On the Regge limit of polygonal WLs

Benjamin Basso
ENS Paris

Amplitudes 2016
Nordita

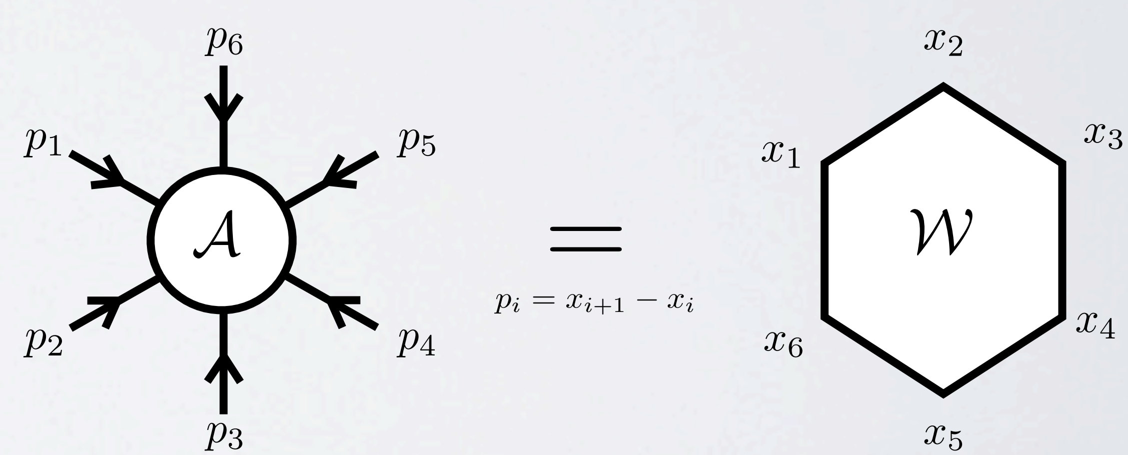
work in progress with Simon Caron-Huot and Amit Sever

Two important corners

Collinear limit (WL side)

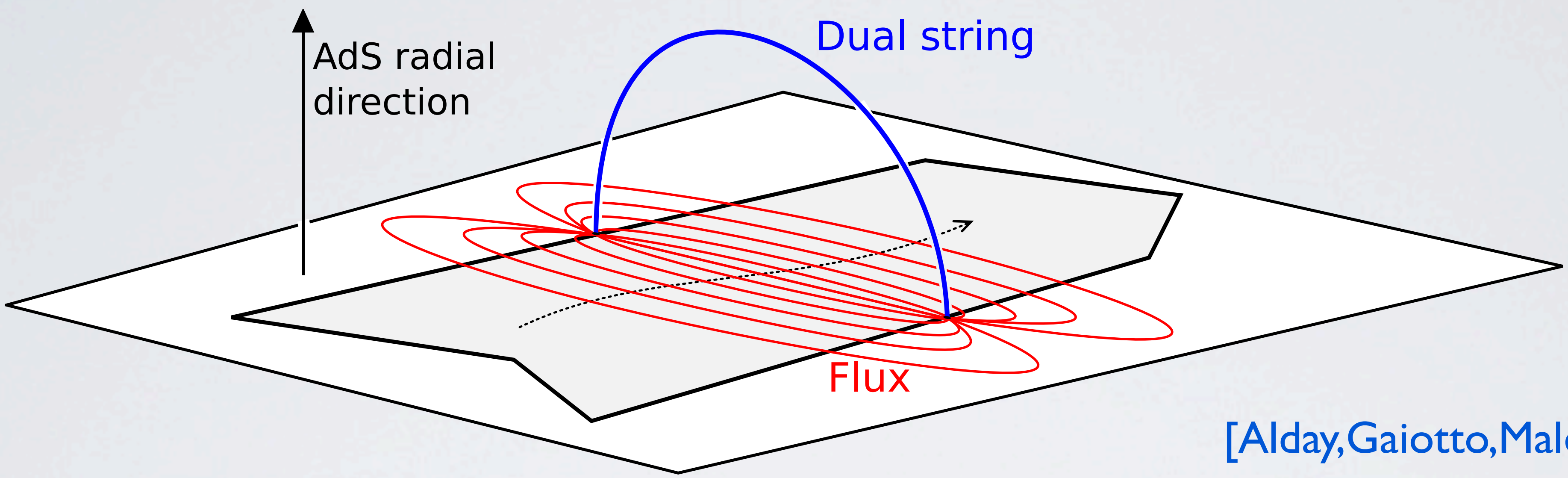
Regge limit (SA side)

Rich interplay



[Alday,Maldacena'07]
[Drummond,Korchemsky,Sokatchev'07]
[Brandhuber,Heslop,Travaglini'07]
[Drummond,Henn,Korchemsky,Sokatchev'07]

Collinear / OPE regime



[Alday,Gaiotto,Maldacena,Sever,Vieira'09]

1+1d background : flux tube sourced by two parallel null lines
bottom&top cusps excite the flux tube out of its ground state

→ *Sum over all flux-tube eigenstates*

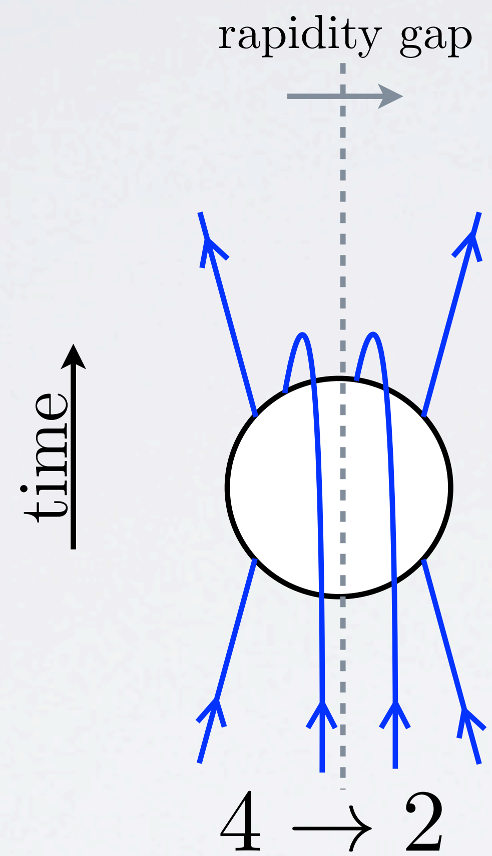
$$\mathcal{W} = \sum_{\text{states } \psi} C_{\text{bot}}(\psi) \times e^{-E(\psi)\tau + ip(\psi)\sigma + im(\psi)\phi} \times C_{\text{top}}(\psi)$$

Regge / BFKL regime

- High energy scattering

- Become very interesting in Mandelstam regions

[Bartels,Lipatov,Sabio Vera'08]
[Bartels,Lipatov,Prygarin'10]



- Picture in terms of Reggeons (pole and cuts) s^{ω}

- Energy dependence governed BFKL eigenvalues $\omega = \omega(m, \nu)$

OPE versus BFKL

OPE $$\mathcal{W}_{\text{hex}} = 1 + \sum_{m\neq 0} (-1)^m e^{im\phi} \int_{-\infty}^{+\infty} \frac{dp}{2\pi}\, \hat{\mu}_m(p)\, e^{ip\sigma - \tau E_m(p)} + \ldots$$

leading twist dominates at large τ

BFKL $$\mathcal{W}_{\text{hex}}^{\circlearrowleft} e^{-i\pi\delta'} = \sum_{m=-\infty}^{\infty} (-1)^m e^{im\phi} \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi}\, \hat{\mu}_{\text{BFKL}}(\nu, m)\, e^{i(\sigma-\tau)\nu + (\sigma+\tau)\omega(\nu,m)} + \ldots$$

leading spin dominates at large $\tau + \sigma$

Both are valid at any coupling

Two different pictures, two different expansions, two different kinematics

Leading term in one expansion $\Longleftrightarrow$ Resummation of **infinitely many** terms in the other

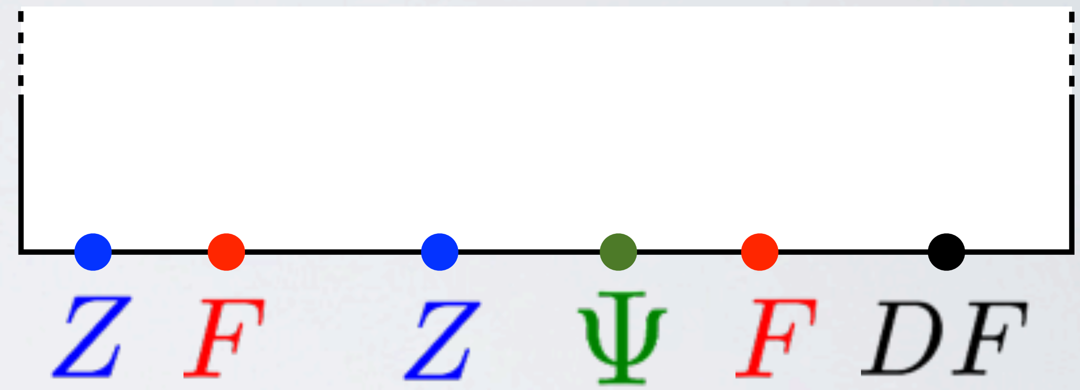
Strikingly similar. Why? How far does the parallelism go?

One side (OPE) much more well understood / developed

Flux-tube spectrum

Well understood....

as field insertions along a light-ray:
create/annihilate state on the flux tube



or discretized version of light-ray:
bath of covariant derivatives

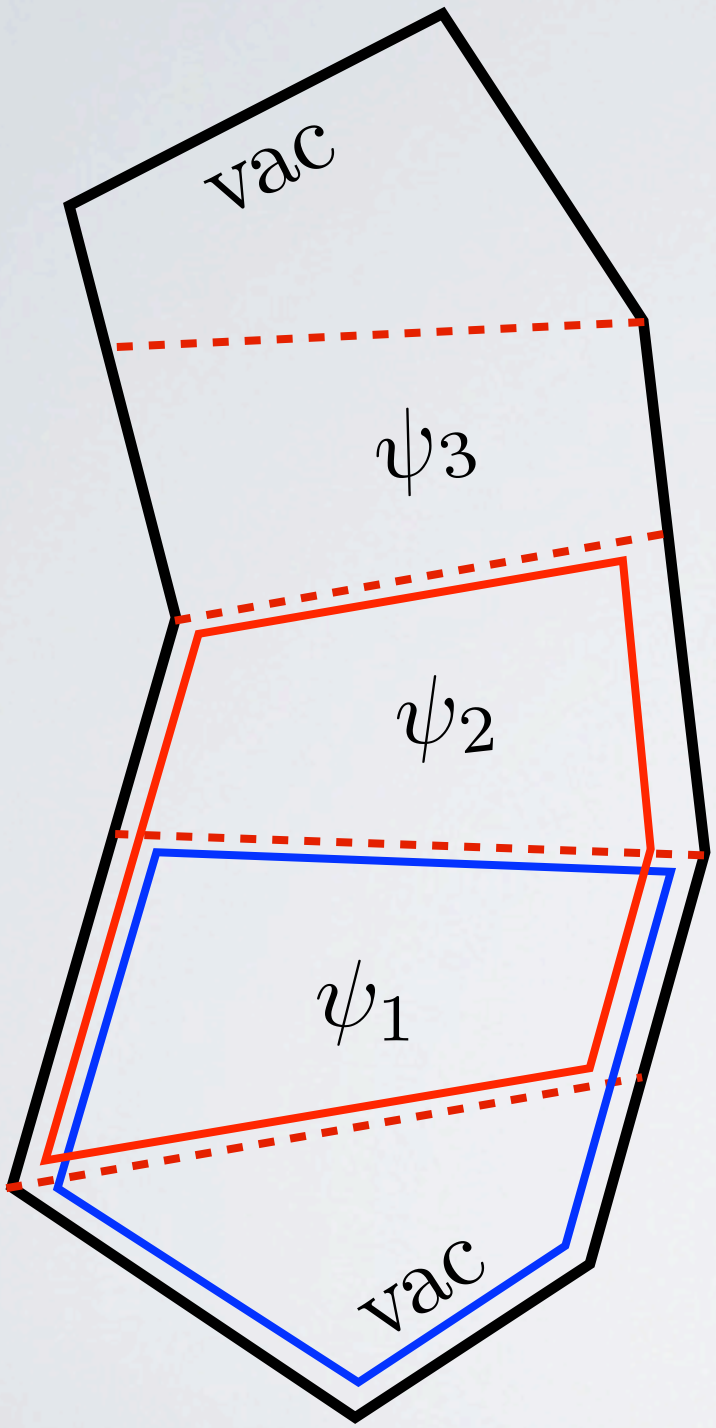
$$\mathcal{O} = \mathrm{tr}\,(Z\,DDDD\ldots DDDD\,\overset{\xrightarrow{p_1}}{F}\,DDDD\ldots DDDD\,\overset{\xrightarrow{p_2}}{F}\,DDDD\ldots DDDD\,Z)$$

flux tube states $\longleftrightarrow$ large spin operators

Both pictures support integrable structures; well described at all loops in the spin chain approach

Can get dispersion relations, flux tube S-matrix, etc. from that $E = E(p)$

Pentagon OPE



Much is known about OPE at any coupling thanks to integrability

[BB,Sever,Vieira'13]

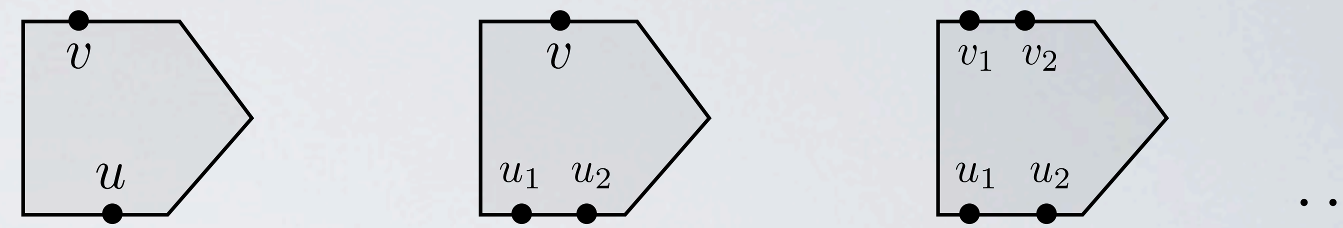
$$= \sum_{\psi_i} \left[\prod_i e^{-E_i \tau_i + i p_i \sigma_i + i m_i \phi_i} \right] \times$$

$$P(0|\psi_1) P(\psi_1|\psi_2) P(\psi_2|\psi_3) P(\psi_3|0)$$

- Flux-tube states ψ
- Pentagon transitions $P(\psi_1|\psi_2)$

Pentagon OPE

All transitions are known



$P(u|v)$ $\quad$ $P(u_1, u_2|v)$ $\quad$ $P(u_1, u_2|v_1, v_2)$

Main ingredients are the elementary transitions :
multi-particle transitions are believed to factorize

[BB,Sever,Vieira'13]
[Belitsky,Derkachov,Manashov'13]
[Belitsky'15]

$$P(\{u_i\}|\{v_i\}) = \frac{\prod_{i,j} P(u_i|v_j)}{\prod_{i>j} P(u_i|u_j) \prod_{i<j} P(v_i|v_j)}$$

All pentagon transitions

$$P_{A|B}(u|v)^2 = \mathcal{F}_{A|B}(u|v) \times \frac{S_{AB}(u,v)}{S_{AB}(u^\gamma,v)}$$

ϕ : scalar

ψ : fermion

F : gluon

[BB,Sever,Vieira'13'14]
[BB,Caetano,Cordova,Sever,Vieira'15]
[Belitsky'14'15]

$$\mathcal{F}_{\phi F}(u|v) = 1\,,$$

$$\mathcal{F}_{\phi\psi}(u|v) = -\frac{1}{(u-v+\frac{i}{2})}\,,$$

$$\mathcal{F}_{\phi\phi}(u|v) = \frac{1}{(u-v)(u-v+i)}\,,$$

$$\mathcal{F}_{FF}(u|v) = \frac{(x^+y^+-g^2)(x^+y^--g^2)(x^-y^+-g^2)(x^-y^--g^2)}{g^2x^+x^-y^+y^-(u-v)(u-v+i)}\,,$$

$$\mathcal{F}_{F\psi}(u|v) = -\frac{(x^+y-g^2)(x^-y-g^2)}{g\sqrt{x^+x^-}y(u-v+\frac{i}{2})}\,,$$

$$\mathcal{F}_{F\bar{\psi}}(u|v) = -\frac{g\sqrt{x^+x^-}y(u-v+\frac{i}{2})}{(x^+y-g^2)(x^-y-g^2)}\,,$$

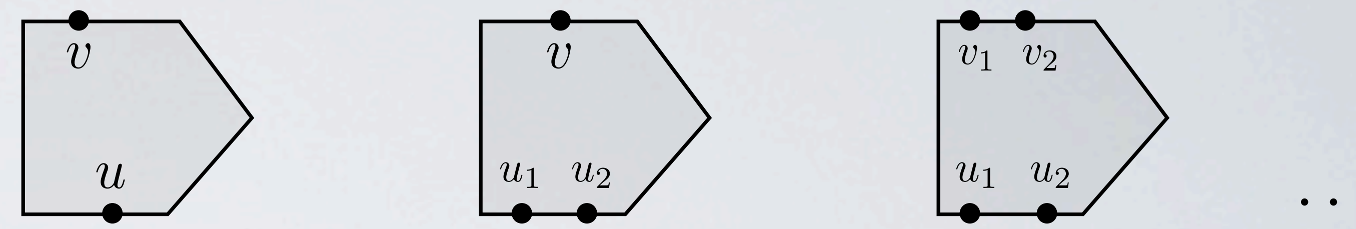
$$\mathcal{F}_{F\bar{F}}(u|v) = \frac{g^2x^+x^-y^+y^-(u-v)(u-v+i)}{(x^+y^+-g^2)(x^+y^--g^2)(x^-y^+-g^2)(x^-y^--g^2)}\,,$$

$$\mathcal{F}_{\psi\psi}(u|v) = -\frac{(xy-g^2)}{\sqrt{gxy}(u-v)(u-v+i)}\,,$$

$$\mathcal{F}_{\psi\bar{\psi}}(u|v) = -\frac{\sqrt{gxy}}{(xy-g^2)}\,,$$

Pentagon OPE

All transitions are known



$P(u|v)$ $\quad$ $P(u_1, u_2|v)$ $\quad$ $P(u_1, u_2|v_1, v_2)$

Main ingredients are the elementary transitions :
multi-particle transitions are believed to factorize

Simple rules generalizating all this to non-MHV amplitudes

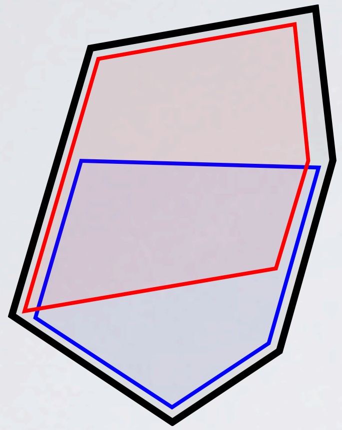
[BB,Caetano,Cordova,Sever,Vieira'15]
[Belitsky'14'15]
[BB,Coronado,Sever,Vieira' to appear]

Systematic expansion around collinear limit (euclidean)

Full 6-gluon amplitude

[BB, Sever, Vieira'15]

OPE series :

$\mathcal{W}_{\text{hex}} =$  $= \sum_n \frac{1}{S_n} \int \frac{du_1 \ldots du_n}{(2\pi)^n} \Pi(\{u_i\})$

Flux tube integrand :

(everything here is known at any coupling)

$$\Pi(\{u_i\}) = \Pi_{\text{dyn}} \times \Pi_{\text{mat}}$$

$$\Pi_{\text{dyn}} = \prod_i \mu(u_i)\, e^{-E(u_i)\tau + ip(u_i)\sigma + im_i\phi} \prod_{i<j} \frac{1}{|P(u_i|u_j)|^2}$$

What about MRK limit?

What are the main ingredients?

Can we find their expression at finite coupling?

Is there a systematic expansion in that regime?

Today : see how much we can learn / guess about all that starting / using OPE / pentagons

Plan

Crossing the kinematics from OPE to Regge
Review of hexagon

Application to heptagon
Regge pentagons

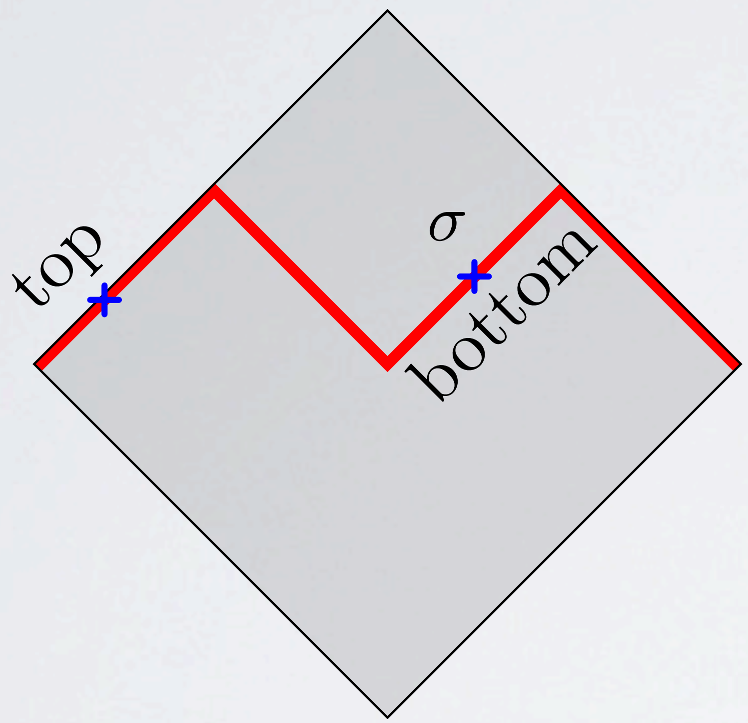
Conjecture for higher polygons

The route to the Regge limit

Passing to the real kinematics

Hexagon in collinear limit

$$\mathcal{W}_{\text{hex}} = 1 + \sum_m e^{im\phi} \int \frac{du}{2\pi} \mu_m(u) e^{ip_m(u)\sigma - E_m(u)\tau}$$



- Leading twist approximation : bottom/top cusps are replaced by insertions of field strength tensor

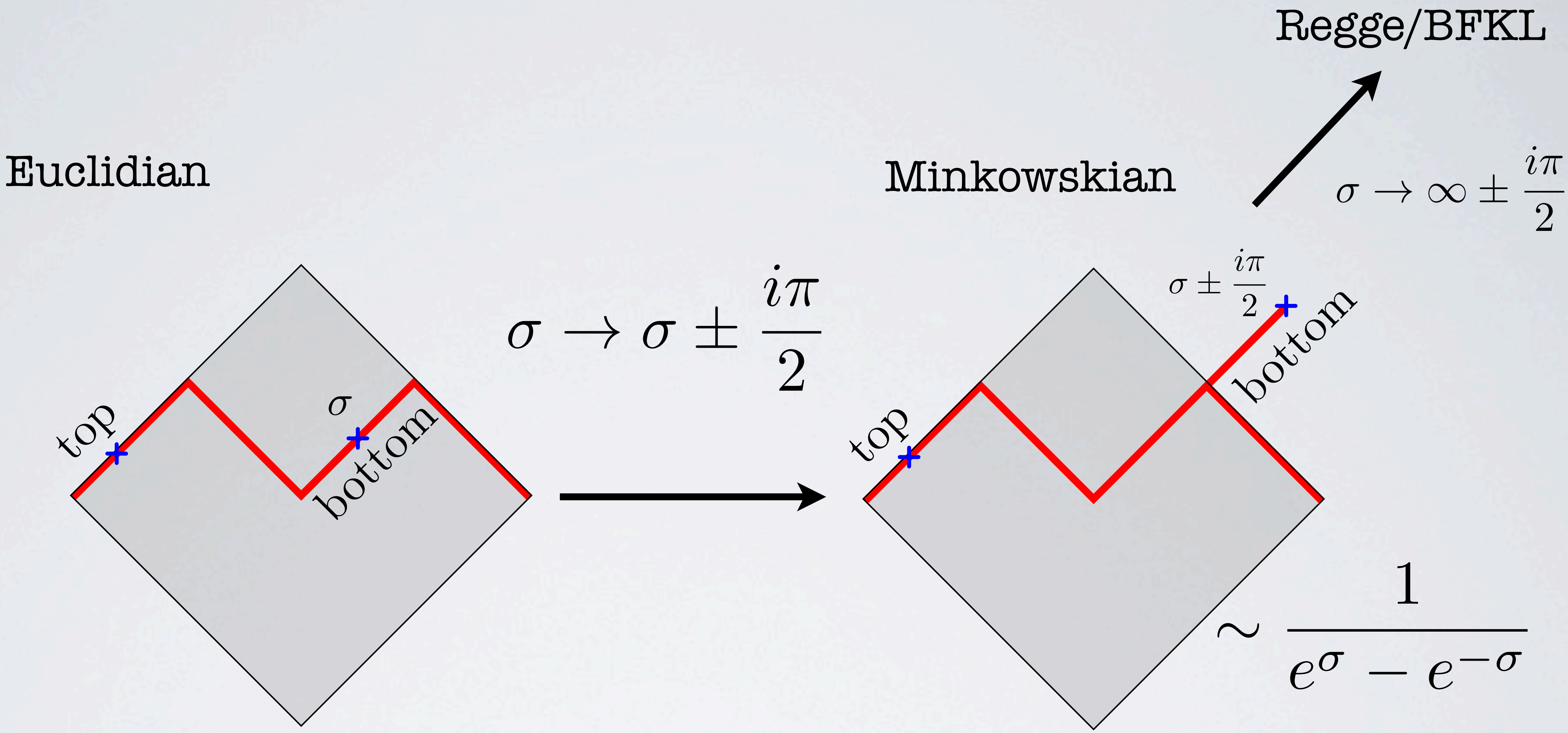
- Insertions are spacelike separated

$$\sim \frac{1}{e^{\sigma} + e^{-\sigma}}$$

At $\sigma = \pm \frac{i\pi}{2}$ the flat cusps are null separated : a cut starts there

$$F_1(\sigma, \tau) = g^2 e^{-\tau} \left[-(e^{\sigma} + e^{-\sigma}) \log(1 + e^{2\sigma}) + 2\sigma e^{\sigma} \right] + O(g^4)$$

Passing to the real kinematics



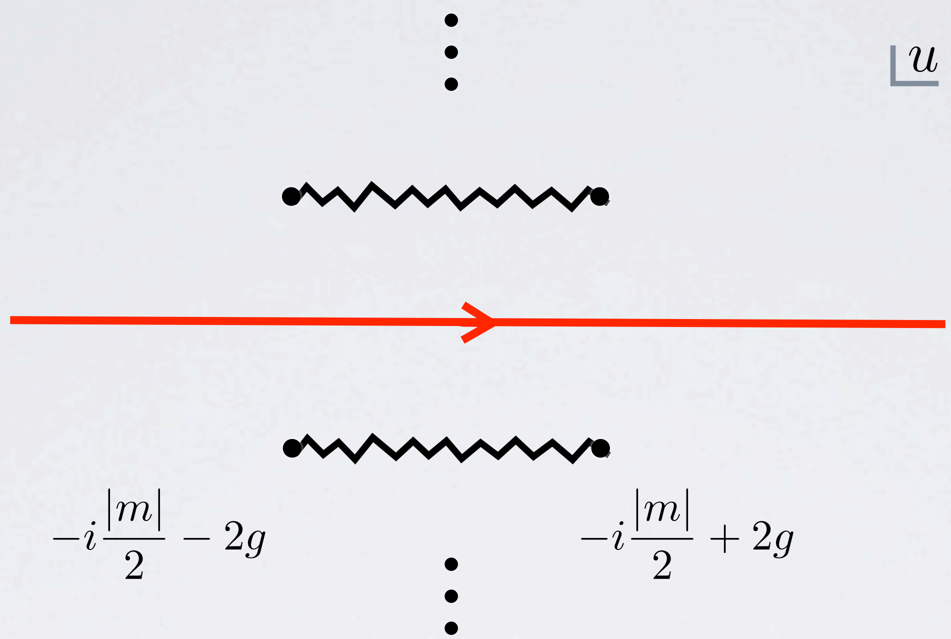
Insertions are spacelike separated

Insertions are timelike separated

BFKL computes the discontinuity through the cut in the large σ limit

Discontinuity and contour deformation

OPE contour :



- Contour along real line in rapidity plane

- There is an infinite tower of Zhukowski cuts both in lower and upper half planes

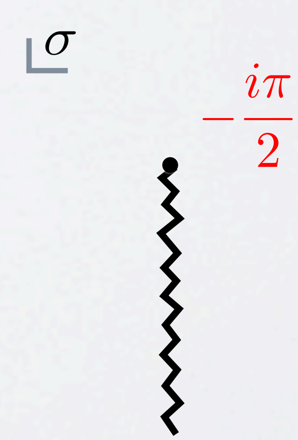
After shifting

$$\sigma \to \sigma - \frac{i\pi}{2} \qquad e^{ip(u)\sigma} \to e^{\pi u} \times e^{ip(u)\sigma}$$

The integral becomes marginally convergent

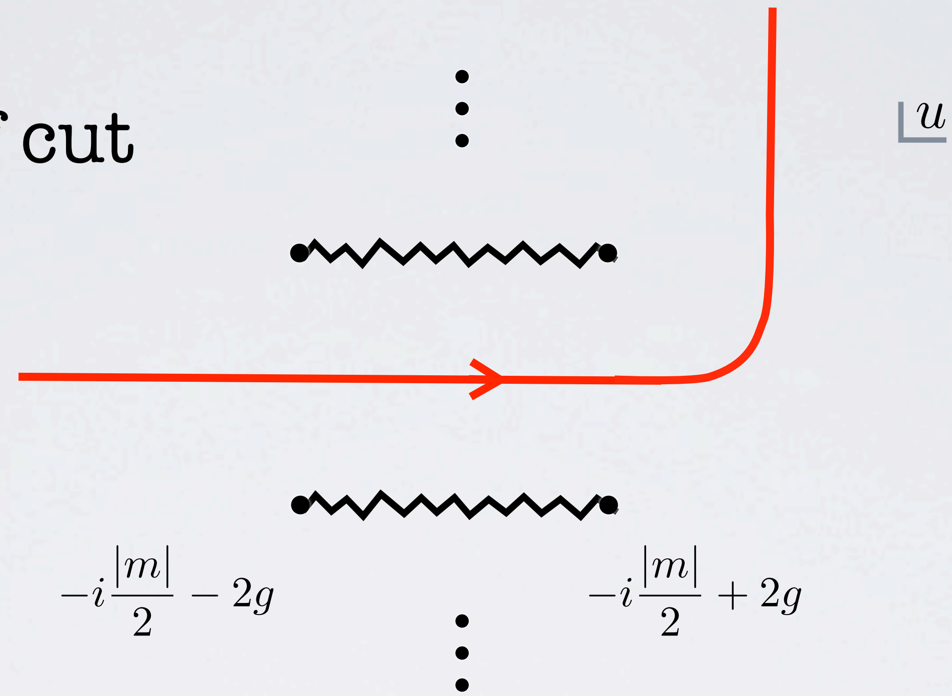
Singularity at

$$\sigma = -\frac{i\pi}{2}$$

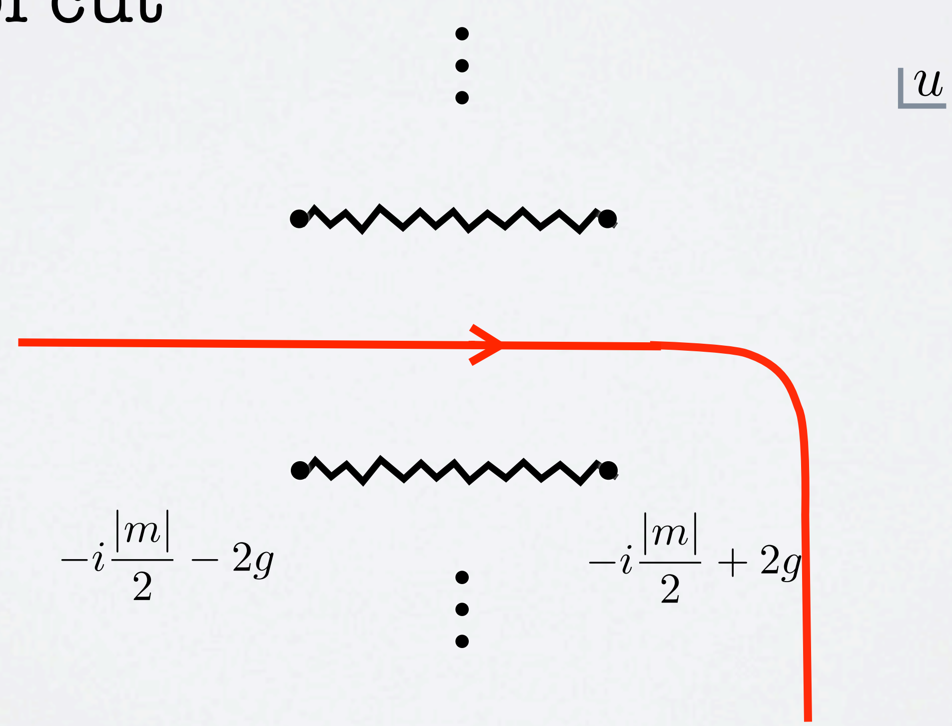


Discontinuity and contour deformation

Move to left side of cut

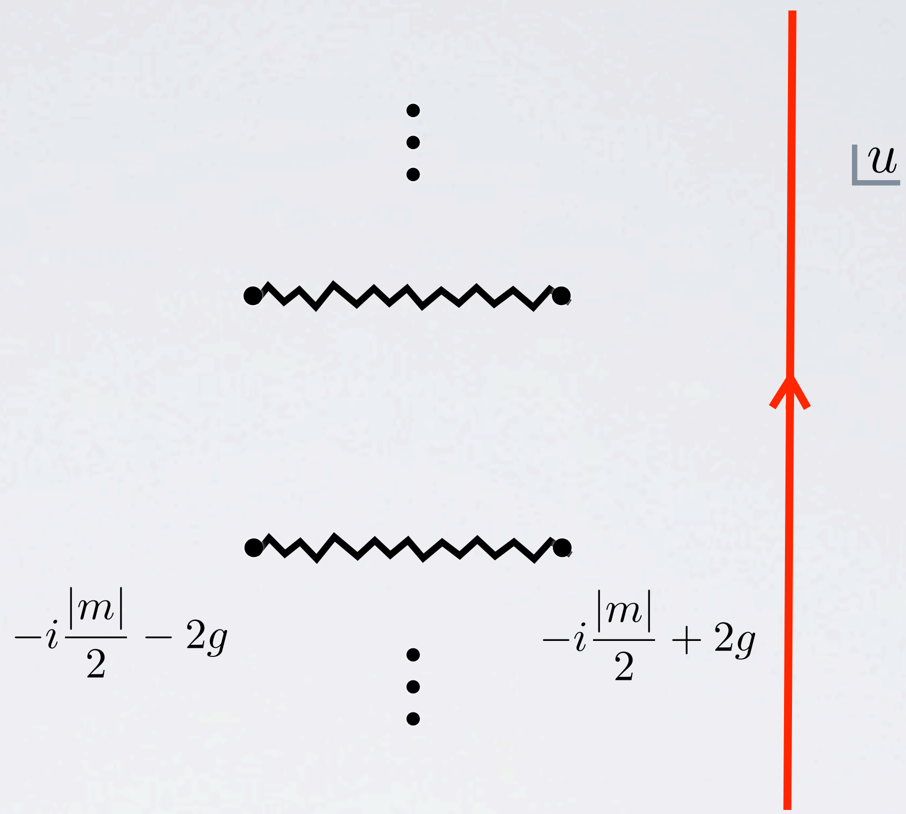


Move to right side of cut



Discontinuity and contour deformation

Difference = discontinuity

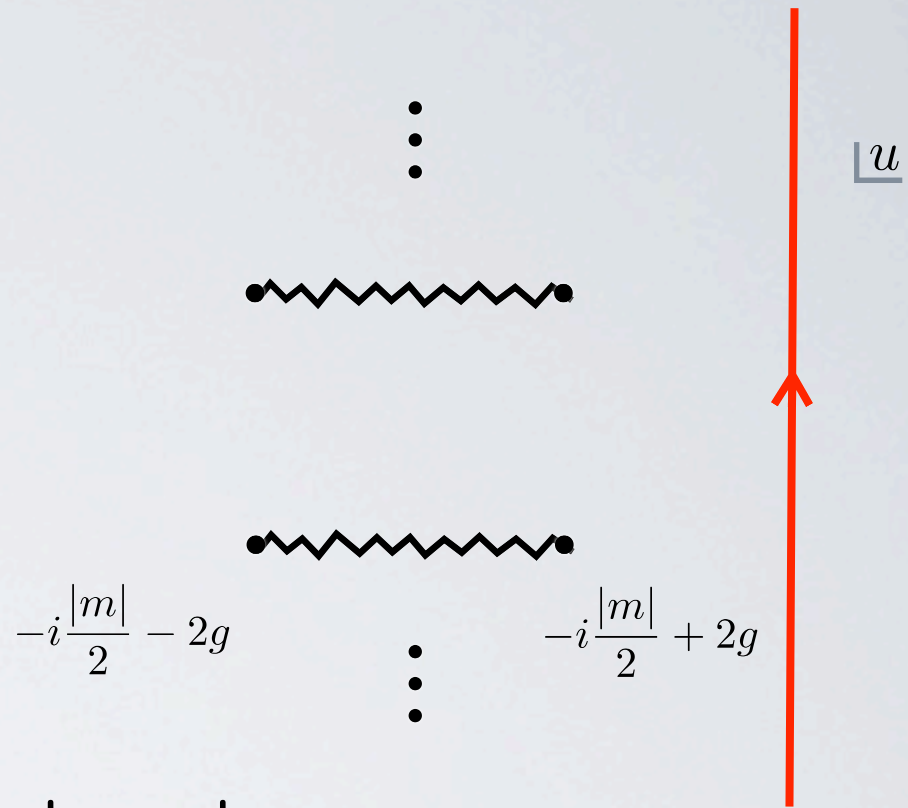


Main message : the discontinuity through the cut is controlled by the same OPE integrand but with a vertical (inverse Laplace like) contour

Discontinuity and contour deformation

Regge limit has to do with enhancement of discontinuity in limit

$$\sigma \to \pm\infty$$



To study this regime we must rotate the contour to lower/upper half plane

Problem : Must avoid infinite sequence of cuts there

Remedy : Redefine the OPE integrand such that this sequence of cuts terminates

Freedom : Vertical contour allows us to add/remove exponentially small terms at large rapidity

[BB,Caron-Huot,Sever'14]

This give rise to a new object : the sister trajectory

Sister map

One loop example : Take energy

$$\psi(1+\frac{|m|}{2}+iu)+\psi(1+\frac{|m|}{2}-iu)-2\psi(1)$$

Use reflection property and drop exponentially small terms

$$\psi(1+\frac{|m|}{2}\pm iu)\to\psi(-\frac{|m|}{2}\mp iu)\pm i\pi+O(e^{-2\pi u})$$

It gives rise to the sister energy in lower/upper half plane

$$\psi(1+\frac{|m|}{2}\pm iu)+\psi(-\frac{|m|}{2}\pm iu)\mp i\pi$$

with infinite sequence of cuts (here poles) in upper/lower plane

Sister map

It is easy to generalize that to all loops since the flux tube data is expressed in terms of psi-function and its derivatives

Before map : all loop dispersion relation for flux tube gluon [BB'10]

$$E_\ell(u) = \ell + \int_0^\infty \frac{dt}{t} K(t) \left(\cos(ut) e^{-\ell t/2} - 1 \right)$$

$$p_\ell(u) = 2u + \int_0^\infty \frac{dt}{t} K(-t) \sin(ut) e^{-\ell t/2}$$

$$K(t) = \frac{2}{1-e^{-t}} \sum_{n\geq 1} (2n)\gamma_{2n} J_{2n}(2gt) - \frac{2}{e^t - 1} \sum_{n \geq 1} (2n-1) \gamma_{2n-1} J_{2n-1}(2gt)$$

(kernel of BES equation)

After map : all loop sister dispersion relation

[BB,Caron-Huot,Sever'14]

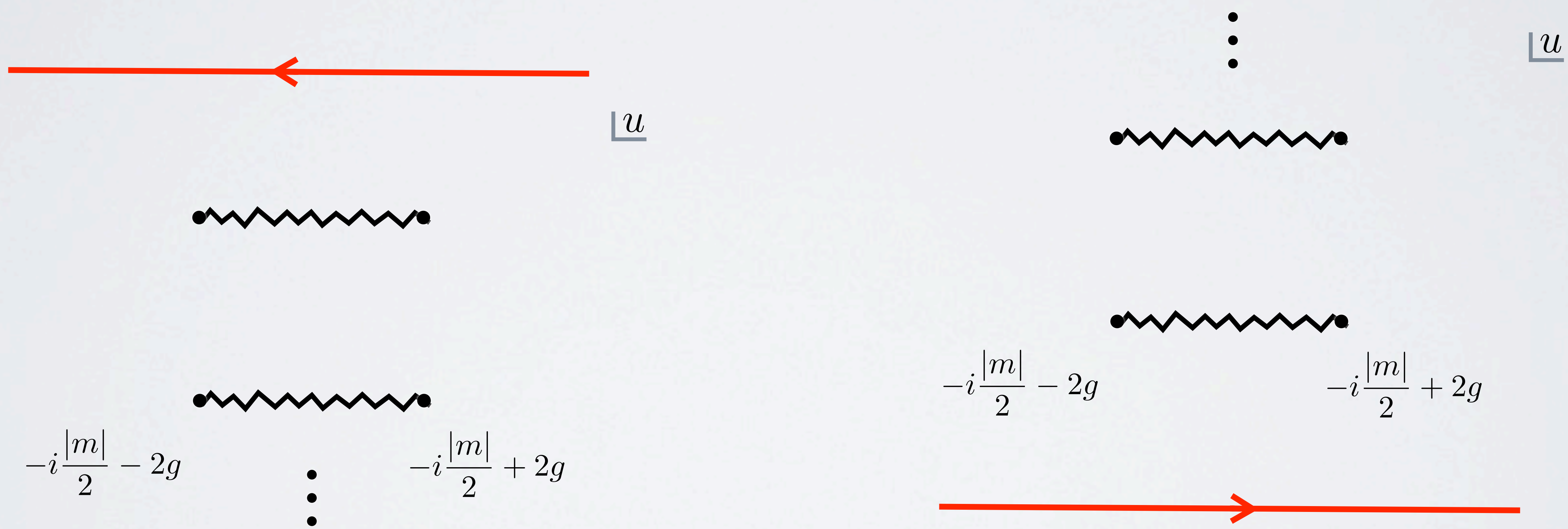
$$\check{E}_\ell(u) = \ell + \frac{i\pi}{2}\Gamma_{\text{cusp}} + \int_0^\infty \frac{dt}{t} \left[K(t) \frac{e^{-iut-\ell t/2} - 2}{2} + K(-t) \frac{e^{-iut+\ell t/2}}{2} \right]$$

$$\check{p}_\ell(u) = 2u + \frac{\pi}{2}\Gamma_{\text{cusp}} - i \int_0^\infty \frac{dt}{t} \left[K(t) \frac{e^{-iut+\ell t/2}}{2} - K(-t) \frac{e^{-iut-\ell t/2}}{2} \right]$$

Sister map

It is easy to generalize that to all loops since the flux tube data is expressed in terms of psi-function and its derivatives

It allows us to write an all loop integral representation for the discontinuity



The integrand is just the sister version of the OPE one

Comment : so far all steps can be done order by order in PT

From sister to BFKL

- OPE for discontinuity (sister trajectory : energy, measure etc.)

$$\mathcal{W}^{\updownarrow}_{\text{hex}} = \sum_m (-1)^m e^{im\phi} \int_{-\infty}^{+\infty} \frac{dp}{2\pi} \hat{\mu}^{\updownarrow}_{|m|}(p) e^{i\sigma p - \tau \check{E}_{|m|}(p)} + \ldots$$

τ large

It captures terms of the type

(leading twist, all conformal spins)

$$e^{-|m|(\tau-\sigma)}(1 + e^{-2\sigma} + \ldots)$$

- BFKL integral

$\tau + \sigma$ large

$$\mathcal{W}^{\updownarrow}_{\text{hex}} = e^{\frac{1}{2} i\pi(\sigma-\tau)\Gamma_{\text{cusp}}} \sum_m (-1)^m e^{im\phi} \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi} \hat{\mu}_{\text{BFKL}}(\nu, m) e^{i(\sigma-\tau)\nu} e^{(\sigma+\tau)\omega(m,\nu)} + \ldots$$

It captures terms of the type

(leading spin, all dimensions)

$$e^{-|m|(\tau-\sigma)}(1 + e^{-2(\tau-\sigma)} + \ldots)$$

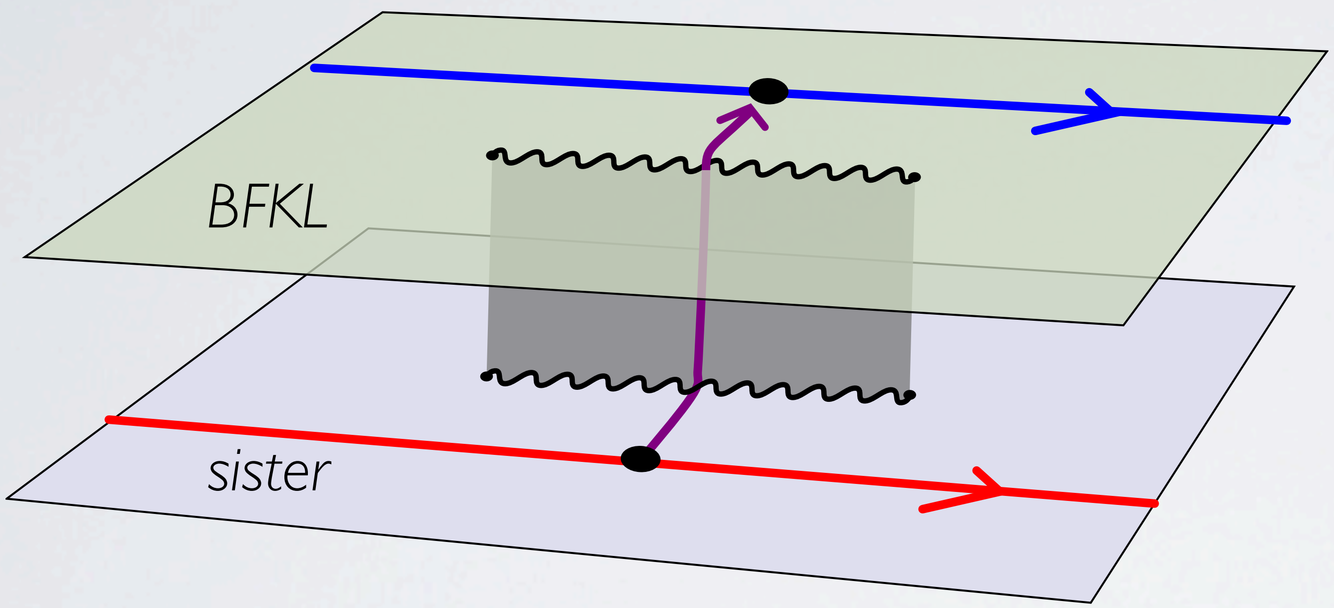
- They operate in different regimes but they should agree in the overlap of their respective domains of validity

Cross-over : $\tau \sim \sigma \gg 1$

From sister to BFKL

Follow saddle from sister to BFKL regime

[BB,Caron-Huot,Sever'14]



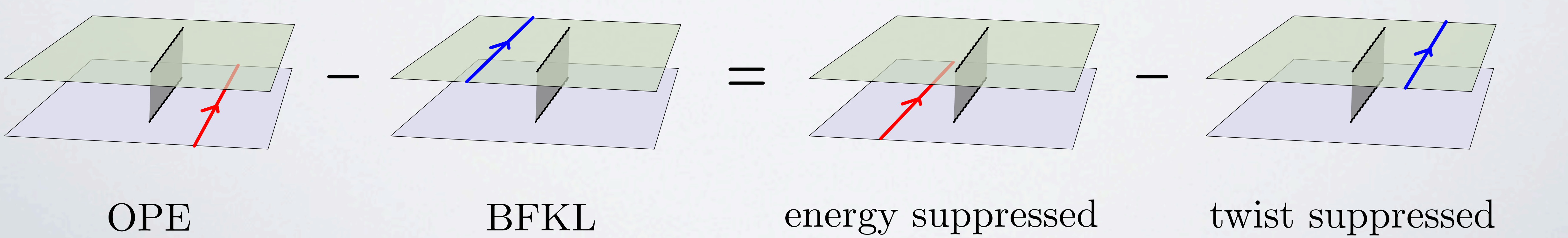
$$\frac{\sigma}{\tau} = \frac{1}{i}\frac{d\check{E}}{dp}(p_*)$$

$$\frac{\tau-\sigma}{\tau+\sigma} = \frac{1}{i}\frac{d\omega}{d\nu}(\nu_*)$$

$$\omega(\nu_*) = \frac{i}{2}\left[p_* + i\check{E}(p_*)\right]$$

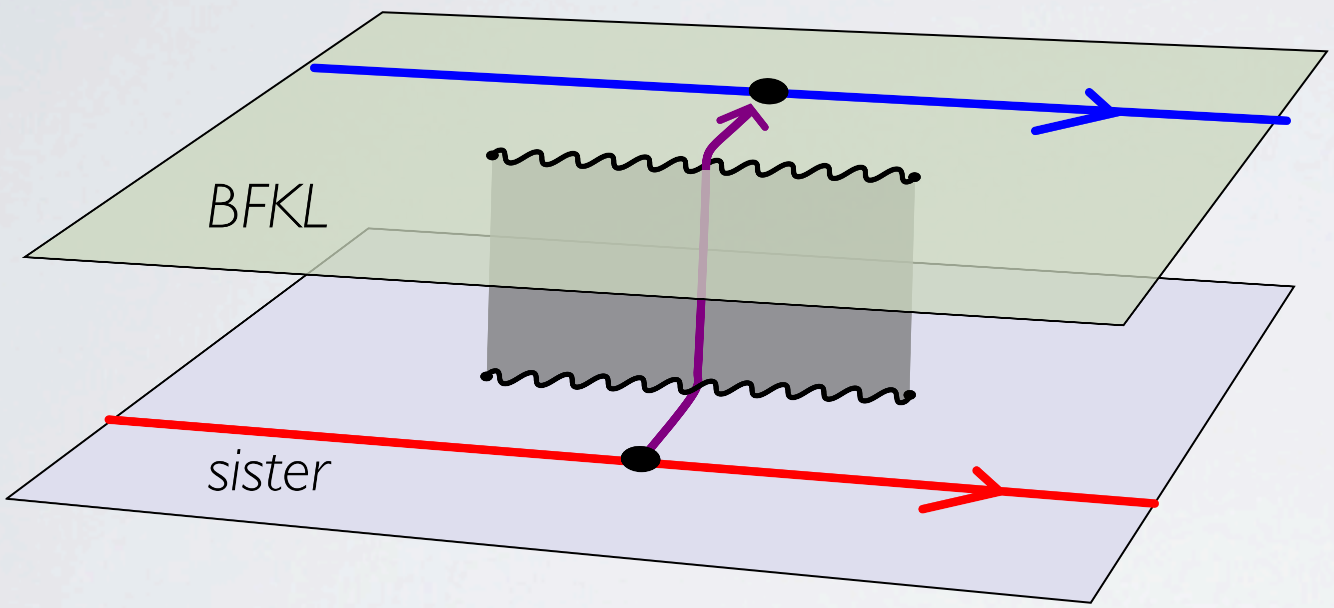
$$\nu_* = \frac{1}{2}\left[p_* - i\check{E}(p_*)\right] - \frac{\pi}{2}\Gamma_{\text{cusp}}$$

Equivalently : wrap contour around the cut and move it to 2nd sheet



From sister to BFKL

Follow saddle from sister to BFKL regime



At finite coupling it is easy to navigate between the two pictures

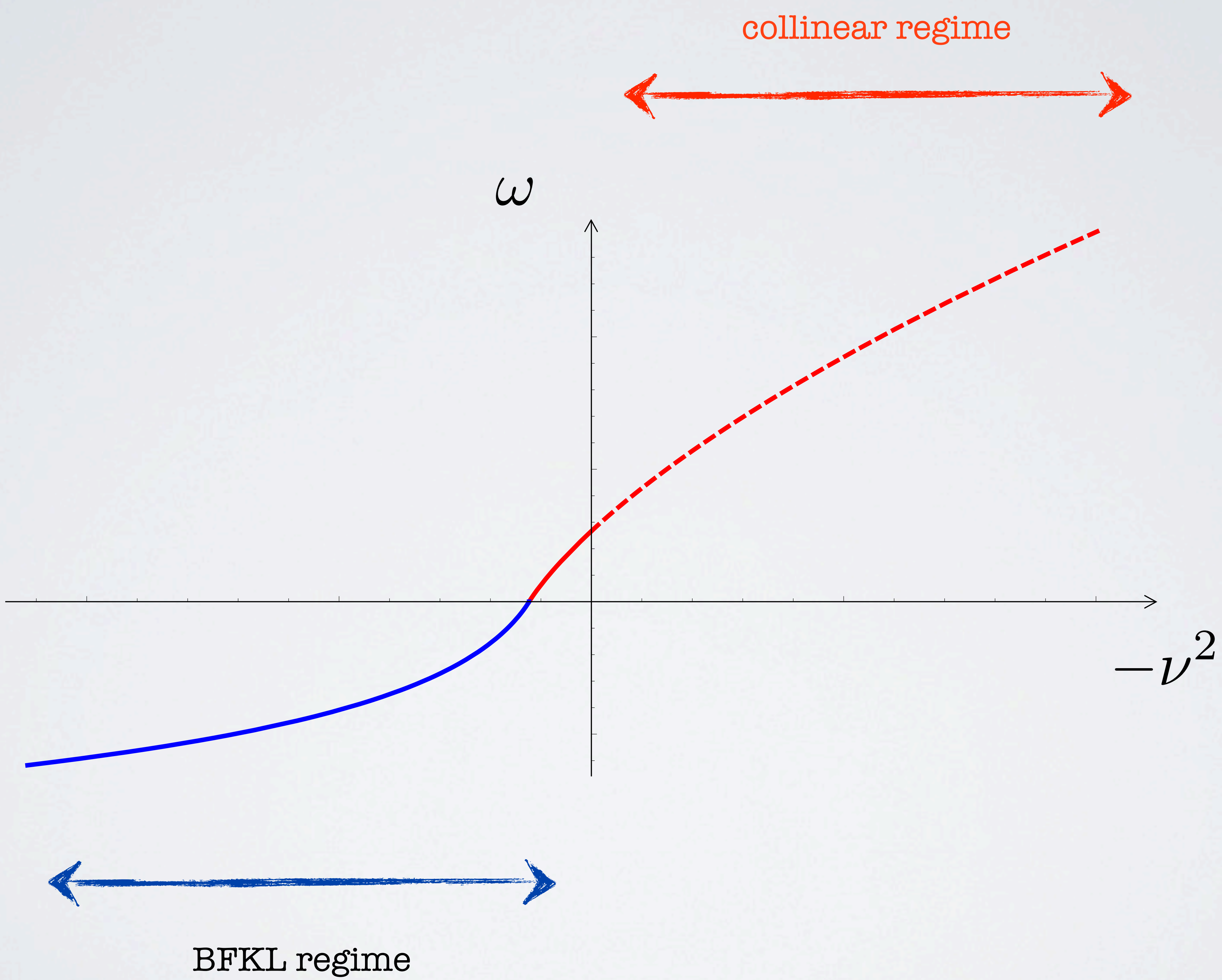
Give BFKL eigenvalue from the sister flux tube energy to all loops

K = BES kernel

$$\omega(u,m) = \int_0^\infty \frac{dt}{t}\left(K(t) - \frac{K(-t)+K(t)}{2}\cos(ut)e^{-|m|t/2}\right)$$

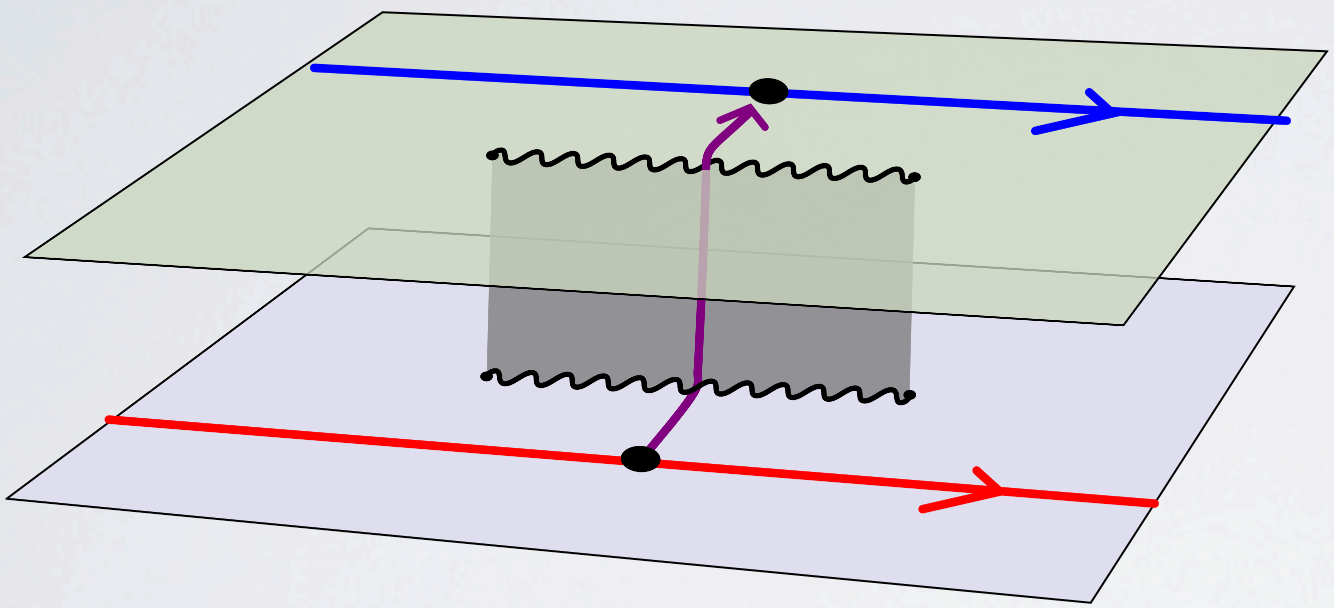
$$\nu(u,m) = 2u + \int_0^\infty \frac{dt}{t}\frac{K(-t)-K(t)}{2}\sin(ut)e^{-|m|t/2}$$

Chew-Frautschi plot



From sister to BFKL

Follow saddle from sister to BFKL regime



At finite coupling it is easy to navigate between the two pictures

Same applies to measure a.k.a impact factor

$$\mu_m^{\mathrm{OPE}}(u) \quad \to \quad \mu_m^{\updownarrow}(u) \quad \to \quad \mu_m^{\mathrm{BFKL}}(u)$$

Tests

- Weak coupling

$$-\omega(\nu,m) = 2g^2\left\{-\frac{2|m|}{\nu^2+m^2}+\psi\left(1+\frac{|m|+i\nu}{2}\right)+\psi\left(1+\frac{|m|-i\nu}{2}\right)-2\psi(1)\right\}+O(g^4)$$

+ higher loop matches [Bartels,Lipatov,Sabio Vera'08],[Fadin,Lipatov], [Dixon,Duhr,Pennington'12],[Dixon,Drummond,Duhr,Pennington'14]

- Finite coupling

constraint from collinear limit $\Rightarrow$ $\omega(\nu = \pm\frac{\pi}{2}\Gamma_{\rm cusp}, m=0) = 0$

[Caron-Huot]

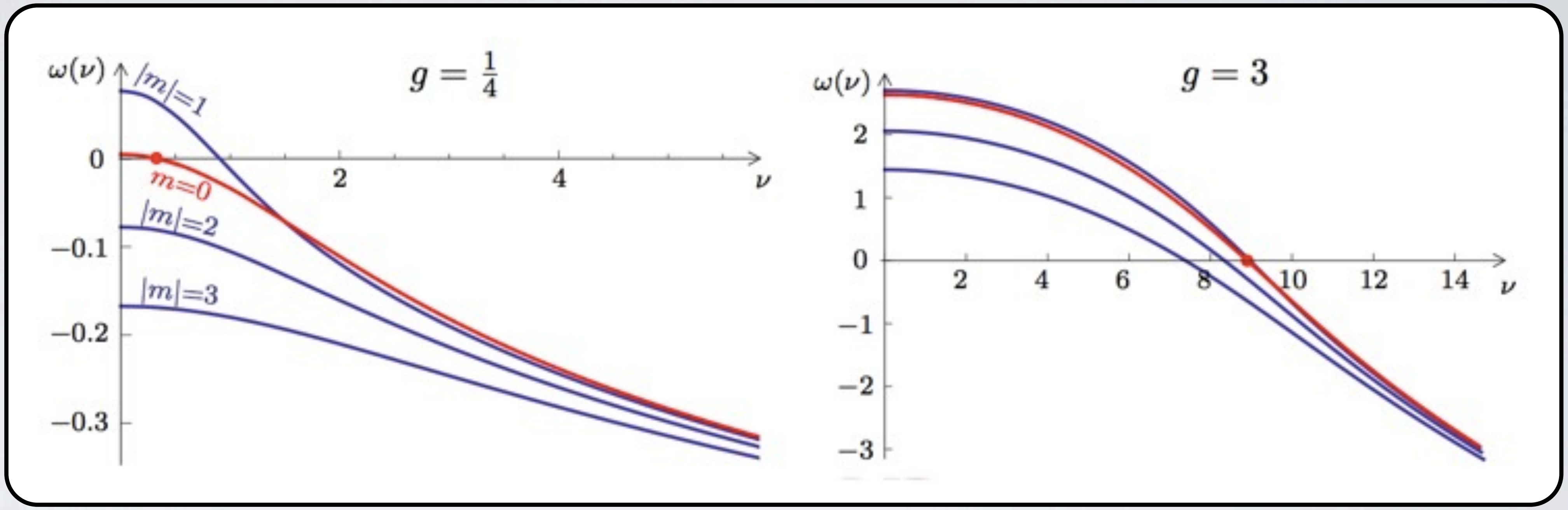
- Strong coupling

$$\omega(\nu) = \frac{\sqrt{\lambda}}{2\pi}(\sqrt{2}-\log(1+\sqrt{2}))+O(1)$$

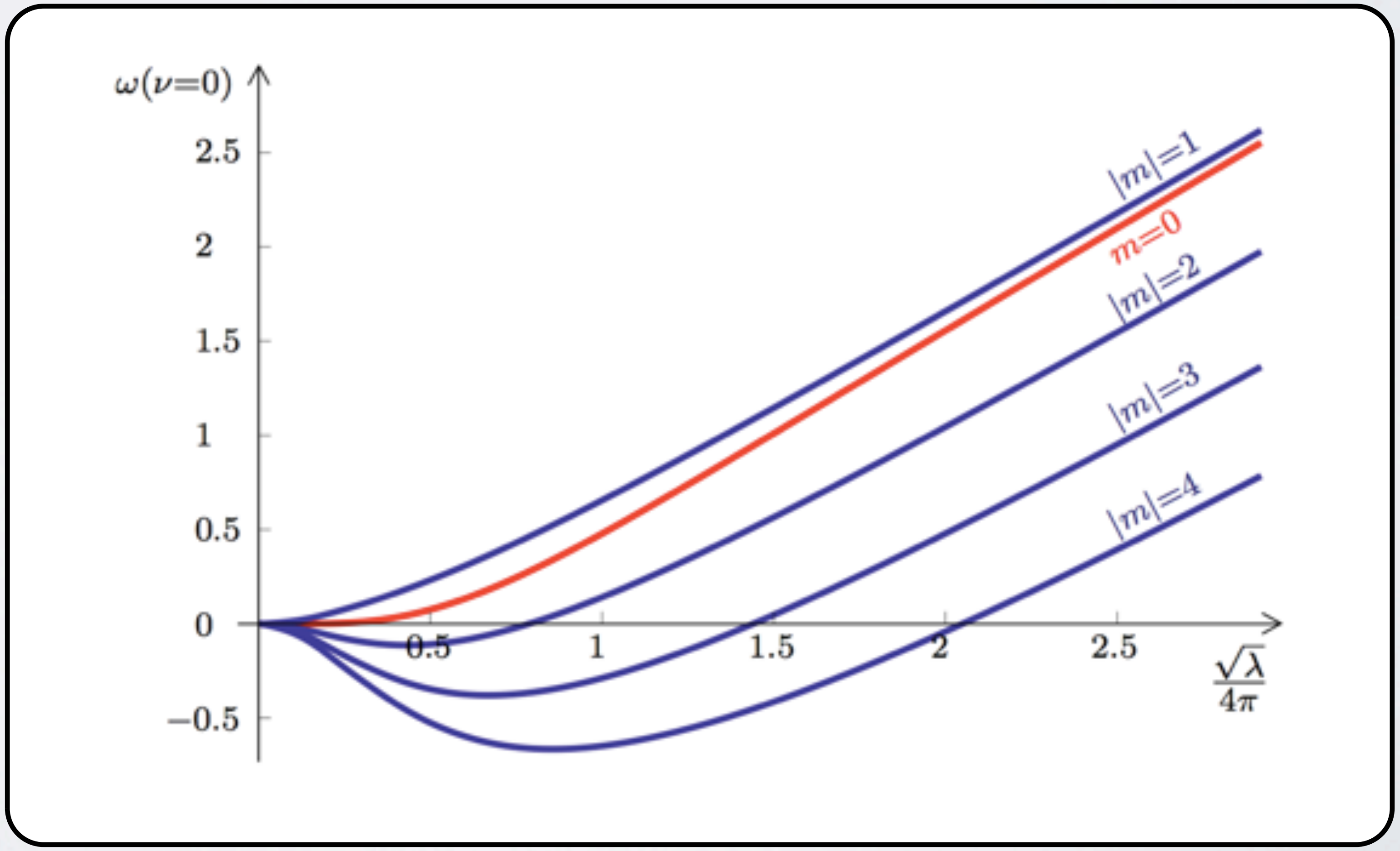
[Bartels,Kotanski,Schomerus,Sprenger]

Adjoint eigenvalues at finite coupling

Eigenvalues :

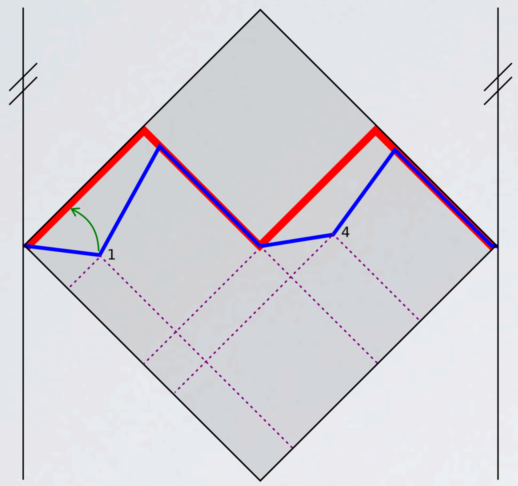


Intercepts :



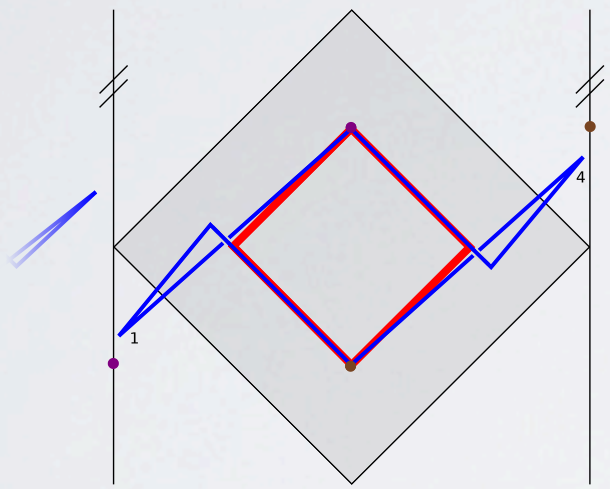
Summary

OPE



$$= 1 + \sum_{m\neq 0} (-1)^m e^{im\phi} \int_{-\infty}^{+\infty} \frac{dp}{2\pi}\, \hat{\mu}_m(p)\, e^{ip\sigma - \tau E_m(p)} + \dots$$

BFKL



$$= -2\pi i \sum_{m=-\infty}^{\infty} (-1)^m e^{im\phi} \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi}\, \hat{\mu}_{\text{BFKL}}(\nu, m)\, e^{i(\sigma-\tau)\nu + (\sigma+\tau)\omega(\nu,m)} + \dots$$

See [Bartels et al.],[Hatsuda]
for related ideas
See [Drummond,Papathanasiou]
for a direct path at function level

Two-step analytic continuation

1. In external momenta $\tau,\ \sigma,\ \phi \quad \Rightarrow$ OPE for discontinuity
2. In flux tube momentum p : crossing the cut between Regge and OPE

Higher polygons

Heptagon

OPE pentagon transition

$$\int \frac{dudv}{(2\pi)^2}\mu(u)\mu(v)e^{-ip(u)\sigma_1+ip(v)\sigma_2}P(u-i0|v+i0)$$

Gluon transition

$$P(u|v) = -\frac{(\frac{1}{2}-iu)\Gamma(iu-iv)(\frac{1}{2}+iv)}{g^2\Gamma(\frac{1}{2}+iu)\Gamma(\frac{1}{2}-iv)}$$

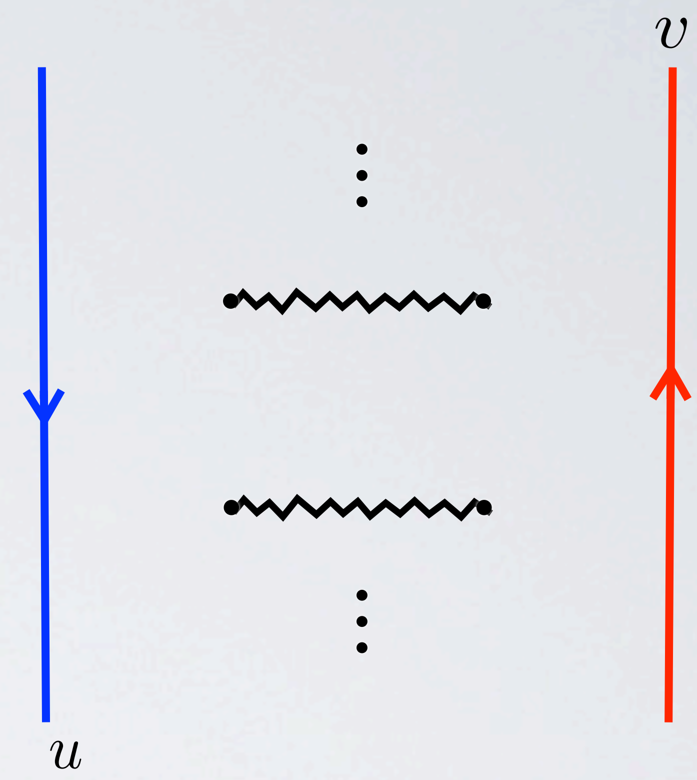
Position space

$$P(\sigma_1|\sigma_2) = \frac{e^{\sigma_1+\sigma_2}}{2}\log\frac{\left(e^{2\sigma_1}+1\right)\left(e^{2\sigma_2}+1\right)}{e^{2\sigma_1}+e^{2\sigma_2}+e^{2\sigma_1+2\sigma_2}}$$
$$+e^{\sigma_2-\sigma_1}\log\frac{e^{2\sigma_2}\left(e^{2\sigma_1}+1\right)}{e^{2\sigma_1}+e^{2\sigma_2}+e^{2\sigma_1+2\sigma_2}}+(\sigma_1\leftrightarrow\sigma_2)$$

Heptagon

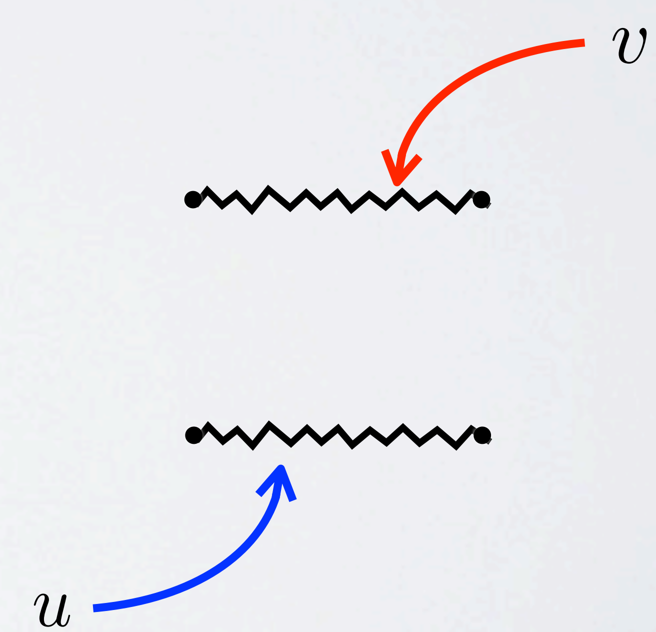
Double discontinuity

$$\sigma_i \to \sigma_i - \frac{i\pi}{2}$$



Sister map both
and take Regge limit

$$\sigma_i \to -\infty \qquad \tau_i \to \infty \qquad \tau_i + \sigma_i \ \text{fixed}$$



We get the Regge pentagon a.k.a
central emission vertex at an coupling

known at weak coupling from
[Bartels,Kormilitzin,Lipatov,Prygarin'12]

Regge pentagon

Structure is essentially the same as for pentagon transitions

$$\text{tree} \times \exp\left[\text{bilinears in } \psi \text{ functions and derivatives}\right]$$

Few important properties :

- Decoupling pole

$$P(u|v) \sim \frac{1}{i\mu(u)(u-v)}$$

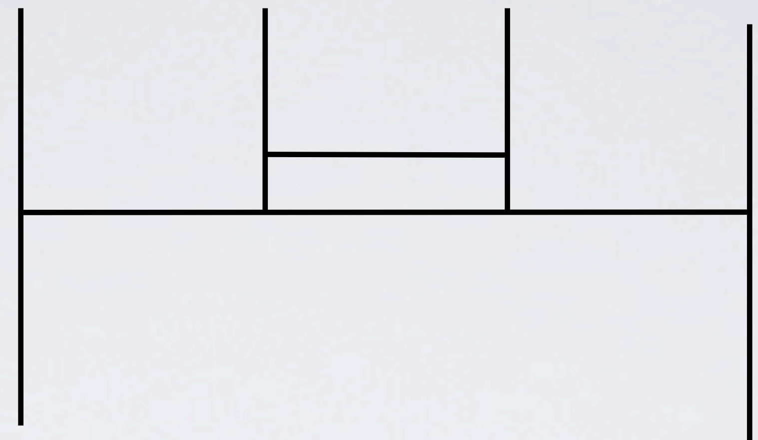
- Reggeon zeroes for mode zero

$$\lim_{\nu(u)\to\frac{\pi}{2}\Gamma_{\mathrm{cusp}}} P(u|v) = 0$$

$$\lim_{\nu(v)\to-\frac{\pi}{2}\Gamma_{\mathrm{cusp}}} P(u|v) = 0$$

Regge picture

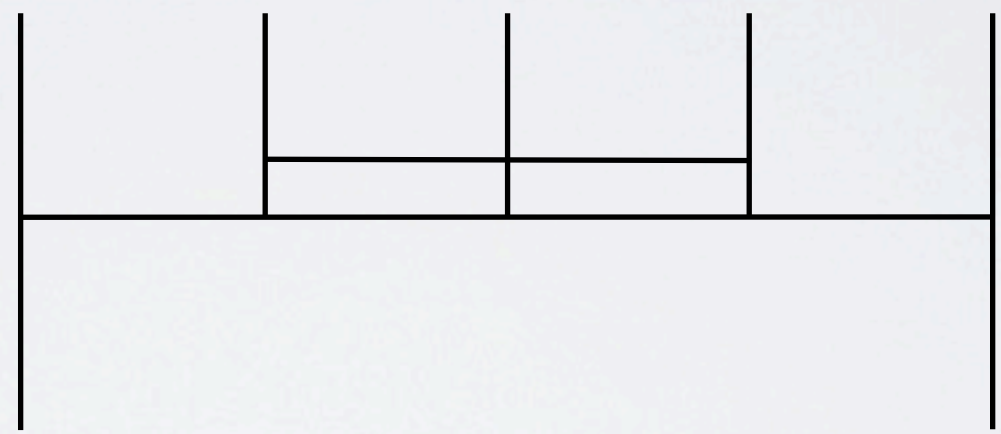
Hexagon gives measure a.k.a impact factor



Diagram

$\mu(u)$

Heptagon gives pentagon transition a.k.a central emission vertex



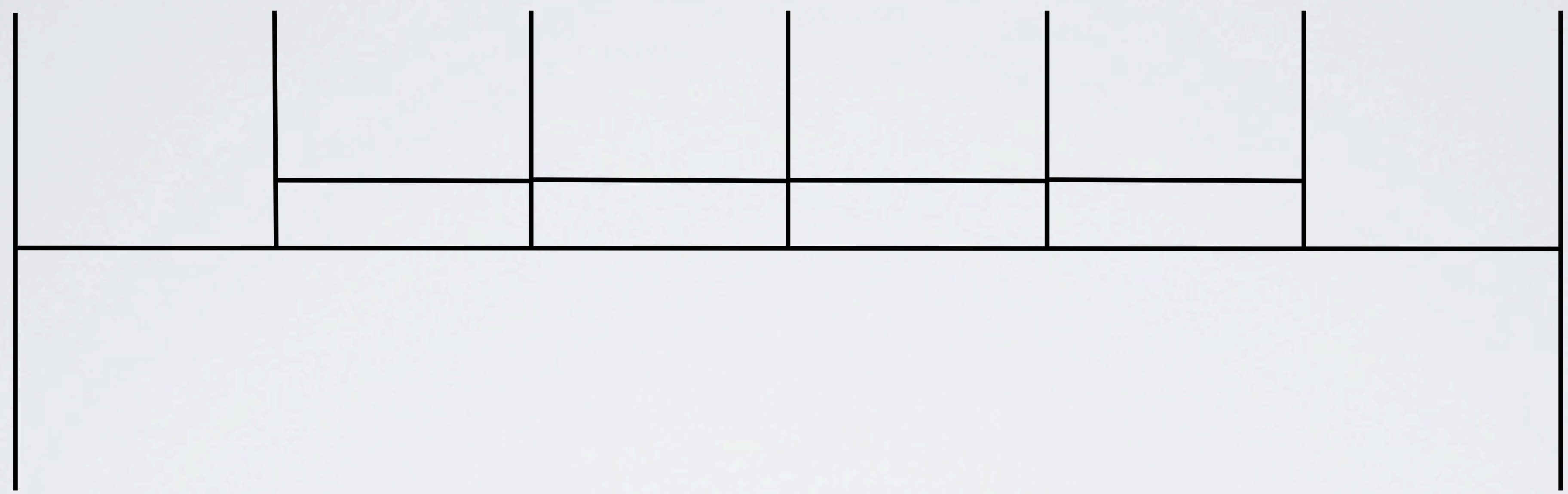
Diagram

$$\mu(u) \times P(u|v) \times \mu(v)$$

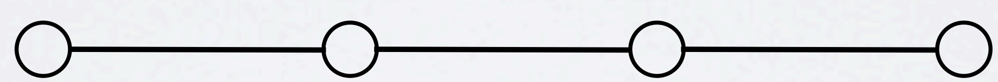
Higher polygons

recent discussions :
[Bargheer'16],
[Del Duca et al.'16]

Linear sequence



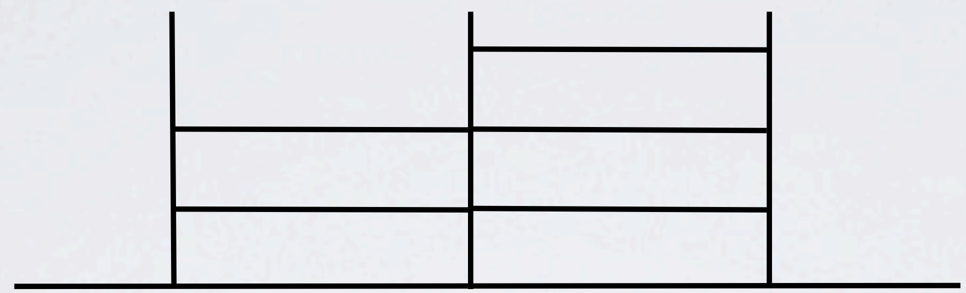
Diagram



$$\mu(u) \times P(u|v) \times \mu(v) \times P(v|w) \times \mu(w) \times P(w|z) \times \mu(z)$$

Conjectures for higher cuts

Multi-cut transition

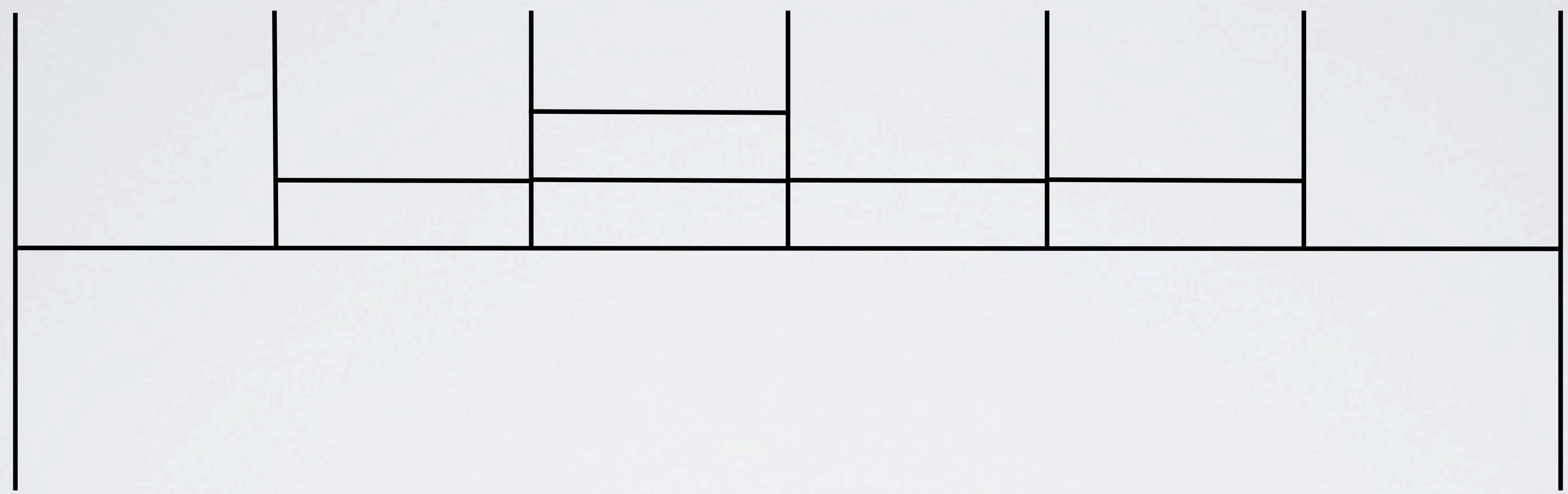


Conjecture (following factorization of OPE transitions) :

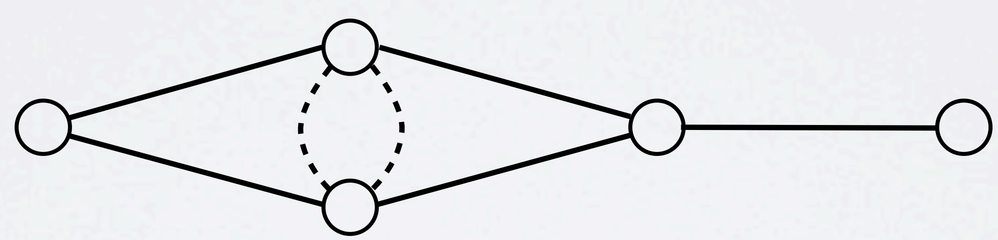
$$\frac{\prod_{i,j} P(u_i|v_j)}{\prod_{i<j} P(u_i|u_j) \prod_{i>j} P(v_i|v_j)}$$

Conjectures for higher cuts

Example of a sequence with up to 3 Reggeons



Diagram



Integrand

$$P(u|v_1)P(u|v_2) \times \frac{1}{P(v_1|v_2)P(v_2|v_1)} \times P(v_1|w)P(v_2|w) \times P(w|z)$$

Weak coupling estimate

- A blob (measure) costs one loop

- A link (transition) costs minus one loop

- A linear sequence has N blobs and N-1 links : it thus appears at one loop

Generalization : if n-1 is the max number of Reggeons in the sequence, then the diagram starts at n loops

Caveat : Generically true up to mode zero contributions

Mode zero measure has pole at $\nu = \pm\pi\Gamma_{\text{cusp}}/2$

- Residue at the pole relates to one Reggeon contribution (OPE vacuum)
- It produces disconnected terms

(integrated with a Feynman like prescription)

Possible tests of this conjecture

Consistency checks : Is the factorized ansatz compatible with everything we know (collinear / soft limits, transcendentality, etc.)?

String coupling saddle point for higher n-gon : looks doable; we could test the factorizability and the presence or not of new stuff (bound states of n>2 Reggeons)

heptagon strong coupling study : [Bartels,Schomerus,Sprenger'14]

First higher cut effect at weak coupling : 2 loop 8 points; comparison with Simon's symbol? or with a function?

recent progress : [Bargheer,Papathanasiou,Schomerus'15], [Bargheer'16],[Broedel,Sprenger,Torres Orjuela'16], [Del Duca et al.'16]

Direct weak coupling analysis using integrable spin chain?

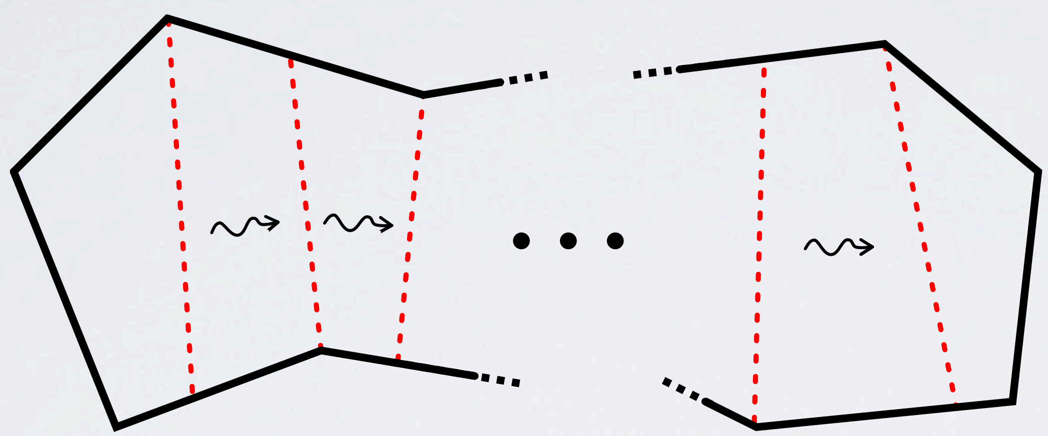
[Bartels,Lipatov,Prygarin'11]

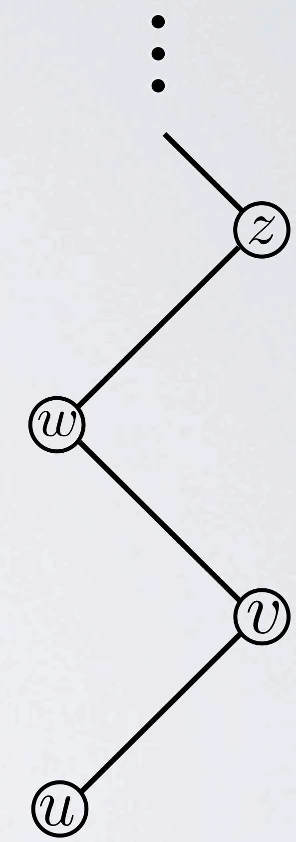
Integrable system is very similar to the one for the flux tube : expect structure of multi-particle wave functions and pentagons transitions to be the same

[BB,Sever,Vieira'13]
[Belitsky,Derkachov,Manashov'14]

Derivation from OPE?

OPE best formulated / understood for a zig-zag sequence :





$$P(u|v) \times P(w|v) \times P(w|z) \times P(z|t) \times \ldots$$

Not obvious if that configuration admits higher cut version (i.e. that we stack more Reggeons in middle channels)

To find higher cuts we must explore other discontinuities....

Generalization to non-MHV amplitudes

[BB,Sever,Vieira'13]
[BB,Caetano,Cordova,Sever,Vieira'15]
[Belitsky'14'15]

NMHV form factors :

$$P(u|v) \to \frac{x^+x^-}{y^+y^-}P(u|v)$$

(these are zero-modes of the pentagon bootstrap)

In the Regge domain :

$$P(u|v) \to \frac{x^+y^-}{x^-y^+}P(u|v)$$

- They do not change the weak coupling counting

- They break symmetry between positive and negative mode numbers

- Mode zero quantities are unaffected - they are the same for MHV and non MHV

Conclusion

Regge and OPE regimes are the two sides of a same story

BFKL and collinear eigenvalues the two “branches” of a same function

This only becomes manifest and fully tractable at finite coupling

Crossing the kinematics then becomes equivalent to crossing a cut in internal momentum / rapidity plane

Conclusion

Clear route from OPE to Regge; OPE / Regge dictionnary

Following it we derive the eigenvalue, impact factor and emission vertex directly from the OPE / pentagon data at any coupling

Pushing the analogy further hints at higher cuts with totally factorized structure, like for multiparticle pentagon transitions

Many questions remain : Completeness of states? Can new states appear for n>=8? Can we go all the way from collinear to Regge for higher n-gon? Are there Regge islands we cannot reach?

THANK YOU!