A Reanalysis of Ultraviolet Divergences in Gravity

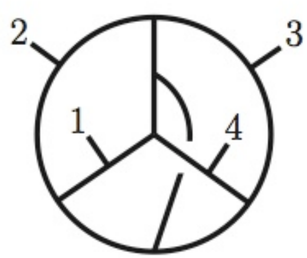
Amplitudes 2016

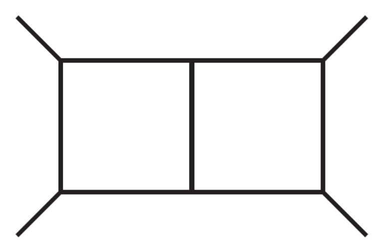
Zvi Bern,

July 8, 2016

UCLA The Mani L. Bhaumik Institute for Theoretical Physics

ZB, Clifford Cheung, Huan-Hang Chi, Scott Davies, Lance Dixon, Josh Nohle: arXiv:1507.06118
ZB, Huan-Hang Chi, Lance Dixon and Alex Edison: to appear.
Also new work with Enciso, Hermann, Kosower, Parra-Martinez, Stankowicz, Trnka, Zeng





UV in Gravity

Most theorists believe that UV properties of quantum field theories of gravity are "well understood", up to "minor" details, e.g. the precise loop order where divergences occur.

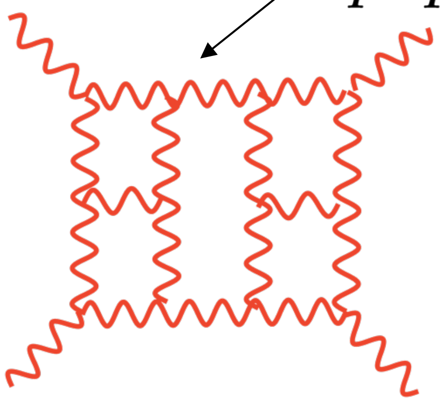
The main purpose of my talk is to try to convince you that the UV structure of gravity is strange and surprising and most certainly *not* "well understood".

1. When UV divergences are present in pure (super) gravity, properties are strange and unexpected.

2. Examples of no divergences even when no known symmetry arguments prevent them. "Enhanced cancellations". Not based on Lagragian symmetries.

Non-Renormalizability of Gravity?

$\kappa = \sqrt{32\pi G_N}$ ← Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

UV: Large momenta in loop integrals

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- **Will reexamine Einstein gravity.**
- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**

Einstein Gravity

Standard argument for 1 loop finiteness of pure gravity:

't Hooft and Veltman (1974)

$\cancel{R^2}$ $\cancel{R_{\mu\nu}^2}$ **Divergences vanish by equation of motion and can be eliminated by field redefinition.**

$\cancel{R_{\mu\nu\rho\sigma}^2}$ **In *D* = 4 topologically trivial space, Gauss-Bonnet theorem eliminates Riemann square term.**

$$\int d^4x \sqrt{-g}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) = 32\pi^2\chi$$

Euler characteristic

Pure gravity divergence with nontrivial topology:

$$\mathcal{L}^{\mathrm{GB}} = -\frac{1}{(4\pi)^2}\frac{53}{90\epsilon}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2)$$

Capper and Duff (1974)
Tsao (1977); Critchley (1978)
Gibbons, Hawking, Perry (1978)
Goroff and Sagnotti (1986)
Bornsen and van de Ven (2009)

- **Euler characteristic vanishes in flat space.** 't Hooft and Veltman (1974)
- **Dimensional regularization makes it subtle.** Capper and Kimber (1980)

The Trace Anomaly

Capper and Duff (1974); Tsao (1977); Critchley (1978); Gibbons, Hawking, Perry (1978); Duff and van Nieuwenhuizen (1980); Siegel (1980); Grisaru, Nielsen, Siegel, Zanon (1984); Goroff and Sagnotti (1986); Bornsen and van de Ven (2009); Etc.

The Gauss-Bonnet divergence exactly corresponds to trace anomaly.

$$D = 4 - 2\epsilon$$

$$\mathcal{L}^{\rm GB} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(4\cdot 53 + 1 + 91 - 180\Big)\left(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right)$$

graviton **scalar** **2 form** **3 form** **Gauss-Bonnet**

$$T^\mu{}_\mu = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)\left(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right)$$

Duff and van Nieuwenhuizen (1980);

Referred to as trace, conformal, trace or Weyl anomaly.

Quantum Inequivalence?

$D = 4 - 2\epsilon$

$D \to 4$

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)\left(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right)$$

graviton scalar 2 form 3 form Gauss-Bonnet

two form dual to scalar **three form not dynamical**

$$\partial_\mu \phi \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \qquad \Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} \qquad D = 4$$

Classically equivalent. But is it quantum mechanically equivalent?

- **Quantum *in*equivalence under duality transformations.**
  Duff and van Nieuwenhuizen (1980)
- **Quantum equivalence under duality. Gauge artifact.**
  Siegel (1980)
- **Quantum equivalence of effective action (ignoring trace anomaly).**
  Fradkin and Tseytlin (1984)
- **Quantum equivalence of susy 1 loop effective action (with Siegel's argument for higher loops)**
  Grisaru, Nielsen, Siegel, Zanon (1984)
- **Quantum *in*equivalence and boundary modes.**
  Finn Larsen and Pedro Lisbao (2015)

Two-Loop Pure gravity

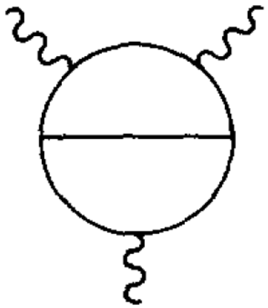
By two loops there is a valid R^3 divergence.

Goroff and Sagnotti (1986); Van de Ven (1992)

Divergence in pure gravity:

UV divergence

$$\mathcal{L}^{R^3} = \frac{209}{2880} \frac{1}{(4\pi)^4} \frac{1}{2\epsilon} R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}$$

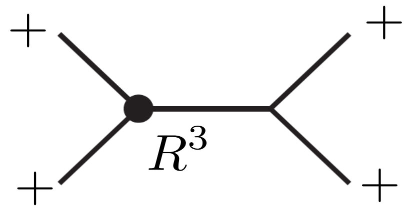


$$D = 4 - 2\epsilon$$

- **Based on dimensional regularization.**
- **On surface nothing weird going on.**
- **The Goroff and Sagnotti result for divergence in standard dimensional regularization is correct.**

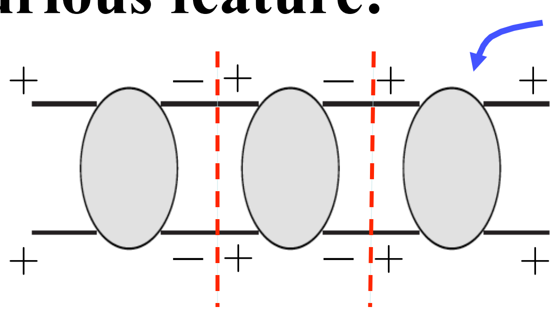
However, a goal of this talk is to show you that UV divergences in pure (super)gravity is subtle and weird, once you probe carefully.

Two-Loop Identical Helicity Amplitude



Four-graviton identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

Curious feature:



Naïve $D = 4$ unitarity arguments show amplitude vanishes!

- Gravity amplitude proportional to 0/0, resolved in dim reg.
- Bardeen and Cangemi pointed out nonvanishing of identical helicity is connected to an anomaly in self-dual sector.
- $N = 4$ sugra 4 loop divergence relies relies on same anomaly, which manifests itself as anomaly in U(1) subgroup of duality

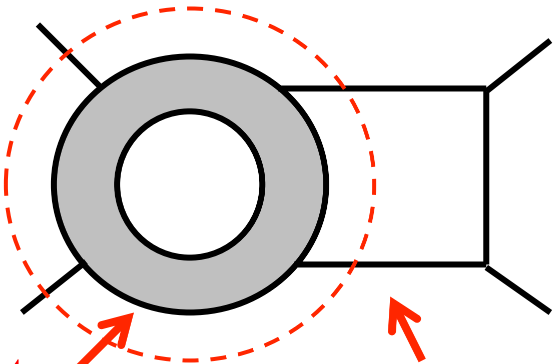
ZB, Davies, Dennen, Smirnov, Smirnov; Carrasco, Kallosh, Tseytlin and Roiban

A surprise:

Divergence is *not* generic but tied to anomaly-like behavior.

A Dimensional Regularization Subtlety

No one-loop divergences
but the integrand
has subdivergences



Representative diagram.

Gauss-Bonnet
subdivergence

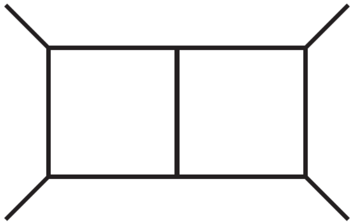
$D \neq 4$, subdivergences!
$D = 4$, no subdivergences

A strange phenomenon: no one-loop divergences,
yet there are one-loop subdivergences to subtract!

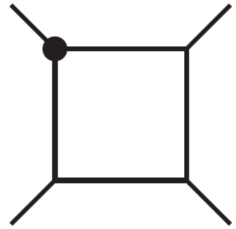
- To match the G&S result we need to subtract subdivergences as they needed to do.
- Using modern methods we can track the pieces.

Two Loop Identical Helicity Divergence

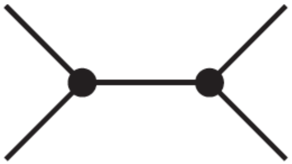
ZB, Cheung, Chi, Davies, Dixon and Nohle



2 loop bare



single GB subtraction



double GB subtraction

$$3431 = 47 \cdot 73$$
$$689 = 13 \cdot 53$$
$$5618 = 2 \cdot 53^2$$
$$209 = 11 \cdot 19$$

$$\mathcal{M}_4^{2\text{-loop}}\Big|_{\text{div.}} = -\frac{1}{\epsilon}\frac{3431}{5400}\mathcal{K}$$

$$\mathcal{M}_4^{1\text{-loop GB}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{689}{675}\mathcal{K}$$

$$\mathcal{M}_4^{\text{tree GB}^2}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{5618}{675}\mathcal{K}$$

$$\mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{209}{24}\mathcal{K}$$

Goroff and Sagnotti divergence reproduced

Trace anomaly, and evanescent operators play central role in the G&S result.

Meaning of Divergence?

What does the divergence mean? $\Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$

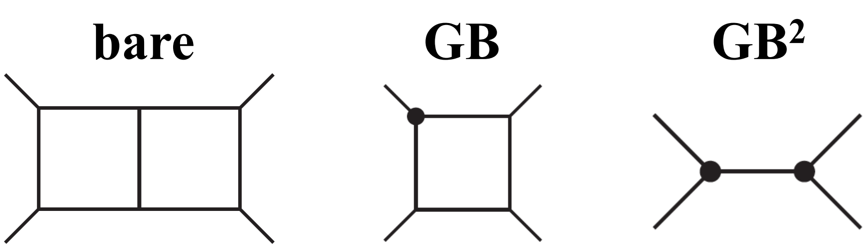
Adding n_3 3-form field offers good way to understand this:

- **On the one hand, no degrees of freedom in $D = 4$, so no change in divergence expected.**
- **On the other hand, the trace anomaly is affected, so expect change in divergence.**

- **Note that 3 form proposed as way to dynamically neutralize cosmological constant.**

Brown and Teitelboim; Bousso and Polchinski

	$1/\epsilon$
bare	$-\frac{3431}{5400} - \frac{199 n_3}{30} + 6 n_3^2$
GB	$\frac{4\cdot 53 - 180 n_3}{360} \cdot \frac{2\cdot(13+180 n_3)}{15}$
GB^2	$24\left(\frac{4\cdot 53 - 180 n_3}{360}\right)^2$
total	$\frac{209}{24} - \frac{15}{2} n_3$



Divergence depends on nondynamical 3-form fields. Quantum inequivalence??

But wait: what about finite parts? Need physical quantity!

Scattering Amplitudes

Pure Gravity:

$$M^{\text{2-loop}}(1^+,2^+,3^+,4^+) = 2\mathcal{K}\Big[stu\left(\frac{209}{24\epsilon}+\frac{117617}{21600}\right)$$
$$-\frac{1}{240}(s^2+t^2+u^2)\left[\frac{2}{\epsilon}s\ln\left(\frac{-s}{\mu^2}\right)-s\ln^2\left(\frac{-s}{\mu^2}\right)+\text{perms}\right]$$
$$+\frac{1}{120}\left(s(6tu-s^2)\ln\left(\frac{-s}{\mu^2}\right)+\text{perms}\right)\Big]$$

Gravity + 3 Form:

$$M^{\text{2-loop}}(1^+,2^+,3^+,4^+) = 2\mathcal{K}\Big[stu\left(\frac{29}{24\epsilon}+\frac{411617}{21600}\right)$$
$$-\frac{1}{240}(s^2+t^2+u^2)\left[\frac{2}{\epsilon}s\ln\left(\frac{-s}{\mu^2}\right)-s\ln^2\left(\frac{-s}{\mu^2}\right)+\text{perms}\right]$$
$$+\frac{1}{120}\left(s(6tu-s^2)\ln\left(\frac{-s}{\mu^2}\right)+\text{perms}\right)\Big]$$

divergences differ.
logarithms identical!

- **Value of divergence not physical. Renormalize away.**
- **3 form is a Cheshire Cat field: physical scattering unaffected.**

- **Results consistent with quantum equivalence under duality.**
- **For carefully defined physically measurable quantities it seems that duality transformations do not alter the physics.**

Divergences and Duality

ZB, Cheung, Chi, Davies, Dixon and Nohle

$$\mathcal{M}_4 = \left[\frac{1}{\epsilon}\left(\frac{209}{24} - \frac{15}{2} n_3\right) - \frac{1}{4} \ln \mu^2\right] \mathcal{K} + \text{finite} + \text{IR}$$

divergence

number of 3 forms

independent of 3 forms

Renormalization scale

$$\mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

- **Weird that renorm. scale and UV divergence not linked! Happens because of Gauss-Bonnet subdivergence.**
- **The UV divergence depends on the details of the regularization prescriptions.**
- **The renormalization scale dependence is robust.**

Focus on renormalization scale dependence *not* divergences!
In QCD these are the same. In gravity *not* related!

Simple Two-Loop Formula

ZB, Cheung, Chi, Davies, Dixon and Nohle

Looking at various theories, we wind up with a simple 2 loop formula:

$$\mathcal{M}_4^{2\text{-loop}}\Big|_{\ln\mu^2}^{\mathrm{UV}} = -\mathcal{K}\frac{N_b - N_f}{8}\ln\mu^2$$

N_b is number of bosonic states.
N_f is number of fermionic states.

- **This appears to be robust and does not depend on dimensional regularization or details of theory.**
- **Vanishes at two loops in susy theory, as expected.**
- **Unless $\ln\mu^2$ dependence vanishes, theory should still be considered nonrenormalizable.**

Simple formula for 2-loop UV properties in any minimally coupled gravity theory! Who ordered this??

Some Questions

$$\left.\mathcal{M}_4^{\text{2-loop}}\right|_{\ln\mu^2}^{\text{UV}} = -\mathcal{K}\,\frac{N_b - N_f}{8}\,\ln\mu^2$$

1. If the trace anomaly drops out in physical quantities, why do we even need it in the first place?

2. Why do we obtain such a simple result from a relatively complicated and subtle calculation?

3. Why do we get a universal renormalization-scale dependence, independent of the details of the theory?

$D = 4$ Cuts

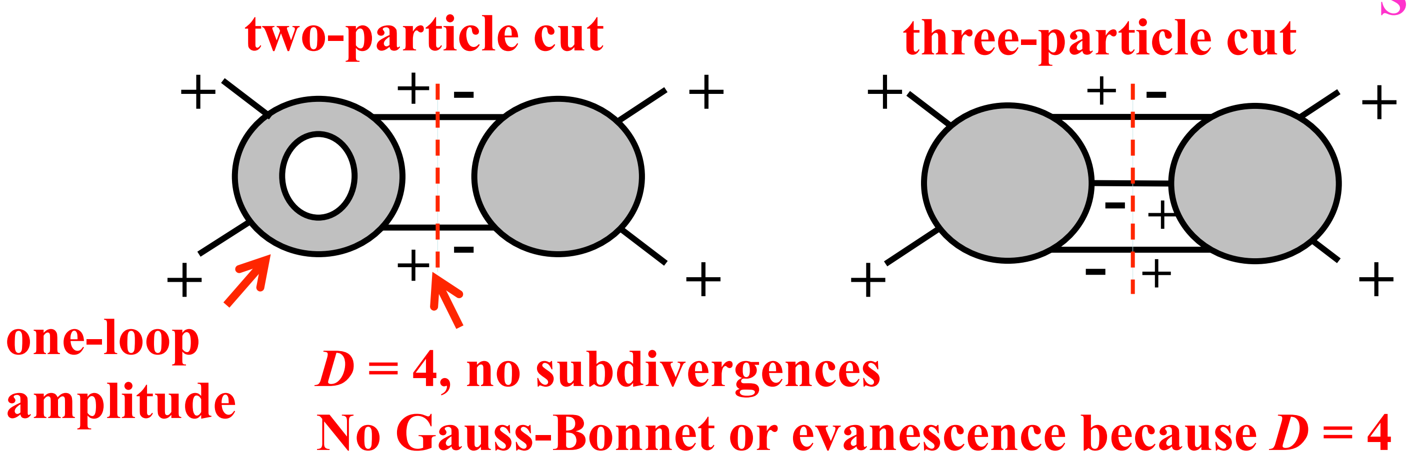
ZB, Chi, Dixon, Edison (to appear)

We can obtain logs from $D = 4$ cuts. Dunbar and Norridge

Similar to YM identical helicity two-loop amplitudes.

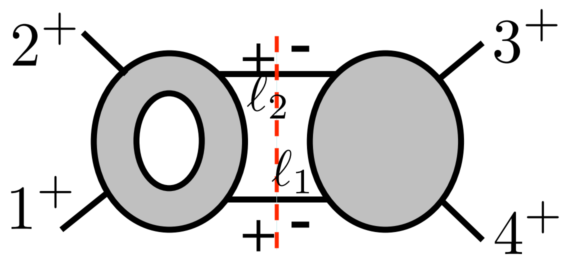
Dunbar and Perkins

See Dunbar's talk



- **One loop four-point amplitude is simple. Rational.**
- **Two-particle cut is simple!**
- **$D = 4$ three-particle cut vanishes.**

Two-Particle $D = 4$ Cut



This cut and permutations are only contributions.

$$\ell_1 = p \qquad \ell_2 = p - k_1 - k_2$$

$$\mathcal{M}^{\text{1-loop}}(1^+, 2^+, \ell_2^+, \ell_1^+) = -\frac{i}{(4\pi)^2}(N_b - N_f)\frac{1}{240}\left(\frac{\kappa}{2}\right)^4\left(\frac{[12][-\ell_1\ell_2]}{\langle 12\rangle\langle -\ell_1\ell_2\rangle}\right)^2(s_{12}^2 + s_{1\ell_1}^2 + s_{2\ell_1}^2)$$

Dunbar and Norridge (1994)

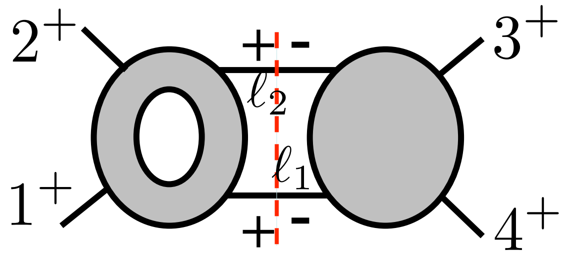
$$\mathcal{M}^{\text{tree}}(3^+, 4^+, \ell_1^-, \ell_2^-) = -i\left(\frac{\kappa}{2}\right)^2 s_{12}\frac{\langle \ell_1(-\ell_2)\rangle^3}{\langle(-\ell_2)3\rangle\langle 34\rangle\langle 4\ell_1\rangle}\,\frac{\langle \ell_1(-\ell_2)\rangle^3}{\langle(-\ell_2)4\rangle\langle 43\rangle\langle 3\ell_1\rangle}$$

Kawai, Lewellen, Tye

$$\int \frac{d^D p}{(2\pi)^D}\,\mathcal{M}^{\text{1-loop}}(1^+, 2^+, \ell_2^+, \ell_1^+) \times \mathcal{M}^{\text{tree}}(3^+, 4^+, \ell_1^-, \ell_2^-)$$

Two-loop problem reduced to a one-loop problem!
Captures the logarithms.
Dim. reg. reintroduced to deal with IR and UV singularities.

Two-Particle Cut



Dunbar and Norridge;
ZB, Chi, Dixon, Edison (to appear)

Result of one-loop integration after summing over permutations.

$$\mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^D} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

$$M^{\text{2-loop}}(1^+,2^+,3^+,4^+) = \mathcal{K}(N_b - N_f)\Big[-stu\left(\frac{1}{8\epsilon}+\frac{3}{10}\right) - \frac{1}{240}(s^2+t^2+u^2)\left[\frac{2}{\epsilon} s\ln\left(\frac{-s}{\mu^2}\right) - s\ln^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\right] + \frac{1}{120}\left(s(6tu - s^2)\ln\left(\frac{-s}{\mu^2}\right) + \text{perms}\right)\Big]$$

- **Reproduces the amplitude (after renormalization), except no evanescent operators or trace anomaly.**
- **The UV divergence calculated here is directly linked to renormalization scale.**

$$\boxed{\mathcal{M}_4^{\text{2-loop}}\Big|_{\ln\mu^2}^{\text{UV}} = -\mathcal{K}\frac{N_b - N_f}{8}\ln\mu^2}$$

Simple result has a simple derivation!

Exhanced UV Cancellations
in
Supergravity Theories

Supergravity and Ultraviolet Divergences

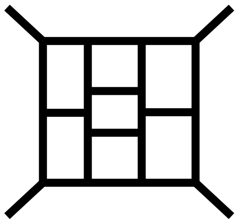
Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

- **First quantized formulation of Berkovits' pure-spinor formalism.** Bjornsson and Green
- **Unitarity method.** ZB, Davies, Dennen

Key point: *all* supersymmetry cancellations are exposed.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense" Bjornsson and Green

- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ?
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.** ?
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗



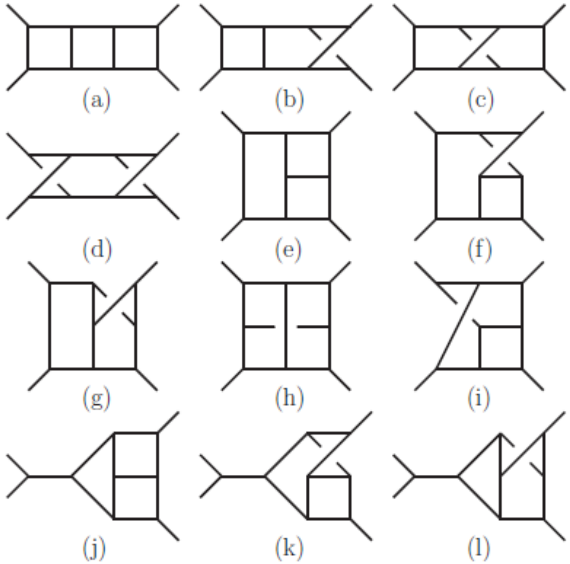
Consensus agreement from all methods

These new types of cancellations do exist: "enhanced cancellations".

ZB, Davies, Dennen

$\mathcal{N} = 4$ Supergravity UV Cancellation

$D = 4 - 2\epsilon$ ZB, Davies, Dennen, Huang



Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

All **three-loop divergences and subdivergences cancel completely!**

Still no standard-symmetry explanation, despite valiant attempt.

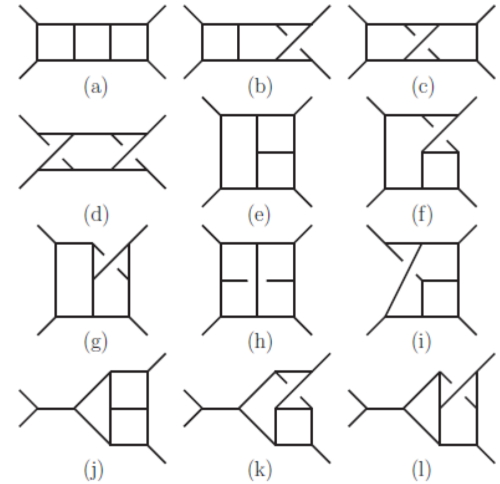
Bossard, Howe, Stelle; ZB, Davies, Dennen

Prediction based on supergravity imply divergences.
A nontrivial example of "enhanced cancellations".

Enhanced UV Cancellations

ZB, Davies, Dennen

Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.

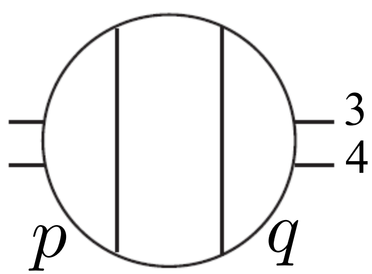


- By definition this is an enhanced cancellation.
- Not the way gauge theory works.

already log divergent

$N = 4$ sugra: pure YM x $N = 4$ sYM

$N = 4$ sugra



$$n_i \sim s^3 t A_4^{\text{tree}} \, (p \cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

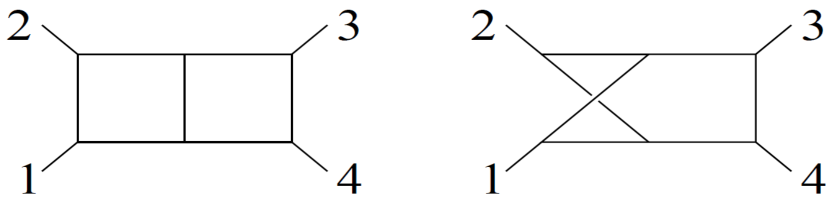
- 3 loop UV finiteness of $N = 4$ supergravity proves existence of "enhanced cancellation" in supergravity theories.
- No known standard symmetry explanation.

Where does new magic come from?

ZB, Davies, Dennen, Huang; Bossard, Howe, Stelle

To analyze we need a simpler example: Half-maximal supergravity in *D* = 5 at 2 loops. No known symmetry explanation in this case.

Similar to *N* = 4, *D* = 4 sugra at 3 loops, except much simpler.



$D = 5$ half max sugra
$N = 4$ sYM x $N = 0$ YM

Quick summary:

- **— Finiteness in *D* = 5 tied to double-copy structure.**
- **— Cancellations in certain forbidden gauge-theory color structures imply hidden UV cancellations in supergravity.**

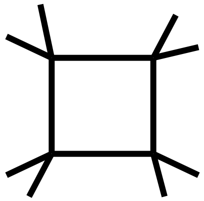
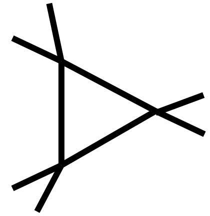
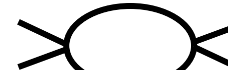
Double copy structure implies extra cancellations!

Unfortunately, argument relies on special two-loop property: integrals of *N* = 4 sugra are identical to those of QCD.

Need a more general approach

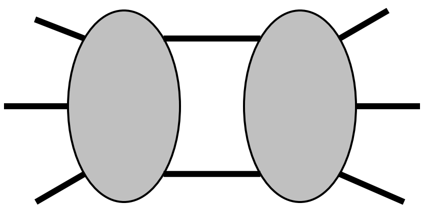
Enhanced Cancellations at One Loop

ZB, Carrasco, Forde, Ita, Johansson (2007)

$A_n = \sum_i d_i$  $+ \sum_i c_i$  $+ \sum_i b_i$ 

Integrand level reduction.

Ossalo, Papadopolous, Pittau; Forde



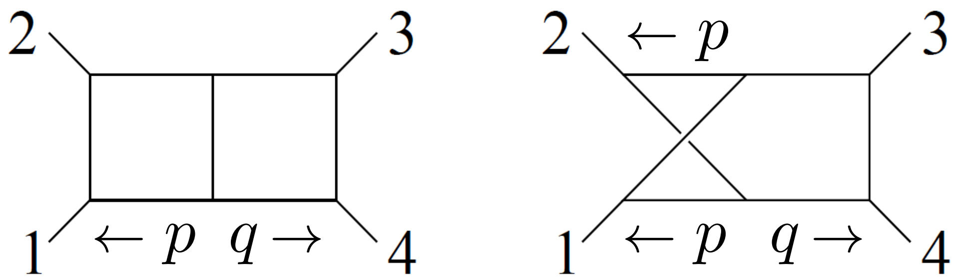
Gravity tree amplitudes have excellent large *z* behavior under BCFW shifts.

For $N \geq 5$ supergravity Forde's formalism was used to demonstrate vanishing coefficients for bubbles and triangles.

Excellent large *z* behavior implies residues at infinity determining bubble and triangle functions vanish.

Can we do the same at higher loops?

A Multiloop Annoyance



route momenta in different ways

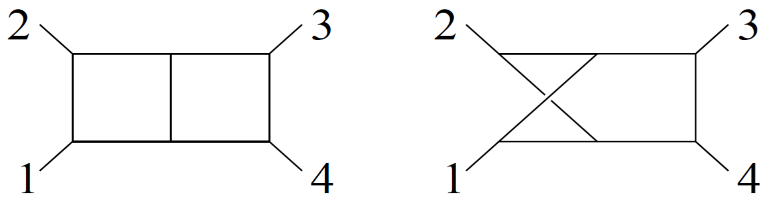
There are no global variables for both planar and nonplanar.

Annoying problem:
How can you find cancellations at integrand level between planar and nonplanar contributions when variables are not the same?

We are looking for all cancellations, including those due to ibp identities. We want to do this at the integrand level.

A More General Approach

ZB, Enciso, Kosower, Parra-Martinez, Zeng
Also Trnka, Hermann, Stankowicz
in progress



Half maximal supergravity in $D = 4$

Requirements:

- **Need integrand based cancellations between planar and nonplanar integrals.**
- **Identify (UV divergent) total derivatives from the integrand.**

We use the algebraic geometry formalism of Ita and of Larsen and Zhang.

See Yang Zhang's talk

- **Sort integrand into a basis composed of:**
  1. **UV divergent pieces**
  2. **UV finite pieces**
  3. **total derivatives**

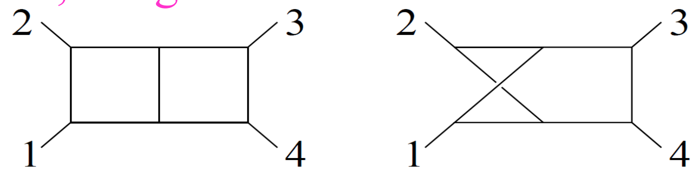
  corresponds to master integrals

Analog of Forde's or OPP's 1 loop methods, but at 2 loops.

An Integrand Basis

ZB, Enciso, Kosower, Parra-Martinez, Zeng

See Yang Zhang's talk

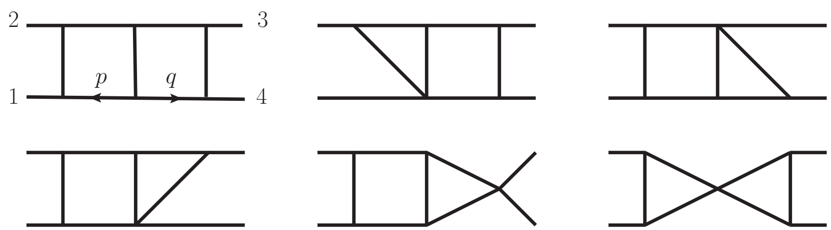


Use algebraic geometry integrand-level reduction methods.

Gluza, Kajda, Kosower ; Johansson, Kosower and Larsen; Ita; Larsen and Zhang

$$\int \prod_{i=1}^{L} \frac{\mathrm{d}^D \ell_i}{\pi^{D/2}} \sum_{j=1}^{L} \frac{\partial}{\partial \ell_j^\mu} \frac{v_j^\mu P}{D_1^{a_1} \cdots D_t^{a_k}} = 0$$

Systematically organizes IBP relations. Syzygy method.

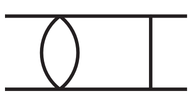


Double box master Integral numerators:

$1,\ p\cdot k_4,\ p\cdot k_4,\ (p\cdot k_4)^2,$

$q\cdot k_1,\ (q\cdot k_1)^2,\ p\cdot k_4\, q\cdot k_1$

$D = 5$



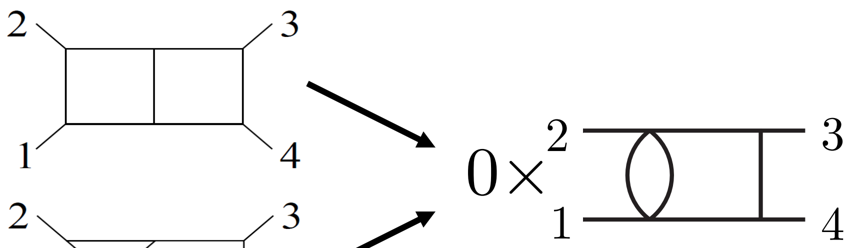
only UV divergent master integral in *D* = 5

Planar 12 master integrals + 703 surface terms.

Nonplanar similar but with 16 masters + 699 surface terms.

For above planar and nonplanar double box diagrams choose basis with only a single UV divergent integral.

$D = 5$ Half Maximal Supergravity



+ total derivatives
+ finite

- **Planar contribution cancels nonplanar ones.**
- **No coefficient has $1/(D - 5)$.**
- **$D = 5$ UV divergent contribution vanish.**

Using Larsen and Zhang's formalism, we have obtained an explicit (complicated) integrand in $D = 5$ for an independent amplitude of half max sugra with following property: Ita; Larsen and Zhang

Divergent terms in integrand either vanish or total derivatives.

- **Enhanced cancellations manifest at integrand level.**
- **Offers solution to problem of no nonplanar global variables.**
- **Formalism compatible with unitarity method. Large z behavior of tree amplitudes is likely the key just as at 1 loop.**

Need to understand systematics from unitarity cuts.

Summary

1. Standard view of gravity UV too naive:
   — New phenomenon: "Enhanced" UV cancellations in gravity.
   — So far divergences of pure (super)gravity theories appear to be due to anomalous behavior!
   — Duality transformations change divergences but not the renormalized amplitudes.
   — Focus on renormalization scale dependence rather than divergences. Equivalent in gauge theory, but not in gravity.
2. Simple two-loop formula for renormalization scale behavior in gravity theories. Best obtained from $D = 4$ unitarity cuts.
3. Confusion on duality transformations and evanescent operators banished, if you look at the problem properly.
4. A path to understanding enhanced cancellations at integrand level reduction to masters + surface terms.

Expect many more surprises as we probe gravity theories using modern perturbative tools.

UCLA The Mani L. Bhaumik Institute for Theoretical Physics

On July 1, 2016, thanks to the generosity of Mani L. Bhaumik, a new Institute for Theoretical Physics was founded at UCLA.

- **It will begin primarily in theoretical high-energy physics, with a goal toward unification of forces and particles.**
- **It will grow into multiple areas of theoretical physics, in line with available resources.**

In the coming years, I hope to see you at UCLA at conferences and workshops hosted by the Institute.

Thank you to the organizers!

Paolo Di Vecchia
Monica, Guică
Henrik, Johansson
Joseph Minihan
Konstantin Zarembo

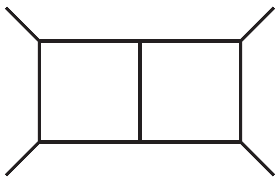
EXTRA SLIDES

Renormalization Scale Dependence

Simple rule for tracking renormalization scale:

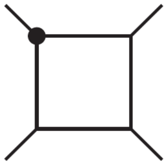
$$D = 4 - 2\epsilon$$

bare



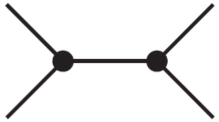
$$(\mu^2)^{2\epsilon}\frac{c_2}{\epsilon} = \frac{c_2}{\epsilon} + 2c_2 \ln \mu^2 + \mathcal{O}(\epsilon)$$

1 counterterm



$$(\mu^2)^{\epsilon}\frac{c_1}{\epsilon} = \frac{c_1}{\epsilon} + c_1 \ln \mu^2 + \mathcal{O}(\epsilon)$$

GB²



$$(\mu^2)^{0\epsilon}\frac{c_0}{\epsilon} = \frac{c_0}{\epsilon}$$

Total: $(c_2 + c_1 + c_0)\dfrac{1}{\epsilon} + (2c_2 + c_1)\ln \mu^2$

$N = 4$ Supergravity at Four Loops

ZB, Davies, Dennen, Smirnov, Smirnov

We also calculated four-loop divergence in $N = 4$ supergravity.

Using BCJ duality:

$$N = 4 \text{ sugra: } (N = 4 \text{ sYM}) \times (N = 0 \text{ YM})$$

Integration uses state-of-the-art software developed for QCD. Industrial strength software needed: FIRE5 and special purpose C++ code.

The 4 loop Divergence of $N = 4$ Supergravity

ZB, Davies, Dennen, A.V. Smirnov, V.A. Smirnov

dim. reg. UV pole

$$\boxed{\mathcal{M}^{4\text{-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}}$$

kinematic factor

$D = 4 - 2\epsilon$

It diverges but it has strange properties:

- **Contributions to helicity configurations that vanish were it not for an anomaly in $U(1)$ subgroup of duality symmetry.**
- **These helicity configuration have vanishing integrands in $D = 4$. Divergence is 0/0. Anomaly-like behavior not found in $N \geq 5$ sugra.**

Carrasco, Kallosh, Tseytlin and Roiban

Motivates closer examination of divergences.
Want simpler example: Pure Einstein gravity is simpler.

Some New Directions in Gravity Loops

If you want to solve a difficult problem get an army of energetic young people to help with new ideas:

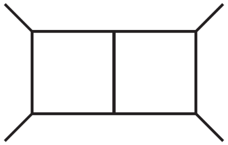
- **Better understanding and applications of BCJ duality.**
  Chiodaroli, Gunaydin, Johansson and Roiban,; Johannsson, Ochirov; O'Connell, Montiero, White; ZB, Davies, Nohle; Boels, Isermann, Monteiro, and O'Connel; Mogull and O'Connell, He, Monteiro, and Schlotterer
- **Scattering equations and double-copy relations.**
  Cachazo, He, Yuan
- **Twistor strings now at loop level for $N = 8$ supergravity.**
  Adamo, Casali and Skinner; Geyer, Mason, Monteiro and Tourkine
- **New ideas on unitarity cuts based on Feynman Tree Theorem**
  Baadsgaard, Bjerrum-Bohr, Bourjaily, Caron-Huot, Damgaard and Feng
- **Important advances in related string theory amplitudes.**
  Carlos Mafra and Oliver Schlotterer
- **Nonplanar analytic hints from Grassmanian and Amplituhedron.**
  ZB, Hermann, Litsey, Stankowicz, Trnka; Hermann and Trnka
- **Awesome equation solver. Millions of equations encountered at 5 loops can be dealt with! Very cool algorithm!**
  Schabinger and von Manteuffel

$N = 1$ Supergravity

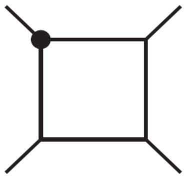
ZB, Chi, Dixon, Edison (to appear)

Divergence violates susy ward identity even though regulator should be supersymmetric! Due to trace anomaly.

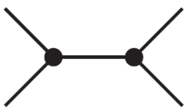
Result for $N = 1$ supergravity with 1 matter multiplet



$$\mathcal{M}_4\Big|_{\rm div} = \frac{1}{\epsilon}\,\frac{81871}{21600}\,\mathcal{K} + 0\ln(\mu^2)\mathcal{K}$$



Very strange, but no stranger than earlier results.



Have no fear: no physical effect! Local counterterm eats the divergence restoring susy.

Still working on case with no matter multiple, but no reason to expect different outcome.