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The black hole as a hydrogen atom

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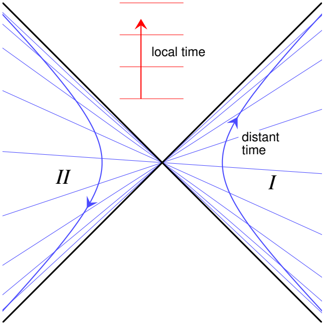
> Without such an identification, *firewalls* would arise violating the basic principles of GR.

Using an algebra derived 25 years ago, we had all knowledge needed to derive these facts.

All needed to be done, was to handle the black hole the way the hydrogen atom was handled when Schrödinger discovered his equation. Partial waves of matter have to be inserted into this algebra to make exactly clear what happens.

But we do end up discovering new physics as well!

Hawking 1975: BH emits particles. However:
BH consists of interior part and exterior part.
Particles entering the interior part cannot come out.
Their quantum information is lost. Therefore, particles emerging from BH are in a **mixed state**.



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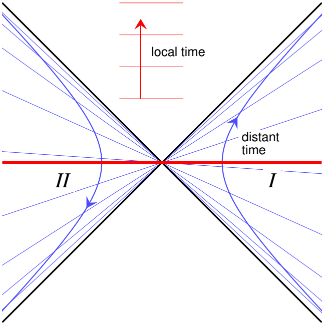

Hartle-Hawking vacuum:

$$|\,HH\,\rangle \;=\; C\sum_{E,n} e^{-\frac{1}{2}\beta E}\,|E,n\rangle_I\,|E,n\rangle_{II}$$

$I \;=\;$ outside
$II \;=\;$ inside [?]

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so, people start guessing:

. . . What does superstring theory say?

They find something that looks like a black hole,
and it is described in terms of pure quantum states !
"microstates". But it is not understood . . .

How are these microstates related to vacuum fluctuations?

But then:

Difficulties regarding entanglement and no-cloning:

Almheiri, Marolf, Polchinski, Sully (AMPS): *If we start in a pure state, and consequently Hawking radiation is in a pure state, then that pure state must be entangled also with earlier radiation. Therefore it can't be in the state originally used by Hawking. This produces a curtain of infinitely energetic particles along the future event horizon: a firewall . . .*

This cannot be right.
As we discovered, you can do better.

And yes, the price is:

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And yes, the price is:
there will be new physics.

We begin with getting the maximum out of *standard Einstein General Relativity* and *standard Quantum Mechanics*

Then, one has to discover 3 important things:

(i) Particles going into a black hole, interact *gravitationally* with particles going out. If you want to describe these as pure quantum states, you cannot ignore that (only if they are in mixed states, you may) because this grav. force is **strong**

(ii) This force generates an algebra that is *linear* in the coordinates & momenta of the in- and out-particles, and therefore, you can superimpose solutions!
$\longrightarrow$ Make an expansion in spherical harmonics.

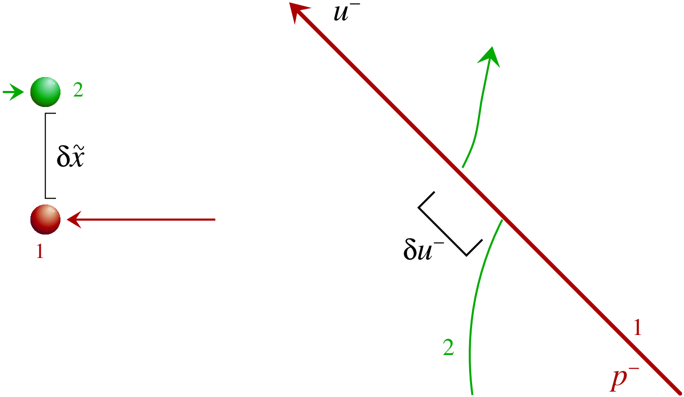
(iii) We have always been wrong in thinking that the mirror particles of the Hawking particles, going into the BH, will get lost. The mirror particles reappear at the other side!

If we ignore any **one** these **3** points, we fail to understand what happens

The gravitational backreaction:

calculate the grav. field due to a fast moving particle,

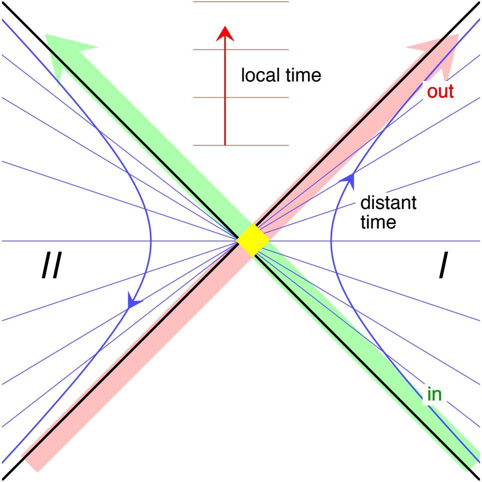
simply Lorentz boost the field of a particle at rest:



$$\boxed{\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}\,') \log|\tilde{x} - \tilde{x}\,'|\ .}$$

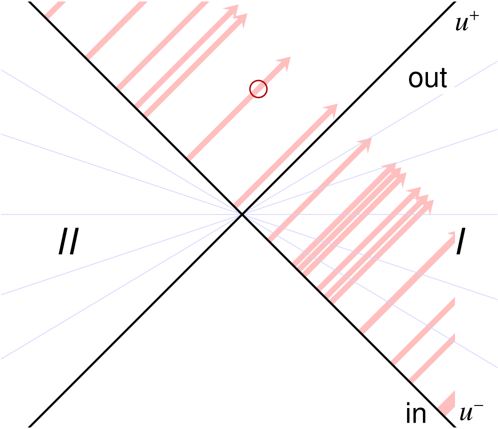
(Rindler space simplification)

Classically: out-going particles are independent of in-going ones.



But not in black holes. This used to be the BH information paradox.

An extra particle $\delta(\mathrm{in})$ with momentum δp^- going in interacts gravitationally with the out-going particles, causing a shift δu^-:



$$\delta u^-(\Omega) \approx -4G\,\delta p^-(\Omega')\,\log|\Omega - \Omega'|\ , \qquad \Omega \equiv (\theta,\,\varphi)\ .$$

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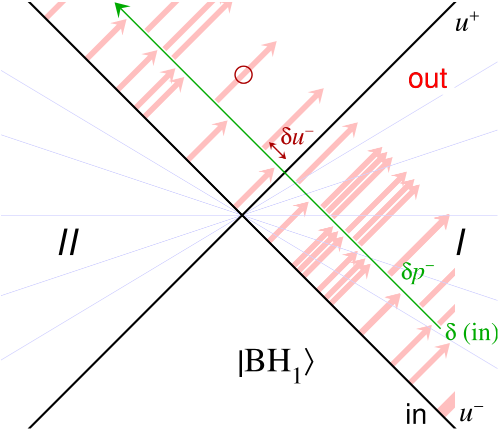

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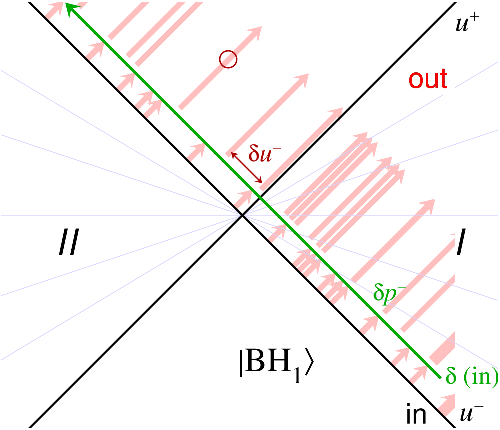


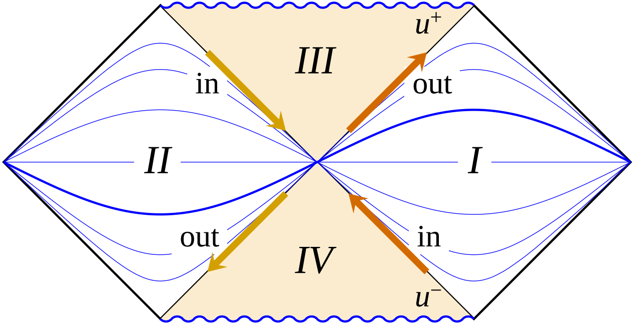
$\delta u^-(\Omega) \approx -4G\,\delta p^-(\Omega')\,\log|\Omega - \Omega'|$.

We argue as follows:

1. Consider ONE pure quantum state of a black hole, defined by ALL of its initial, in-going particles. Call this state $|BH(1)\rangle_{\rm in}$.
2. Assume that this quantum state evolves into a pure final state of out-going particles, $|BH(1)\rangle_{\rm out}$.
   Of course, $|BH(1)\rangle_{\rm out}$ consists of a superposition of a very large number of elementary particles states. But, it is pure.
3. Now add a single particle entering the horizon with momentum $p^-_{\rm in}$ at$\Omega = (\theta, \varphi)$, so that we modify the in-state:
   $|BH(2)\rangle_{\rm in} = |BH(1) + p^-_{\rm in}\rangle_{\rm in}$.
4. All particles in $|BH(1)\rangle_{\rm out}$ are dragged by an amount $\delta u^-_{\rm out}$, given by our drag formula:
$$|BH(2)\rangle_{\rm out} = e^{-i\int P^+(\Omega)\delta u^-_{\rm out}(\Omega)}\,|BH(1)\rangle_{\rm out}.$$
5. Repeat this by adding or removing in-particles, as many as you want. This way, we get the $|BH(3)\rangle_{\rm out}$ that correspond to *any* conceivable in-state. Call the new in-states $|BH(1),\ p^-_{\rm in}(\theta,\varphi)\rangle_{\rm in}$. Then
$$|BH(3)\rangle_{\rm out} = e^{-i\int {\rm d}^2\Omega{\rm d}^2\Omega' P^+_{\rm out}(\Omega)\,f(\Omega,\Omega')\,p^-_{\rm in}(\Omega')}|BH(1)\rangle_{\rm out}.$$
6. Thus, notice that this gives a unitary(?)* S-matrix, in terms of the states$|p^-(\Omega)\rangle_{\rm in}$ and $|p^+(\Omega)\rangle_{\rm out}$.

* You must allow **all** values for $u^\pm(\Omega)$ and $p^\pm(\Omega)$!!

The Penrose diagram



The
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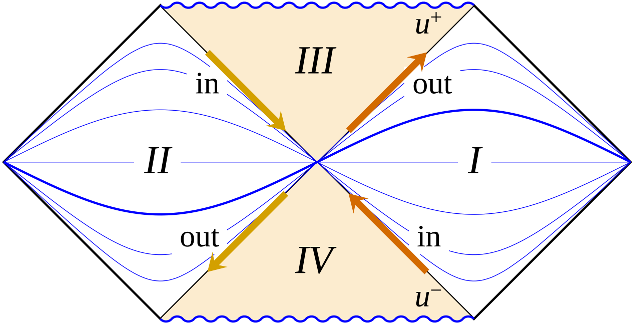
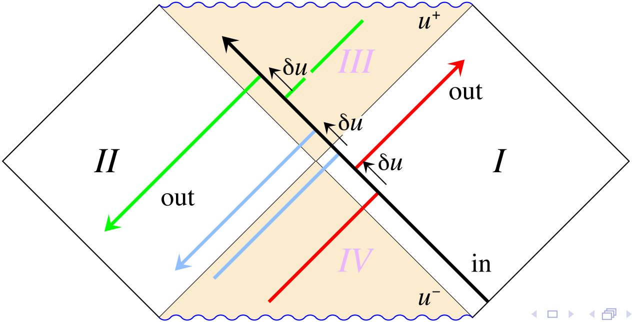




The sum of all momenta going in, drag the positions of all out-going things.

This leads to a *linear* algebra relating the momenta of the in-going particles to the positions of the out-going ones.

These obey simple commutation relations,

$$[\,\frac{1}{N}\sum_i u_i\,,\ \sum_j p_j\,] = i$$

Integrate all particles over the horizon.

A local algebra for the total momentum densities and the average position operators, as distributions on the horizon:

$$u^-_{\rm out}(\Omega) \;=\; \int \mathrm{d}^2\Omega'\, f(\Omega,\,\Omega')\, p^-_{\rm in}(\Omega')\ ,$$

This equation allows us to <u>transform</u> in-going particles into out-going particles

$$(\Delta_\Omega - 1)\, f(\Omega,\,\Omega') \;=\; -8\pi G\, \delta^2(\Omega,\,\Omega')$$

$$\begin{aligned}
(\Delta_\Omega - 1)u^-_{\rm out}(\Omega) &= -\,8\pi G\, p^-_{\rm in}(\Omega)\ ; && (1)\\
[u^-_{\rm out}(\Omega),\, p^+_{\rm out}(\Omega')] &= [u^+_{\rm in}(\Omega),\, p^-_{\rm in}(\Omega')] \;=\; i\,\delta^2(\Omega,\,\Omega')\ . && (2)\\
[u^+_{\rm in}(\Omega), u^-_{\rm out}(\Omega')] &= if(\Omega,\,\Omega')\ ; && (3)\\
(\Delta_\Omega - 1)u^+_{\rm in}(\Omega) &= +\,8\pi G p^+_{\rm out}(\Omega)\ . && (4)
\end{aligned}$$

This algebra is extremely simple, but also tricky.
How to interpret the sign switch $\mathrm{in} \leftrightarrow \mathrm{out}$?
(it *is* correct, in our notation)

All states, both in the initial and the final black hole, are a representation of this algebra.

The relation between in- and out- is now not much more than a Fourier transformation: $u^{\pm} \leftrightarrow p^{\mp}$, with $p = -i\frac{\partial}{\partial u}$

Is that all ? **NO !**

There is a complication.

Let's calculate the representation:

Do the partial wave expansion

Partial waves on the spherical black hole:

$$\begin{aligned}\begin{pmatrix} u \\ p \end{pmatrix}(\theta,\varphi) &= \sum_{\ell,m} \begin{pmatrix} u_{\ell,m} \\ p_{\ell,m} \end{pmatrix} Y_{\ell,m}(\theta,\varphi) \\ [u^{\pm}_{\ell,m}\,,\ p^{\mp}_{\ell',m'}] &= i\delta_{\ell\ell'}\,\delta_{mm'} \\ (1-\Delta_\Omega)f(\Omega) = 8\pi G\,\delta^2(\Omega) &\to (\ell^2+\ell+1)f_{\ell,m} = 8\pi G\ .\end{aligned}$$

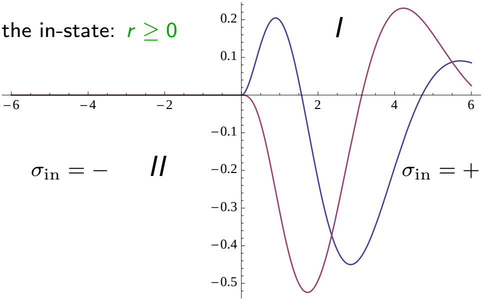
$$\boxed{u^{\pm}_{\ell,m} = \mp\frac{8\pi G/R^2}{\ell^2+\ell+1}\,p^{\pm}_{\ell,m}}$$

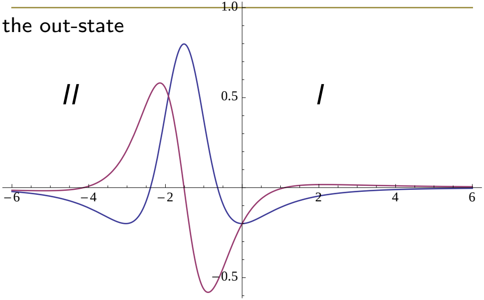
The out-going wave is the Fourier transform of the in-going wave.

Indeed, if $u_{\rm in}$ approaches horizon at $u=0$ as: $u_{\rm in}\to\lambda u_{\rm in}$, then $u_{\rm out}$ will be driven out: $u_{\rm out}\to\lambda^{-1}u_{\rm out}$

Partial wave expansion in black hole: at given ℓ, m, the out-state is the Fourier transform of the in-state.

However, Radius $r \geq 0$ and $r < 0$ both occur!





In the 1920s, partial, spherical wave expansions showed how the quantum dynamics of the hydrogen atom can be understood.

Now, we see that partial, spherical wave expansions solve the quantum dynamics of a black hole.

The underlying mathematics is almost the same,
but the physics is different:

In an atom, we expand the wave function;
in the black hole we expand the local momentum density.

The connection between regions I and II is called

the Einstein - Rosen bridge.

The ER bridge seems to connect *two* black holes.

To restore unitarity for a single black hole, we have only one option:

region II describes the points on the horizon that are *antipodal* to the points in region I.

Sanchez-Whiting 1986/87

This gives us the Schrödinger equation (generates dilatations):

$$H = \{u^+ p^-\} = -\{u^- p^+\}$$

with boundary conditions as given in the algebra.

$$\boxed{\begin{pmatrix}\psi_+^{\rm out}\\ \psi_-^{\rm out}\end{pmatrix} = \frac{1}{\sqrt{2\pi}}\Gamma(\tfrac12 - i\kappa)\, e^{i\phi_\ell(\kappa)} \begin{pmatrix} e^{-\frac12\pi\kappa} & ie^{+\frac12\pi\kappa}\\ ie^{+\frac12\pi\kappa} & e^{-\frac12\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\rm in}\\ \psi_-^{\rm in}\end{pmatrix}}$$

To be read as follows: If κ is the new momentum variable, at each ℓ, m, we have a wave function $\psi_\sigma^{\rm in}(\kappa)$, and $\psi_\sigma^{\rm out}(\kappa)$ related by this unitary transformation matrix

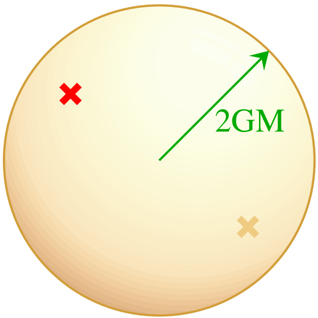
the two signs, $\sigma = \pm 1$ *mix.*

This means that our particles fill region *I* as well as region *II* in the Penrose diagram.

This BH universe has two asymptotic regions.

I and *II* *talk* to each other ! What does this mean?

If you could move faster than light, and you entered a black hole at one side,



you would re-emerge at the other side, CPT inverted!

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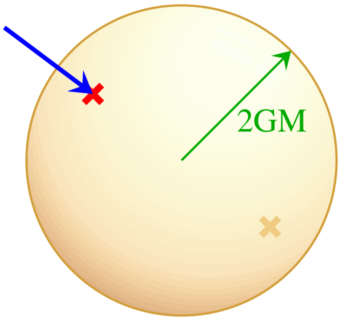

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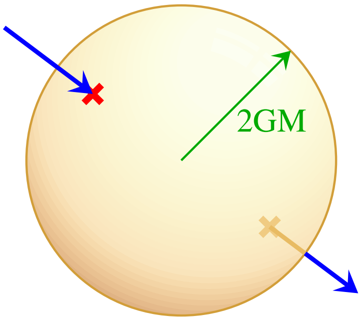

you would re-emerge at the other side, *CPT* inverted!

This has deep and interesting implications for the nature of space-time

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but more work must be done . . . we will have to be much more precise in our analysis !

$— \quad \sim \quad —$

ℓ must be odd!

$$\begin{aligned} u^{\pm}(\theta,\varphi) &= -u^{\pm}(\pi-\theta,\,\varphi+\pi) \\ p^{\pm}(\qquad) &= -p^{\pm}(\qquad\qquad\qquad) \end{aligned}$$

only obeyed at odd ℓ:

$$Y_{\ell m}(\pi-\theta,\,\varphi+\pi) = (-1)^{\ell}\,Y_{\ell m}(\theta,\varphi)$$

Note:

Antipodal identification of points on the horizon (*only* on the horizon, not in the bulk!) leads to *100% entanglement* of the Hawking particles at antipodal ponts, in principle an observable property.

We can calculate the BH microstates – except for the cut-off at $\ell \approx M_{\rm Planck}\, R_{BH}$. Taking discrete points (θ, φ) on the horizon, black holes have hair: one hair at every θ, φ, with end points on the tortoise coordinates: $u_{\rm out}$ describes exponentially growing hair, $u_{\rm in}$ describes exponentially shrinking hair.

At every point on the horizon, there is only one hair.

On the scalp, there is also a stationary (*fermionic*) degree of freedom, the sign function $\sigma_{\rm in}$ or $\sigma_{\rm out}$. These obey conservation laws, but $\sigma_{\rm in}$ and $\sigma_{\rm out}$ do not commute.

$\sigma_{\rm in}(\ell, m)$ and $\sigma_{\rm out}(\ell, m)$ form fermionic hair at fixed lengths.

There is no black hole interior; region *II* of the Penrose diagram – with causality reflected in time – describes the antipodal part. When black hole forms, the interior region seems to be physical but of course it is invisible. Only when the black hole decays, one realises that the interior was never there.

Particles crossing the black hole interchange
position with momentum and back.

The essential observation that the spherical partial waves of matter all return their quantum information independently, allows for new assessments of space-time properties that was not possible before.

All this could have been discovered decades ago.
String theory was not used – but may enter later –

Discussion.

We think that the apparent entanglement of regions I and II of the Penrose diagram is an important discovery. Before, we had been content with abstract functional integral expresions, which did not disclose so clearly the fact that one cannot ignore region II. Lesson learned: Whenever more explicit calculations are possible, we should do them; they yield much more understanding of what goes on.
Previous calculations did not lead to answers as explicit as what we have now.

Yet our work is far from finished:

(1) We now have the microstates. They are in a basis forced upon us by the demand of black hole unitarity. A cut-off is needed limiting ℓ to a maximum value ($\ell < \mathcal{O}(R)$ in Planck units). How can we understand this cut-off? Ans how is our basis expressed in terms of the Fock space states of a "Standard Model"?

(2) Some authors suggest that ours is "merely a semiclassical calculation", and that we ignored "quantum corrections" but we disagree, as the operators $p^-(\tilde{x})$ at different $\tilde{x}$ all commute. At small values of ℓ, our expressions should be very precise, but at ℓ close to the Planck length, we do expect deviations due to shifts arising from the transverse momenta.

(3) The representation of our algebra is different from Fock space, and therefore difficult to match with the Standard Model states. Trying to do this properly will be extremely important. It could lead to constraints on the SM coming from quantum gravity.

(4) Other forces between in- and out- particles can be considered: electro-magnetism and non-Abelian forces. At the functional integral level, this was done in Ref. (4). We could try to do this more explicitly now.

References:

The importance of the gravitational shift (Shapiro delay) was already emphasized in:
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Supertranslations:
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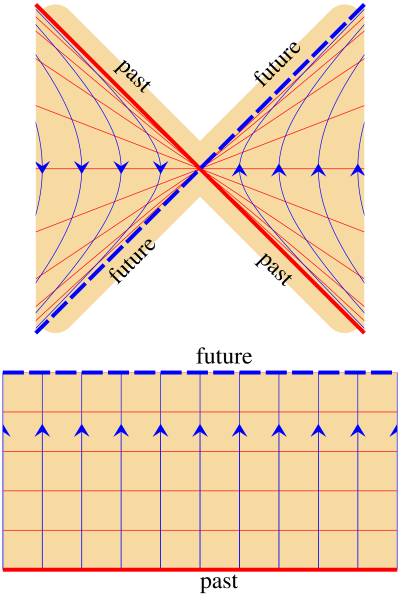
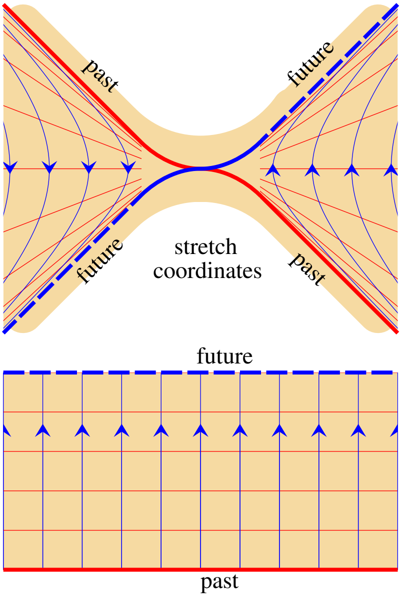
(6) Diagonalizing the Black Hole Information Retrieval Process, arXiv:1509.01695
(7) Black hole unitarity and antipodal entanglement, arXiv:1601.03447[gr-qc] v3
(8) L. Mersini-Houghton, Entropy of the Information Retrieved from Black Holes, arXiv:1511.04795 [hep-th]
(9) The Quantum Black Hole as a Hydrogen Atom: Microstates without Strings Attached, arXiv:1605.05119

<u>THE END</u>



Black Hole Formation

How to connect flat space-time to a black hole space-time to describe gravitational collapse.



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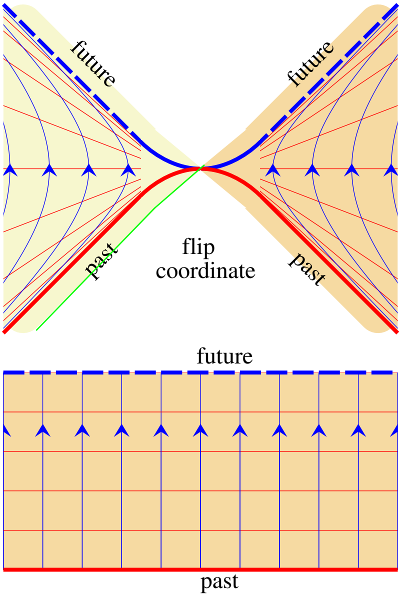

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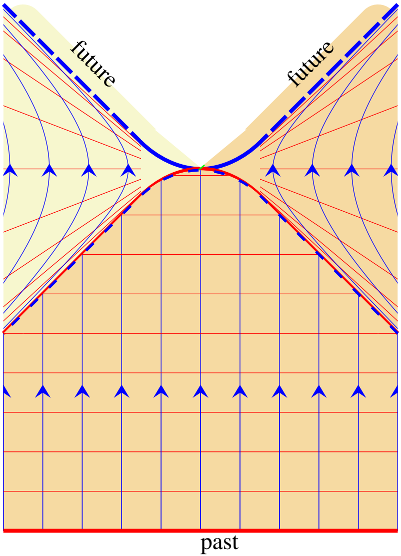

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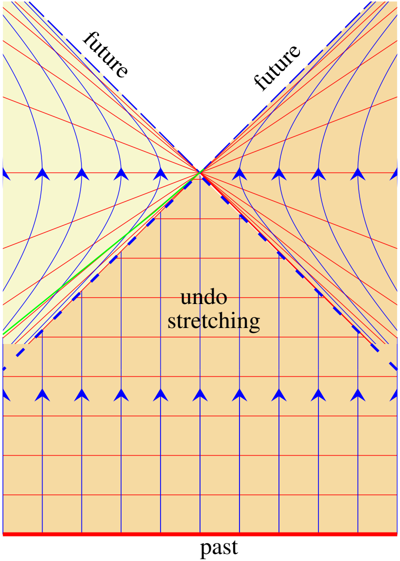

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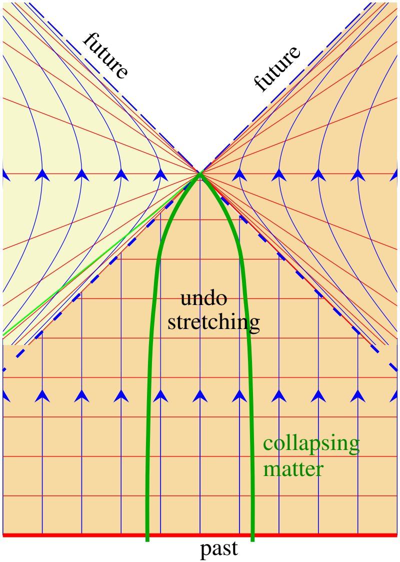

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