

Thermal Hall conductivity of a chiral nodal superconductor in zero magnetic field

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Nodal

universal thermal conductivity $T \rightarrow 0$

K_{xx}

Chiral

K_{xy}, K_{yx}

Topological

edge states

Motivation: Sr_2RuO_4

suggested to be *chiral*

earliest suggestion: $\Delta(\hat{p}) = \Delta_A(\hat{p}_x + i\hat{p}_y)\hat{z}$ $\sim 2\text{D}$

*muSR, Kerr, Josephson interference, ...
(also against: absence of surface current ...)*

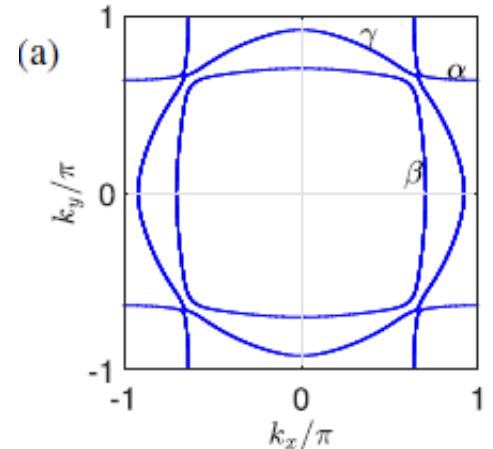
Nodal: evidenced by

specific heat Deguchi PRL 04

thermal conductivity
Suderow et al, JPCM 98,
Suzuki et al PRL 02
Hassinger et al, arXiv: 1606.04936

penetration depth
Bonalde et al, PRL 00

....



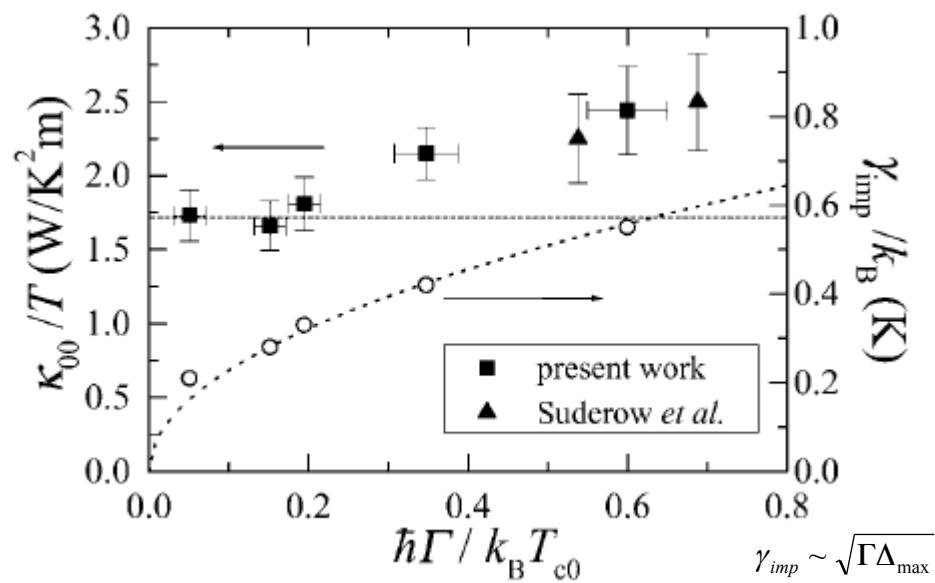
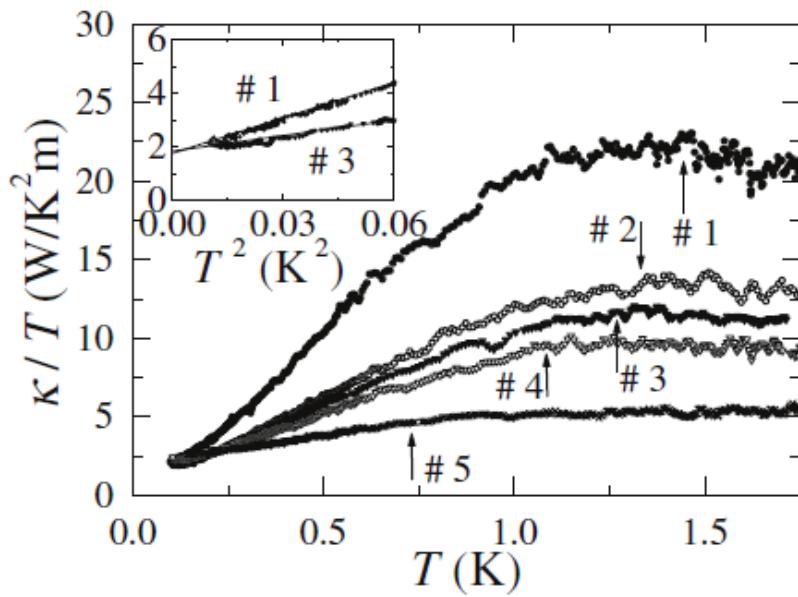
Suggestions for line nodes:

near $(\pi,0)$, $(0,\pi)$ (odd parity) (Yanase 03, ...)

...

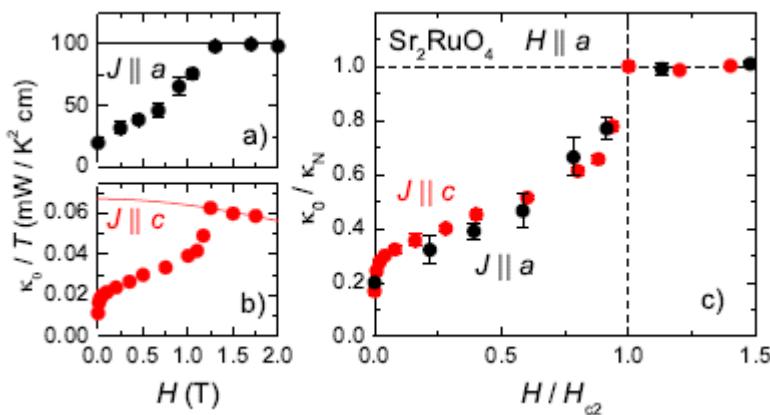
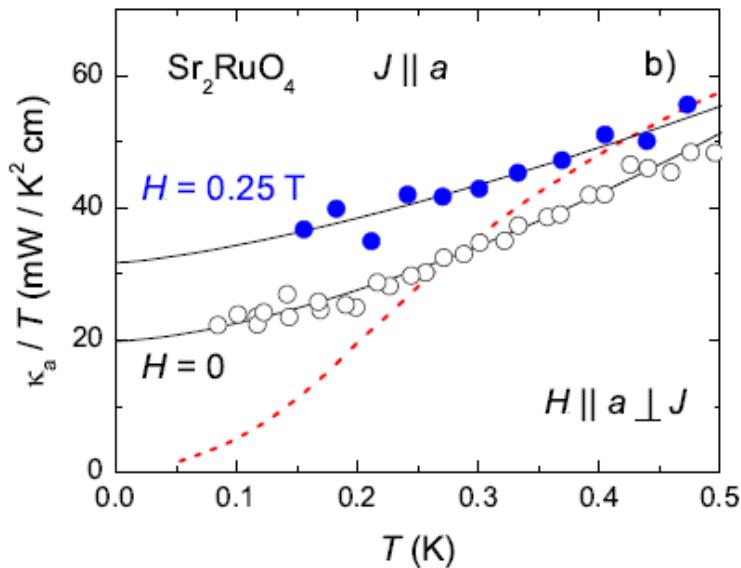
near where $\alpha \beta \gamma$ crosses (Scaffidi and Simon 15)

...



“universal thermal conductivity”
as $T \rightarrow 0$

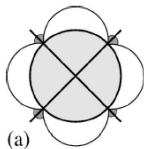
$$\ln\left(\frac{T_{c0}}{T_c}\right) = \Psi\left(\frac{1}{2} + \frac{\hbar\Gamma}{2\pi k_B T_c}\right) - \Psi\left(\frac{1}{2}\right)$$



their conclusion:
all Fermi surfaces have nodes

line nodes

c.f. High Tc, $d_{x^2-y^2}$



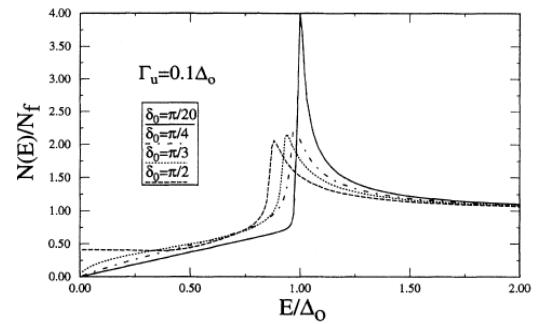
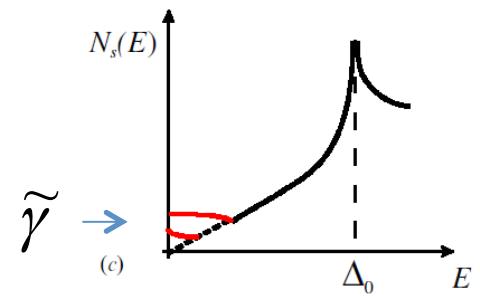
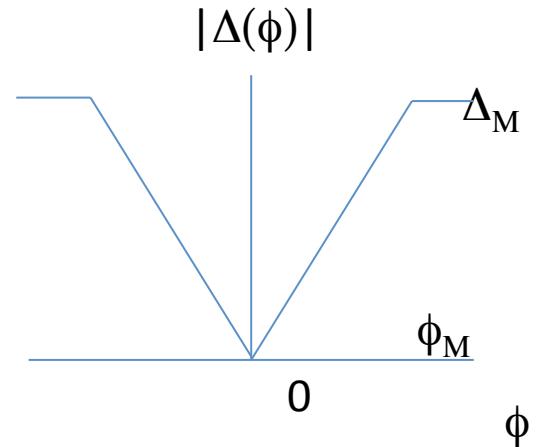
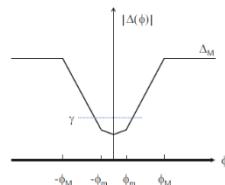
Graf, Yip, Rainer, Sauls (1996)

$$K_{xx}^{univ} \rightarrow N_f v_f^2 \frac{\pi^2}{3} T \times \frac{2\phi_M}{\pi\Delta_M}$$

independent of impurity concentration ,
scattering phase shift
(up to typically small vertex corrections)

contribution from *impurity band*

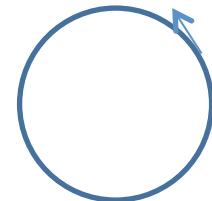
same results expected if nodes/ near nodes
and $\langle \Delta(\phi) \rangle = 0$
(but not extended-s)



Xu et al PRB 94

Chiral:

$$\Delta(\hat{p}) = \Delta_A(\hat{p}_x + i\hat{p}_y)\hat{z}$$



$$Q_x = -K_{xx}(\nabla T)_x - K_{xy}(\nabla T)_y$$

$$K_{xy}, K_{yx} \neq 0$$

$$Q_y = -K_{yx}(\nabla T)_x - K_{yy}(\nabla T)_y$$

(symmetry allowed)

tetragonal symmetry

$$K_{yx} = -K_{xy} \neq 0$$

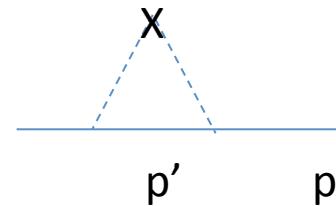
thermal Hall conductivity finite (but see below)

(c.f. electrical Hall conductivity)

Effect vanishes in Born limit

$$\begin{pmatrix} u_p \\ v_p \end{pmatrix} \rightarrow \begin{pmatrix} u_{p'} \\ v_{p'} \end{pmatrix}$$

$$\left| \begin{pmatrix} u_{p'}^* & v_{p'}^* \\ 0 & 0 \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & -U \end{pmatrix} \begin{pmatrix} u_p \\ v_p \end{pmatrix} \right|^2 \propto |u_{p'}^* u_p - v_{p'}^* v_p|^2$$



$$\propto |real - e^{i(\phi_{p'} - \phi_p)}|^2$$

$$\propto |real - \cos(\phi_{p'} - \phi_p)|^2 + \sin^2(\phi_{p'} - \phi_p)$$



also vanishes in resonance limit

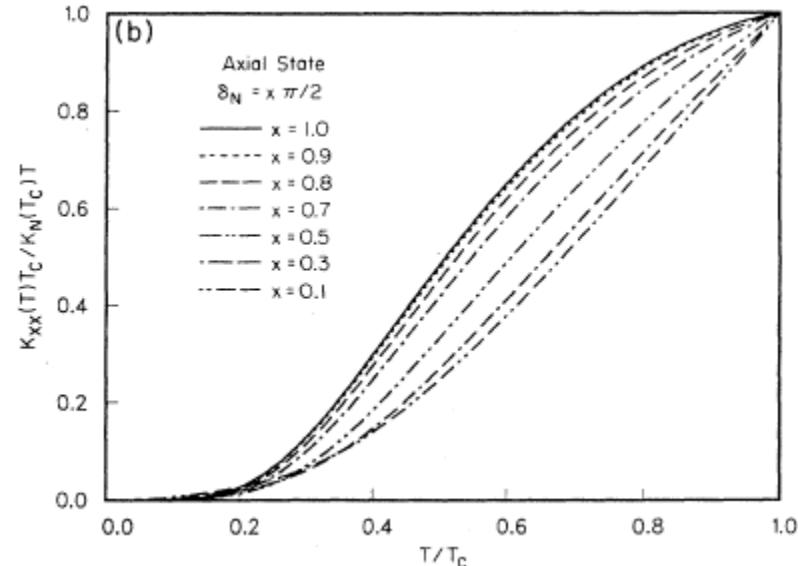
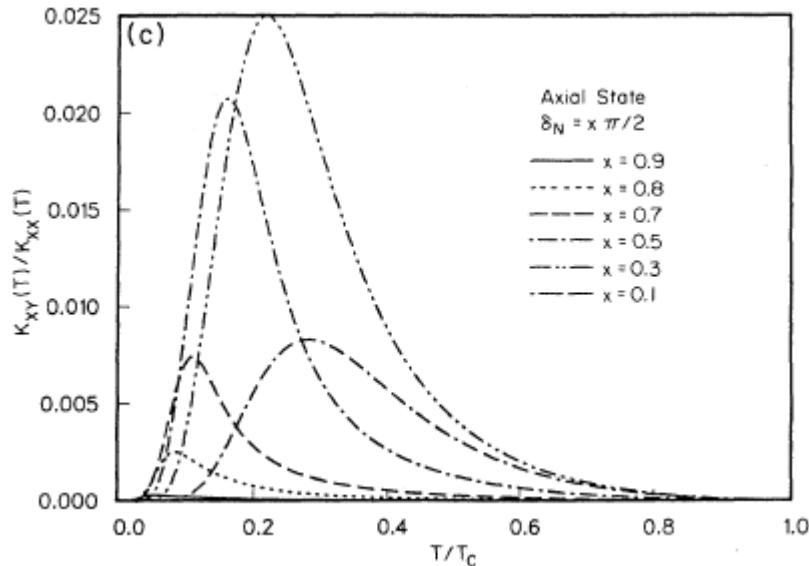
$$\left| \begin{pmatrix} u_{p'}^* & v_{p'}^* \\ 0 & 0 \end{pmatrix} \begin{pmatrix} t_{11} & 0 \\ 0 & t_{22} \end{pmatrix} \begin{pmatrix} u_p \\ v_p \end{pmatrix} \right|^2 \propto |u_{p'}^* u_p + v_{p'}^* v_p|^2$$

$$t_{11} = t_{22}$$

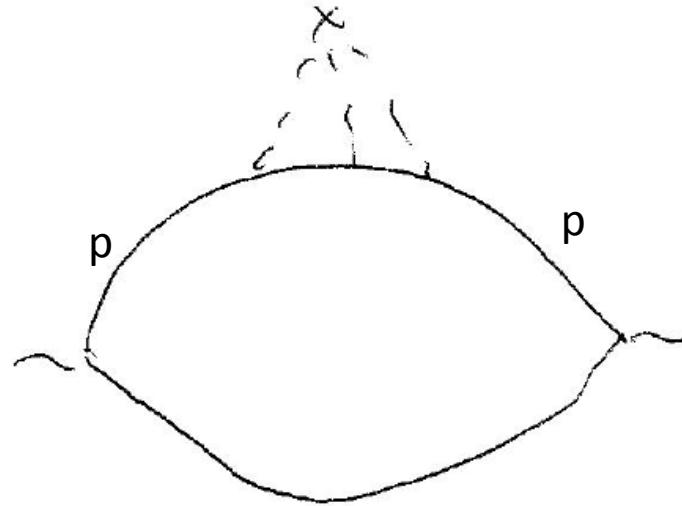
needs to break
“p-h symmetry”

$$\Delta(\hat{p}) = \Delta_A (\hat{p}_x + i\hat{p}_y) \hat{z}$$

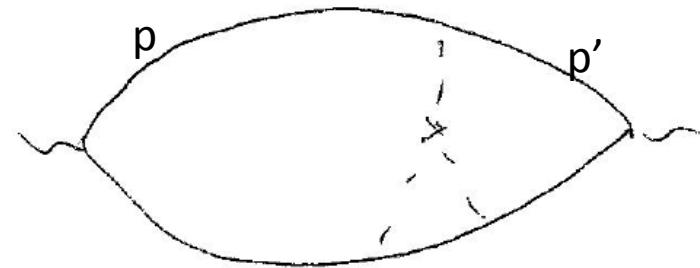
axial/ABM



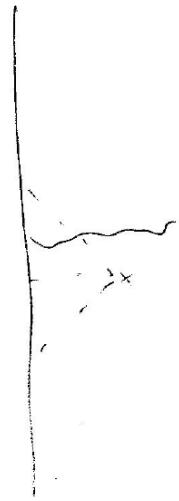
assumed well-defined quasiparticles,
scalar kinetic equation (after Bogoliubov transf)
vanishes in Born ($\delta \sim 0$) and resonance limits ($\delta \sim \pi/2$)
 K_{xy} from “in-scattering”
note $K_{xy} > 0$ if $\tan \delta > 0$



scattering out from p



scattering into p from p'



$$\sigma_{1,imp}(\hat{p})$$

$$\hat{\Lambda} \equiv \langle \frac{\hat{\Delta} \mathbf{v}_f \cdot \nabla T}{(|\Delta(\phi)|^2 + \gamma^2)^{3/2}} \rangle$$

Thermal Hall conductivity of a nodal chiral superconductor

$$\Delta(\hat{p}) \sim \Delta_A (\hat{p}_x + i\hat{p}_y) \hat{z}$$

single band

$T \rightarrow 0$

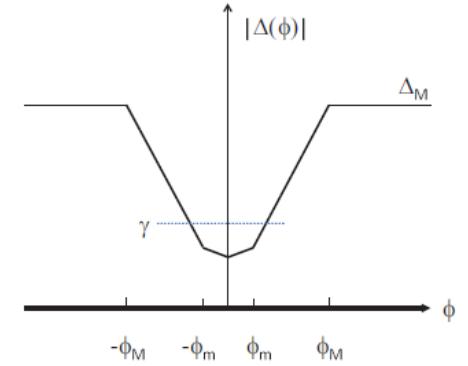
$\varepsilon \ll \gamma$ (impurity self-energy / band width)

point, isotropic impurities

$$\check{\sigma}_{\text{imp}} = n_{\text{imp}} \check{t}$$

$$\check{t} \equiv u(1 - N_f u \langle \check{g} \rangle)^{-1}$$

$$\cot \delta = -1/(\pi N_f u)$$



δ =impurity phase shift

normal state: $\hat{t}^R = -\frac{1}{\pi N_f} \sin \delta \times e^{\pm i \delta}$

superconducting state:

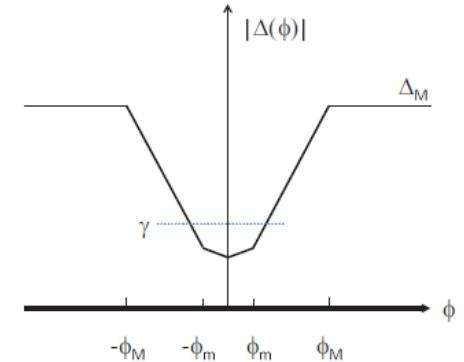
$$\hat{g}_0^R(\hat{\mathbf{p}}, \epsilon) = -\pi \frac{\tilde{\epsilon}^R \tau_3 - \hat{\Delta}}{D^R}$$

$$\tilde{\epsilon}^R \rightarrow \epsilon + i\gamma \quad \text{as } \epsilon \rightarrow 0$$

life time for $\epsilon \rightarrow 0$

equilibrium

$$D^R \equiv \sqrt{|\Delta(\phi)|^2 - (\tilde{\epsilon}^R)^2}.$$



$$\hat{g}_0^R \rightarrow -\pi \frac{i\gamma \tau_3 - \hat{\Delta}}{(\gamma^2 + |\Delta(\phi)|^2)^{1/2}}$$

$$\gamma = \Gamma_u \frac{\tilde{\gamma}}{\cot^2 \delta + \tilde{\gamma}^2}$$

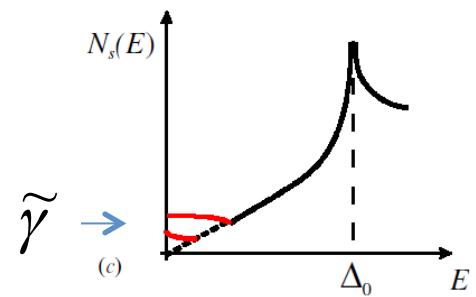
$$\tilde{\gamma} \equiv \left\langle \frac{\gamma}{(|\Delta(\phi)|^2 + \gamma^2)^{1/2}} \right\rangle$$

density of states at $\epsilon \rightarrow 0$

$$\left(\approx \frac{4\phi_M}{\pi} \frac{\gamma}{\Delta_M} \ln \frac{2\Delta_M}{\gamma} \right)$$

$$\Gamma_u = n_{\text{imp}} / \pi N_f$$

$$(\gamma \sim \sqrt{\Gamma_u \Delta_{\max}} \text{ if } |\cot \delta| \ll \tilde{\gamma})$$



gradient expansion

quasiclassical equations: (c.f. Andreev equation)

$$[\epsilon\tau_3 - \hat{\Delta} - \check{\sigma}_{\text{imp}}, \check{g}] + \underline{i\mathbf{v}_f \cdot \nabla \check{g}} = 0 \quad \check{g}(\hat{p}, \epsilon, \vec{r})$$
$$\check{g}^2 = -\pi^2 \check{1} \quad \hat{\Delta}(\hat{p})$$

First order in space gradient

no p-derivatives, formally higher order in Δ/E_F

$$\hat{g}_0^K = (\hat{g}_0^R - \hat{g}_0^A) \tanh\left(\frac{\epsilon}{2T(x)}\right)$$

$$\check{g} = \check{g}_0 + \check{g}_1$$

....

First order in gradients:

$$\hat{g}_1^K = (\hat{g}_1^R - \hat{g}_1^A) \tanh\left(\frac{\epsilon}{2T(x)}\right) + \hat{g}_{1a}^K$$


only piece needed for heat current

$$\hat{g}_{1a}^K = \hat{g}_{1a}^{K,ns} + \hat{g}_{1a}^{K,V}$$

$$J_i^E = 2N_f \int \frac{d\phi}{2\pi} v_{f,i} \int \frac{d\epsilon}{4\pi i} \epsilon \frac{1}{4} \text{Tr} g_{1a}^K$$

“non-self-consistent”

$$\frac{\hat{g}_{1a}^{K,ns}}{\pi} = \frac{i\gamma^2 + \gamma\tau_3\hat{\Delta}}{(|\Delta(\phi)|^2 + \gamma^2)^{3/2}} \left(\frac{\epsilon}{2T^2} \operatorname{sech}^2 \frac{\epsilon}{2T} \right) \mathbf{v}_f \cdot (-\nabla T)$$



universal thermal conductivity

(γ cancels if line nodes)

“vertex correction”

$$\hat{g}_{1a}^{K,V} \propto \hat{\sigma}_{imp,1a}^{K,V}$$



correction to K_{xx}
generates finite K_{yx}

$$\sigma_{1a,\text{imp}}^K = \Gamma_u \left(\cot\delta + \langle \frac{\hat{g}_0^R}{\pi} \rangle \right)^{-1} \langle \frac{\hat{g}_{1a}^K}{\pi} \rangle \left(\cot\delta + \langle \frac{\hat{g}_0^A}{\pi} \rangle \right)^{-1}$$

isotropic scattering,
p-wave only

$$\frac{K_{yx}}{K_{xx}^{univ}} \approx \frac{2\phi_M}{\pi} \times \frac{\gamma}{\Delta_M} \times \frac{2 \cot \delta}{\cot^2 \delta + \left(\frac{4\phi_M}{\pi} \frac{\gamma}{\Delta_M} \ln \frac{2\Delta_M}{\gamma} \right)^2}$$

$\tilde{\gamma}$

not universal, vanishes if δ integral multiple of $\pi/2$,
but

if $\cot \delta \sim \left(\frac{4\phi_M}{\pi} \frac{\gamma}{\Delta_M} \ln \frac{2\Delta_M}{\gamma} \right)$ then

$$\frac{K_{yx}}{K_{xx}^{univ}} \sim \frac{1}{2 \ln \frac{2\Delta_M}{\gamma}}$$

$$K_{xx}^{univ} = N_f v_f^2 \frac{\pi^2}{3} T \times \frac{2\phi_M}{\pi \Delta_M}$$

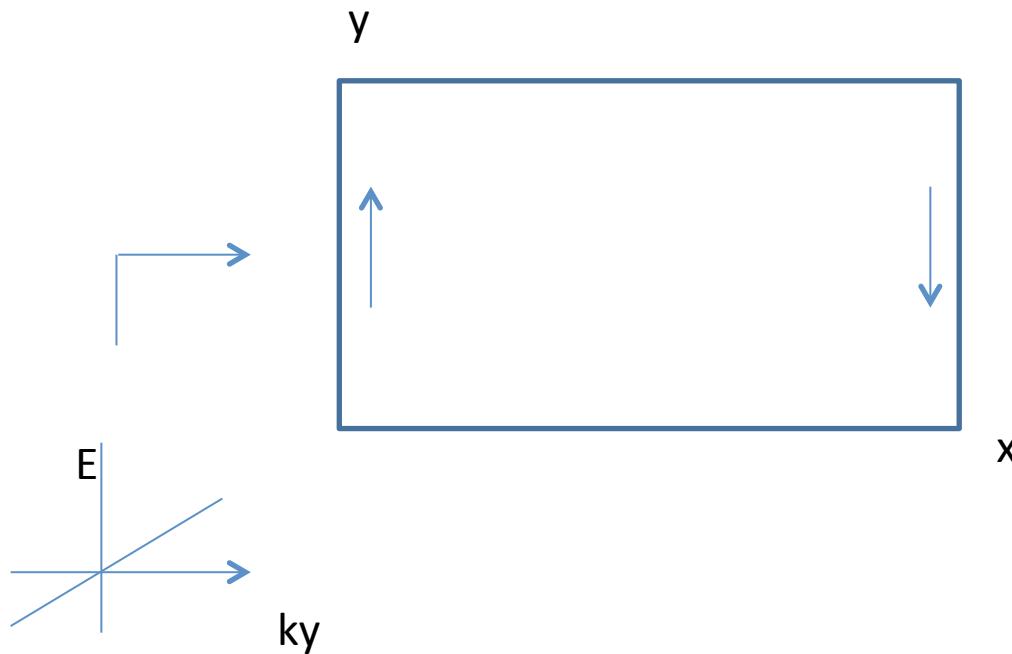
$$\frac{K_{yx}}{T} \sim \frac{\pi^2}{6} k_B^2 \frac{E_f}{T_c} \frac{\frac{2\phi_M}{\pi}}{\ln \frac{2\Delta_M}{\gamma}}$$

Thermal Hall conductivity via (topological) surface state

Senthil and Fisher 99 (d+id')
Read and Green 00

....

Scaffidi+Simon, PRL 15
Imai et al, arXiv:1601.0345



universal topological edge state contribution

$$\frac{K_{yx}}{T} = \frac{C}{2\pi} \frac{\pi^2}{6} k_B^2$$

where C is the Chern number

universal topological edge state contribution

$$\frac{C}{2\pi} \frac{\pi^2}{6} k_B^2$$

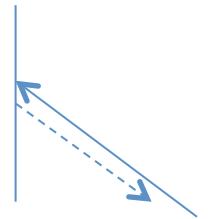
Impurity band contribution typically *much larger**

$$\frac{K_{yx}}{T} \sim \frac{\pi^2}{6} k_B^2 \frac{E_f}{T_c} \frac{\frac{2\phi_M}{\pi}}{\ln \frac{2\Delta_M}{\gamma}} \times \frac{2\tilde{\gamma} \cot \delta}{\cot^2 \delta + \tilde{\gamma}^2}$$

since $E_f / T_c \gg 1$

(unless δ very close to integral multiple of $\pi/2$ or very small γ)

[if quasiclassical/ Andreev approx, surface bound state has NO contribution to thermal conduction, so small factor T_c / E_F
Andreev 1964!]

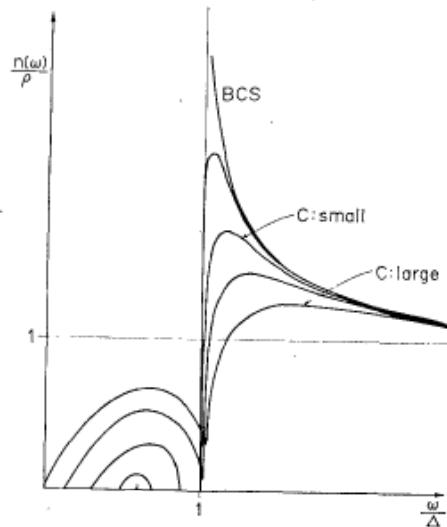


*though thermal Hall angle $\tan^{-1}(K_{yx} / K_{xx})$ much smaller

Fully gapped:

$$\frac{K_{yx}}{N_f v_f^2 \frac{\pi^2}{3} T} \approx \frac{2 \cot \delta}{\cot^2 \delta + \frac{\gamma^2}{\Delta_M^2}} \frac{\gamma^3}{\Delta_M^4}$$

$$\frac{K_{yx}}{T} \sim k_B^2 \frac{E_f}{T_c} \frac{\left(\frac{\gamma}{\Delta}\right)^2 \cot \delta}{\cot^2 \delta + \left(\frac{\gamma}{\Delta}\right)^2}$$



sign: obtained

$$K_{yx} > 0, K_{xy} < 0 \text{ if } \tan\delta > 0$$

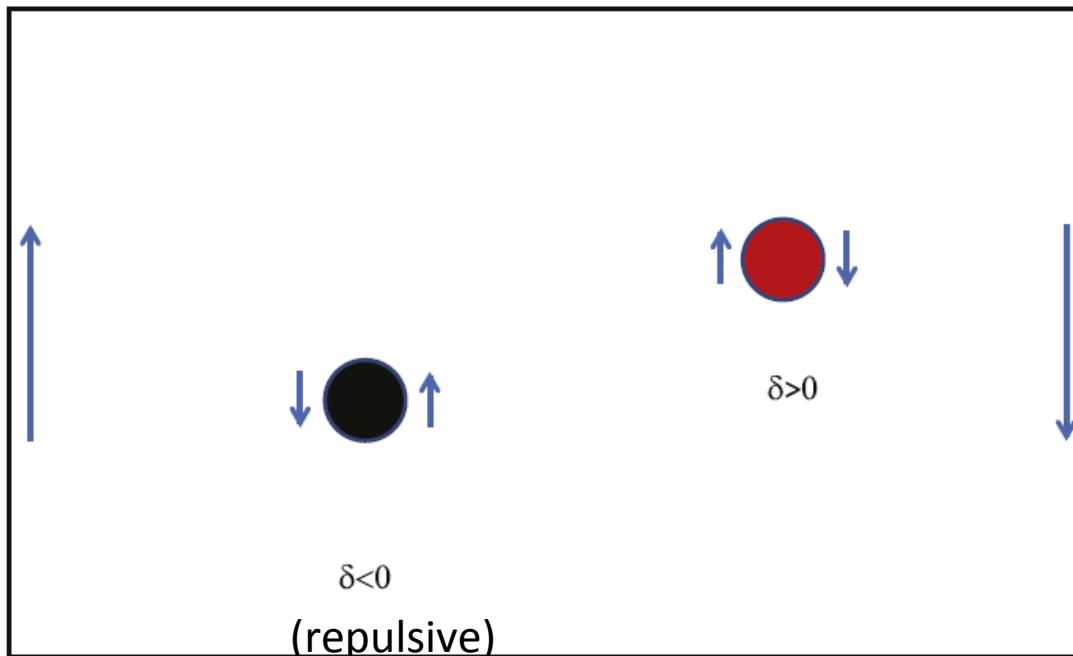
opposite sign to Arfi et al

found no mechanism of
sign changes with T

But:

z-axis out

mode propagation direction



sign: obtained

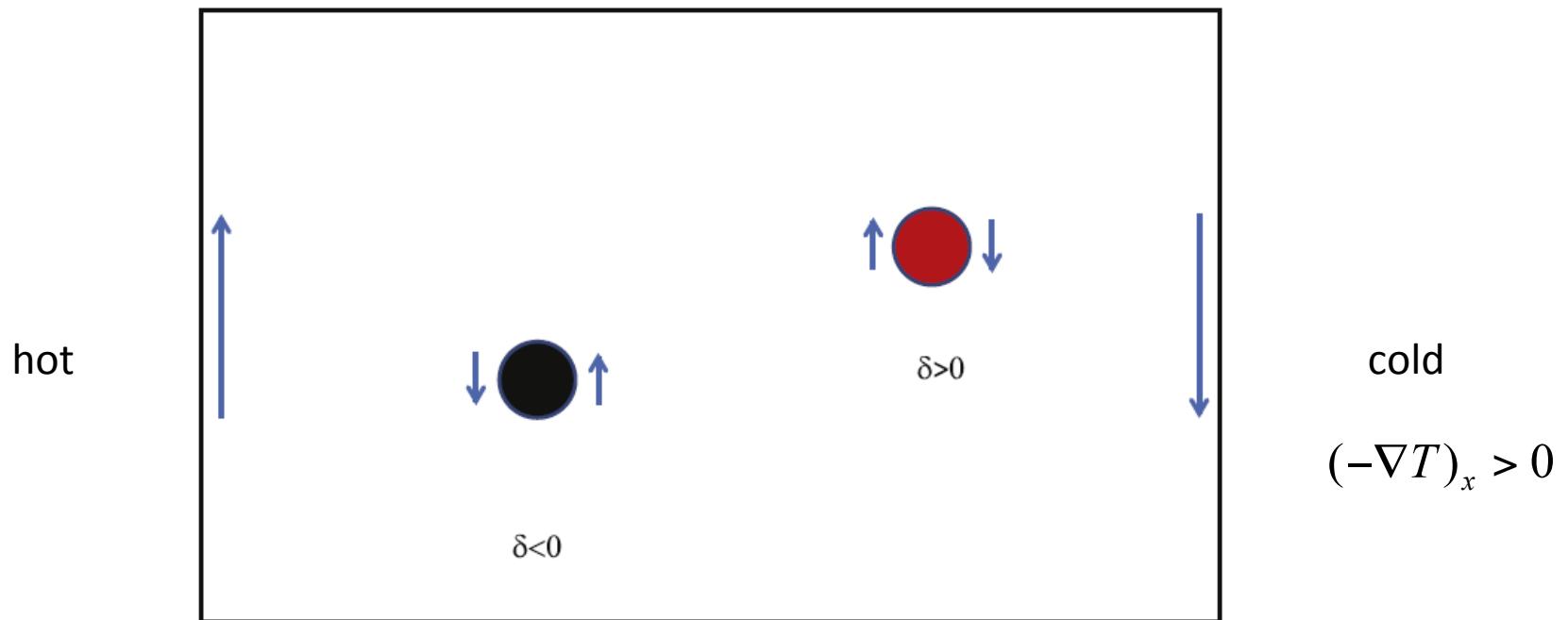
$$K_{yx} > 0, K_{xy} < 0 \text{ if } \tan\delta > 0$$

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But:

z-axis out

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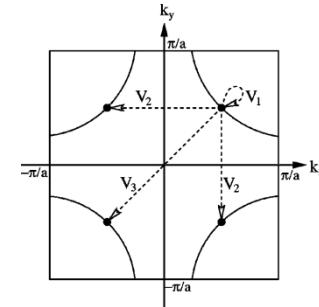
$K_{yx} < 0$ if $\tan\delta < 0$, in agreement with mine

Back to line nodes:

vertex correction to K_{xx}

only investigated by Durst and Lee PRB 00

scattering between nodes



I get: $K_{xx}^V / K_{xx}^{univ} \sim \text{coef} / \ln \frac{2\Delta_M}{\gamma}$

same form as Durst and Lee, but
coef ~ 1 (Born) $\rightarrow -1/3$ (resonance)

Zero field thermal Hall

spontaneous symmetry breaking (time-reversal / reflection)

previous work: He3-A ([th: U. Eckern](#))

Others:

Hc1 asymmetry

field and current around impurities (muSR?)

circular dichroism / Kerr ([expr: Kapitulnik](#))

currents near surfaces/ domain walls (?)

Hall mobility of electron bubble ([expr: Kono et al; th: Shevtsov+Sauls](#))

Considered only isotropic scatterers

single band

spin-orbit coupling?

anisotropic scattering?

$$K_{xy}=-K_{yx}=\frac{36}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\Delta dE\frac{\Delta^2}{T^2}\left\{-\frac{\partial n^0}{\partial E}\right\}b^2Lc_2\langle D\tilde{v}^2\mu_x^2\rangle^2$$

$$\begin{aligned} \overline{|t_{\mathbf{p}'\mathbf{p}}^s|^2} = & \tfrac{1}{2} \operatorname{tr}[|t_{11}|^2 u_{\mathbf{p}'}^\dagger u_{\mathbf{p}'} u_{\mathbf{p}}^\dagger u_{\mathbf{p}} + |t_{22}|^2 v_{\mathbf{p}'}^\dagger v_{\mathbf{p}'} v_{\mathbf{p}}^\dagger v_{\mathbf{p}} \\ & + 2 \operatorname{Re}(t_{11} t_{22}^* v_{\mathbf{p}'} u_{\mathbf{p}'}^\dagger u_{\mathbf{p}} v_{\mathbf{p}}^\dagger)] , \end{aligned}$$

$$\begin{aligned} t_{\mathbf{p}'\mathbf{p}}^s = & u_{\mathbf{p}'}^\dagger(t_{11})_{\mathbf{p}'\mathbf{p}} u_{\mathbf{p}} + u_{\mathbf{p}'}^\dagger(t_{12})_{\mathbf{p}'\mathbf{p}} v_{\mathbf{p}} \\ & + v_{\mathbf{p}'}^\dagger(t_{21})_{\mathbf{p}'\mathbf{p}} u_{\mathbf{p}} + v_{\mathbf{p}'}^\dagger(t_{22})_{\mathbf{p}'\mathbf{p}} v_{\mathbf{p}} \end{aligned}$$

$$\begin{aligned} \overline{|t_{\mathbf{p}'\mathbf{p}}^s|^2} = & \left[a \left(1 + \frac{\xi_{\mathbf{p}} \xi_{\mathbf{p}'} }{E_{\mathbf{p}} E_{\mathbf{p}'}} \right) + b \left(\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} + \frac{\xi_{\mathbf{p}'}}{E_{\mathbf{p}'}} \right) \right. \\ & \left. + 2 \operatorname{Re} \left[c \frac{\Delta_{\mathbf{p}} \cdot \Delta_{\mathbf{p}'}^*}{E_{\mathbf{p}} E_{\mathbf{p}'}} \right] \right] \frac{|t_N|^2}{2} , \end{aligned}$$

$$\begin{aligned} c = & -\tfrac{1}{2}\{ \cos^2\delta_N - |g(x)|^2 \sin^2\delta_N \\ & - 2i \operatorname{Re}[g(x)] \sin\delta_N \cos\delta_N \}^{-1} \end{aligned}$$

$$\operatorname{Re}(g(x))>0$$

$$c=c_1+ic_2$$

$$\operatorname{sgn}(c_2)=-\operatorname{sgn}(\tan\delta)?$$

$$\operatorname{sgn}(K_{xy})=-\operatorname{sgn}(\tan\delta)?$$

$$\text{then same as mine}$$

