# Higgs hadroproduction: a laboratory for perturbative QCD.

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work in collaboration with
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Franz Herzog, Achilleas Lazopoulos, Bernhard Mistlberger

and

Simone Lionetti, Bernhard Mistlberger, Andrea Pelloni,
Caterina Spechia

# Breakthroughs in phenomenology

Perturbative Methods in QCD

LHC PRECISION MEASUREMENTS and HIGGS DISCOVERY

# LHC precision physics

SM theory

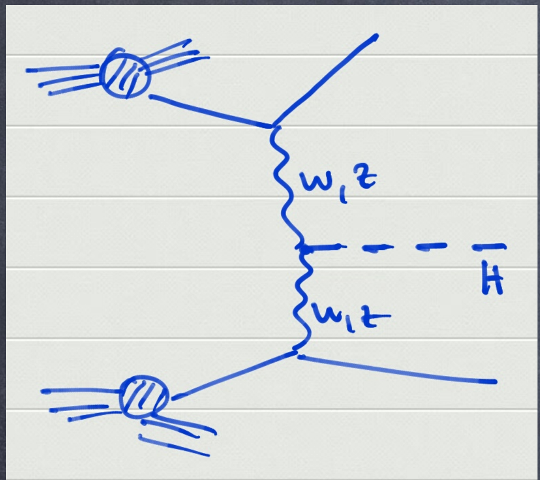
BSM theories

Compute consequences

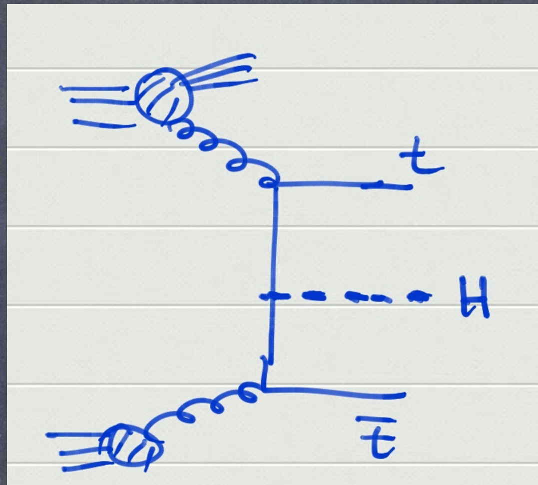
LHC

A "Feynman diagram" of finding New Laws

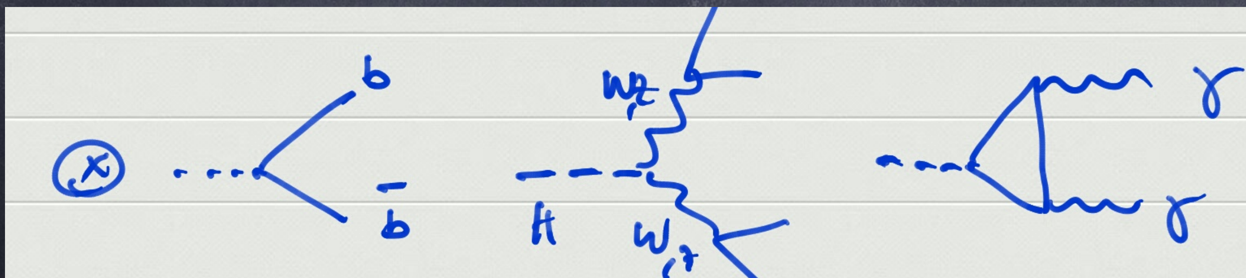
# Higgs hadroproduction




Weak boson fusion

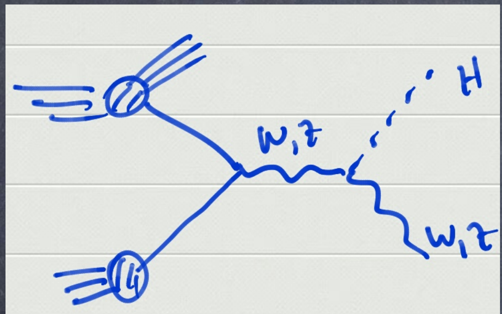



Associated with top-pair



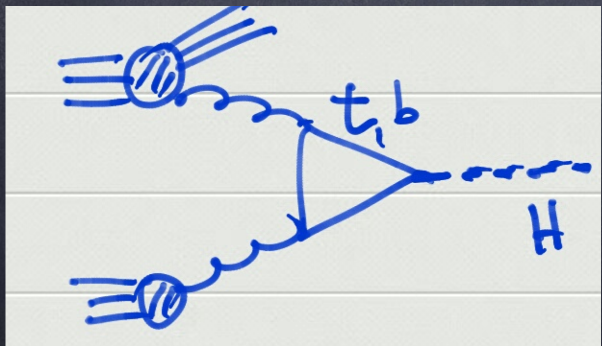

Complicated final states with many legs for signals and even more legs for background processes

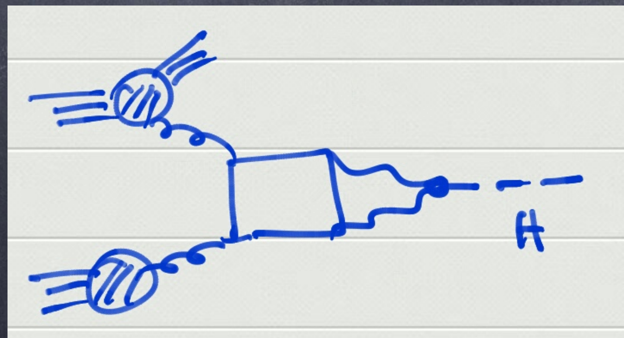
# Higgs hadroproduction




Associated with W or Z

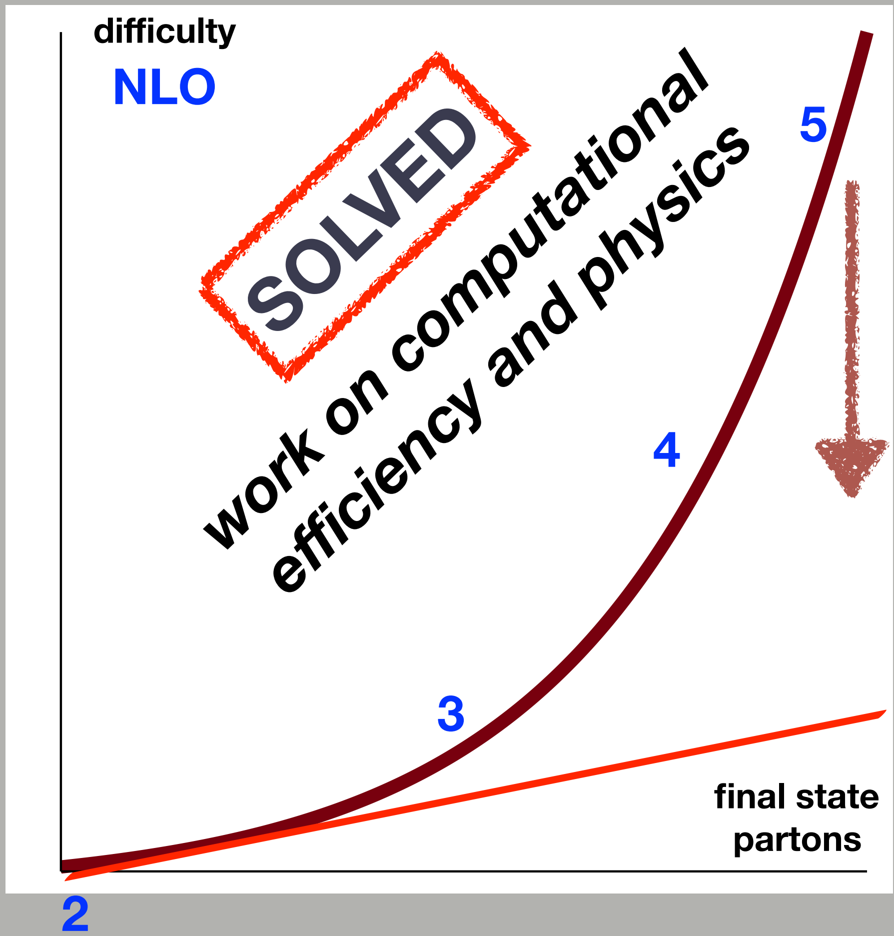
2 to 1 processes, opportunity for multi-loop calculations and higher precision.

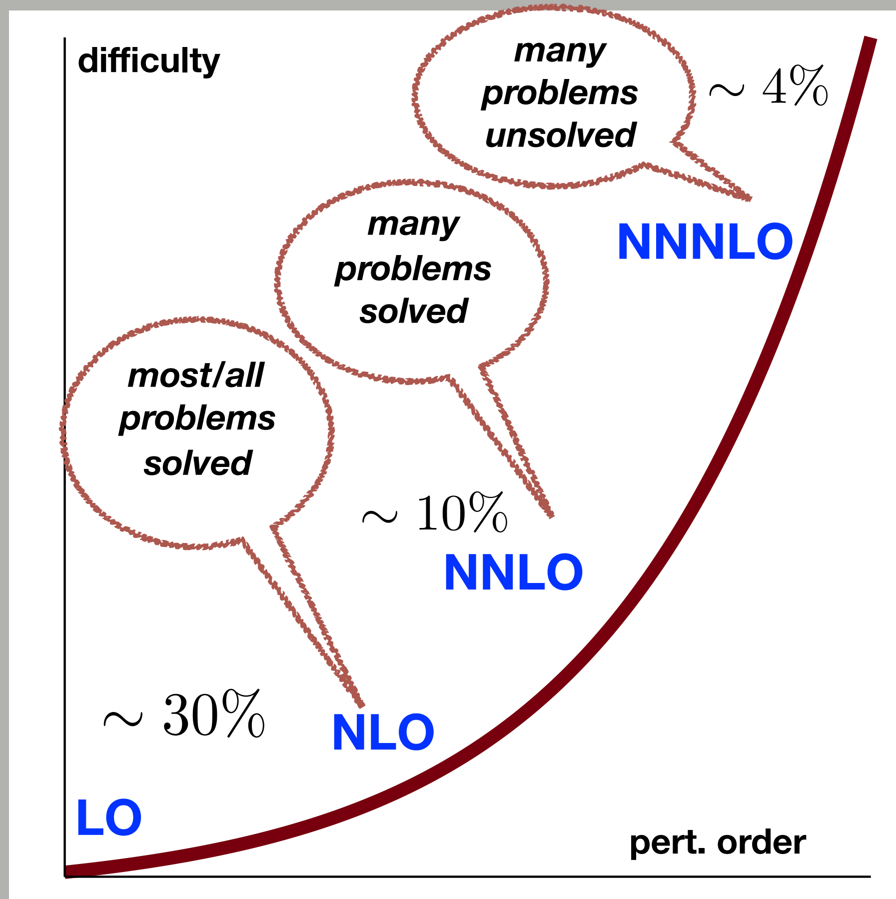



Gluon fusion

# Perturbative QCD




$$\sigma = \sigma_0 \alpha_s^n + \sigma_1 \alpha_s^{n+1} + \sigma_2 \alpha_s^{n+2} + \ldots$$

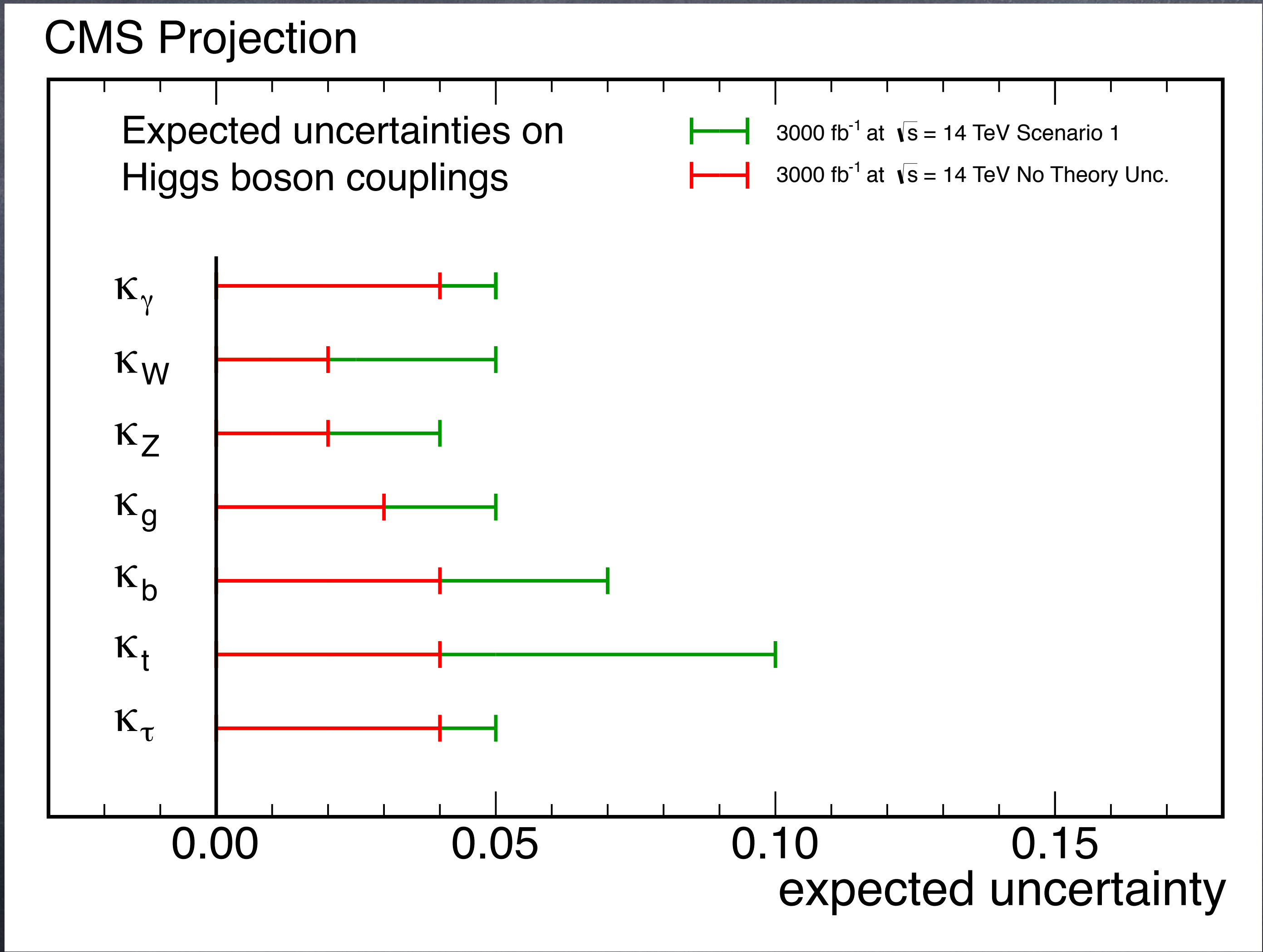
# In this talk

- Motivation for high precision determination of the gluon-fusion cross-section
- Techniques for a 2->1 computation through N3LO. Status of the computation.
- Results on the inclusive Higgs cross-section
- Challenges for sifferential cross-sections at N3LO.

# How many Higgs bosons at the LHC?

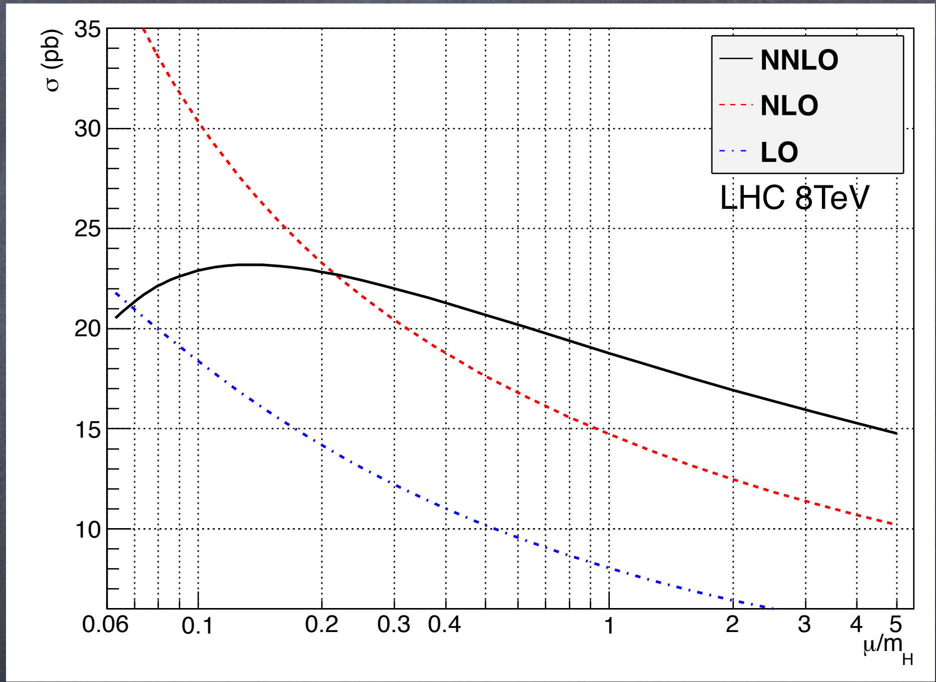
- Important test of the Standard Model Higgs sector
- Theoretical input needed for Higgs coupling extractions
- Precise measurements

# N3LO will have a very important impact in Higgs coupling measurements



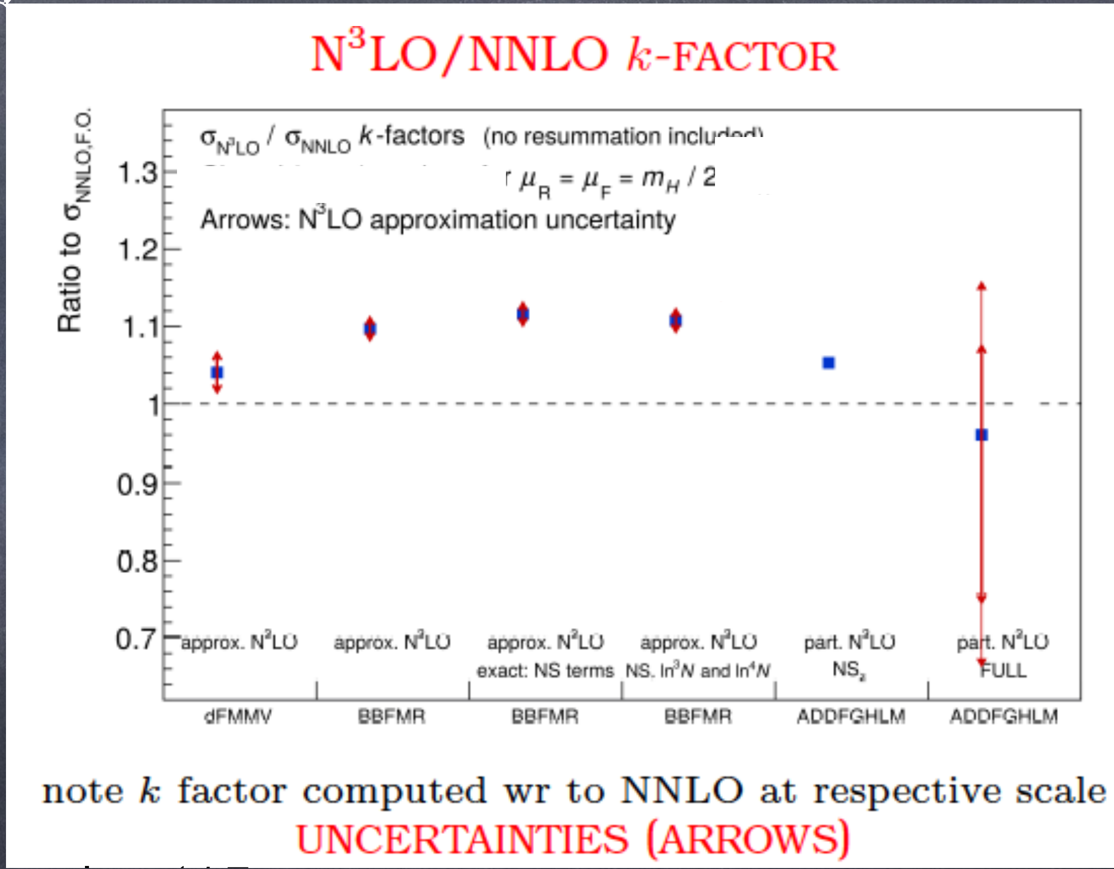
# NNLO

- Convergence through NNLO is slow...
- but acceptable with a judicious scale choice (mu=mh/2).
- O(10%) scale uncertainty
- Indications that corrections beyond NNLO are small from some flavours of resummation, but...



# ...and estimates beyond

- some estimates of beyond NNLO corrections were large.
- N3LO necessary not only to reduce scale variation
- but to also prove the validity of perturbation theory

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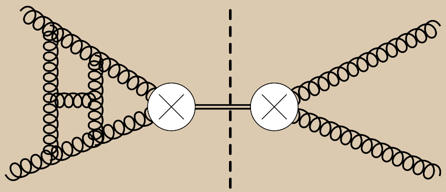
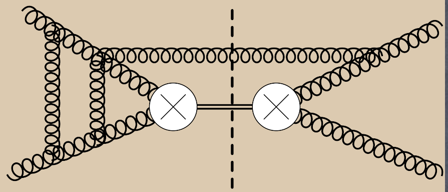
# From NNLO to N3LO

- going one order higher in perturbation theory is a big challenge
- NNLO has been a big challenge on its own, not very far in the past...
- ...strategy and division of the problem is crucial!

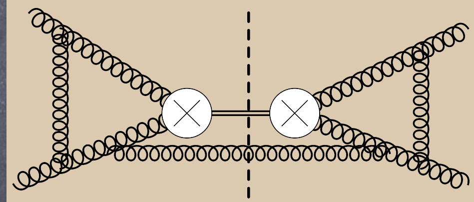
# A natural division



VVV



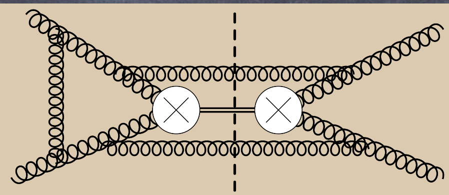
RVV



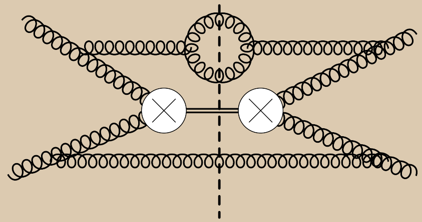
(RV)x(RV)

$$\frac{P_{gg}^{(1)}}{\epsilon} \otimes \sigma^{\mathrm{NNLO}}(\epsilon)$$

IR+UV



RRV



RRR

# From NNLO to N3LO

- learn from the experience at NNLO and do a "soft expansion" for the partonic cross-sections first

$$\sigma = \sum_n (1-z)^n C_n$$

with $z = \frac{m_H^2}{s}$

# From NNLO to N3LO

| 2005 | 2014 | 2014 | 2015 | 201x |
| --- | --- | --- | --- | --- |
| soft | soft -virtual | next to soft | next to next to … next to soft | closed functional form |

# From NNLO to N3LO

- Wilson coefficient
- Three-loop splitting functions
- Collinear and UV counterterms
- Triple virtual
- Soft expansion for triple real
- Exact (real-virtual)^2
- Exact real-virtual-virtual
- Soft expansion real-real-virtual
- Expansion using the differential equation method
- Exact quark channels
- Exact real-real-virtual

Schroder, Steinhauser
Chetyrkin, Kuhn, Sturm
Moch, Vermaseren, Vogt
Hoeschele, Hoff, Pak, Steinhauser, Ueda
Bueller, Lazopoulos
CA, Buehler, Duhr, Herzog
Baikov, Chetyrkin, Smirnov, Steinhauser
Gehrmann, Glover, Huber, Ikizlerli, Studerus
CA, Duhr, Dulat, Mistlberger
CA, Duhr, Dulat, Furlan, Herzog, Mistlberger
Kilgore
Gehrmann, Jaquier, Glover, Koukoutsakis
Duhr, Gehrmann
Dulat, Mistlberger
Ye Li, Hua Xing Zhu
CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger
Ye Li, von Manteuffel, Schwinger, Hua Xing Zhu
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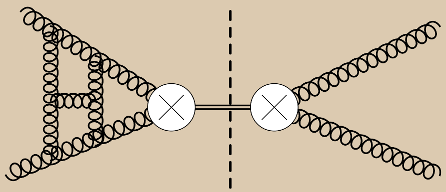
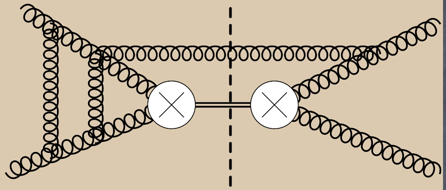
CA, Duhr, Dulat, Herzog, Mistlberger
CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger
Anzai, Kasselhuhn, Hoff, Kilgore, Steinhauser, Ueda
Duhr, Dulat, Mistlberger
Mistlberger et al

# What is now known for the N3LO correction



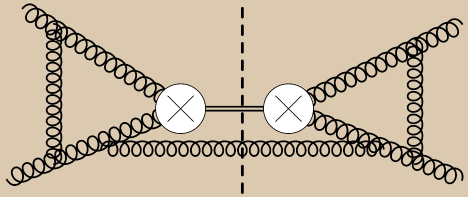
VVV

exact



RVV

exact



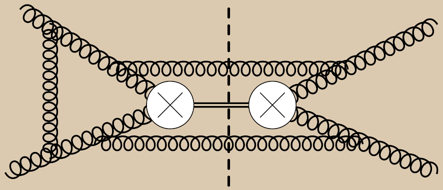
(RV)x(RV)

exact

$$\frac{P_{gg}^{(1)}}{\epsilon} \otimes \sigma^{\mathrm{NNLO}}(\epsilon)$$

IR+UV

exact



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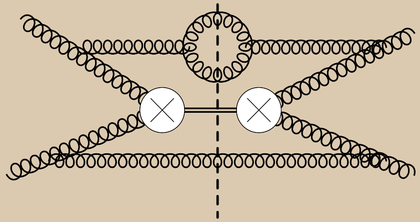
exact



RRR

expansion

# What is now known for the N3LO correction

$$\sigma(z) = \delta(1-z)\, C_\delta + \sum_{\alpha=0}^{5} \log^\alpha(1-z)\, f_\alpha(z)$$

| $C_\delta, f_5(z), \ldots, f_1(z)$ | $f_0(z)$ |
|---|---|
| Exact | Expansion<br>$\sim 37$ terms |

# Why now?

- Since the NNLO computations in 2002 a lot has changed.
- Could this computation had happened earlier?
- Some techniques and ideas have been present for quite some time now
- but, recent progress in the field of loop computations and new ideas were also crucial.

# Old and new

- Reverse unitarity: map phase space integrals on loop integrals with Cutkosky rules:

$$2\pi\,\delta\left(p_4^2-m^2\right) \longrightarrow \frac{i}{p_4^2-m}$$

- expand around the threshold limit:
"Cutkosky rules can be differentiated with respect to masses and kinematic parameters"

$$2\pi\,\delta\left(p_4^2-m^2\right) \to \frac{i}{p_4^2-m^2} = \sum_{n=0}^{\infty} \frac{\left[2(p_1+p_2)\cdot p_3\right]^n}{\left[s-m_4^2\right]^{n+1}}$$

# Old and new

- Laporta algorithm: Gauss elimination of linearly dependent integrals and reduction of amplitudes to master integrals.
- New implementation of the algorithm with great efficiency optimisations.

# Old and new

- Dimensional shifts, Mellin-Barnes, multi-dimensional integrations, polylogarithms
- New criteria to chose the order of integrations
- Clever representations of phase-space integrals
- From Mellin-Barnes to Euler type representations
- Symbol/coproduct and algebraic techniques for iterative integrations

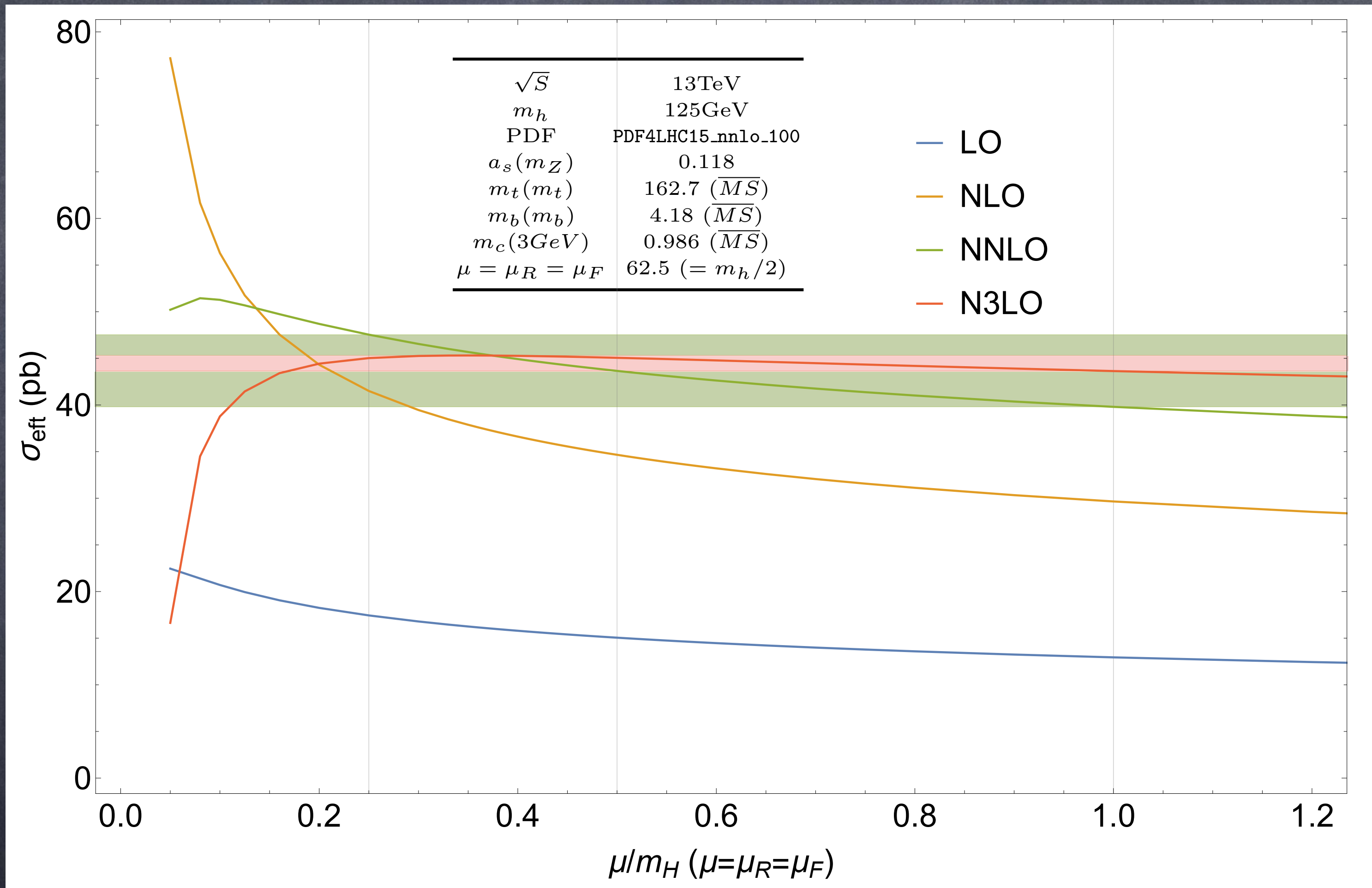
# Old and new

- Differential equations method
- Finding Henn canonical forms
- Strategy of regions to determine boundary conditions
- Expansion of differential equations around the threshold limit turning their solution into an algebraic problem

# How tough of a problem?

- Two orders of magnitude more Feynman diagrams than NNLO
- 1028 N3LO master integrals (27 at NNLO)
- 72 boundary conditions for the N3LO master integrals (5 at NNLO)

# From NNLO to N3LO




N3LO result is very precise and within the NNLO scale variation.

# Composition of the inclusive cross-section

$$
\begin{aligned}
48.58\,\text{pb} = \quad & 16.00\,\text{pb} & (+32.9\%) & \quad (\text{LO, rEFT}) \\
& +20.84\,\text{pb} & (+42.9\%) & \quad (\text{NLO, rEFT}) \\
& -\ \ 2.05\,\text{pb} & (-4.2\%) & \quad ((t,b,c),\ \text{exact NLO}) \\
& +\ \ 9.56\,\text{pb} & (+19.7\%) & \quad (\text{NNLO, rEFT}) \\
& +\ \ 0.34\,\text{pb} & (+0.7\%) & \quad (\text{NNLO},\ 1/m_t) \\
& +\ \ 2.40\,\text{pb} & (+4.9\%) & \quad (\text{EW, QCD-EW}) \\
& +\ \ 1.49\,\text{pb} & (+3.1\%) & \quad (\text{N}^3\text{LO, rEFT})
\end{aligned}
$$

- N3LO QCD for infinite Mtop limit — CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger
- Finite quark-mass corrections at
  - NLO exact — Dawson; Djouadi, Gtaudenz, Spira, Zerwas; Harlander, Kant; CA,Beerli, Bucherer, Daleo, Kunszt; Bonciani, Degrassi, Vicini
  - NNLO 1/mtop expansion — Harlander, Mantler, Marzani, Ozeren; Pak, Rogal, Steinhauser
- Two-loop electroweak corrections — Actis, Passarino, Sturm, Uccirati; Aglietti, Bonciani, Degrassi, Vicini
- Mixed QCD-electroweak corrections — CA, Boughezal, Petriello

# Theoretical Uncertainties

| $\delta$(scale) | $\delta$(trunc) | $\delta$(PDF-TH) | $\delta$(EW) | $\delta(t,b,c)$ | $\delta(1/m_t)$ |
|---|---|---|---|---|---|
| $^{+0.10\text{ pb}}_{-1.15\text{ pb}}$ | $\pm 0.18$ pb | $\pm 0.56$ pb | $\pm 0.49$ pb | $\pm 0.40$ pb | $\pm 0.49$ pb |
| $^{+0.21\%}_{-2.37\%}$ | $\pm 0.37\%$ | $\pm 1.16\%$ | $\pm 1\%$ | $\pm 0.83\%$ | $\pm 1\%$ |

- Small uncertainties O(1% - 2%)...but quite a few of them
- missing N3LO pdfs
- missing exactly computed mixed QCD+EWK
- missing N3LO partonic cross-sections in closed functional form
- missing top-bottom interference effects at NNLO

# From NNLO to N3LO

$$\sigma^{NNLO} = 47.02 \text{ pb } {}^{+5.13 \text{ pb } (10.9\%)}_{-5.17 \text{ pb } (11.0\%)} \text{ (theory)}^{+1.48 \text{ pb } (3.14\%)}_{-1.46 \text{ pb } (3.11\%)} \text{ (PDF+}\alpha_s\text{)}$$

A doubling of the "theory" precision...

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)} \text{ (theory)} \pm 1.56\,\text{pb}\,(3.20\%) \text{ (PDF+}\alpha_s\text{)}$$

# Differential cross-sections

in collaboration with Simone Lionetti, Bernhard Mistlberger, Andrea Pelloni, Caterina Specchia

# from inclusive to differential

- Experiments impose cuts and reject many of the Higgs signal events.
- The inclusive cross-section is an idealized observable.
- It is important to predict with high accuracy differential distributions.
- Ideally, we would like a fully differential Higgs cross-section calculation at N3LO.

# from inclusive to differential...challenges

- Inclusive = Integral over differential, but
- An one-scale problem (Higgs mass) becomes a multi scale problem (...energies and angles of all particles) with complicated infrared divergence structure.
- Integration variables are physically measured quantities. We are not allowed to "integrate them out"

$$\int_0^1 \frac{dx dy}{(2-x-y)^a} J(x,y)$$

- Integral with an overlapping singularity when the "physical" variables x,y->1.
- J(x,y) is a phase-space selection function. Need the answer for any J(x,y)
- Non-linear mappings or sector decomposition or other subtraction methods can be used to calculate the singular part analytically and the remainder numerically
- These techniques have not been extended or used at such a high perturbative order as N3LO....

$$\int_0^1 \frac{dx dy}{(2-x-y)^a}$$

$$= \int_{-i\infty}^{i\infty} \frac{dw_1 dw_2}{(2\pi i)^2} \frac{\Gamma(-w_1)}{1+w_1} \frac{\Gamma(-w_2)}{1+w_2} 2^{-a-w_1-w_2}$$

- For the inclusive cross-section calculation, there are additional techniques, which integrate out the original phase-space.
  - Reverse-unitarity
  - Feynman parameters
  - Differential equations
  - Mellin-Barnes

We must now live without them!

# from inclusive to differential..challenges

- The inclusive cross-section has been computed as series in a threshold expansion.
- A complicated technique...relies on reverse unitarity, strategy of regions and crucially on the differential equations method.
- Especially challenging to extend it to a more differential computation:

$$\int_0^\infty dx dy \frac{x^\epsilon y^\epsilon}{(\delta + x + y)^a}$$

- Assume delta -> 0, the expansion parameter
- Inclusively, there is only one "region": x,y ~ delta
- Let's now be differential in x

$$x^\epsilon \int_0^\infty dy \frac{y^\epsilon}{(\delta + x + y)^a}$$

- We now have more "regions", accounting for all possible hierarchies of x and delta.

small

$$\delta(p_h \boxed{+ g_1 + g_2 + \ldots} - p_1 - p_2) J(p_h, g_1, g_2, \ldots)$$

- Momentum conservation is important for the cancelation of infrared divergences. In a threshold expansion, this cancelation happens order by order.

- But, generic experimental observables in the selection function J(ph,...) cannot be defined analytically, as an order by order threshold expansion

- Cancelation of poles needs to be numerical, but also incomplete...hard to control that the correctness of the computation.

# Higgs differntial, radiation inclusive

- As a first step, we can be differential only on the components of the Higgs momentum:

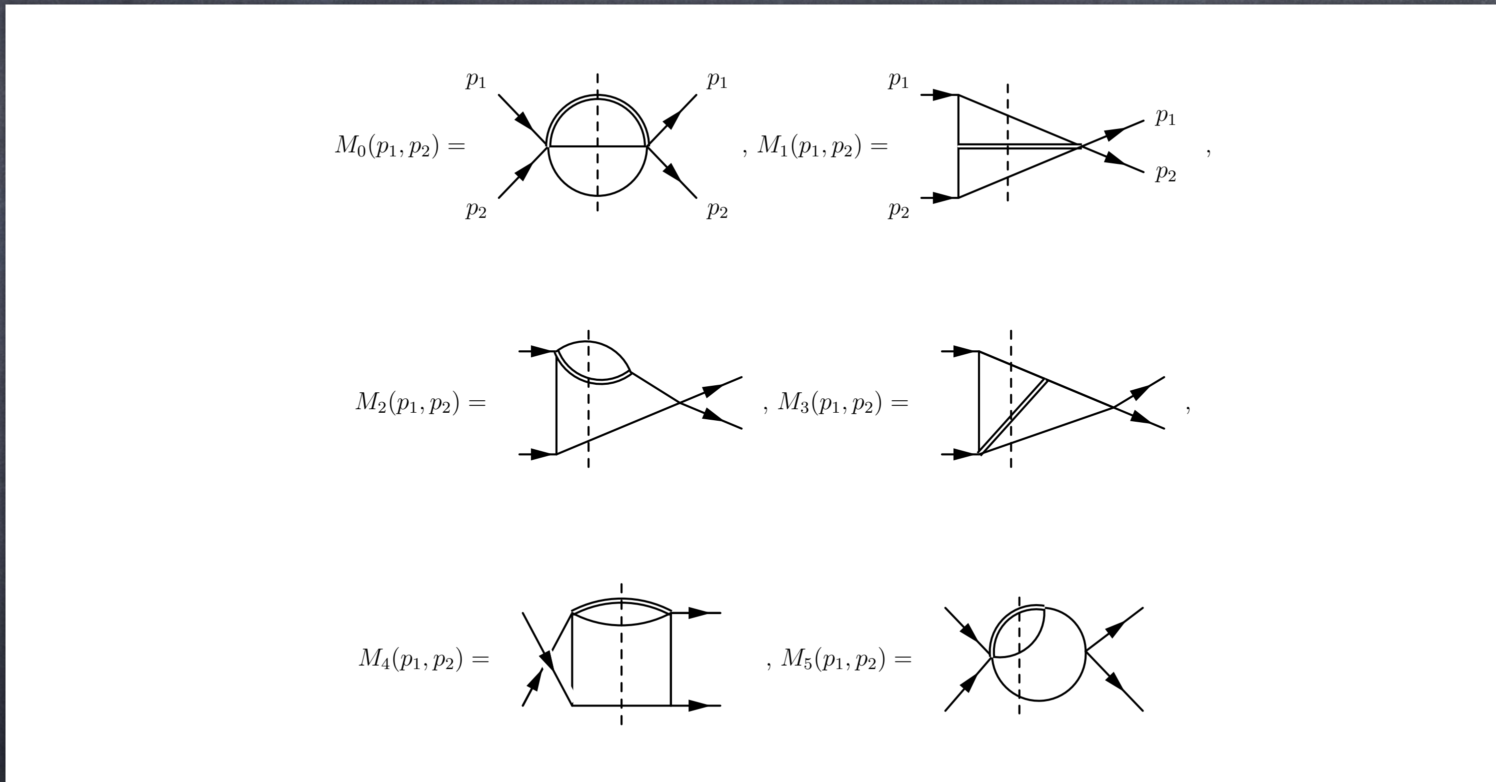
$$\hat{\sigma}_{ij} = \sum_f \int dPS_{ij\to f}\mathcal{J}(p_h)\mathcal{M}_{ij\to f}$$

- Treat numerically the Higgs phase-space and analytically the "radiation" phase-space:

$$\hat{\sigma}_{ij} = \sum_f \int \frac{dQ^2 dW}{2^{3-2\epsilon}s^{1-\epsilon}} d\Omega_{d-2}\mathcal{J}(p_h)\,\left(\Delta_3(m_h^2, s, Q^2) - W^2\right)^{-\epsilon}\mathcal{F}_{ij\to f}\left(W, Q^2, m_h^2, s\right)$$

$$\mathcal{F}_{ij\to f}\left(W, Q^2, m_h^2, s\right) = \int \left[\prod_{k=0}^{f} d^D g_k \delta^{(+)}(g_k^2)\right]\delta(g - \sum_{k=0}^{f} g_i)\mathcal{M}_{ij\to k}$$

# Test case: double real radiation at NNLO




very few especially simple master integrals...

# Amazing simplicity at NNLO

- All master integrals can be written in terms of a single Appel hypergeometric function:

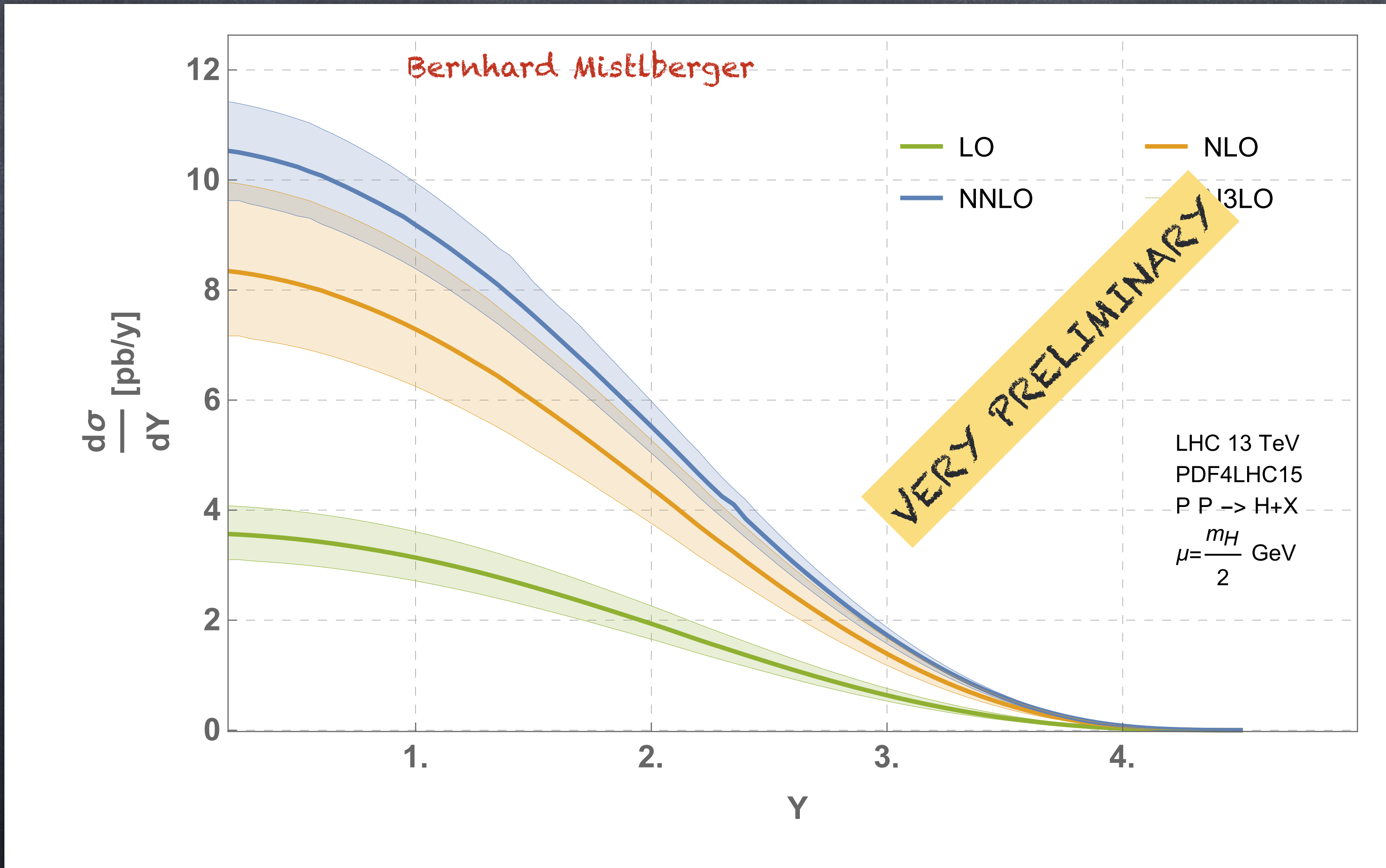
$$F_1(1,-\epsilon,-\epsilon,1-2\epsilon,x,y) = 1+\epsilon(\log(1-x)+\log(1-y)) \\ -\epsilon^2\left(2\mathrm{Li}_2(x)+2\mathrm{Li}_2(y)+\frac{1}{2}(\log(1-x)-\log(1-y))^2\right)+O(\epsilon^3)$$

- Originates from a single type of angular integrations which fits all

- Could be more general than 2 ->1 processes...

- Unclear if similarly simple results hold at N3LO...

# Very preliminary first results



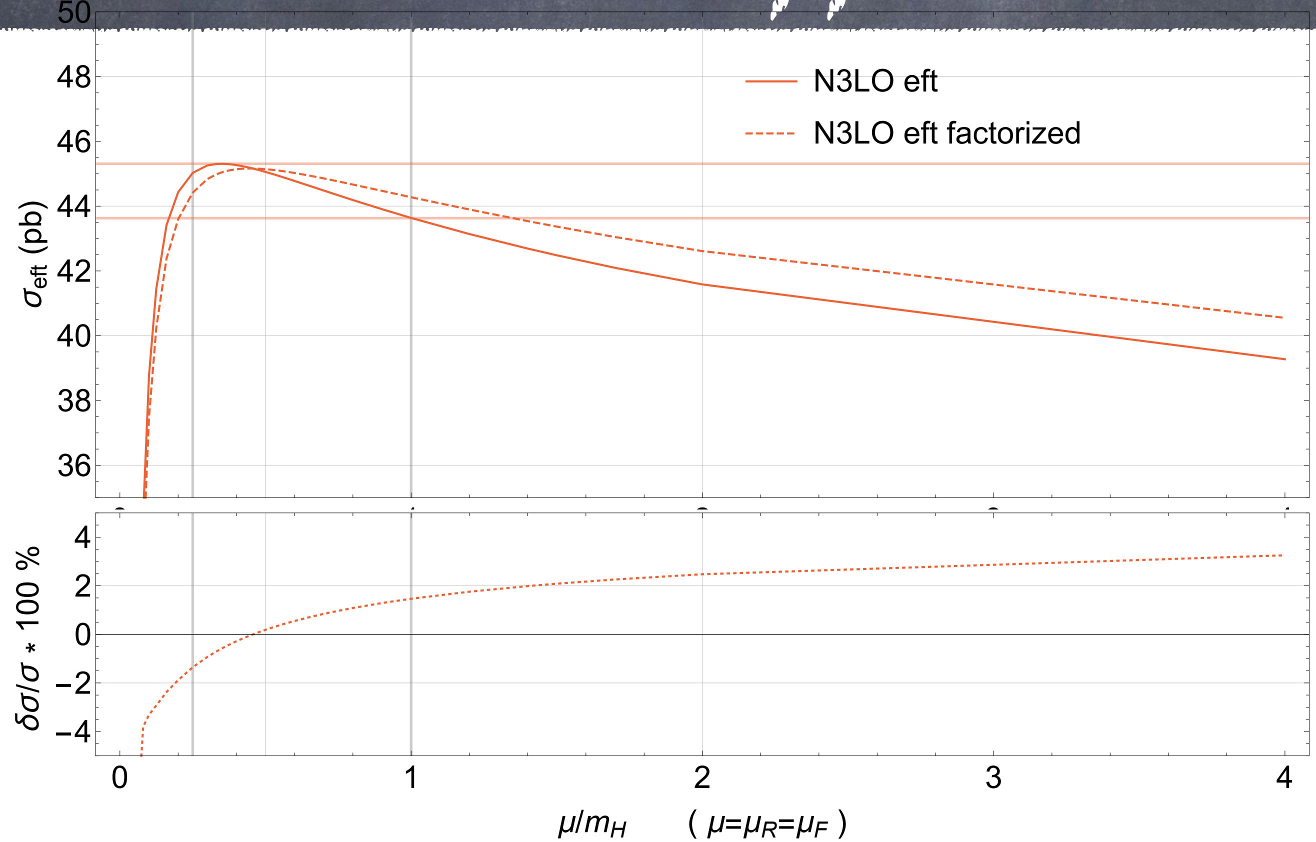
# Conclusions/ Outlook

- First N3LO computation for a hadron collider process
- Results to the most precise determination of the Higgs and a BSM CP-even scalar.
- Further improvements can come with further cutting edge calculations: exact quark mass dependence at NLO, exact EWK-QCD corrections, more NNLO and N3LO processes for PDF fits
- Tempting next theoretical challenge: can we do differential distributions?

# Επιπλέον Υλικό

Extra slides

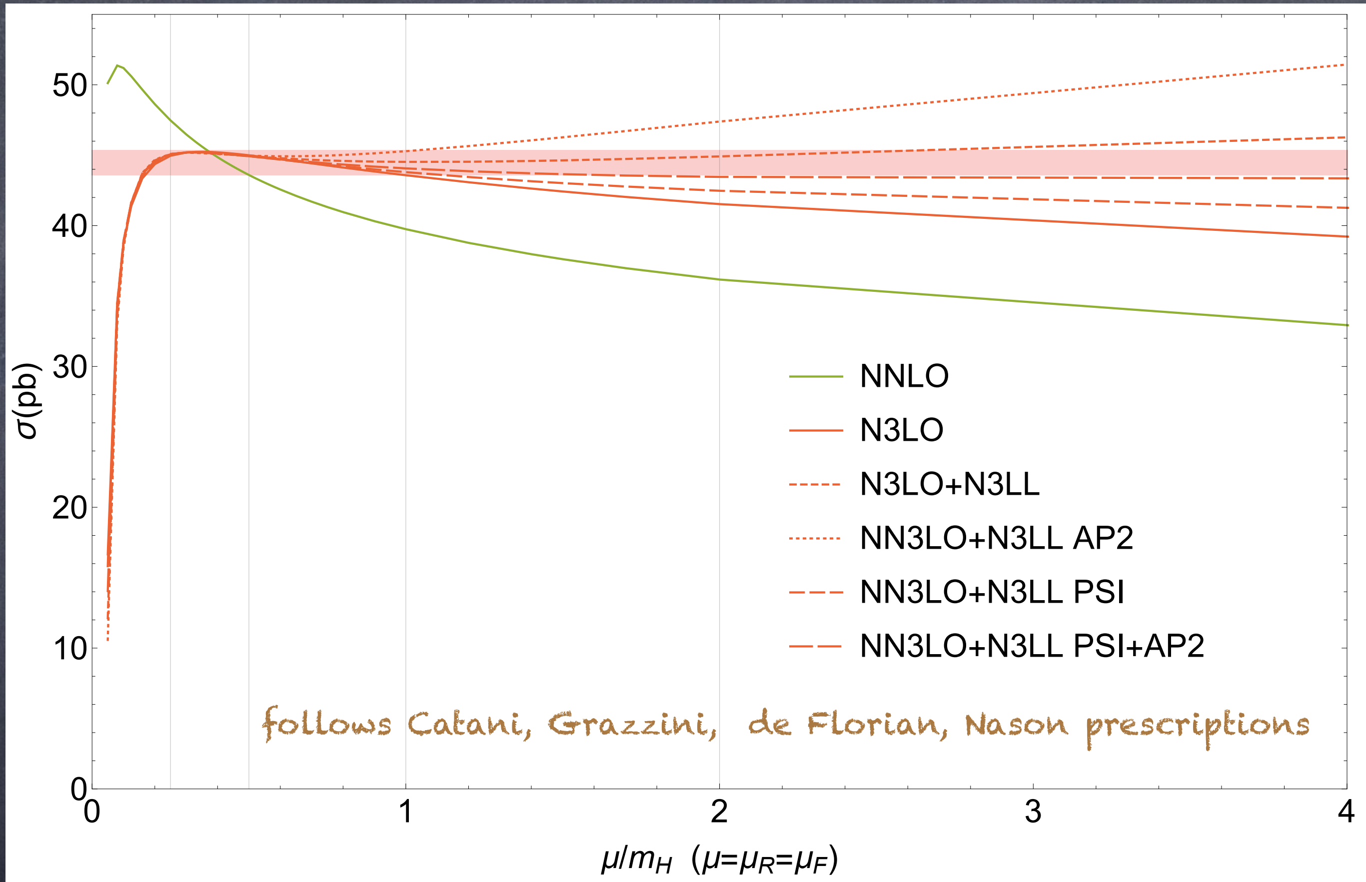
# Factorisation of the Wilson coefficient




$$\frac{\hat{\sigma}_{ij,EFT}}{z} = \sigma_0 \, |1 + \ldots + a_s^3 \, C_3 + \mathcal{O}(a_s^4)|^2 \sum_{i,j} (1 + \ldots + a_s^3 \, \eta_{ij}^{(3)}(z) + \mathcal{O}(a_s^4))$$

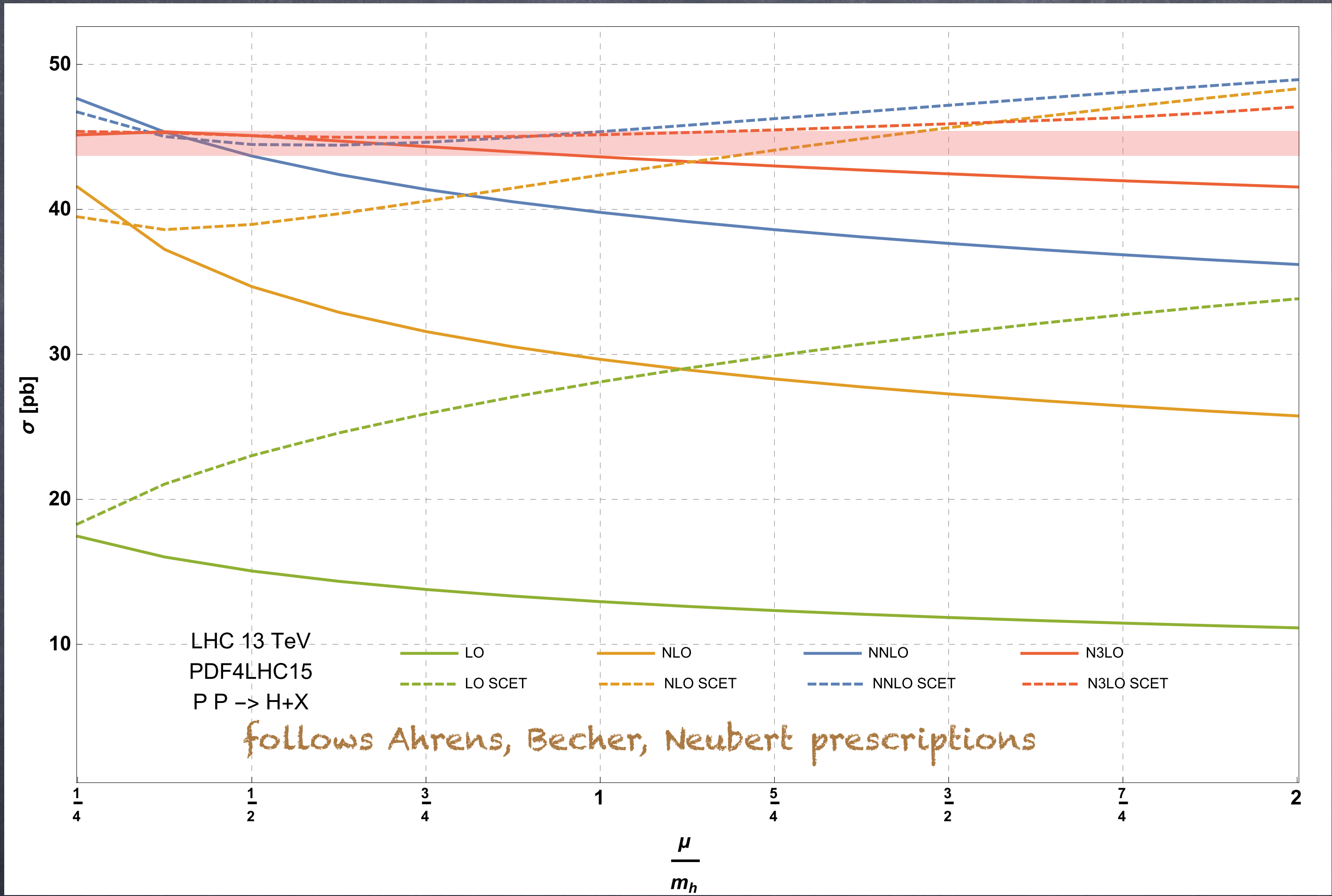
Excellent agreement between expanding the product and expanding the factors separately

# Resummation (I)




Traditional QCD threshold resummation agrees with N3LO

# Resummation (II)




SCET renormalisation group improvement agrees with N3LO

# PDF Uncertainties

$$\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\mathrm{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\mathrm{PDF}{+}\alpha_s)$$

Discrepancies between PDFs exist.

$$\sigma_{\mathrm{ABM12}} = 45.07\,\mathrm{pb}^{+2.00\,\mathrm{pb}\,(+4.43\%)}_{-2.88\,\mathrm{pb}\,(-6.39\%)}\ (\mathrm{theory}) \pm 0.52\,\mathrm{pb}\,(1.17\%)\ (\mathrm{PDF}{+}\alpha_s)$$

Cross-section with ABM pdfs and alphas differs from PDF4LHC beyond the level of the quoted accuracy.