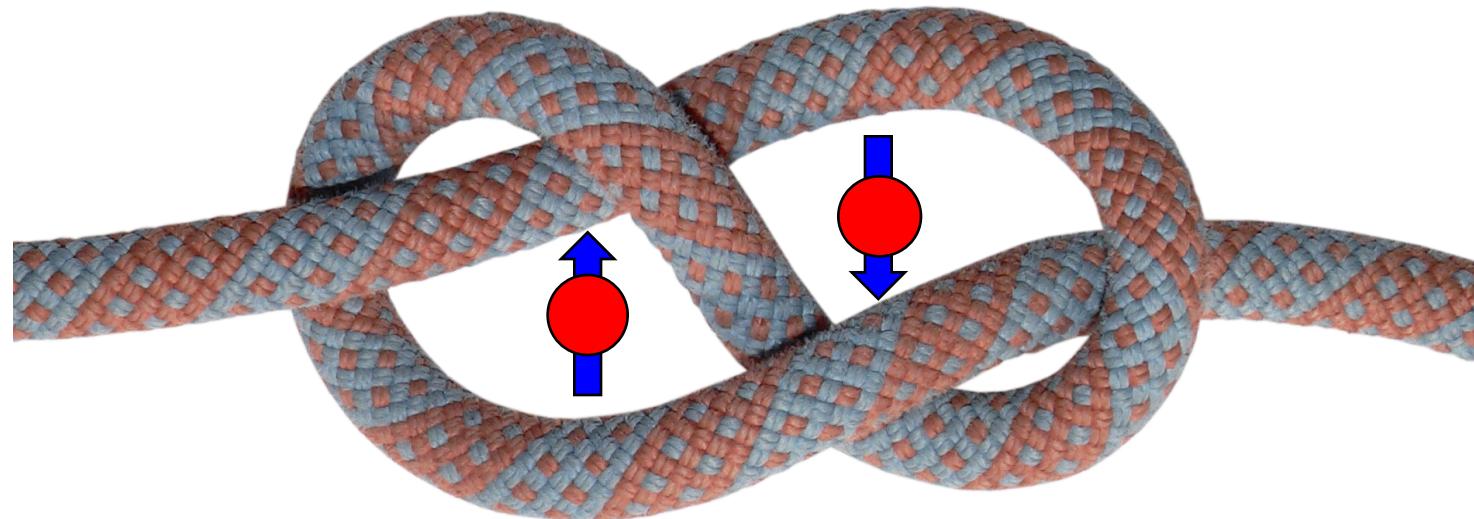


Oxide Heterostructures

A route to engineering correlated topological phases of matter

Arun Paramekanti
(University of Toronto)



Multicomponent Superconductors (Nordita, 22 July, 2016)

Funding:

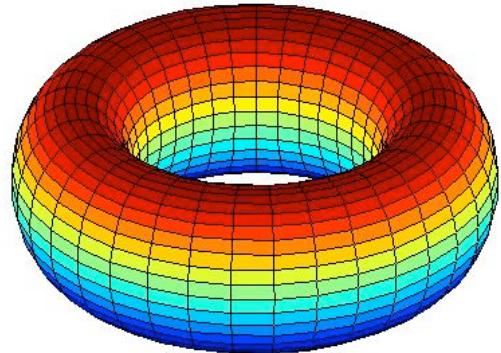


UNIVERSITY OF
TORONTO

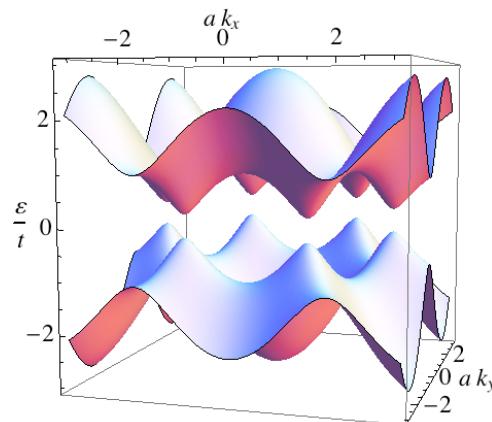
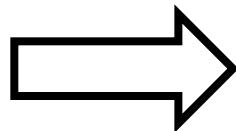


Bands in Crystals – Momentum Space Topology

Crystal momentum



2D Brillouin zone: Torus



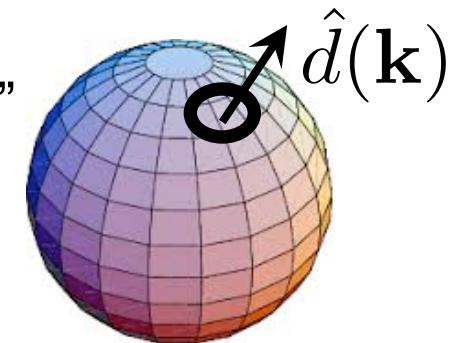
k-space energy bands: $E_n(\mathbf{k})$
Bloch wavefunctions: $|\psi_n(\mathbf{k})\rangle$

Two-Band System

$$H(\mathbf{k}) = \vec{d}(\mathbf{k}) \cdot \vec{\sigma} \xrightarrow{\text{pseudospin}}$$

$$E_{\pm}(\mathbf{k}) = \pm |\vec{d}(\mathbf{k})|$$

“Bloch sphere”



$\hat{d}(\mathbf{k})$: Information about wavefunction

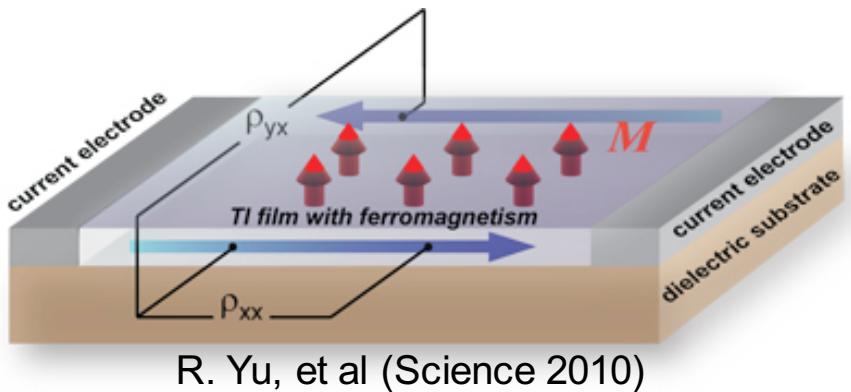
$$\int \frac{dk_x dk_y}{4\pi} \hat{d}(\mathbf{k}) \cdot \partial_x \hat{d}(\mathbf{k}) \times \partial_y \hat{d}(\mathbf{k})$$

→ Topological invariant (Chern number)

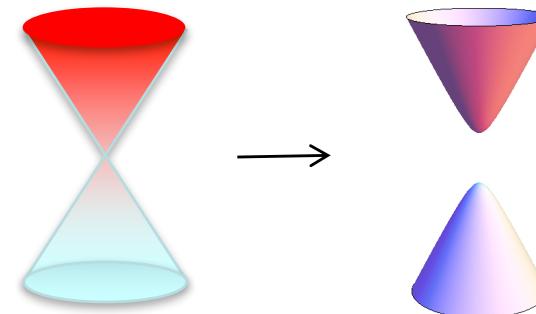
Sample boundary: Change of topology leads to gapless edge states

Quantum anomalous Hall effect Magnetically doped topological insulators

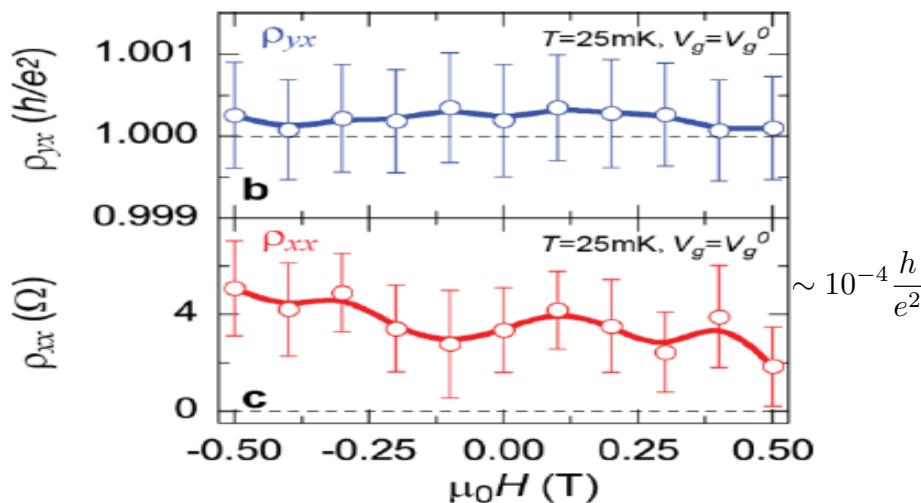
- Consider TIs near a band inversion transition (e.g., thin films)
- Breaking time-reversal strongly with dopant magnetization: QAH effect
- Spin orbit coupling is crucial



R. Yu, et al (Science 2010)



Surface Dirac fermions get gapped



$(\text{Bi},\text{Sb})_2\text{Te}_3$ film doped with Cr or V atoms
Ferromagnetic $T_c \sim 10\text{-}15\text{K}$

C.Z. Chang et al, Science 2013 (Xue group, Tsinghua)
C.Z. Chang et al, arXiv (M. Chan + J. Moodera groups, PSU/MIT)
A. J. Bestwick et al, arXiv (Goldhaber-Gordon group, Stanford)
A. Kandala, et al, arXiv (N. Samarth and C.X. Liu groups, PSU)

- How can we get a “high temperature” QAH effect?

Outline

- Strong correlations lead to a high temperature scale for magnetism
- But strong correlations also leads to Mott localization!
- How can we bring together strong correlations and band topology?
 - most natural in Kondo type systems (two-fluid picture)
- Can Kondo-type materials display “high temperature” QAH effect?
- Does the correlations+topology interplay lead to exotic phases?

Collaborators



Ashley Cook
(PhD:Toronto -> U. Zurich)



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(PhD: Univ. Toronto)



Lukasz Cincio
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- Toronto: K. Plumb, J.P. Clancy, **Young-June Kim**
- SNU: B.-C. Jeon, T.-W. Noh
- ORNL: **A.A. Aczel**, G. Cao, T. J. Williams, S. Calder, A. Christianson, D. Mandrus. A. Kolesnikov
- India: **Santu Baidya**, **Tanusri-Saha Dasgupta**, Umesh Waghmare

Outline

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(1) Strong correlations

- Brink of Mott localization or deep Mott insulator regime
- Common in 3d oxides: Kinetic energy ~ Interactions
- “High” energy/temperature scales for correlations/magnetism

Periodic Table of the Elements

The Periodic Table of the Elements displays the following information for each element:

- Atomic Number:** The element's position in the sequence.
- Symbol:** The standard one- or two-letter symbol used in chemistry.
- Name:** The element's name.
- Atomic Mass:** The element's mass number.
- Groups:** Elements in the same group share similar chemical properties. The groups are labeled as follows:
 - IA, IIA, IIIA, IVA, VA, VIA, VIIA, VIIIA (top row)
 - IIIB, IVA, VVA, VIVA, VIIA (middle row)
 - VIB, VIIB, VIIIB (bottom row)
- Periods:** Elements in the same period have similar electronic configurations. The periods are numbered 1 through 7.
- Series:** The Lanthanide Series (Ce to Lu) and Actinide Series (Ac to Lr) are shown as rows below the main table.

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(2) Band topology

- Nontrivial band topology: SOC + conducting electron fluid
- Strong SOC needs heavy elements
- Expected in 4d/5d oxides (eg: Rhenium, Osmium, Iridium)



Increasing
SOC

Periodic Table of the Elements																			
Atomic Number	Symbol	Name	Atomic Mass																
1	H	Hydrogen	1.008	2	He	Helium	4.003	3	Li	Lithium	6.941	4	Be	Beryllium	9.012	5	B	Boron	10.811
11	Na	Sodium	22.990	12	Mg	Magnesium	24.305	3	Sc	Scandium	44.956	4	Ti	Titanium	47.88	5	V	Vanadium	50.942
19	K	Potassium	39.098	20	Ca	Calcium	40.078	21	Sc	Scandium	44.956	22	Ti	Titanium	47.88	23	Cr	Chromium	51.996
37	Rb	Rubidium	85.468	38	Sr	Strontrium	87.62	39	Y	Yttrium	88.906	40	Zr	Zirconium	91.224	41	Nb	Niobium	91.923
55	Cs	Ceasium	132.955	56	Ba	Barium	137.327	57-71	Hf	Hafnium	178.490	72	Ta	Tantalum	180.907	73	W	Tungsten	183.840
87	Fr	Franium	223.020	88	Ra	Radium	226.025	89-103	Rf	Rutherfordium	[261]	104	Dub	Dubium	[262]	105	Sg	Seaborgium	[266]
Lanthanide Series																			
57	La	Lanthanum	138.905	58	Ce	Cerium	140.115	59	Pr	Praseodymium	140.908	60	Nd	Neodymium	144.24	61	Pm	Promethium	144.913
89	Ac	Actinium	227.028	90	Th	Thorium	232.038	91	Pa	Protactinium	231.036	92	U	Uranium	238.029	93	Np	Neptunium	237.048
Actinide Series																			
105	Dub	Dubium	[262]	106	Sg	Seaborgium	[266]	107	Bh	Bohrium	[264]	108	Hs	Hassium	[269]	109	Mt	Methylmerium	[268]
110	Ds	Darmstadtium	[269]	111	Rg	Roentgenium	[272]	112	Cn	Copernicium	[277]	113	Uut	Ununtrium	unknown	114	Fl	Flerovium	[289]
115	Uup	Ununpentium	unknown	116	Lv	Livermorium	[298]	117	Uus	Ununseptium	unknown	118	Uuo	Ununoctium	unknown	101	Fm	Einsteinium	[254]
102	Md	Mendelevium	258.1	103	No	Nobelium	259.101	104	Fr	Fermium	257.095	105	Md	Mendelevium	258.1	106	Lu	Lawrencium	[262]
Alkali Metal		Alkaline Earth		Transition Metal		Basic Metal		Semimetal		Nonmetal		Halogen		Noble Gas		Lanthanide		Actinide	

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Engineering topology + correlations in solids

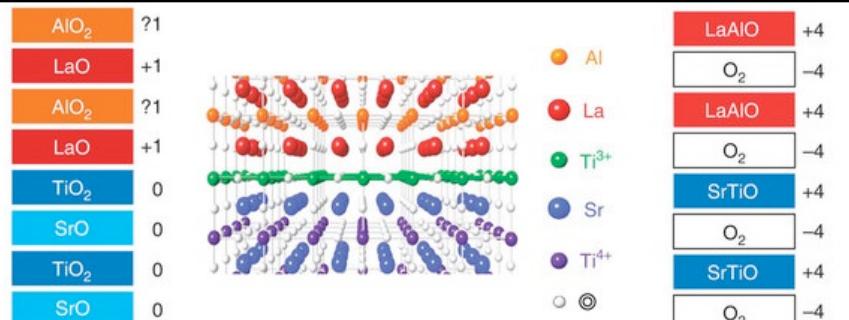
Oxide heterostructures and superlattices

LaAlO₃/SrTiO₃ interface [100]

H. Y. Hwang, J.M. Triscone, J. Mannhart,
R. Ashoori, K. A. Moler, ...

Basic physics: Magnetism + Superconductivity

Applications: Write/Erase circuits using electric field



3d-3d Superlattices along [111]

nature
materials

LETTERS

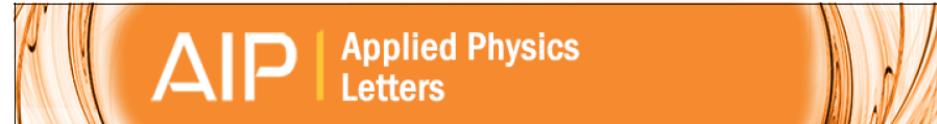
PUBLISHED ONLINE 22 JANUARY 2012 | DOI: 10.1038/NMAT3224

Exchange bias in LaNiO₃-LaMnO₃ superlattices

Marta Gibert^{1*}, Pavlo Zubko¹, Raoul Scherwitzl¹, Jorge Íñiguez² and Jean-Marc Triscone¹

3d/5d Superlattices along [111]

H. Takagi group (APL 2015)



Local electronic and magnetic studies of an artificial La₂FeCrO₆ double perovskite

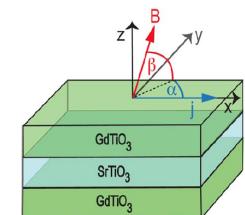
Benjamin Gray, Ho Nyung Lee, Jian Liu, J. Chakhalian, and J. W. Freeland

Citation: Applied Physics Letters 97, 013105 (2010); doi: 10.1063/1.3455323

Quantum Wells, Modulation Doping

S. Stemmer group (UCSB)

Confined 2DEGs, high mobilities, magnetism



Prediction of topological insulators in simple TMO bilayers

ARTICLE

Received 20 Jun 2011 | Accepted 18 Nov 2011 | Published 20 Dec 2011

DOI: 10.1038/ncomms1602

Interface engineering of quantum Hall effects in digital transition metal oxide heterostructures

Di Xiao¹, Wenguang Zhu^{1,2}, Ying Ran³, Naoto Nagaosa^{4,5} & Satoshi Okamoto¹

Rapid Communication

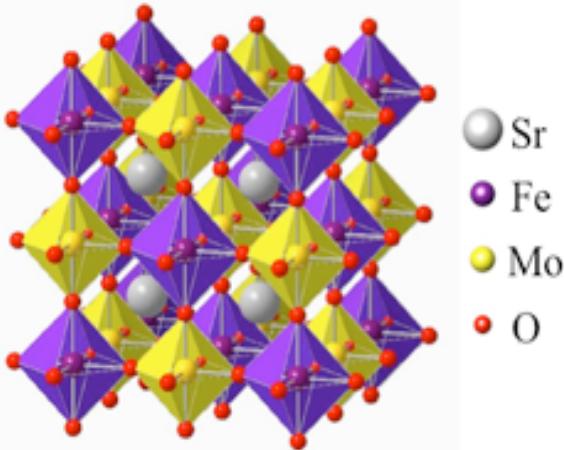
Topological insulators from complex orbital order in transition-metal oxides heterostructures

Andreas Rüegg and Gregory A. Fiete
Phys. Rev. B 84, 201103(R) – Published 14 November 2011

Double Perovskites: Mixing correlations and SOC

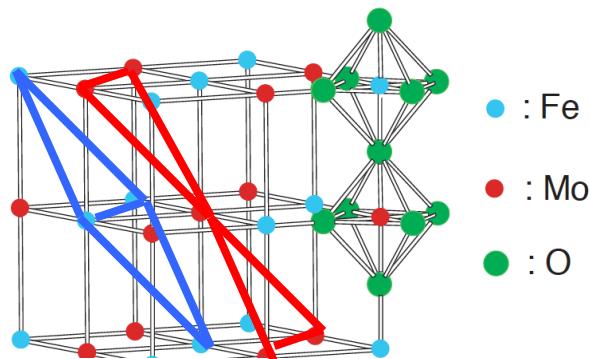
General formula: $A_2BB'O_6$ ($B, B' = 3d, 4d, 5d$)

Double perovskite lattice



Metallic systems

- B: Magnetism and B':Conduction electrons
- . Half metallic ferrimagnets (eg: Sr_2FeMoO_6 , $T_C= 420K$)
 - . Large polarization: good for spin injection
 - . Interplay of **Magnetism, SOC, Metallicity**



Layered along [111]

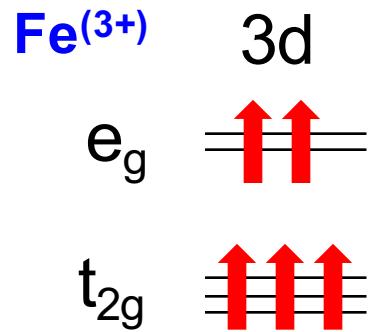
Mott insulators

- B=magnetism, B'=inert or magnetism
- . **Well isolated** TM-oxygen octahedra
 - . **Frustrated** fcc lattice (eg: Ba_2YReO_6)
 - . Unusual spin-orbit coupled liquids?
 - . Insulating ferrimagnets (eg: Sr_2CrOsO_6 , $T_C=725K$)

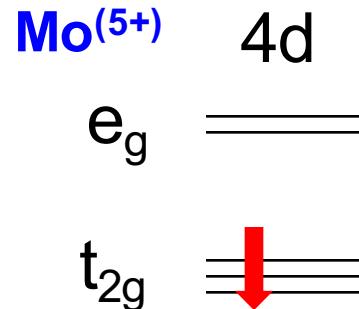
Single atom physics

Nominal valence: $\text{Ba}_2^{(2+)}\text{Fe}^{(3+)}\text{Re}^{(5+)}\text{O}_6^{(2-)}$

$\text{Sr}_2^{(2+)}\text{Fe}^{(3+)}\text{Mo}^{(5+)}\text{O}_6^{(2-)}$



$S=5/2$



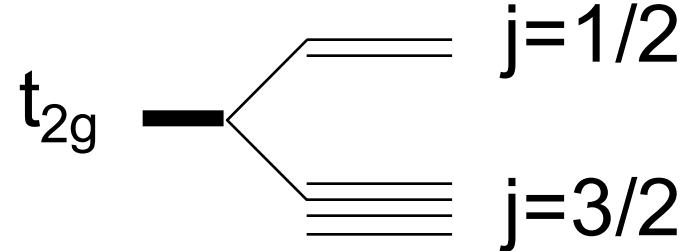
Hund's coupling > crystal field

SOC: $\lambda \sim 100\text{meV}$

Re: Spin orbit coupling in t_{2g} ($L=1$)

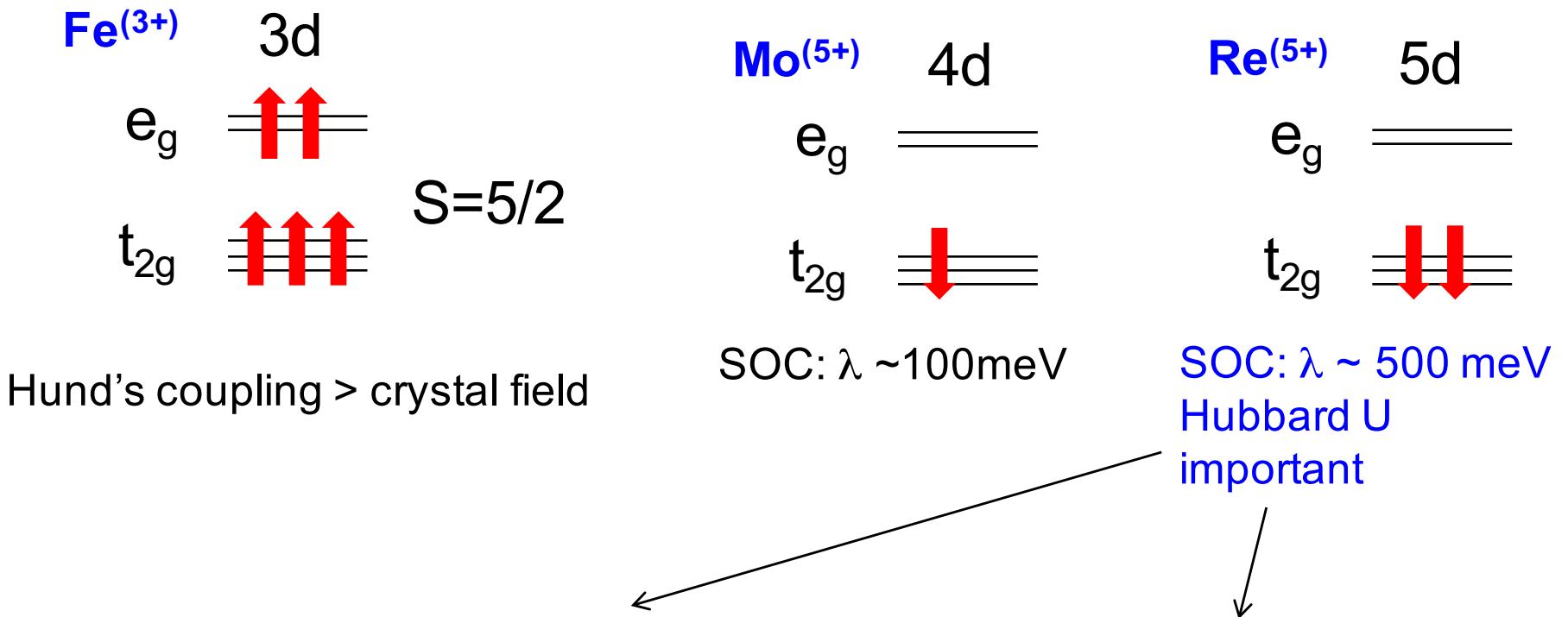
$$P_{t_{2g}} \vec{L} P_{t_{2g}} = -\vec{\ell} \quad (\ell = 1)$$

$$H_{\text{S.O.}} = -\lambda \vec{\ell} \cdot \vec{s}$$



Single atom physics

Nominal valence: $\text{Ba}_2^{(2+)}\text{Fe}^{(3+)}\text{Re}^{(5+)}\text{O}_6^{(2-)}$
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$$P_{t_{2g}} \vec{L} P_{t_{2g}} = -\vec{\ell} \quad (\ell = 1)$$

$$H_{\text{S.O.}} = -\lambda \vec{\ell} \cdot \vec{s}$$

Re: Interactions in t_{2g}

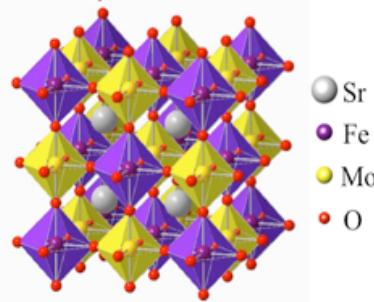
$$H_{\text{int}} = U \sum_{\alpha} n_{\alpha\uparrow} n_{\alpha\downarrow} + \left(U - 5 \frac{J_H}{2}\right) \sum_{\alpha<\beta} n_{\alpha} n_{\beta}$$

$$-2J_H \sum_{\alpha<\beta} \vec{S}_{\alpha} \cdot \vec{S}_{\beta} + J_H \sum_{\alpha \neq \beta} d_{\alpha\uparrow}^{\dagger} d_{\alpha\downarrow}^{\dagger} d_{\beta\downarrow} d_{\beta,\uparrow}$$

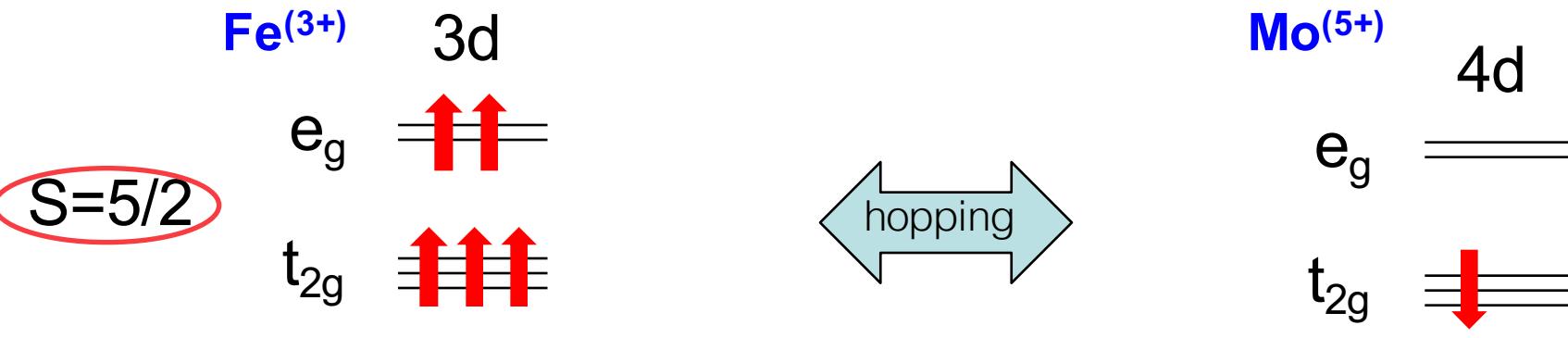
(Kanamori)

Double perovskites: Itinerant perspective

Double perovskite lattice

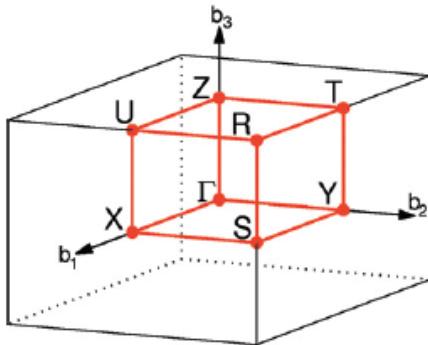


Origin of half-metallicity



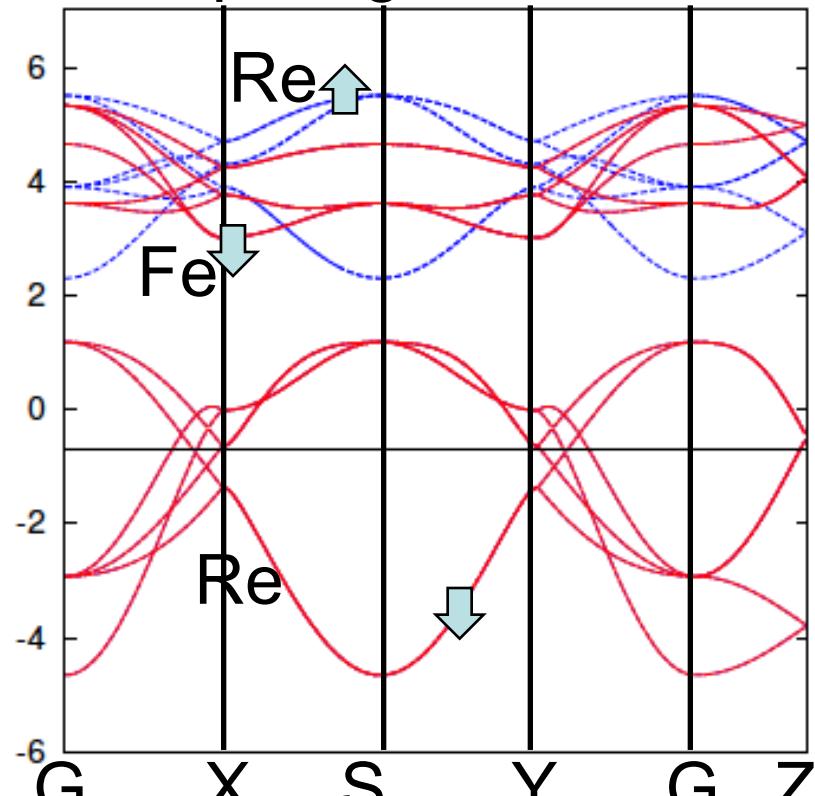
Hund's coupling > crystal field

Electronic model for $\text{Ba}_2\text{FeReO}_6$: Hartree theory

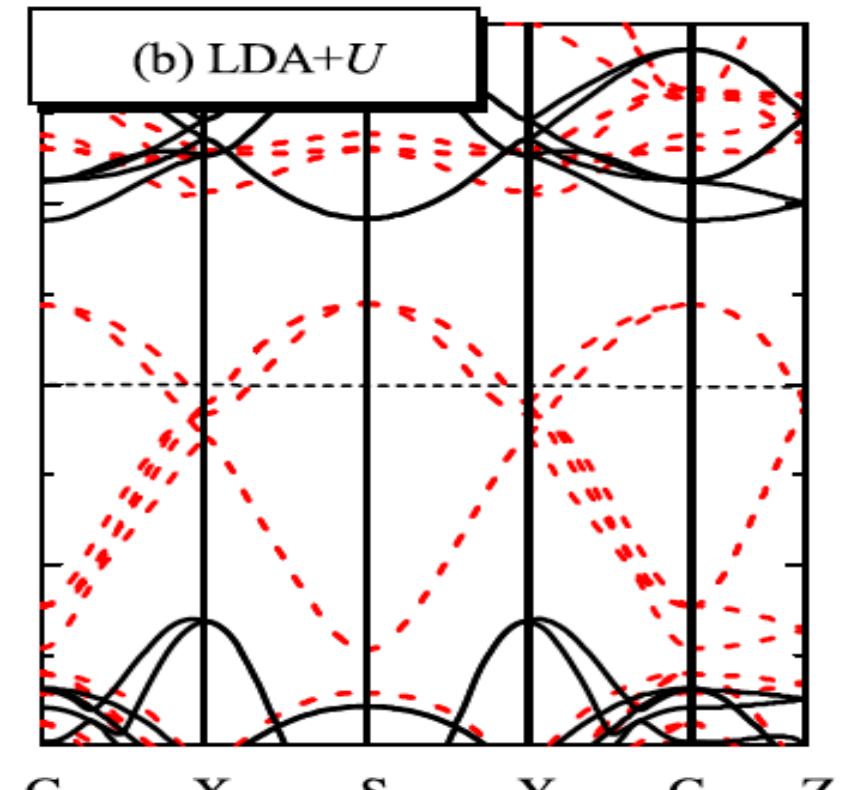


Correlation on Re: Stabilizes half-metallic state
Keep only intra-orbital U
 $t_{\text{Fe-Re}} \sim 330 \text{ meV}$, $U \sim 2.5 \text{ eV}$, $\Delta_{\text{CT}} \sim 1 \text{ eV}$
 $t_{\text{Re-Re}} \sim 100 \text{ meV}$, Other hoppings small $< 50 \text{ meV}$

Comparing Hartree-corrected dispersion with LDA+U



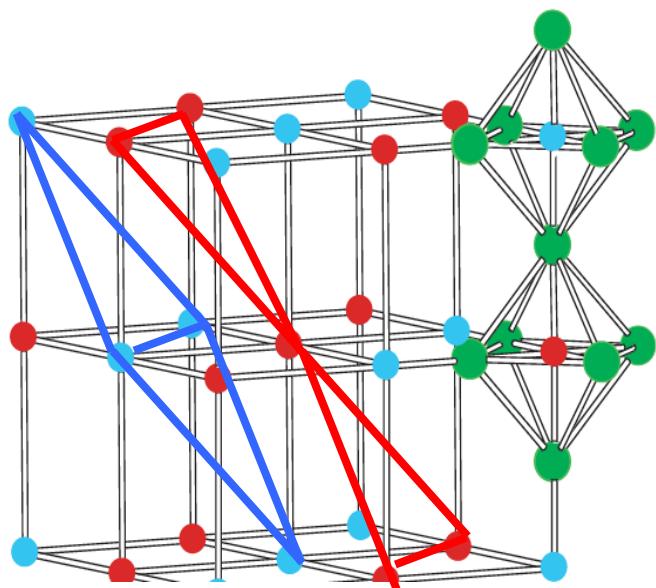
A. Cook, AP (PRB 2013)



B.C. Jeon, T.W. Noh, et al, (JPCM, 2010)

Ultrathin double perovskite films

- What if we dimensionally confine the half-metal to 2D?



Exchange bias in LaNiO₃-LaMnO₃ superlattices

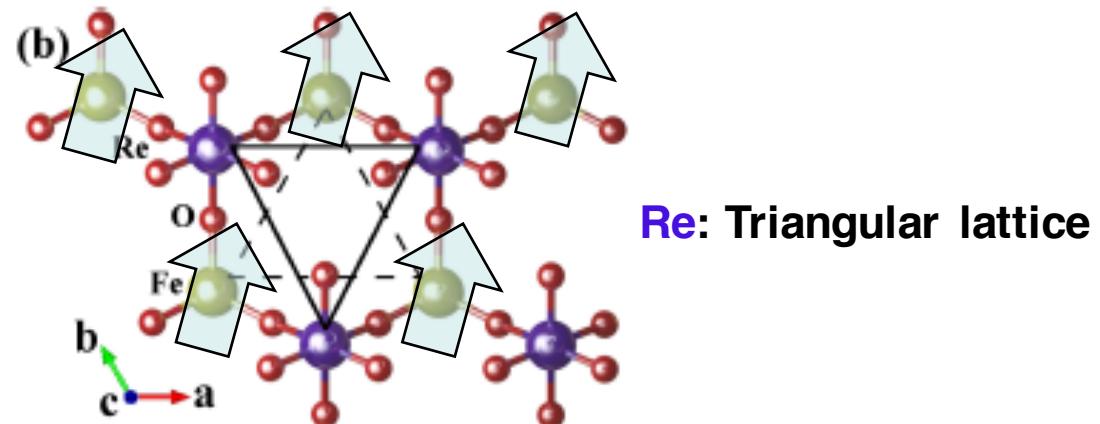
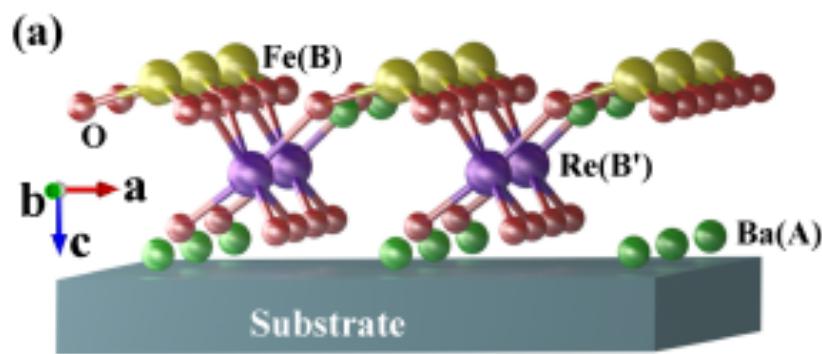
Marta Gibert^{1*}, Pavlo Zubko¹, Raoul Scherwitzl¹, Jorge Íñiguez² and Jean-Marc Triscone¹



Local electronic and magnetic studies of an artificial La₂FeCrO₆ double perovskite
Benjamin Gray, Ho Nyung Lee, Jian Liu, J. Chakhalian, and J. W. Freeland

Citation: *Applied Physics Letters* 97, 013105 (2010); doi: 10.1063/1.3455323

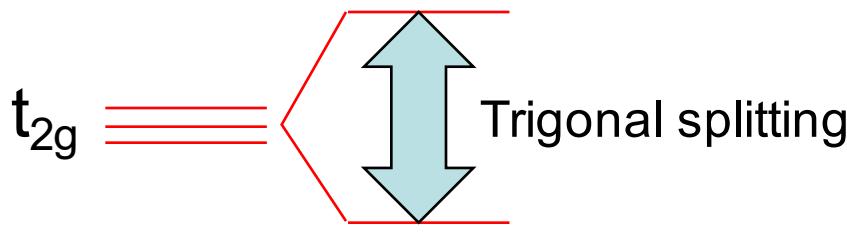
- Bilayer: Buckled honeycomb lattice



Ultrathin double perovskite films

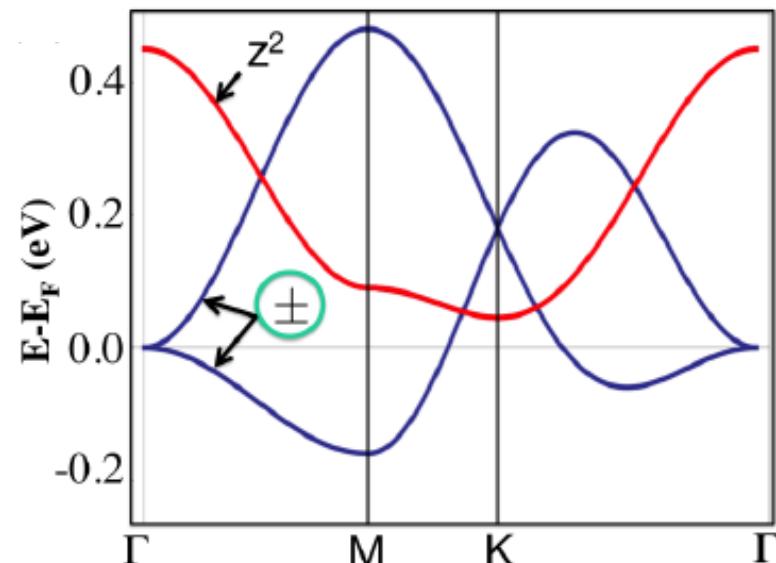
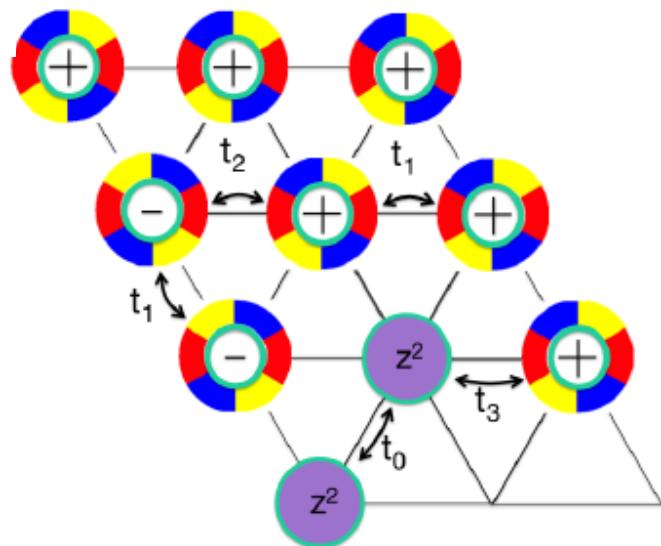
- Nominally Fe^{4+} ($3d^6$) and Re^{4+} ($5d^1$)
- Triangular half-metal

$$L_z = 0 : d_{3z^2-r^2}$$



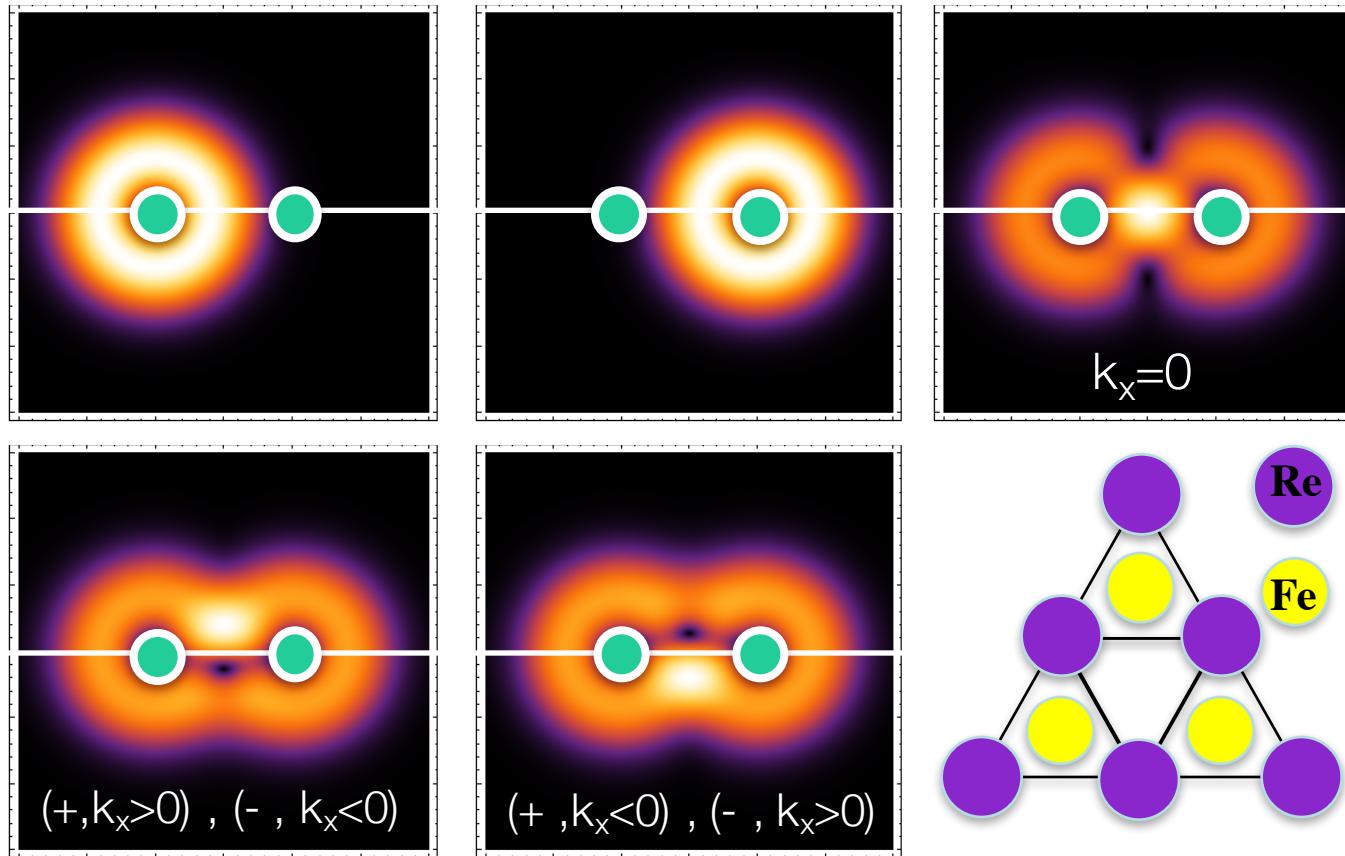
$$L_z = \pm 2 : d_{x^2-y^2} \pm id_{xy}$$

Non-Kramers doublet



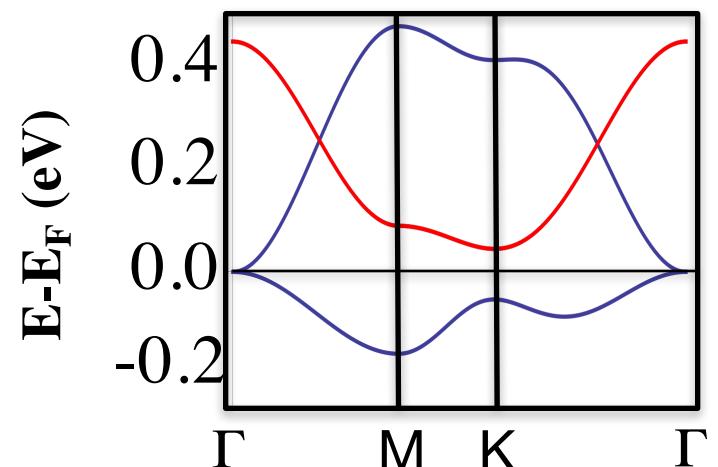
Dirac points at K: Inversion + “Time-reversal”
Quadratic band touching at Γ : C3 + “Time reversal”

“Orbital Dipoles” and the “Orbital Rashba effect”

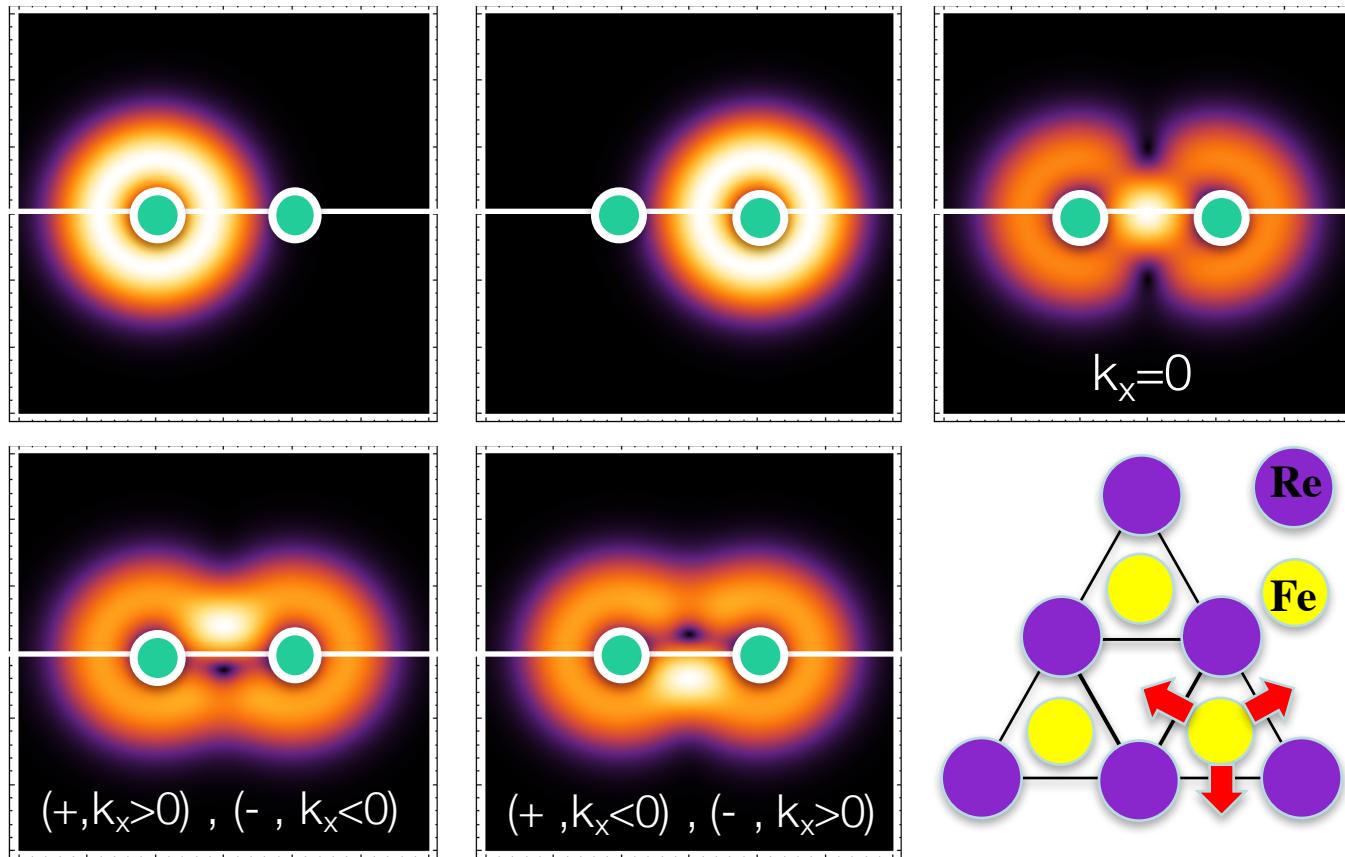


Inversion-symmetry breaking

- “Orbital Rashba” effect gaps out K Dirac point
- Half-semimetal

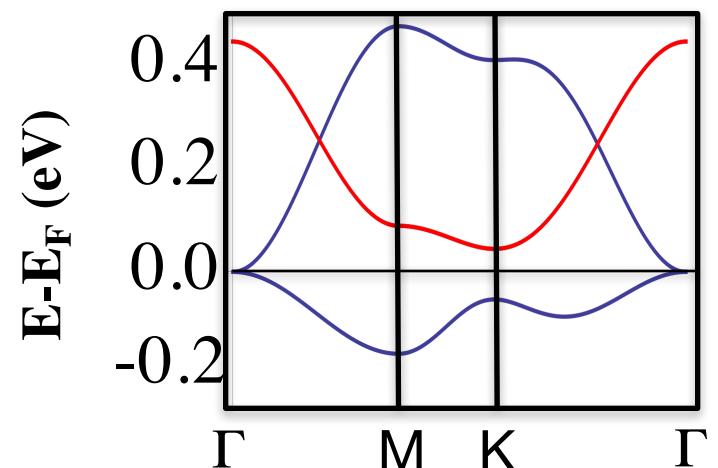


“Orbital Dipoles” and the “Orbital Rashba effect”

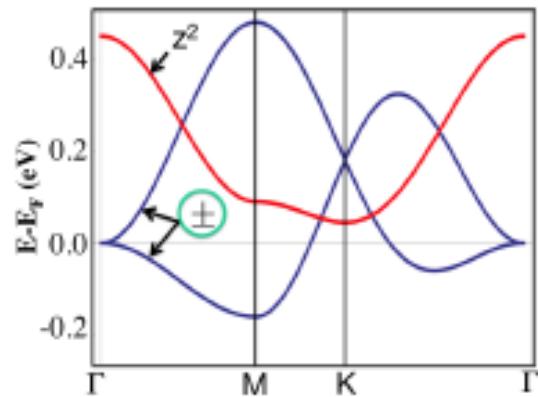


Inversion-symmetry breaking

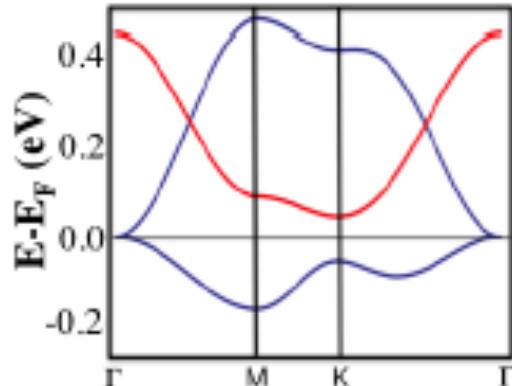
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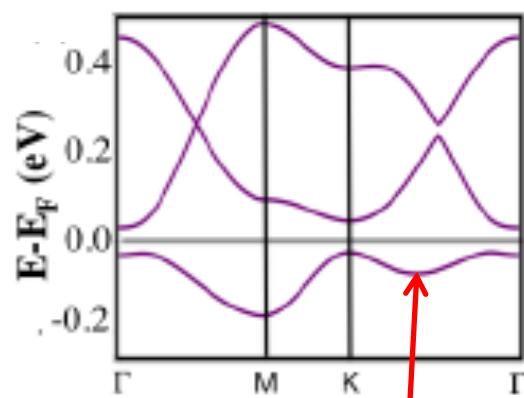
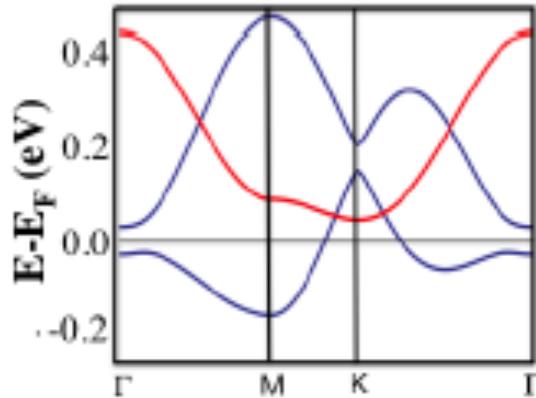
Large gap quantum anomalous Hall insulator



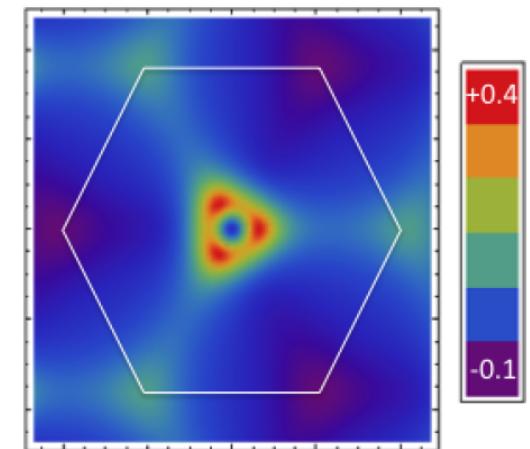
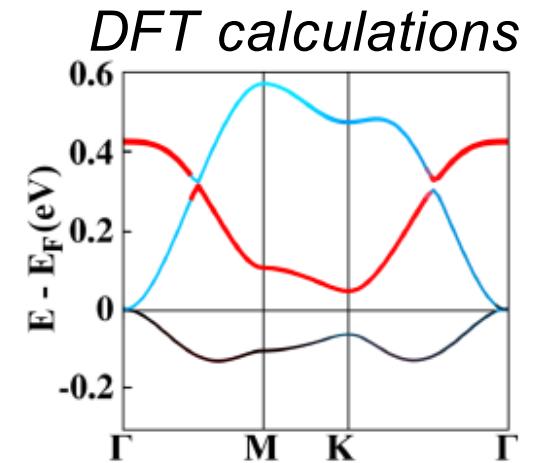
“Orbital
Rashba”
→



↓ SOC



QAH gap ~ 100 meV
Ferromagnetic Tc ~ 300 K



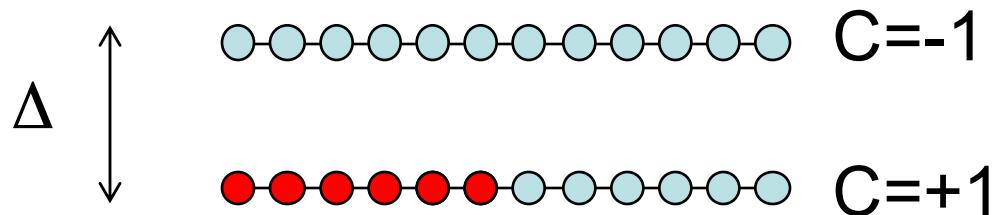
Berry curvature

Outline

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- But strong correlations also leads to Mott localization!
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Exotic phases from topology + interactions

Correlations in Flat Chern Bands

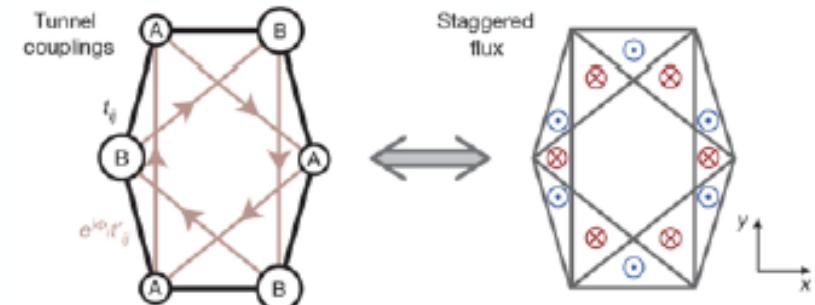
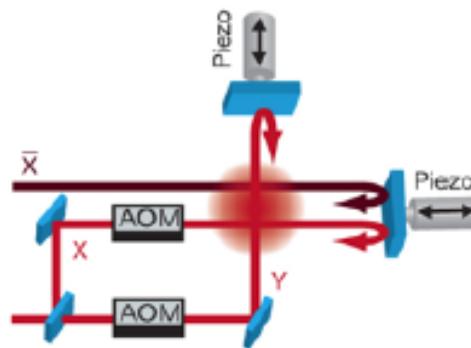


Bosons in nearly flat Chern bands \sim Landau levels

- Bandwidth W : $W \ll \Delta$
- Interaction U : $W \ll U \ll \Delta$
- $v=1/2$ fractional QHE of bosons

D.N. Sheng, et al, Nat Comm 2011, PRL 2011, PRL 2012
T. Neupert, et al, PRL 2011

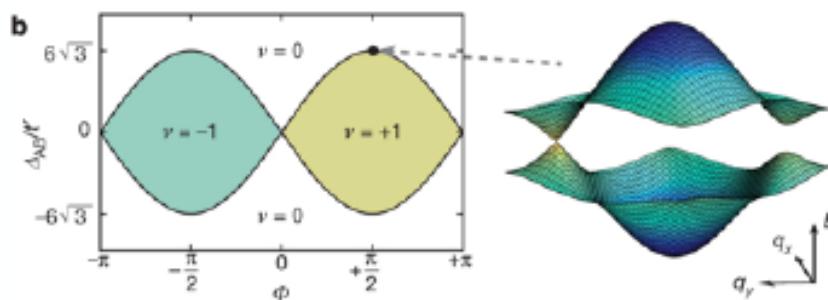
Chern bands for ultracold atoms: Atomic Haldane model



Periodic Lattice Shaking

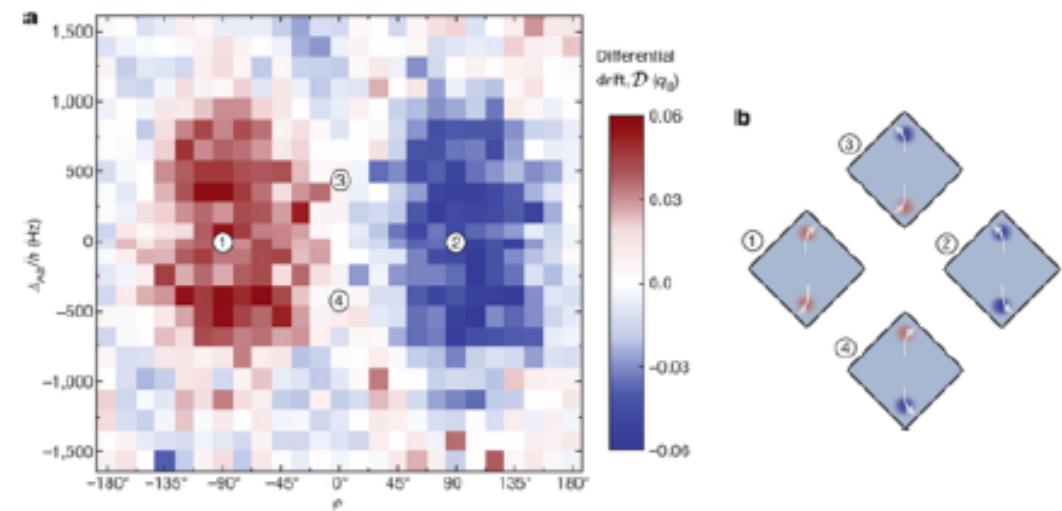


Effective Staggered Flux Pattern
Second neighbor hopping = $t_2 e^{i\phi}$



Location of gap closing points
measured using Bloch oscillations

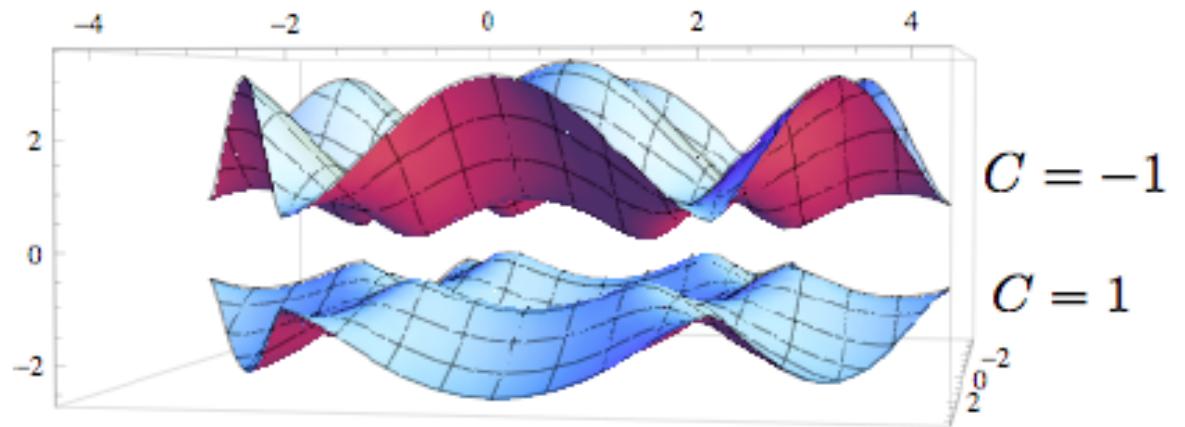
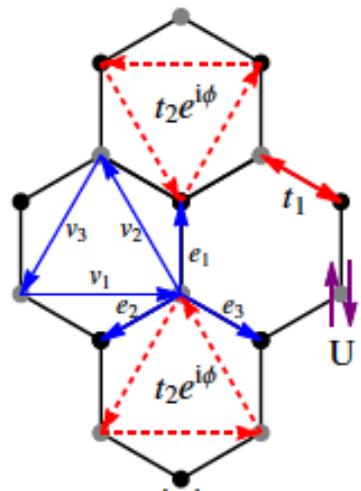
What happens with interactions?



Berry curvature at Dirac points measured
using differential drift

Esslinger group: G. Jotzu, et al, Nature 2014

Equilibrium Haldane-Hubbard model of spin-1/2 fermions

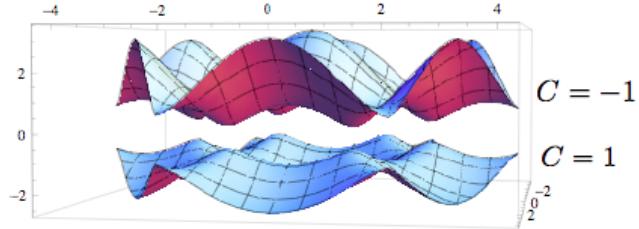


- Consider 2 flavors: \uparrow, \downarrow
- Fill up lower Chern band: $\sigma_{xy} = 2e^2/h$ (“2” due to spin)
- Local interactions between \uparrow and \downarrow : “Haldane-Hubbard” model

$$H_{\text{HH}} = -t_1 \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) - t_2 \sum_{\langle\langle ij \rangle\rangle \sigma} (e^{i\nu_{ij}\phi} c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- What happens as we crank up interactions?
- What is the nature of magnetism in the Mott insulator?

Is the Mott Insulator an exotic spin liquid?



Mott insulator: View Spin \uparrow = Boson, Spin \downarrow = Empty

Chiral Spin Liquid = Bosonic Laughlin QH state

$$\Psi \sim \prod (z_i - z_j)^2 \quad z \equiv \text{location of } \uparrow$$

Theoretical proposals –

Kalmeyer, Laughlin (PRL 1987)

X. G. Wen, F. Wilczek, A. Zee (1989)

D. F. Schroeter, E. Kapit, R. Thomale, M. Greiter (PRL 2007, 2009)

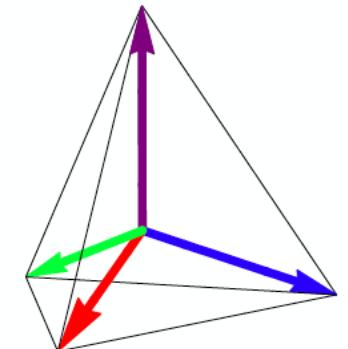
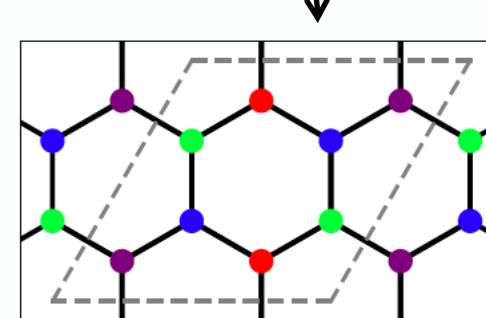
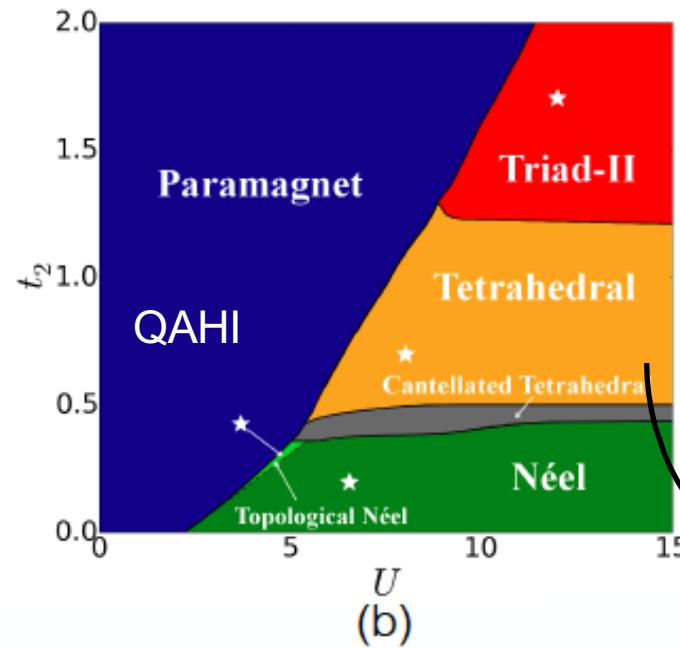
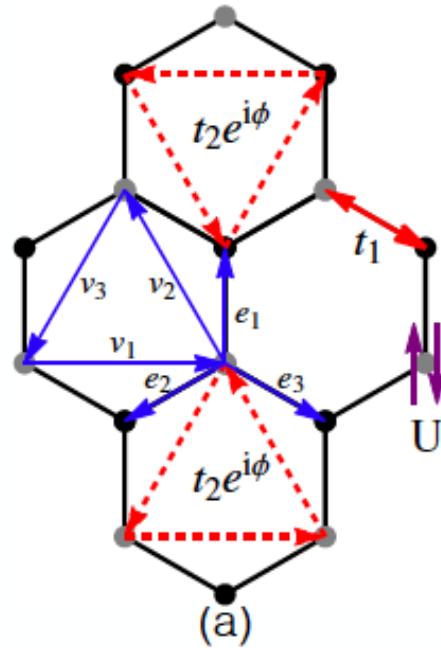
Y. Zhang, T. Grover, A. Vishwanath PRB 2011

J. He, et al, PRB 2011, 2012 - Slave rotor mean field theory: Chiral spin liquid ($v=1/2$ bosonic Laughlin state)

J. Maciejko, A. Ruegg, PRB 2013 - Exotic Z_2 CI* phase at intermediate U

D. Prychynenko, S. Huber arXiv:1410.2001 – Topological SDW order (with sublattice imbalance)

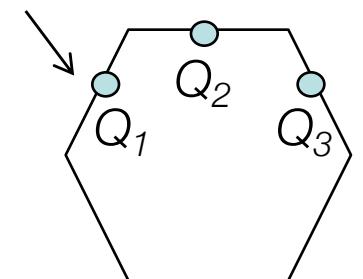
Hartree Mean Field Theory



- Triple-Q noncoplanar state
- Large scalar spin chirality: “Skyrmion crystal”
- “Regular” Magnetic Order

V.S.Arun, R. Sohal, C. Hickey, AP (PRB 2016)

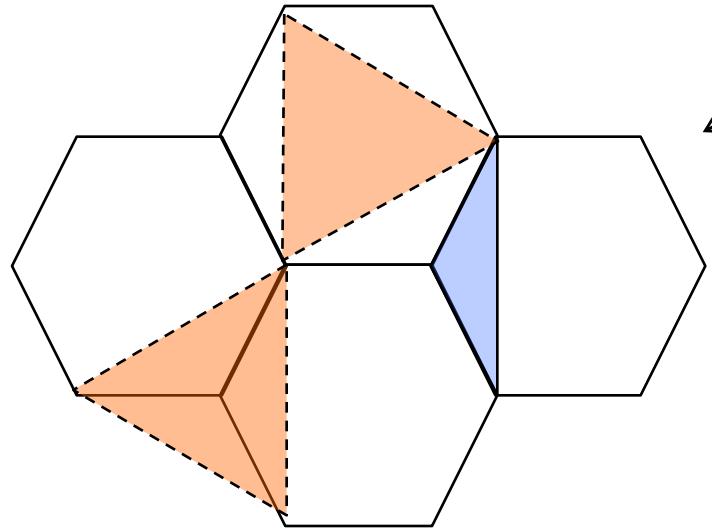
L. Messio, C. Lhuillier, G. Misguich (PRB 2011)



Strong coupling limit: Chiral spin interactions

$$H_{\text{spin}} = \frac{4t_1^2}{U} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{4t_2^2}{U} \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$-\frac{24t_1^2 t_2}{U^2} \sum_{\text{small}-\Delta} \hat{\chi}_{\Delta} \sin \Phi_{\Delta} - \frac{24t_2^3}{U^2} \sum_{\text{big}-\Delta} \hat{\chi}_{\Delta} \sin \Phi_{\Delta}$$

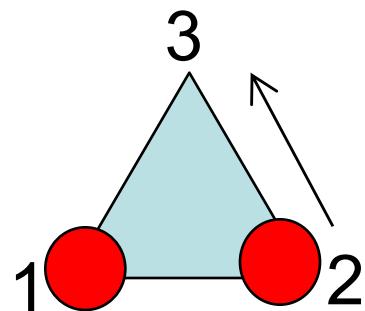


$$\chi_{\Delta} = \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3$$

Scalar spin chirality

Boson language: Correlated hopping

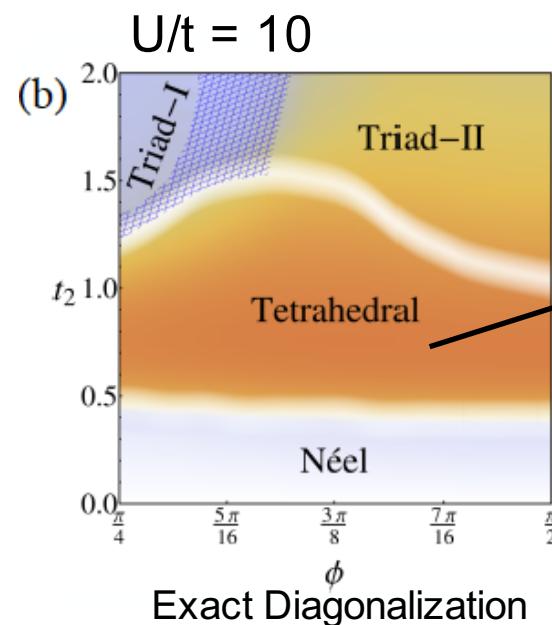
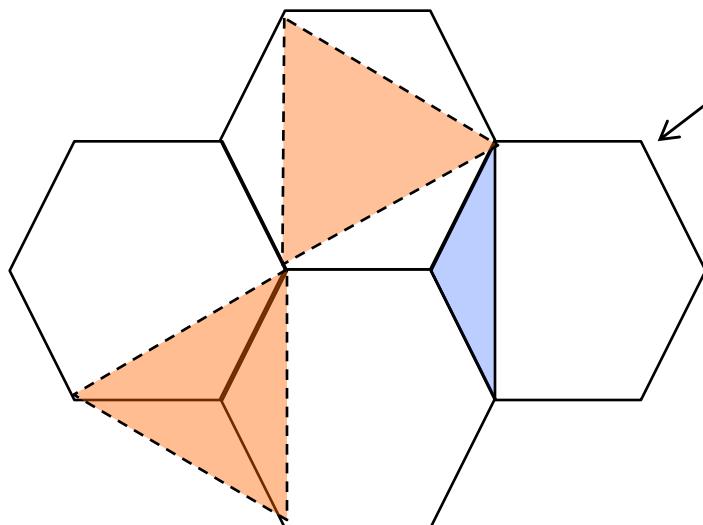
$$in_1(b_2^\dagger b_3 - b_3^\dagger b_2)$$



Strong coupling limit: Chiral spin interactions

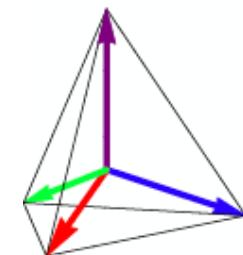
$$H_{\text{spin}} = \frac{4t_1^2}{U} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{4t_2^2}{U} \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$-\frac{24t_1^2 t_2}{U^2} \sum_{\text{small}-\Delta} \hat{\chi}_{\Delta} \sin \Phi_{\Delta} - \frac{24t_2^3}{U^2} \sum_{\text{big}-\Delta} \hat{\chi}_{\Delta} \sin \Phi_{\Delta}$$



$$\chi_{\Delta} = \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3$$

Scalar spin chirality



Tetrahedral state
= Boson supersolid

- Classical ground states
- Exact diagonalization
- Ground state: magnetically ordered!

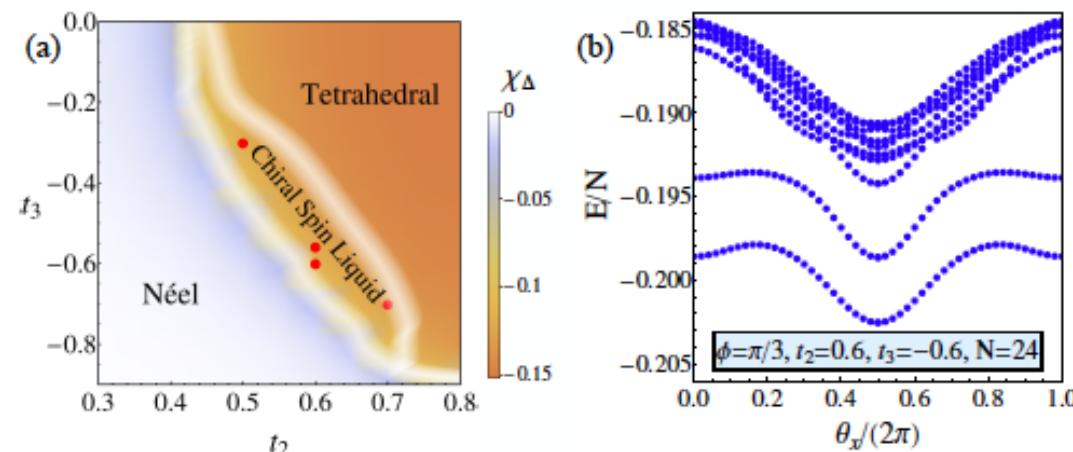
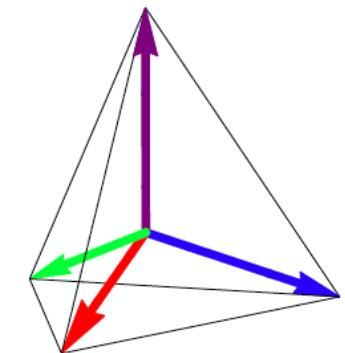
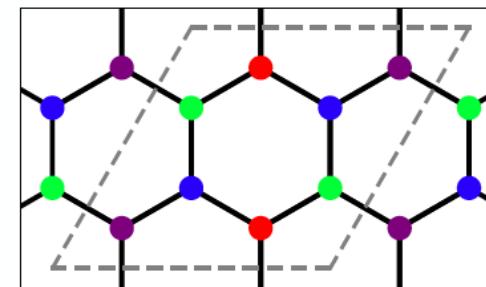
C. Hickey, L. Cincio, Z. Papić, AP (PRL 2016)

Cf: Nielsen, Sierra, Cirac, Nat. Comm. 2013 – Square lattice Chern insulator yields a CSL Mott insulator

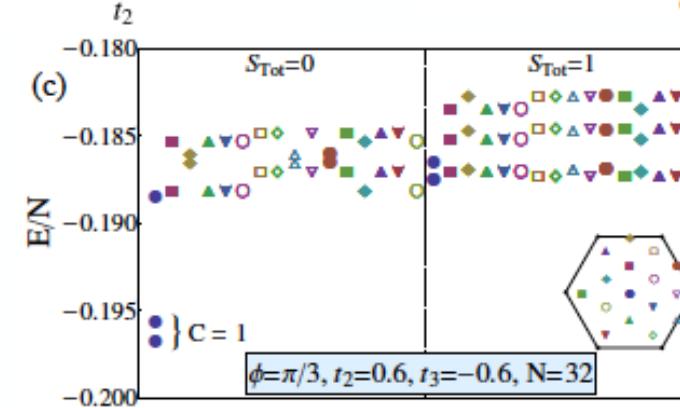
Cf: B. Bauer, et al, Nature Communications 5, 5137 (2014) – Kagome chiral terms give CSL Mott insulator

Melting the Tetrahedral Order

Add 3rd neighbor AFM interaction
Frustrates Tetrahedral order
Creates a CSL retaining large chirality!



Exact diagonalization
results: N=18,24,32



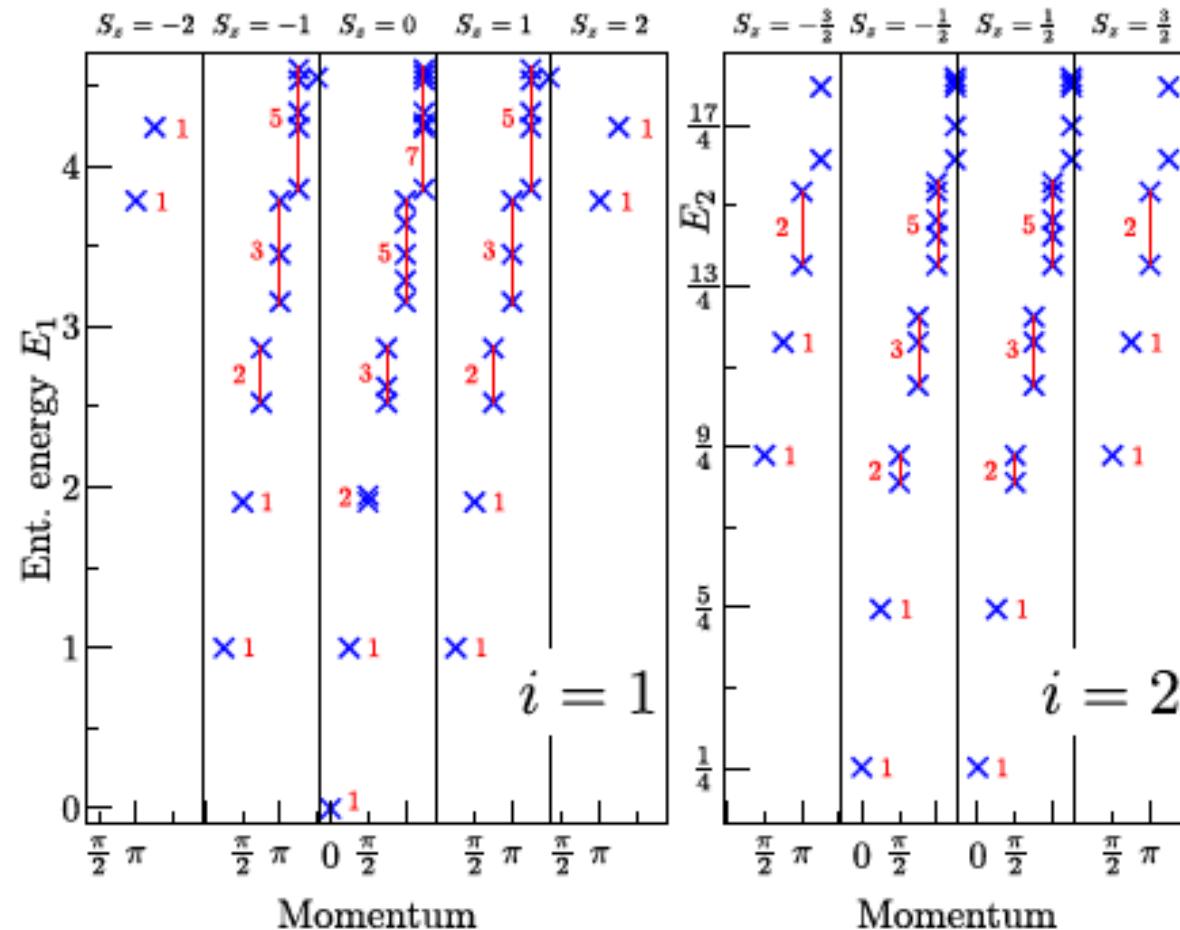
Bose supersolid -> FQH liquid

C. Hickey, L. Cincio, Z. Papić, AP (PRL 2016)

Infinite cylinder DMRG results

Y. Zhang, et al PRB 2013

L. Cincio and G. Vidal, PRL 2013

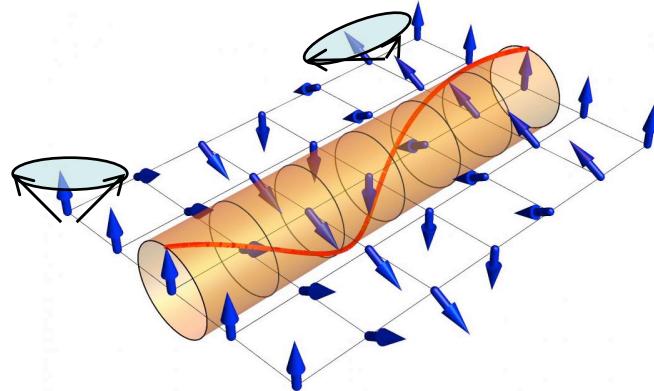
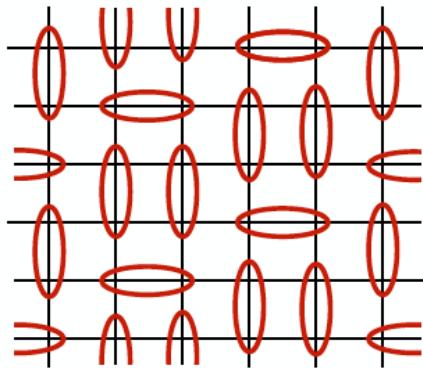


Agrees with chiral semion

Routes to Quantum Spin Liquids

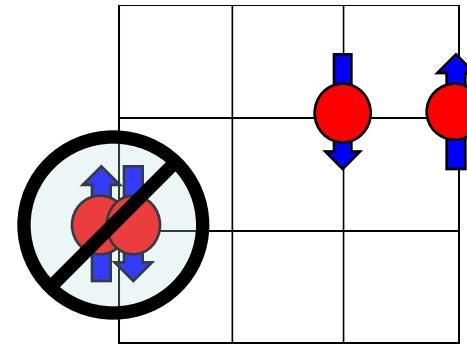
Resonating valence bonds

- Quantum dimer models
- Melting valence bond crystals



Disordering magnetically ordered states

Gutzwiller projected band electrons
1 electron per site -> Variational spin wavefunction



Variational Monte Carlo studies or
slave particle gauge theory approaches

Could there be a continuous
Tetrahedral – Chiral Spin Liquid
quantum phase transition?

Spin crystallization transition out of the CSL

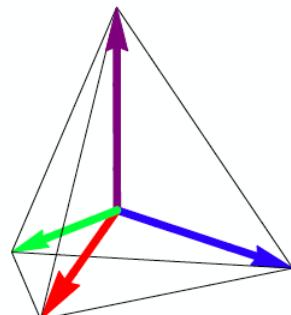
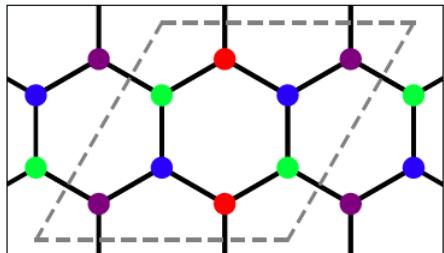
- Low energy theory of CSL: Pure topological Chern Simons theory

$$\mathcal{L}_{\text{CS}} = \frac{1}{2\pi} \epsilon^{\alpha\beta\lambda} a_\alpha \partial_\beta a_\lambda \quad (\text{Torus degeneracy}=2)$$

- Need gapped excitations in CSL: Semions
- Like to examine continuous transition to a magnetically ordered state

Minimally couple bosonic spinons to CS gauge field

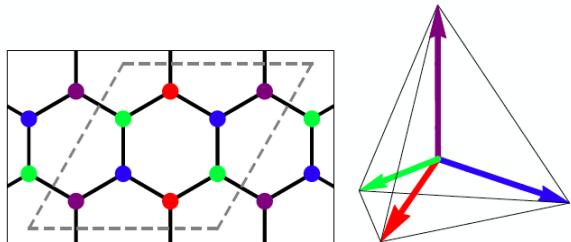
- In the CSL: Bosonic spinons bind π -flux to give semions
- Out of CSL: Condensing spinons can yield magnetic order



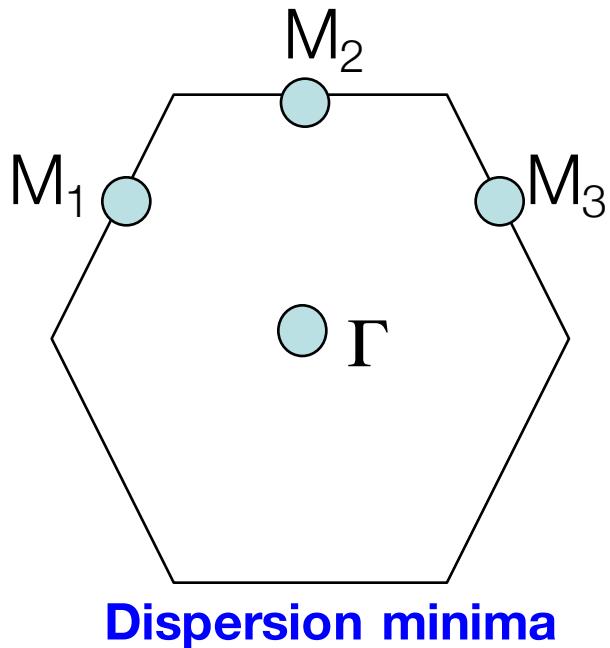
Adiabatic spinon transport: Berry Fluxes

Triangular loops: $\pi/2$
Hexagonal loops: π

Spin crystallization transition out of the CSL



Hofstadter problem
Triangular loops: $\pi/2$
Hexagonal loops: π

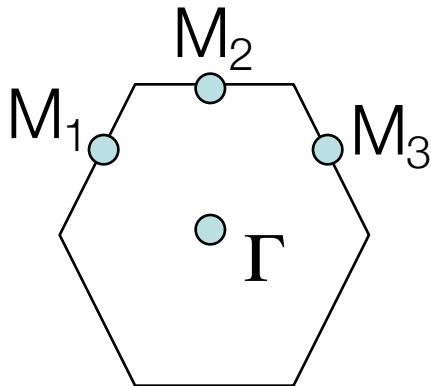


Suffices to focus on one Δ sublattice
Define spinon fields: $\phi_{i,\sigma}$
 i = Mode label (1-4)
 σ = Spin label

$$\begin{aligned}\mathcal{L}_{CS,\phi} = & \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \phi_{i\alpha}^* (\partial_\tau - ia_0) \phi_{i\alpha} + r |\phi_{i\alpha}|^2 \\ & + |(\vec{\nabla} - i\vec{d}) \phi_{i\alpha}|^2\end{aligned}$$

- Spinon condensates are “frustrated”: Lots of ways to condense into 4 minima
- Degeneracy will be broken by interaction effects

Spin crystallization transition out of the CSL



$$\mathcal{L}_{\text{CS},\phi} = \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \phi_{i\alpha}^* (\partial_\tau - ia_0) \phi_{i\alpha} + r |\phi_{i\alpha}|^2 + |(\vec{\nabla} - i\vec{d}) \phi_{i\alpha}|^2$$

$$\rho_i = \phi_{i\alpha}^* \phi_{i\alpha}$$

$$\vec{\mathcal{S}}_i = \phi_{i\alpha}^* \vec{\sigma}_{\alpha\beta} \phi_{i\beta}$$

$u_2 < 0$ favors multimode

Fourth order interactions:

$$\begin{aligned} \mathcal{L}_{\text{int}}^{(1)} = & u_1 \left(\sum_i \rho_i \right)^2 + u_2 \sum_{i \neq j} \rho_i \rho_j + u_3 \sum_{i \neq j} \vec{\mathcal{S}}_i \cdot \vec{\mathcal{S}}_j \\ & + u_4 \sum_{[ijkl]} \phi_{i\alpha}^* \phi_{j\beta}^* \phi_{k\alpha} \phi_{l\beta} + u_5 \sum_{i \neq j} \phi_{i\alpha}^* \phi_{i\beta}^* \phi_{j\alpha} \phi_{j\beta} \end{aligned}$$

Sixth order interactions:

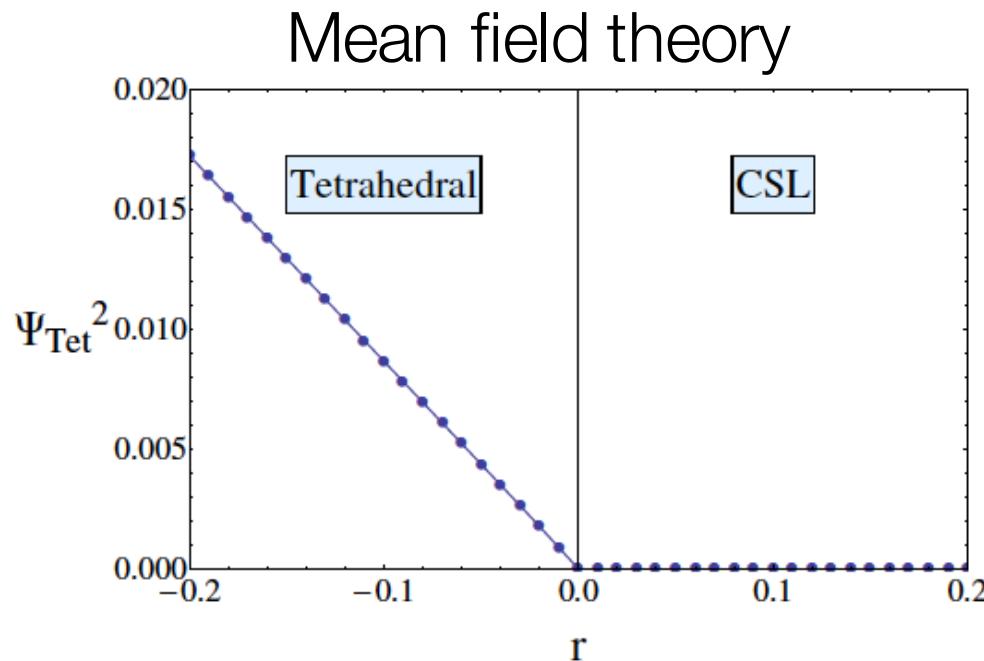
Favors tetrahedral order

$$\mathcal{L}_{\text{int}}^{(2)} = w_1 \left(\sum_i \rho_i \right)^3 + w_2 \sum_{i,j,k} \epsilon^{ijk} \vec{\mathcal{S}}_i \cdot (\vec{\mathcal{S}}_j \times \vec{\mathcal{S}}_k) + \dots$$

“Momentum space skyrmion crystal”

Spin crystallization transition out of the CSL

Chern-Simons-Higgs theory



- The CSL arises from frustration induced melting of tetrahedral order
- We formulate a Chern-Simons-Higgs theory of the phase transition

Summary

- 2D double perovskites: $\text{Sr}_2\text{FeMoO}_6$, $\text{Ba}_2\text{FeReO}_6$
 - Topological phases including emergent Chern bands
 - C=1 quantum anomalous Hall insulators
- Chern bands and interactions
 - Novel magnetic orders
 - Melting noncoplanar order: route to chiral spin liquid