$\mathcal{N} = 2$ supergravities as double copies of gauge theories

Marco Chiodaroli

Max-Planck-Institut
für Gravitationsphysik
Albert-Einstein-Institut

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Based on work with M. Günaydin, H. Johansson and R. Roiban
(arXiv:1408.0764, arXiv:1511.01740 and arXiv:1512.09130)

Introduction: **Double-copy construction**

[Bern, Carrasco and Johansson, 2008]

$$(1) \qquad \mathcal{A}_n^L = i^L g^{n-2+2L} \sum_{i \in \mathcal{G}_3} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$\downarrow$$

color/kinematics duality:

$$C_i + C_j + C_k = 0 \qquad \Leftrightarrow \qquad n_i + n_j + n_k = 0$$

- Critical for multi-loop calculations for maximal and half-maximal sugras
  [Bern, Carrasco, Dixon, Johansson, Davies, Dennen, Smirnov, Smirnov ...]
- Rests at the nexus of various recent ideas/research directions
  - string amplitudes [Stieberger 2009; Bjerrum-Bohr, Damgaard, and Vanhove 2009; Mafra, Schlotterer, and Stieberger 2011...]
  - scattering equations [Cachazo, He, and Yuan, 2013-16]
  - ambitwistor strings [Mason and Skinner 2013; Casali, Geyer, Mason, Monteiro, Roehrig 2015 ...]
  - classical double copies [O'Connell, Luna, Monteiro, Nicholson, White 2014-16]
  - YM origin of gravitational symmetries [Anastasiou, Borsten, Duff, Hughes, and Nagy 2014]

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DC property has been established for large classes of gravities:

- $\mathcal{N} \geq 4$ supergravities [Bern, Carrasco, Johansson 2008-10]
- self-dual gravity [Monteiro and O'Connell 2011]
- $\mathcal{N} < 4$ pure gravities [Johansson and Ochirov 2014]
- theories with higher-dimension operators [Broedel and Dixon 2012]
- several families of $\mathcal{N} < 4$ gravities with matter [Carrasco, MC, Günaydin and Roiban 2012; Bern, Davies, Dennen, Huang and Nohle 2013; Johansson and Ochirov 2014 ...]
- $\mathcal{N} = 16, 12, 10, 8$ sugras in $3D$ [Bargheer, He and McLoughlin 2012; Huang and Johansson 2013]

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[see also Ochirov's talk]

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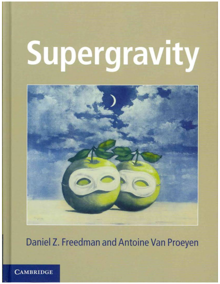
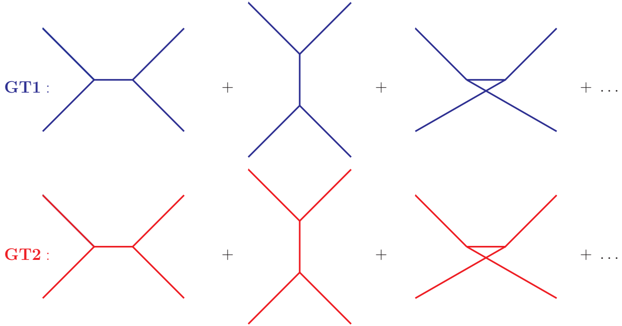
Q: **Is the double copy a general feature of gravitational interactions?**

Given a supergravity theory, find the gauge theories entering the construction:



$\longrightarrow$



Focus on double copies with $\mathcal{N} = 2$ supersymmetry:

- $\mathcal{N} = 2$ sugras are natural testing ground for ideas about theories with reduced SUSY
- easier to study UV properties (divergencies expected already at one loop for generic theories with matter)

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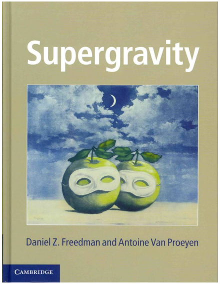

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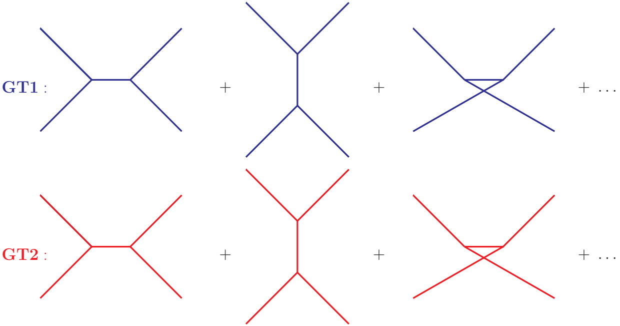

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$\mathcal{N} = 2$ theories are more difficult to chart than their $\mathcal{N} \geq 4$ relatives. Matter content alone no longer specifies theory.

General $5D$ Maxwell-Einstein $\mathcal{N} = 2$ supergravity
[Günaydin, Sierra, Townsend 1984]

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}(\varphi)F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}(\varphi)\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

- n_V vector multiplets $\left(A^x_\mu, \lambda^x_i, \varphi^x\right)$ $\quad x, y = 1, \ldots, n_V$
  graviton multiplet $\left(h_{\mu\nu}, \psi_\mu, A^0_\mu\right)$ $\quad I, J = 0, \ldots, n_V$
- All quantities in the Lagrangian expressed in terms of sym tensors C_{IJK} through associated cubic polynomial

$$N(\xi) = \left(\frac{2}{3}\right)^{3/2} C_{IJK}\xi^I\xi^J\xi^k \quad \leftarrow \quad \text{Ambient space coordinates } \xi^I$$

$$\mathring{a}_{IJ} = -\frac{1}{2}\partial_I\partial_J \ln N(\xi)\Big|_{N(\xi)=1} \qquad g_{xy} = \mathring{a}_{IJ}\partial_x\xi^I\partial_y\xi^J$$

$\Rightarrow$ Theory is uniquely specified by cubic interactions $\Leftarrow$

$\mathcal{N}=2$ theories are more difficult to chart than their $\mathcal{N}\geq 4$ relatives. Matter content alone no longer specifies theory.

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[Günaydin, Sierra, Townsend 1984]

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$\mathcal{N} = 2$ very special sugras in $4D$ – a panoramic view

symmetric theories:

- Generic Jordan family, $\frac{SO(n,2)}{SO(n)\times U(1)} \times \frac{SU(1,1)}{U(1)}$
- Magical theories, $L(q,1), \quad q = 1, 2, 4, 8$

homogeneous theories:

- $L(q,p)$ target spaces $q = -1, 0, 1, \ldots$ $p = 0, 1, \ldots$

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→

- Yang-Mills-Einstein sugras (gauged isometry group)
  ⋆ compact
  ⋆ non-compact
  always Minkowski vacuum
- Gauged supergravities (gauged R-symmetry) Minkowski or AdS vacuum

↓

- Spontaneously-broken supergravities

Generic Jordan family:

[Günaydin, Sierra, Townsend, 1984]

$$N(\xi) = \sqrt{2}\big(\xi^0(\xi^1)^2 - \xi^0(\xi^i)^2\big)\,, \qquad i = 2, 3, \ldots, n_V$$

Homogeneous supergravities:

[de Wit and van Proeyen, 1991]

$$N(\xi) = \sqrt{2}\big(\xi^0(\xi^1)^2 - \xi^0(\xi^i)^2\big) + \xi^1(\xi^\alpha)^2 + \tilde{\Gamma}^i_{\alpha\beta}\xi^i\xi^\alpha\xi^\beta\,,$$

$i = 2, 3, \ldots q + 2$; α, β indices with range $r = \mathcal{D}_q(P + \dot{P}) \Rightarrow n_V = 2 + q + r$

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$\tilde{\Gamma}^i$ form a representation of the Clifford algebra $\mathcal{C}(q+1, 0)$

q	$\mathcal{D}_q$	metric-preserving group $S_q(P, \dot{P})$
-1	1	$SO(P)$
0	1	$SO(P)\times SO(\dot{P})$
1	2	$SO(P)$
2	4	$U(P)$
3	8	$USp(2P)$
4	8	$USp(2P)\times USp(2\dot{P})$
5	16	$USp(2P)$
6	16	$U(P)$
$k+8$	as for k	as for k

General philosophy: introduce gauge-group representations other than adjoint

[MC, Jin and Roiban 2013; Johansson and Ochirov 2014]

Divide gauge-theory spectrum as $\mathcal{V}_G \oplus V_{R_1} \oplus V_{R_2} \oplus \ldots$

		GT1: $\mathcal{V}_G$	GT1: $V_{\bar{R}_1}$	GT1: $V_{\bar{R}_1}$
GT2	$\mathcal{V}_G$	$\mathcal{V}_G \otimes \mathcal{V}_G$	X	X
GT2	V_{R_1}	X	$V_{R_1} \otimes V_{\bar{R}_1}$	X
GT2	V_{R_2}	X	X	$V_{R_2} \otimes V_{\bar{R}_2}$

$\Rightarrow$ Sugra states associated to gauge-invariant bilinears of gauge-theory states

Color factors built out of invariant symbols, e.g. $f^{\hat{a}\hat{b}\hat{c}}$, $T^{\hat{a}}_R$. Color relations:

$$\left.\begin{array}{c} f^{\hat{a}\hat{b}\hat{e}} f^{\hat{c}\hat{d}\hat{e}} - f^{\hat{a}\hat{c}\hat{e}} f^{\hat{b}\hat{d}\hat{e}} + f^{\hat{a}\hat{d}\hat{e}} f^{\hat{b}\hat{c}\hat{e}} = 0 \\ [T^{\hat{a}}_R, T^{\hat{b}}_R] - i f^{\hat{a}\hat{b}\hat{c}} T^{\hat{c}}_R = 0 \end{array}\right\} \quad \Rightarrow \quad c_s + c_t + c_u = 0$$

Need to impose corresponding numerator relations.

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DC construction 1: **generic Jordan family**

$$\mathcal{M}_4 = \frac{SO(n_v,2)}{SO(n_v)\times SO(2)} \times \frac{SU(1,1)}{U(1)}$$

- $\mathcal{N}=2$ sYM with no additional matter multiplet

- $\mathcal{N}=0$ YM-scalar theory

$$\mathcal{L} = -\frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu} + \frac{1}{2}(D_\mu\phi^a)^{\hat{a}}(D^\mu\phi^a)^{\hat{a}} - \frac{g^2}{4}f^{\hat{a}\hat{b}\hat{c}}f^{\hat{a}\hat{b}'\hat{c}'}\phi^{\hat{b}b}\phi^{\hat{c}c}\phi^{\hat{b}'b}\phi^{\hat{c}'c}$$
$$+\frac{g\lambda}{3!}f^{\hat{a}\hat{b}\hat{c}}F^{abc}\phi^{\hat{a}a}\phi^{\hat{b}b}\phi^{\hat{c}c} \qquad a,b=1,\ldots,n_V$$

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Dimensional reduction of $(4+n_v)$-dimensional pure YM theory. Explicit $SO(n_V)$ isometry.

[MC, Günaydin, Johansson and Roiban, 2014]

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c/k duality forces the F^{abc} appearing
in cubic couplings to obey Jacobi relations

[MC, Günaydin, Johansson and Roiban, 2014]

$$\text{DC state map:} \quad \begin{cases} \text{spin 2 :} & h_{--} = A_-|_{\mathcal{N}=0} \otimes A_-|_{\mathcal{N}=2} \\ \text{spin 1 :} & A_-^{-1} = A_-|_{\mathcal{N}=0} \otimes \phi|_{\mathcal{N}=2} \\ & A_-^{0} = A_-|_{\mathcal{N}=0} \otimes \bar{\phi}|_{\mathcal{N}=2} \\ & A_-^{a} = \phi^a|_{\mathcal{N}=0} \otimes A_-|_{\mathcal{N}=2} \\ \text{spin 0 :} & i\bar{z}^a = \phi^a|_{\mathcal{N}=0} \otimes \bar{\phi}|_{\mathcal{N}=2} \\ & i\bar{z}^0 = A_-|_{\mathcal{N}=0} \otimes A_+|_{\mathcal{N}=2} \end{cases}$$

- Nonzero supergravity amplitude:

$$\mathcal{M}_3^{(0)}\left(1A_-^a, 2A_-^b, 3A_+^c\right) = -\left(\frac{\kappa}{2}\right)\frac{\lambda}{\sqrt{2}}\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}F^{abc}$$

The resulting supergravity is of the Yang-Mills-Einstein class!

global sym in $\mathcal{N}=0$ gauge-theory factor $\Rightarrow$ local sym in resulting sugra

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DC Construction 2: **Homogeneous supergravities**

[MC, Günaydin, Johansson and Roiban 2015]

Gauge theory 1: $\mathcal{N} = 2$ sYM with half-hypermultiplet in representation R

half-hyper matter content: $(\chi_+ \, , \, \varphi_1 \, , \, \varphi_2 \, , \, \chi_-)$

not necessary to add CPT-conjugate states if representation is pseudo-real

$$(\exists\, V \text{ s.t. } V T_R^{\hat{a}} V^\dagger = -(T_R^{\hat{a}})^* \ , \ V\, V^* = -1)$$

Gauge theory 2: YM-scalar theory with fermions in representation R

$$\mathcal{L} = -\frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu} + \frac{1}{2}(D_\mu\phi^a)^{\hat{a}}(D^\mu\phi^a)^{\hat{a}} + \frac{i}{2}\overline{\lambda}^\alpha D_\mu\gamma^\mu\lambda_\alpha$$
$$+\frac{g}{2}\phi^{a\hat{a}}\Gamma^{a}_{\alpha}{}^{\beta}\overline{\lambda}^\alpha\gamma_5 T^{\hat{a}}\lambda_\beta - \frac{g^2}{4}f^{\hat{a}\hat{b}\hat{e}}f^{\hat{c}\hat{d}\hat{e}}\phi^{a\hat{a}}\phi^{b\hat{b}}\phi^{a\hat{c}}\phi^{b\hat{d}}, \qquad a,b = 1,\dots q+2$$

Constraint from c/k duality:

$$n_s + n_t + n_u = 0 \quad \rightarrow \quad \{\Gamma^a, \Gamma^b\} = 2\delta^{ab}$$

Can be seen as $(q+6)$-dimensional YM theory with P copies (flavors) of the irreducible fermions

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R: $\overline{\lambda} = \lambda^t \mathcal{C}_q \mathcal{C}_4 V$

q	$\mathcal{D}_q$	4D fermions	conditions	flavor group G_f
-1	1	P	R	$SO(P)$
0	1	$P+\dot{P}$	RW	$SO(P)\times SO(\dot{P})$
1	2	$2P$	R	$SO(P)$
2	4	$4P$	R/W	$U(P)$
3	8	$8P$	PR	$USp(2P)$
4	8	$8P+8\dot{P}$	PRW	$USp(2P)\times USp(2\dot{P})$
5	16	$16P$	PR	$USp(2P)$
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$k+8$	$16\,\mathcal{D}_k$	$16\,r(k,P)$	as for k	as for k

Gauge theory 2: YM-scalar theory with fermions in representation R

$$\mathcal{L} = -\frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu} + \frac{1}{2}(D_\mu\phi^a)^{\hat{a}}(D^\mu\phi^a)^{\hat{a}} + \frac{i}{2}\overline{\lambda}^{\alpha}D_\mu\gamma^\mu\lambda_\alpha$$
$$+\frac{g}{2}\phi^{a\hat{a}}\Gamma^{a}_{\alpha}{}^{\beta}\overline{\lambda}^{\alpha}\gamma_5 T^{\hat{a}}\lambda_\beta - \frac{g^2}{4}f^{\hat{a}\hat{b}\hat{e}}f^{\hat{c}\hat{d}\hat{e}}\phi^{a\hat{a}}\phi^{b\hat{b}}\phi^{a\hat{c}}\phi^{b\hat{d}}\,, \qquad a,b = 1,\ldots q+2$$

PR: $\overline{\lambda} = \lambda^t \mathcal{C}_q \mathcal{C}_4 \Omega V$

q	$\mathcal{D}_q$	$4D$ fermions	conditions	flavor group G_f
-1	1	P	R	$SO(P)$
0	1	$P+\dot{P}$	RW	$SO(P)\times SO(\dot{P})$
1	2	$2P$	R	$SO(P)$
2	4	$4P$	R/W	$U(P)$
3	8	$8P$	PR	$USp(2P)$
4	8	$8P+8\dot{P}$	PRW	$USp(2P)\times USp(2\dot{P})$
5	16	$16P$	PR	$USp(2P)$
6	16	$16P$	R/W	$U(P)$
$k+8$	$16\,\mathcal{D}_k$	$16\,r(k,P)$	as for k	as for k

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-1	1	P	R	$SO(P)$
0	[illegible]	[illegible]	[illegible]	[illegible]
1				
2				
3				
4				
5				
6				
$k+$				

Supergravity states realized as double-copies:

$$A^{-1}_{-} = \bar{\phi}\big|_{\mathcal{N}=2}\otimes A_{-}\big|_{\mathcal{N}=0} \qquad h_{-} = A_{-}\big|_{\mathcal{N}=2}\otimes A_{-}\big|_{\mathcal{N}=0}$$

$$A^{0}_{-} = \phi\big|_{\mathcal{N}=2}\otimes A_{-}\big|_{\mathcal{N}=0} \qquad i\bar{z}^{0} = A_{+}\big|_{\mathcal{N}=2}\otimes A_{-}\big|_{\mathcal{N}=0}$$

$$A^{a}_{-} = A_{-}\big|_{\mathcal{N}=2}\otimes \phi^a\big|_{\mathcal{N}=0} \qquad i\bar{z}^{a} = \bar{\phi}\big|_{\mathcal{N}=2}\otimes \phi^a\big|_{\mathcal{N}=0}$$

$$A_{\alpha-} = \chi_{-}\big|_{\mathcal{N}=2}\otimes (U\lambda_{-})_\alpha\big|_{\mathcal{N}=0} \qquad i\bar{z}_{\alpha} = \chi_{+}\big|_{\mathcal{N}=2}\otimes (U\lambda_{-})_\alpha\big|_{\mathcal{N}=0}$$

[MC, Günaydin, Johansson and Roiban 2015]

DC construction 3: **Higgsed supergravities**

- Consider gauge theories with a different matter representation for each mass in the spectrum $\Rightarrow \mathcal{V}_G \oplus V_{m_1} \oplus V_{m_2} \oplus \dots$
- Take spontaneously-broken sYM theory as first gauge-theory factor

$$A_\mu^{\hat{A}} = \left(A_\mu^{\hat{a}}, W_{\mu\hat{\alpha}}, \overline{W}_\mu^{\hat{\alpha}}\right) , \qquad \phi^{\hat{A}i} = \left(\phi^{\hat{a}i}, \varphi^i{}_{\hat{\alpha}}, \overline{\varphi}^{\hat{\alpha}i}\right)$$

- Extra color identities satisfied by representation matrices and Clebsh-Gordan coefficients;
- Second gauge theory: YM-scalar theory with extra massive scalars and trilinear interactions; impose corresponding numerator identities.

$$\begin{aligned} A_-^{-1} &= \bar{\phi}\big|_{\mathcal{N}=2} \otimes A_-\big|_{\mathcal{N}=0} & h_- &= A_-\big|_{\mathcal{N}=2} \otimes A_-\big|_{\mathcal{N}=0} \\ A_-^0 &= \phi\big|_{\mathcal{N}=2} \otimes A_-\big|_{\mathcal{N}=0} & i\bar{z}^0 &= A_+\big|_{\mathcal{N}=2} \otimes A_-\big|_{\mathcal{N}=0} \\ A_-^a &= A_-\big|_{\mathcal{N}=2} \otimes \phi^a\big|_{\mathcal{N}=0} & i\bar{z}^a &= \bar{\phi}\big|_{\mathcal{N}=2} \otimes \phi^a\big|_{\mathcal{N}=0} \\ \varphi_\alpha &= \varphi\big|_{\mathcal{N}=2} \otimes \varphi_\alpha\big|_{\mathcal{N}=0} & W_\alpha &= W\big|_{\mathcal{N}=2} \otimes \varphi_\alpha\big|_{\mathcal{N}=0} \end{aligned}$$

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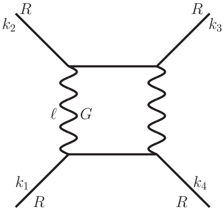
[MC, Günaydin, Johansson and Roiban 2015]

Quick peek at one loop for homogeneous supergravities

- Consider matter amplitudes with external vectors

$$\mathcal{M}_4^{1\ell}(A_+^{\alpha}, A_-^{\beta}, A_-^{\gamma}, A_+^{\delta}) = \mathcal{A}_4^{1\ell}(\chi_+^{\hat\alpha}, \chi_-^{\hat\beta}, \chi_-^{\hat\gamma}, \chi_+^{\hat\delta}) \otimes \mathcal{A}_4^{1\ell}(\lambda_+^{\alpha\hat\alpha}, \lambda_-^{\beta\hat\beta}, \lambda_-^{\gamma\hat\gamma}, \lambda_+^{\delta\hat\delta})$$

- Master graph in BCJ representation for $\mathcal{N} = 2$ gauge theory



$$\begin{cases} C_1 = (T^{\hat a} T^{\hat b})_{\hat\alpha\delta} (T^{\hat a} T^{\hat b})_{\hat\beta\hat\gamma} \\ n_1 = \dfrac{s^2}{\langle 12\rangle\langle 34\rangle}\delta^{(4)}\left(\sum_i \eta_i^a |i\rangle\right) \end{cases}$$

Other numerators obtained using permutation symmetry and dual Jacobi relations; no dependence from loop momenta

Vertex and propagator corrections in the $\mathcal{N}=0$ gauge theory are related to gravity divergence:

$$\mathcal{M}^{1\ell}\Big|_{\text{div}} = 2is\Big\{tA^{\text{tree}}_{t,\phi}\Big(\tilde{\mathcal{V}}^{1-\text{loop}}_{\phi\lambda\bar{\lambda}}\Big|_{T(G),\text{div}} - \frac{1}{2}\tilde{\Pi}^{1-\text{loop}}_{\phi}\Big|_{T(G),\text{div}} + \frac{1}{4}\tilde{\Pi}^{1-\text{loop}}_{\phi}\Big|_{T(R),\text{div}}\Big) + (uA^{\text{tree}}_{u,A} + sA^{\text{tree}}_{s,A})\Big(\tilde{\mathcal{V}}^{1-\text{loop}}_{A\lambda\bar{\lambda}}\Big|_{T(G),\text{div}} - \frac{1}{2}\tilde{\Pi}^{1-\text{loop}}_{A}\Big|_{T(G),\text{div}} + \frac{1}{4}\tilde{\Pi}^{1-\text{loop}}_{A}\Big|_{T(R),\text{div}}\Big)\Big\}$$

with $A^{\text{tree}} = A^{\text{tree}}_{s,A} + A^{\text{tree}}_{t,\phi} + A^{\text{tree}}_{u,A}$

Consider amplitude with two identical vectors and the CPT-conjugate states:

$$\mathcal{M}^{1\ell}_4(1A^{\alpha}_{+}, 2A_{-\alpha}, 3A_{-\alpha}, 4A^{\alpha}_{+})\Big|_{\text{div}} = \frac{2i}{(4\pi)^2}\Big(\frac{\kappa}{2}\Big)^4\frac{1}{\epsilon}\Big(\frac{11}{3} - \frac{n_s}{6} + \frac{n_f}{3}\Big)\langle 23\rangle^2[14]^2$$

[MC, Günaydin, Johansson and Roiban, work in progress]

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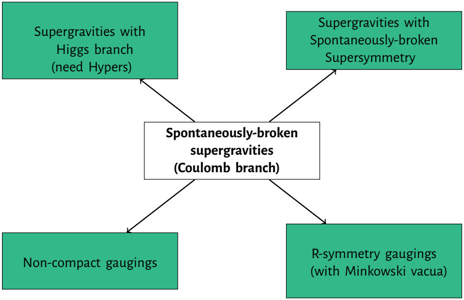
Summary of $(\mathcal{N}=2)\otimes(\mathcal{N}=0)$ double copies

[MC, Günaydin, Johansson and Roiban, 2014-16]

Gauge theories	Double copy
GT1: Pure $\mathcal{N}=2$ sYM theory **GT2:** YM+scalar theory from dim. red.	Generic Jordan Family of Maxwell-Einstein Sugras
GT1: As before **GT2**: Add trilinear scalar couplings	Compact isometry gaugings Yang-Mills-Einstein theories
GT1: Spontaneous gauge sym breaking **GT2:** Add massive scalars	Spontaneously-broken Yang-Mills-Einstein theories
GT1: Add Hypers in representation R **GT2**: Add fermions in representation R with Yukawa couplings	Homogeneous supergravities, Magical supergravities
GT1: Add Hypers in representation R **GT2**: Add scalars in representation R	Supergravities with matter hypermultiplets

What next?

⋆ Double-copy extensions so far give key stepping stones to capture more general supergravities and suggest that the construction has a fundamental role to play in generic theories with reduced supersymmetry



⋆ Possible to replace $\mathcal{N}=2$ gauge theory factor with either $\mathcal{N}=4$ or non supersymmetric theory

⋆ Also interesting $(\mathcal{N}=1)\otimes(\mathcal{N}=1)$ double copies [Nagy 2014; Schreiber 2016]

⋆ Computation at one loop finds hints of simple structure. Calculations at two loops and/or higher point may be feasible

Thank You!

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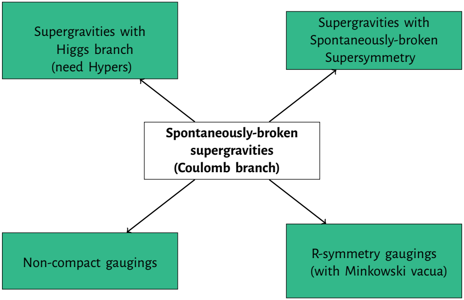

⋆ Possible to replace $\mathcal{N} = 2$ gauge theory factor with either $\mathcal{N} = 4$ or non supersymmetric theory

⋆ Also interesting $(\mathcal{N} = 1) \otimes (\mathcal{N} = 1)$ double copies [Nagy 2014; Schreiber 2016]

⋆ Computation at one loop finds hints of simple structure. Calculations at two loops and/or higher point may be feasible

Thank You!

Extra Slides

$\mathcal{N}=2$ truncations of $\mathcal{N}=8$ supergravity

Factorized theories			
$\mathcal{N}$	**Factors**	**Scalar Manifold**	**Supergravity**
2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_2}$	$\frac{SO(6,2)\times SU(1,1)}{SO(6)\times SO(2)\times U(1)}$	$\mathcal{N}=2$ sugra, 7 vectors
2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_4}$	$\frac{SO(4,2)\times SU(1,1)}{SO(4)\times SO(2)\times U(1)}$	$\mathcal{N}=2$ sugra, 5 vectors
2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_2\times\mathbf{Z}_2}$	$\frac{SU(1,1)\times SU(1,1)\times SU(1,1)}{U(1)\times U(1)\times U(1)}$	$\mathcal{N}=2$ sugra, 3 vectors
2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_4}$	$\frac{SU(1,1)}{U(1)}$	$\mathcal{N}=2$ sugra, 1 vectors
2	$\mathcal{N}=1\vert_{\mathbf{Z}_3}\times\mathcal{N}=1\vert_{\mathbf{Z}_3}$	$\frac{SU(2,1)}{SU(2)\times U(1)}$	$\mathcal{N}=2$ sugra, 1 hyper

Non-factorized theories			
$\mathcal{N}$	**Orbifold**	**Scalar Manifold**	**Matter**
2	$\mathbf{Z}_2$	$\frac{SO^*(12)}{U(6)}$	15 vectors
2	$\mathbf{Z}_3$	$\frac{E_{6(2)}}{SU(6)\times SU(2)}$	10 hypers
2	$\mathbf{Z}_4$	$\frac{SU(3,3)}{S(U(3)\times U(3))}$	9 vectors
2	$\mathbf{Z}_3$	$\frac{SO(8,2)\times SU(1,1)}{SO(8)\times U(1)\times U(1)}\times\frac{SU(2,1)}{U(2)}$	9 vecs, 1 hyper
2	$\mathbf{Z}_6$		pure
2	$\mathbf{Z}_6$	$\frac{SO(4,4)}{SO(4)\times SO(4)}$	4 hypers
2	$\mathbf{Z}_6$	$\frac{SO(3,2)\times SU(1,1)}{SO(3)\times U(1)\times U(1)}$	4 vectors
2	$\mathbf{Z}_6$	$\frac{SO(6,4)}{SO(6)\times SO(4)}$	6 hypers
...	...	...	...

$\mathcal{N}=2$ truncations of $\mathcal{N}=8$ supergravity

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2	$\mathcal{N}=2\vert_{\mathbb{Z}_2}\times\mathcal{N}=0\vert_{\mathbb{Z}_2\times\mathbb{Z}_2}$	$\frac{SU(1,1)\times SU(1,1)\times SU(1,1)}{U(1)\times U(1)\times U(1)}$	$\mathcal{N}=2$ sugra, 3 vectors
2	$\mathcal{N}=2\vert_{\mathbb{Z}_2}\times\mathcal{N}=0\vert_{\mathbb{Z}_4}$	$\frac{SU(1,1)}{U(1)}$	$\mathcal{N}=2$ sugra, 1 vectors
2	$\mathcal{N}=1\vert_{\mathbb{Z}_3}\times\mathcal{N}=1\vert_{\mathbb{Z}_3}$	$\frac{SU(2,1)}{SU(2)\times U(1)}$	$\mathcal{N}=2$ sugra, 1 hyper

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2	$\mathbb{Z}_6$		
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2	$\mathbb{Z}_6$	$\frac{SO(6,4)}{SO(6)\times SO(4)}$	
...	...	...	...

- scalar manifolds are symmetric spaces; family with target spaces $\frac{SO(n,2)}{SO(n)\times U(1)}\times\frac{SU(1,1)}{U(1)}$;
- Interesting amplitudes with matter multiplets as external states; different couplings possible.

[Carrasco, MC, Günaydin, and Roiban 2012]

[Damgaard,Huang, Sondergaard, Zhang, 2012]

Amplitudes from double copy:

$$\mathcal{M}_3(1^{\mathcal{H}_-},2^{\mathcal{H}_-},3^{\mathcal{H}_+}) = i\frac{\kappa}{2}\frac{\langle 12\rangle^2}{\langle 23\rangle^2\langle 31\rangle^2}\,\mathcal{Q}_3^{34}\,\delta^{(4)}\Big(\sum\eta_i^\alpha|i\rangle\Big)$$

$$\mathcal{M}_3(1^{\mathcal{V}^a},2^{\mathcal{V}^b},3^{\mathcal{V}^c}) = i\frac{\kappa}{2\sqrt{2}}\frac{\lambda F^{abc}}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\,\mathcal{Q}_3^{34}\,\delta^{(4)}\Big(\sum\eta_i^\alpha|i\rangle\Big)$$

$$\mathcal{M}_3(1^{\mathcal{V}^a},2^{\mathcal{V}^b},3^{\mathcal{H}_-}) = i\frac{\kappa}{2}\frac{\delta^{ab}}{\langle 12\rangle^2}\,\mathcal{Q}_3^{34}\,\delta^{(4)}\Big(\sum\eta_i^\alpha|i\rangle\Big)$$

use spinor-helicity variables, $k_{(i)\mu}\sigma^\mu_{a\dot a} = |i]_a\langle i|_{\dot a}$

$$\delta^{(4)}\Big(\sum\eta_i^\alpha|i\rangle\Big) = \prod_{\alpha=1,2}\sum_{i<j}\langle ij\rangle\eta_i^\alpha\eta_j^\alpha \qquad \mathcal{Q}_3^{34} = \sum_{i<j}\langle ij\rangle^2(\eta_i^3\eta_i^4)(\eta_j^3\eta_j^4)$$

$\mathcal{N}=4$ constrained on-shell superfields ($\mathcal{N}=2$ m-plets packed together)

$$\mathcal{H}_+ = h_{++} + \eta_\alpha\psi_+^\alpha + \eta^1\eta^2A_+^{-1} + \eta^3\eta^4A^0{}_+ + \eta^3\eta^4\eta_\alpha\tilde{\zeta}_+^\alpha - i\eta^1\eta^2\eta^3\eta^4z^0$$

$$\mathcal{V}^a = A_+^a + \eta_\alpha\zeta_+^{a\alpha} - i\eta^1\eta^2z^a + i\eta^3\eta^4\bar{z}^a + \eta^3\eta^4\eta_\alpha\zeta_-^{a\alpha} + \eta^1\eta^2\eta^3\eta^4A_-^a$$