Singularity structure of gravity amplitudes

with Enrico Herrmann (Caltech) 1604.03479 + in progress

related work by Paul Heslop and Arthur Lipstein 1604.03046

Jaroslav Trnka

Center for Quantum Mathematics and Physics (QMAP), UC Davis

Amplitudes 2016, Nordita, July 5, 2016

Mainz Institute for Theoretical Physics

ACTIVITIES 2017

SCIENTIFIC PROGRAMS JGU CAMPUS MAINZ

Amplitudes: Practical and Theoretical Developments
Fabrizio Caola CERN, Herbert Gangl Univ. Durham, Jaroslav Trnka UC Davis, Johannes Henn, Stefan Müller-Stach, Stefan Weinzierl JGU
February 6-17, 2017

Quantum Vacuum and Gravitation: Testing General Relativity in Cosmology
Manuel Asorey Univ. Zaragoza, Emil Mottola LANL, Ilya Shapiro Univ. Juiz de Fora, Andreas Wipf Univ. Jena
March 13-24, 2017

Low-Energy Probes of New Physics
Peter Fierlinger, Martin Jung TU Munich, Susan Gardner Univ. of Kentucky
May 2-24, 2017

The TeV Scale: a Threshold to New Physics?
Csaba Csaki Cornell, Christophe Grojean DESY, Andreas Weiler TU Munich, Pedro Schwaller JGU
June 12 - July 7, 2017

Diagrammatic Monte Carlo Methods for QFTs in Particle-, Nuclear-, and Condensed Matter Physics
Christoph Gattringer Univ. Graz, Shailesh Chandrasekharan Duke Univ., Dean Lee Univ. North Carolina State
September 18-29, 2017

TOPICAL WORKSHOPS JGU CAMPUS MAINZ

Quantum Methods for Lattice Gauge Theories Calculations
Ignacio Cirac MPI for Quantum Optics, Simone Montangero Univ. Ulm, Peter Zoller Univ. Innsbruck
February 6-10, 2017, Schloss Waldthausen

Women at the Intersection of Mathematics and High Energy Physics
Katrin Wendland Univ. Freiburg, Sylvie Paycha Univ. Potsdam, Kasia Rejzner Univ. York, Gabriele Honecker JGU
March 6-10, 2017

Geometry, Gravity and Supersymmetry
Vicente Cortés Univ. Hamburg, José Figueroa-O'Farrill Univ. Edinburgh, George Papadopoulos King's College London
April 24-28, 2017

Foundational and Structural Aspects of Gauge Theories
Claudio Dappiaggi Univ. Pavia, Klaus Fredenhagen DESY, Marco Benini Univ. Potsdam
May 29 - June 6, 2017

Supernova Neutrino Observations: What can we learn and do?
Hans-Thomas Janka MPI for Astrophysics, Georg Raffelt MPI for Physics, Lutz Köpke, Michael Wurm JGU
October 9-13, 2017

MITP SUMMER SCHOOL

August, 2017, Erbacher Hof Mainz
Joachim Kopp, Felix Yu, Anna Kaminska, Maikel De Vries, Matthias Neubert JGU

For more details: http://www.mitp.uni-mainz.de

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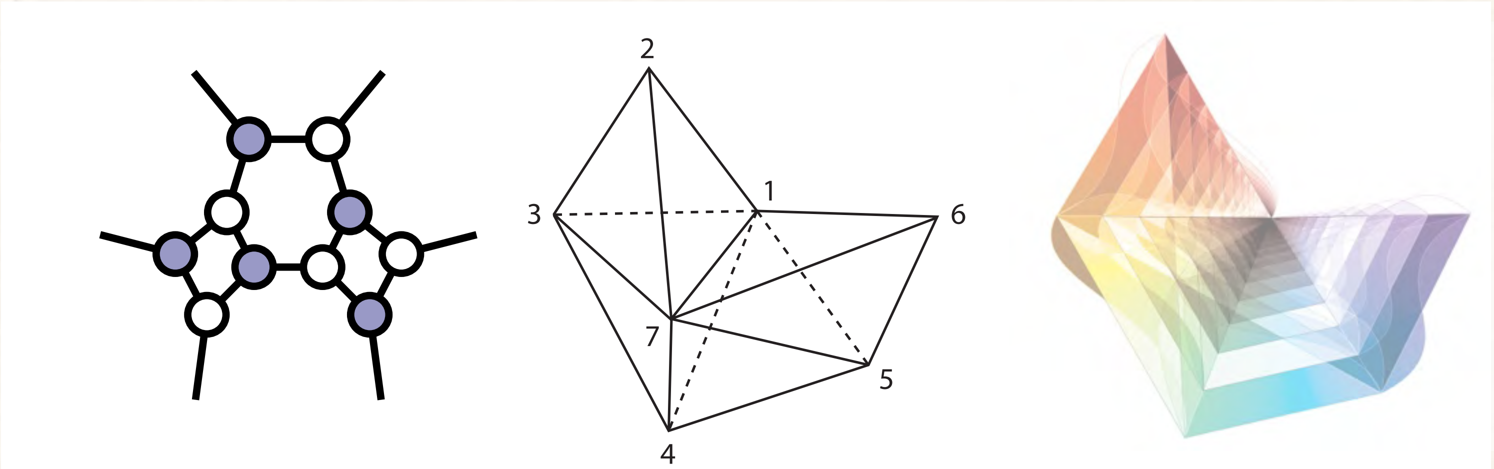
Goal

Mathematical structures in planar N=4 SYM



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Non-planar N=4 SYM

↓

N=8 SUGRA

Questions

- Do non-planar integrands have special properties?
- N=8 supergravity: singularity structure, from IR to UV

Singularities of amplitudes

- Scattering amplitudes of massless particles in D=4

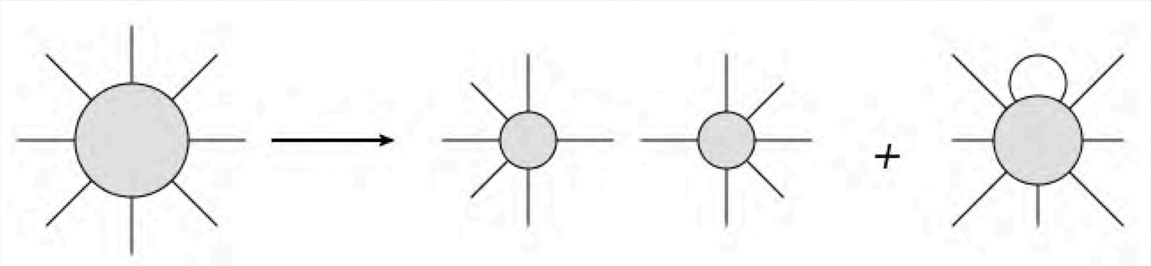
Fixed by singularities

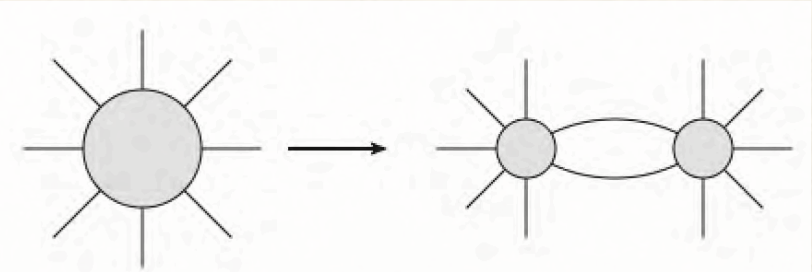
$$\mathcal{A} = \sum_j \int d\mathcal{I}_j = \int d\mathcal{I}$$

No loop momenta, complicated functions

Loop momenta, rational function

Powerful unitarity

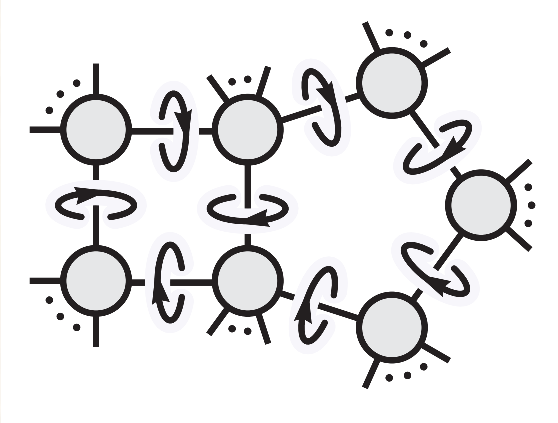




Generalized unitarity

(Bern, Dixon, Kosower) (Britto, Cachazo, Feng)

- Iterative use of cut equation



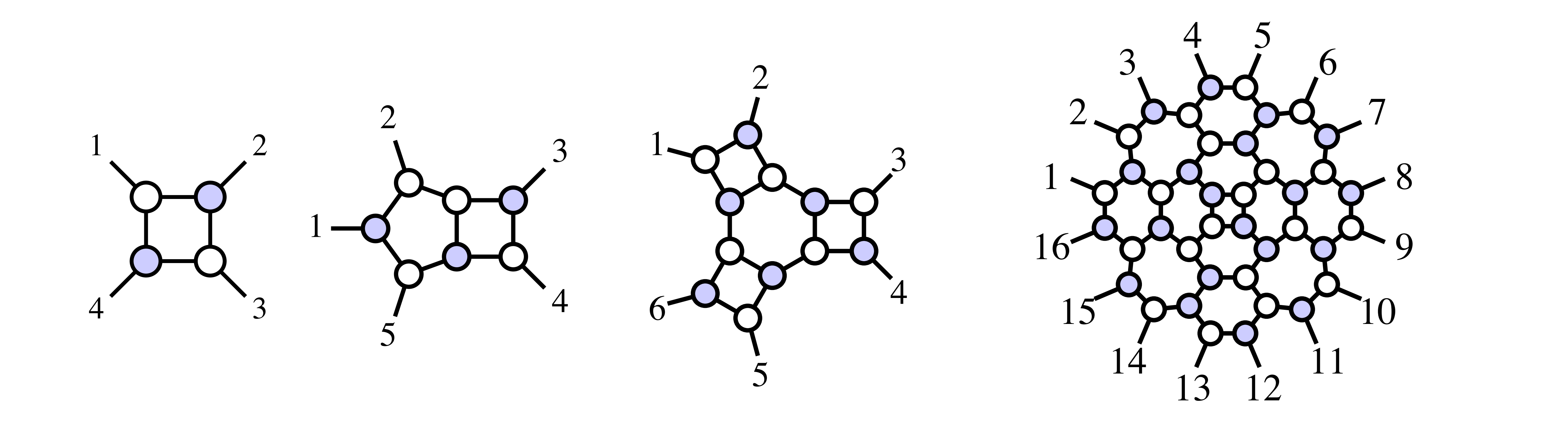
- Cuts of loops are products of tree-level amplitudes
- Cut everything you can: maximal cuts
- Factorization into three point amplitudes

On-shell diagrams

On-shell diagrams

(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

- Draw planar graph with three point vertices



Cuts of loop integrands

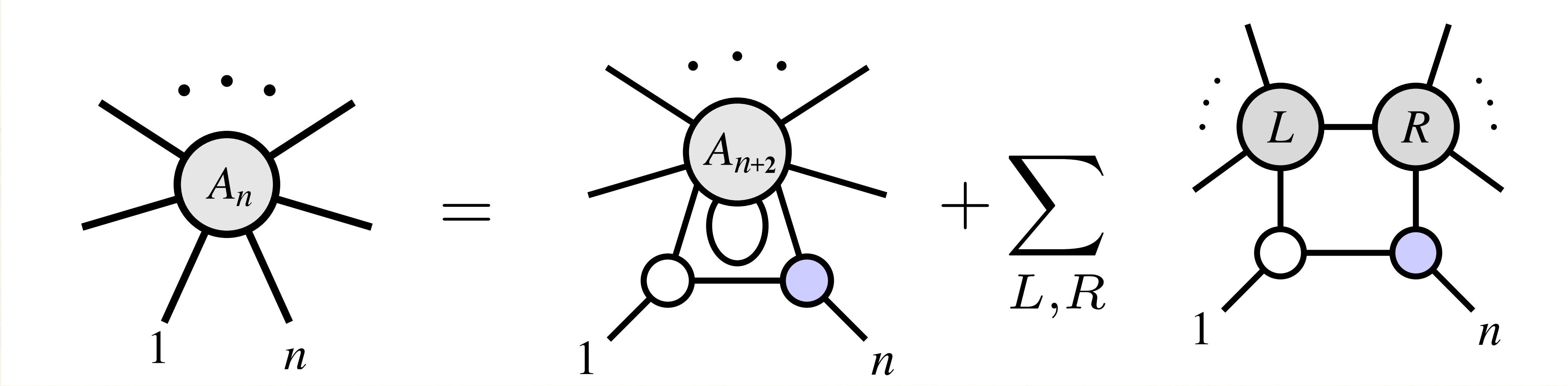
Product of 3pt amplitudes

- Exist in all theories, special in planar N=4 SYM

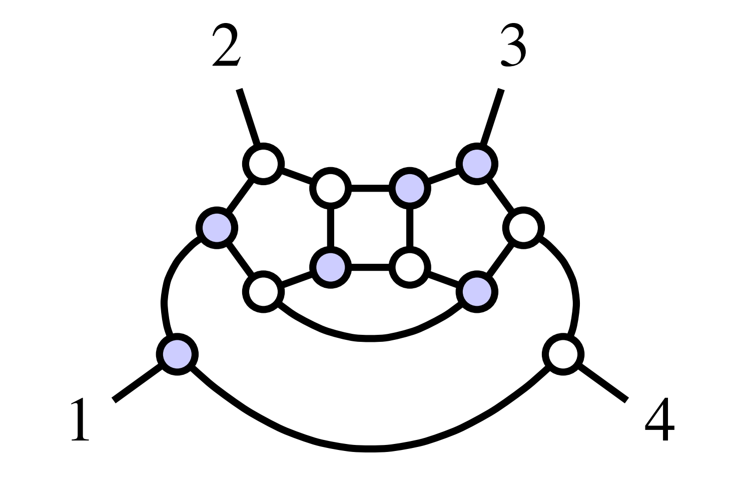
Recursion relations

(Britto, Cachazo, Feng, Witten, 2005) (Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, JT, 2010)

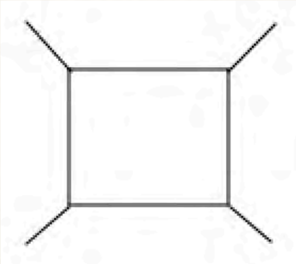
- Recursion relations for ℓ-loop integrand



- Example: 4pt 1-loop



5-loop on-shell diagram =
1-loop off-shell box



Momentum conservation

- Deep connection: on-shell diagrams vs Grassmannian
- Simple motivation: linearize momentum conservation

$$\delta(P) = \delta\left(\sum_a \lambda_a \widetilde{\lambda}_a\right)$$

- The Grassmannian as matrix of coefficients

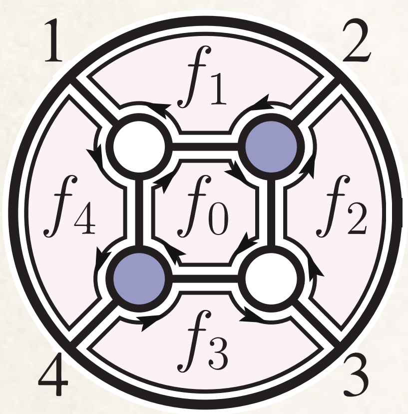
(Arkani-Hamed, Cachazo, Cheung, Kaplan, 2009) (Mason, Skinner, 2009)

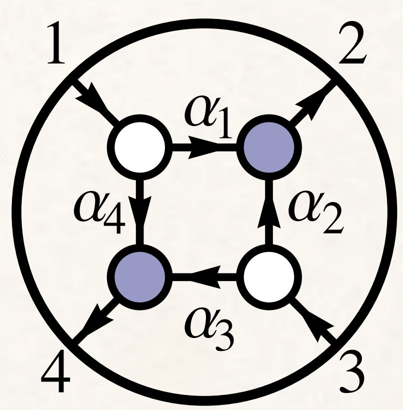
$$\delta\left(C_{ab}\widetilde{\lambda}_b\right)\delta\left(C^{\perp}_{ab}\lambda_b\right)$$

Positive Grassmannian

(Postnikov, 2006)

- Building positive matrix: face or edge variables





$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- Connection to mathematics for planar diagrams

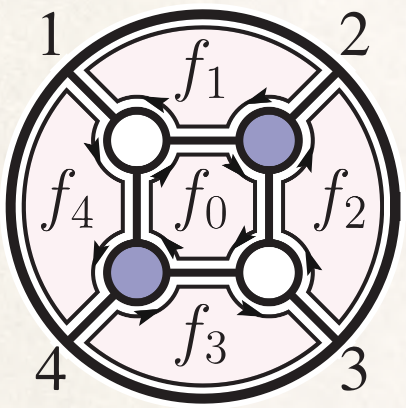
  Choose $\alpha_i > 0$: positive minors -> Positive Grassmannian

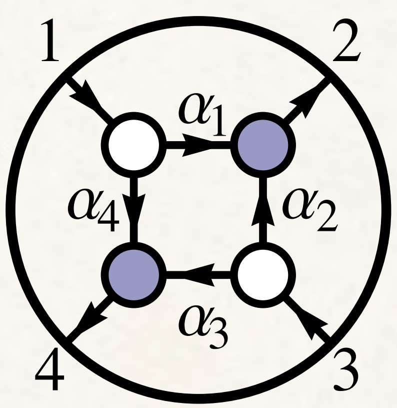
  Area of research in algebraic geometry, combinatorics

Connection to amplitudes

(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT, 2012)

- Building positive matrix: face or edge variables





$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- Same function as a product of 3pt amplitudes equal to

$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\delta(C\cdot Z)$$

→ Solves for α_i in terms of $\lambda_i, \widetilde{\lambda}_i$ and gives $\delta(P)\delta(Q)$

$$\delta(C\cdot Z) = \delta(C\cdot\widetilde{\lambda})\delta(C^\perp\cdot\lambda)\delta(C\cdot\widetilde{\eta})$$

Hidden properties

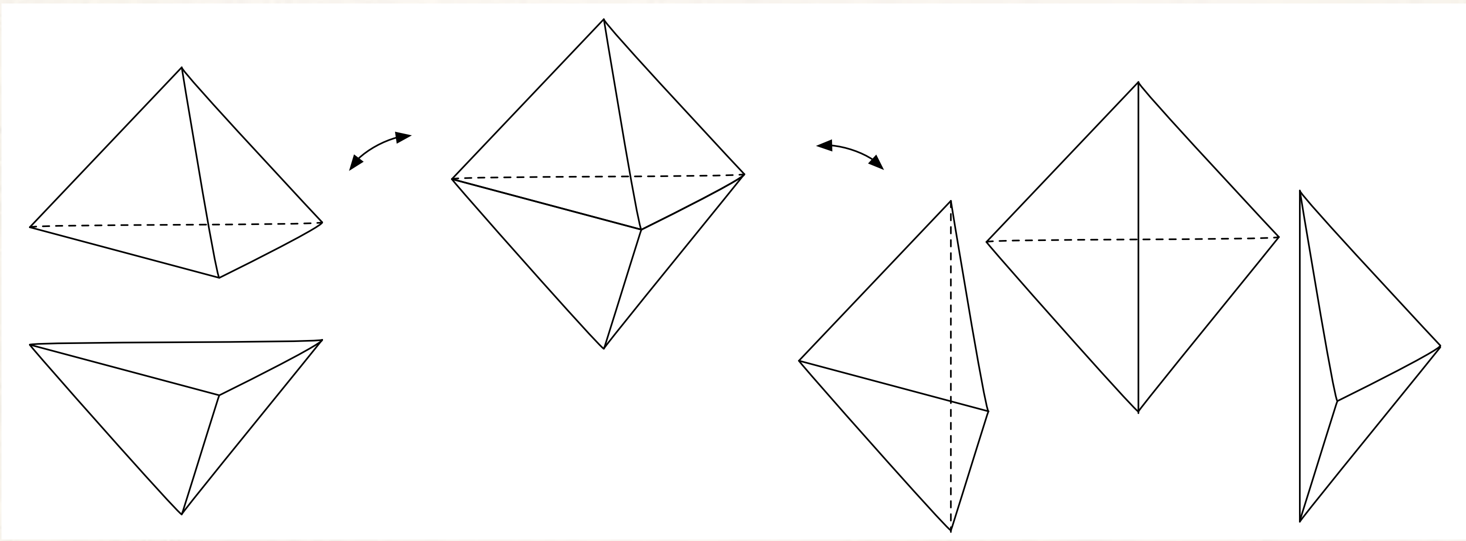
- Dual conformal symmetry: absence of poles at infinity
  - Cuts never localized at $\ell \to \infty$
  - Relation to UV behavior
- Logarithmic singularities $\frac{dx}{x}$
  - Statement about types of poles in the cut structure
  - Link to the uniform transcendentality
- Recursion relations, complete geometry picture?

Amplituhedron

(Arkani-Hamed, JT, 2013)

- Motivation: Grassmannian + polytope picture

(Hodges 2009)



Amplitudes are volumes!

- Definition of the space $Y = C \cdot Z$

external data: momentum twistors

Grassmannian and generalizations

- Loop integrand = logarithmic volume on this space

Scattering inequalities

- Amplitudes are fixed by cuts: unique object
- Amplituhedron: provides the list of all legal cuts in the form of inequalities (for both loops and helicity)

$$P_j(z_k) \geq 0$$ z_k parametrize loop momenta

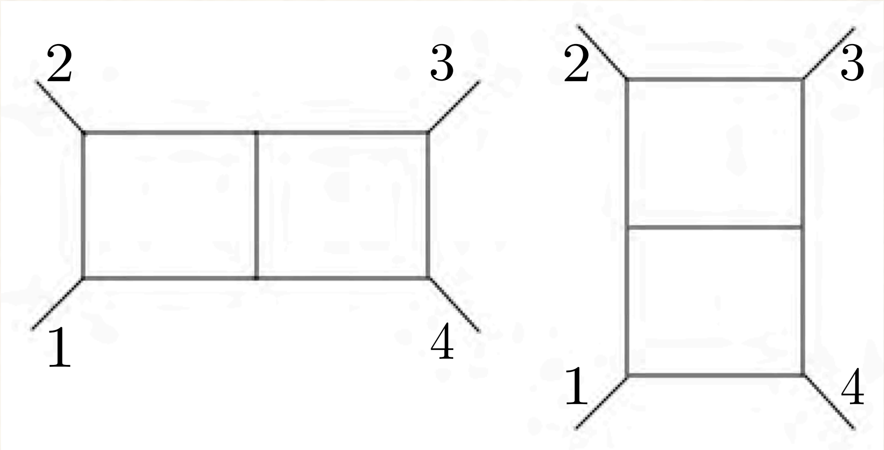
Furthermore the inequalities define a nice region in Grassmannian

- Implication: homogeneous equations $\text{Cut}\, I = 0$

(Arkani-Hamed, Hodges, JT, 2014)

Example of inequalities

- Consider 4pt two-loop amplitude

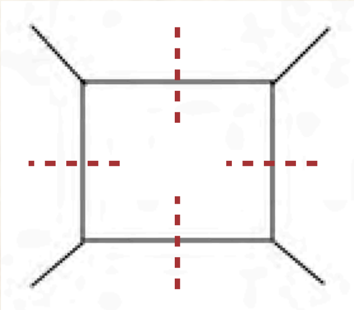


- Inequalities: $z_1, z_2, z_3, z_4 \geq 0$

$$z_5, z_6, z_7, z_8 \geq 0$$

$$(z_1 - z_5)(z_6 - z_2) + (z_3 - z_7)(z_8 - z_4) \geq 0$$

- Check: one-loop cut



$$z_1 = 0$$
$$z_2 = 0$$
$$z_3 = 0$$
$$z_4 = 0$$

$$-z_5 z_6 - z_7 z_8 \geq 0$$

Ω vanishes on this cut

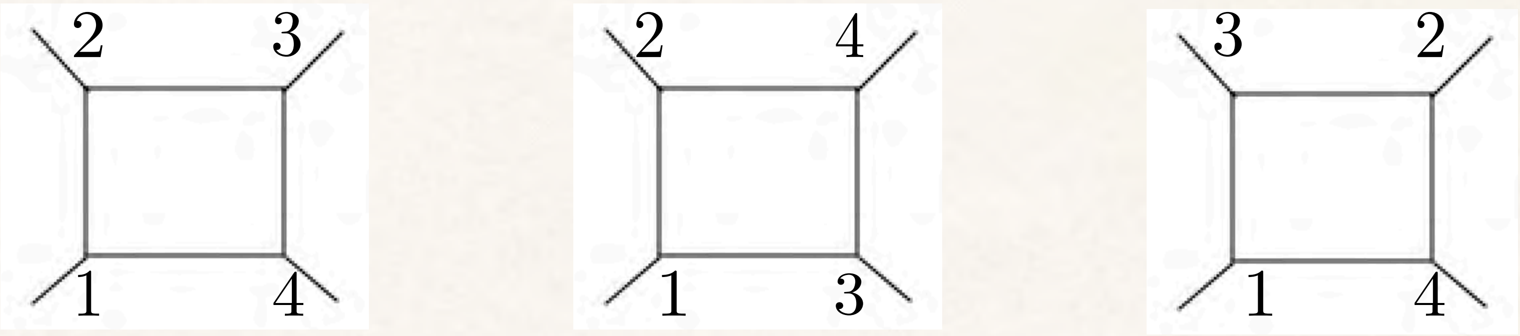
Non-planar amplitudes in N=4 SYM

(Arkani-Hamed, Bourjaily, Cachazo, JT, 2014)

(Bern, Herrmann, Litsey, Stankowicz, JT, 2014, 2015)

Non-planar problems

- No unique integrand, labeling problem

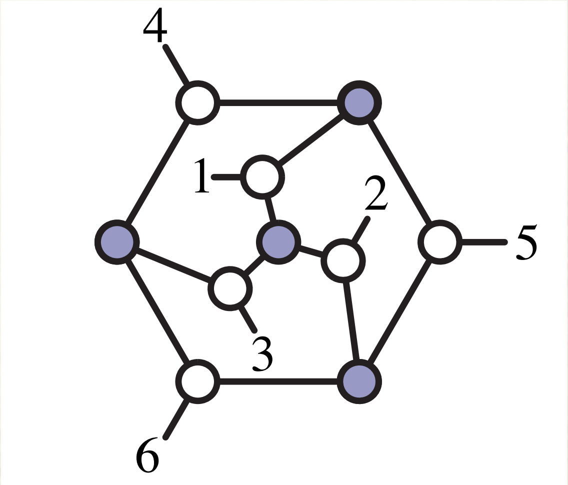


What is ℓ ?

- No momentum twistors, no known hidden symmetries

- On-shell diagrams for singularities

(Arkani-Hamed, Bourjaily, Cachazo, Postnikov, JT, 2014) (Franco, Galloni, Penante, Wen 2015)



Connection to Grassmannian, logarithmic form

$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\delta(C\cdot Z)$$

see Jake's talk

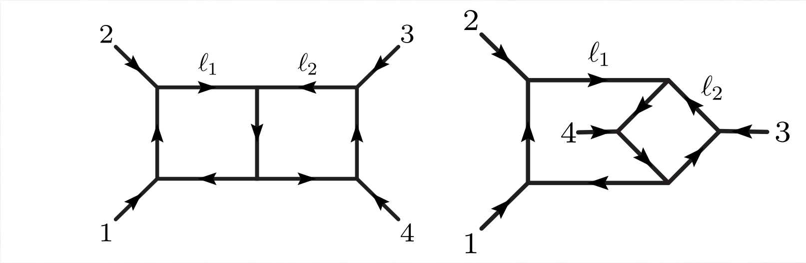
Non-planar amplitudes

- No unique integrand, no recursion relations
- On-shell diagrams: cuts of amplitudes
- Conjecture: amplitudes have the same properties
  - Logarithmic singularities → Volumes of something?
  - No poles at infinity → Analogue od DCI?
  - Only homogeneous cuts → Scattering inequalities? Amplituhedron?

Non-planar amplitudes

- Conservative approach: sum of integrals

$$A = \sum_i a_i \cdot C_i \cdot I_i$$



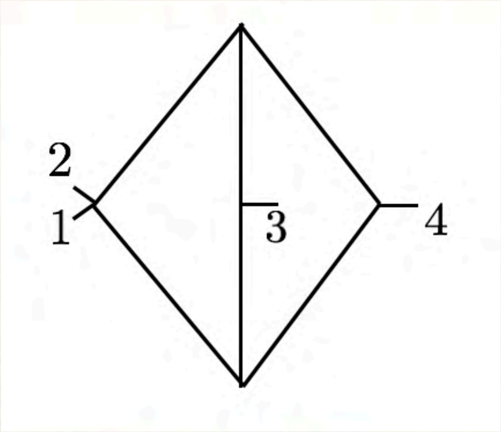
Fix by homogeneous conditions

$f^{1ab} f^{bcd} \dots f^{4ef}$

Basis of integrals:

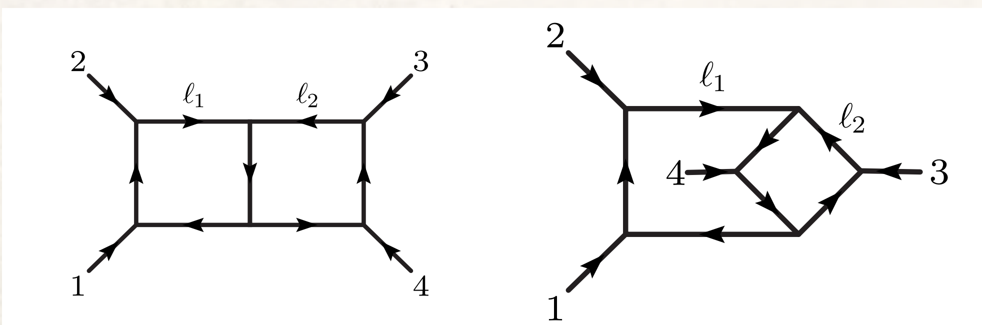
- Logarithmic singularites
- No poles at infinity

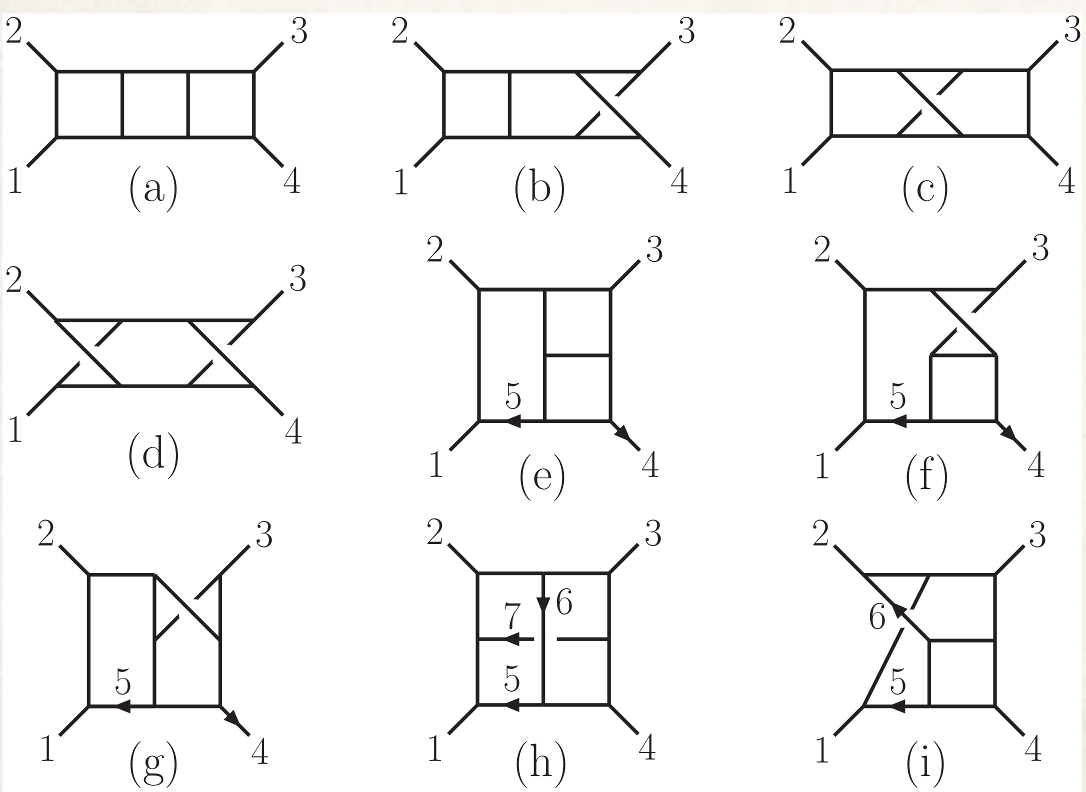
- Some diagrams forbidden

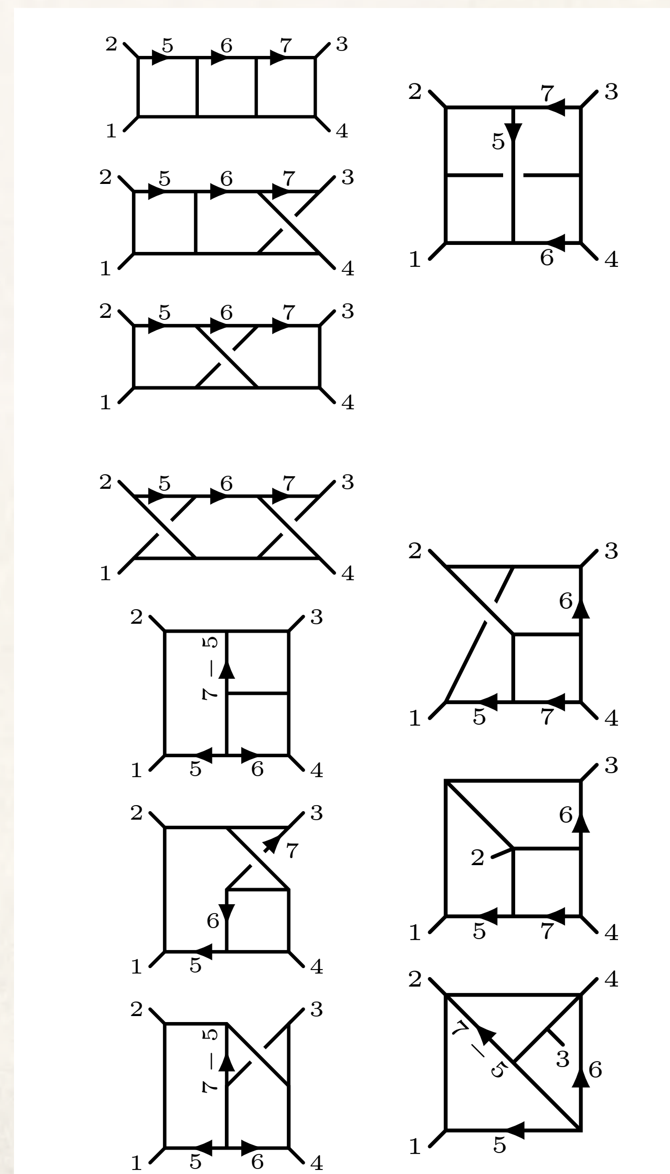


Explicit checks

- Construct basis for 4pt at 2-loop and 3-loop, 5pt 2-loop







Expand the amplitude: ✓

Non-planar conclusion

- Same properties of amplitudes as in planar sector
- Open questions: identify new symmetries, role of color factor, complete geometric formulation…..
- What about gravity?

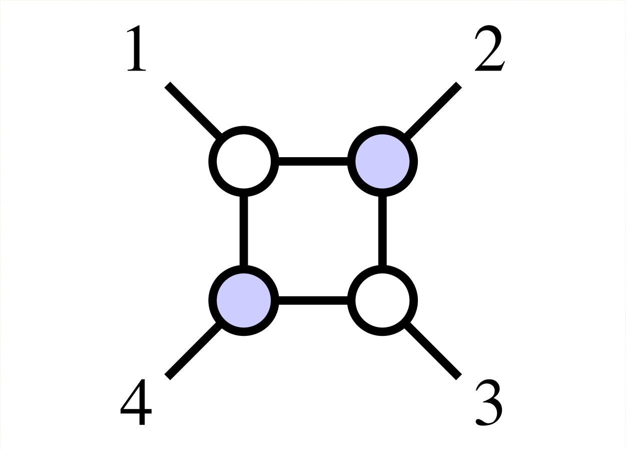
Gravity on-shell diagrams

(Herrmann, JT 2016)

(Heslop, Lipstein 2016)

First look: gravity on-shell diagrams

- Let us just get some data
- Simplest class: MHV leading singularities



Yang-Mills

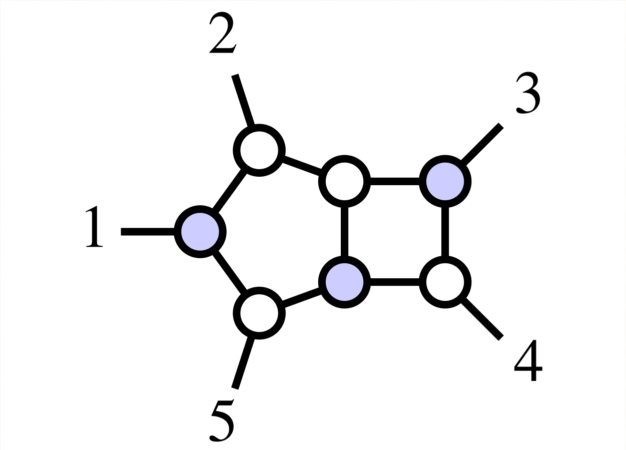
$$\frac{1}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

Gravity

$$\frac{[13][24]}{\langle 12\rangle\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle}$$

First look: gravity on-shell diagrams

- Let us just get some data
- Simplest class: MHV leading singularities



Yang-Mills

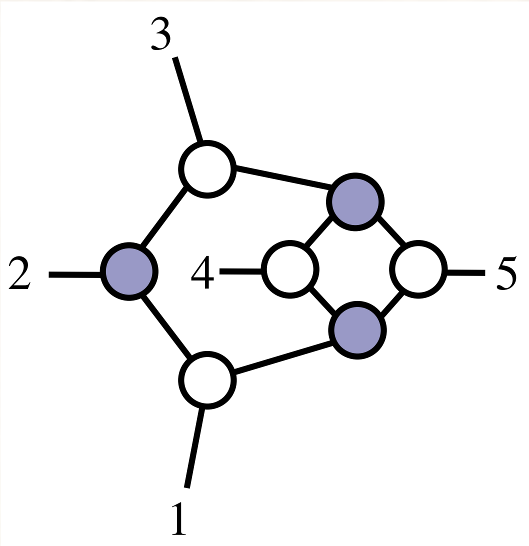
$$\frac{1}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Gravity

$$\frac{[12][23][45]^2}{\langle 12\rangle\langle 13\rangle\langle 15\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle}$$

First look: gravity on-shell diagrams

- Let us just get some data
- Simplest class: MHV leading singularities



Yang-Mills

$$\frac{\langle 13\rangle}{\langle 12\rangle\langle 23\rangle\langle 14\rangle\langle 15\rangle\langle 34\rangle\langle 35\rangle}$$

Gravity

$$\frac{[12][23][45]^2}{\langle 12\rangle\langle 14\rangle\langle 15\rangle\langle 23\rangle\langle 34\rangle\langle 35\rangle}$$

First look: gravity on-shell diagrams

- Based on these results: in Yang-Mills we could already conjecture the structure
- Natural conjecture for gravity

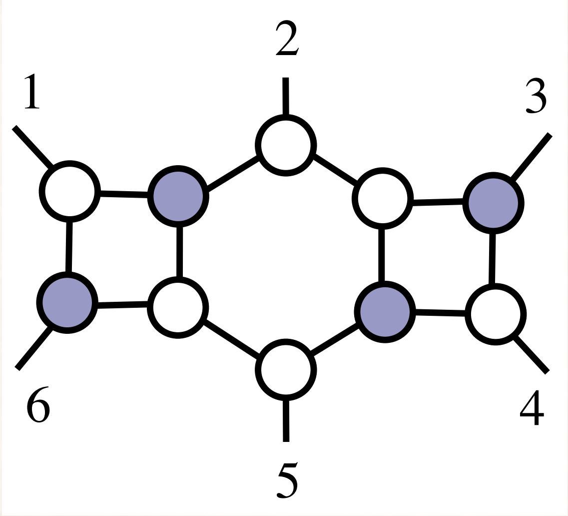
anti-holomorphic numerator

$$\frac{[\ldots][\ldots][\ldots][\ldots]}{\langle\ldots\rangle\langle\ldots\rangle\langle\ldots\rangle\langle\ldots\rangle\langle\ldots\rangle}$$

single poles in the denominator

First look: gravity on-shell diagrams

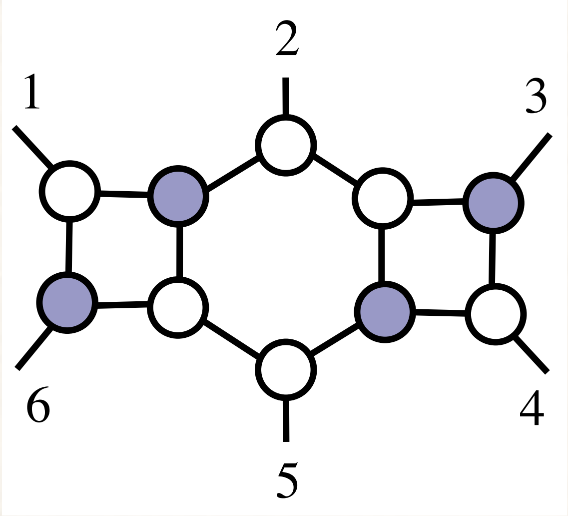
- Higher poles, more complicated numerators



$$\frac{\langle 5|6+1|2]\langle 2|3+4|5][16]^2[34]^2}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle{\color{red}\langle 25\rangle}^2}$$

First look: gravity on-shell diagrams

- Higher poles, more complicated numerators



$$\frac{\langle 5|6+1|2]\langle 2|3+4|5][16]^2[34]^2}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle\langle 25\rangle^2}$$

Each numerator factor:
collinear condition in the vertex

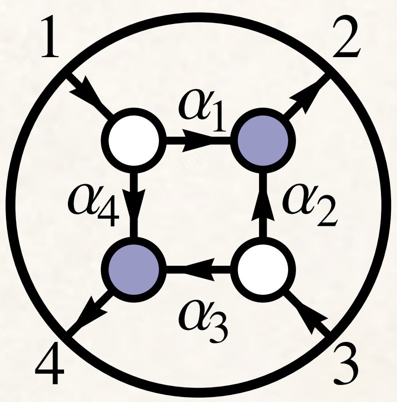
Higher poles:
infinite momentum

$$\langle 25\rangle = 0$$

blows up the loop in hexagon

Grassmannian formula

- Edge variables for each edge



$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- The value of the diagram in Yang-Mills

$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\dots\frac{d\alpha_m}{\alpha_m}\cdot\delta(C\cdot Z)$$

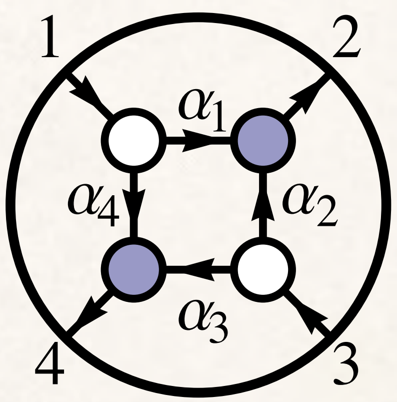
→ Solves for α_i in terms of $\lambda_i, \widetilde{\lambda}_i$ and gives $\delta(P)\delta(Q)$

$$\delta(C\cdot Z) = \delta(C\cdot\widetilde{\lambda})\delta(C^{\perp}\cdot\lambda)\delta(C\cdot\widetilde{\eta})$$

Grassmannian formula

(Herrmann, JT 2016)

- Edge variables for each edge



$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- The value of the diagram in gravity

$$\Omega = \frac{d\alpha_1}{\alpha_1^3}\frac{d\alpha_2}{\alpha_2^3}\dots\frac{d\alpha_m}{\alpha_m^3}\prod_v \Delta_v \cdot \delta(C\cdot Z)$$

Special numerator: factor in each vertex

$$\delta(C\cdot Z) = \delta(C\cdot\widetilde{\lambda})\delta(C^\perp\cdot\lambda)\delta(C\cdot\widetilde{\eta})$$

Grassmannian formula

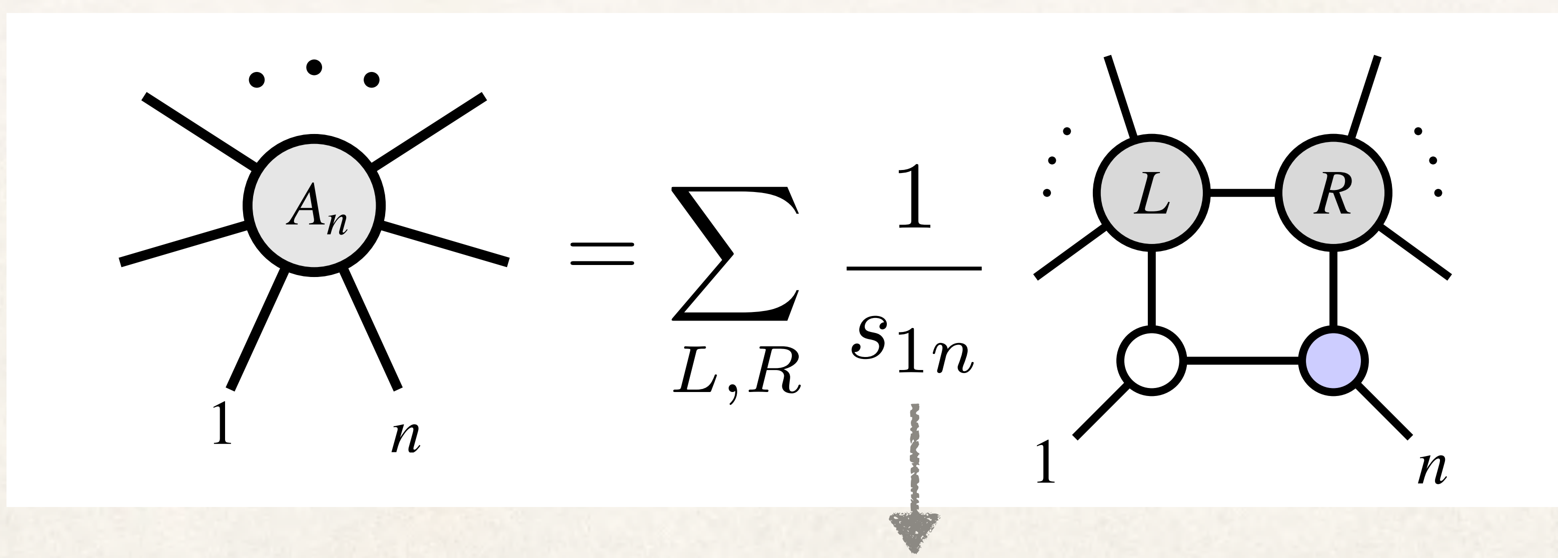
(Herrmann, JT 2016)

- Analyzing: $$\Omega = \frac{d\alpha_1}{\alpha_1^3}\frac{d\alpha_2}{\alpha_2^3}\dots\frac{d\alpha_m}{\alpha_m^3}\prod_v \Delta_v \cdot \delta(C\cdot Z)$$
  - Higher poles at infinity
  - Single poles for finite momentum
  - Vanishes if for collinear momenta in vertex
- Similar formula for N<8 SUGRA

Relation to other work

(Heslop, Lipstein 2016)

- Gravity on-shell diagrams in the context of BCFW recursion relations
- Gravity tree amplitudes not just sum of on-shell diagrams



Extra kinematical factors

Singularities of gravity amplitudes

(Herrmann, JT 2016)

Conjectures for the amplitude

- No loop recursion using on-shell diagrams
  - Absence of variables, generic non-planar problem
- Conjectures for loop integrands
  - Logarithmic poles for cuts
  - Collinearity conditions
  - Poles at infinity

Logarithmic poles for cuts, Collinearity conditions → IR

Poles at infinity → UV

Logarithmic poles

- On-shell diagrams: all finite cuts logarithmic

$$\text{Integral} \rightarrow \text{Cut} \rightarrow \text{Cut} \rightarrow \frac{d\alpha}{\alpha^2}$$

never happens

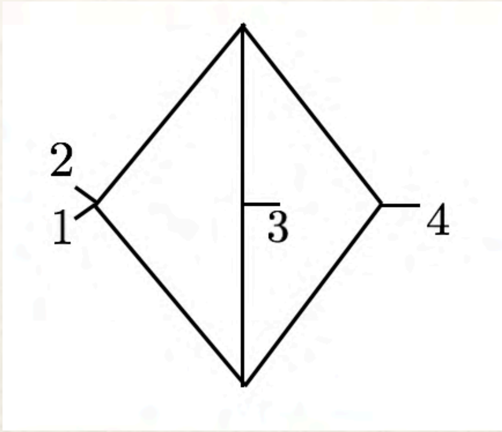
- Strong hint from BCJ relations

$$A^{(YM)} = \sum_i \frac{n_i^{(BCJ)} c_i}{s_i} = \sum_i \frac{n_i^{(dlog)} c_i}{s_i}$$

Some diagrams prohibited

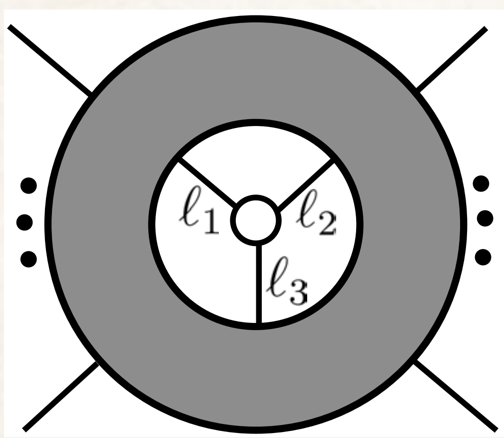
cancels double poles

$$A^{(GR)} = \sum_i \frac{n_i^{(dlog)} n_i^{(BCJ)}}{s_i}$$

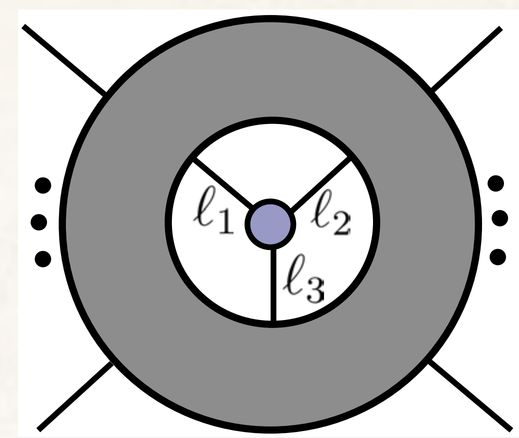


Collinearity conditions

- On-shell diagrams: any cut of the form



$\sim [\ell_1\, \ell_2]$



$\sim \langle \ell_1\, \ell_2 \rangle$

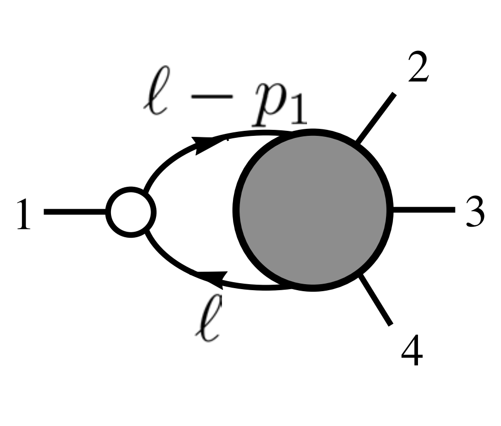
- In special case of external legs: collinear limit

$$A \sim \frac{[12]}{\langle 12 \rangle} \cdot (\dots)$$

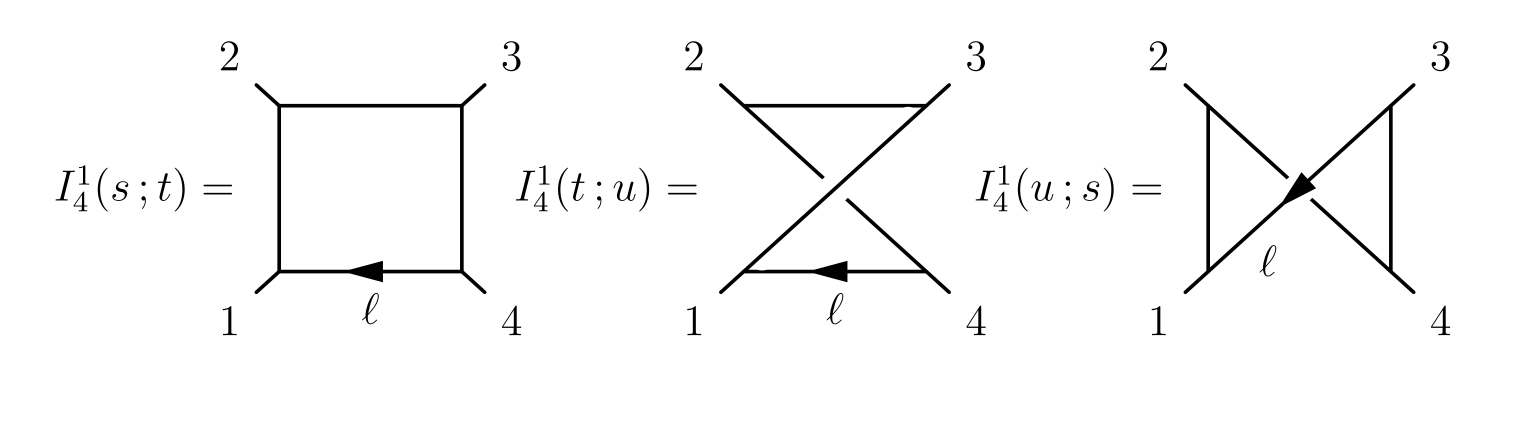
Collinearity conditions

- Example: prediction for cut

IR behavior of gravity

$\ell = \lambda_1 \widetilde{\lambda}_\ell$  $\sim [\ell\, 1]$ $\longrightarrow$ $A_{GR}^{1-loop} \sim \frac{1}{\epsilon}$

- Expansion of the 4pt 1-loop amplitude: three boxes

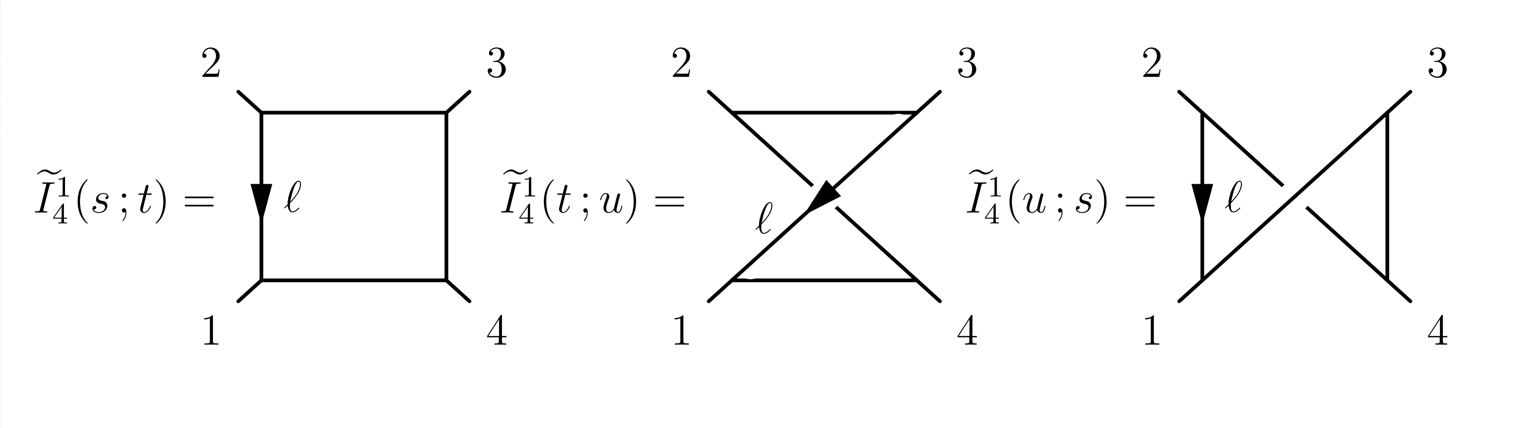


Each scales $\sim \frac{1}{[\ell\, 1]}$

Sum ~ 1

Collinearity conditions

- We made a choice in labeling diagrams
- Consider another labels, and sum over both options



Total sum

$\sim [\ell\, 1]$

- Two loop checks more complicated, symmetrization!

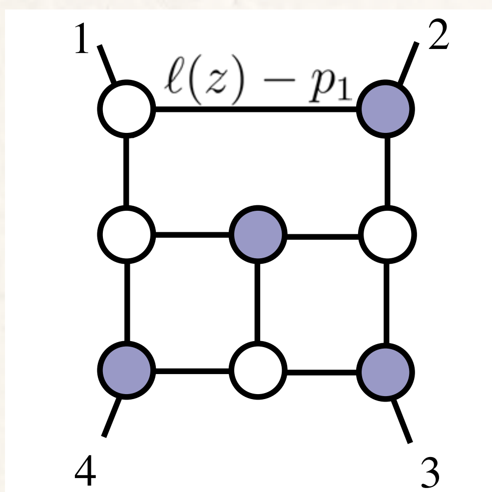
Also important in the pre-integrand story

(Carrasco, 2015)

Poles at infinity and UV behavior of N=8 supergravity

Poles at infinity

- They are present starting at 3-loops



$$\sim \frac{dz}{z}$$

Pole at

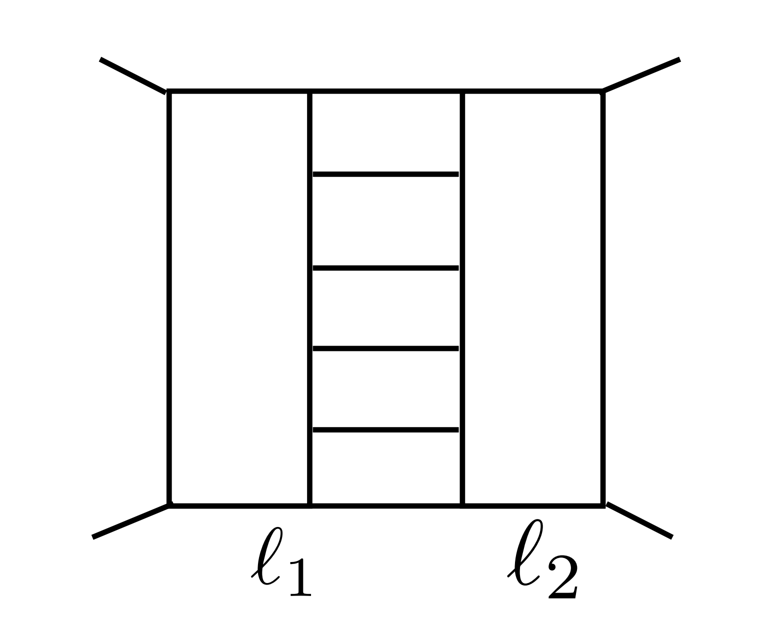
$$z \to \infty$$
$$\ell(z) \to \infty$$

- Higher poles at higher loops

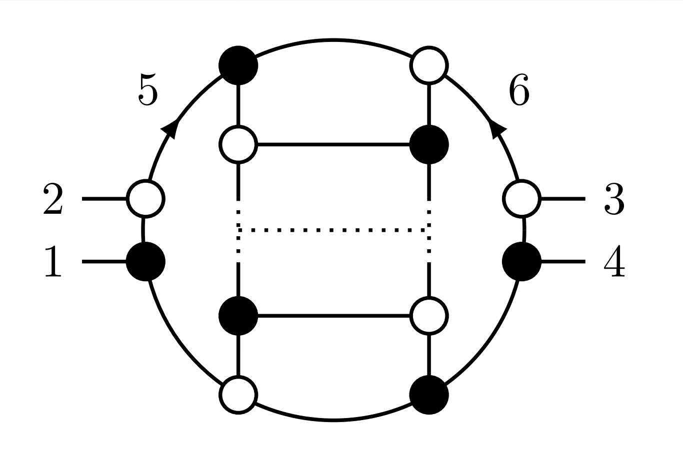
- Generically everything complex, the detailed description of space of poles at infinity needed

Poles at infinity

- Expected divergence at 7-loops



matches the on-shell diagram with pole at infinity

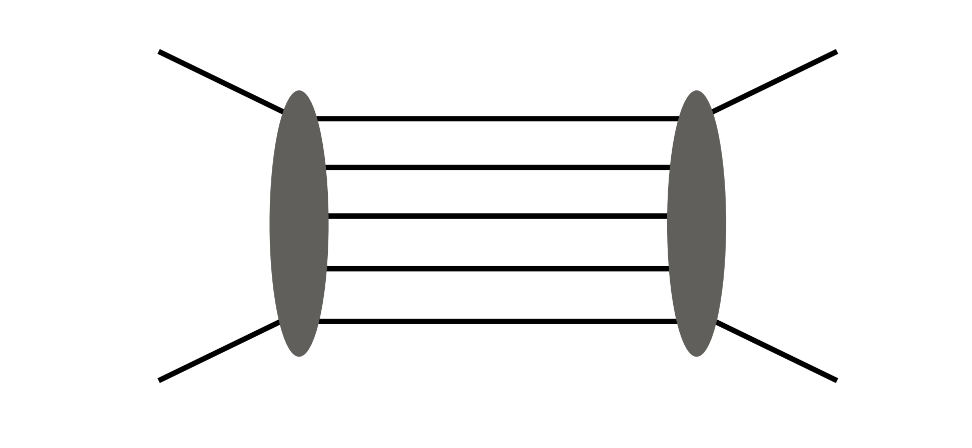


- What is the relation between poles at infinity and UV?

- The problem: real vs complex kinematics

Poles at infinity

- Lower cuts: real kinematics, no poles at infinity



- Requires cancelations between different diagrams
- Higher cuts: kinematics complex, poles at infinity

Cancelations of UV divergencies

- Expansion of the amplitude in terms of integrals

  Enhanced cancelations

- Understand these cancelations at the integrand level

  (Bern, Herrmann, Martinez, Stankowicz, JT, in progress)

  - Relation to poles at infinity
  - Lower supersymmetry -> lower loops

- We saw these cancelation in IR in the collinear regions

Rigidity of N=8 SUGRA

- Related to the UV question:

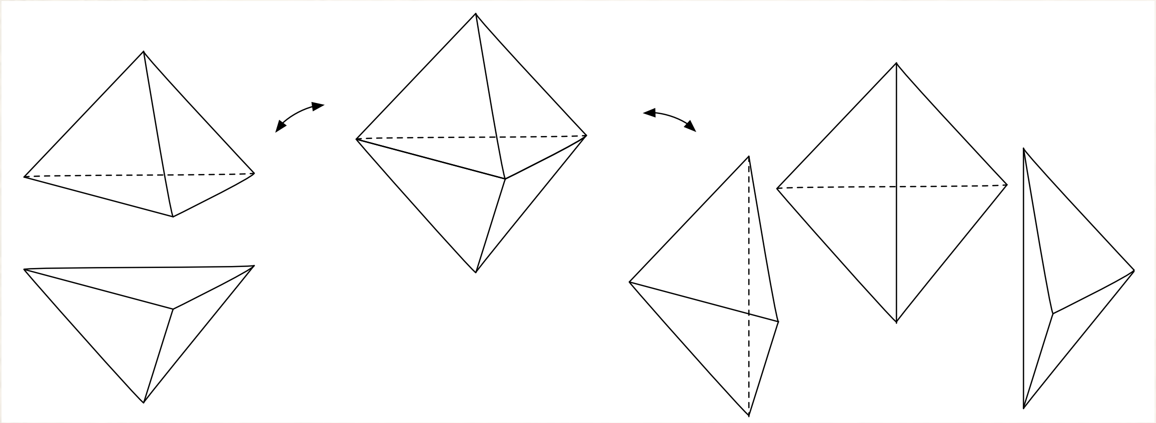
What is special about loop integrands in N=8 SUGRA?

- For N=4 SYM: simple singularity structure (logarithmic) + all singularities captured by inequalities = Amplituhedron
- If N=8 SUGRA is special, it must be the rigidity of poles at infinity which are absent in N=4 SYM

Gravitational polytopes

(Herrmann, JT, in progress)

- The key to gravity is hidden in tree-level amplitudes
- Amplituhedron: first sign in NMHV tree amplitudes



Translate to momentum space

Look at gravity: different than BCJ

- Encouraging preliminary results: gravity volume forms

$$\frac{(\langle 12\rangle[45]\langle 3|4+5|6])^2}{s_{345}\cdot s_{34}s_{45}\cdot s_{61}s_{12}} \rightarrow \frac{(\langle 12\rangle[45]\langle 3|4+5|6])^4}{s_{345}\cdot s_{34}s_{45}s_{53}\cdot s_{61}s_{12}s_{26}}$$

Conclusion

Conclusion

- Grassmannian formula for gravity on-shell diagrams
- Conjecture for IR properties of the gravity integrands
- Poles at infinity: key to UV behavior
- First signs of new structures in tree-level amplitudes

Thank you for your attention