Odd-Frequency Superconductivity in Topological Insulators and Multiband Superconductors

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Outline

- Introduction to odd-frequency pairing
- Odd-frequency pairing in topological insulator-superconductor hybrid structures
  - Spin-singlet *s*-wave superconductor
  - Spin-triplet *p*-wave superconductor
- Odd-frequency pairing in multiband superconductors

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Superconducting Symmetries

The superconducting order parameter is fermionic:

$$\Delta_{\alpha\beta}(\mathbf{k}) = -\Delta_{\beta\alpha}(-\mathbf{k})$$

orbital

$$\Delta_{\alpha\beta}(\mathbf{k}) = \Delta_0 e^{i\varphi}\eta(\mathbf{k})\chi_{\alpha\beta}$$

spin

spin-singlet *s*-wave or spin-triplet *p*-wave

The order parameter can also be odd in time/frequency: [1]

$$\Delta_{\alpha\beta}(\mathbf{k},\omega) = -\Delta_{\beta\alpha}(-\mathbf{k},-\omega)$$

odd-frequency spin-triplet *s*-wave

[1]: Berezinskii, JETP Lett. 20, 287 (1974)

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Odd-frequency (ω) Pairing

BCS order parameter: $F(\mathbf{r}, t; \mathbf{r}', t' \to t) = \langle \psi(\mathbf{r}, t)\psi(\mathbf{r}', t' \to t)\rangle$

vanishes for an odd-frequency component

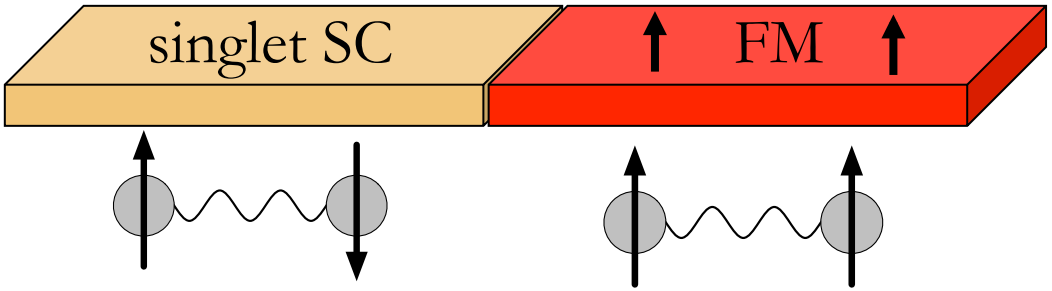
Equal-time odd-frequency order parameter: [1,2]

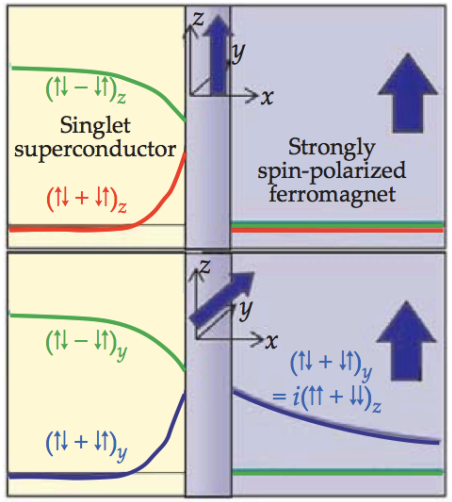
$$\left.\frac{\mathrm{d}F(\mathbf{r}, t; \mathbf{r}', t')}{\mathrm{d}t}\right|_{t\to t'}$$

Theory proposals for odd-frequency bulk superconductors exists [1,2]
but only found so far at interfaces

[1]: Abrahams et al, PRB 52, 1271 (1995), [2]: Dahal et al, NJP 11, 065005 (2009)

S | F Interface





[1]

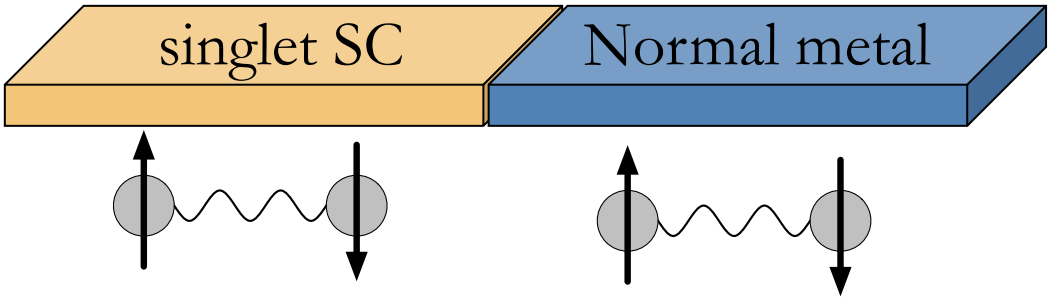
Spin-singlet *s*-wave pairing in SC converted into

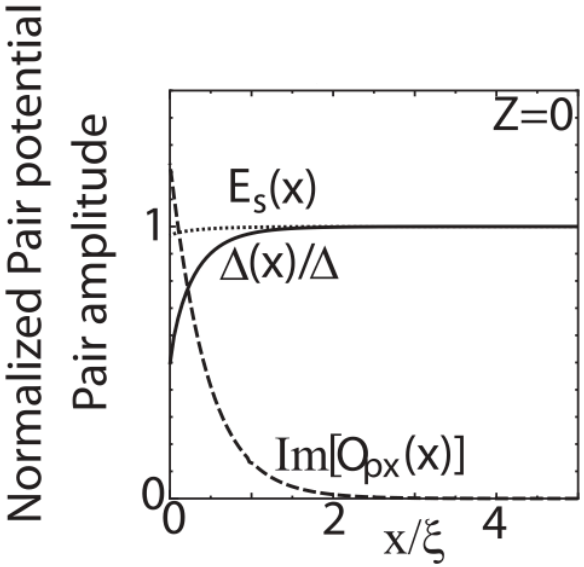
odd-frequency spin-triplet *s*-wave pairing in FM

- Long-range superconducting proximity effect in the FM
- *s*-wave = robust against impurities

Bergeret et al, RMP, 77, 1321 (2005), [1]: Eschrig, Phys. Today 64, 43 (2011)

S | N Interface





[1]

Spin-singlet *s*-wave pairing in SC converted into

odd-frequency spin-singlet p-wave pairing

- Only high-transparency junctions
- *p*-wave = only ballistic systems

[1]: Tanaka et al, PRL 99, 037005 (2007)

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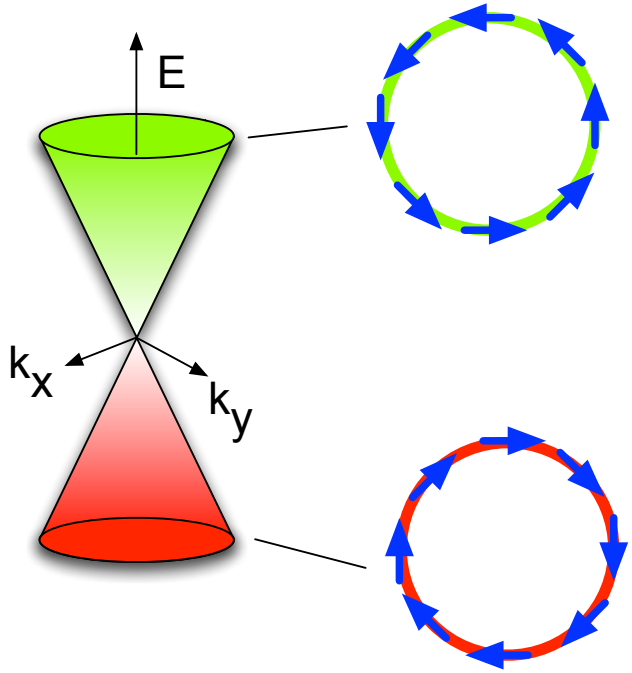
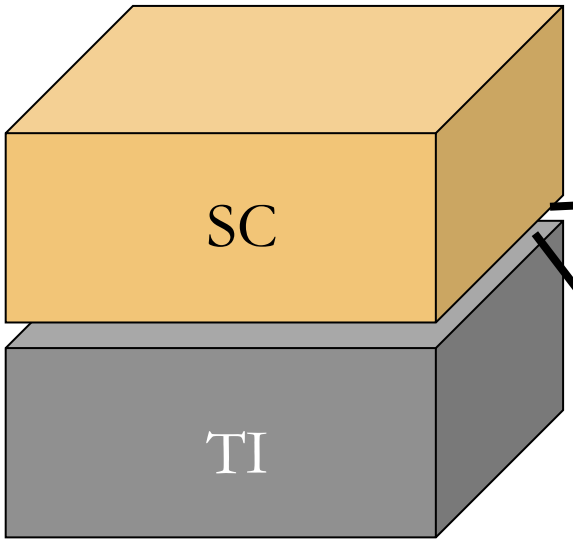
Topological Insulator (TI)

Surface state of a topological insulator

- Dirac spectrum
- Momentum locked to spin: H ~ $\mathbf{k} \cdot \boldsymbol{\sigma}$



TI – SC Hybrid Structure



$$H_{\mathrm{SC}} = \sum_{\mathbf{k},\alpha,\beta} \varepsilon(\mathbf{k}) d^{\dagger}_{\alpha,\mathbf{k}} d_{\alpha,\mathbf{k}} + \sum_{i,\alpha,\beta} \Delta(i)_{\alpha\beta} d^{\dagger}_{\alpha,i} d^{\dagger}_{\beta,i} + \mathrm{H.c.}$$

Conventional s-wave SC with position (i) dependent order parameter

$$H_T = \sum_{\alpha} T_i c^{\dagger}_{\alpha,i} d_{\alpha,i} + \mathrm{H.c.}$$

Local tunneling

$$H_{\mathrm{TI}} = \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\alpha,\mathbf{k}} \mathbf{k} \cdot \sigma_{\alpha\beta} c_{\beta,\mathbf{k}}$$

TI surface state

ABS and Balatsky, PRB 86, 144506 (2012)

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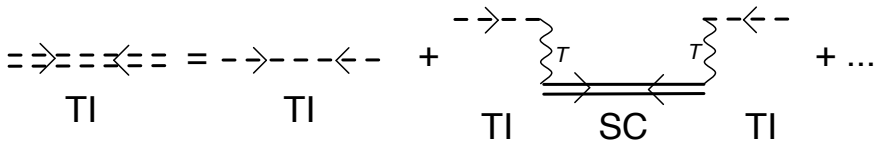
Analytic Derivation

Anomalous Green's function in TI:



$$\hat{F}_{\mathrm{TI}}(\omega_n|i,i) = -|T|^2 \sum_{j,l} \hat{G}^0(\omega_n|i,j)\hat{F}(j,l)\hat{G}^0(\omega_n|l,i)$$

$$\left(\begin{aligned} \hat{G}^0(\omega_n,\mathbf{k}) &= \frac{\mathbf{k}\cdot\sigma - i\omega_n}{\mathbf{k}^2+\omega_n^2} \\ \hat{F}(\omega_n,\mathbf{k}) &= \frac{\hat{\Delta}(i)}{\omega_n^2+\varepsilon^2(\mathbf{k})+\hat{\Delta}(i)^2} \\ \Delta(\mathbf{k}) &= \Delta_0\delta_{\mathbf{k},0} + i\frac{\partial\Delta}{\partial x}|_0\partial_{\mathbf{k}_x} \end{aligned}\right)$$

Order parameter for *s*-wave odd-frequency pairing:

$$\begin{aligned} \hat{F}_{\mathrm{TI}}(\omega_n|i{=}0) &= \sum_{\mathbf{k}} \frac{|T|^2\omega_n\hat{\sigma}\partial_x\hat{\Delta}|_0}{2[\omega_n^2+\varepsilon(\mathbf{k})^2+\Delta^2(0)](\omega_n^2+\mathbf{k}^2)^2} \\ &\sim |T|^2\omega_n\sigma^z\partial_x\Delta|_0/(E_F^2|\omega_n|^2) \end{aligned} \quad \Rightarrow \quad \partial_\tau\hat{F}_{\mathrm{TI}}(\tau|i)|_0 \sim \frac{\partial\Delta}{\partial x}$$

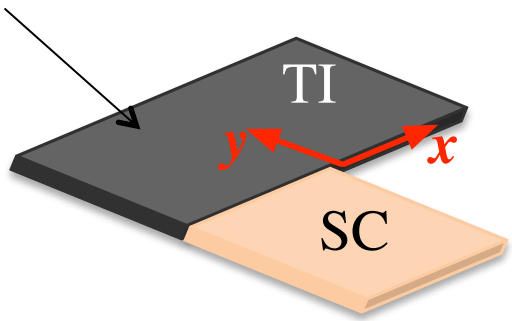
→ Odd-frequency spin-triplet *s*-wave pairing:

- Spatially inhomogeneous SCs

ABS and Balatsky, PRB 86, 144506 (2012)

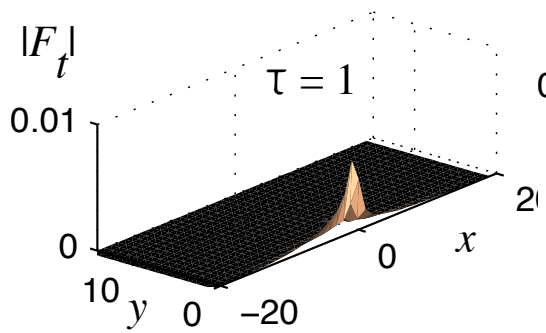
S | N Junction in a 2D TI

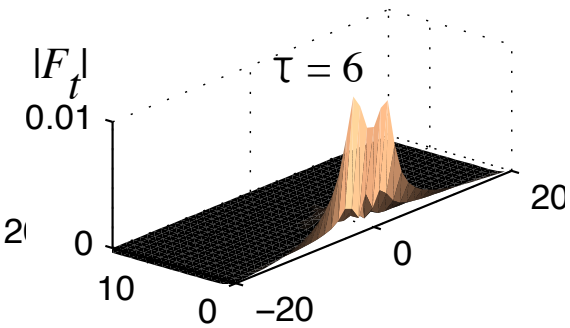
Kane-Mele 2D TI



Spin-triplet s-wave pairing:

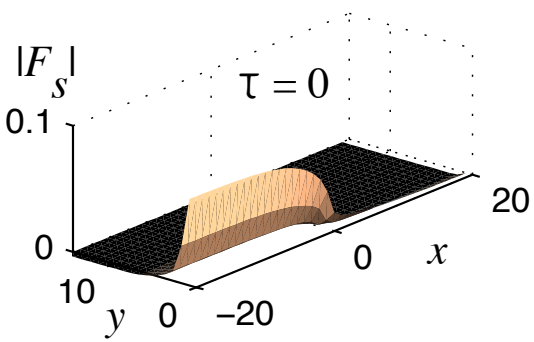
$$F_t(\tau|i) = (\langle c_{i\downarrow}(\tau)c_{i\uparrow}(0) + c_{i\uparrow}(\tau)c_{i\downarrow}(0)\rangle)/2$$

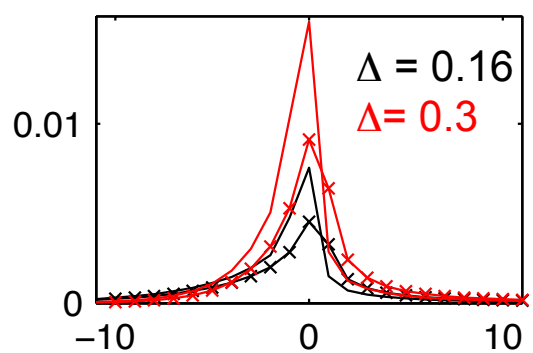




Spin-singlet s-wave pairing:

$$F_s(\tau|i) = (\langle c_{i\downarrow}(\tau)c_{i\uparrow}(0) - c_{i\uparrow}(\tau)c_{i\downarrow}(0)\rangle)/2$$





$\times$: $\partial_\tau F_t|_0$

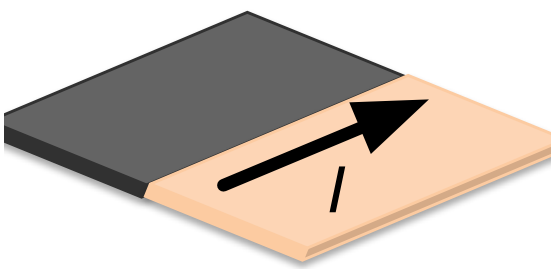
$-$: $\dfrac{\partial F_s}{\partial x}$

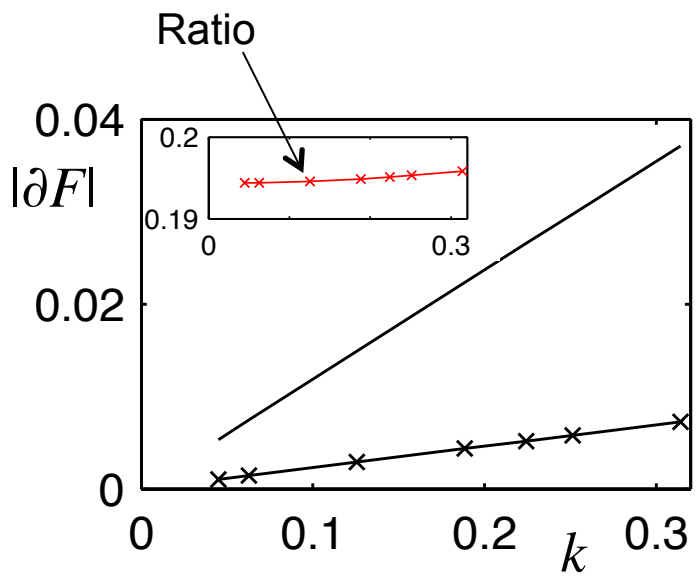
$$\boxed{\partial_\tau F_t|_0 \sim \frac{\partial F_s}{\partial x}}$$

ABS and Balatsky, PRB 86, 144506 (2012)

In-surface Supercurrent

In-surface supercurrent: $\Delta = |\Delta| e^{\imath k x}$





$\times : \partial_\tau F_t|_0$

$- : \dfrac{\partial F_s}{\partial x}$

$$\partial_\tau F_t|_0 \sim \frac{\partial F_s}{\partial x}$$

ABS and Balatsky, PRB 86, 144506 (2012)

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Gradient-Induced Odd-ω Pairing

- Electric field induced sublattice staggering in silicene and stanene

- Linear **k**-dependence of the pairing in a *p*-wave superconductor:

$$\hat{F}_{\mathrm{TI}}(\mathbf{k},\omega_n) = \frac{-4i|T|^2\Delta_0\overbrace{[\mathbf{k}\times\mathbf{d}(\mathbf{k})]}^{\text{Effective }\mathbf{d}\text{-vector}}\cdot\boldsymbol{\sigma}\sigma_y}{(\omega_n^2+k^2)^2(\omega_n^2+E_k^2)}\omega_n$$

Odd-frequency *s*-wave spin-triplet pairing

Kuzmanovski and ABS (in preparation), ABS and Balatsky, PRB 87, 220506(R) (2013),

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Odd-frequency pairing in TIs

Odd-frequency pairing in TI-SC hybrid structures

- Spin-singlet *s*-wave SC with in-plane gradient →

  Odd-frequency spin-triplet *s*-wave pairing

  - SN junctions
  - Supercurrents
  - Sublattice staggering

- Spin-triplet *p*-wave SC →

  Odd-frequency spin-triplet *s*-wave pairing

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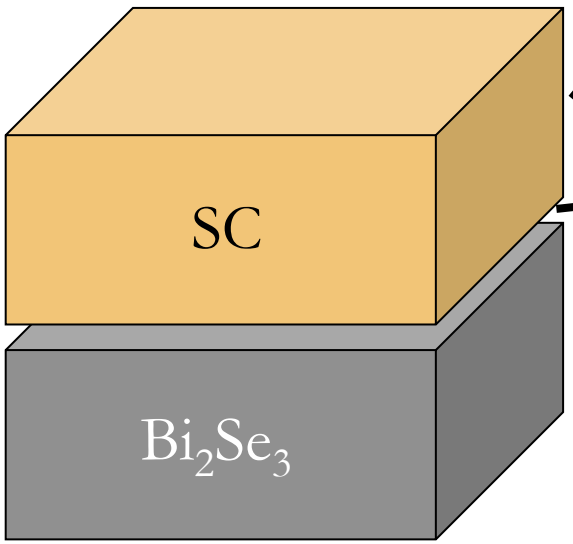
Bi_2Se_3 – SC Hybrid Structure



$$H_{\mathrm{SC}} = \sum_{\mathbf{k},\sigma} (-2\cos(k_x a) - 2\cos(k_y a) + \mu_{\mathrm{SC}}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{2} \sum_{\mathbf{k},\sigma,\sigma'} \Delta_{\sigma\sigma'}(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c^\dagger_{-\mathbf{k}\sigma'} - \Delta^*_{\sigma\sigma'}(-\mathbf{k}) c_{-\mathbf{k}\sigma} c_{\mathbf{k}\sigma'}$$

2D superconductor

$$H_T = -\sum_{\mathbf{k},\sigma} T_1 c^\dagger_{\mathbf{k}\sigma} b_{1\mathbf{k}\sigma} + T_2 c^\dagger_{\mathbf{k}\sigma} b_{2\mathbf{k}\sigma} + \mathrm{H.c.}$$

Local tunneling

$$H_{\mathrm{TI}} = \gamma_0 - 2\sum_{\mathbf{k},i} \gamma_i \cos(k_i a) + \sum_{\mathbf{k},\mu} d_\mu \Gamma_\mu$$

Bi_2Se_3 (τ orbital Pauli matrix) [1]

$$\left(\begin{array}{l} d_0 = \epsilon - 2\sum_i t_i \cos(k_i a), d_i = -2\lambda_i \sin(k_i a) \\ \Gamma_0 = \tau_x \otimes \sigma_0, \Gamma_x = -\tau_z \otimes \sigma_y, \Gamma_y = \tau_z \otimes \sigma_x, \Gamma_z = \tau_y \otimes \sigma_0 \end{array} \right)$$

ABS and Balatsky, PRB 87, 220506(R) (2013), [1]: Rosenberg and Franz, PRB 85, 195119 (2012)

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Superconductivity in Bi_2Se_3

Classification of all superconducting symmetries in Bi_2Se_3

– Spin-singlet/triplet, spatial ($s/d/p$-wave), even/odd-frequency, even/odd orbital

SC			Proximity-induced superconductivity in Bi_2Se_3			
Superconductor			Even-frequency		Odd-frequency	
Γ	Basis function	J_z	Even-orbital	Odd-orbital	Even-orbital	Odd-orbital
A_{1g}	$\psi = 1$	0	A_{1g} singlet, A_{2u} triplet ($m_s = \pm 1$)	-	-	A_{1g} singlet, A_{2u} triplet ($m_s = \pm 1$)
B_{1g}	$\psi = k_x^2 - k_y^2$	± 2	B_{1g} singlet, B_{2u} triplet ($m_s = \pm 1$)	-	-	B_{1g} singlet, B_{2u} triplet ($m_s = \pm 1$)
B_{2g}	$\psi = 2k_x k_y$	± 2	B_{2g} singlet, B_{1u} triplet ($m_s = \pm 1$)	-	-	B_{2g} singlet, B_{1u} triplet ($m_s = \pm 1$)
A_{1u}	$\mathbf{d} = (k_x, k_y, 0)$	0	A_{1u} triplet ($m_s = \pm 1$)	A_{1g} triplet ($m_s = 0$)	A_{1g} triplet ($m_s = 0$)	A_{1u} triplet ($m_s = \pm 1$)
A_{2u}	$\mathbf{d} = (k_y, -k_x, 0)$	0	A_{2u} triplet ($m_s = \pm 1$), A_{1g} singlet	-	-	A_{2u} triplet ($m_s = \pm 1$), A_{1g} singlet
B_{1u}	$\mathbf{d} = (k_x, -k_y, 0)$	± 2	B_{1u} triplet ($m_s = \pm 1$), B_{2g} singlet	B_{1g} triplet ($m_s = 0$)	B_{1g} triplet ($m_s = 0$)	B_{1u} triplet ($m_s = \pm 1$), B_{2g} singlet
B_{2u}	$\mathbf{d} = (k_y, k_x, 0)$	± 2	B_{2u} triplet ($m_s = \pm 1$), B_{1g} singlet	B_{2g} triplet ($m_s = 0$)	B_{2g} triplet ($m_s = 0$)	B_{2u} triplet ($m_s = \pm 1$), B_{1g} singlet
E_{2u}^+	$\mathbf{d} = (0, 0, k_x + ik_y)$	1	E_{2u}^+ triplet ($m_s = 0$)	A_{1g} triplet ($m_s = 1$), $B_{1g}+iB_{2g}$ triplet ($m_s = -1$)	A_{1g} triplet ($m_s = 1$), $B_{1g}+iB_{2g}$ triplet ($m_s = -1$)	E_{2u}^+ triplet ($m_s = 0$)
E_{2u}^-	$\mathbf{d} = (0, 0, k_x - ik_y)$	-1	E_{2u}^- triplet ($m_s = 0$)	A_{1g} triplet ($m_s = -1$), $B_{1g}-iB_{2g}$ triplet ($m_s = 1$)	A_{1g} triplet ($m_s = -1$), $B_{1g}-iB_{2g}$ triplet ($m_s = 1$)	E_{2u}^- triplet ($m_s = 0$)

ABS and Balatsky, PRB 87, 220506(R) (2013)

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Frequency and Interband Index

Complete reciprocity between oddness in frequency and orbital index

Superconductor			Even-frequency		Odd-frequency	
Γ	Basis function	J_z	Even-orbital	Odd-orbital	Even-orbital	Odd-orbital
A_{1g}	$\psi = 1$	0	A_{1g} singlet, A_{2u} triplet ($m_s = \pm 1$)	-	-	A_{1g} singlet, A_{2u} triplet ($m_s = \pm 1$)
B_{1g}	$\psi = k_x^2 - k_y^2$	± 2	B_{1g} singlet, B_{2u} triplet ($m_s = \pm 1$)	-	-	B_{1g} singlet, B_{2u} triplet ($m_s = \pm 1$)
B_{2g}	$\psi = 2k_x k_y$	± 2	B_{2g} singlet, B_{1u} triplet ($m_s = \pm 1$)	-	-	B_{2g} singlet, B_{1u} triplet ($m_s = \pm 1$)
A_{1u}	$\mathbf{d} = (k_x, k_y, 0)$	0	A_{1u} triplet ($m_s = \pm 1$)	A_{1g} triplet ($m_s = 0$)	A_{1g} triplet ($m_s = 0$)	A_{1u} triplet ($m_s = \pm 1$)
A_{2u}	$\mathbf{d} = (k_y, -k_x, 0)$	0	A_{2u} triplet ($m_s = \pm 1$), A_{1g} singlet	-	-	A_{2u} triplet ($m_s = \pm 1$), A_{1g} singlet
B_{1u}	$\mathbf{d} = (k_x, -k_y, 0)$	± 2	B_{1u} triplet ($m_s = \pm 1$), B_{2g} singlet	B_{1g} triplet ($m_s = 0$)	B_{1g} triplet ($m_s = 0$)	B_{1u} triplet ($m_s = \pm 1$), B_{2g} singlet
B_{2u}	$\mathbf{d} = (k_y, k_x, 0)$	± 2	B_{2u} triplet ($m_s = \pm 1$), B_{1g} singlet	B_{2g} triplet ($m_s = 0$)	B_{2g} triplet ($m_s = 0$)	B_{2u} triplet ($m_s = \pm 1$), B_{1g} singlet
E_{2u}^+	$\mathbf{d} = (0, 0, k_x + ik_y)$	1	E_{2u}^+ triplet ($m_s = 0$)	A_{1g} triplet ($m_s = 1$), $B_{1g}+iB_{2g}$ triplet ($m_s = -1$)	A_{1g} triplet ($m_s = 1$), $B_{1g}+iB_{2g}$ triplet ($m_s = -1$)	E_{2u}^+ triplet ($m_s = 0$)
E_{2u}^-	$\mathbf{d} = (0, 0, k_x - ik_y)$	-1	E_{2u}^- triplet ($m_s = 0$)	A_{1g} triplet ($m_s = -1$), $B_{1g}-iB_{2g}$ triplet ($m_s = 1$)	A_{1g} triplet ($m_s = -1$), $B_{1g}-iB_{2g}$ triplet ($m_s = 1$)	E_{2u}^- triplet ($m_s = 0$)

Generic property for multiband superconductors

ABS and Balatsky, PRB 87, 220506(R) (2013)

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Multiband Superconductors

- S: Spin (spin-singlet: S = 0 or spin-triplet: S = 1)
- P: Spatial parity (even: *s*-,*d*-wave or odd: *p*-,*f*-wave)
- T: Time (even or odd-frequency)
- O: Orbital/band parity

S = 0	P	T	O	S = 1	P	T	O
even-ω	+	+	+	even-ω	−	+	+
even-ω	−	+	−	even-ω	+	+	−
odd-ω	+	−	−	odd-ω	+	−	+
odd-ω	−	−	+	odd-ω	−	−	−

Spin-singlet *s*-wave: TO = 1

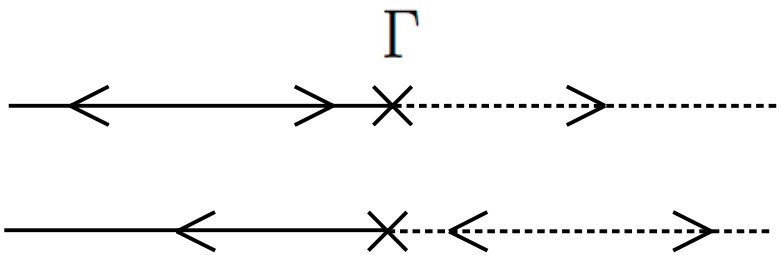
ABS and Balatsky, PRB 88, 104514 (2013)

Two-Band SC with Band Hybridization

Bands (orbitals) a & b with finite interband hybridization/scattering Γ:

$$H = \sum_{k\sigma} \epsilon_{k1} a^\dagger_{k\sigma} a_{k\sigma} + \epsilon_{k2} b^\dagger_{k\sigma} b_{k\sigma} + \boxed{\sum_{k\sigma} \Gamma(k) a^\dagger_{k\sigma} b_{k\sigma} + \text{H.c.}}$$
$$+ \sum_{k} \Delta_1(k) a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow} + \Delta_2(k) b^\dagger_{k\uparrow} b^\dagger_{-k\downarrow} + \text{H.c.}$$

Interband pairing F_{12}:



$F_1 \Gamma G_2$

$-\overleftarrow{G}_1 \Gamma F_2$

ABS and Balatsky, PRB 88, 104514 (2013), Komendova, Balatsky, and ABS, PRB 92, 094517 (2015)

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Interband Pairing

Perturbation theory to infinite order in Γ:
(using a geometric series)

Odd-interband: $$F_{12}^{\text{odd}}(\mathbf{k}, i\omega) = \frac{F_{12} - F_{21}}{2} = i\omega\Gamma(\Delta_1 - \Delta_2)/D$$

Even-interband: $$F_{12}^{\text{even}}(\mathbf{k}, i\omega) = \frac{F_{12} + F_{21}}{2} = \Gamma(\Delta_1\epsilon_{k2} + \Delta_2\epsilon_{k1})/D$$

$$\left(\begin{array}{l} D = (\omega^2+E_1^2)(\omega^2+E_2^2)-\Gamma^2[2(\epsilon_1\epsilon_2-\omega^2)-\Delta_2^*\Delta_1-\Delta_1^*\Delta_2]+\Gamma^4 \\ E_j^2 \;=\; E_{kj}^2 \;=\; \epsilon_{kj}^2 + |\Delta_j|^2 \end{array}\right)$$

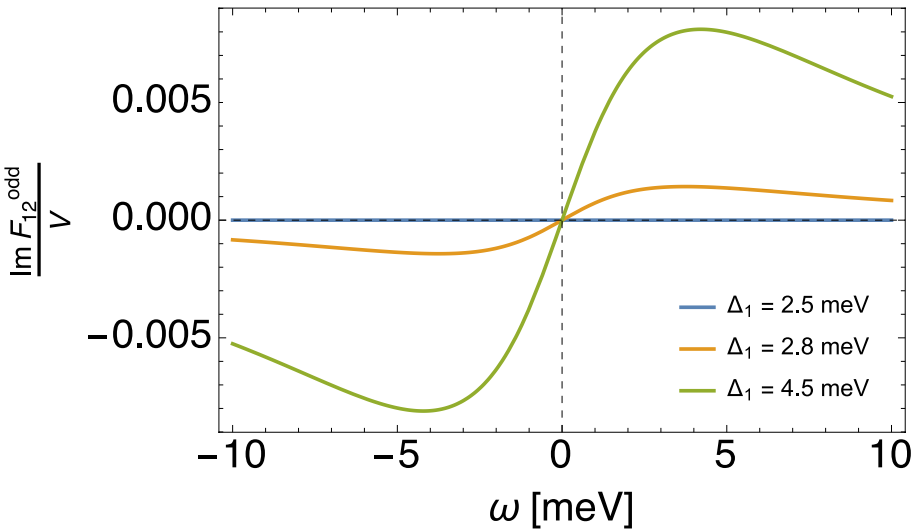
Interband pairing: $\Gamma \neq 0$

Odd-frequency, odd-interband pairing: $\Gamma \neq 0$, $\Delta_1 \neq \Delta_2$

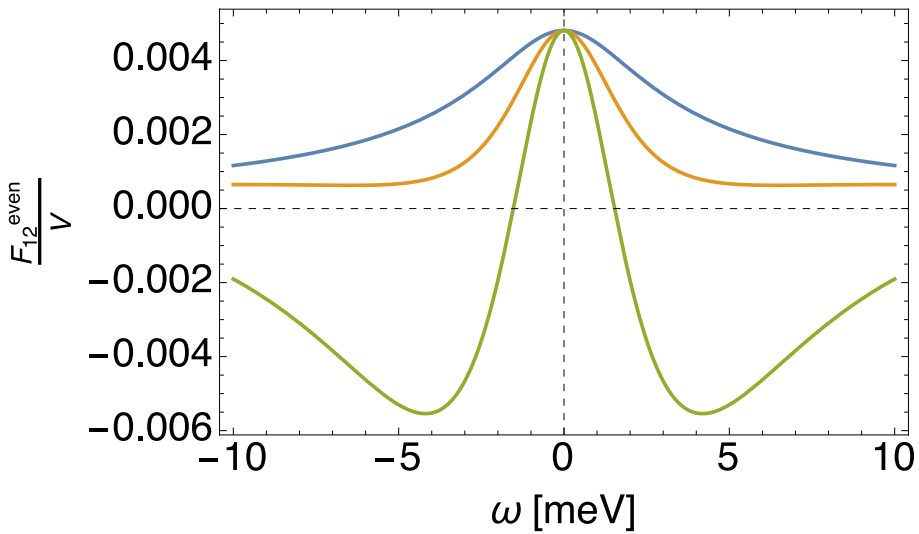
Komendova, Balatsky, and ABS, PRB 92, 094517 (2015)

Interband Frequency Dependence

Odd-frequency



Even-frequency



$\Delta_2 = 2.5$ meV, $\Gamma = 3$ meV

Odd-frequency, odd-interband pairing: $\Gamma \neq 0$, $\Delta_1 \neq \Delta_2$

Komendova, Balatsky, and ABS, PRB 92, 094517 (2015)

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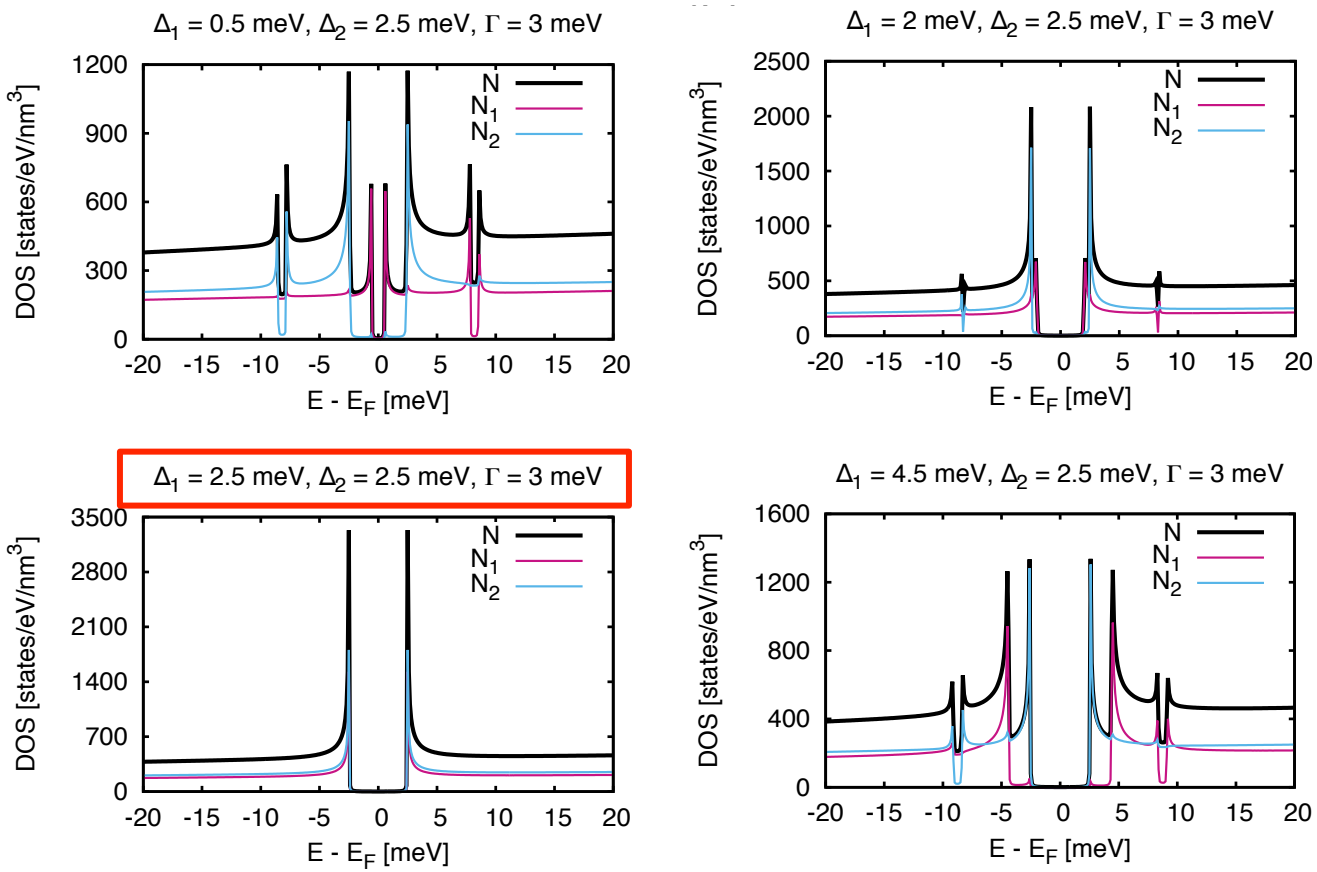
DOS for Two-Band Superconductor



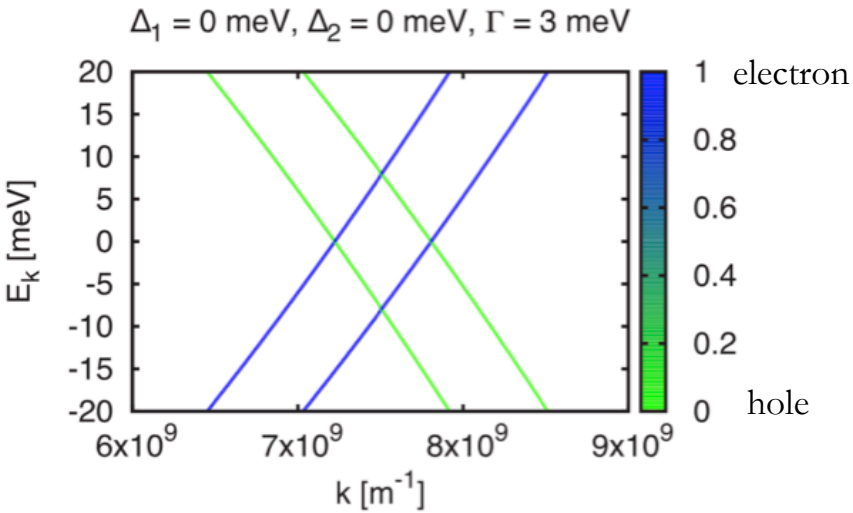
Additional gaps with coherence peaks at high energies

Only appears with odd-frequency pairing

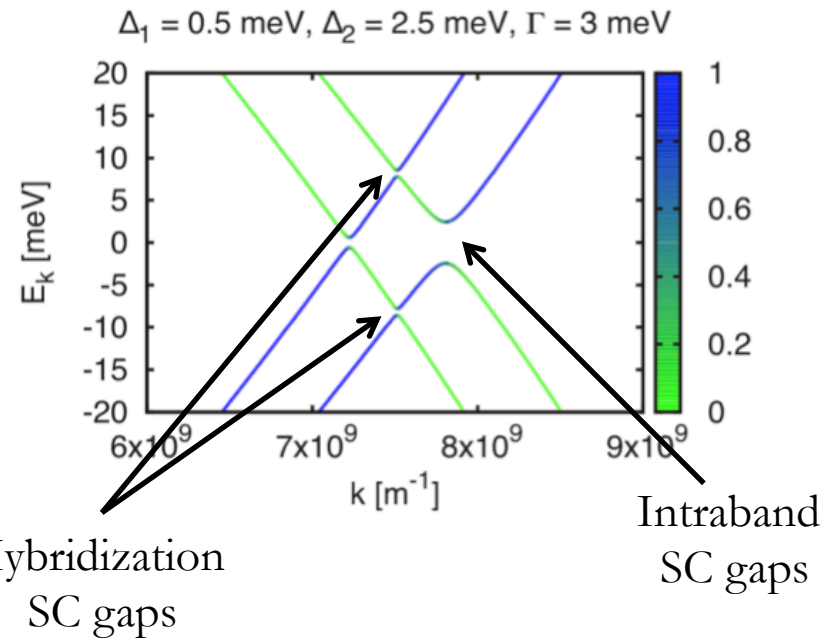
Komendova, Balatsky, and ABS, PRB 92, 094517 (2015)

Hybridization Gaps

Two bands



Two bands + Superconductivity



Hybridization gaps

- High-energy gaps with pronounced coherence peaks
- Only appears for finite odd-frequency pairing

Komendova, Balatsky, and ABS, PRB 92, 094517 (2015)

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Multiband Superconductors

Odd-frequency pairing in multiband superconductors

– Odd-frequency, odd-interband pairing if there exist interband pairing

- Finite interband hybridization (+ non-identical intraband pairing)
- Hybridization gaps only if odd-frequency pairing is present

- TI-SC hybrid structures
- Iron-based superconductors, heavy fermion superconductors, Sr_2RuO_4, MgB_2, …

ABS and Balatsky, PRB 88, 104514 (2013), Komendova, Balatsky, and ABS, PRB 92, 094517 (2015)

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Summary

- Odd-frequency pairing in TI-SC hybrid structures
  - Spin-singlet *s*-wave SC with in-plane gradient →

    Odd-frequency spin-triplet s-wave pairing
  - Spin-triplet *p*-wave SC →

    Odd-frequency spin-triplet s-wave pairing

- Odd-frequency pairing in multiband superconductors
  - Odd-frequency, odd-interband pairing if there is interband pairing
  - Gives hybridization gaps

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