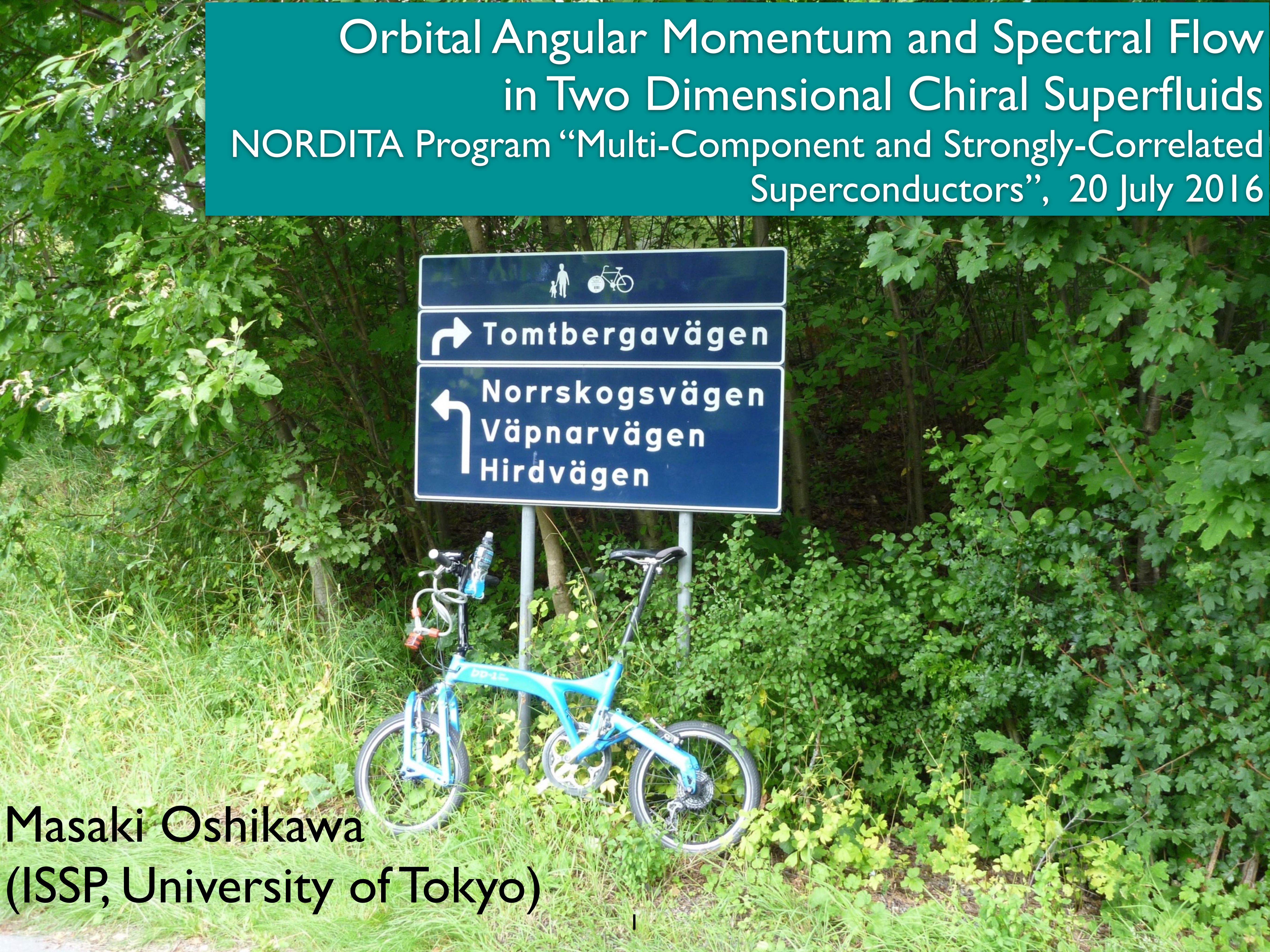
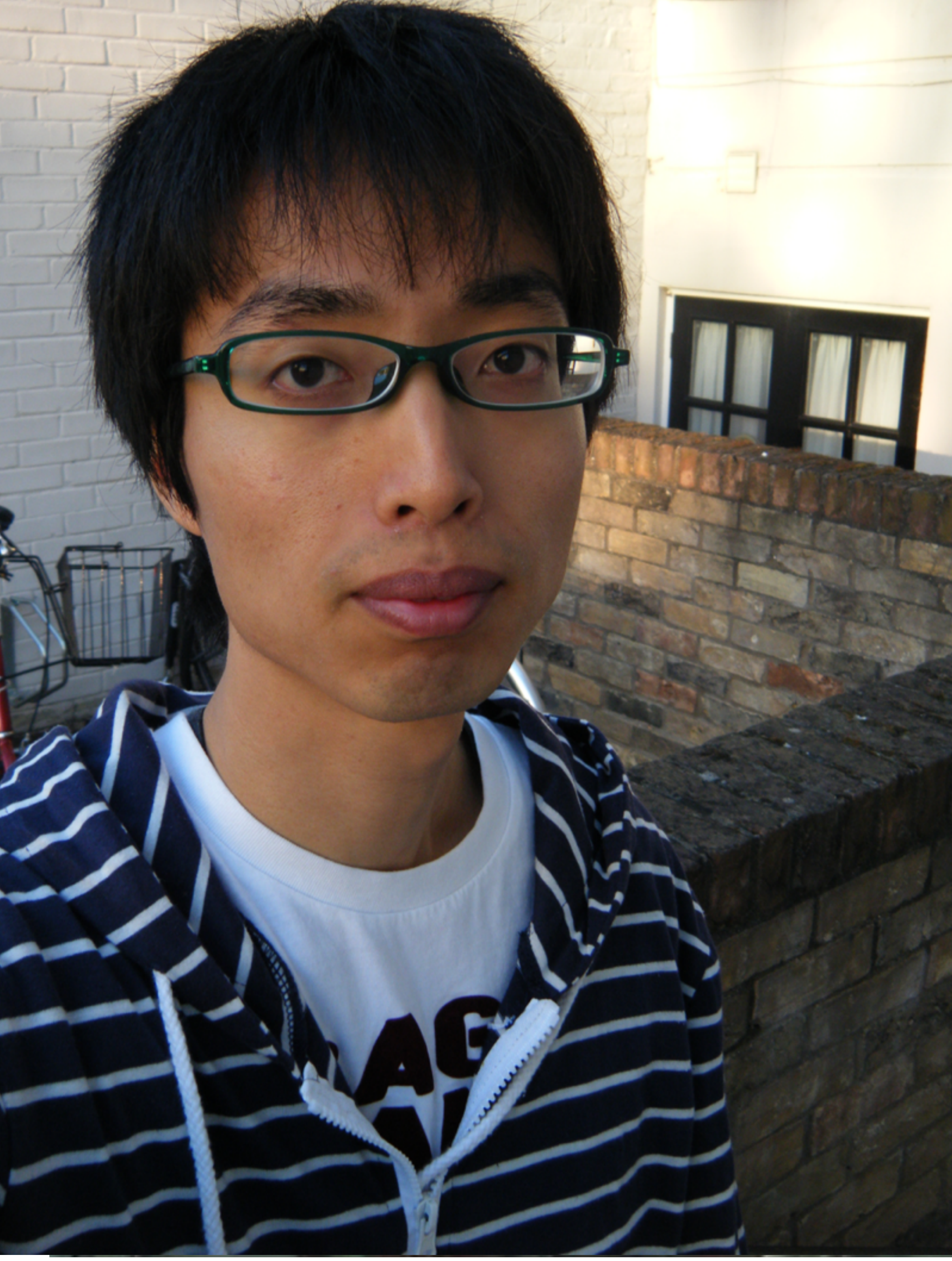
Orbital Angular Momentum and Spectral Flow in Two Dimensional Chiral Superfluids

NORDITA Program “Multi-Component and Strongly-Correlated Superconductors”, 20 July 2016



Masaki Oshikawa
(ISSP, University of Tokyo)



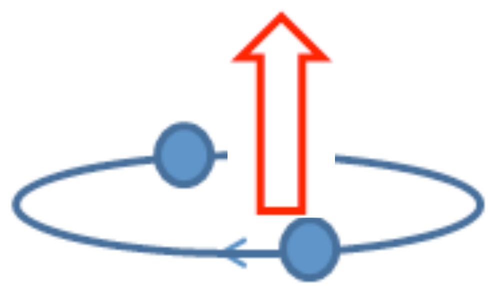
Thanks to:
Yoshi Maeno
for discussion

Yasuhiro Tada

Wenxing Nie

Phys. Rev. Lett. 114, 195301 (2015)

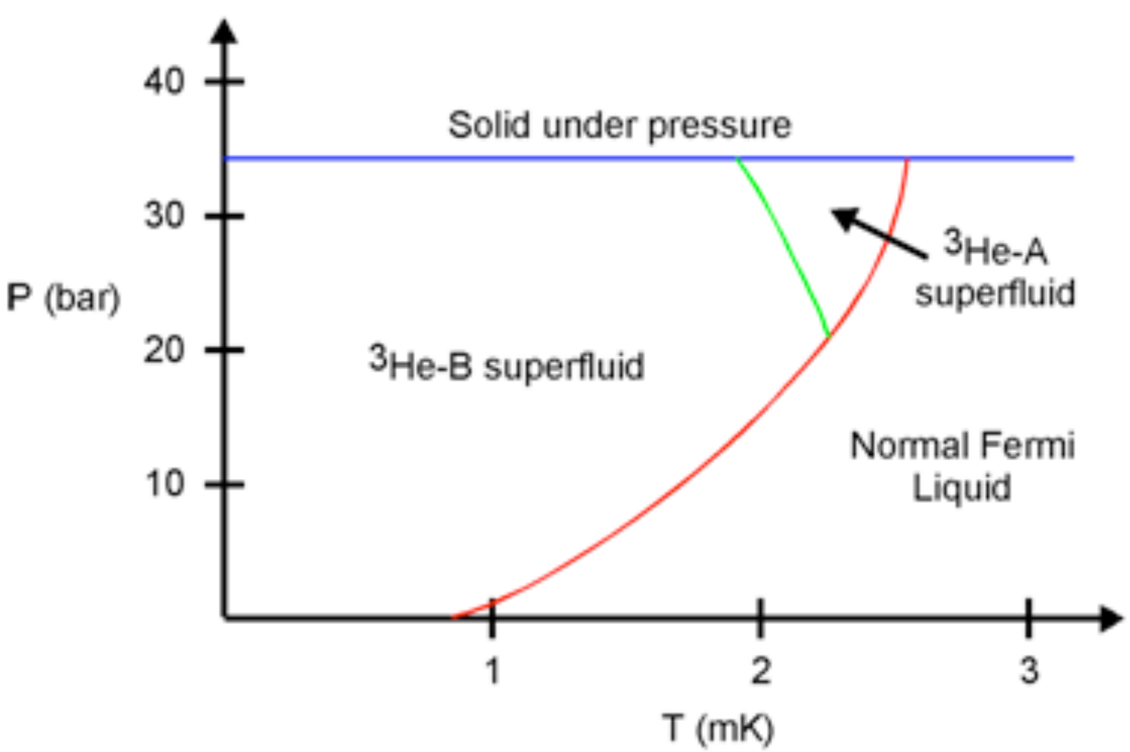
Chiral Superfluid



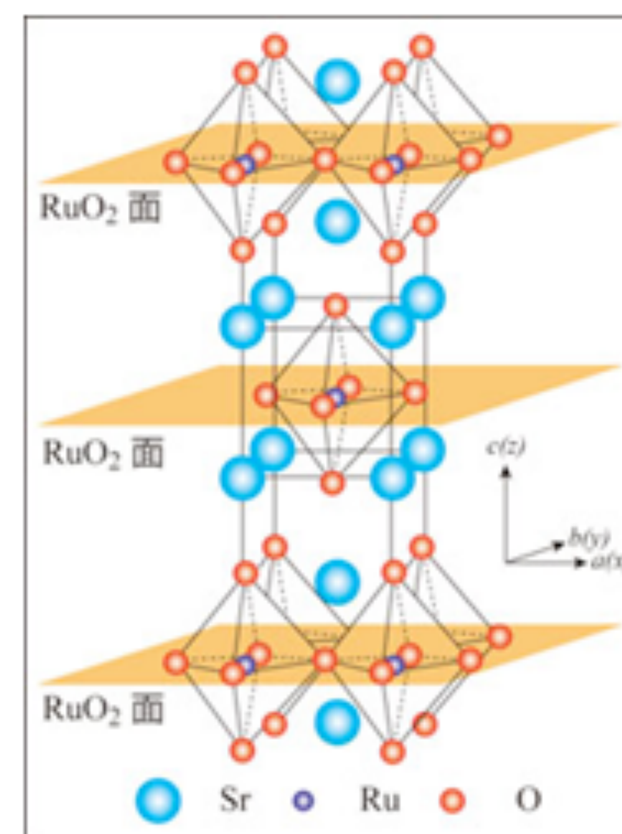
Cooper pair with
definite angular momentum $l_z=\nu$

pairing amplitude $\Delta \sim (p_x + ip_y)^\nu$

A-phase of superfluid ^{3}He



Superconducting phase of Sr_2RuO_4?



d+id

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **89**, 020509(R) (2014)

Chiral *d*-wave superconductivity in SrPtAs

Mark H. Fischer,[1,2] Titus Neupert,[3,4] Christian Platt,[5] Andreas P. Schnyder,[6] Werner Hanke,[5] Jun Goryo,[7]
Ronny Thomale,[5] and Manfred Sigrist[4]

Intrinsic Angular Momentum

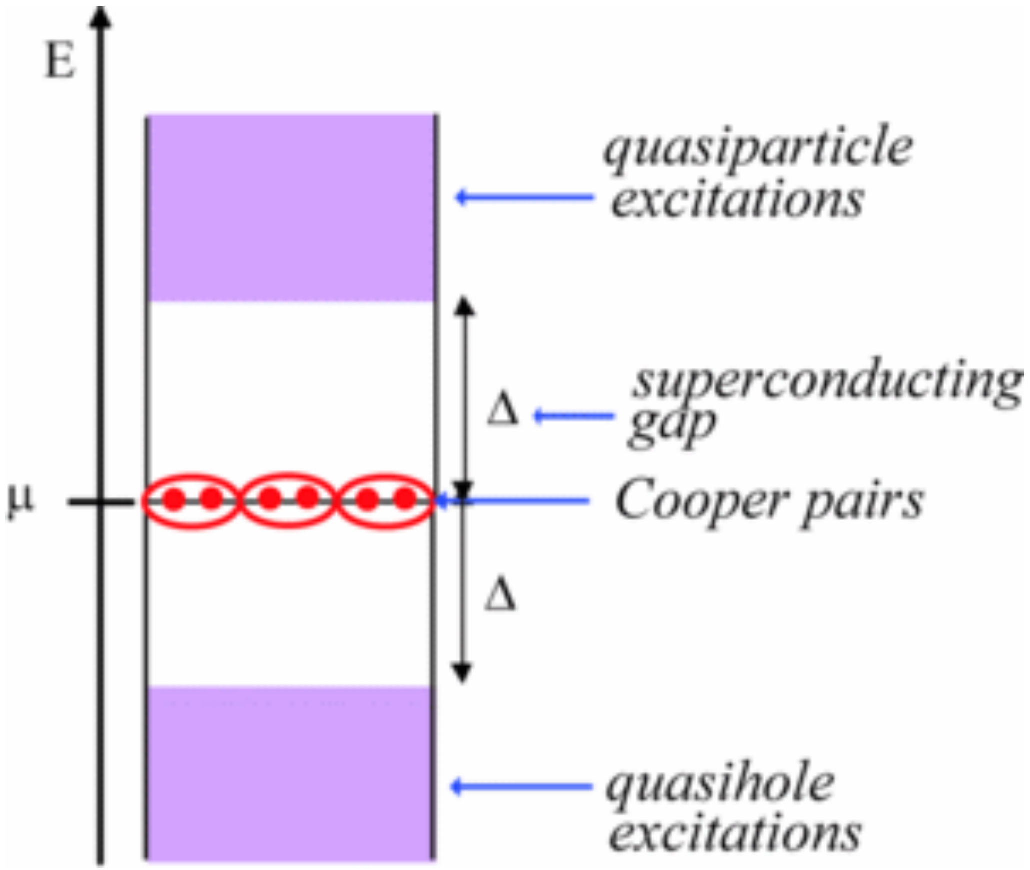
Consider a chiral superfluid in 2D

Q: What is the total angular momentum L of the superfluid consisting of N fermions?

A1: Each Cooper pair has angular momentum ν

Therefore $L = \nu \times N/2 = \nu N/2$

Alternative View



In BCS superconductor, usually

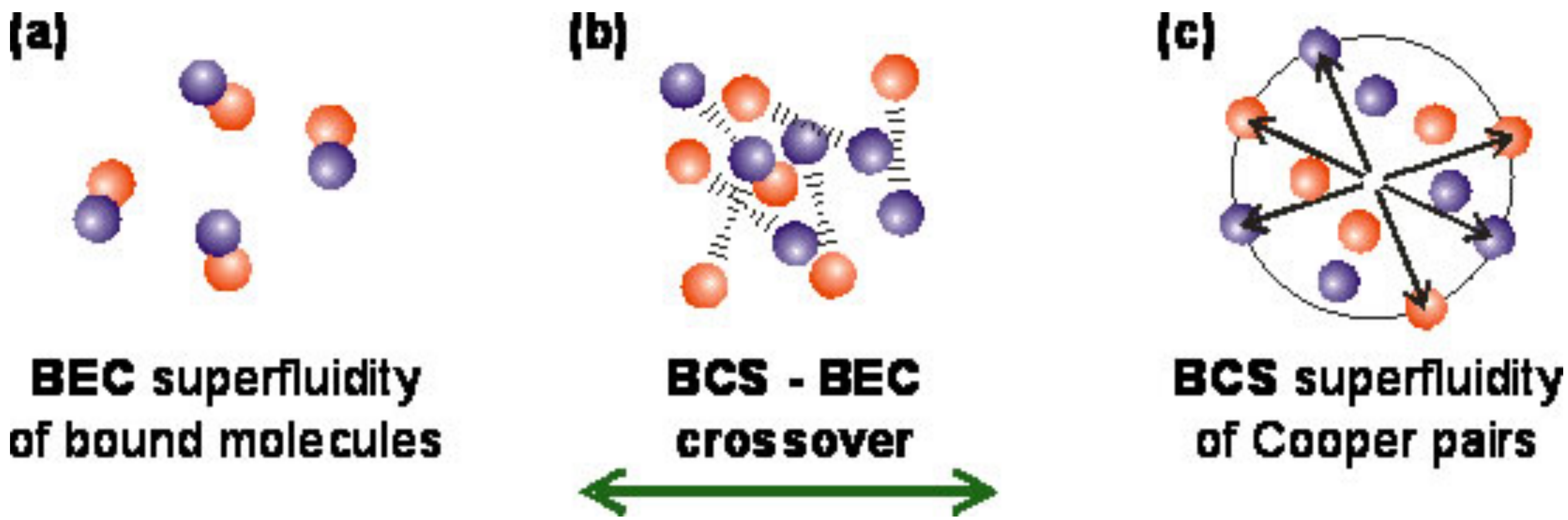
$$\Delta \ll E_F$$

only the fermions near the Fermi surface would be affected?

A2: Only the fermions near the Fermi surface contribute to *L*, so

$$L = \nu \frac{N}{2} \left(\frac{\Delta}{E_F} \right)^{\gamma} \quad \gamma > 0$$

BEC vs BCS limits



[taken from Greiner Lab@Harvard web page]

"BEC" limit: each "molecule" has angular momentum ν
total angular momentum $L = \nu N/2$ naturally expected
"BCS" limit: less clear

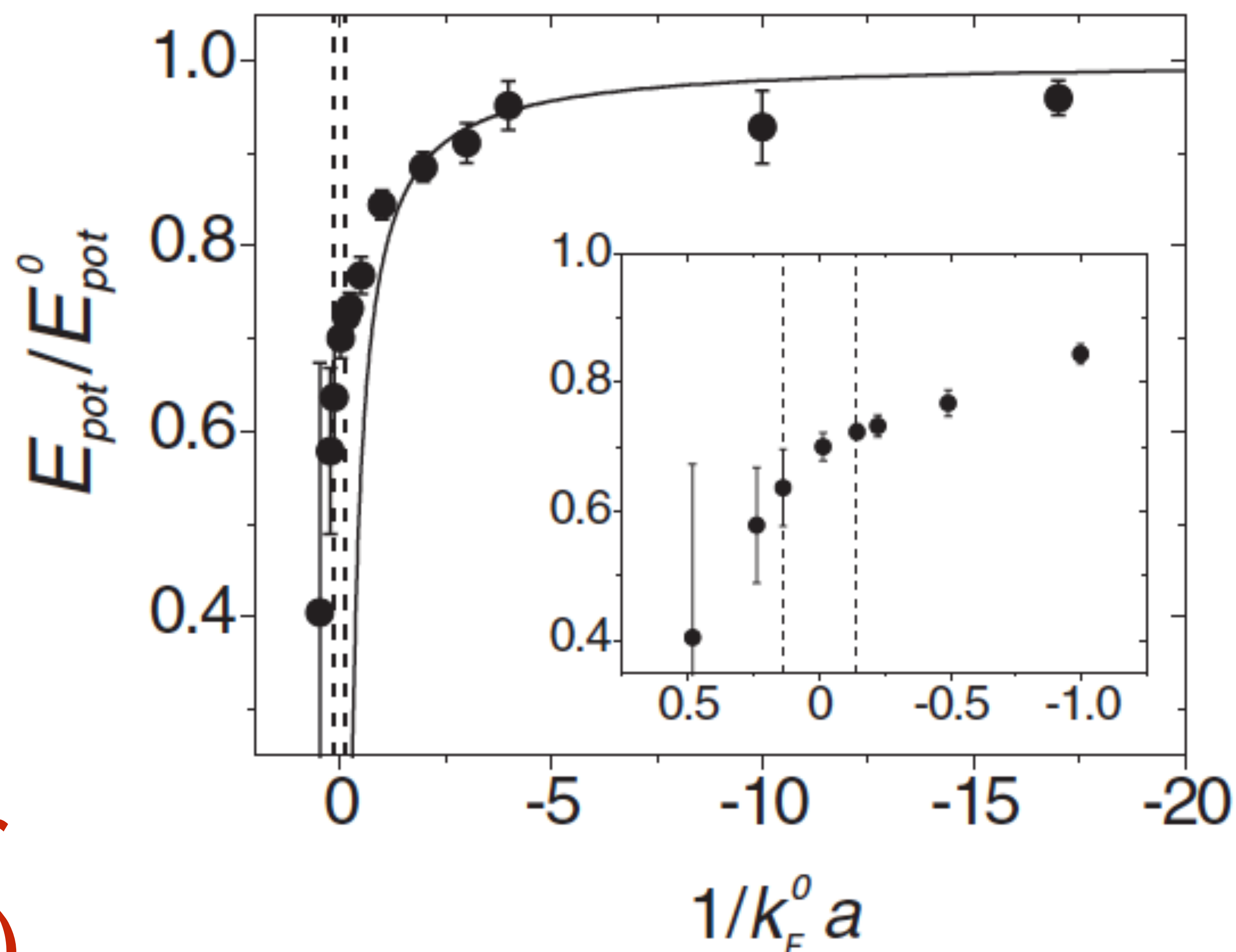
PRL 97, 220406 (2006) PHYSICAL REVIEW LETTERS week ending 1 DECEMBER 2006

Potential Energy of a ^{40}K Fermi Gas in the BCS-BEC Crossover

J. T. Stewart,* J. P. Gaebler, C. A. Regal, and D. S. Jin

JILA, Quantum Physics Division, National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440, USA†

(Received 28 July 2006; published 30 November 2006)



s-wave case:
(zero angular momentum Cooper pairs)

BCS-BEC crossover
(no phase transition)

BCS vs BEC in Chiral SFs

PHYSICAL REVIEW B VOLUME 61, NUMBER 15 15 APRIL 2000-I

Paired states of fermions in two dimensions with breaking of parity and time-reversal symmetries and the fractional quantum Hall effect

N. Read and Dmitry Green
Departments of Physics and Applied Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120
(Received 30 June 1999)

chiral *p+ip* (*d+id*, *f+if*, ...) superfluid in 2D:
quantum phase transition between
BEC ("strong-pairing", non-topological) and
BCS ("weak-pairing", topological) phases

L **could be different in two phases!**

Studies on *p*+*ip*

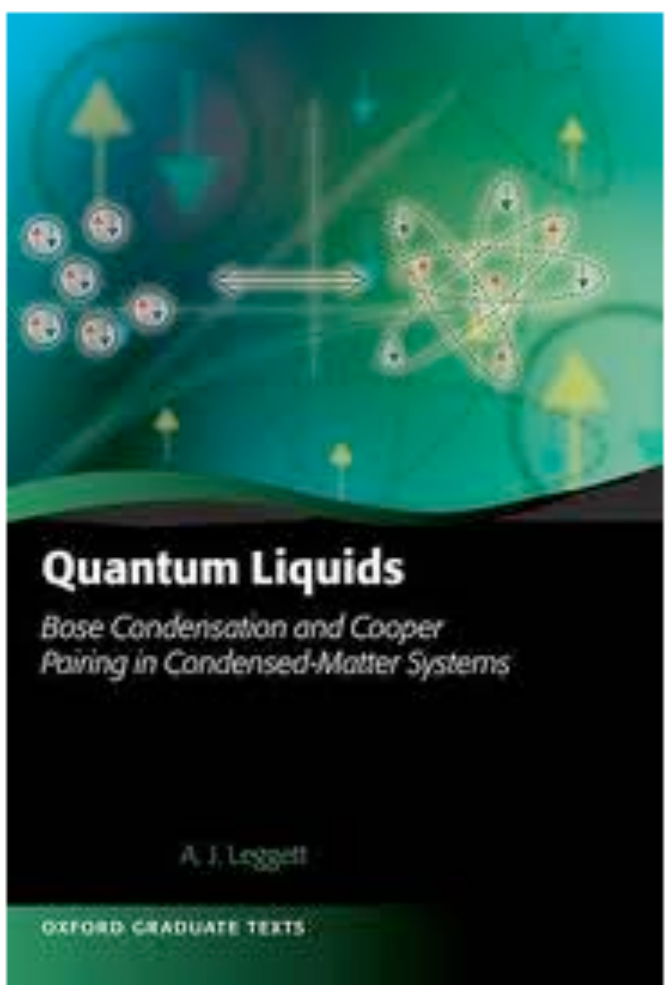
- one particle density matrix
  (M. Ishikawa 1977, T. Kita1996,1998) γ=0
- two particle correlation, density-current correlation function
  (P. W. Anderson, P. Morel, 1961) γ=1
- gradient expansion of the Green's function,
  Eilenberger equation
  (M. Cross, 1975 γ=2;
  Y. Tsutusmi, K. Machida, 2011, 2012 γ=0)
- NMR experiments (O. Ishikawa group, γ=0)

Various approaches give different results, but recent results seem to support γ=0 (i.e. full IAM $L=N/2$)

But why? how general is it? validity of approximations?

Anthony J. Leggett says.....

The question of what is the true expectation value of the angular momentum of superfluid ^{3}He-A in a given container geometry is one which is more than 30 years old and still has apparently not attained a universally agreed resolution,...



in "Quantum Liquids"
(Oxford University Press, 2006)

Why difficult?

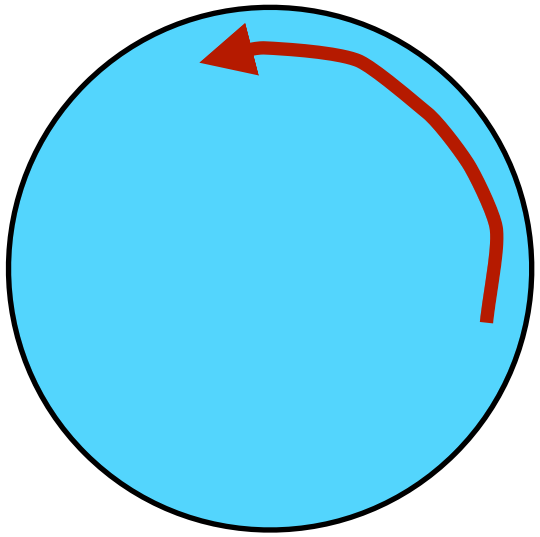
Infinite system: $N=\infty$ $L=\infty$

comparison not well-defined

Finite system on torus (periodic b.c.):

N and *L* are finite, but *L* is not conserved

(torus lacks rotational invariance)



Finite system in a circular potential

✓ *N* and *L* are finite

✓ *L* is conserved

inhomogeneous system with boundary

edge states

Goal of our work

First we would like to clarify IAM of chiral superfluid in an ideal setting:
Bogoliubov Hamiltonian in rotationally invariant potential without any further approximation or assumptions

$$H_{\mathrm{Bogoliubov}} = \int d\vec{r}\psi^{\dagger}(\vec{r})\left(-\frac{\hbar^2\nabla^2}{2m}+V(\vec{r})-\mu\right)\psi(\vec{r})+\frac{\Delta}{2}\int d\vec{r}\psi^{\dagger}(\vec{r})(\hat{p}_x+i\hat{p}_y)\psi^{\dagger}(\vec{r})$$
$$+\frac{\Delta^*}{2}\int d\vec{r}\psi(\vec{r})(\hat{p}_x-i\hat{p}_y)\psi(\vec{r})$$

(for $p+ip$)

This hopefully will lead to deeper understanding of IAM and edge currents, which would be useful in studying more realistic settings

Volovik's observation

Angular momentum L and fermion number N are NOT conserved in Bogoliubov Hamiltonian

However, the combination $Q = L - N/2$ is conserved!

$Q = 0 \Leftrightarrow L = N/2$ (for $p+ip$)

($Q = L - \nu N/2$ for general chiral SF)

But when do we have $Q = 0$?

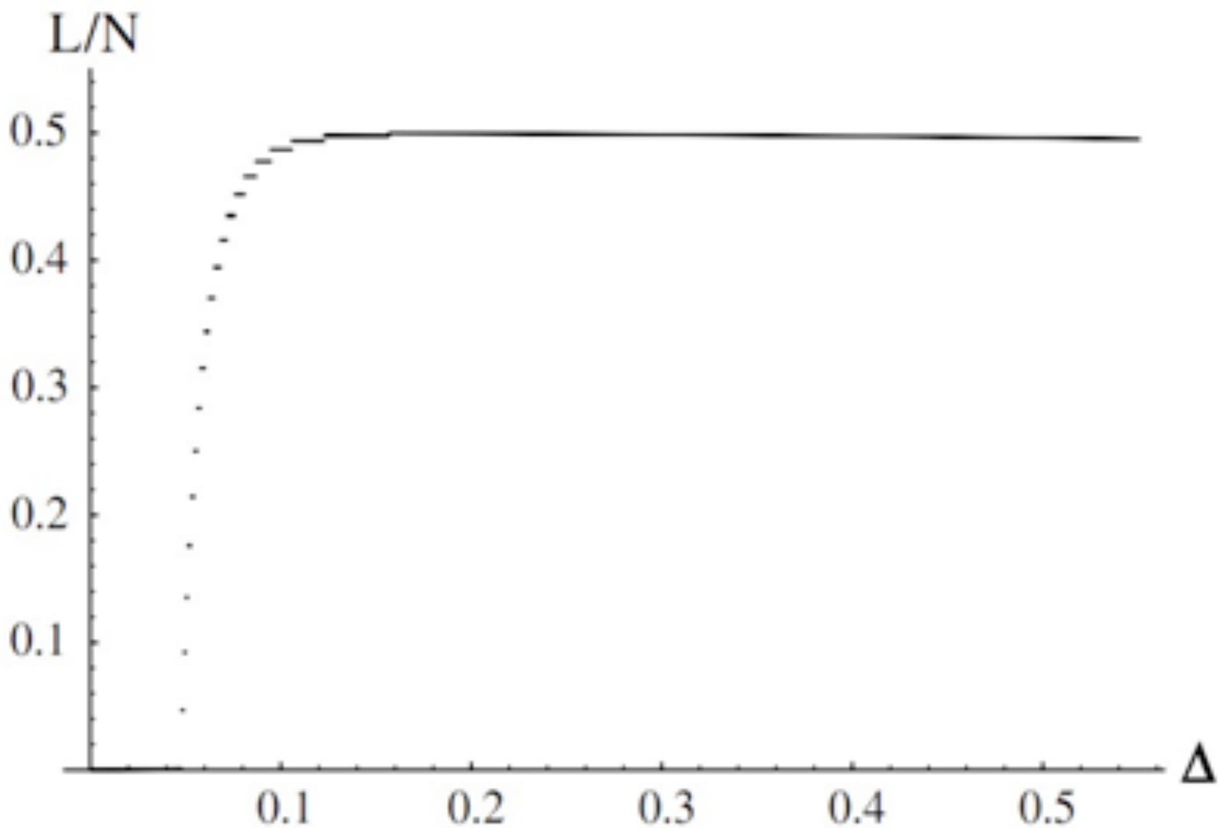
Not always the case, for example in the normal limit $\Delta \to 0$

Stone-Anduaga 2008

Chiral $p+ip$ superfluid in 2D harmonic potential

Bogoliubov Hamiltonian ⇔ tridiagonal matrix

(easily diagonalized numerically)



Found a quick convergence to $L=N/2$

Reason unclear....

There is no simple identity lying behind this fact, and mathematically it results from a quite non trivial rearrangement of spectral weight between the positive and negative E

Our approach

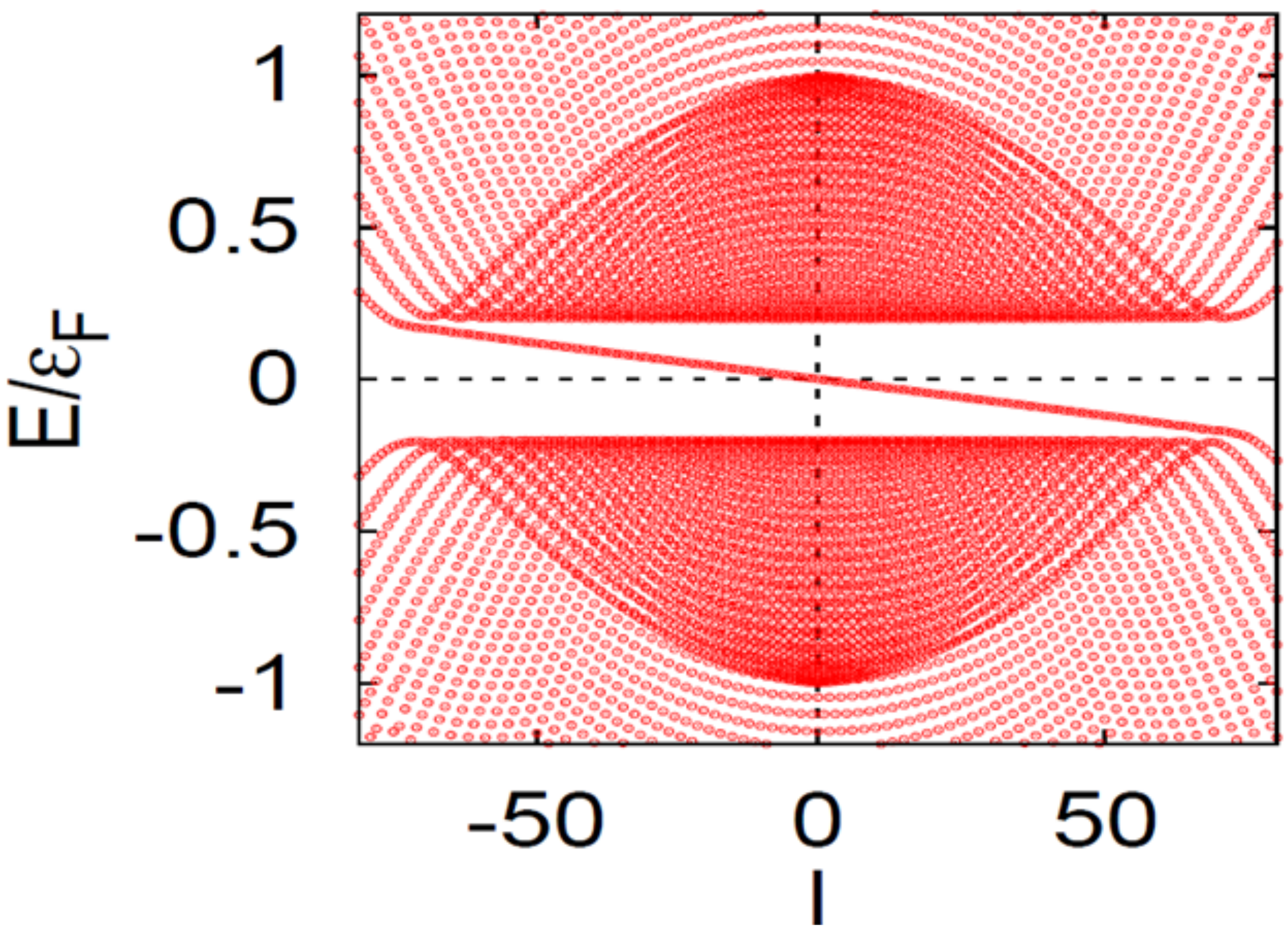
We explicitly solve Bogoliubov Hamiltonian
in 2D circular potentials, following Stone-Anduaga,
but with a reformulation to respect Volovik's conserved
charge Q explicitly

$$H_{\mathrm{Bogoliubov}} = \sum_{l\geq 0}\sum_{\mu\mu'} (\hat{a}^{\dagger}_{l+1,\mu} \quad \hat{a}_{-l,\mu}) \underbrace{\begin{pmatrix} \varepsilon_{l+1,\mu}\delta_{\mu\mu'} & \Delta^{(l)}_{\mu\mu'} \\ \Delta^{(l)*}_{\mu'\mu} & -\varepsilon_{-l,\mu}\delta_{\mu\mu'} \end{pmatrix}}_{H^{(l)}_{\mathrm{BdG}}} \begin{pmatrix} \hat{a}_{l+1,\mu'} \\ \hat{a}^{\dagger}_{-l,\mu'} \end{pmatrix}$$

$$\langle Q \rangle = \langle GS|\hat{Q}|GS\rangle = \sum_{l=0}^{\infty} (l+\frac{1}{2})[(\sum_{m|E_m<0} 1) - M] = \sum_{l=0}^{\infty} [-\frac{l+1/2}{2}\sum_{m} \mathrm{sgn} E^{(l)}_m]$$

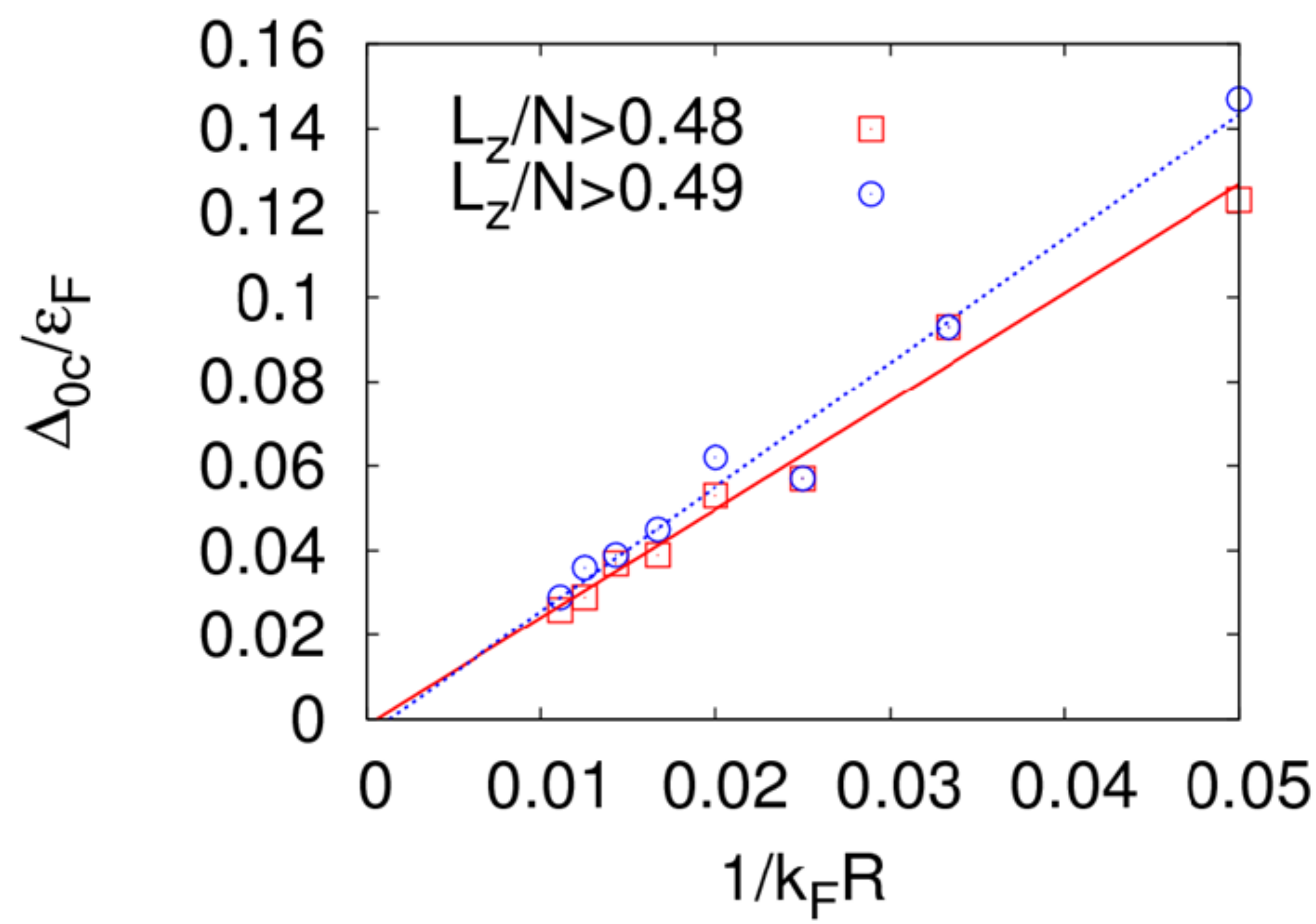
explicitly given as a ***conserved quantity***, which can vary only when $E_m^{(l)}$ changes sign

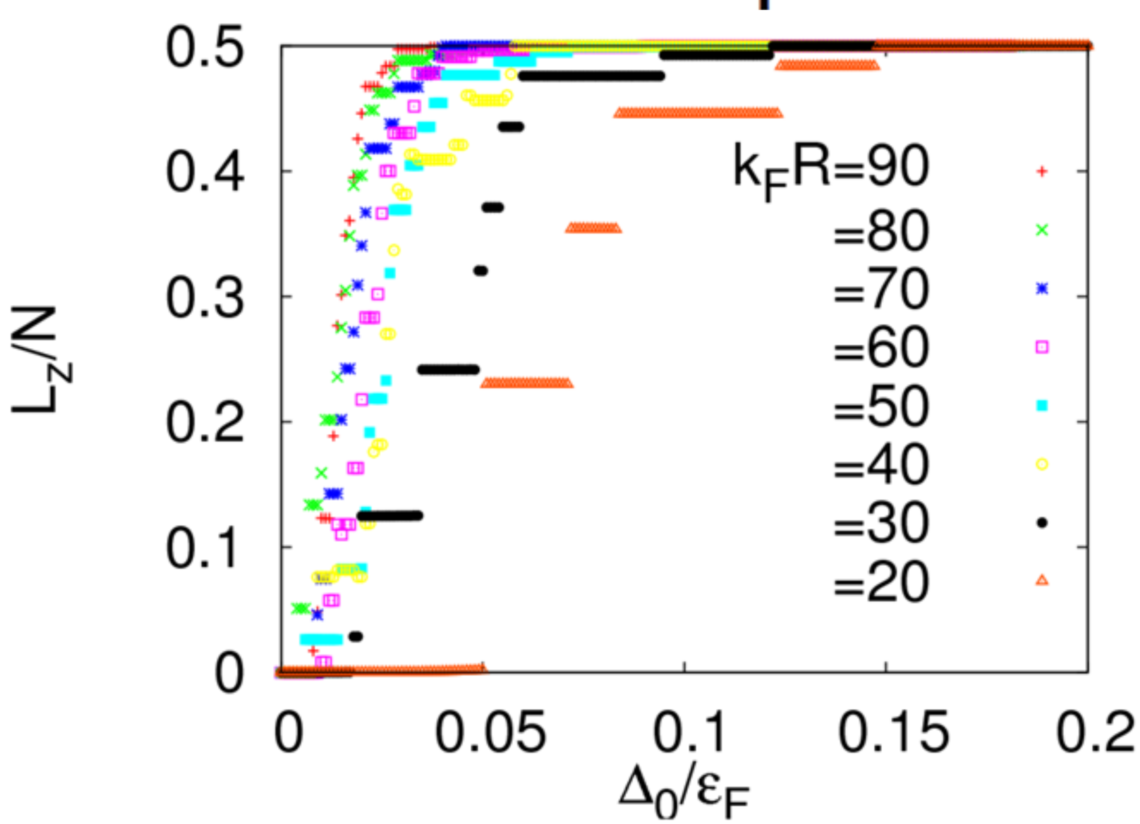
Result for $p+ip$ in circular well



Q=0 (exact) $\Leftrightarrow$ L=N/2

for $\Delta > \Delta_c$
in a finite well



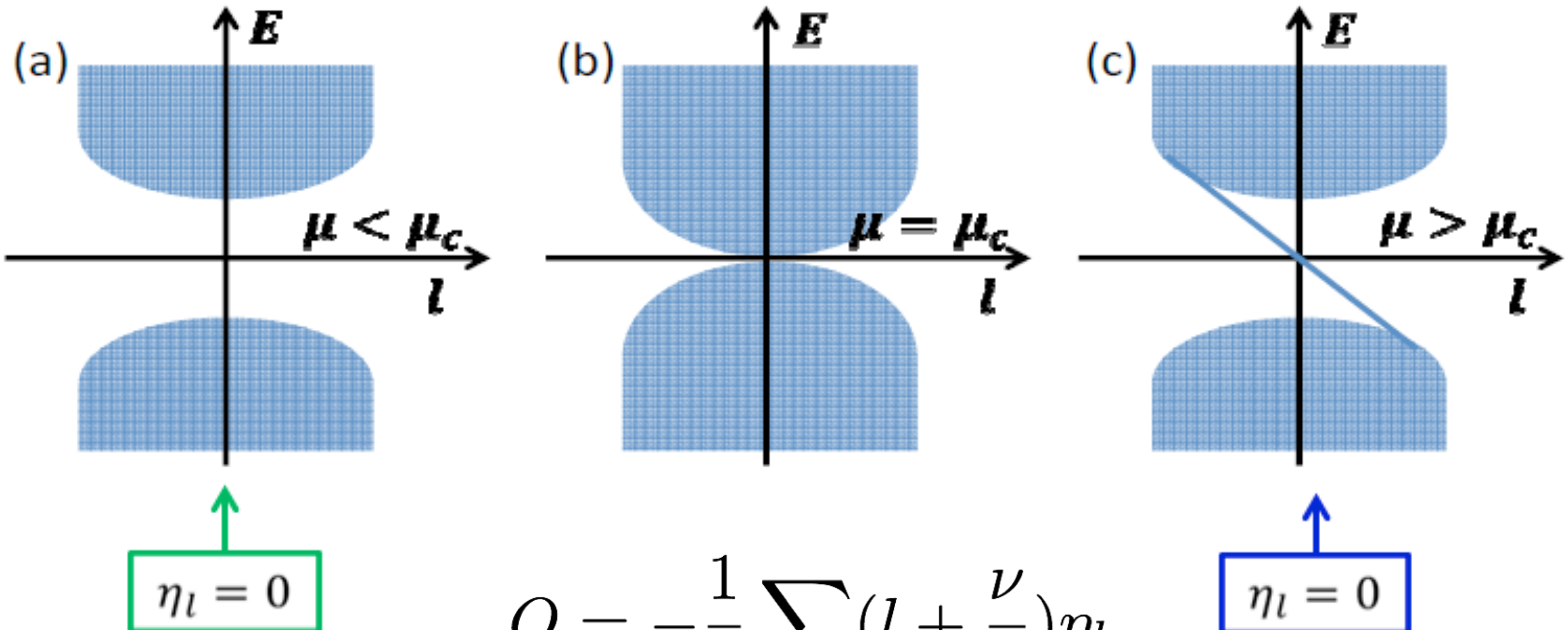


$\Delta c \rightarrow 0$ in the ∞ size limit:
$L \rightarrow N/2$ for arbitrary $\Delta > 0$

From BEC to BCS

Tuning chemical potential

Q=0 BEC regime ⇨ BCS regime Q=0



$$Q = -\frac{1}{2}\sum_l (l + \frac{\nu}{2})\eta_l$$

$$\eta_l = \sum_m \mathrm{sgn} E_m^{(l)}$$

No spectral flow occurs

⇨ ***In the disc geometries, "BEC" and "BCS" are adiabatically connected***

Chiral *d+id*-wave superfluid

ν=2

Same question on the intrinsic angular momentum as in *p*+i*p* : (now full IAM is *N*, instead of *N*/2)

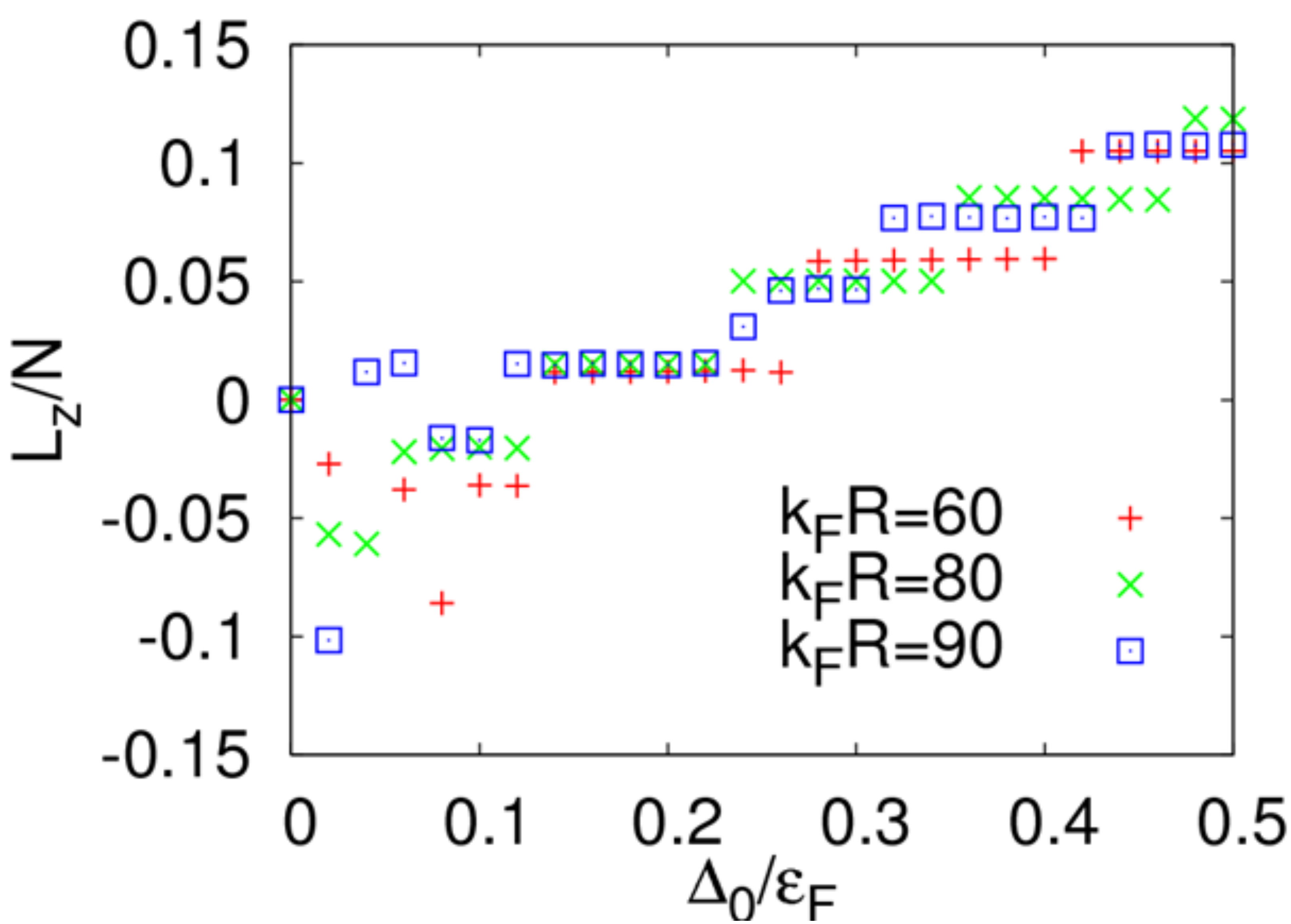
Same answer ($L \to N$) ?

Let's see the results.....

Results for *d+id* in circular well

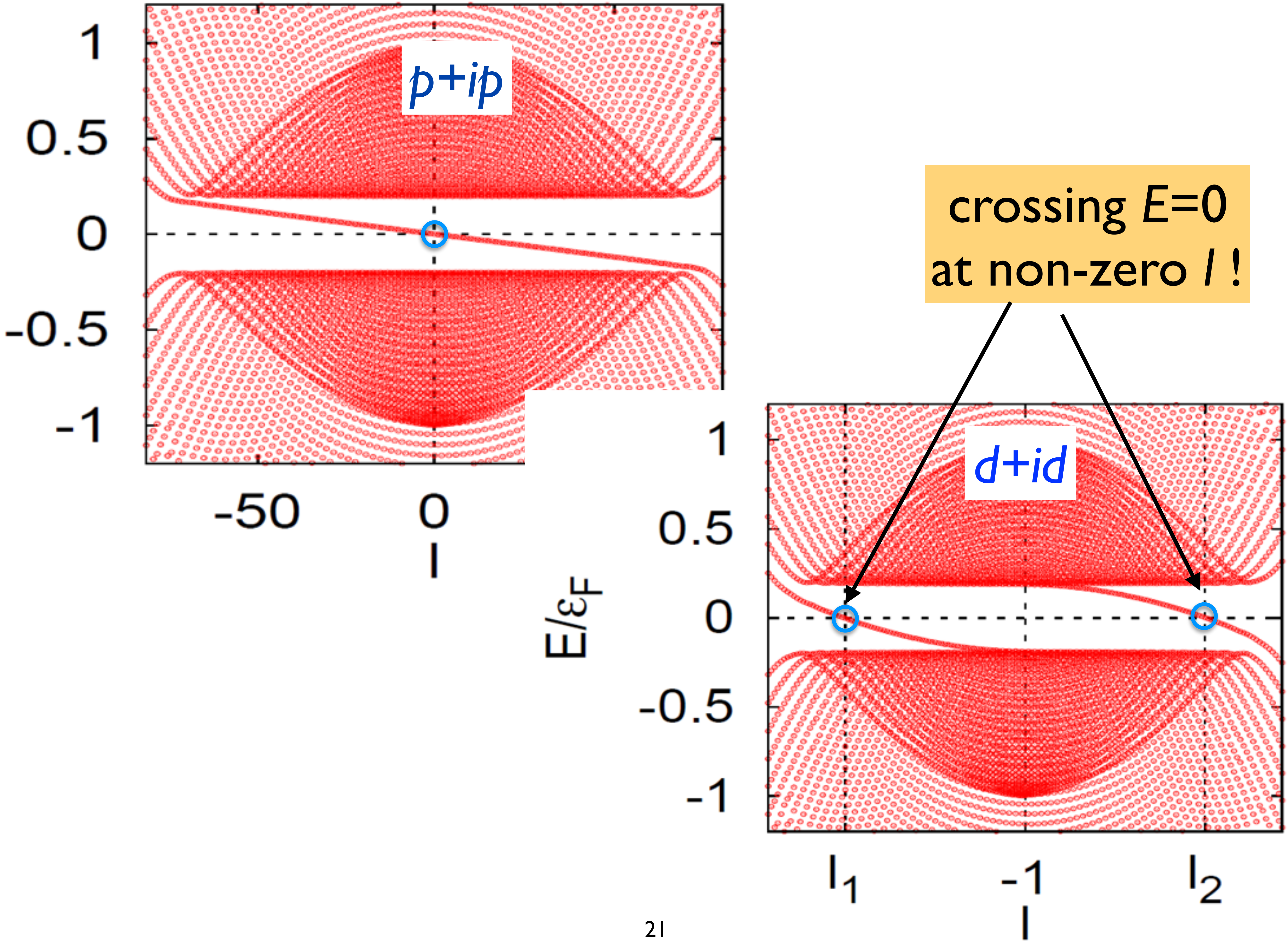
$Q \neq 0$ in general

$Q \rightarrow -\nu N/2$, $L/N \rightarrow 0$ in the thermodynamic limit?!



VERY different from *p+ip*!!

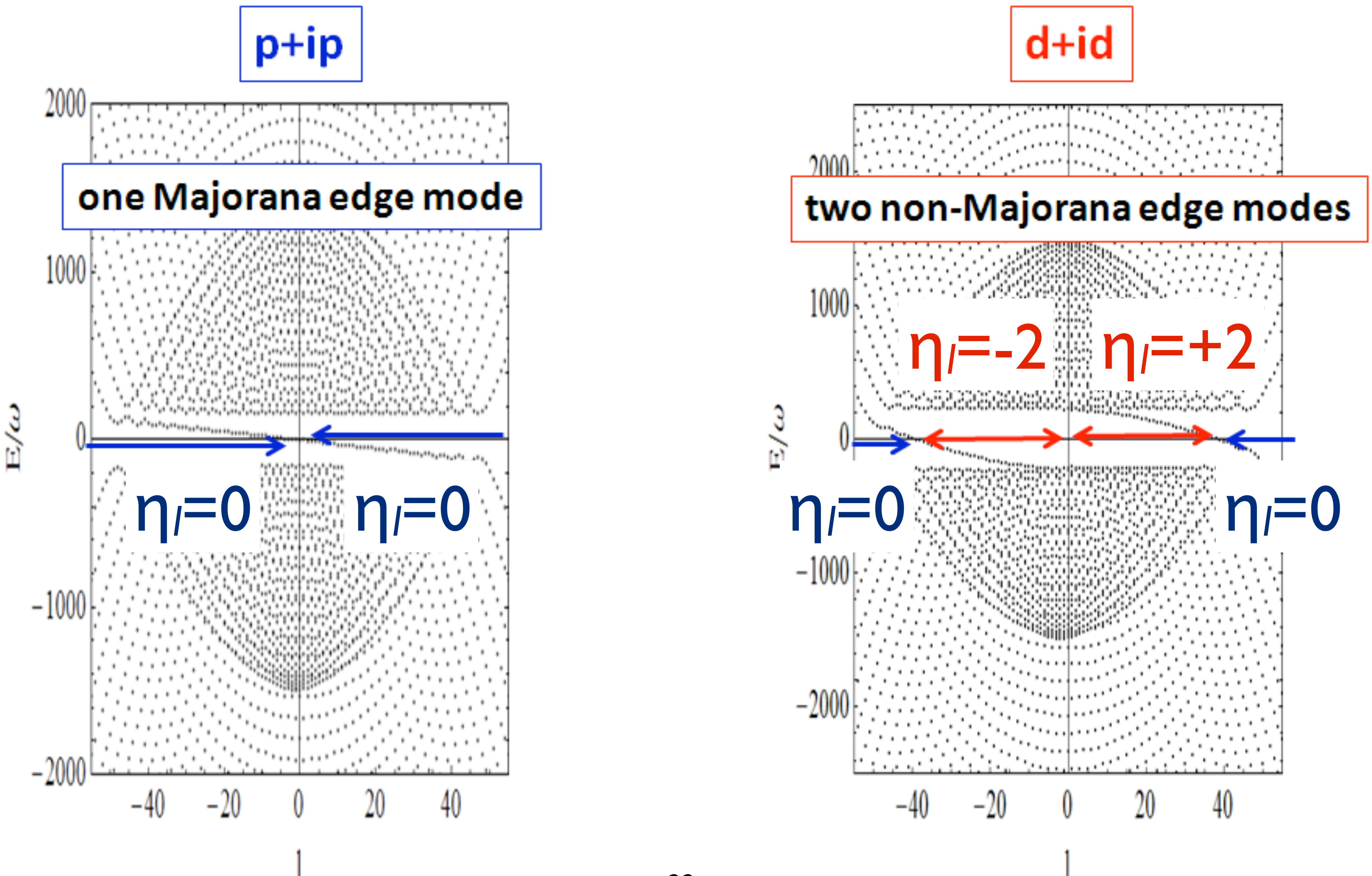
Why?



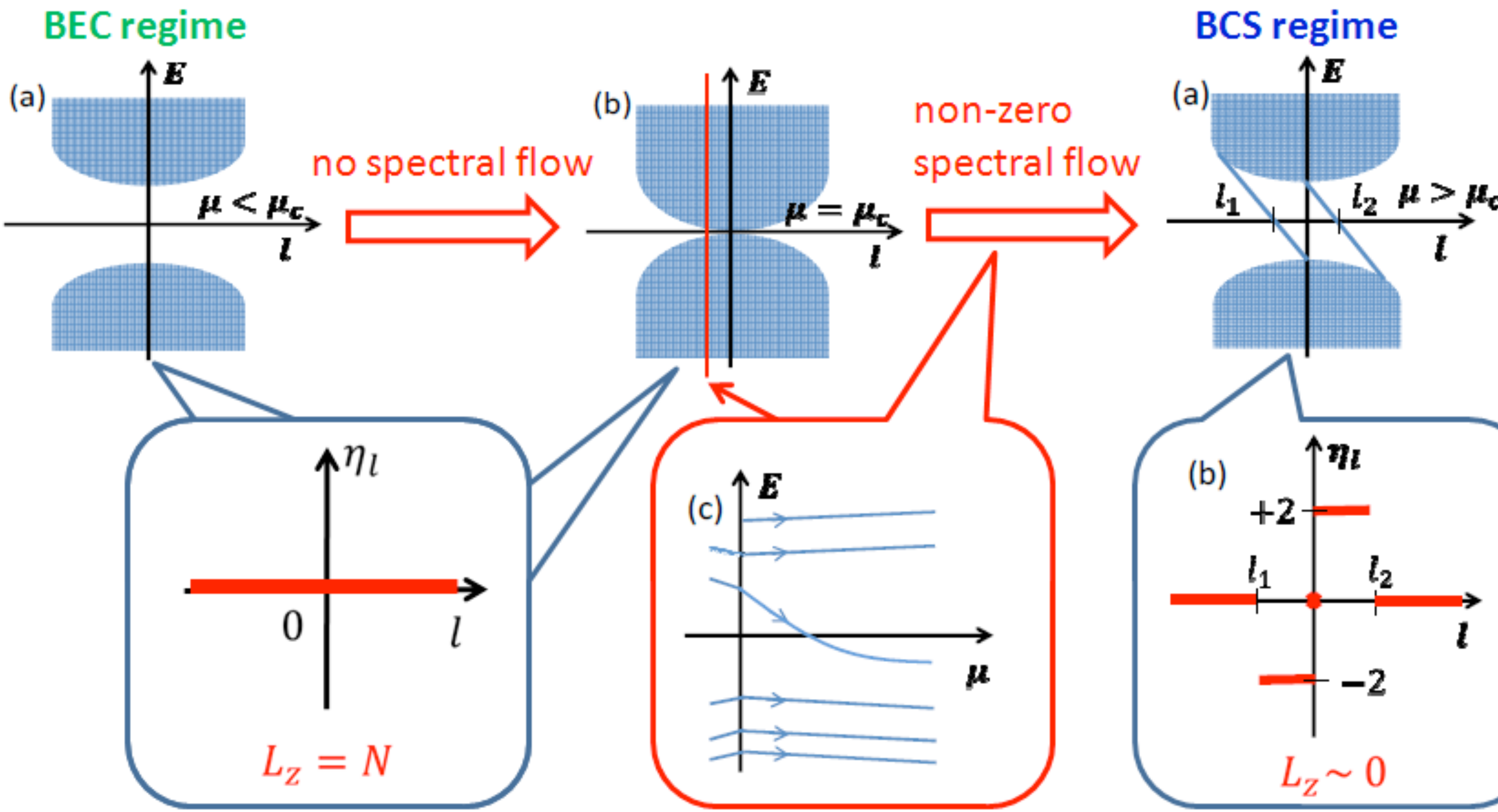
p+ip
1
0.5
0
-0.5
-1
-50
0
l
E/εF
d+id
crossing E=0
at non-zero l !
l1
-1
l2
l

Structure of edge states directly linked to the conserved charge

$$Q = -\frac{1}{2}\sum_l (l + \frac{\nu}{2})\eta_l$$



From BEC to BCS: *d+id* case

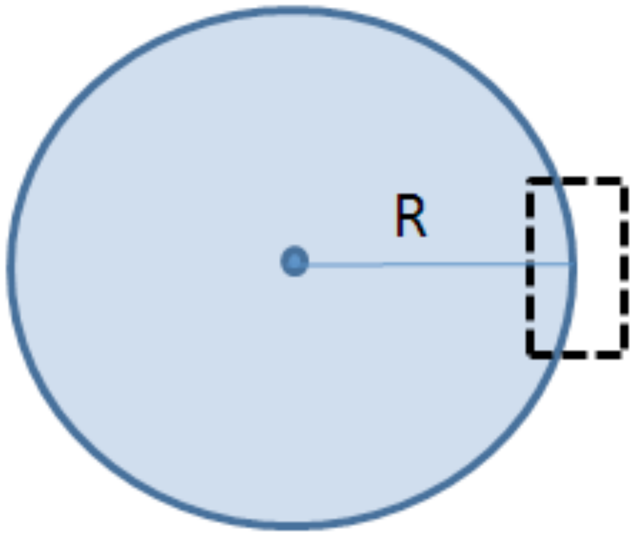


two non-degenerate edge modes

→ non-zero spectral flow between BEC and BCS

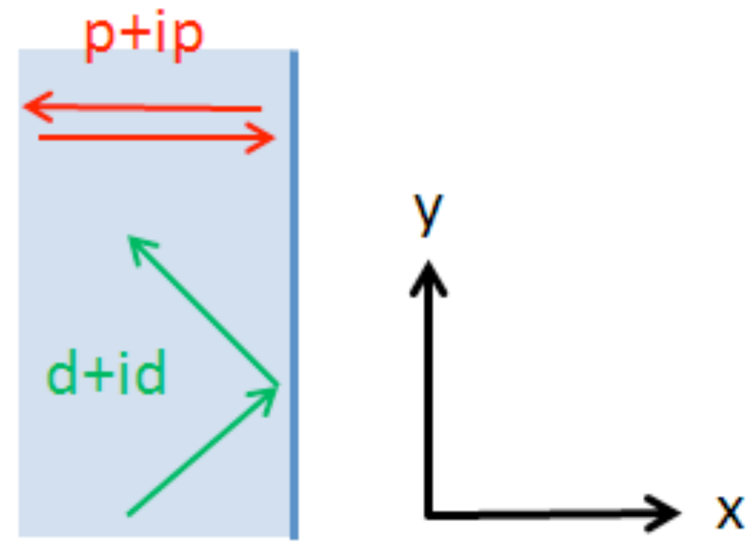
→ change in L_z

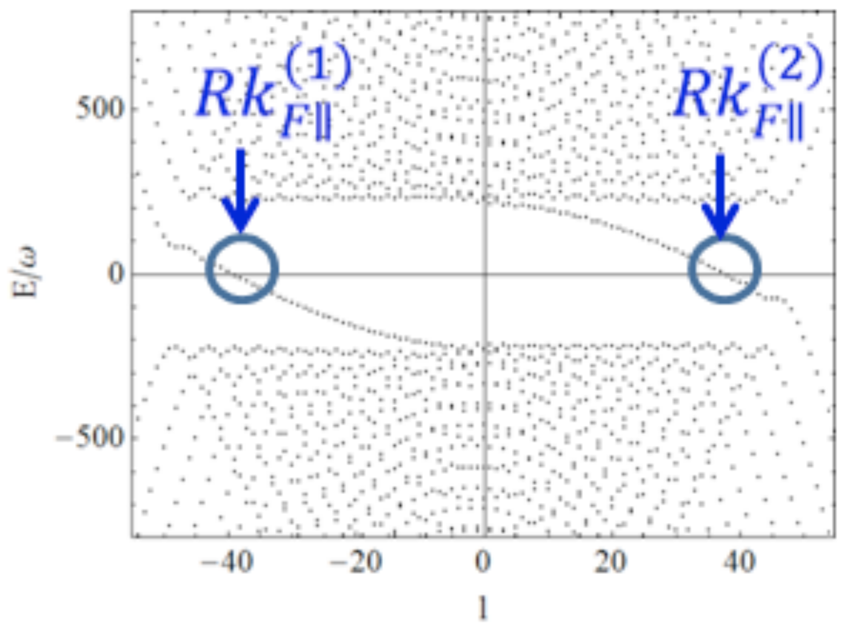
Quasi-classical evaluation in BCS



$$\approx$$

$$\xi << R$$





$$k_{\parallel} = \frac{2\pi l}{2\pi R} = \frac{l}{R}$$ edge mode momentum

$$\frac{1}{\nu}\sum_{j=1}^{\nu}(k_{F\parallel}^{(j)})^2 = \frac{k_F^2}{2} \quad \text{for } \nu \geq 2$$

"isotropic"

$$1/\nu \sum_{j=1}^{\nu}(k_{F\perp}^{(j)})^2 = k_F^2/2$$

$$Q \simeq -\sum_{j=1}^{\nu}(Rk_{F\parallel}^{(j)})^2/2$$

$$= -\nu N/2 + \mathcal{O}(\Delta_0/\varepsilon_F)$$

$$\boxed{L_z \simeq O(\Delta/\varepsilon_F)}$$

$$Q = L - \nu\frac{N}{2}$$

arXiv.org > cond-mat > arXiv:1409.7459

Condensed Matter > Superconductivity

Orbital Angular Momentum and Spectral Flow in Two Dimensional Chiral Superfluids

Yasuhiro Tada, Wenxing Nie, Masaki Oshikawa

(Submitted on 26 Sep 2014)

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G.E. Volovik

(Submitted on 30 Sep 2014 (v1), last revised 30 Oct 2014 (this version, v3))

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in < 48 hours!!

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Vanishing edge currents in non-p-wave topological chiral superconductors

Wen Huang, Edward Taylor, Catherine Kallin

(Submitted on 1 Oct 2014)

The edge currents of two dimensional topological chiral superconductors with nonzero Cooper pair angular momentum---e.g., chiral p-, d-, and f-wave superconductivity---are studied. Bogoliubov-de Gennes and Ginzburg--Landau calculations are used to show that in the continuum limit, \emph{only} chiral p-wave states have a nonzero edge current. Outside this limit, when lattice effects become important, edge currents in non-p-wave superconductors are comparatively smaller, but can be nonzero. Using Ginzburg--Landau theory, a simple criterion is derived for when edge currents vanish for non-p-wave chiral superconductivity on a lattice. The implications of our results for putative chiral superconductors such as Sr2RuO4 and UPt3 are discussed.

For chiral p-wave ($m = 1$), the bulk contribution is half in magnitude as the current carried by the chiral edge states, and flows in the opposite direction: $J_{\text{edge}} = \hbar k_F^2/(8\pi m^*)$ and $J_{\text{bulk}} = -\hbar k_F^2/(16\pi m^*)$[4]. The total edge current per spin component can thus be written as $J = n\hbar/4m^*$, where $n = k_F^2/4\pi$ is the number density per spin component. This value is consistent with numerical BdG calculations in the continuum limit of lattice models[13] (for simple lattice models at least, iterating BdG to full self-consistency has negligible impact on our results). It is also the edge current needed to produce a macroscopic angular momentum $N\hbar/2$ for N fermions in a disc[4] (see below).

On the other hand, (10) and (11) vanish independently *for all* $m > 1$, a fact that can be proved by induction. Thus the total edge current is identically zero for any chiral superconductor with Cooper pair angular momentum $> \hbar$. In the continuum at least, p-wave is special! As noted in the Introduc-

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Note added—As this manuscript was being prepared for submission, a preprint[48] appeared which has some overlap. Focussing on the problem of the total angular momentum in the continuum limit, the authors of Ref. 48 find that the total angular momentum vanishes to order Δ_0/E_F in the weak-coupling BCS limit for all states with $m > 1$, consistent with our results. They also extend these results to the BEC limit of the crossover, where they derive the result given by (12) for all m.

Large Chern Number and Edge Currents in Sr_2RuO_4

Thomas Scaffidi and Steven H. Simon
Rudolf Peierls Centre for Theoretical Physics, Oxford OX1 3NP, United Kingdom

(Received 30 October 2014; revised manuscript received 19 June 2015; published 21 August 2015)

We show from a weak-coupling microscopic calculation that the most favored chiral superconducting order parameter in Sr_2RuO_4 has a Chern number of $|C| = 7$. The two dominant components of this order parameter are given by $\sin(3k_x) + i\sin(3k_y)$ and $\sin(k_x)\cos(k_y) + i\sin(k_y)\cos(k_x)$ and lie in the same irreducible representation E_u of the tetragonal point group as the usually assumed gap function, $\sin(k_x) + i\sin(k_y)$. While the latter gap function leads to $C = 1$, the two former lead to $C = -7$, which is also allowed for an E_u gap function since the tetragonal symmetry only fixes C modulo 4. Since it was shown that the edge currents of a $|C| > 1$ superconductor vanish exactly in the continuum limit, and can be strongly reduced on the lattice, this form of order parameter could help resolve the conflict between experimental observation of time-reversal symmetry breaking and yet the absence of observed edge currents in Sr_2RuO_4.

DOI: 10.1103/PhysRevLett.115.087003 PACS numbers: 74.70.Pq, 74.20.Mn, 74.20.Rp

Structure of the ground state

General Bogoliubov transformation (Lanbote'74)

$$|\mathrm{GS}\rangle \doteq \mathcal{N} \otimes_l |\mathrm{GS}\rangle_l$$

$$|\mathrm{GS}\rangle_l = \left(\prod_{j=1}^{n_\uparrow^{(l)}} \tilde{c}^\dagger_{j,l+\nu,\uparrow}\right)\left(\prod_{j=1}^{n_\downarrow^{(l)}} \tilde{c}^\dagger_{j,-l,\downarrow}\right)$$

← **additional factors**

non-paired creation operators
correspond to ***pair breaking***

$$\times \exp\left(\sum_{j>n_\uparrow^{(l)}} \sum_{j'>n_\downarrow^{(l)}} \tilde{c}^\dagger_{j,l+\nu,\uparrow} F^{(l)}_{jj'} \tilde{c}^\dagger_{j',-l,\downarrow}\right)|0\rangle$$

← **usual BCS form**

$$Q = \sum_l (l+\nu/2)\left(n_\uparrow^{(l)} - n_\downarrow^{(l)}\right)$$

finite Q ⇔ finite $n_\uparrow, n_\downarrow$

$$\begin{array}{c} n_c \\ n_c \end{array}\begin{pmatrix} c_{nl+\nu s} \\ c^+_{n-l\bar{s}} \end{pmatrix} = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}^+ \begin{pmatrix} b_m^{(l)} \\ d_m^{(l)+} \end{pmatrix} \begin{array}{l} n_+ \\ n_- \end{array}$$

$n_+ =$ # of positive eigenvalues

$n_- =$ # of positive eigenvalues

$n_\uparrow =$ # of zero eigenvalues of $M_1^+ M_1$

$n_\downarrow =$ # of zero eigenvalues of $M_4^+ M_4$

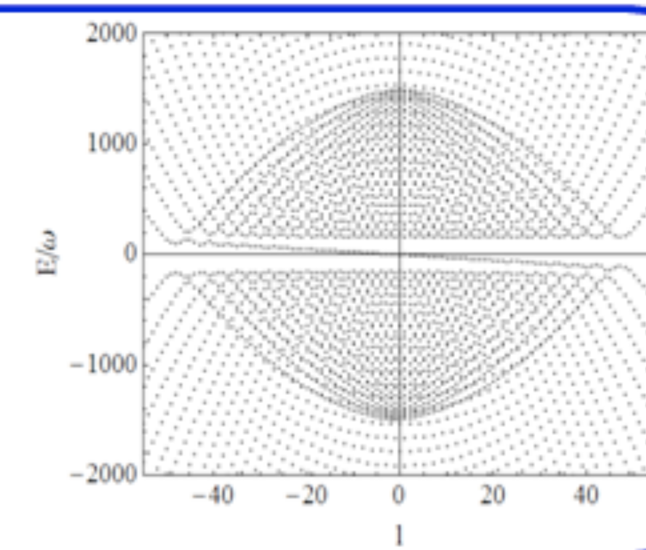
Structure of the ground state

$$|\mathrm{GS}\rangle \doteq \mathcal{N} \otimes_l |\mathrm{GS}\rangle_l$$

p+ip

$$|p\mathrm{GS}\rangle_l = \exp[\tilde{c}^\dagger_{l+1} F^{(l)} \tilde{c}^\dagger_{-l}]|0\rangle$$

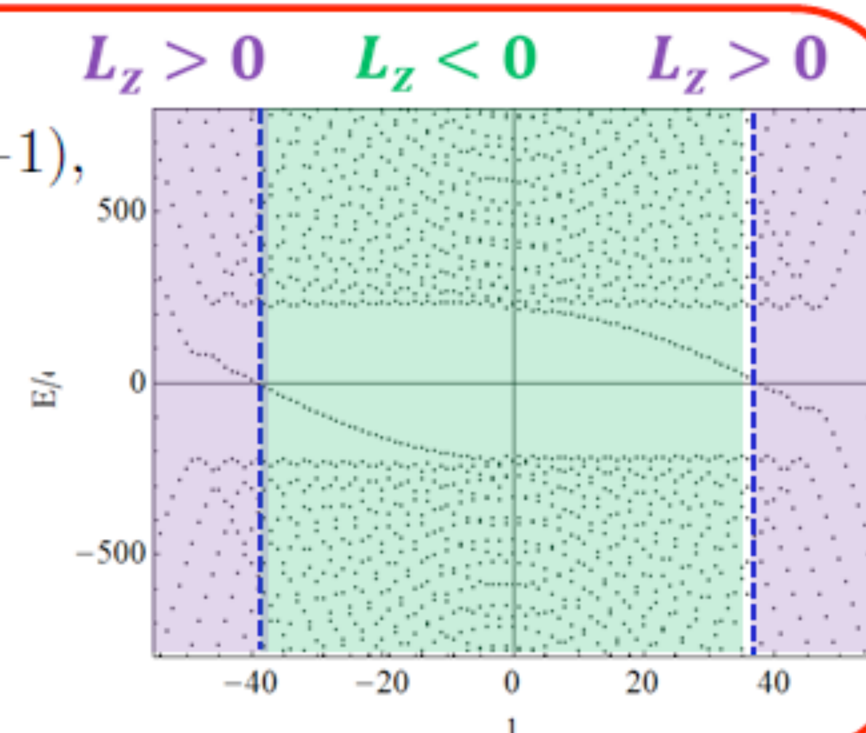
GS wf has same structure as well known BCS form



d+id

$$|d\mathrm{GS}\rangle_l = \begin{cases} \exp[\tilde{c}^\dagger_{l+2} F^{(l)} \tilde{c}^\dagger_{-l}]|0\rangle & (|l| > |l_{1,2}|, l = -1), \\ \tilde{c}^\dagger_{l+2} \exp[\tilde{c}^\dagger_{l+2} F^{(l)} \tilde{c}^\dagger_{-l}]|0\rangle & (l_1 < l < -1), \\ \tilde{c}^\dagger_{-l} \exp[\tilde{c}^\dagger_{l+2} F^{(l)} \tilde{c}^\dagger_{-l}]|0\rangle & (-1 < l < l_2). \end{cases}$$

depaired fermions have
angular momentum *opposite* to the chirality



Summary

- (Numerically) exact solution of Bogoliubov model in rotationally invariant confining potentials
- Formulation respecting Volovik's quantum number Q.
Q=0 $\Leftrightarrow$ Full "Intrinsic Angular Momentum" $L=\nu N/2$

- $p+ip$: $Q \rightarrow 0$ in the thermodynamic limit: full IAM
- $d+id$: $Q \neq 0$ $L \rightarrow O(\Delta/E_F)$ cf. semiclassical analysis
reduced L in the thermodynamic limit

The difference is related to the structure of the edge states. $p+ip$ is special due to symmetry-constrained dispersion relation. Generic chiral superfluids are expected to have nonzero Q (reduced L)