07$ (gain distribution). Note again that the tail exponents $\alpha + 1$ are very close to the "random walk value" of 3/2 for both distributions.

Inverse statistics for single stocks



(a) Boeing Airways

(b) General Electric

Inverse statistics for single stocks

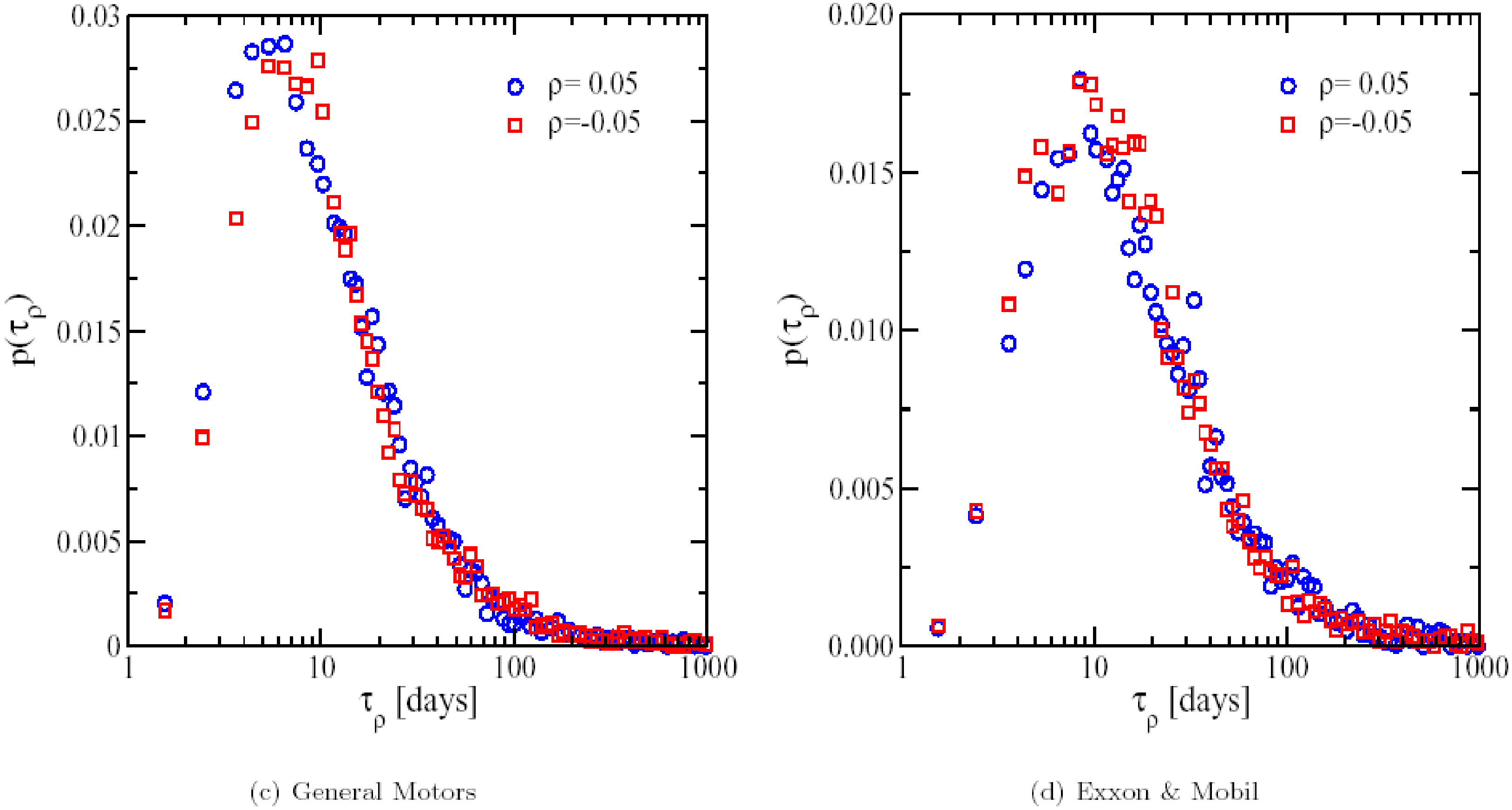


FIG. 7: Same as Fig. 3(a), but for some of the individual companies of the DJIA: (a) Boeing Airways (1962.1–1999.8); (b) General Electric (1970.0–1999.8); (c) General Motors (1970.0–1999.8); (d) Exxon & Mobil, former Standard Oil (1970.0–1999.8).

Averaged over many single stocks

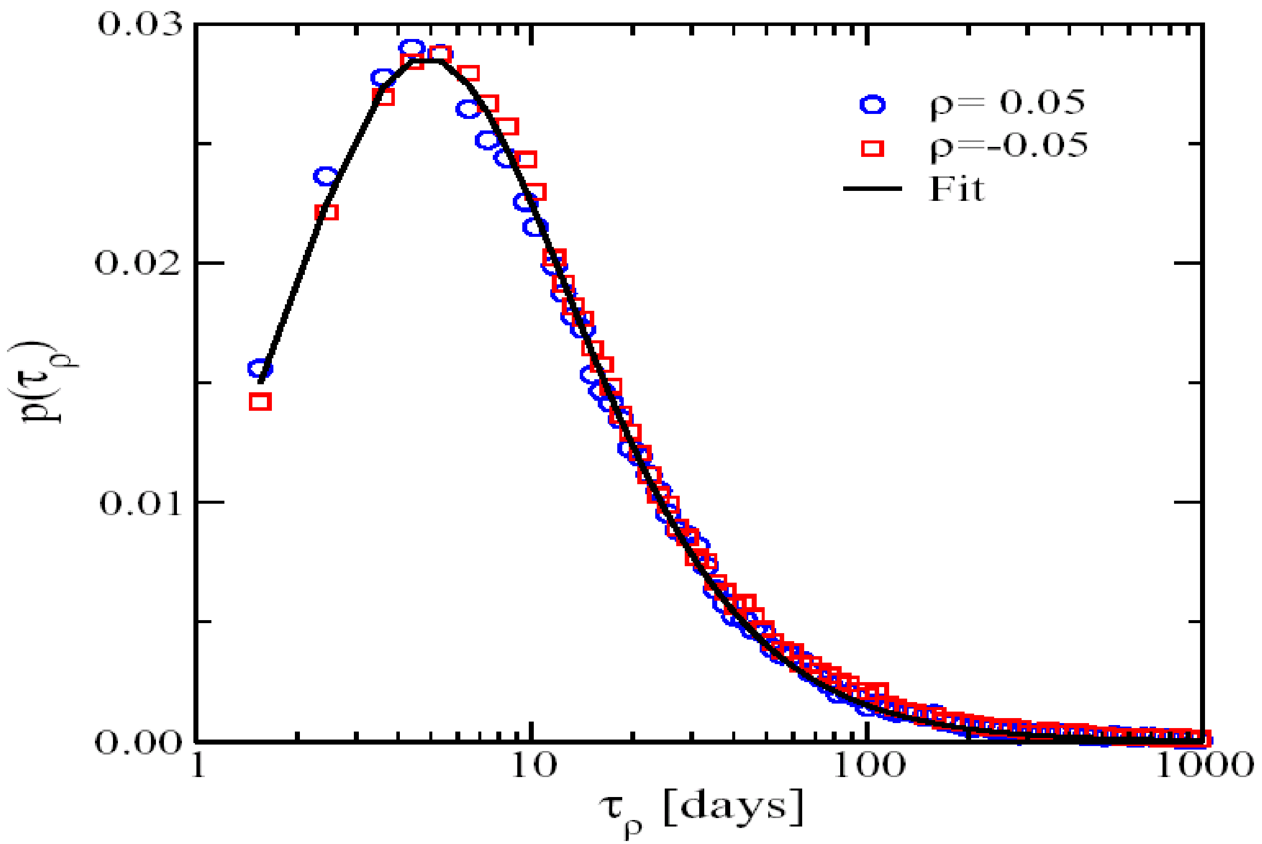
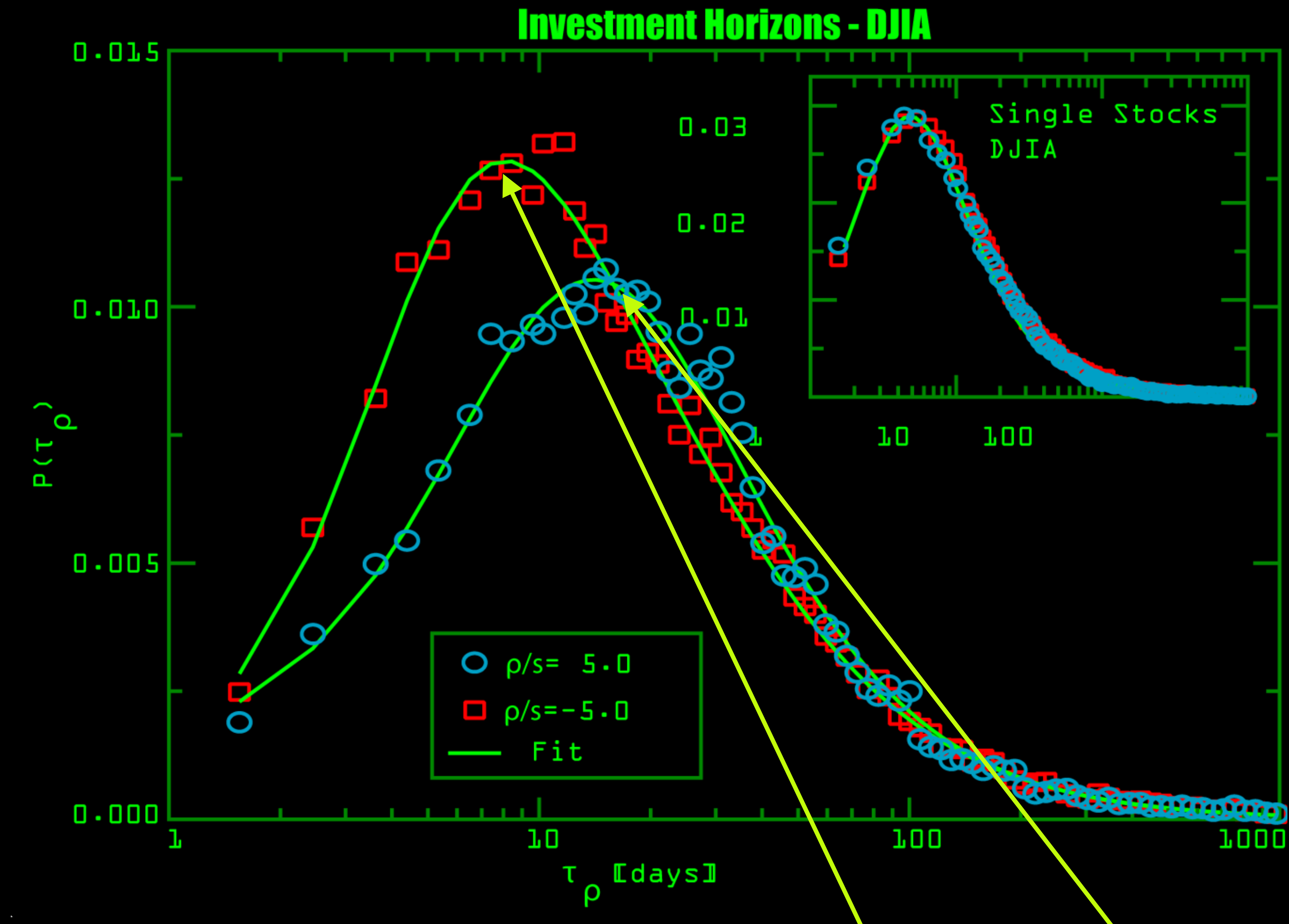


FIG. 8: Averaged gain and loss distribution for the companies listed in table I. The fit is Eq. (2) with values $\alpha \approx 0.60$, $\beta \approx 3.24$, $\nu \approx 0.94$ and $t_0 \approx 1.09$. Note that the tail exponent $\alpha + 1$ is 0.1 above the "random walk value" of 3/2.

NOTE: No asymmetry



This is Gain Loss Asymmetry: - 8 days, + 15 days

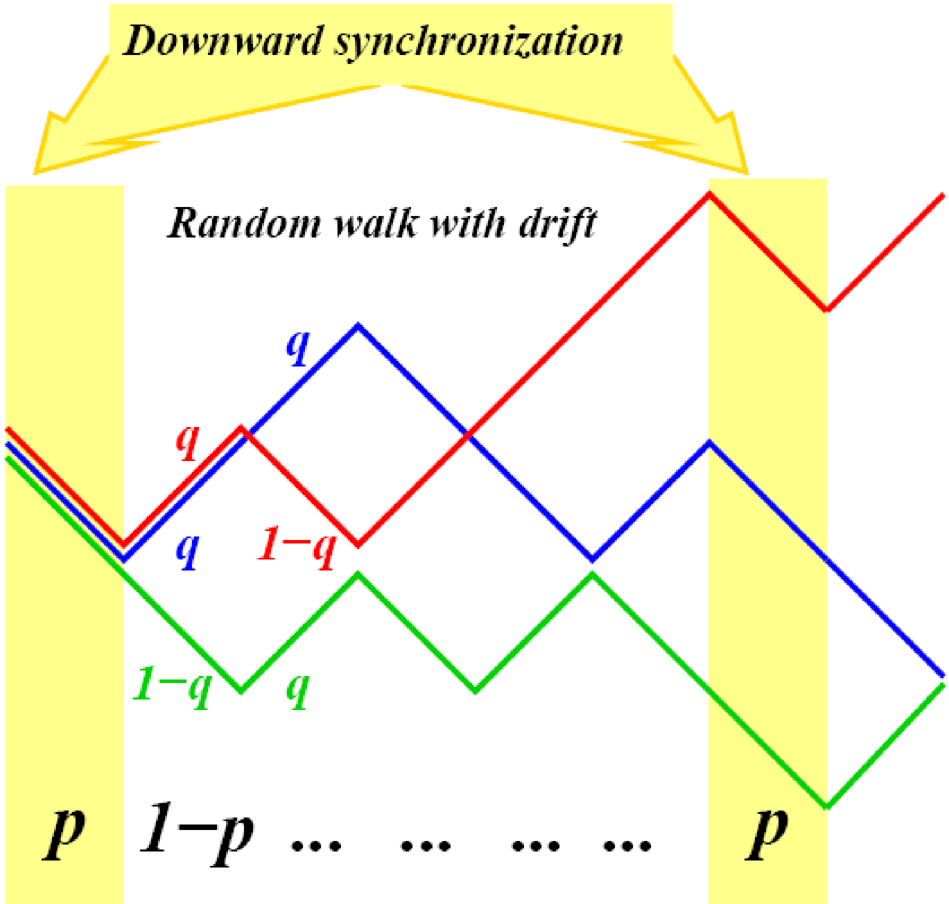
How to explain the asymmetry in the market?

- External events (wars, terror, earthquakes, hurricanes) introduces a fear factor in the market

- Psychology of society/market: When stocks begin to fall they do it synchronously

- Under up-trends stocks move more or less randomly

The Fear Factor Model (FFM)



For the log-price of a stock:

With prob. p : *all* stocks move *downwards synchronously*

With prob. 1-p : they do *independent biased* random walks

- With prob. q : move upward
- With prob. 1-q : move downward

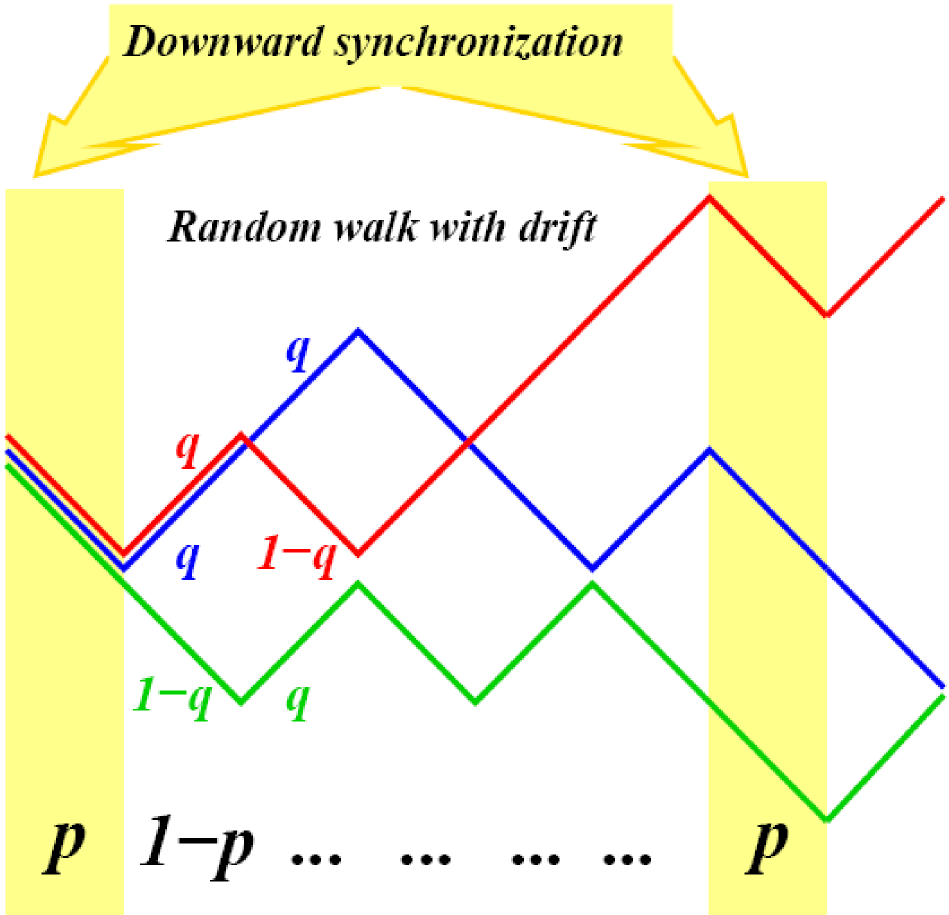
q determined from:

- Requirement : $s_i(t)$ is drift-less

p : fear factor

N : # of stocks in the index

The Fear Factor Model (FFM)



<u>The q-parameter is determined from the assumption: *indiv. stock prices are drift-less*</u>

- $p + (1-p)(1-q) = (1-p)q$

  Price-drop *Price-rise*

- p and q are coupled
- $$q = \frac{1}{2(1-p)}$$

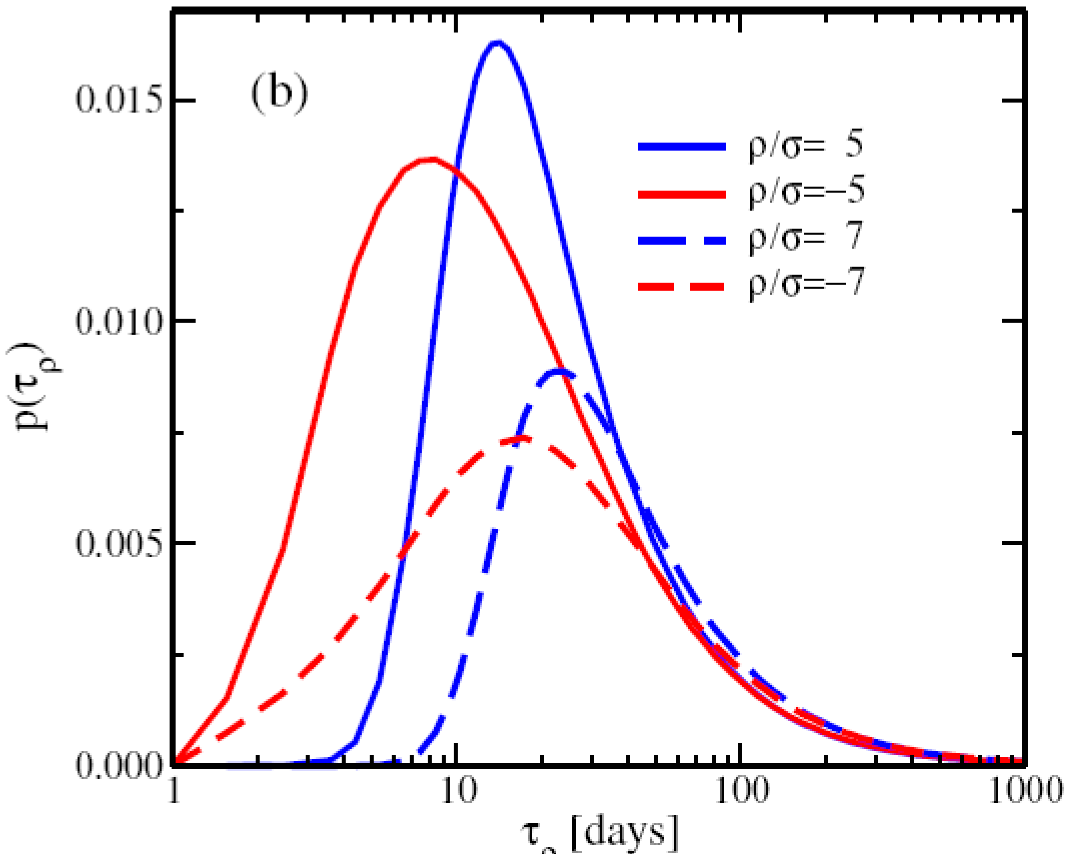
The Fear Factor Model: The assumptions

- Single Stocks
  - The price process of a single stock, S(t), makes a *Geometrical Brownian Motion:*
    - i.e. s(t)=ln[S(t)] is Brownian random process
  - s(t) is un-biased (no drift)

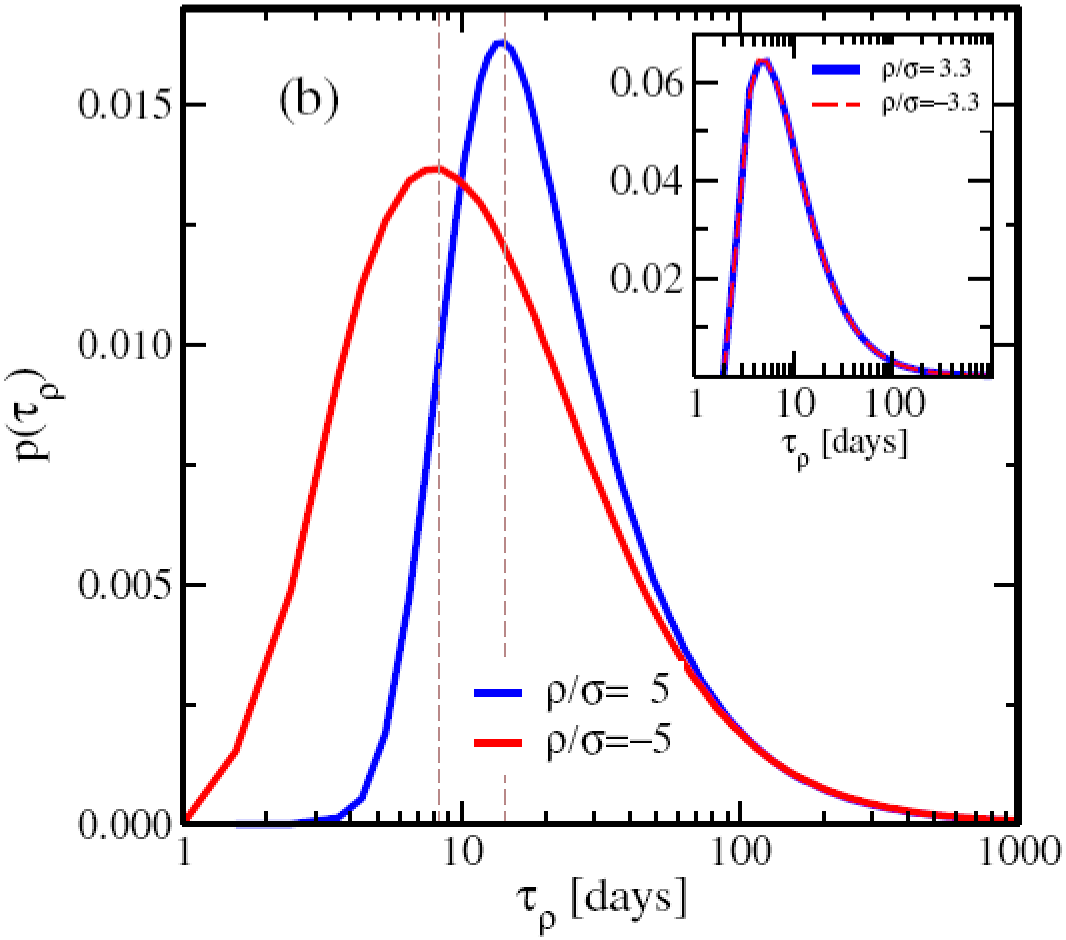
- The Stock Index
  - The stock index consists of N stocks
  - The value of the stock index, I(t), is calculated as:

$$I(t) = \sum_{i=1}^{N} S_i(t) = \sum_{i=1}^{N} \exp\left(s_i(t)\right), \qquad S_i(t) = \ln s_i(t)$$

FFModel



FFModel



(b)
0.015
0.010
0.005
0.000
p(τ_ρ)
1
10
100
1000
τ_ρ [days]
ρ/σ= 5
ρ/σ=−5
0.06
0.04
0.02
ρ/σ= 3.3
ρ/σ=−3.3
1
10
100
τ_ρ [days]

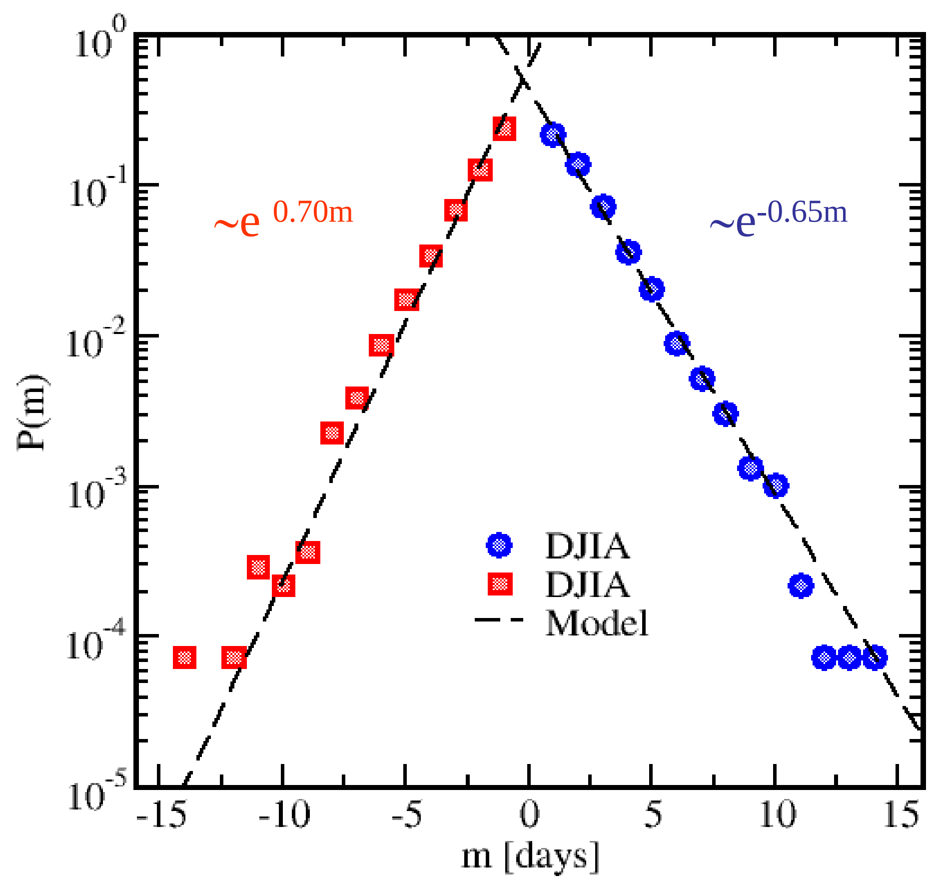
Fear factor p measures probability for stocks going down synchronously

Let us consider the probability that the DJIA index goes down several days in a row

(“mini crashes”)

Model results

NOTE the slight asymmetry

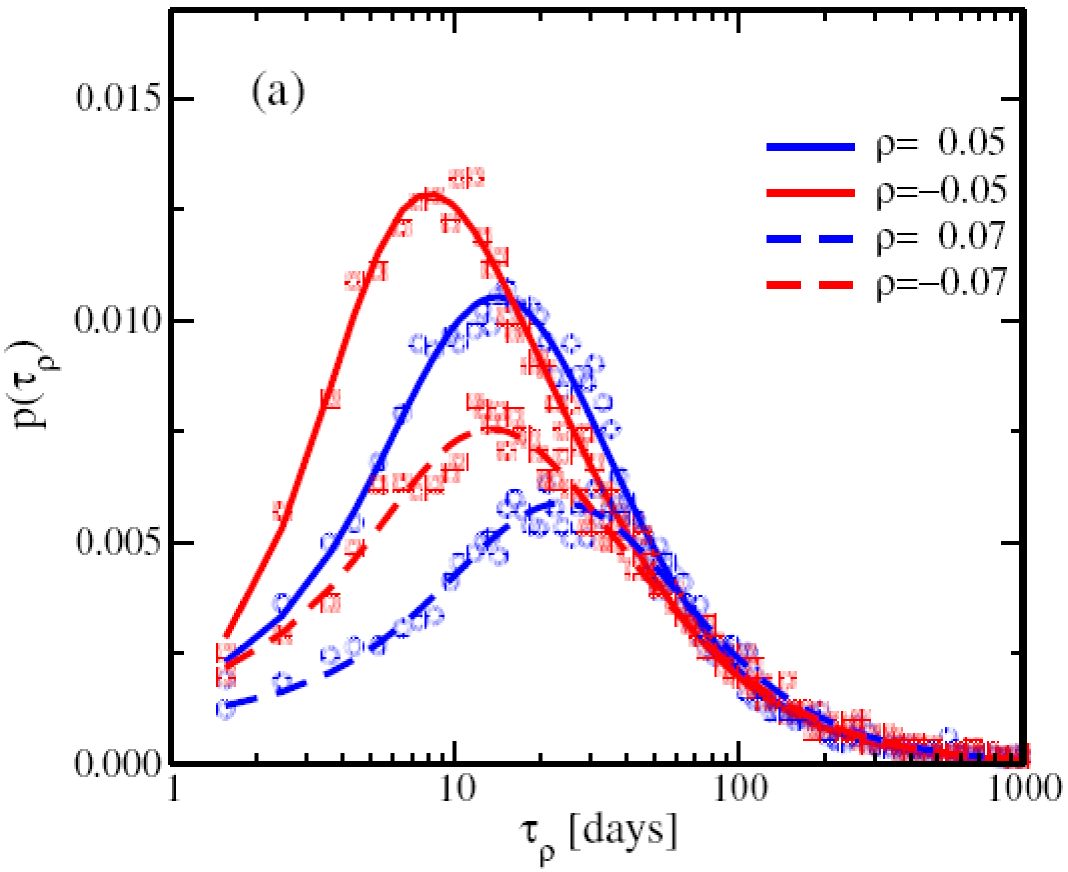


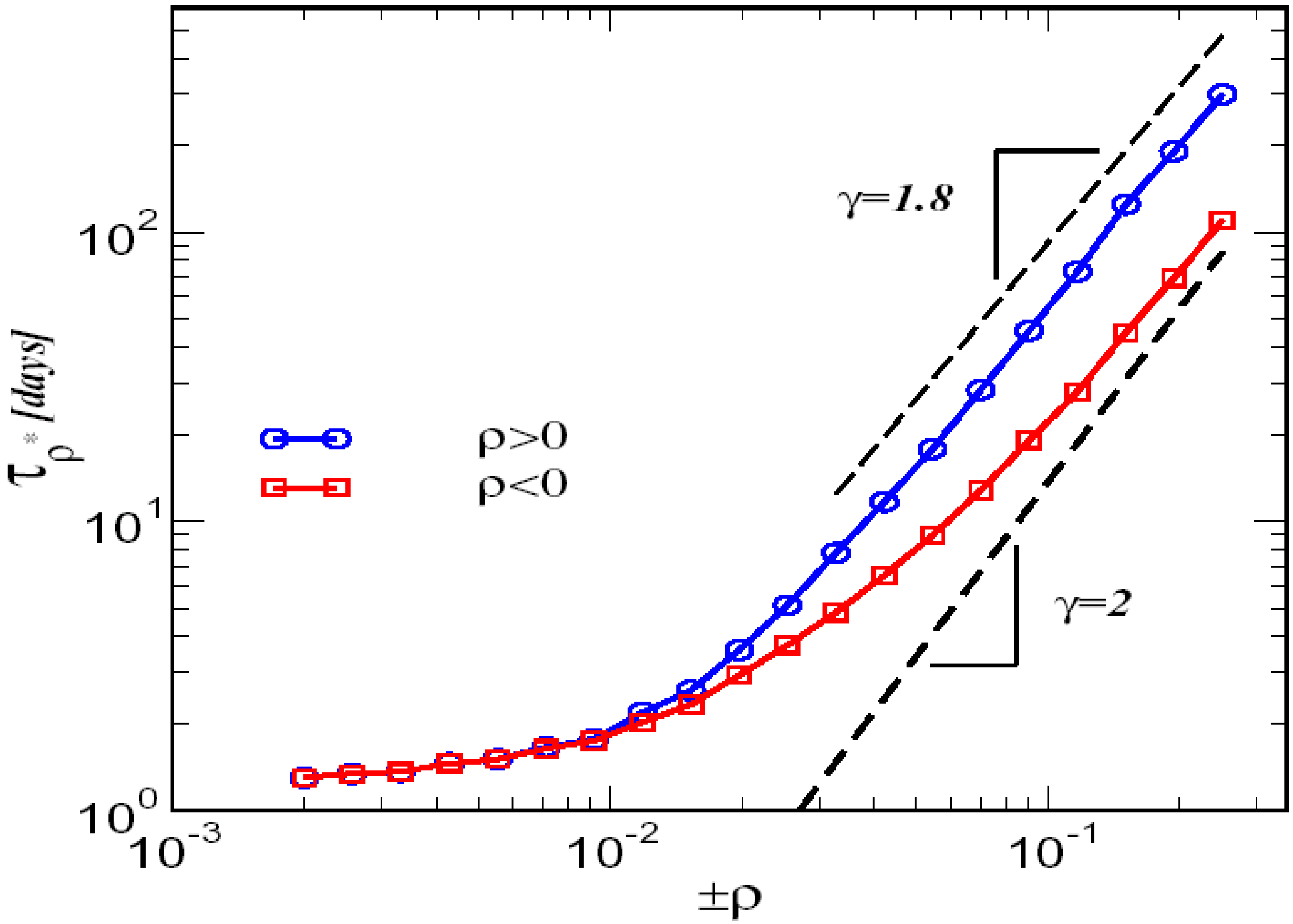
Let us consider the probability that the DJIA index drops (m<0) or rises (m>0) several days (*m*) in a row ("mini crashes/rallies")

m=1 : 10% more likely to have a price drop than a price rise

The model catch *also* this feature of the real market excellently!

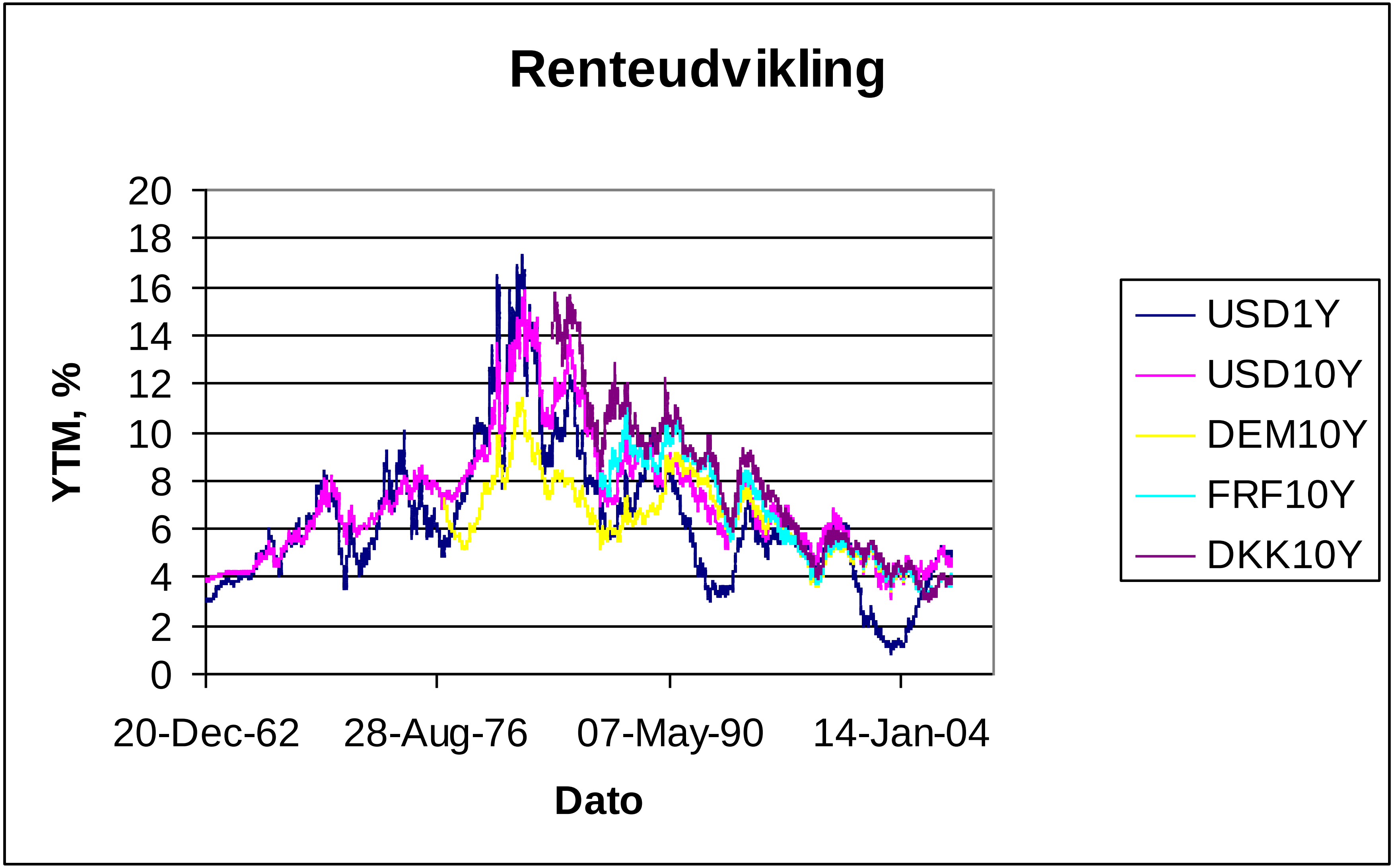
DJIA


(a)
ρ= 0.05
ρ=−0.05
ρ= 0.07
ρ=−0.07
0.015
0.010
0.005
0.000
1
10
100
1000
p(τρ)
τρ [days]

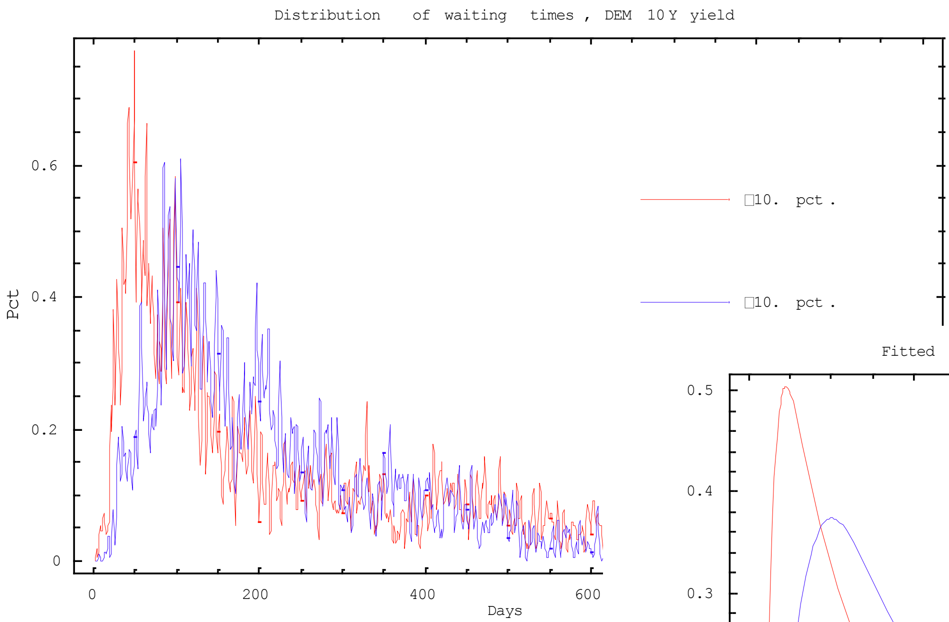


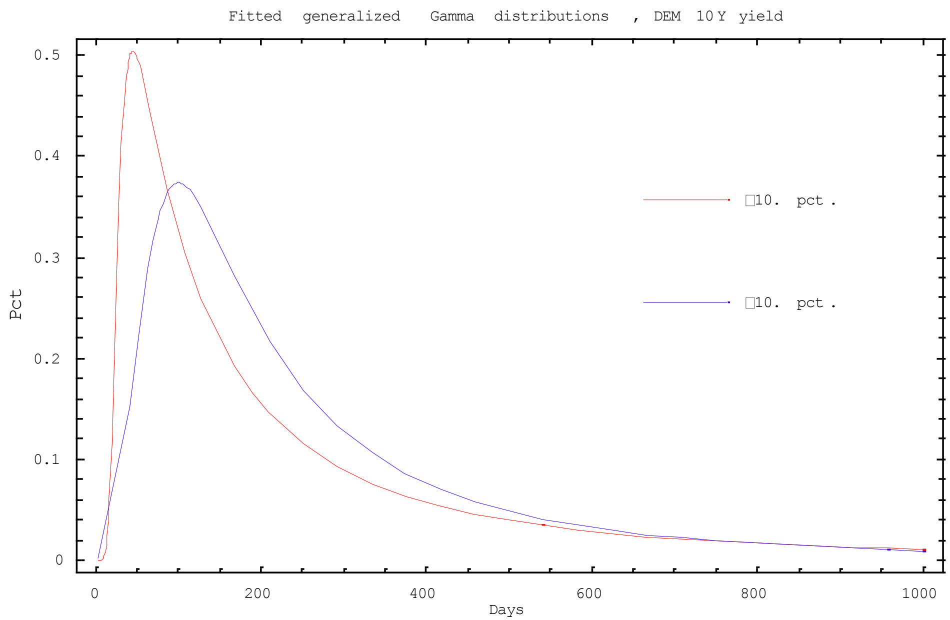
G. 6: The optimal investment horizon τ^*_ρ for positive (open circles) and negative (open squares) levels of return $\pm\rho$ for the JIA. In the case where $\rho < 0$ one has used $-\rho$ on the abscissa for reasons of comparison. If a geometrical Brownian price ocess is assumed, one will have $\tau^*_\rho \sim \rho^\gamma$ with $\gamma = 2$ for all values of ρ. Such a scaling behaviour is indicated by the lower shed line in the graph. Empirically one finds $\gamma \simeq 1.8$ (upper dashed line), only for large values of the return.

Interest rates



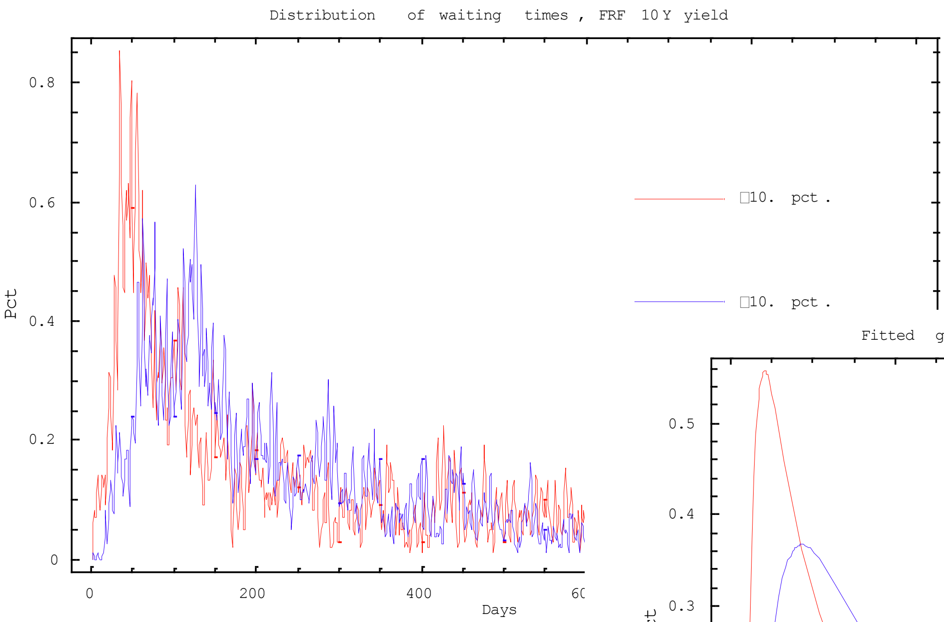
Renteresultater
DEM 10% ændring



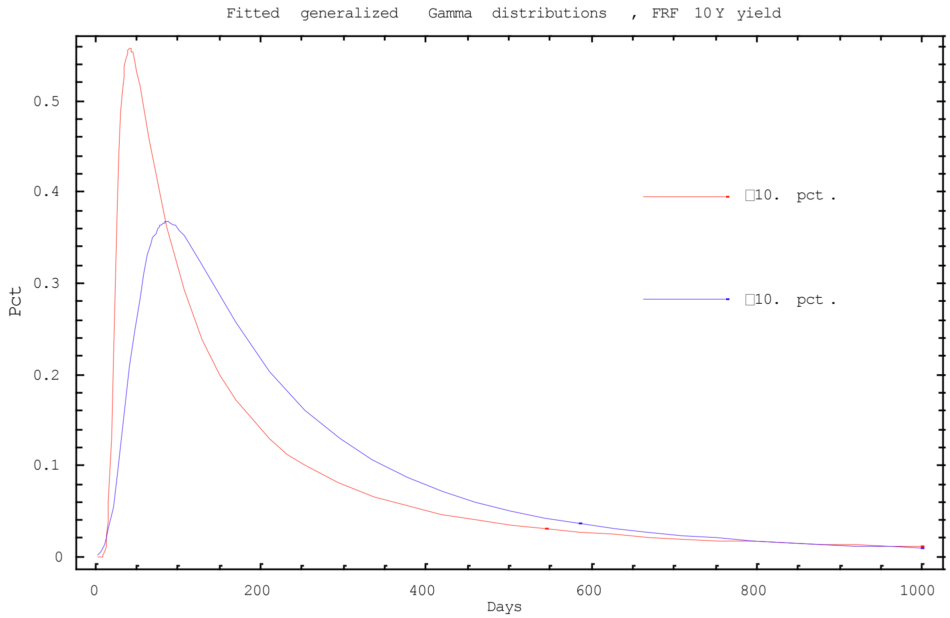


Op/ned
= 45/100

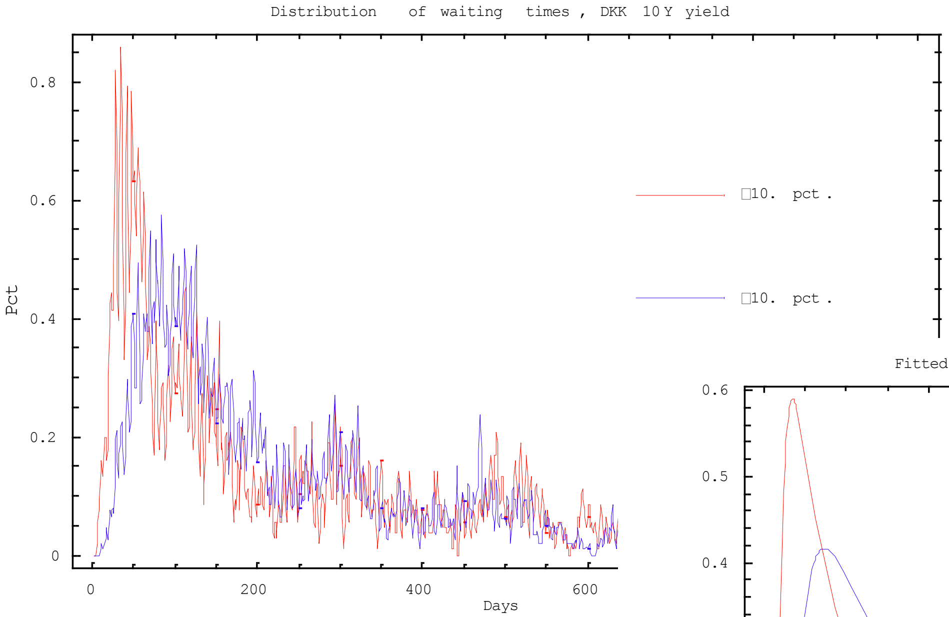
Interest rates FRF 10%



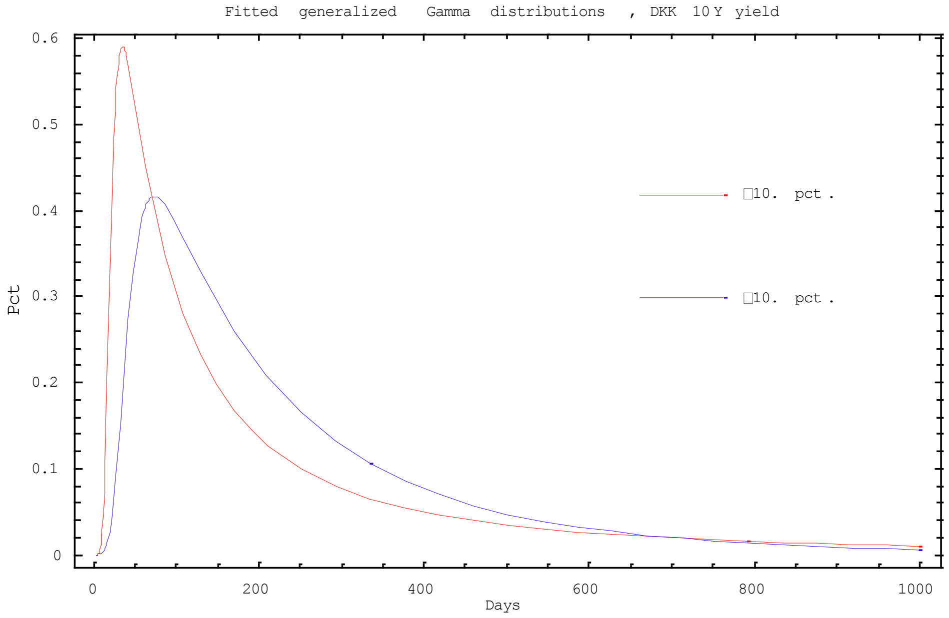
Up/down
= 42/87



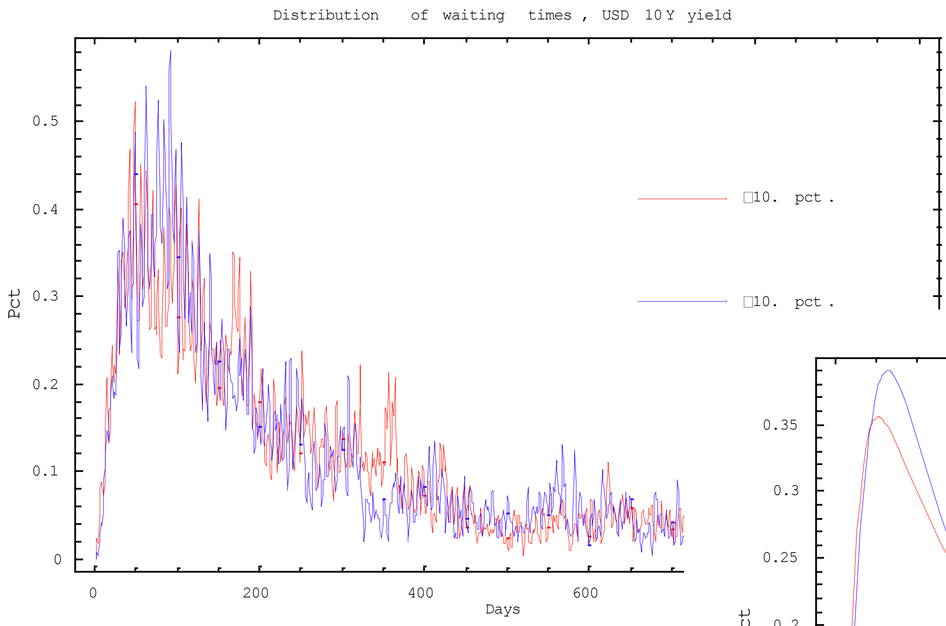
Interest rates
DKK 10%



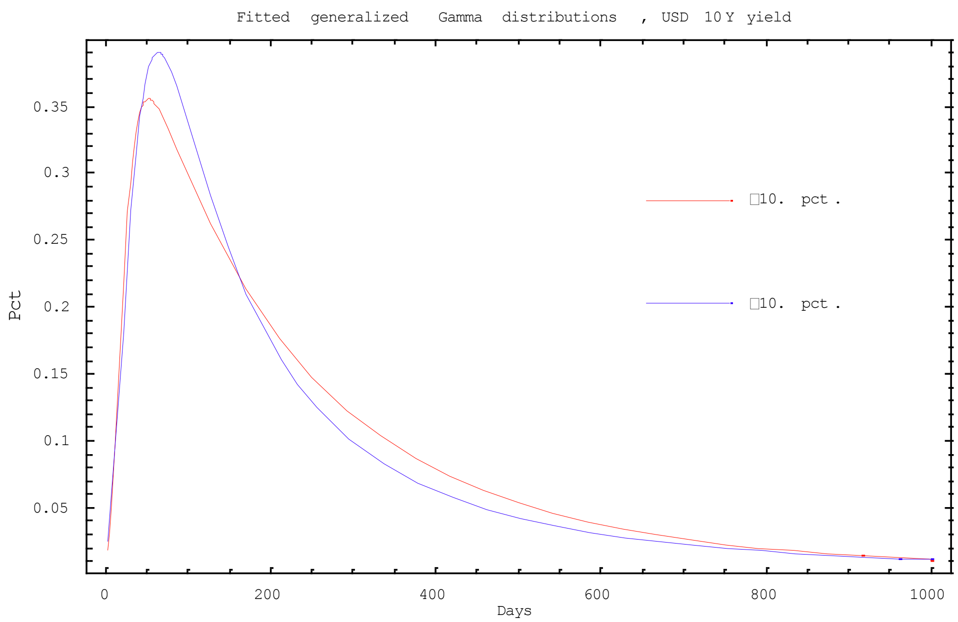
Up/down
= 35/74



Interest rates
USD10Y 10%

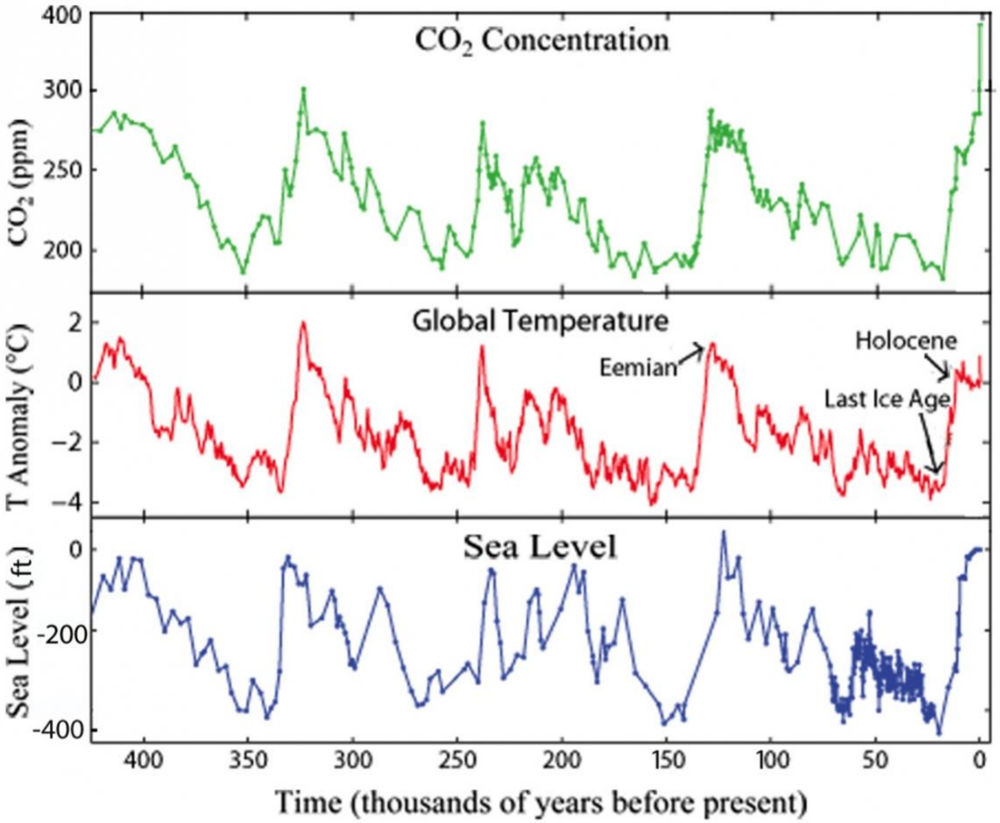


Up/down
= 53/64



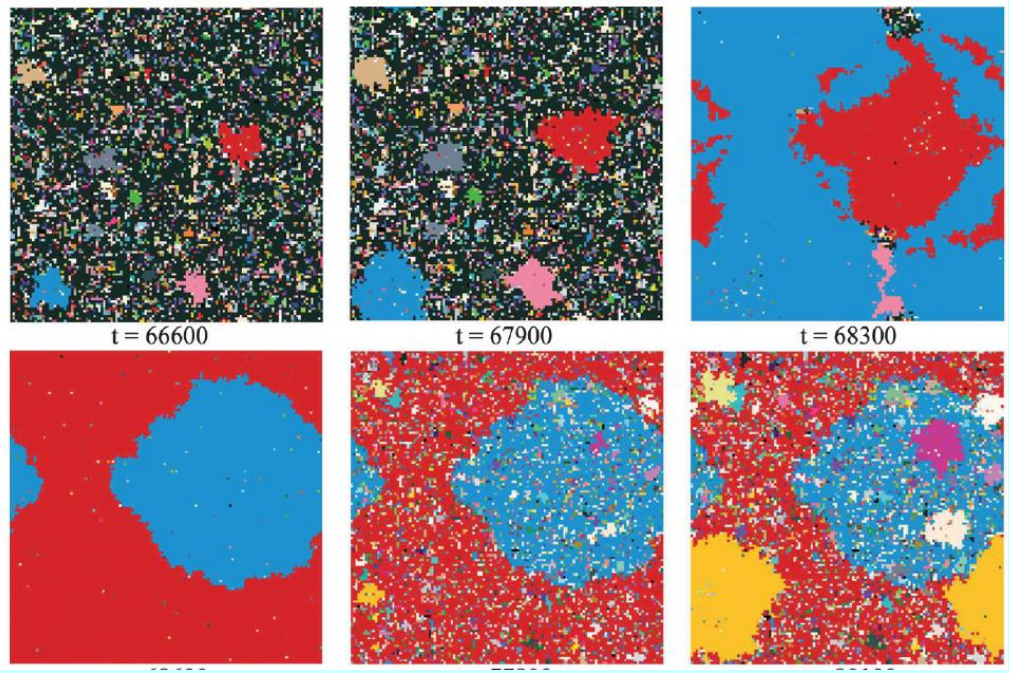
Extreme events: Often fast change -> slow response -> positive feed-back mechanism (bi-stability)

Other examples: Climate changes, paradigm shifts



John Englander / "High Tide on Main Street" adapted from Hansen & Sato

Emergence and Decline of Scientific Paradigms: Another example of Extreme Events !



Scientific paradigms: Fast rise → slow decline

(Kuhn: “Scientific revolutions” or “Paradigm shifts”)

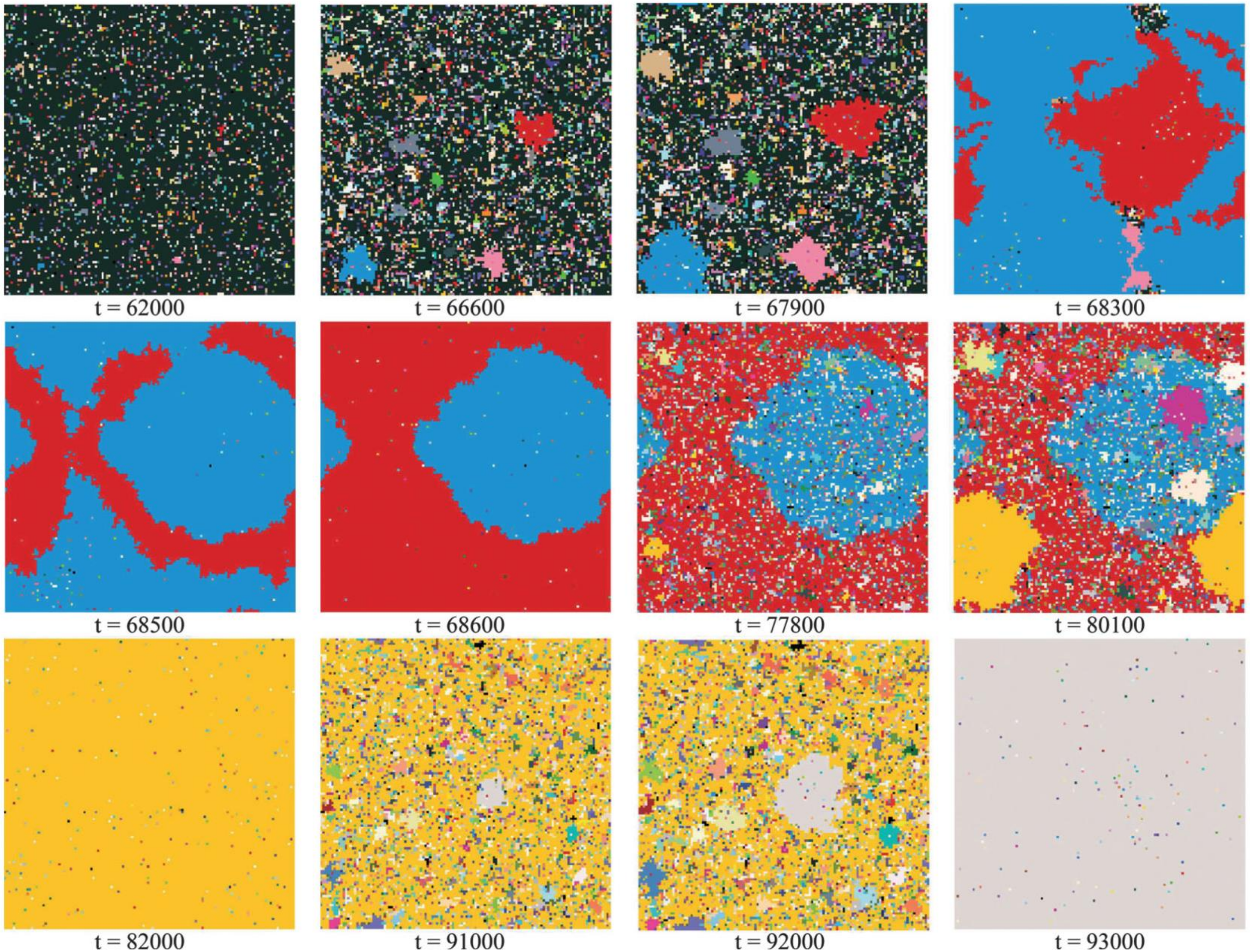
Scientific paradigms

- Single words, concepts: Nano, string theory, systems biology, climate change, chaos
- Rapid growth → Slow decline (asymmetry)
- Scientific paradigms → Global awareness (nanotechnology, climate change)
- How does a person change 'his/her' opinion and concept?

Model ingredients

- An ‘infinity of concepts: A person never ‘returns’ to old concept
- Cooperativity: A person change concept with a weight proportional to the ‘size’ of the paradigm → A paradigm can ‘flush’ through the whole world ! Extreme event !
- Certain small probability α to change concept (innovation rate)
- Technically: N=LxL persons on a lattice, 4 neighbors, $\alpha \sim 10^{-6}$, ‘infinitely’ many different concepts

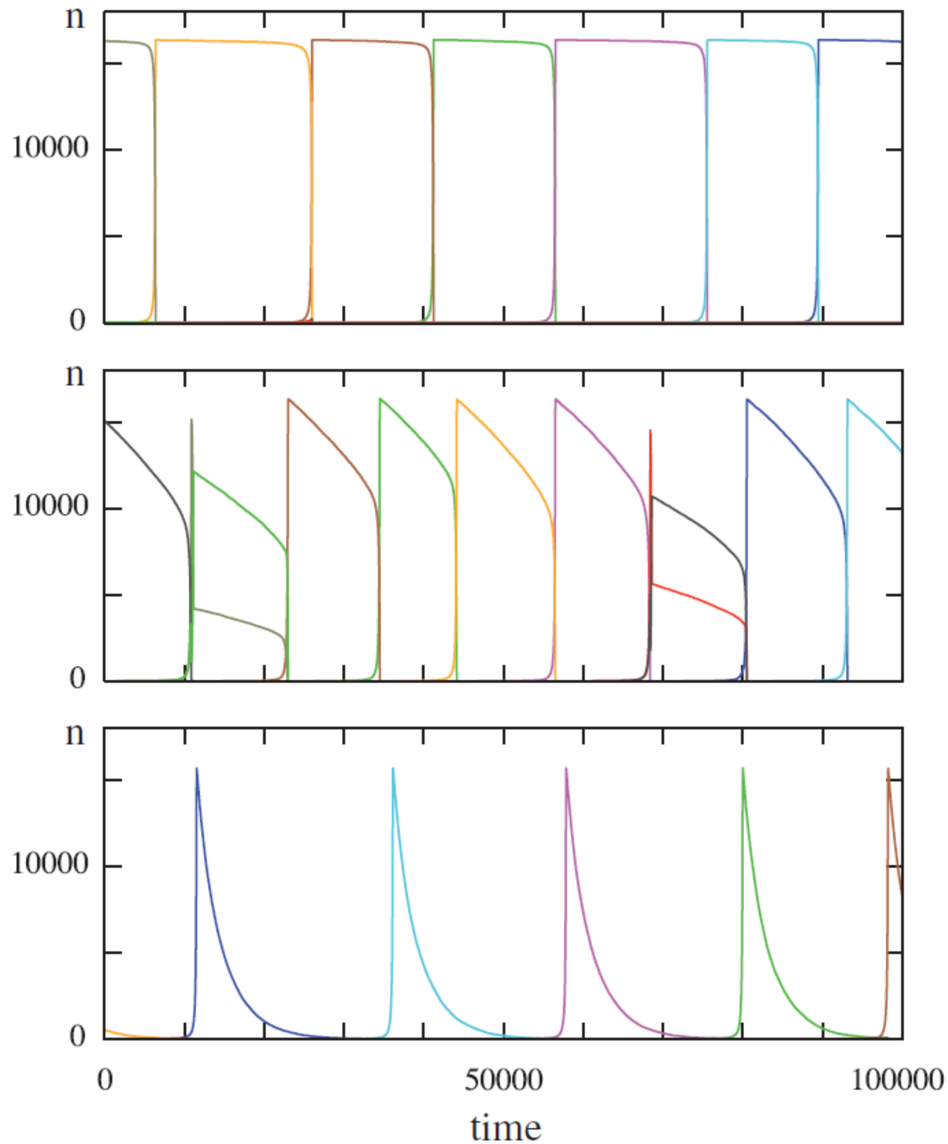
A paradigm/concept has one color



←Erosion!

N = 128 x 128, $\alpha \sim 25 \times 10^{-6}$, choose a neighbor, change according to weight

Time series of the dominant paradigm: Avalanches!



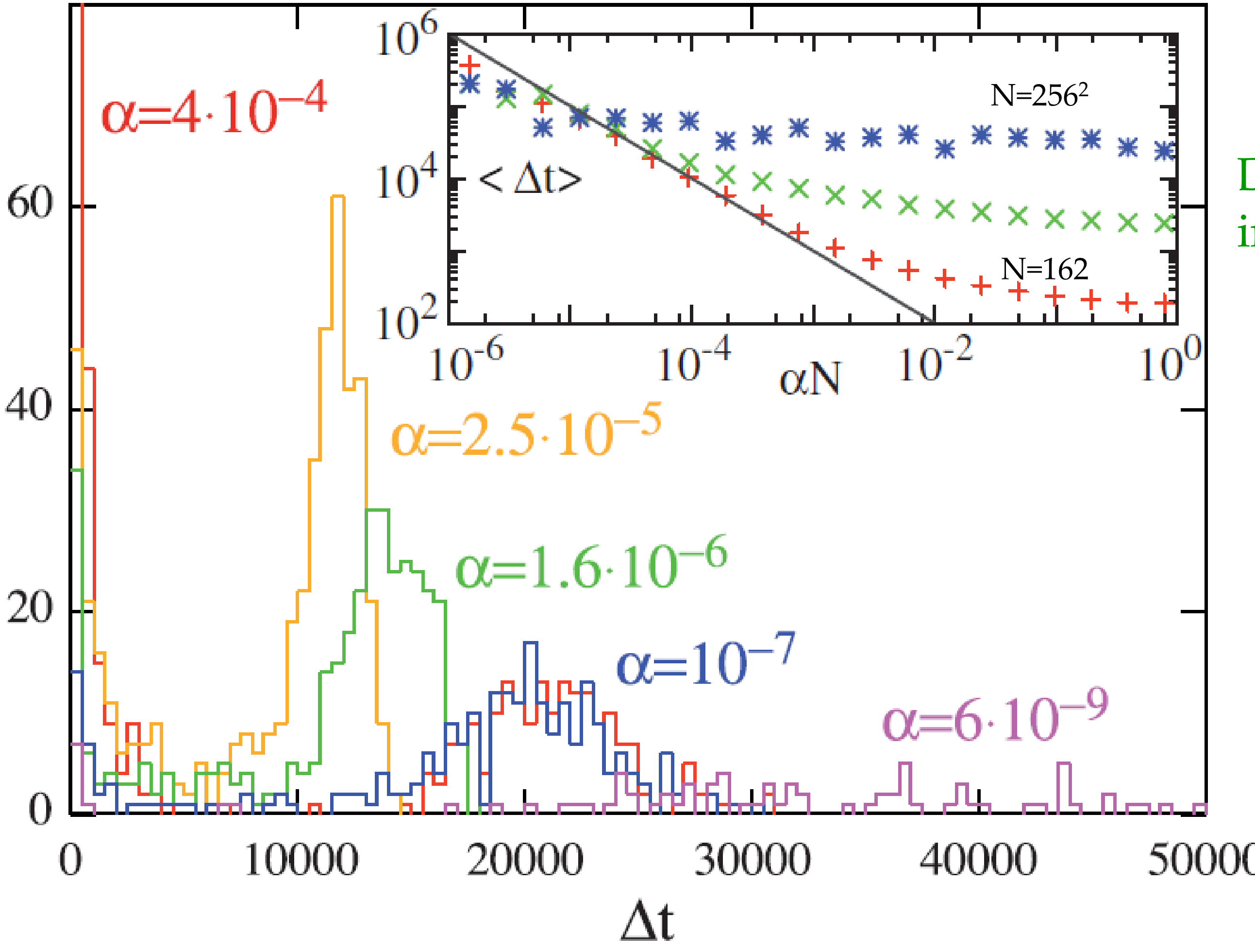
$\alpha = 0.4 \times 10^{-6}$

$\alpha = 25 \times 10^{-6}$

$\alpha = 400 \times 10^{-6}$

Nearly constant temporal ‘epoch’ independent of α

Waiting times between shifts of dominant state



Diagonal: Individual innovations $t = 1/\alpha N$

N= 128 x 128

Number of visited sites

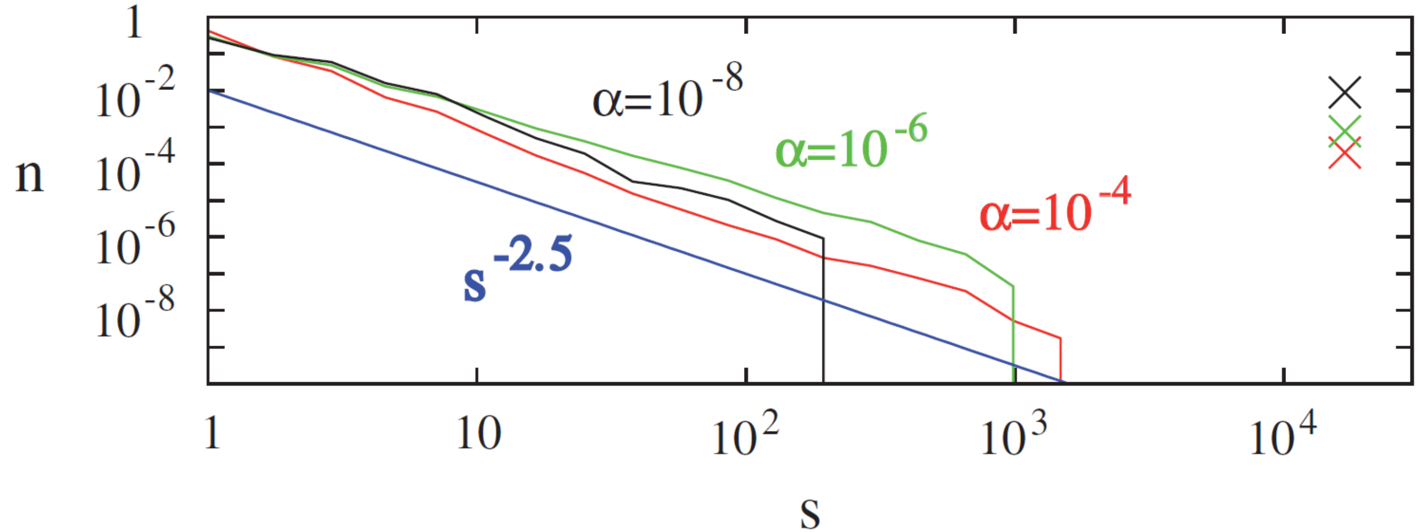


FIG. 4 (color). Distribution of the number of sites s that a particular idea visits during its life span for different values of α. Scaling $\sim 1/s^{2.5}$ for comparison (blue line). Note the gap between the main part of the distribution and the bins counting the ideas with system-wide sweeps (crosses). System size is $N = 128 \times 128$.

Conclusions

- Interplay between dominance of concept vs. inability of defending itself → Vulnerability against competing concepts !
- Takeover a chaotic process with multiple new competing ideas
- Existing paradigm eroded in a ‘pre-paradigm’ phase (Kuhn)
- Takeover on much shorter time scale than the decline

Thomas Kuhn: 5 Paradigm phases

Phase 1- *Pre-paradigm phase*, in which there is no consensus on any particular theory: Several incompatible and incomplete theories.

Phase 2- Normal Science begins, in which puzzles are solved within the context of the dominant paradigm.

Phase 3- If the paradigm proves chronically unable to account for anomalies, the community enters a crisis period.

Phase 4- Scientific revolution: Underlying assumptions of the field are reexamined and a new paradigm is established.

Phase 5- Post-Revolution: Scientists return to normal science, solving puzzles within the new paradigm.

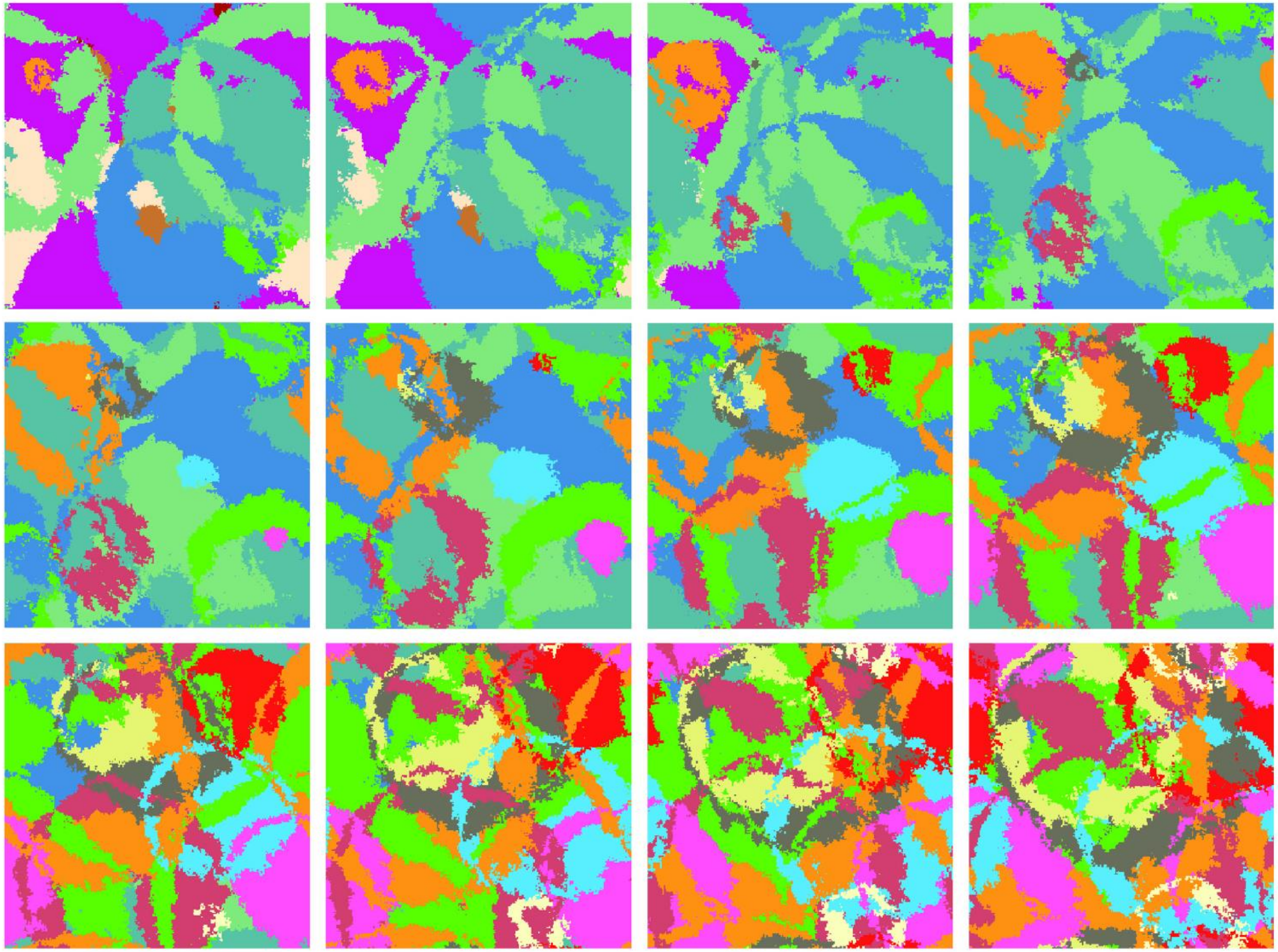
Epidemics and Immunity spreading

- Epidemic spreads ‘person to person’
- Immunization very important process, allows complex life to survive an infinity of pathogens
- Disease spreading: to neighbor before immunization

Model ingredients

- The same disease never returns to same person → Immunized against previous diseases
- A disease is transferred to a random neighbor if he/she is not immunized
- Small probability α to introduce a new disease
- An epidemics moves through the system as a wave
- Technically: N=LxL persons on a lattice, 4 neighbors, $\alpha \sim 10^{-6}$

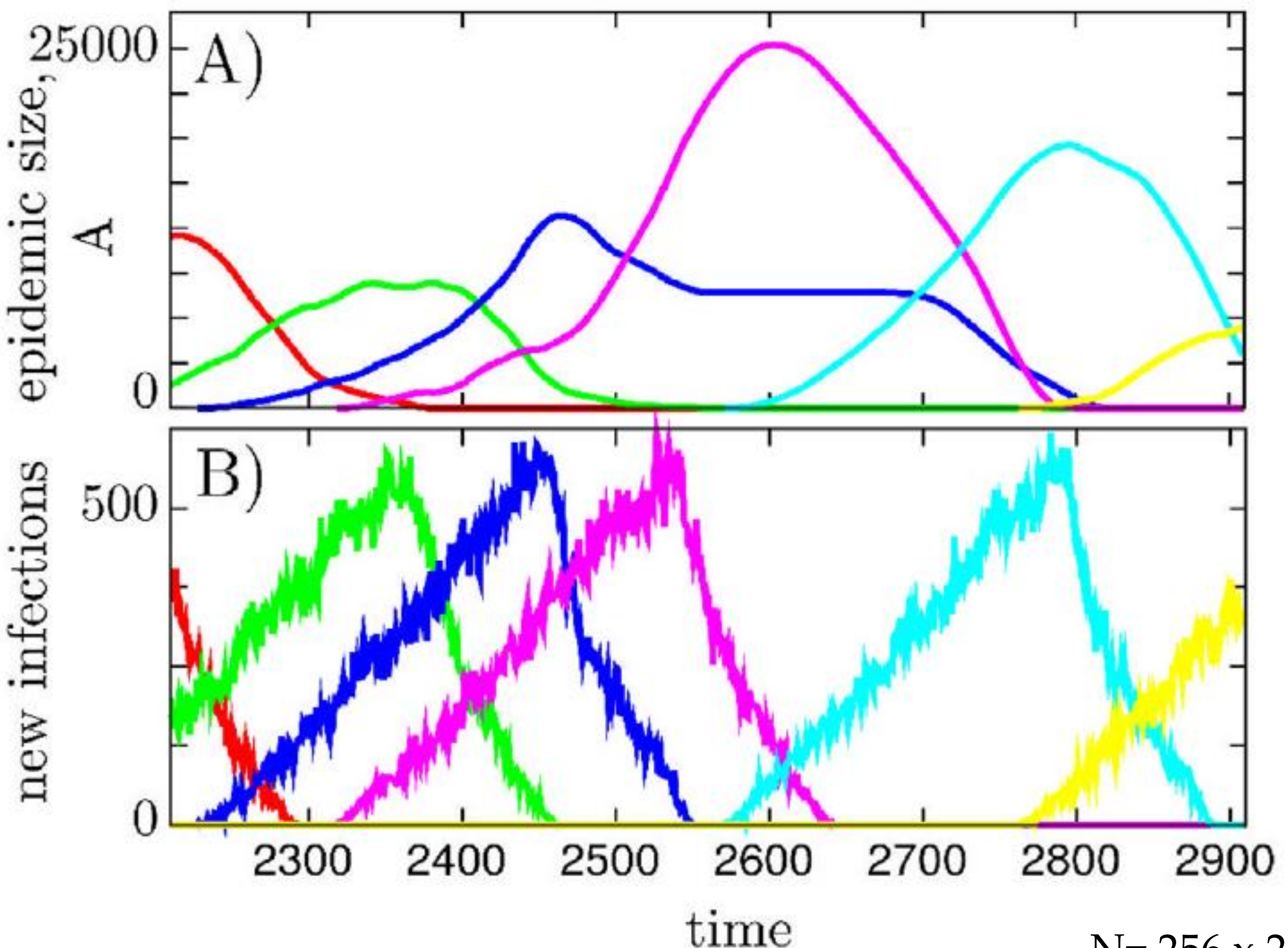
Waves of multiple epidemics



$\alpha = 4 \times 10^{-7}$

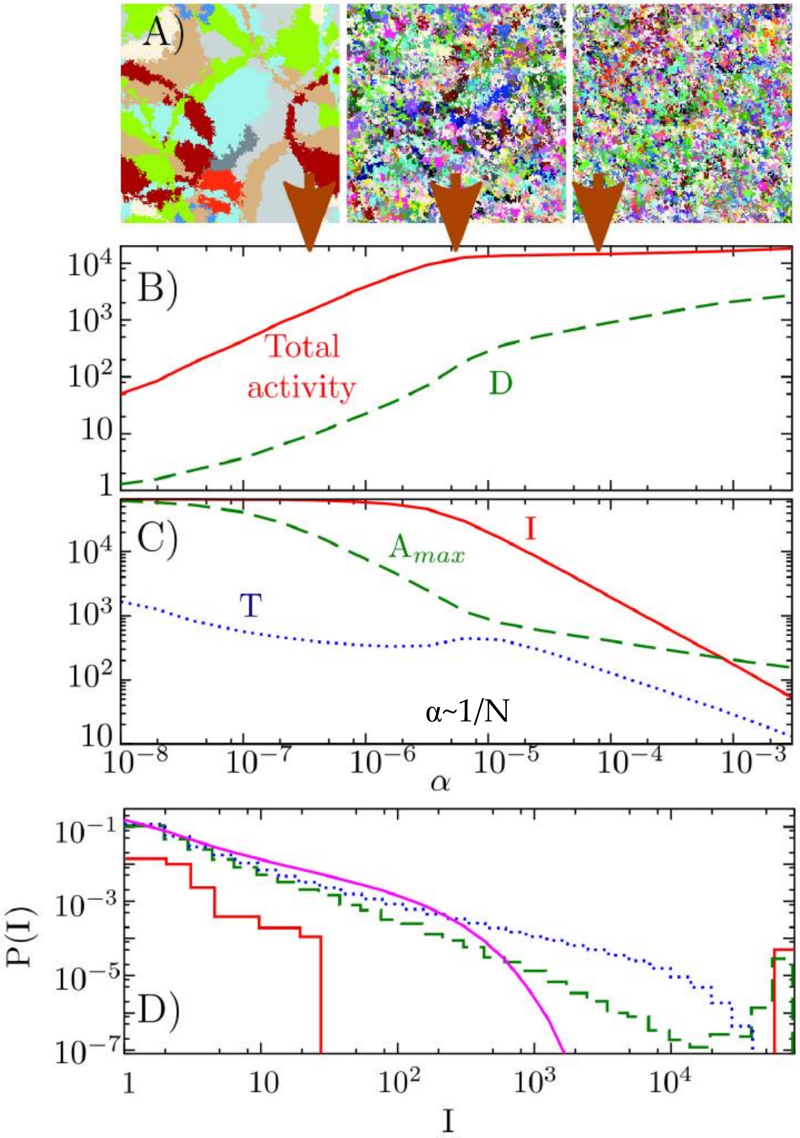
Figure 1. Dynamics of multiple epidemics. 12 consecutive snapshots of a $L = 256$ system with $\alpha = 4 \times 10^{-7}$. There are $\Delta t = 15$ updates per site between the snapshots. Note the wavefronts penetrating each other while the areas left behind the wavefronts are re-colonized from nucleation centers at the colliding fronts.
doi:10.1371/journal.pone.0013326.g001

Rise and fall of epidemics



N= 256 x 256

Steady state behavior versus mutation rate α



Snapshots (256 x 256)

New infections/time step

Diversity

Accu. immunity/disease

Max extension

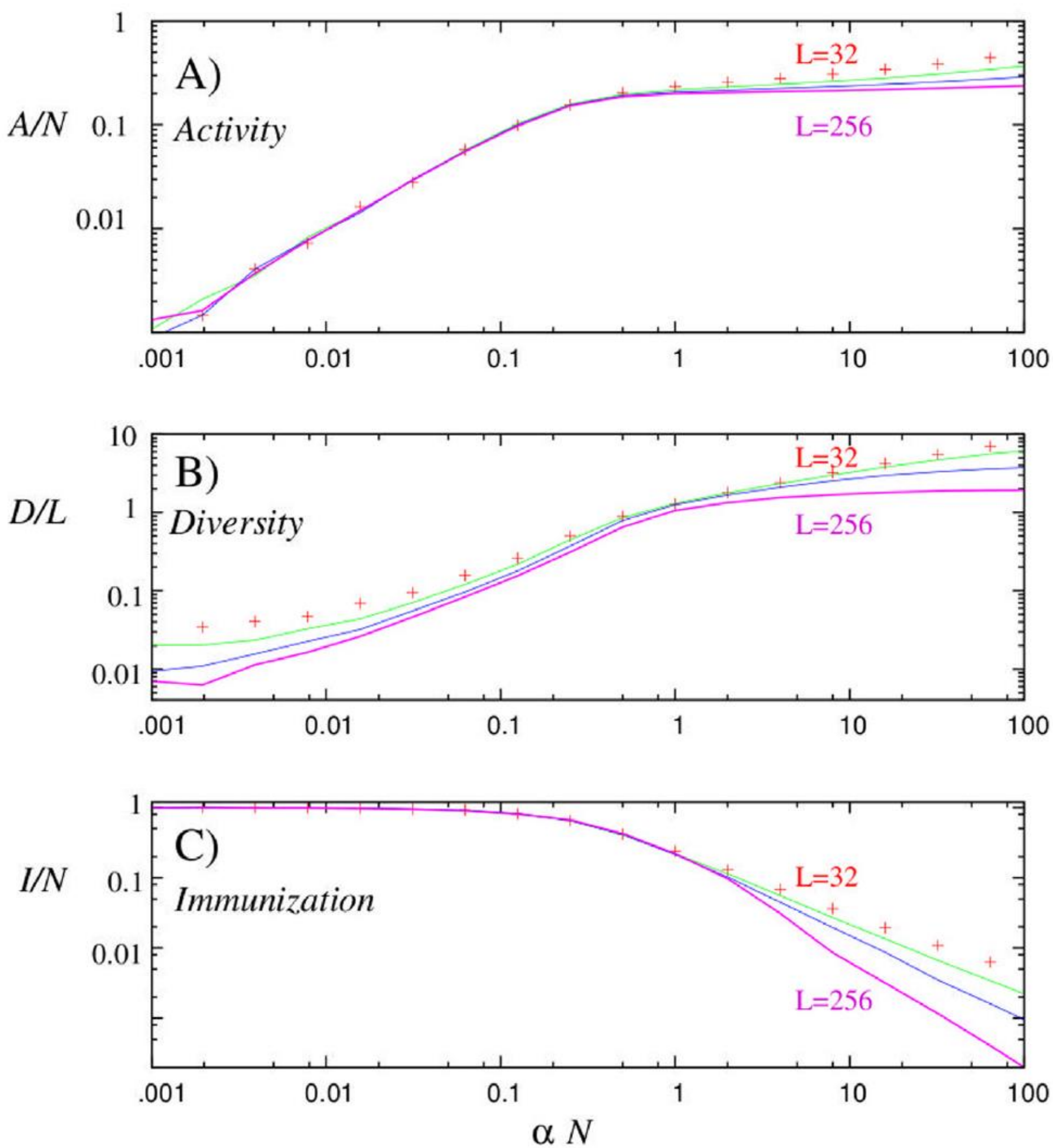
Average duration

Immunity per disease (different α)

$\alpha = 4 \times 10^{-7}$

$\alpha = 1 \times 10^{-4}$

Steady state behavior: data collapse

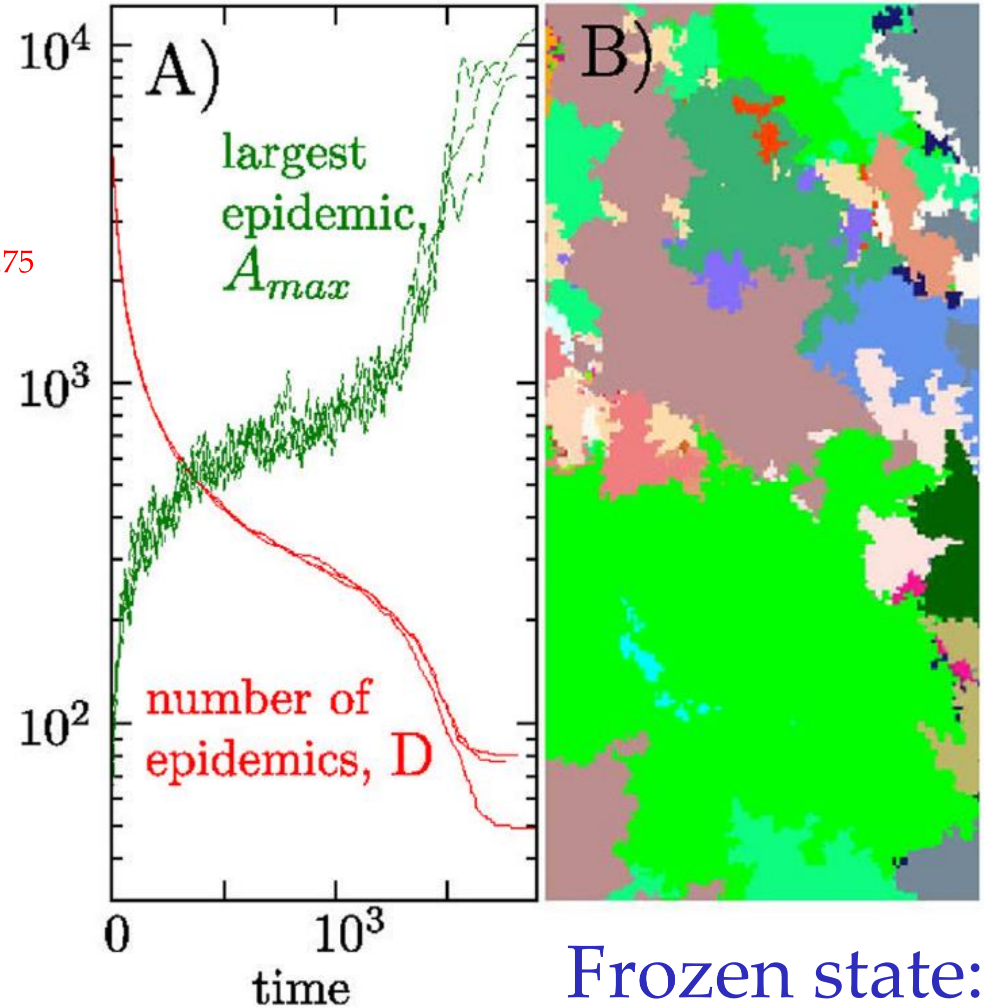


Dynamics to frozen state: NO new diseases

α=0 !

Initial infection:
One out of 6000

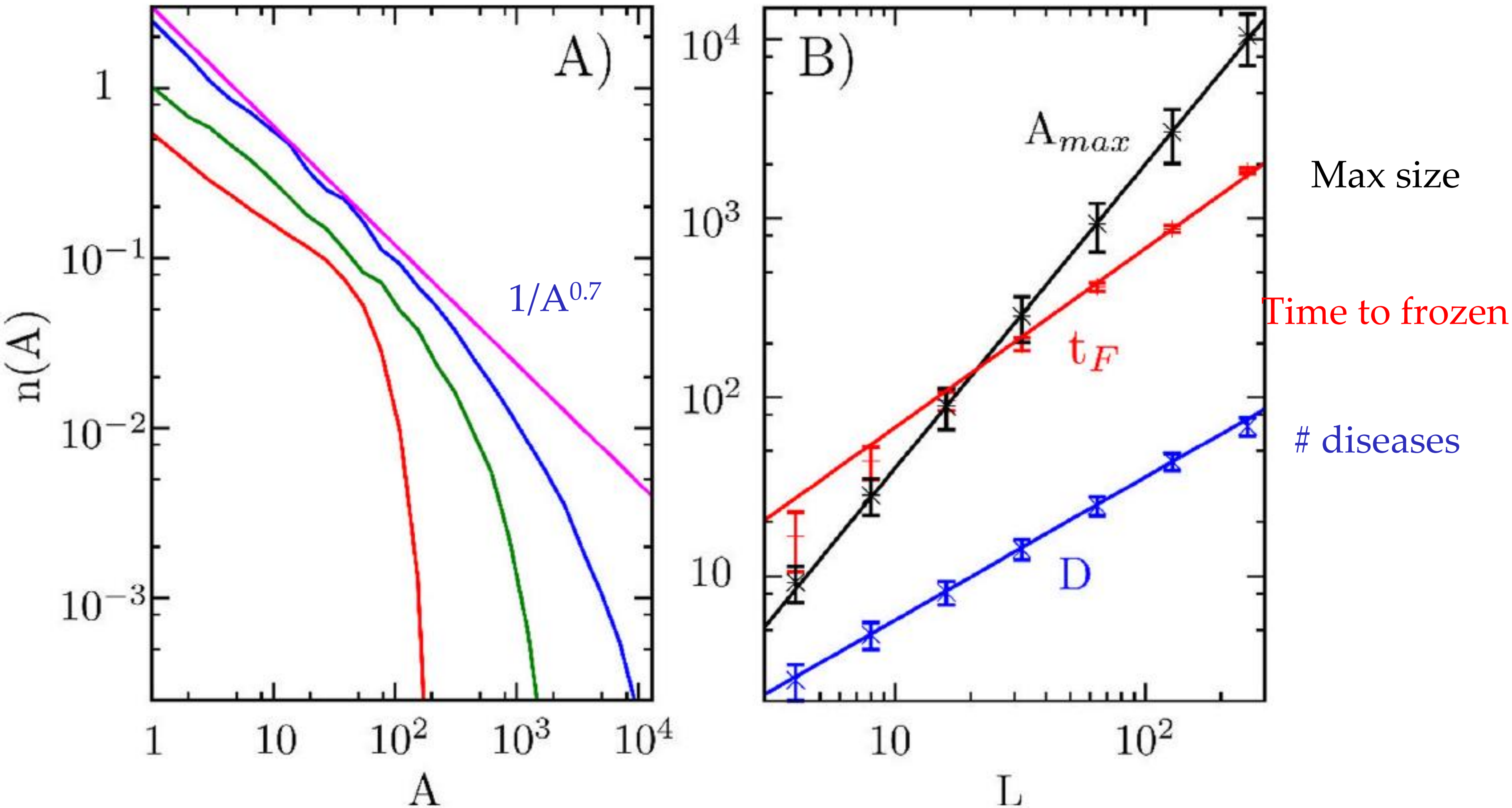
D~1/$t^{0.75}$



Frozen state: Scale free!

Frozen state scaling

L=16, 64, 256



Avg. number of epidemics of size A

Conclusions

- Very simple epidemics/immunity: epidemic waves quite realistic
- Number of diseases small: series of interpenetrating infections
- Number of diseases large: fragmented fronts
- Natural extensions: time for infection, death of host, back-mutations

We now apply kernel smoothing to give increased weight to datapoints with desired properties.

With the Gaussian kernel:

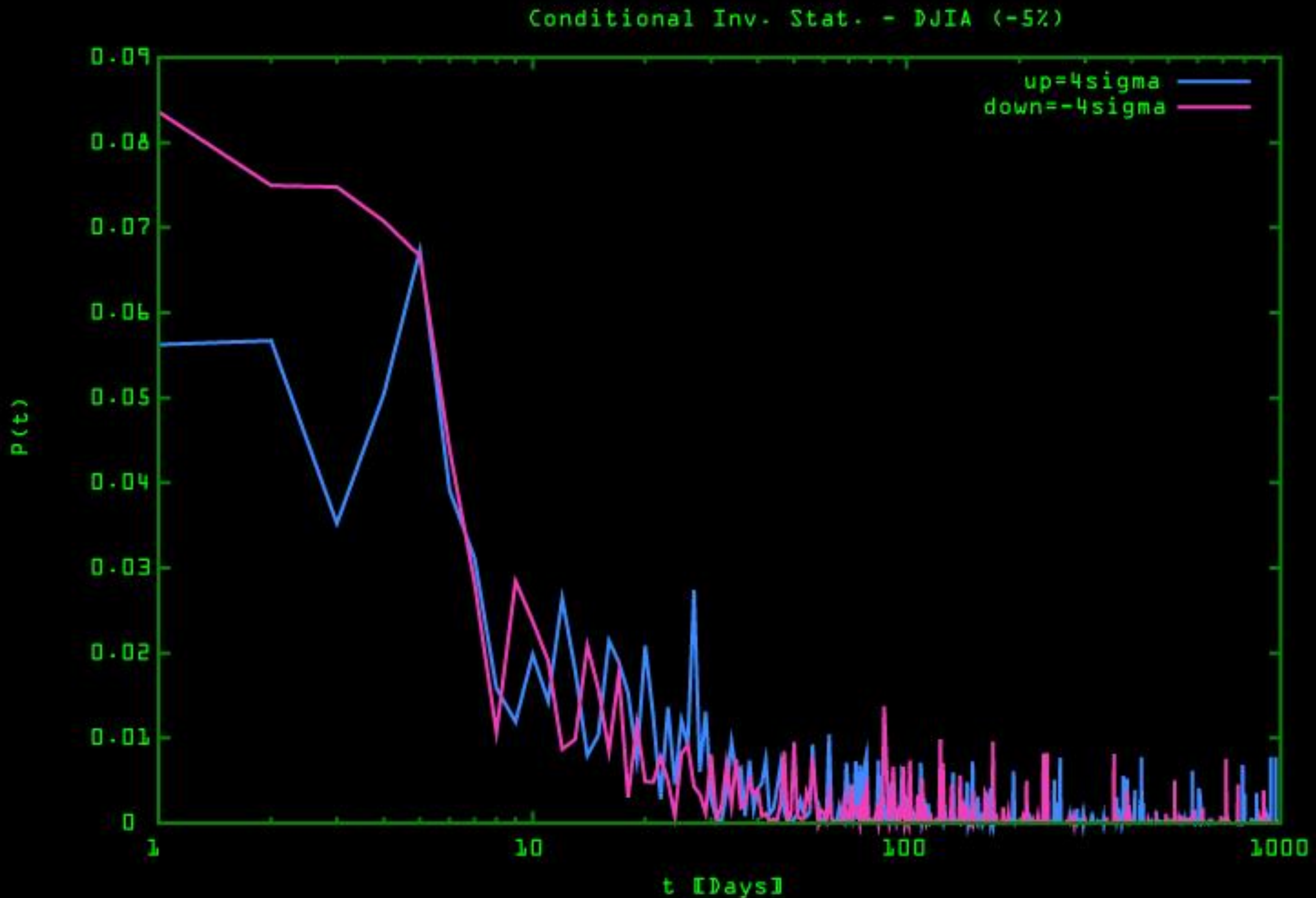
$$\kappa(c,t) \propto \exp\left[\frac{-\left(x(t)-c\right)^2}{\Sigma}\right]$$

where $\mathbf{c}$ is the condition, $\mathbf{x(t)}$ the series to condition on and $\mathbf{\Sigma}$ the covariance matrix of $\mathbf{x(t)}$.

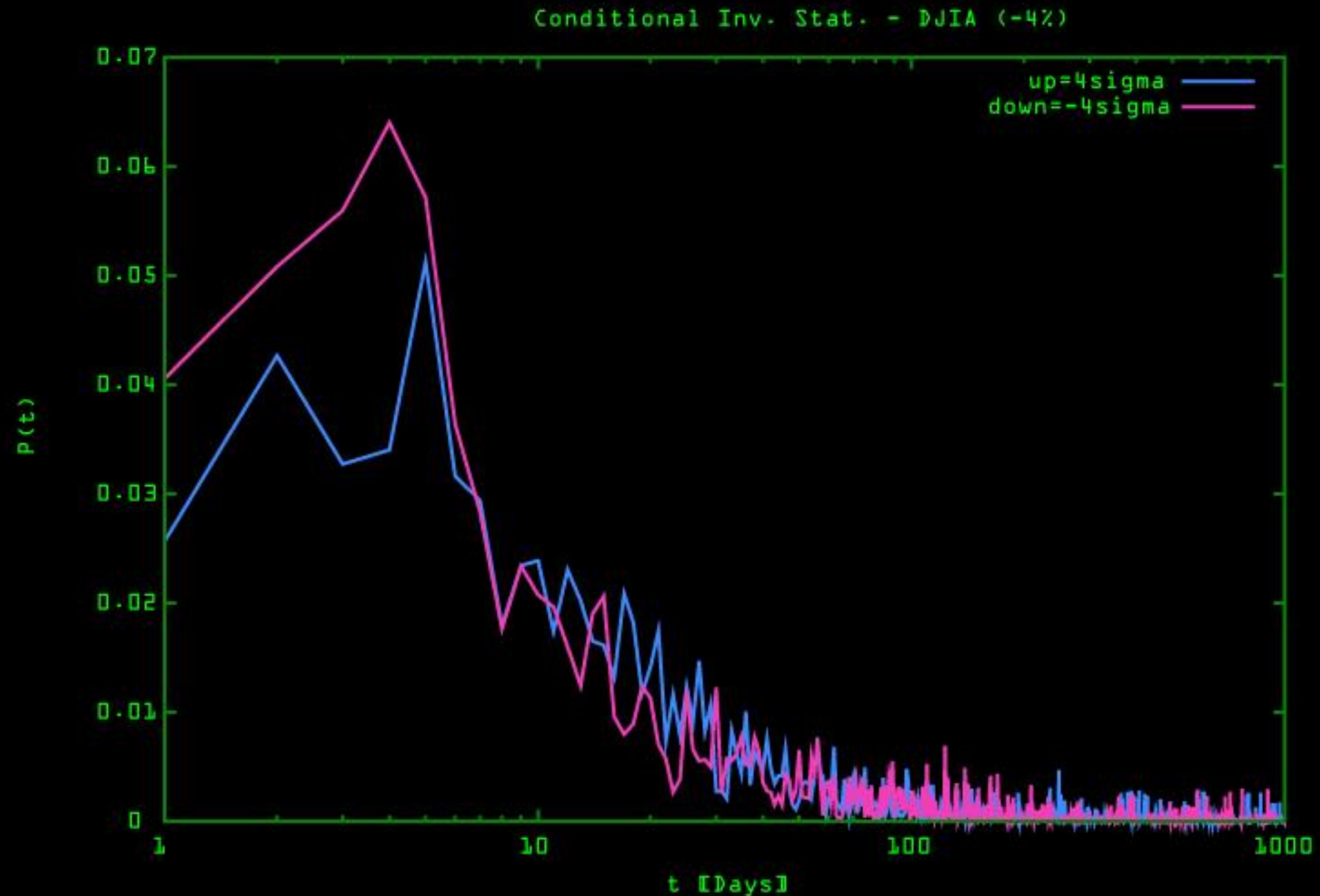
The kernel smoothing approach can be used to give different weight to each datapoint. Hence one can – for example – choose to emphasize inverse statistics originating from days with return of x%.

How does the inverse statistics look, if yesterday gave a return of 3%?

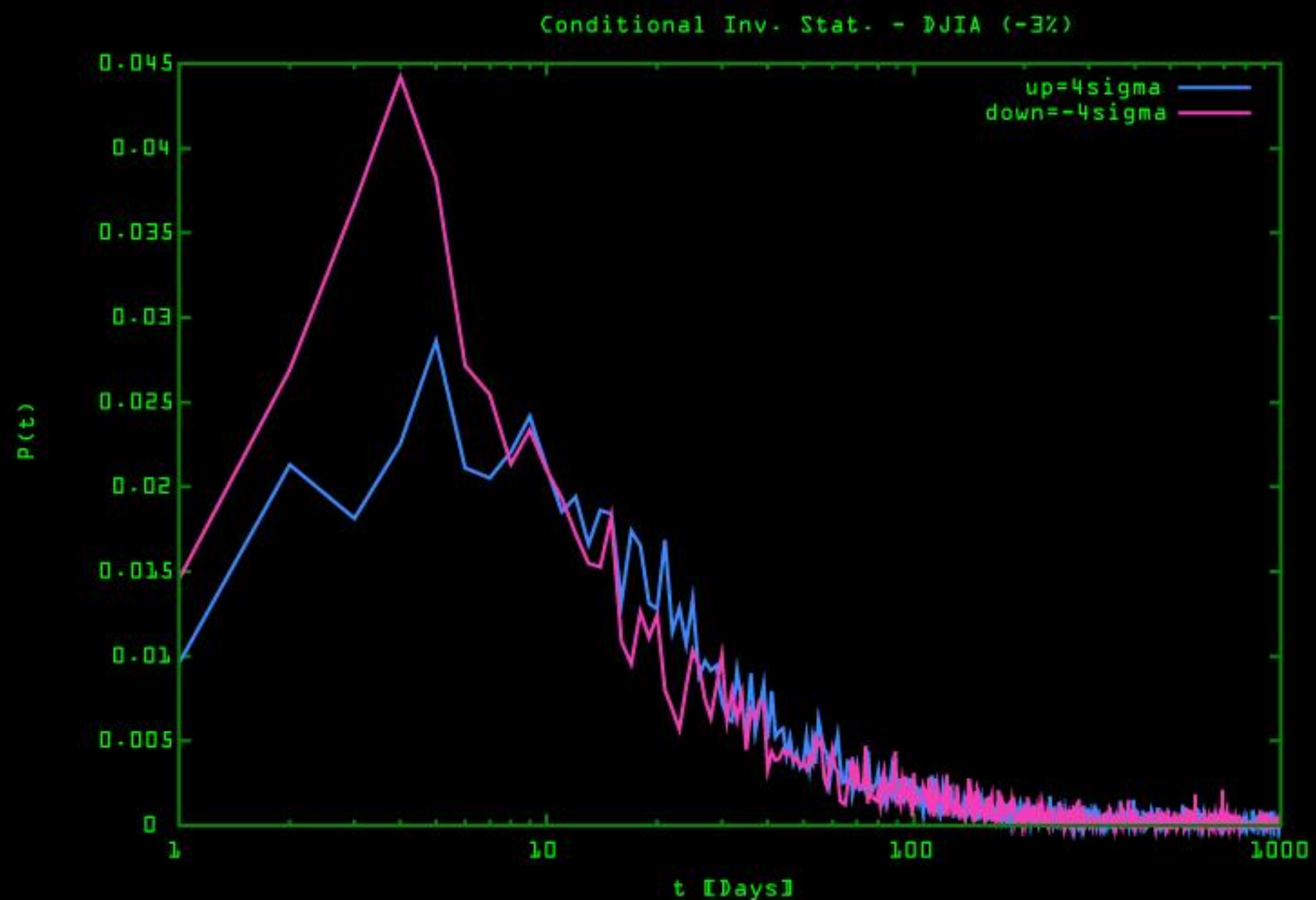
The following plots are inverse statistics with a barrier of 4 times daily volatility, conditioned on the day before the investment having a return of -5, -4,...,5 times daily volatility.



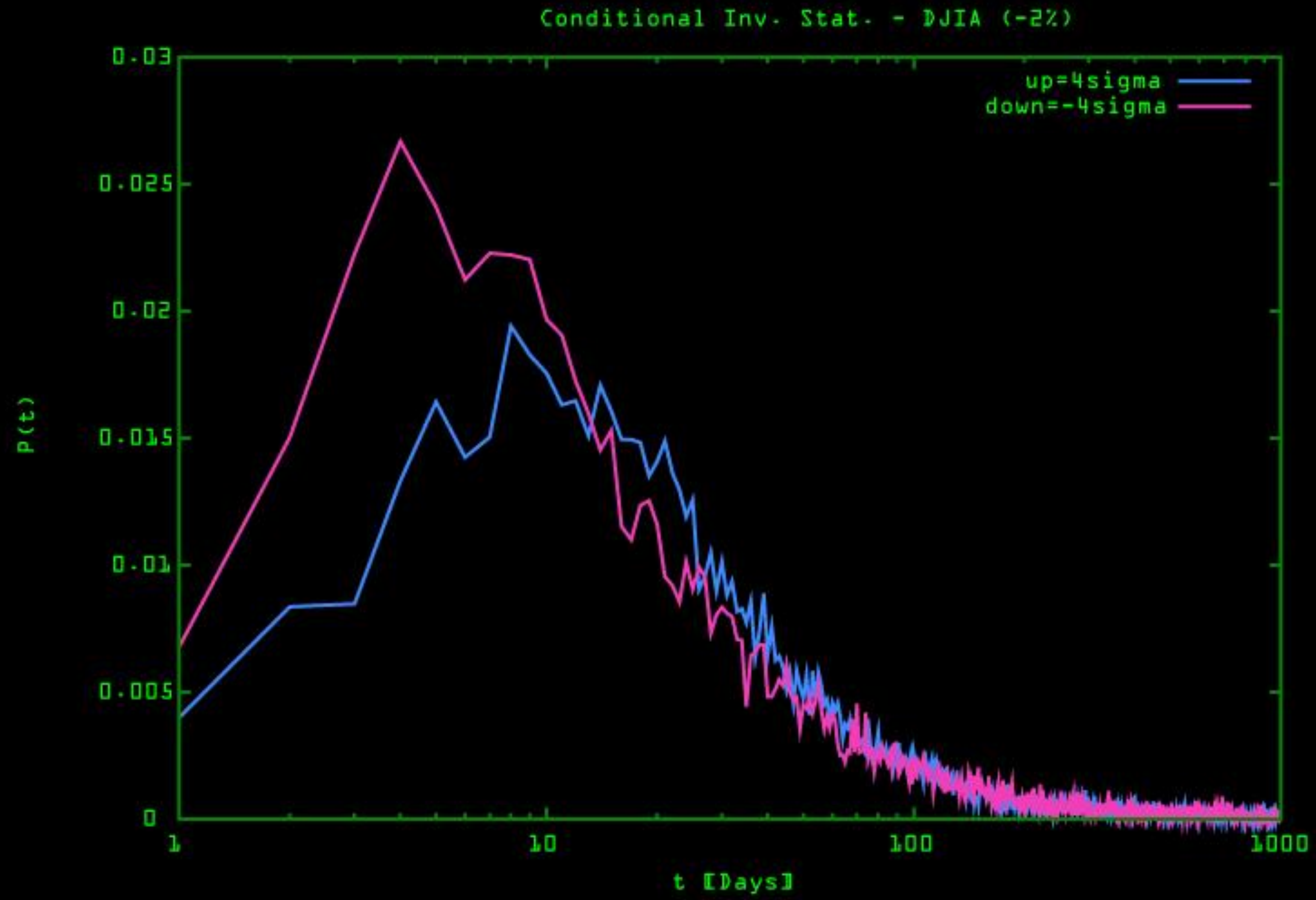
Conditional Inv. Stat. - DJIA (-5%)
up=4sigma
down=-4sigma
P(t)
t [Days]
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000



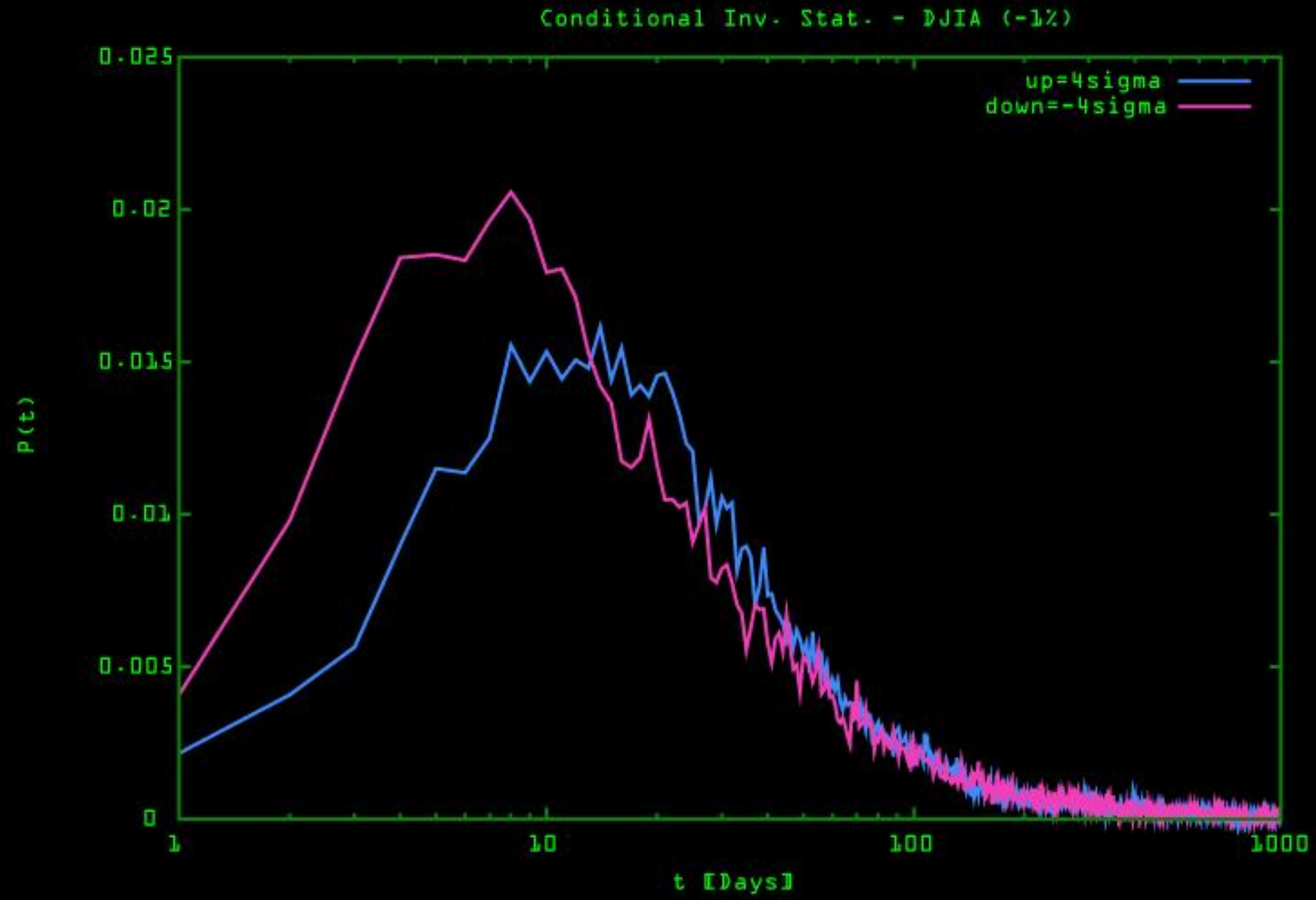
Conditional Inv. Stat. - DJIA (-4%)
up=4sigma
down=-4sigma
P(t)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000
t [Days]



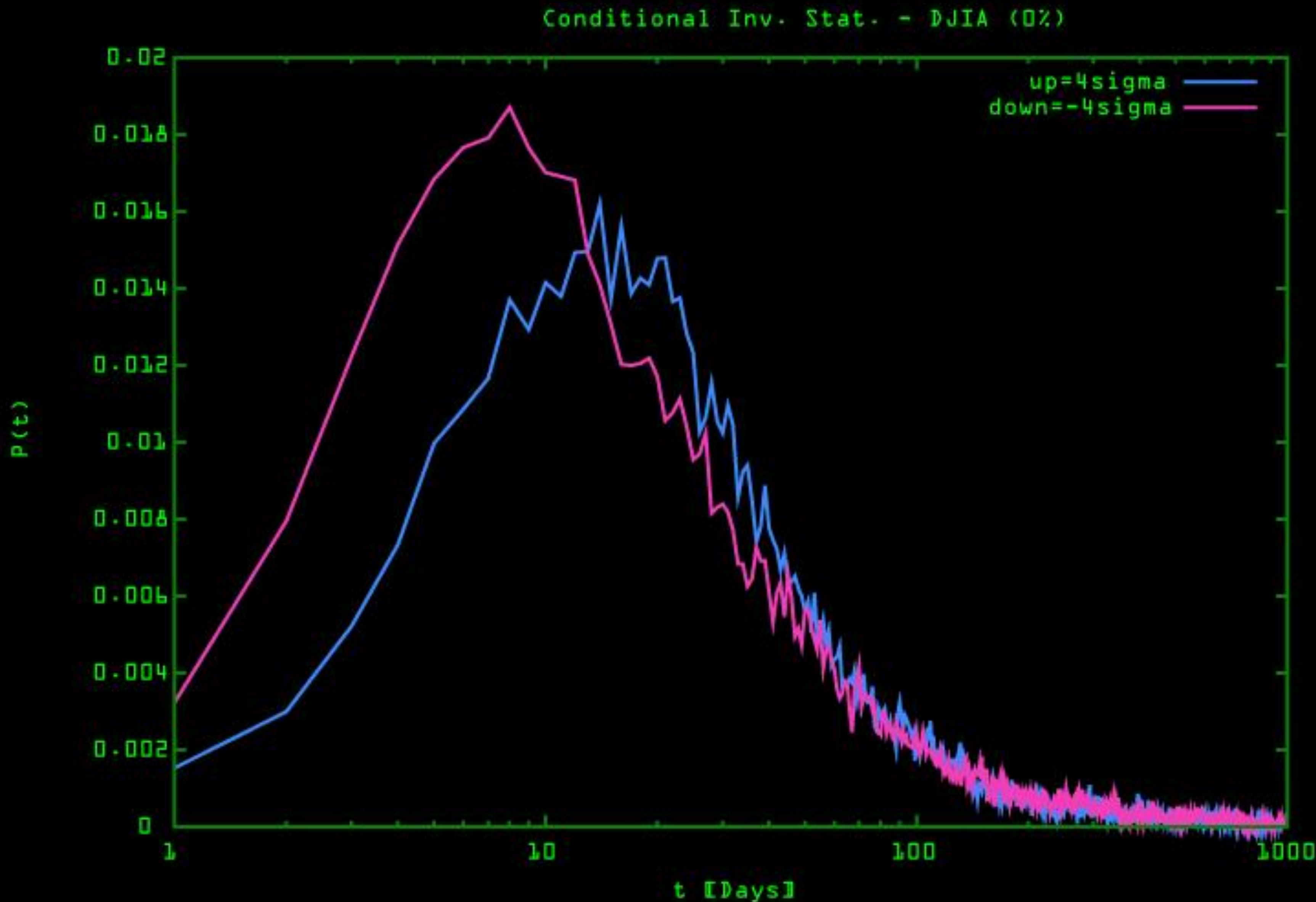
Conditional Inv. Stat. - DJIA (-3%)
up=4sigma
down=-4sigma
P(t)
t [Days]
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000



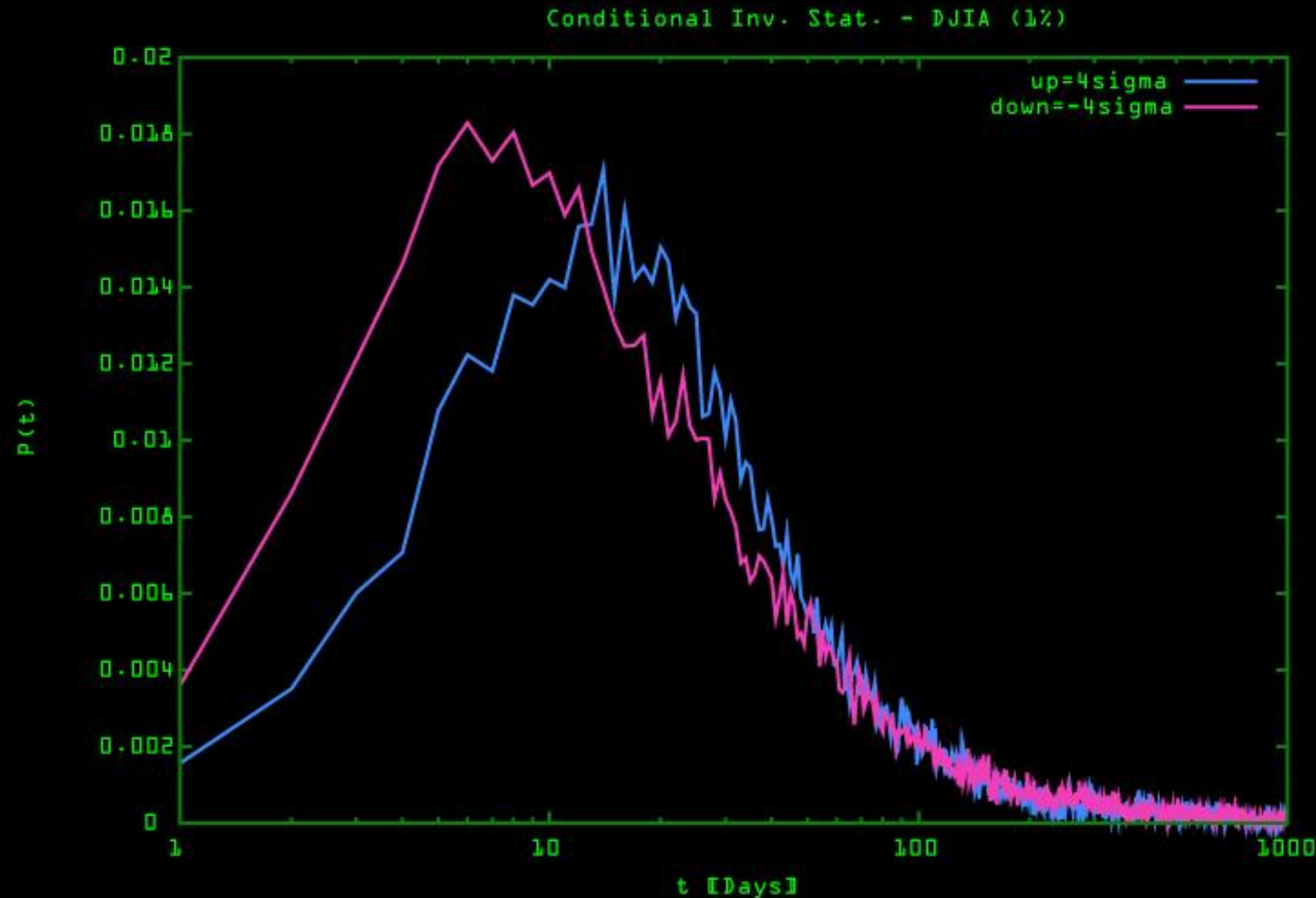
Conditional Inv. Stat. - DJIA (-2%)
up=4sigma
down=-4sigma
P(t)
0
0.005
0.01
0.015
0.02
0.025
0.03
1
10
100
1000
t [Days]



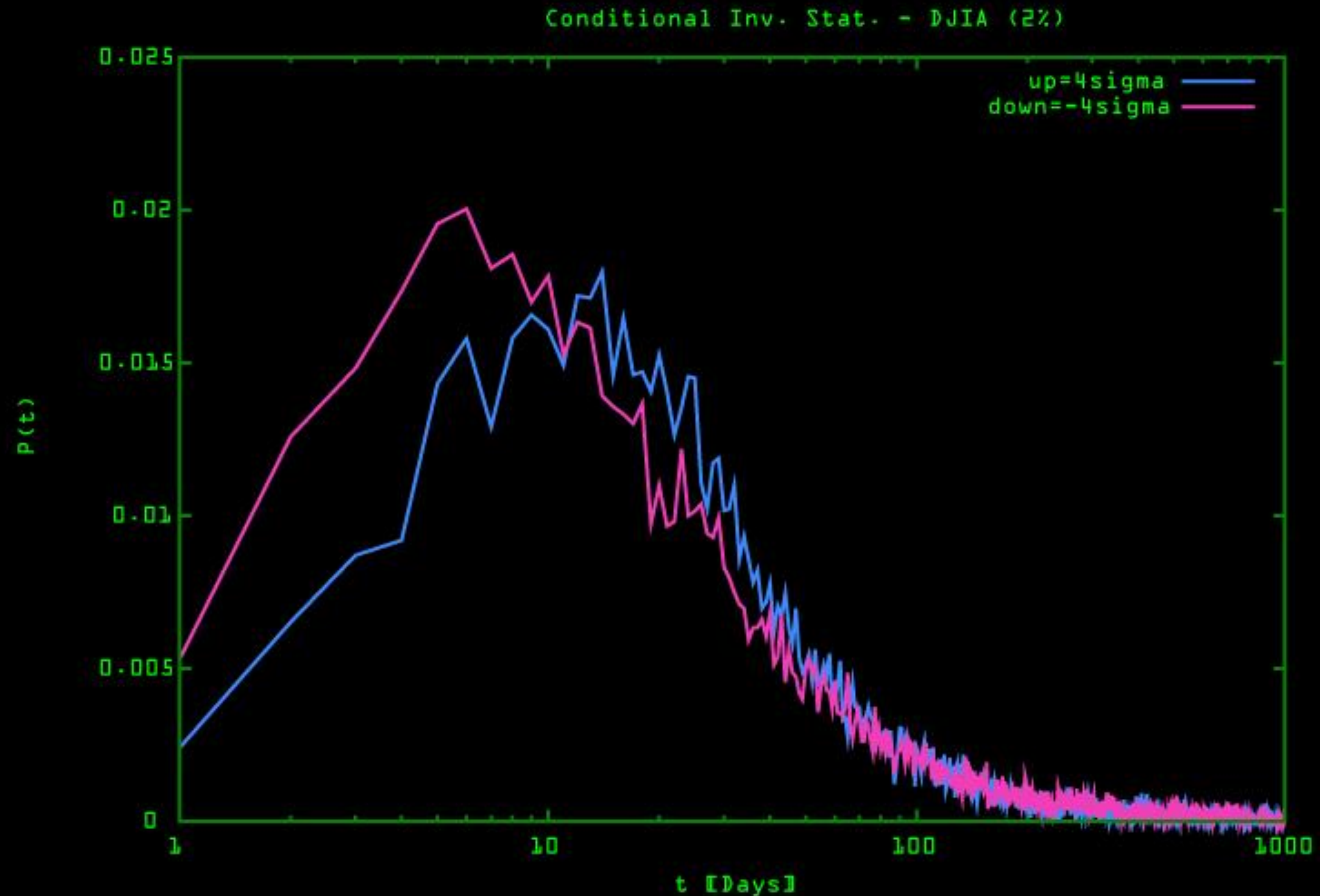
Conditional Inv. Stat. - DJIA (-1%)
up=4sigma
down=-4sigma
0.025
0.02
0.015
0.01
0.005
0
P(t)
1
10
100
1000
t [Days]

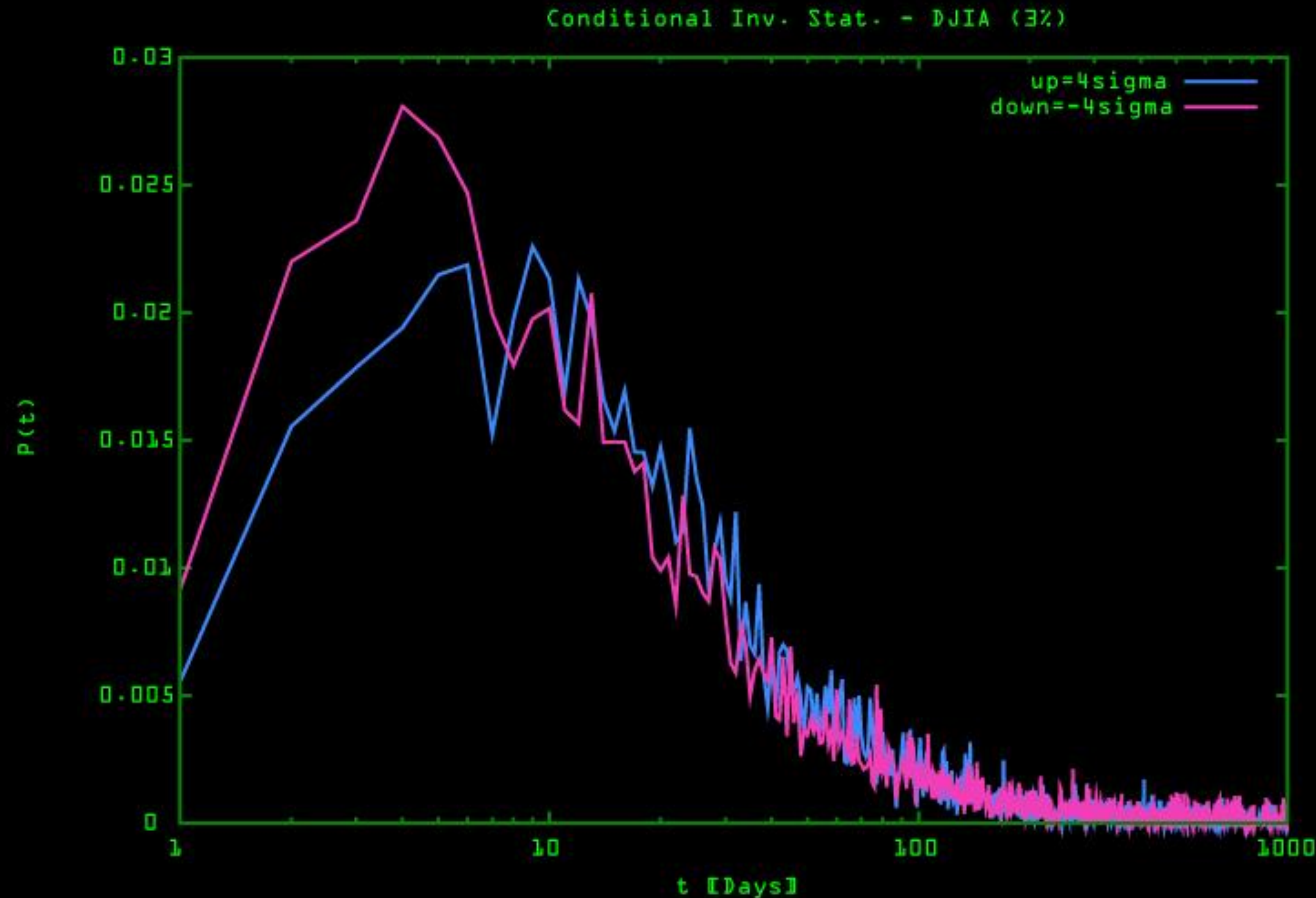


Conditional Inv. Stat. - DJIA (0%)
up=4sigma
down=-4sigma
p(t)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
10
100
1000
t [Days]

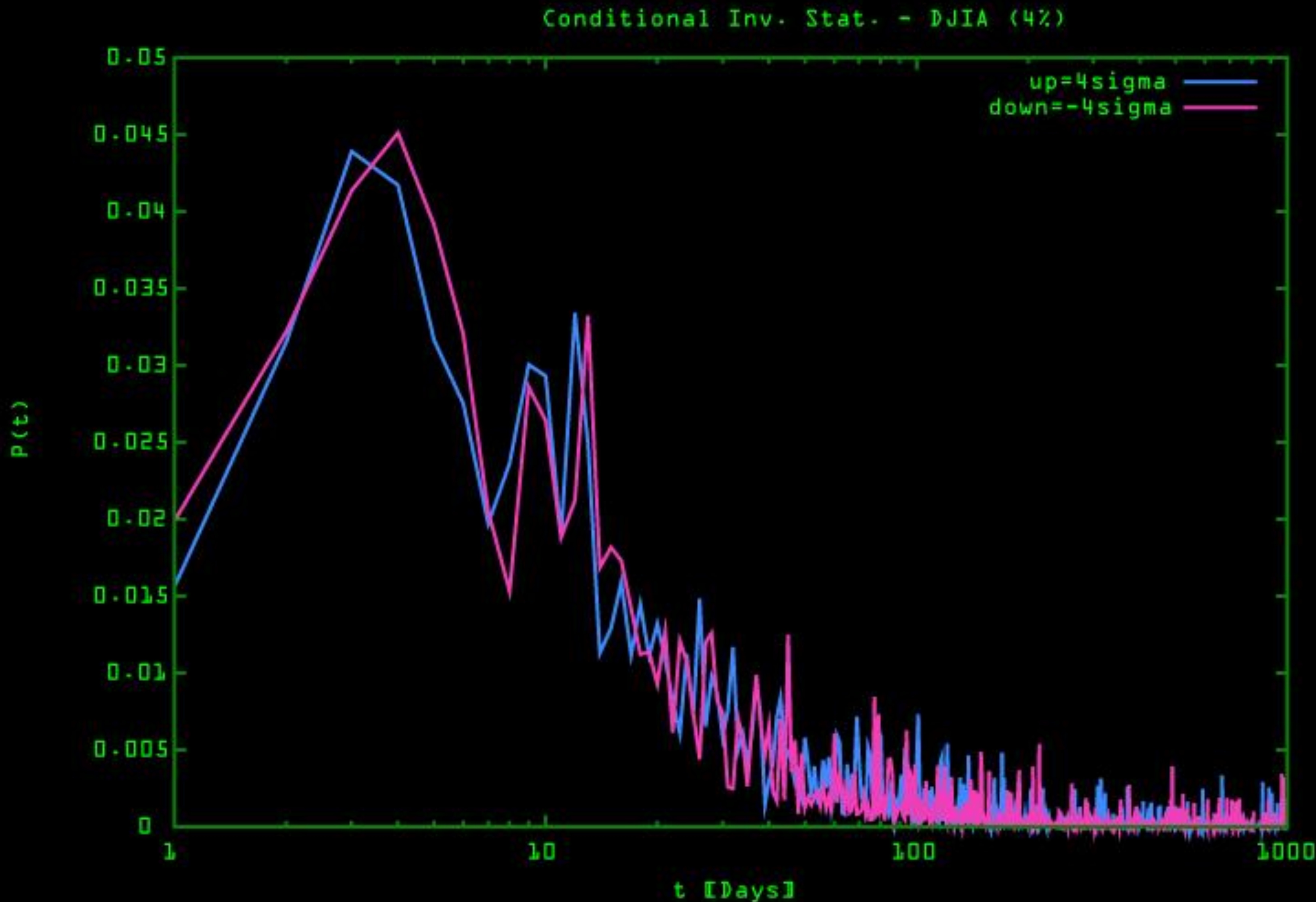


Conditional Inv. Stat. - DJIA (1%)
up=4sigma
down=-4sigma
P(t)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
10
100
1000
t [Days]

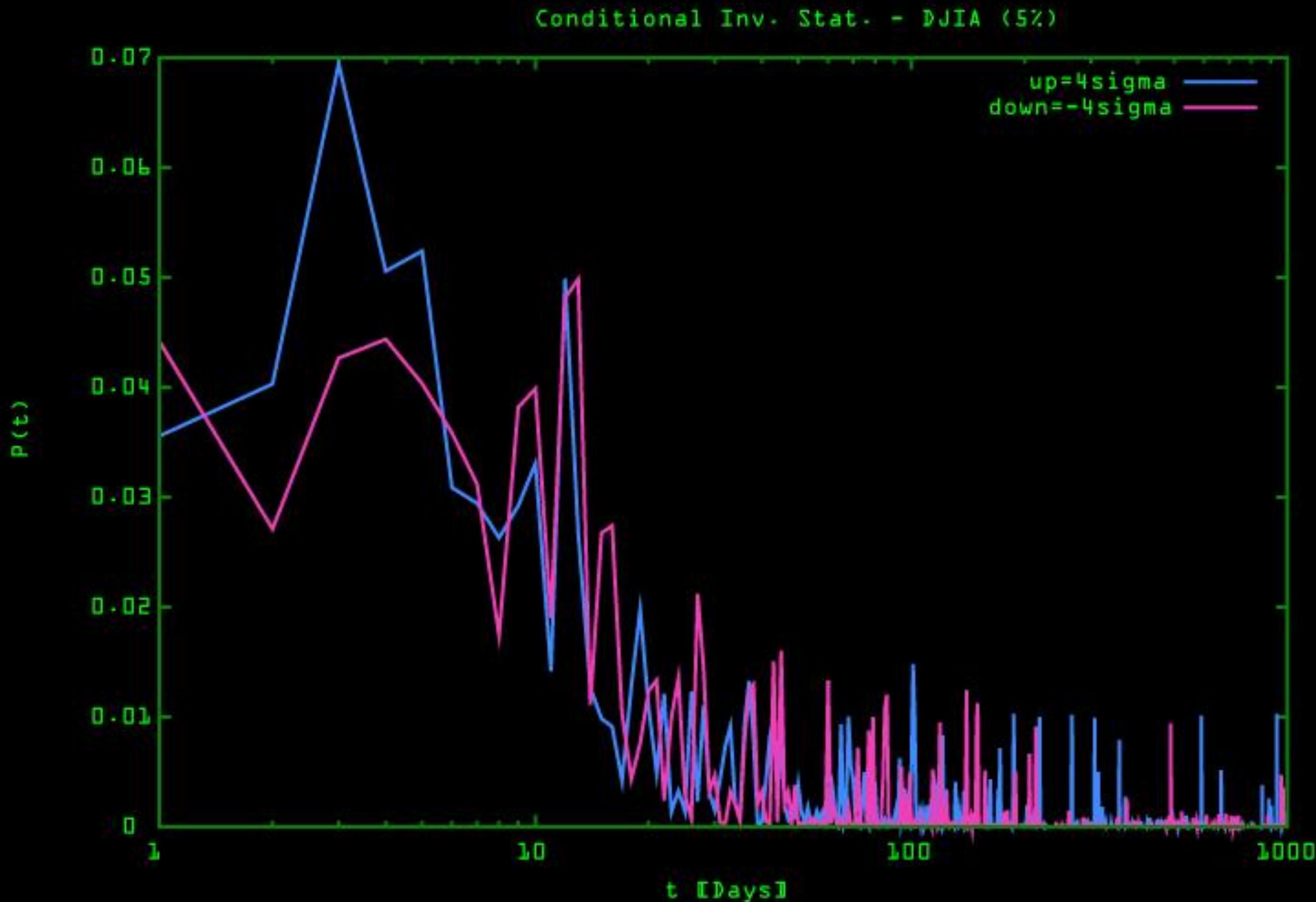

Conditional Inv. Stat. - DJIA (2%)
up=4sigma
down=-4sigma
P(t)
t [Days]
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000



Conditional Inv. Stat. - DJIA (3%)
up=4sigma
down=-4sigma
P(t)
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000
t [Days]

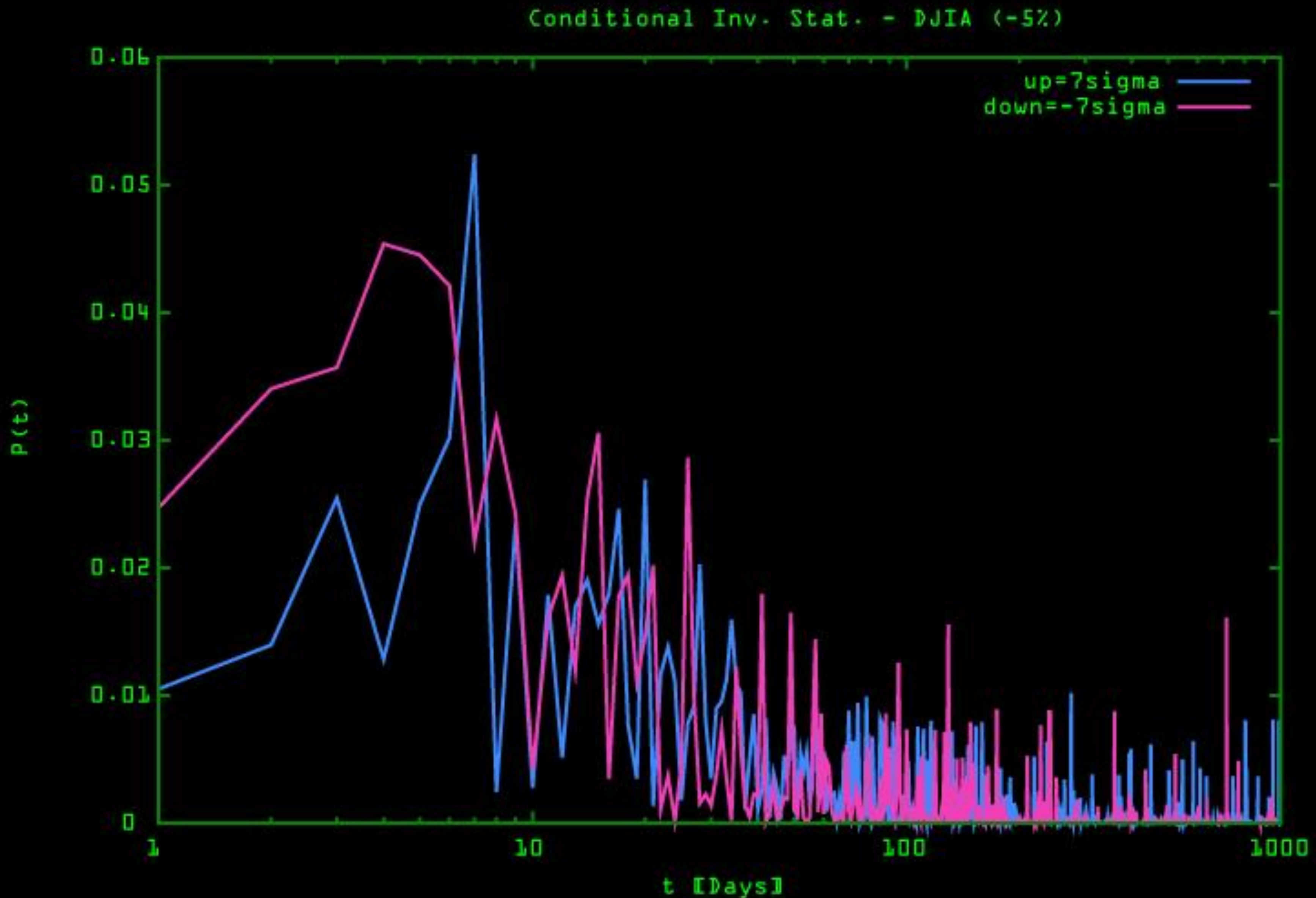


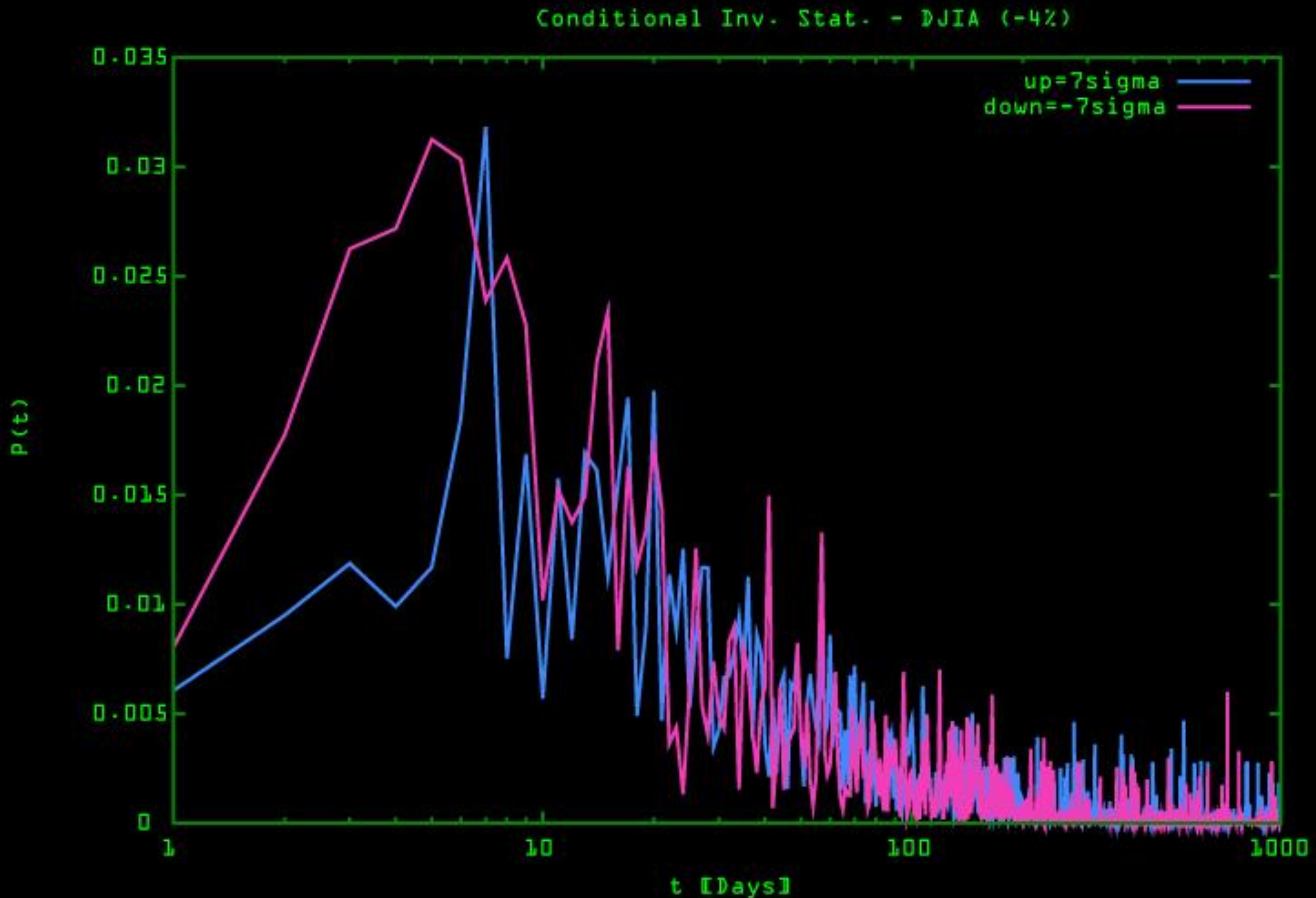
Conditional Inv. Stat. - DJIA (4%)
up=4sigma
down=-4sigma
P(t)
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000
t [Days]



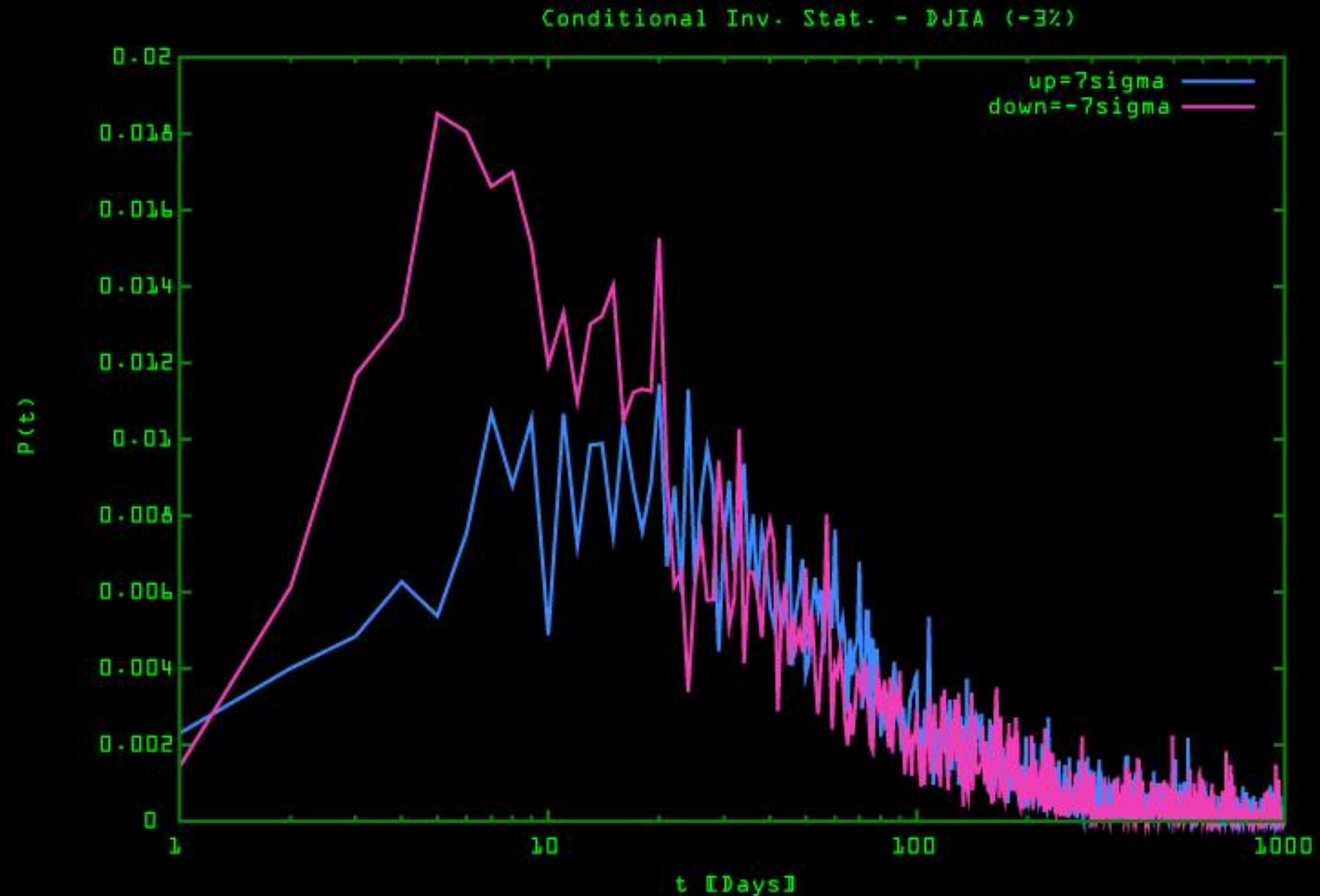
Conditional Inv. Stat. - DJIA (5%)
up=4sigma
down=-4sigma
P(t)
t [Days]
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000

The following plots are inverse statistics with a barrier of 7 times daily volatility, conditioned on the day before the investment having a return of -5, -4,...,5 times daily volatility.

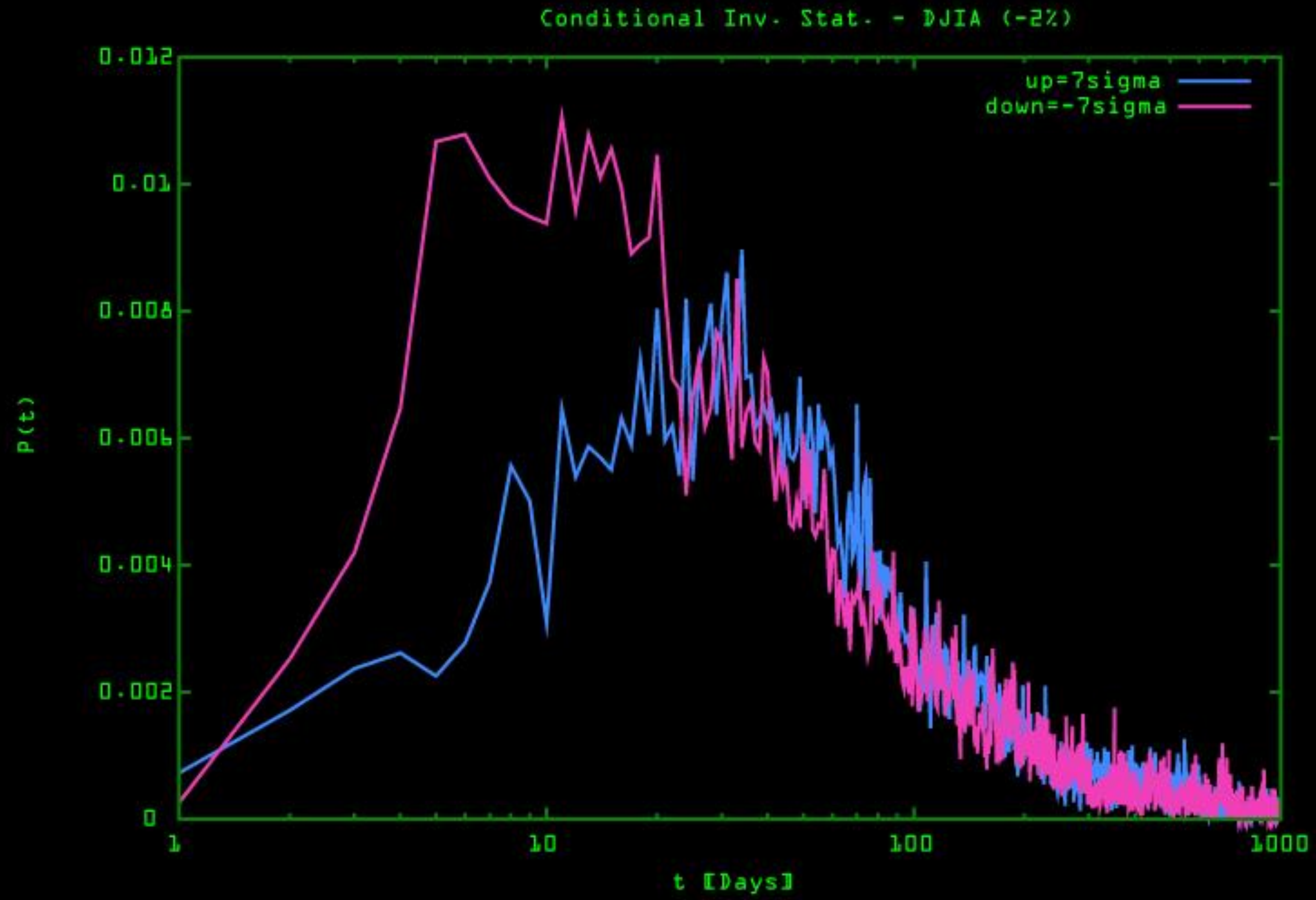

Conditional Inv. Stat. - DJIA (-5%)
up=7sigma
down=-7sigma
P(t)
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000
t [Days]



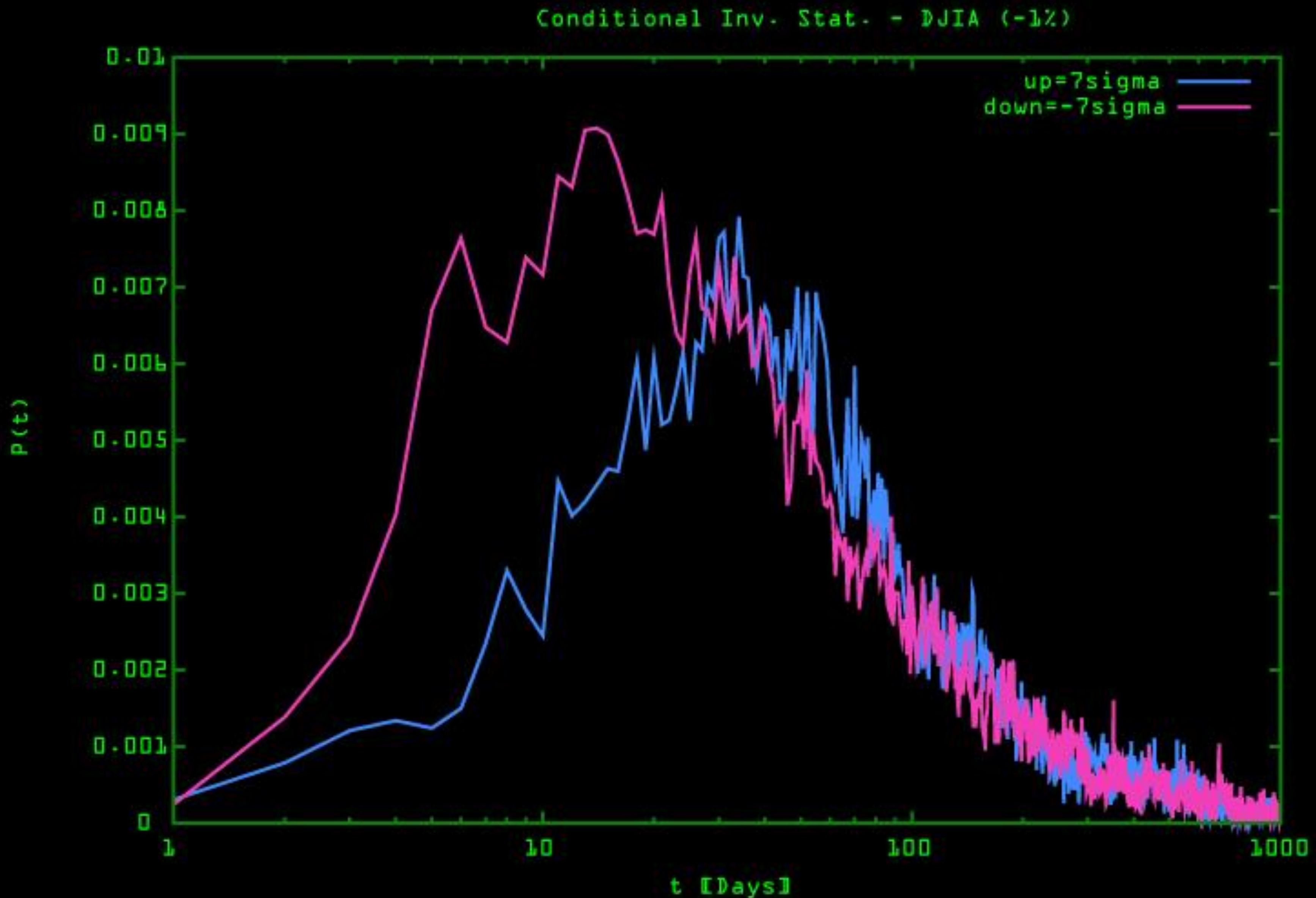
Conditional Inv. Stat. - DJIA (-4%)
up=7sigma
down=-7sigma
P(t)
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000
t [Days]

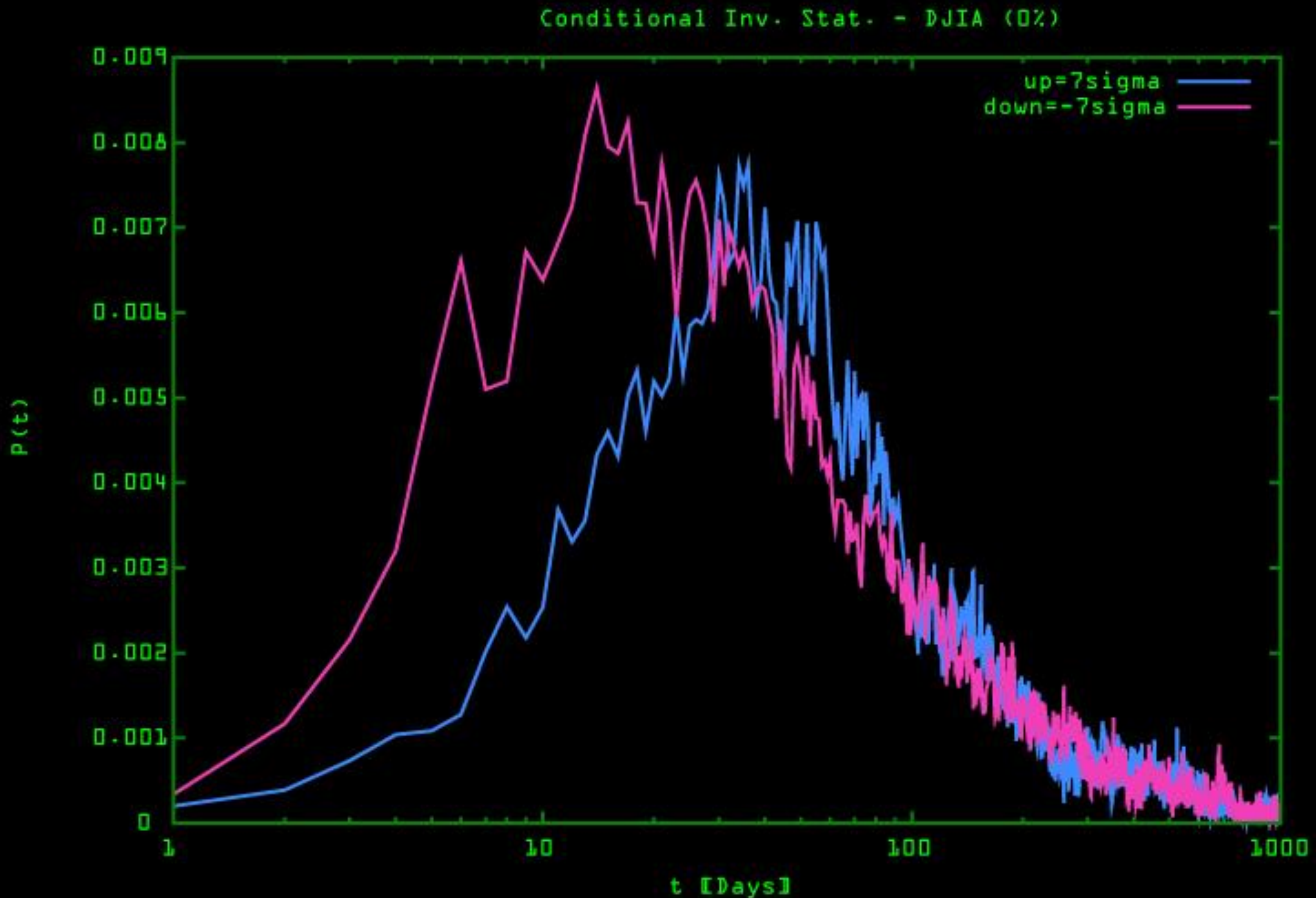


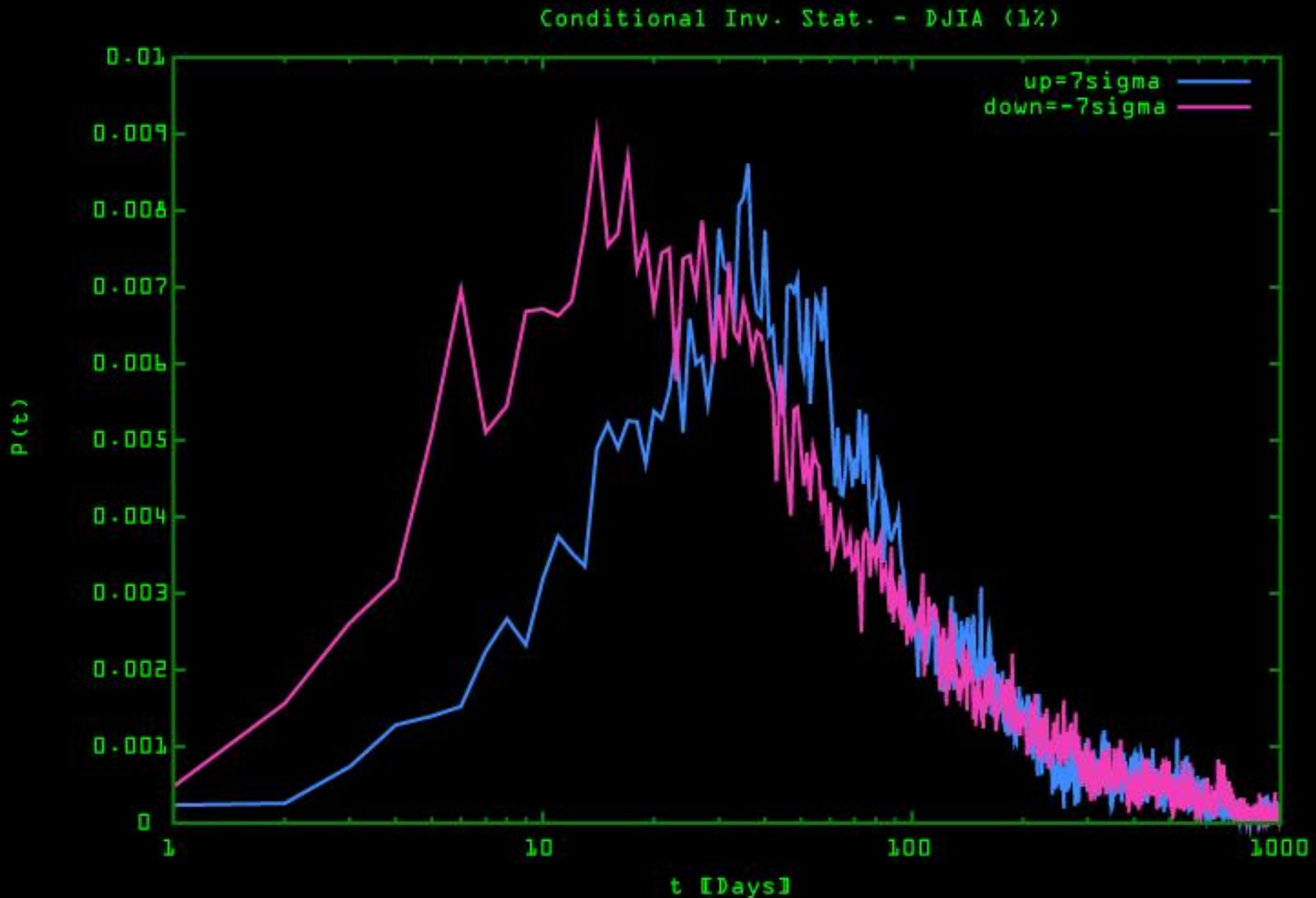
Conditional Inv. Stat. - DJIA (-3%)
up=7sigma
down=-7sigma
P(t)
t [Days]
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
10
100
1000



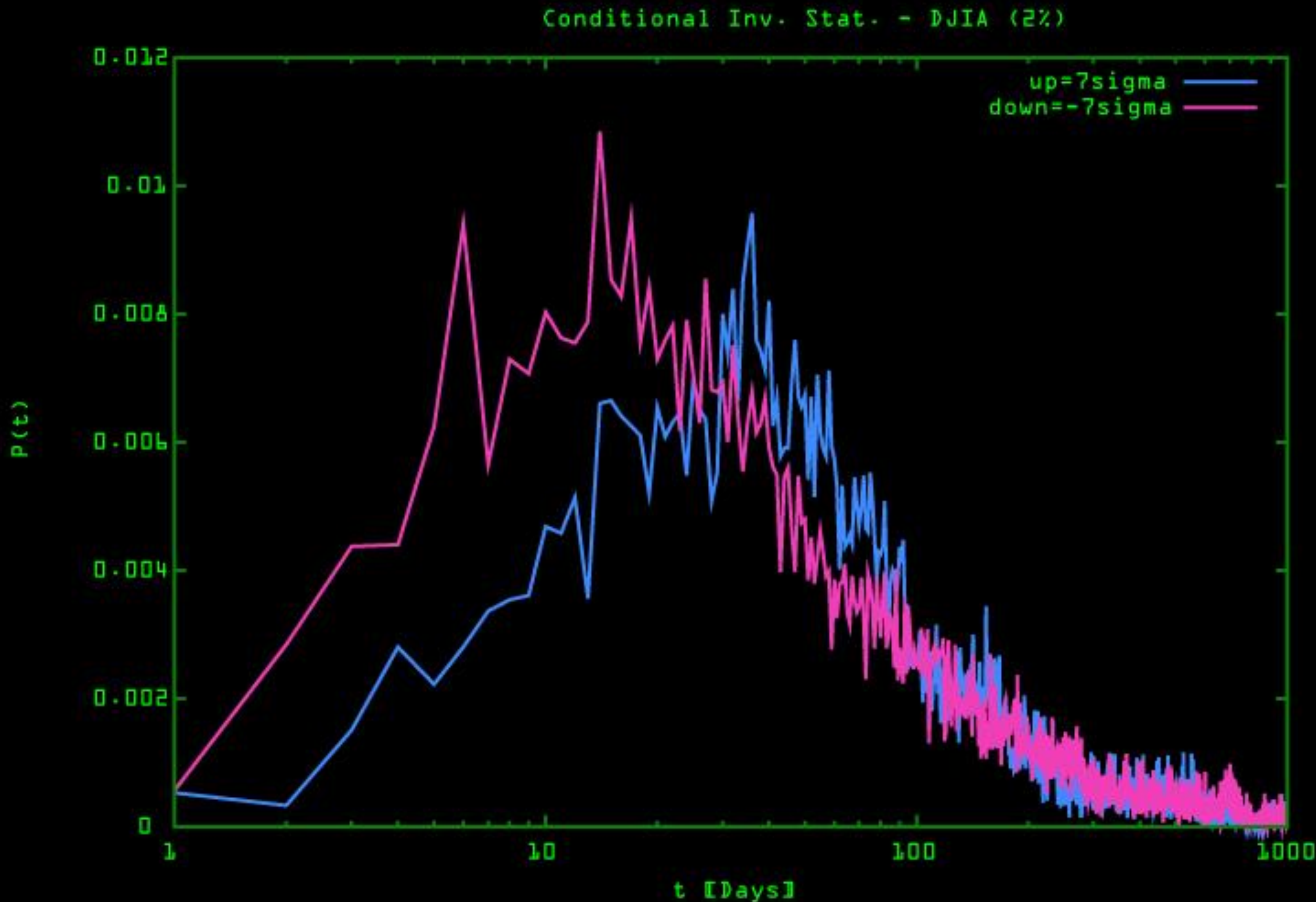
Conditional Inv. Stat. - DJIA (-2%)
up=7sigma
down=-7sigma
P(t)
t [Days]
0.012
0.01
0.008
0.006
0.004
0.002
0
1
10
100
1000


Conditional Inv. Stat. - DJIA (-1%)
up=7sigma
down=-7sigma
P(t)
t [Days]
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
1
10
100
1000

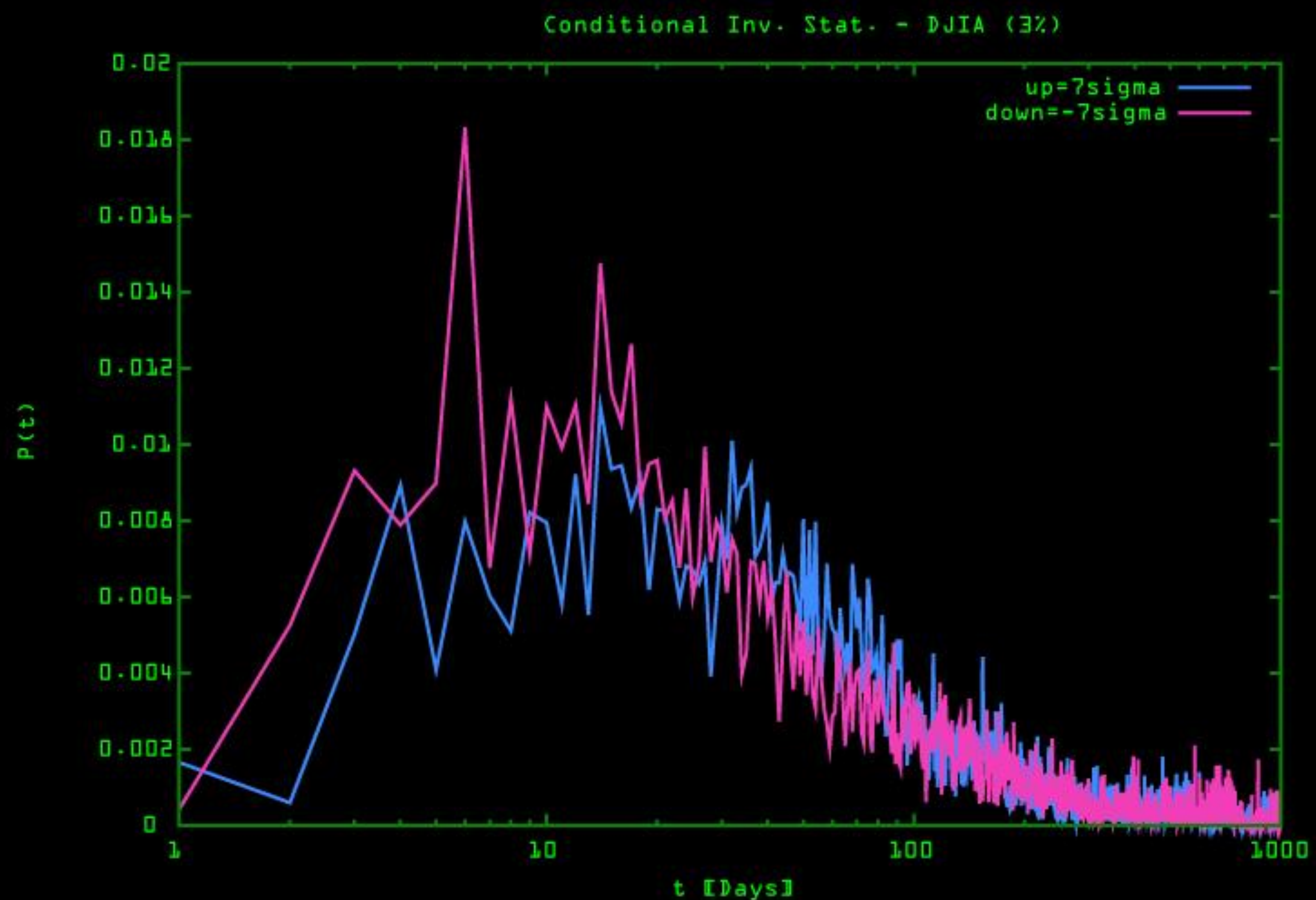

Conditional Inv. Stat. - DJIA (0%)
up=7sigma
down=-7sigma
P(t)
t [Days]
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
1
10
100
1000



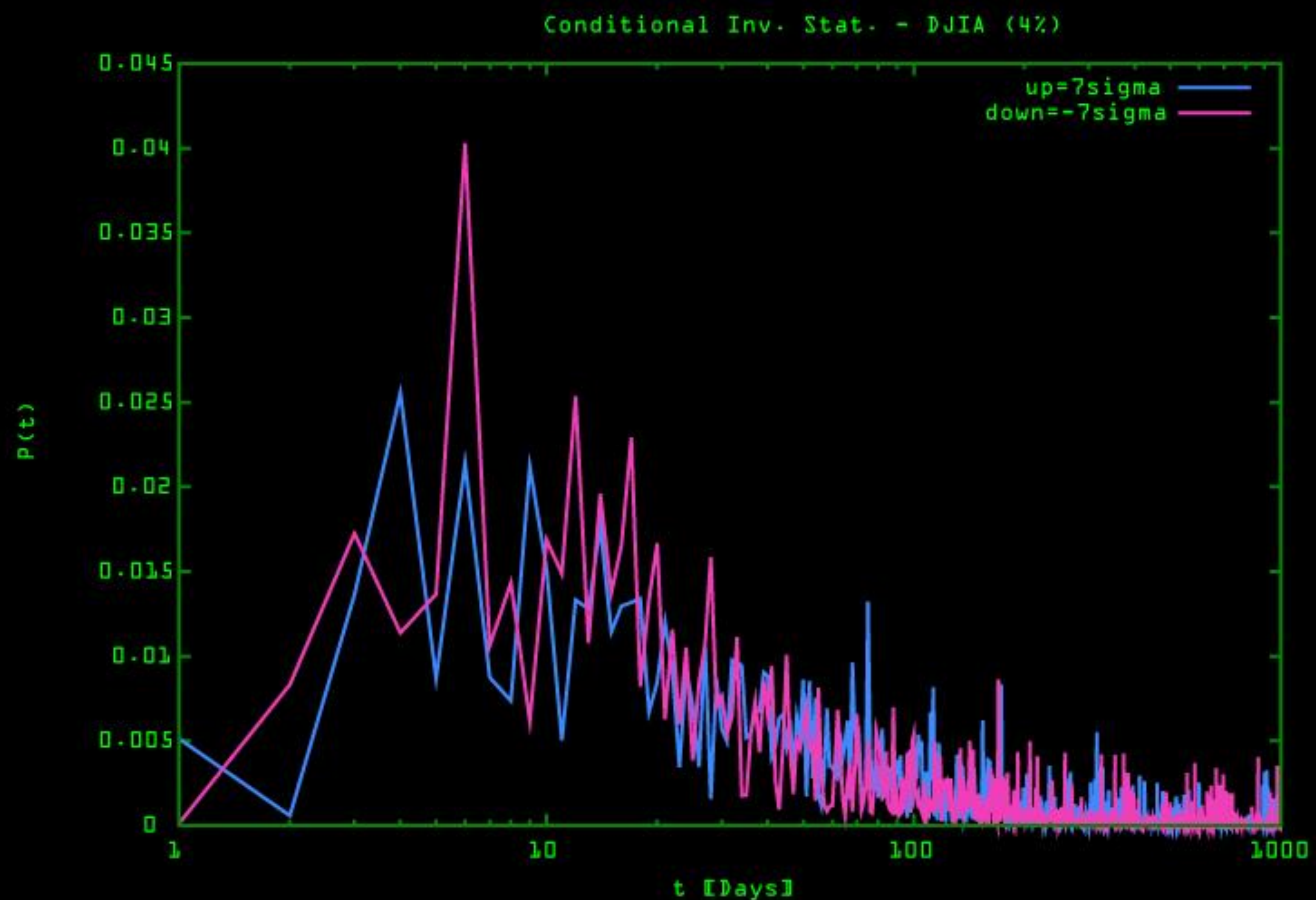
Conditional Inv. Stat. - DJIA (1%)
up=7sigma
down=-7sigma
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
P(t)
1
10
100
1000
t [Days]



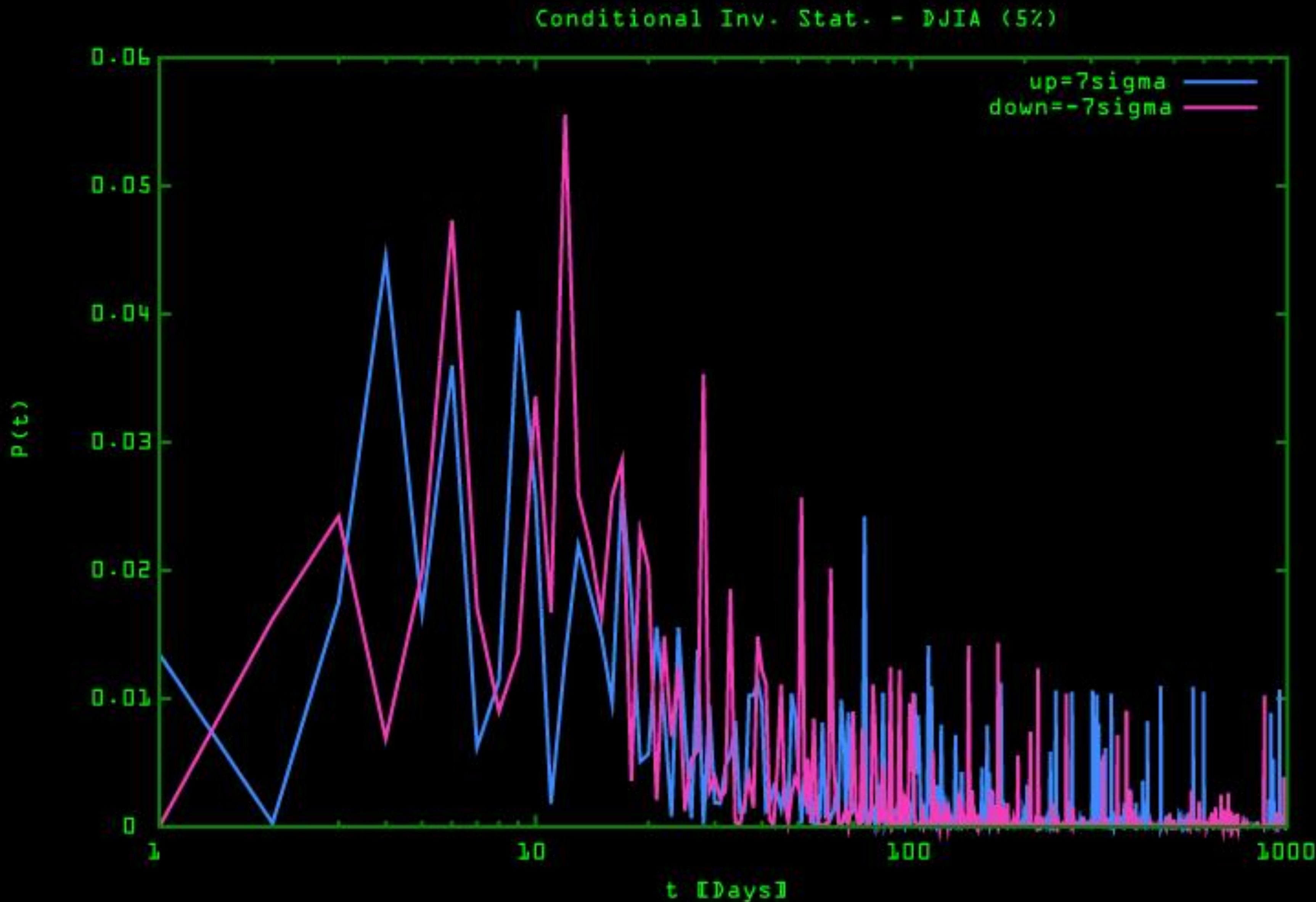
Conditional Inv. Stat. - DJIA (2%)
up=7sigma
down=-7sigma
P(t)
0.012
0.01
0.008
0.006
0.004
0.002
0
1
10
100
1000
t [Days]



Conditional Inv. Stat. - DJIA (3%)
up=7sigma
down=-7sigma
P(t)
t [Days]
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
10
100
1000

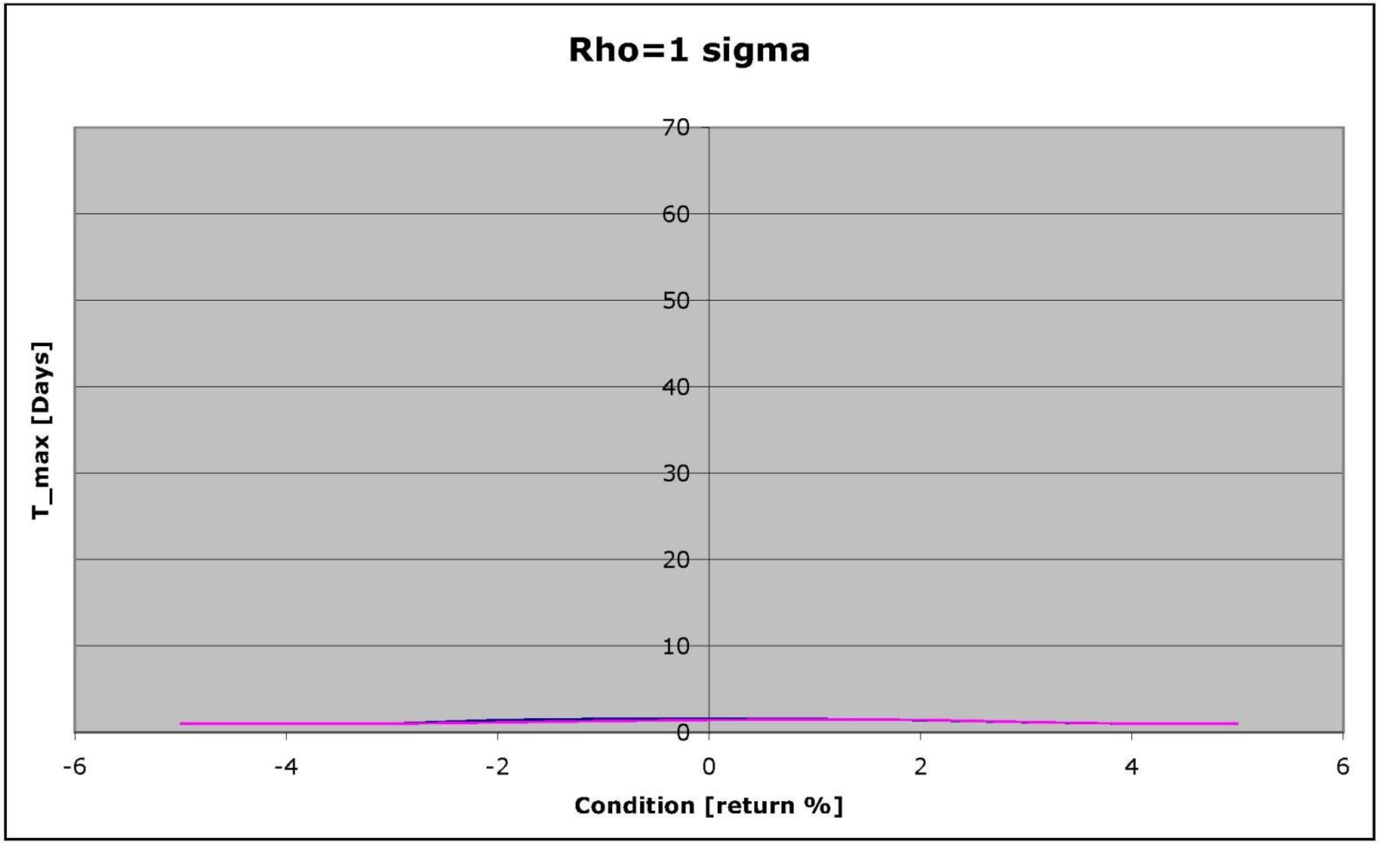


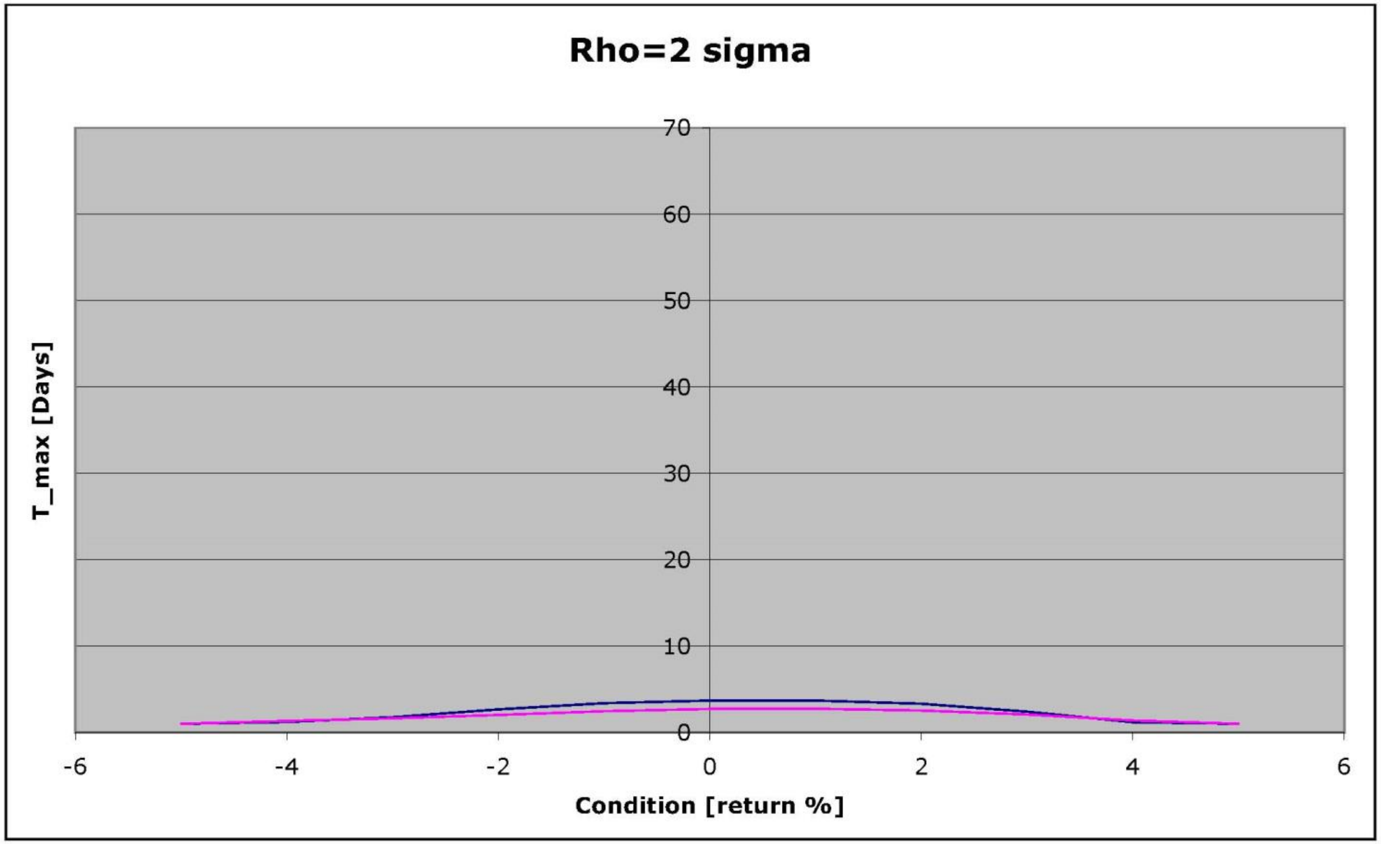
Conditional Inv. Stat. - DJIA (4%)
up=7sigma
down=-7sigma
P(t)
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000
t [Days]

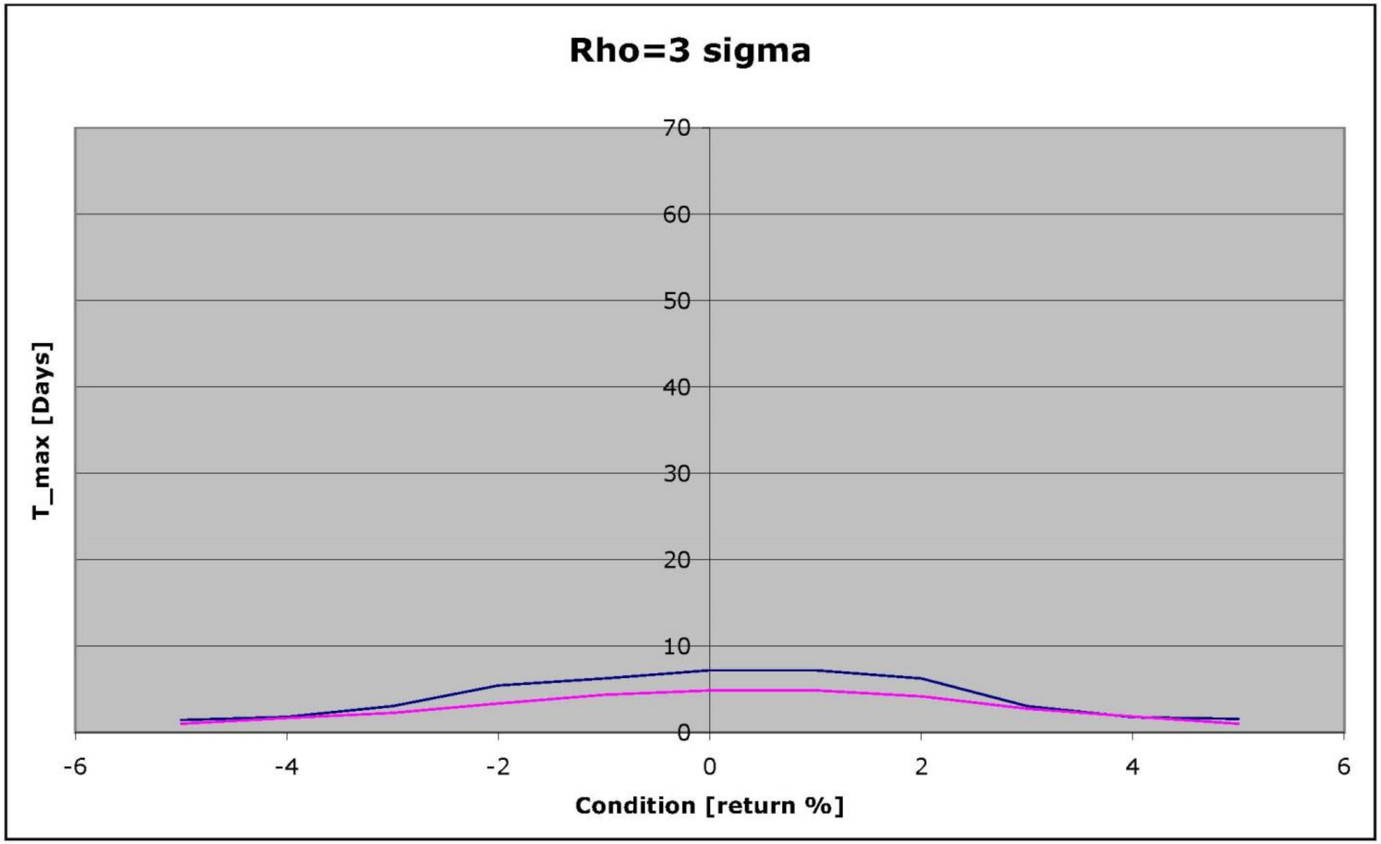


Conditional Inv. Stat. - DJIA (5%)
up=7sigma
down=-7sigma
P(t)
t [Days]
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000

In general we can track the dependence of the distribution maximum on barrier and return-condition…

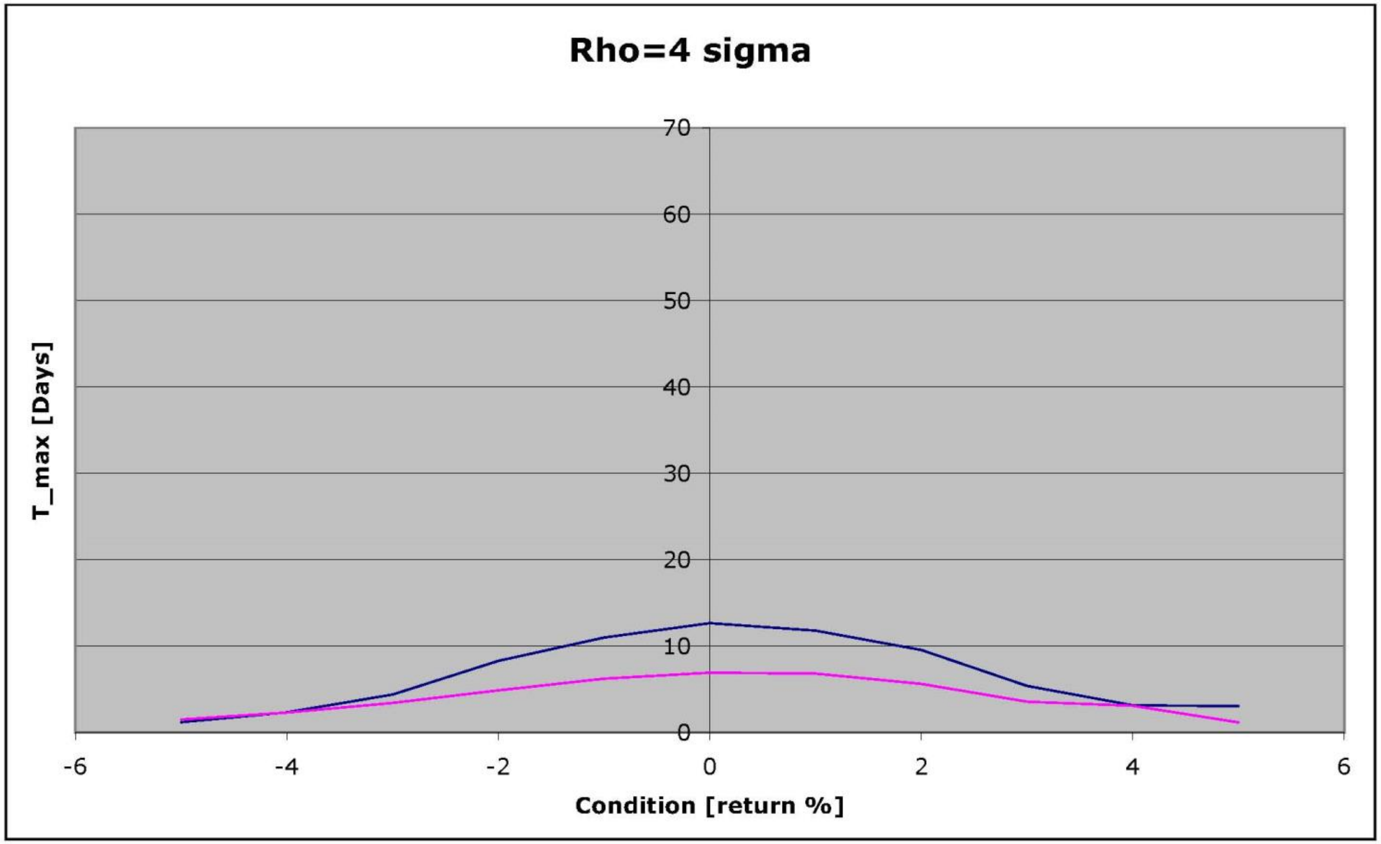

Rho=1 sigma
T_max [Days]
70
60
50
40
30
20
10
0
-6
-4
-2
0
2
4
6
Condition [return %]


Rho=2 sigma
70
60
50
40
30
20
10
0
T_max [Days]
-6
-4
-2
0
2
4
6
Condition [return %]

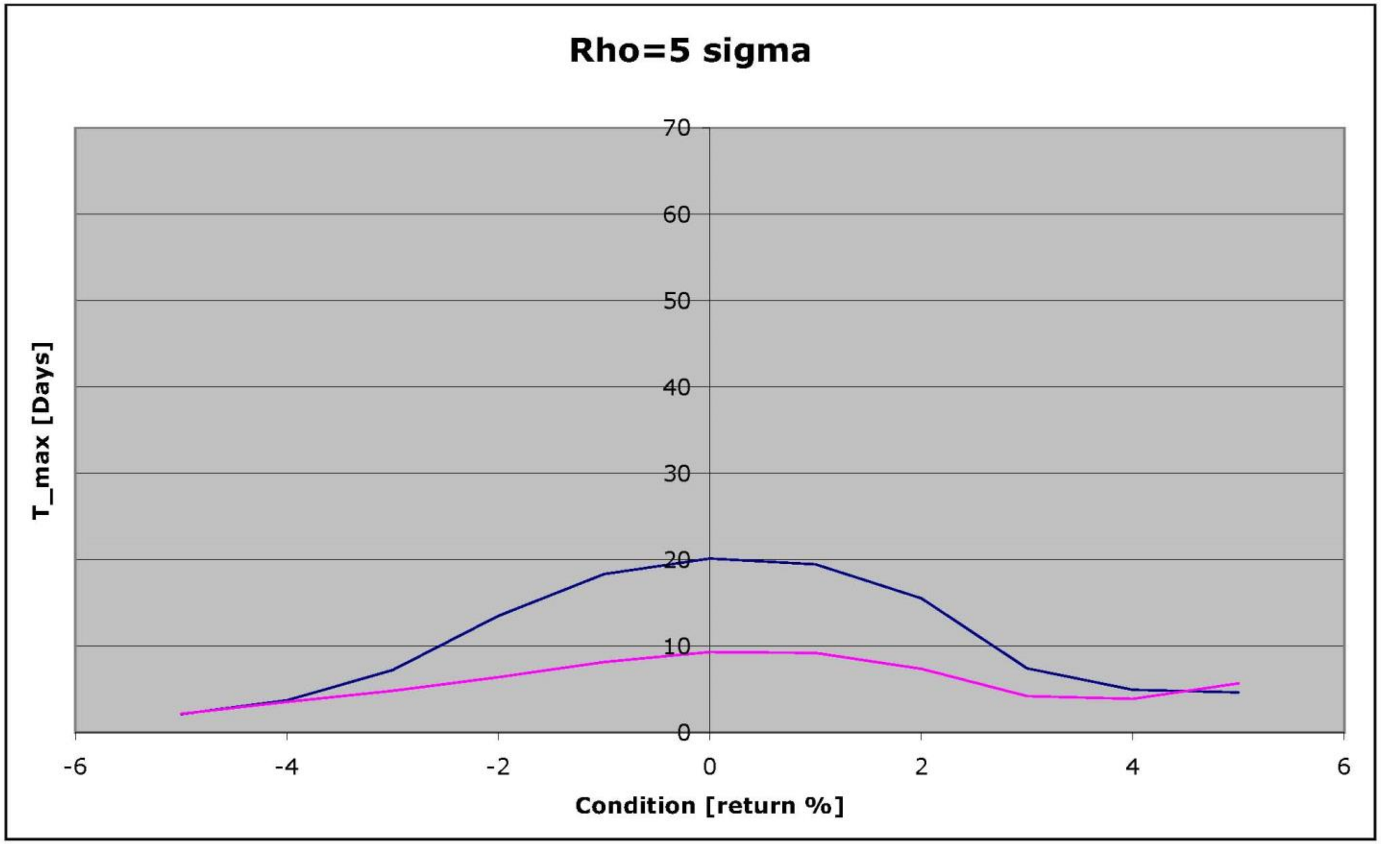


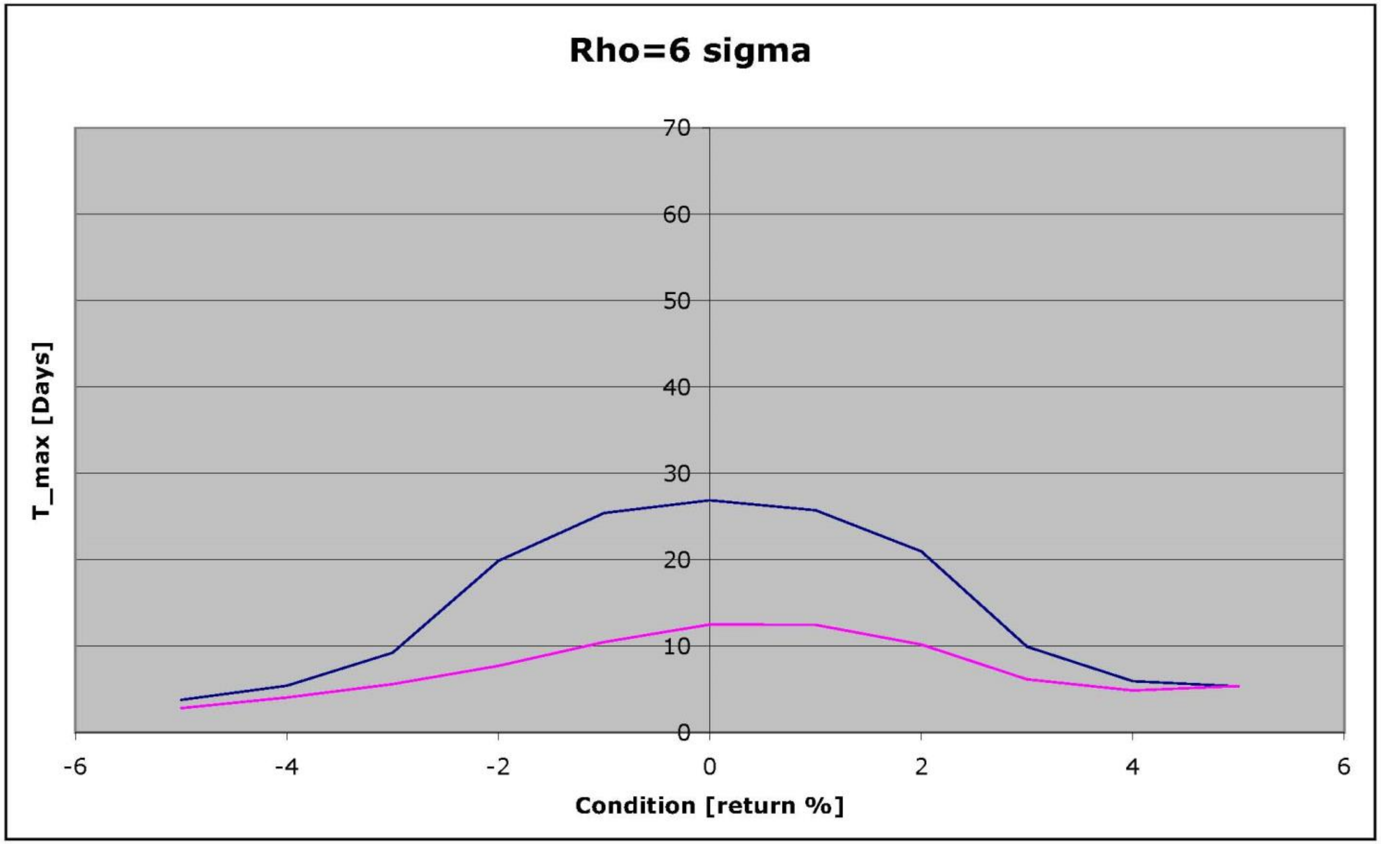
Rho=3 sigma
T_max [Days]
70
60
50
40
30
20
10
0
-6
-4
-2
0
2
4
6
Condition [return %]

Conditional Inverse Statistics

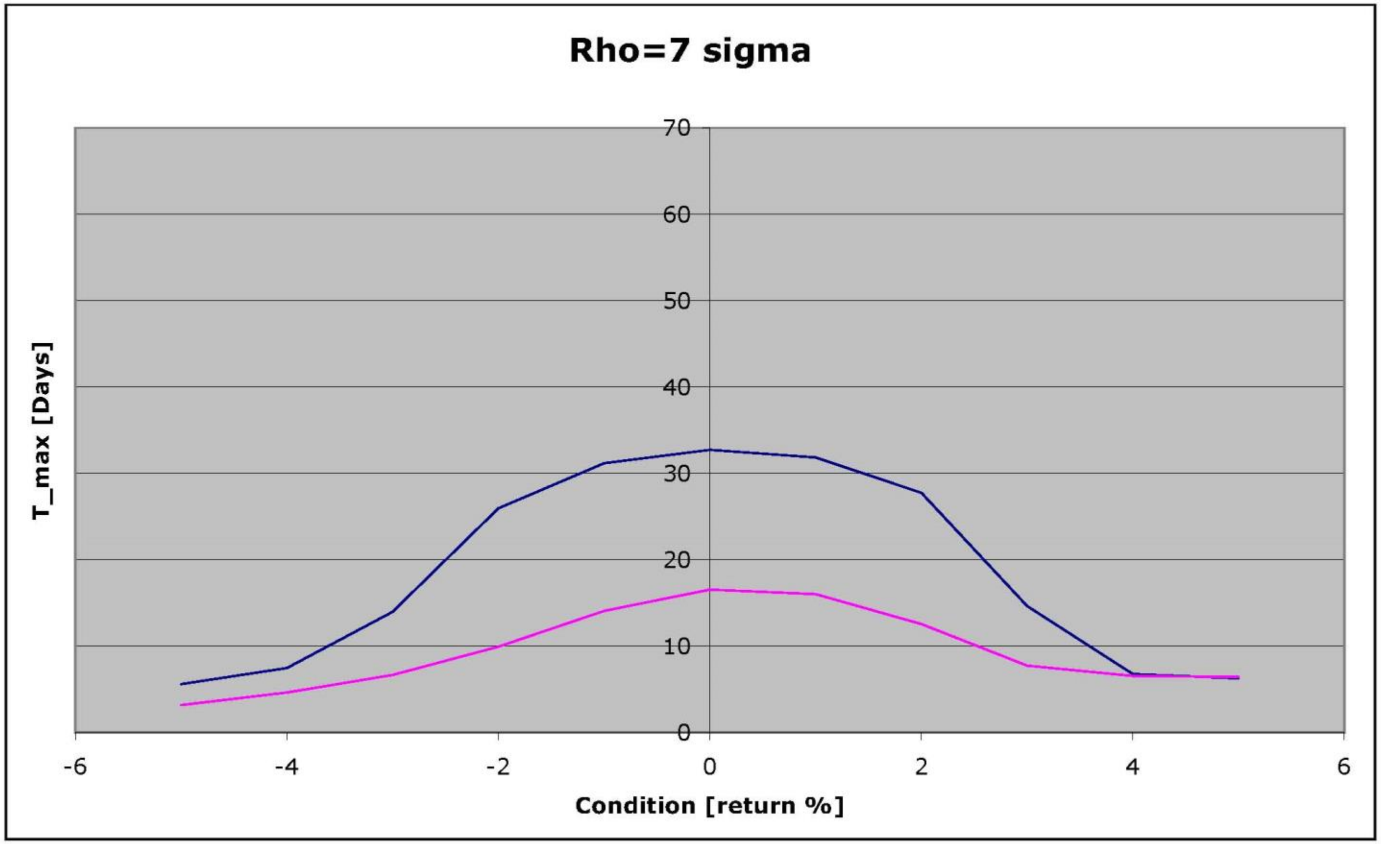


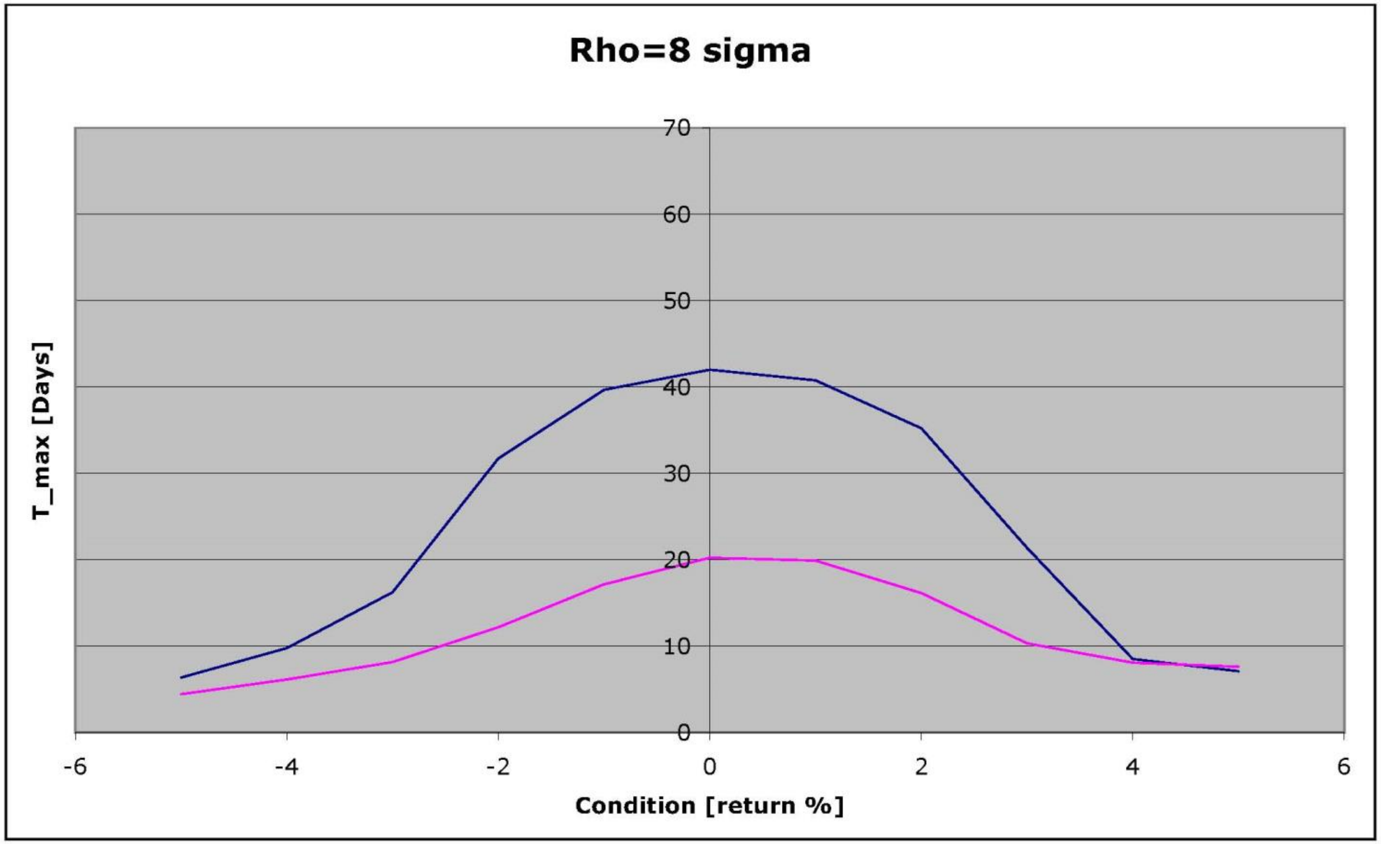
Conditional Inverse Statistics

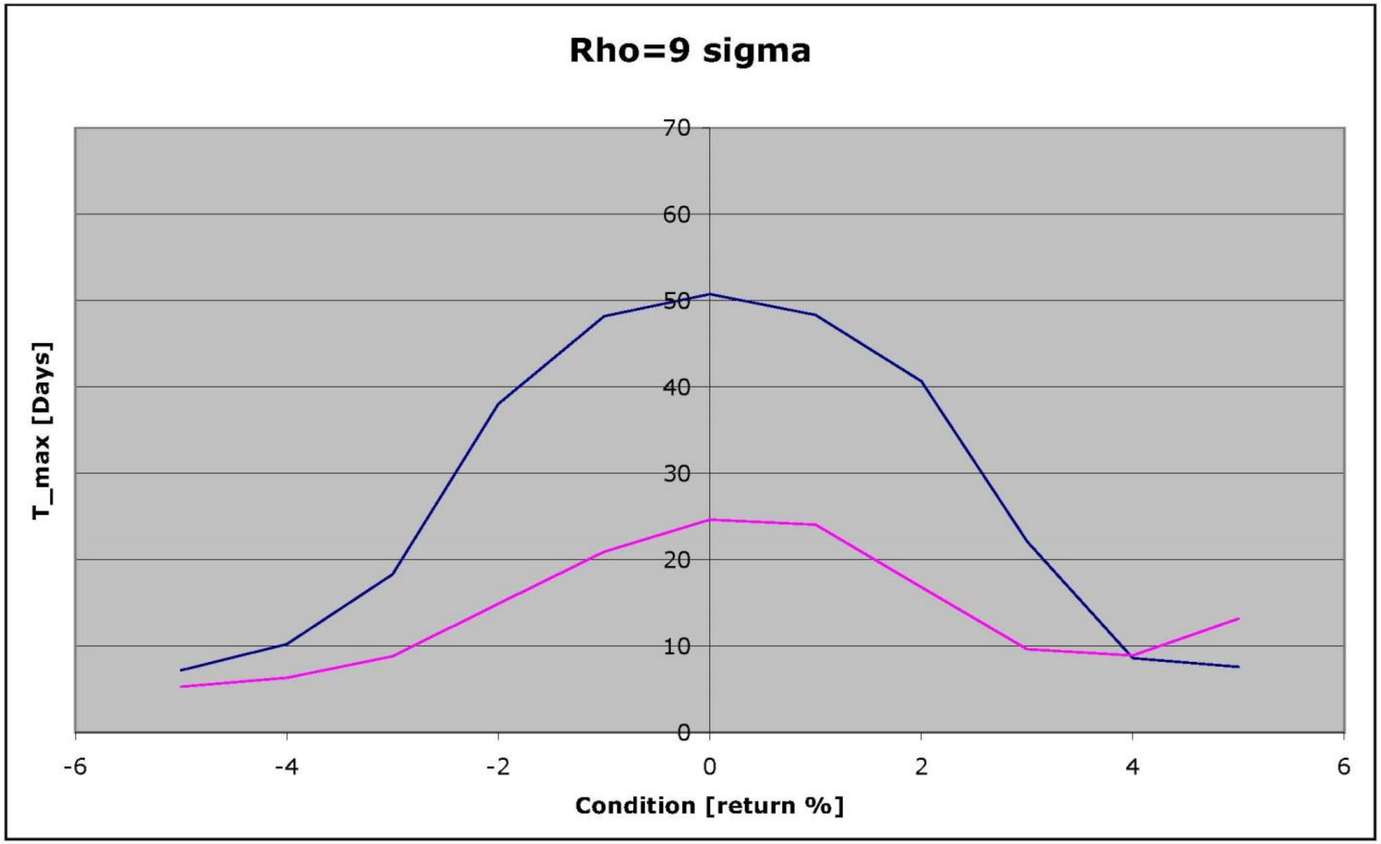




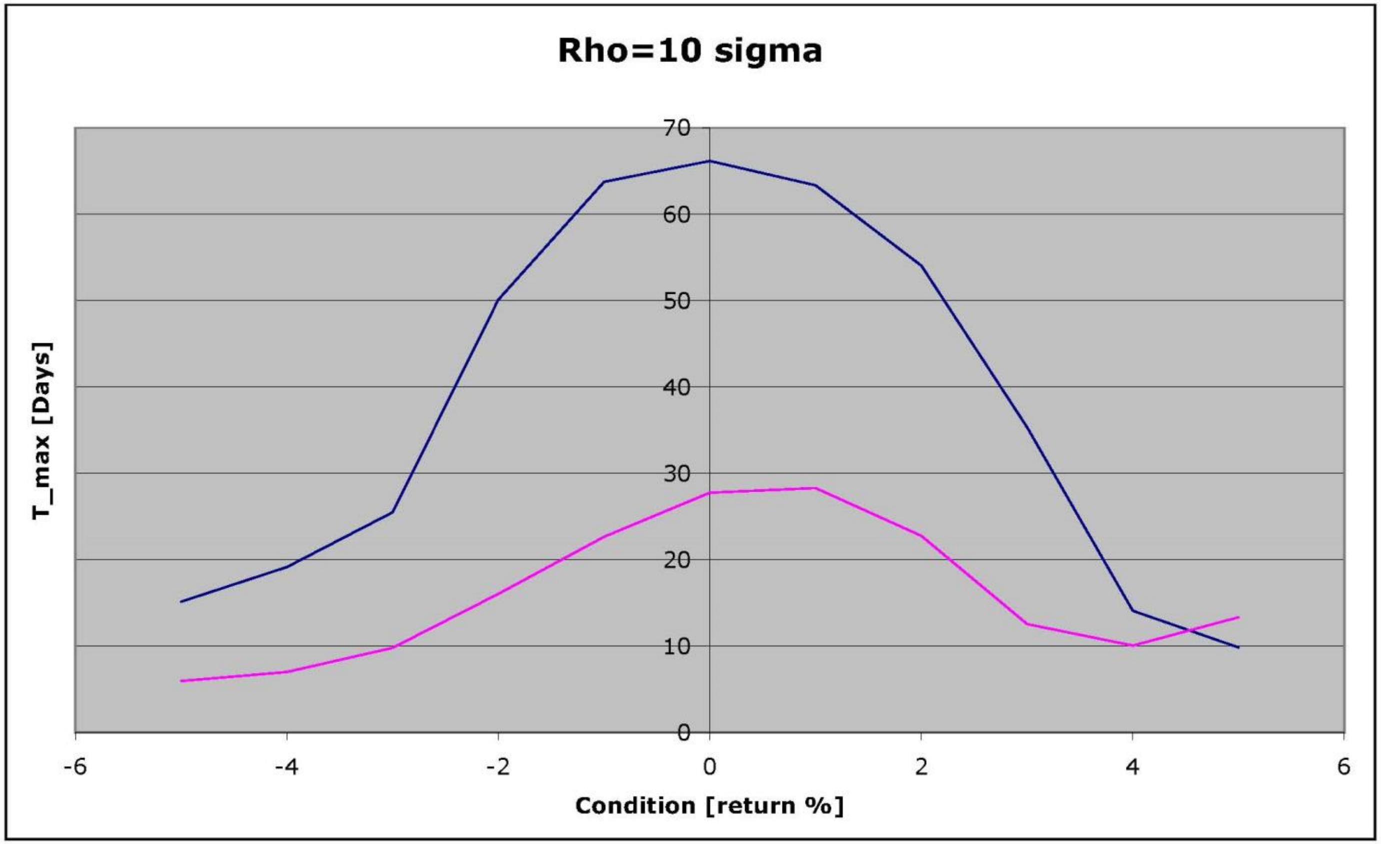
Rho=6 sigma
70
60
50
40
30
20
10
0
T_max [Days]
-6
-4
-2
0
2
4
6
Condition [return %]


Rho=7 sigma
T_max [Days]
70
60
50
40
30
20
10
0
-6
-4
-2
0
2
4
6
Condition [return %]


Rho=8 sigma
T_max [Days]
0
10
20
30
40
50
60
70
-6
-4
-2
0
2
4
6
Condition [return %]



Rho=9 sigma
70
60
50
40
30
20
10
0
-6
-4
-2
0
2
4
6
T_max [Days]
Condition [return %]


Rho=10 sigma
T_max [Days]
0
10
20
30
40
50
60
70
-6
-4
-2
0
2
4
6
Condition [return %]

These priliminary results show us that:

- Due to low volatility, waiting times are largest for small return conditions
- For negative return conditions, the gain-loss asymmetry is conserved
- For positive return conditions, tha gain-loss asymmetry disappears

- Inverse statistics are more pronounced for days following large drops than gains
- Using the technic on finely sampled data opens up the possibility of defining new risk measures
- Inverse statstics does not exist, if one transforms from time to volatility measures. Gain-loss asymmetry is a consequence of volatility dynamics, following a subset of extreme events.

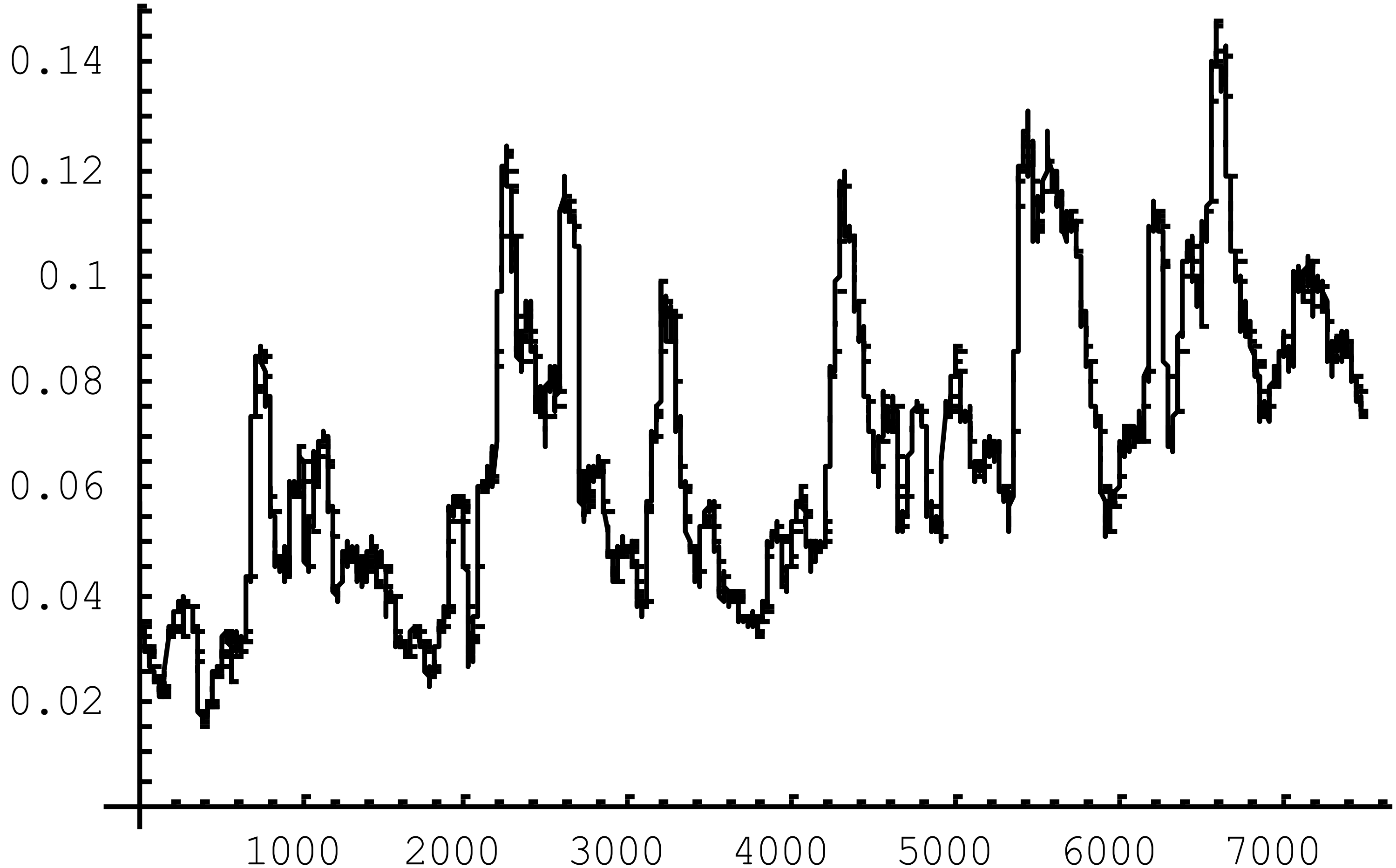
Estimate differences in the DJIA:

$$r_{\Delta t}(t) = s(t + \Delta t) - s(t) \ ,$$

When does it for first time exceed predescribed level ρ :

where $s(t) = \ln S(t)$. Hence the log-return is nothing but the log-price change of the asset. We consider a situation where an investor is aiming for a given return level denoted ρ, which may be both positive (being "long" on the market) or negative (being "short" on the market). If the investment is made at time t, then the investment horizon is defined as the time $\tau_\rho(t) = \Delta t$ so that the inequality $r_{\Delta t}(t) \geqslant \rho$ when $\rho \geqslant 0$, or $r_{\Delta t}(t) \leqslant \rho$ when $\rho < 0$, is satisfied for the *first* time. The investment horizon distribution, $p(\tau_\rho)$, is then the distribution of investment horizons τ_ρ (see Fig. 1) averaged over the data.

Tysk 100-dages rentevolatilitet (ej annualiseret)

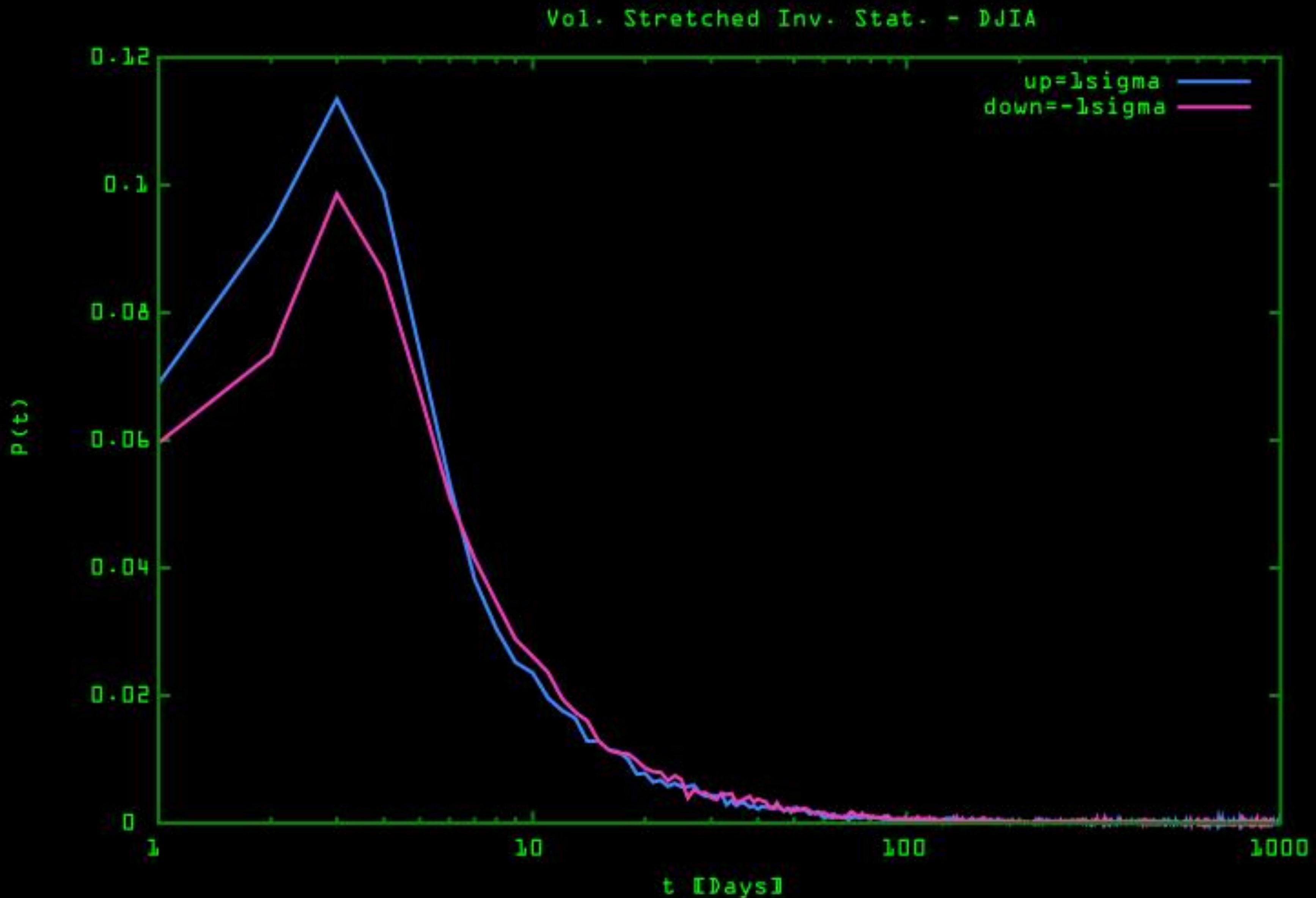


Where does inverse statistics stem from?

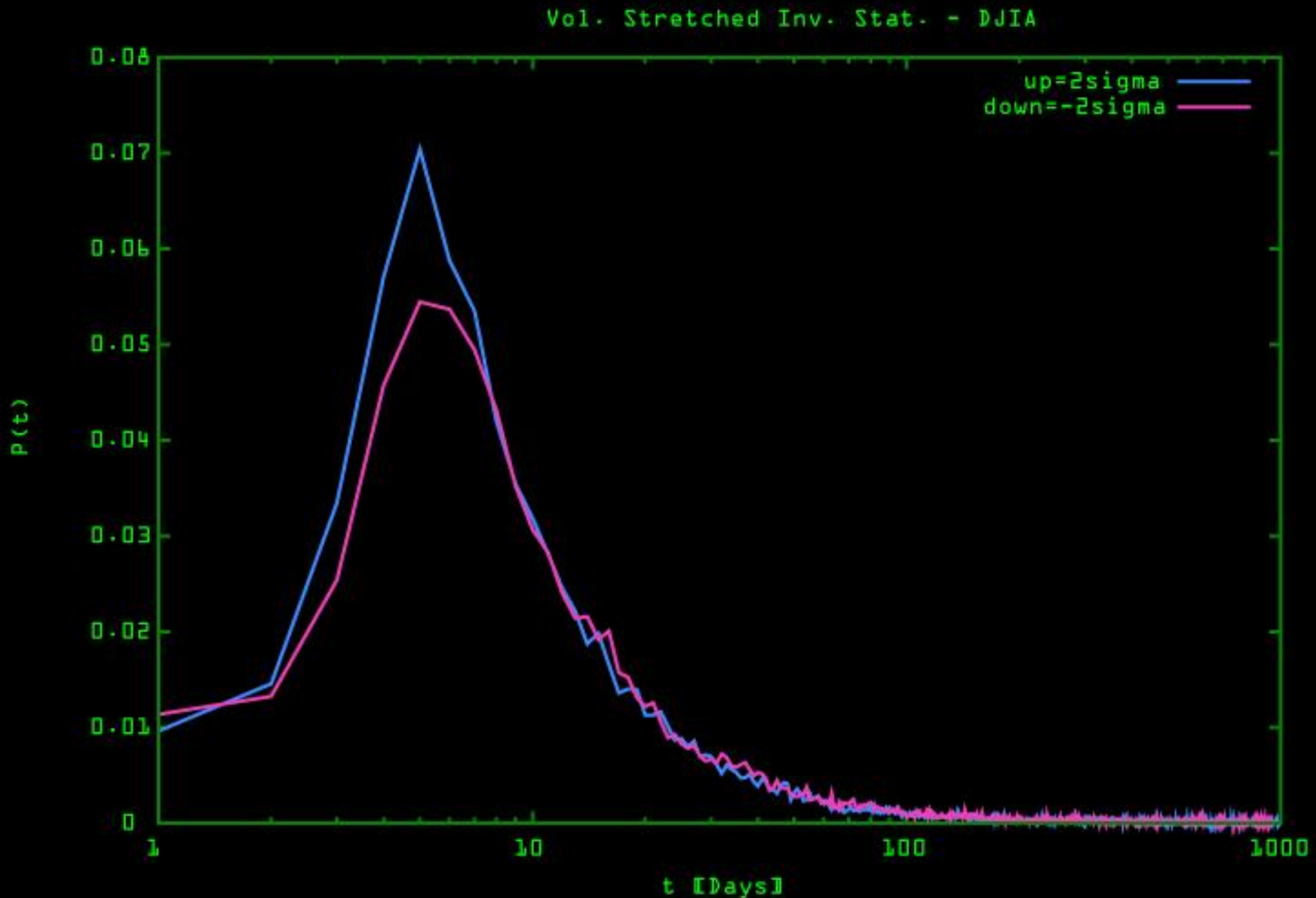
We know it has to do with faster movements for negative than positive returns.

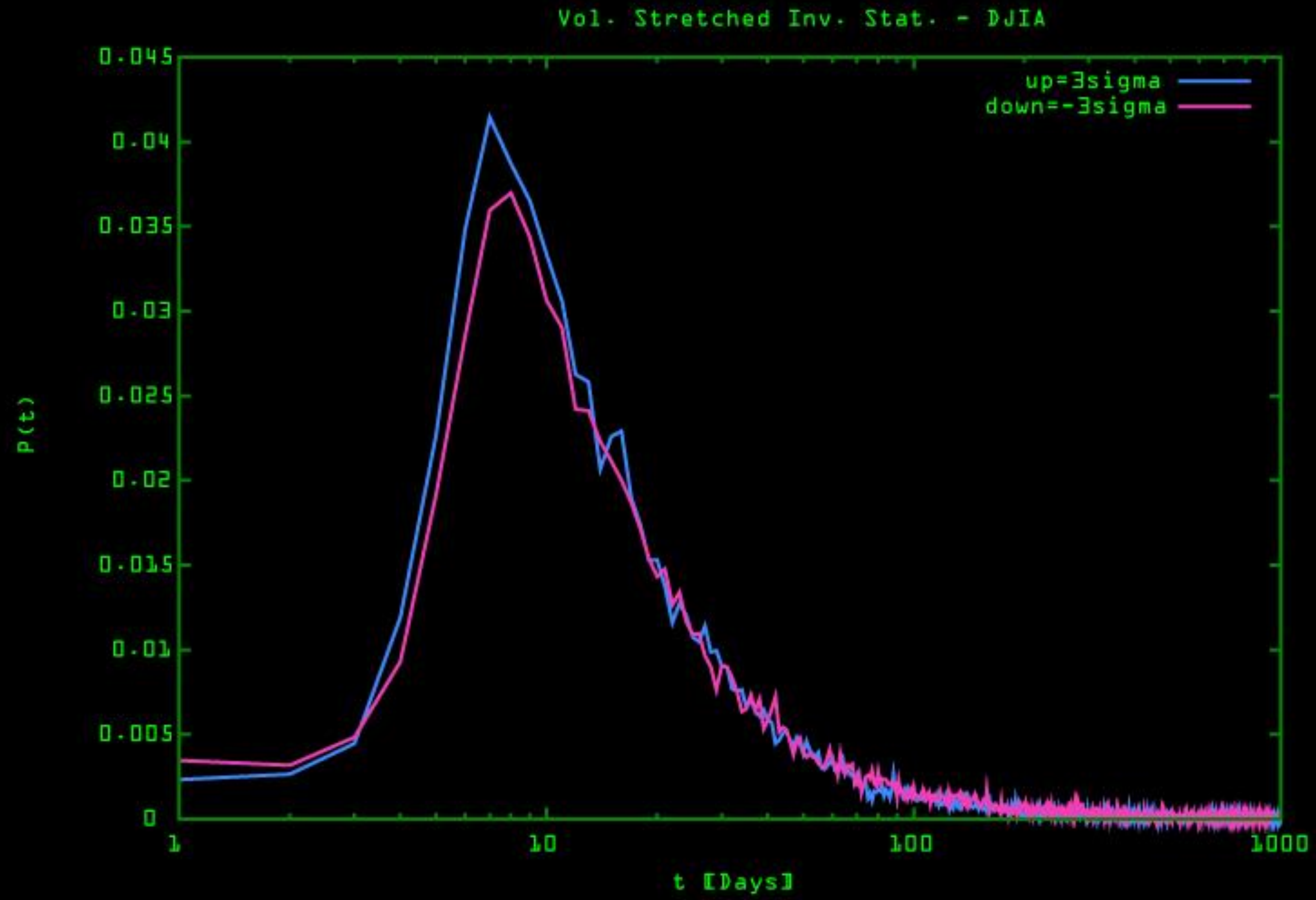
Idea:

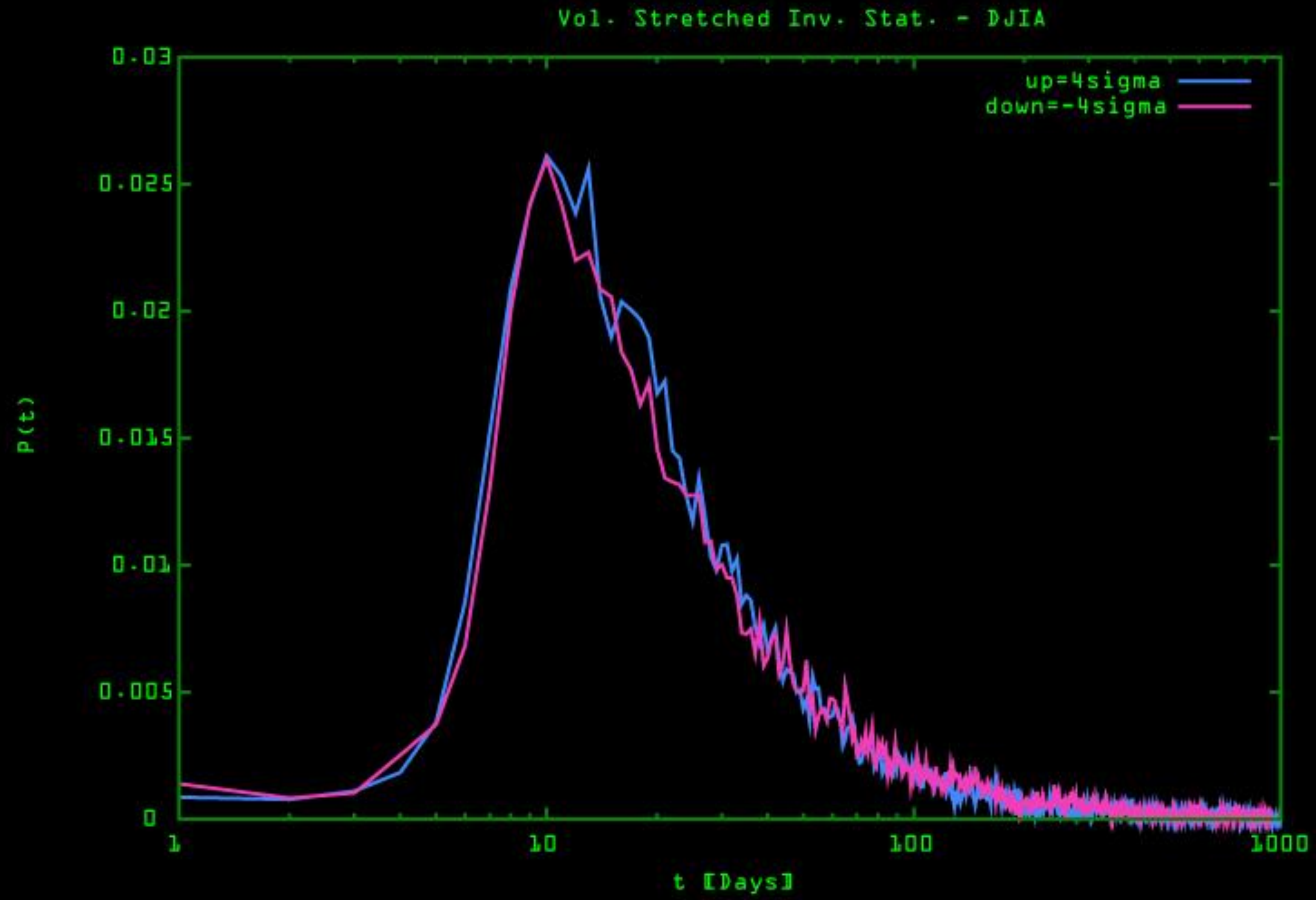
Lets scale time with volatility such that days with high vol. becomes relatively longer; days with low vol. relatively shorter.



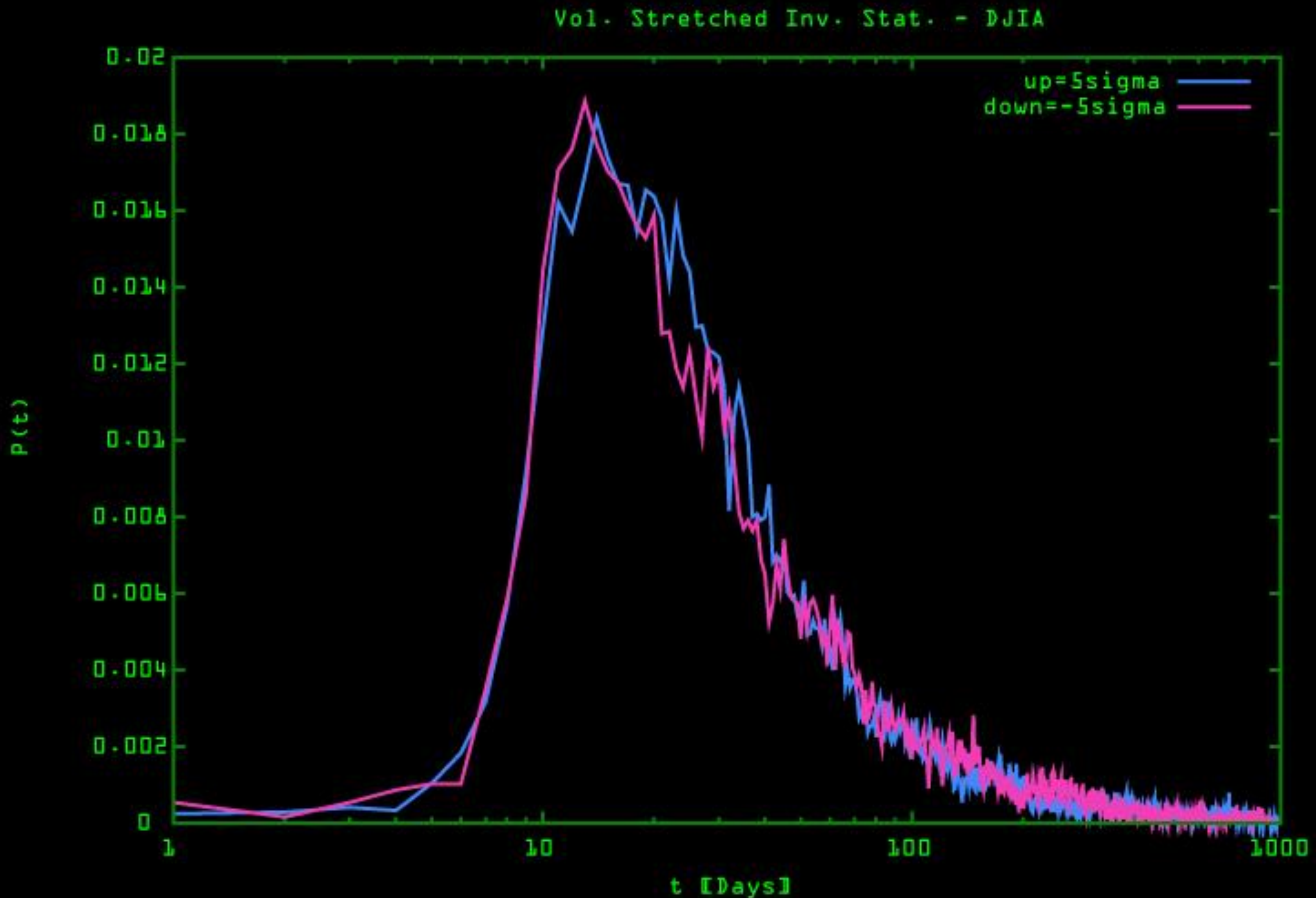
Vol. Stretched Inv. Stat. - DJIA
up=1sigma
down=-1sigma
p(t)
t [Days]
0.12
0.1
0.08
0.06
0.04
0.02
0
1
10
100
1000

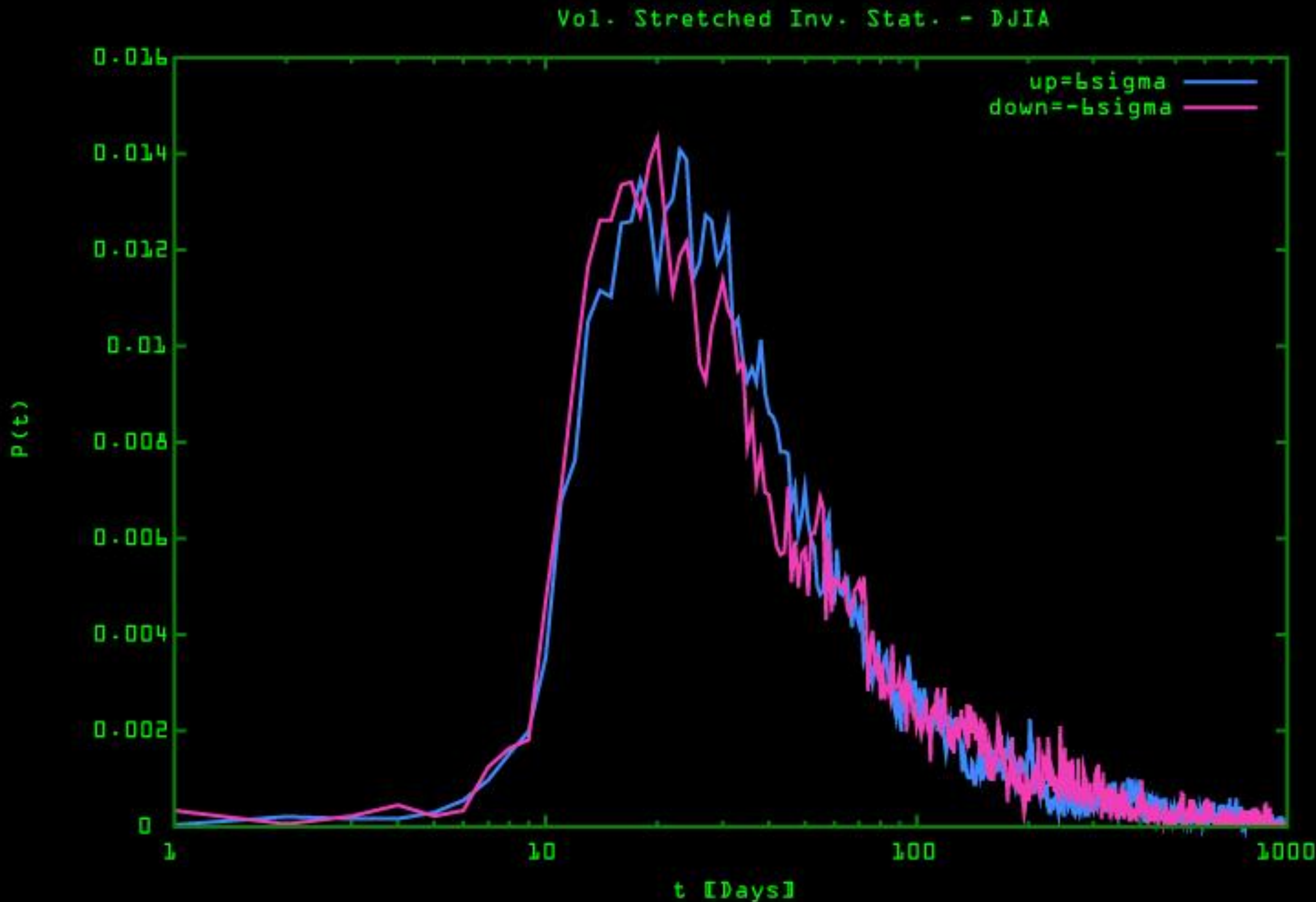

Vol. Stretched Inv. Stat. - DJIA
up=2sigma
down=-2sigma
P(t)
t [Days]
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000


Vol. Stretched Inv. Stat. - DJIA
up=3sigma
down=-3sigma
P(t)
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000
t [Days]

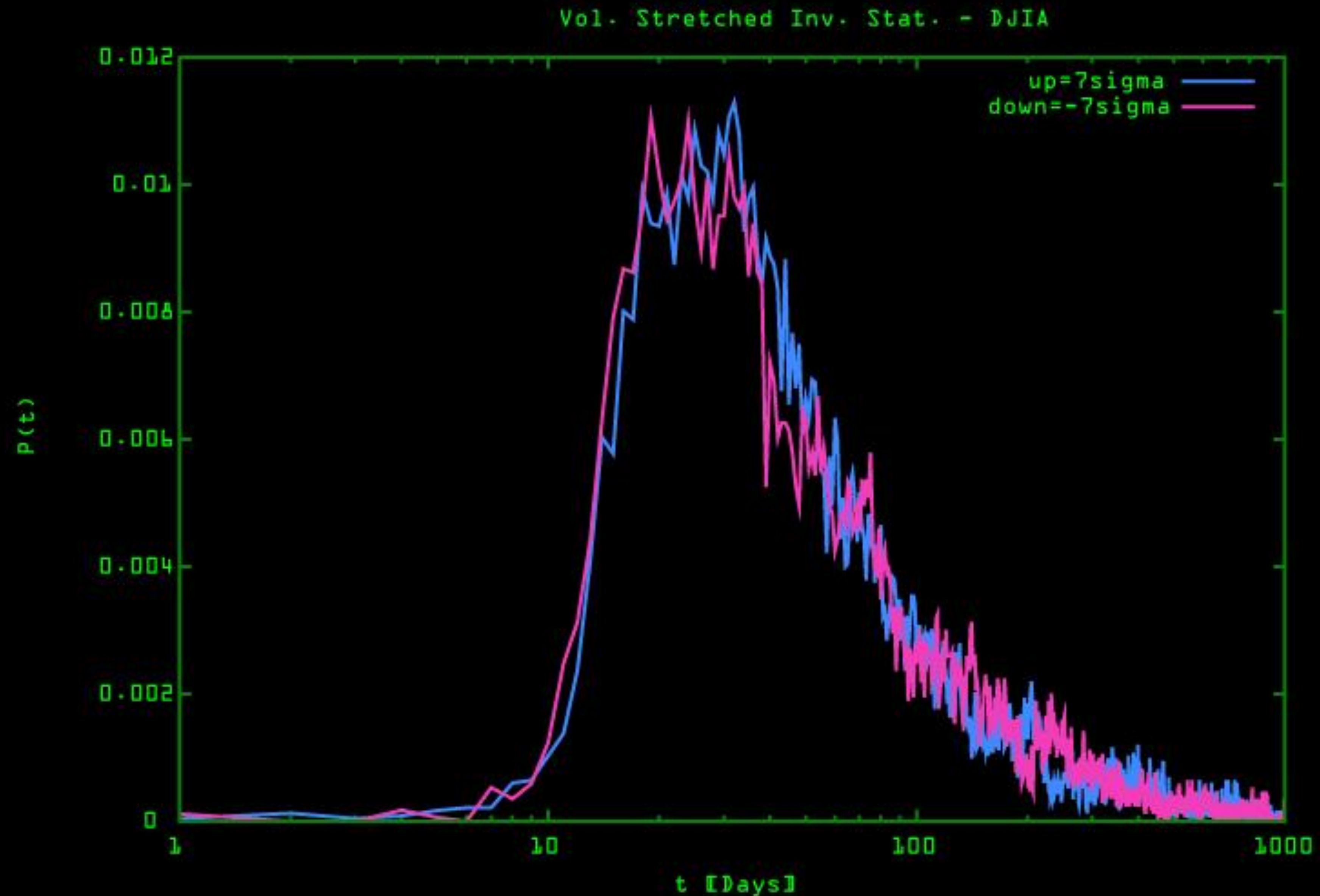


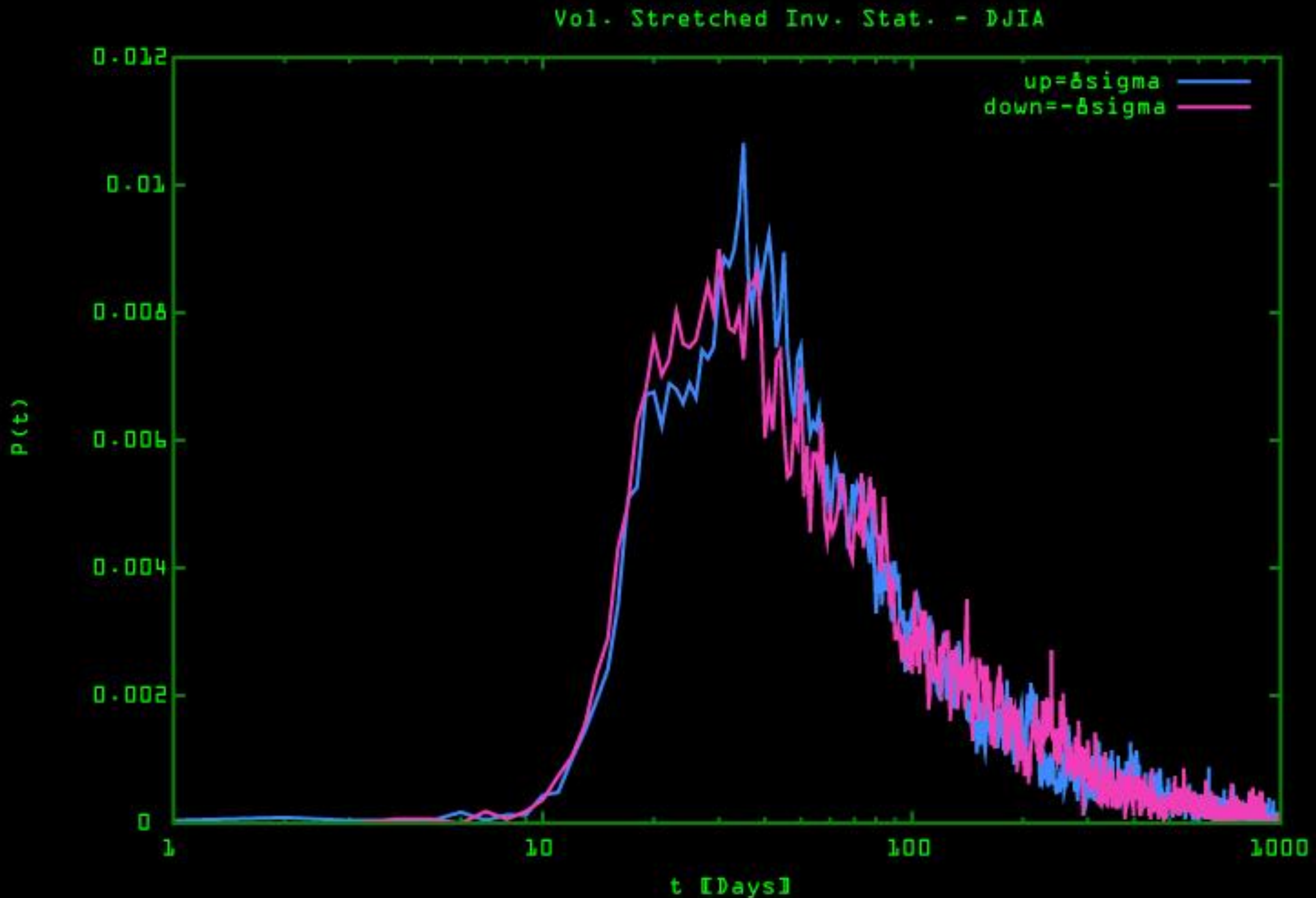
Vol. Stretched Inv. Stat. - DJIA
up=4sigma
down=-4sigma
P(t)
0.03
0.025
0.02
0.015
0.01
0.005
0
1
10
100
1000
t [Days]


Vol. Stretched Inv. Stat. - DJIA
up=5sigma
down=-5sigma
P(t)
t [Days]
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
1
10
100
1000

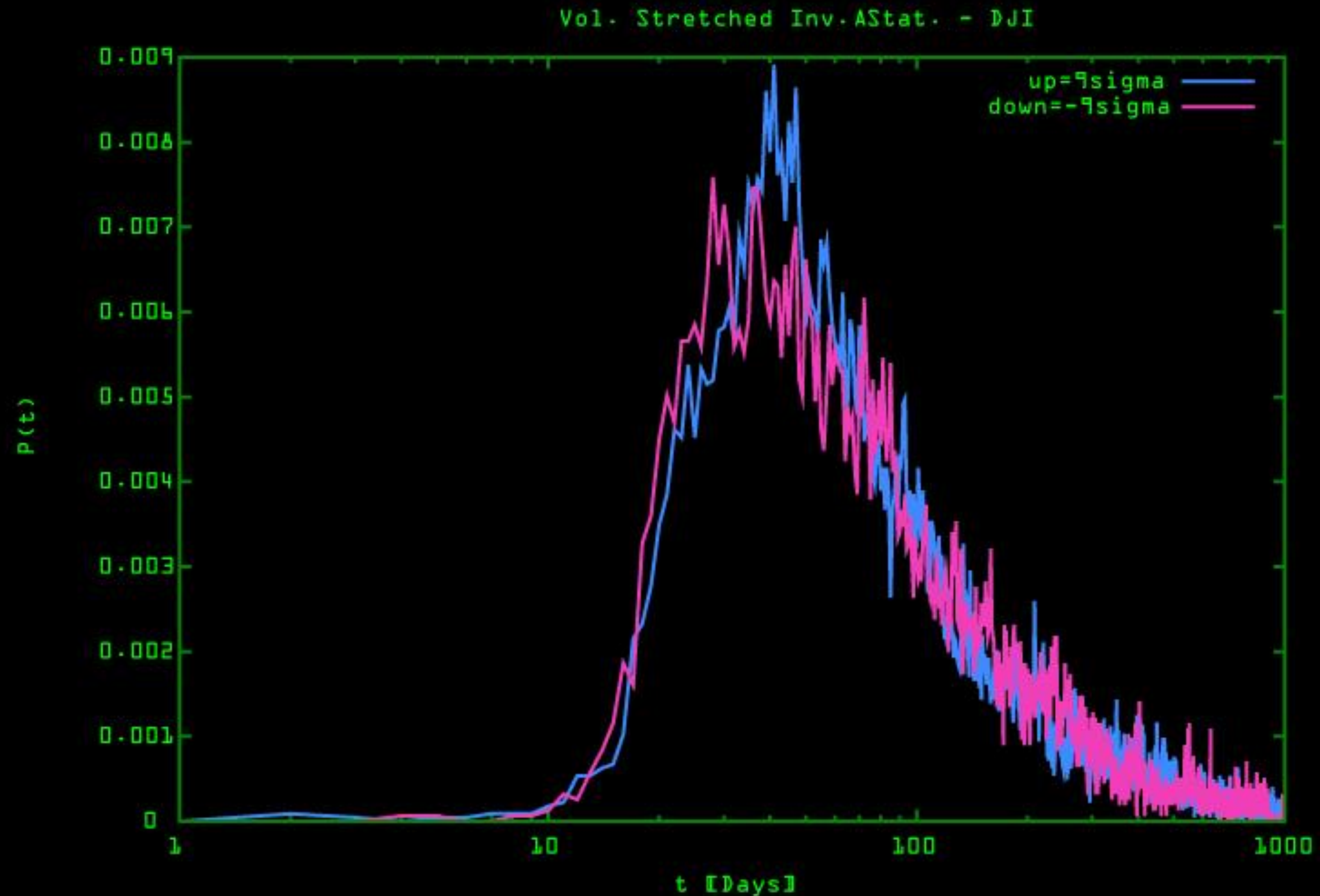


Vol. Stretched Inv. Stat. - DJIA
up=6sigma
down=-6sigma
p(t)
t [Days]
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
1
10
100
1000

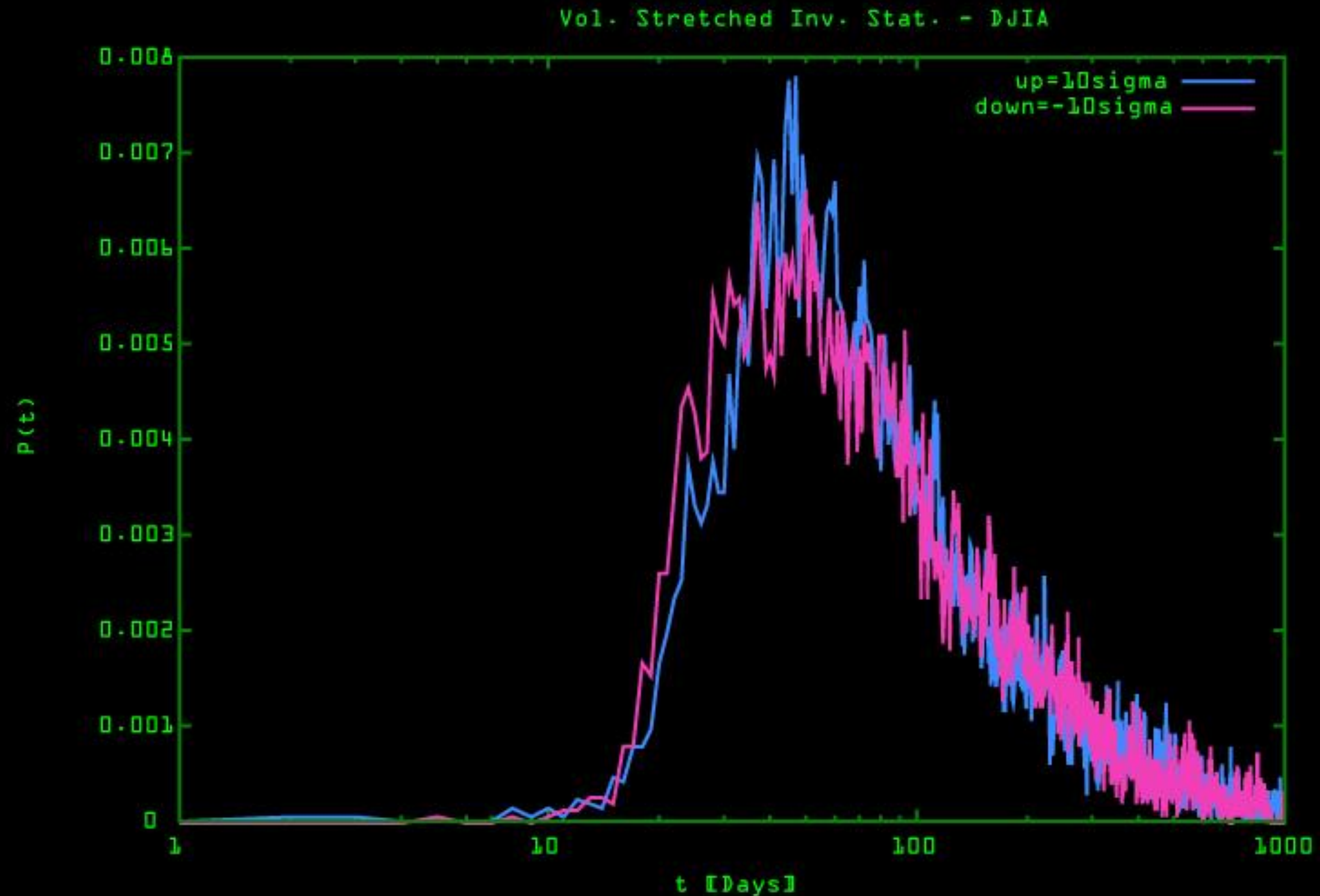

Vol. Stretched Inv. Stat. - DJIA
up=7sigma
down=-7sigma
P(t)
0
0.002
0.004
0.006
0.008
0.01
0.012
1
10
100
1000
t [Days]



Vol. Stretched Inv. Stat. - DJIA
up=&sigma
down=-&sigma
0.012
0.01
0.008
0.006
0.004
0.002
0
p(t)
1
10
100
1000
t [Days]



Vol. Stretched Inv.AStat. - DJI
up=9sigma
down=-9sigma
P(t)
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
1
10
100
1000
t [Days]


Vol. Stretched Inv. Stat. - DJIA
up=10sigma
down=-10sigma
P(t)
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
1
10
100
1000
t [Days]