Effective theories for the dynamics of black branes in supergravity

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Based on:
arXiv:1606.09644 by JA, J.Gath, N.Obers, V. Niarchos, A.V.Perdersen

Motivation

- Construct new perturbative black hole solutions in any gravity theory with black branes

Context

SUPERGRAVITY BACKGROUND

$g_{\mu\nu}(L)$

(1) Same asymptotics $\frac{r_0}{R} \ll 1$

(2) Different asymptotics $\frac{r_0}{L} \ll 1$

Context

arXiv:hep-th/9803131 by E.Witten

arXiv:0712.2456 by S. Bhattacharyya, V.E.Hubeny, S. Minwalla and M. Rangamani

arXiv:0708.1770 by S. Bhattacharyya, S. Lahiri, R. Loganayagam & S. Minwalla

Context

FLUID

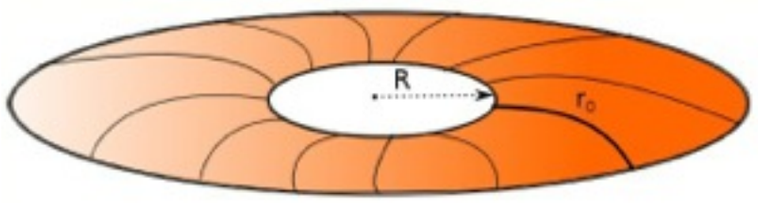
AdS boundary

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

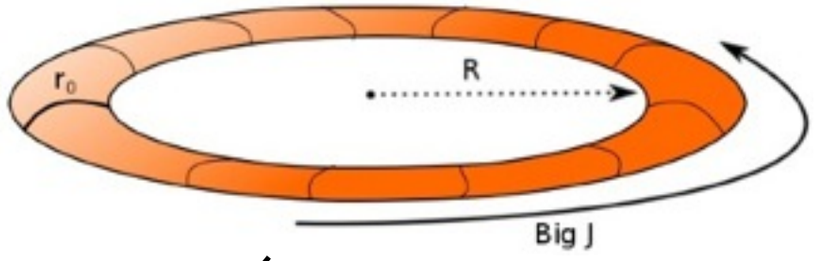
planar black hole (AdS black brane)

arXiv:hep-th/0104066 by Policastro, Son and Starinets.

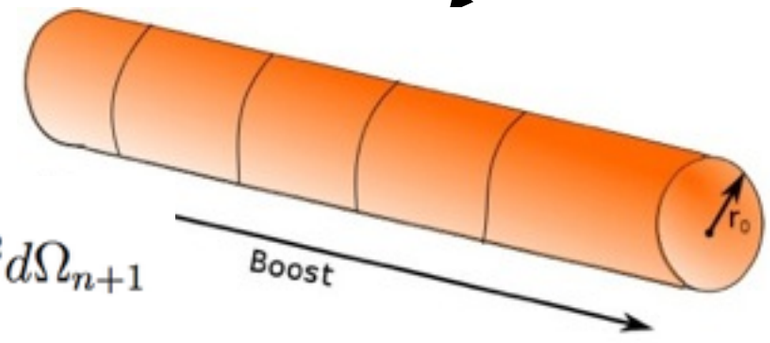
Context



increase J



zoom locally



$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}$$

arXiv:0708.2181 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers & M.Rodriguez

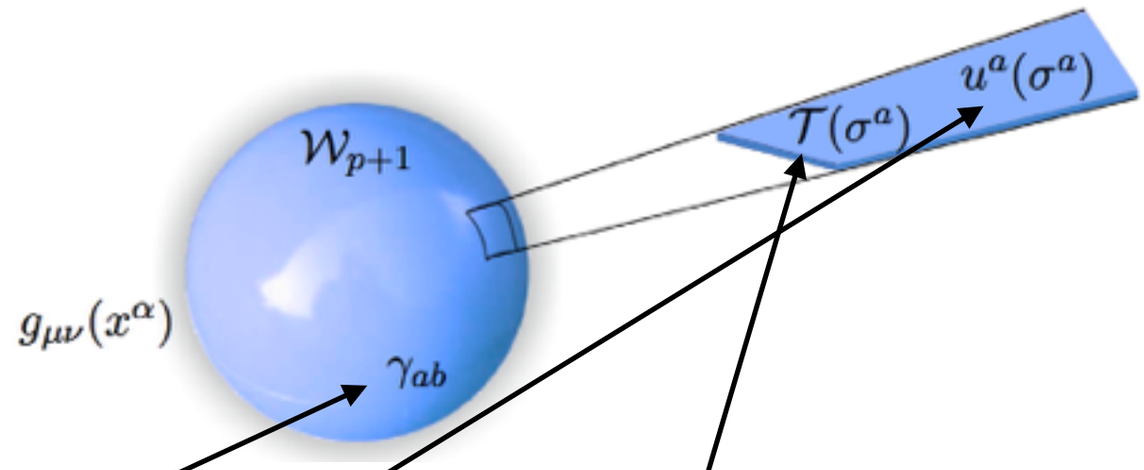
The effective theory

Part I: Review

$$G_{\mu\nu} = 0$$

The effective theory

Neutral black brane:



$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1} \; + \; \ldots$$

The effective theory

Read off the stress tensor from the metric:

$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}$$

$$T^{ab} = P\eta^{ab} + (\epsilon + P)u^a u^b$$

$$T^{\mu\nu} = \int_{\mathcal{W}} \sqrt{-\gamma} T^{ab} u_a^\mu u_b^\nu \delta(x^\alpha - X^\alpha)$$

$$\frac{\epsilon}{n+1} = -P = \frac{\Omega_{(n+1)} r_0^n}{16\pi G}$$

arXiv::0910.1601 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers

The effective theory

Minkowski

$$G_{\mu\nu}|_{\mathbf{lin}} = T_{\mu\nu}$$

$r>>r_o$

$$T^{\mu\nu} = \int_{\mathcal{W}} \sqrt{-\gamma} T^{ab} u_a^\mu u_b^\nu \delta(x^\alpha - X^\alpha)$$

The effective theory

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_a T^{ab} = 0 \qquad T^{ab} K_{ab}{}^{i} = 0$$

The effective theory

Minkowski

$$G_{\mu\nu}|_{\mathbf{lin}} = T_{\mu\nu}$$

$r>>r_0$

$r_0<<r<<\mathcal{R}$

$$G_{\mu\nu} = 0$$

$r<<\mathcal{R}$

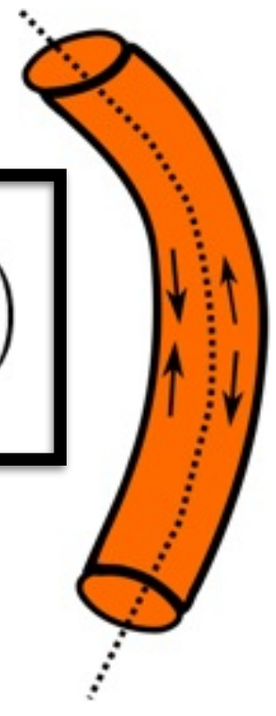
locally Schwarzschild

The effective theory

Perturbing the brane:

$$ds^2_{(1)} = \left(\eta_{ab} - 2K_{ab}{}^{\hat{i}} r\cos\theta + \frac{r_0^n}{r^n}u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1-\frac{r_0^n}{r^n}} + r^2 d\theta^2 + r^2\sin^2\theta d\Omega^2_{(n)}$$
$$+ h_{\mu\nu}(r,\theta)dx^\mu dx^\nu + \mathcal{O}\left(r^2/R^2\right) \quad .$$

$$T^{\mu\nu} = \int_{\partial\mathcal{W}} \sqrt{-\gamma}\left(T^{ab}u_a^\mu u_b^\nu \delta(x) + \mathcal{D}^{abi}u_a^\mu u_b^\nu \partial_i\delta(x) + u_a^{(\mu}\mathcal{S}^{a\nu)i}\partial_i\delta(x)\right)$$



arXiv:0708.2181 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers & M.Rodriguez

arXiv:11104835 by JA. J.Camps, T.Harmark & N.A. Obers

arXiv:1201.3506 by J. Camps & R. Emparan

The effective theory

Effective action:

arXiv:1203.3544 by Bhattacharya et al.

$$I[X^i] = \int_{\mathcal{W}} \sqrt{-\gamma}\Big(P + \boxed{v_1 \mathfrak{a}^2 + v_2 \omega^2 + v_3 \mathcal{R}}$$
$$+\lambda_1 K^i K_i + \lambda_2 K^{abi} K_{abi} + \lambda_3 u^a u^b K_a{}^{ci} K_{bci}$$
$$+\varpi_1 u^a \omega_a + \varpi_2 u^a \omega_a u^b \omega_b \Big)$$

$$\nabla_a \left(e^b u^\mu{}_b \right) = u^{\mu b} \gamma_{ab}{}^c e_c + n^\nu{}_i K_{ab}{}^i e^a \quad ,$$
$$\nabla_a \left(n^i n^\mu{}_i \right) = -u^{\mu b} K_{ab}{}^i n_i - n^\mu{}_j \omega_a{}^{ij} n_i$$

$$\omega_a = \epsilon_{ij} \omega_a{}^{ij}$$

arXiv:1304.7773 and arXiv:1312.0597 by JA
arXiv:1406.7813 by JA & T.Harmark

The effective theory

From here one can extract the bending moment and the spin current:

$$\mathcal{D}^{ab}{}_{i} = \frac{\partial I}{K_{ab}{}^{i}} = \mathcal{Y}^{abcd} K_{cdi} \qquad \mathcal{S}^{a}{}_{ij} = \frac{\partial I}{\omega_{a}{}^{ij}}$$

$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 u^{(a}\gamma^{b)(c}u^{d)}\right)$$

$$\mathcal{S}^{aij} = \varpi_1 u^a \epsilon^{ij} + \varpi_2 u^b \omega_b u^a \epsilon^{ij}$$

arXiv:1304.7773 by JA
arXiv:1312.0597 by JA

The effective theory

The stationary sector: $u^a = \dfrac{\mathbf{k}^a}{\mathbf{k}} \qquad T = \mathcal{T}\mathbf{k}$

$$I = \int_{\mathcal{W}} \sqrt{-\gamma} P$$

Equations of motion: $K^i = n u^a u^b K_{ab}{}^i \qquad \mathbf{k}|_{\partial \mathcal{W}_{p+1}} = 0$

The effective theory

$$K^i = n u^a u^b K_{ab}{}^i \qquad \mathbf{k}|_{\partial \mathcal{W}_{p+1}} = 0$$

In two dimensions, only the plane and the helicoid solve these equations.

$$K_{ab}{}^i = 0$$

$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + d\rho^2 + \rho^2 d\phi^2$$

$$\mathbf{k}^2 = 1 - \Omega^2 \rho^2$$

arXiv:0912.2352 by R. Emparan, T. Harmark, V. Niarchos & N. Obers
arXiv:1503.08834 by JA & M. Blau

The effective theory

The helicoid:

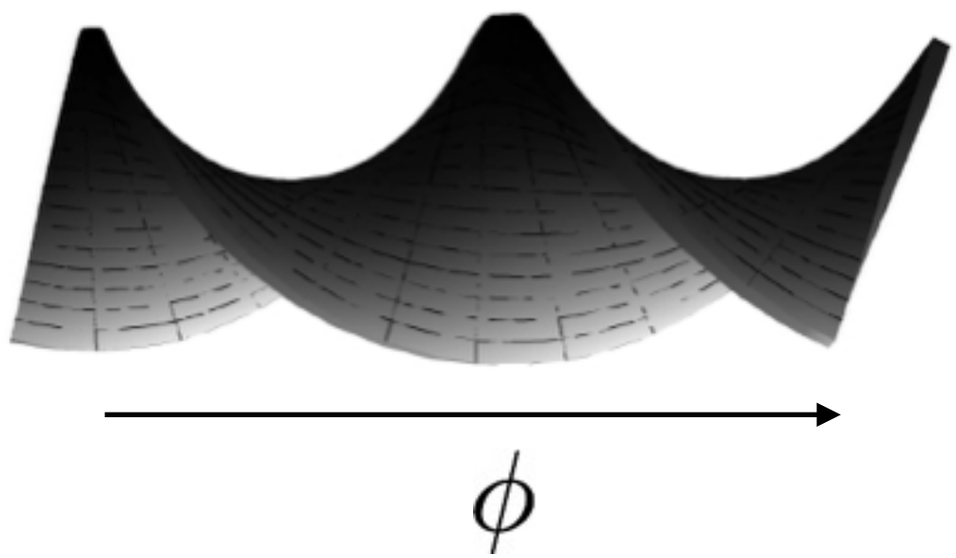
$$\gamma_{ab}d\sigma^a d\sigma^b = -d\tau^2 + d\rho^2 + (\lambda^2 + \rho^2)d\phi^2$$

$$\mathbf{k}^2 = 1 - \Omega^2(\lambda^2 + \rho^2)$$

$$\mathcal{F}[R] = \frac{\Omega_{(n+1)}}{16\sqrt{\pi}G}\frac{r_+^n}{a\Omega}\int d\phi\lambda\Gamma\left(1+\frac{n}{2}\right)(1-\lambda^2\Omega^2)^{\frac{n+1}{2}}\ {}_2\tilde{F}_1\left(-\frac{1}{2},\frac{1}{2};\frac{n+3}{2};1-\frac{1}{\lambda^2\Omega^2}\right)$$

Topology: $\mathbb{R}\times\mathbb{S}^{D-3}$

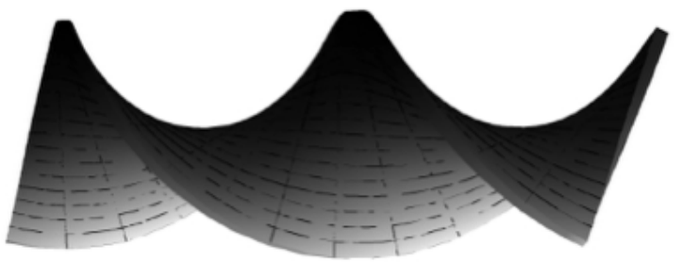
arXiv:1503.08834 by JA & M. Blau

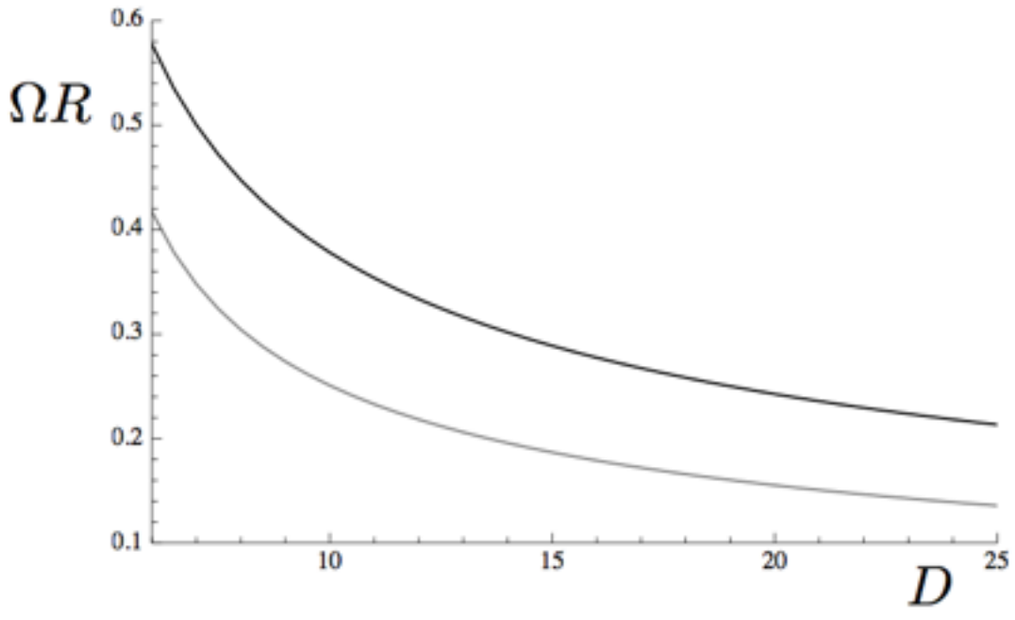


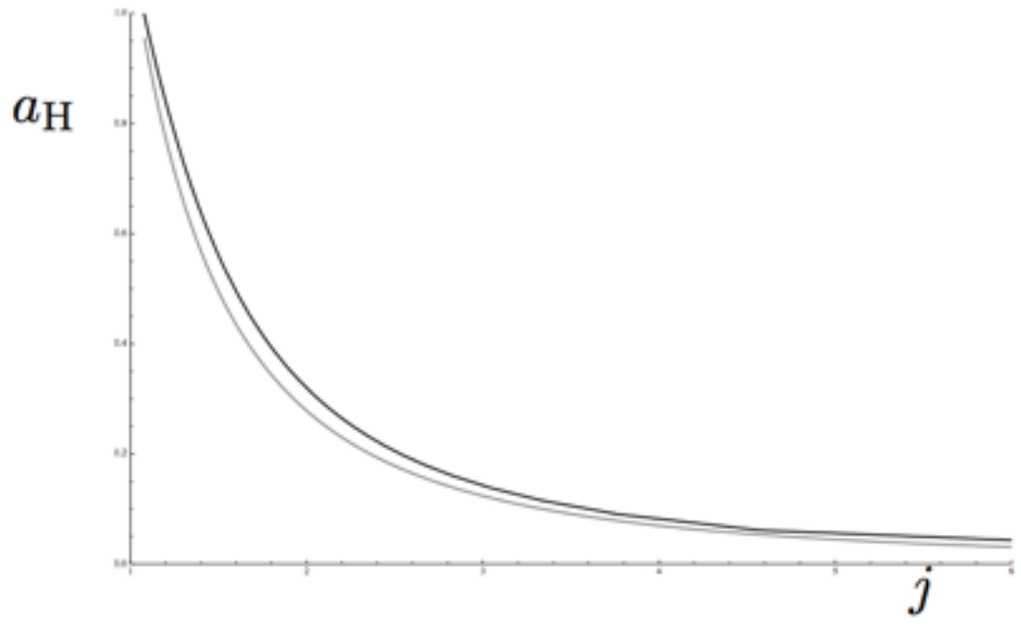
The effective theory

Helicoidal black ring in D>5:

$$\frac{1-(\tilde{n}+1)\Omega^2R^2}{(1-\Omega^2R^2)}-\frac{{}_2\tilde{F}_1\left(\frac{1}{2},\frac{3}{2};\frac{\tilde{n}+4}{2};-\frac{1-\Omega^2R^2}{\Omega^2R^2}\right)}{2\Omega^2R^2\,{}_2\tilde{F}_1\left(-\frac{1}{2},\frac{1}{2};\frac{\tilde{n}+2}{2};-\frac{1-\Omega^2R^2}{\Omega^2R^2}\right)}=0$$





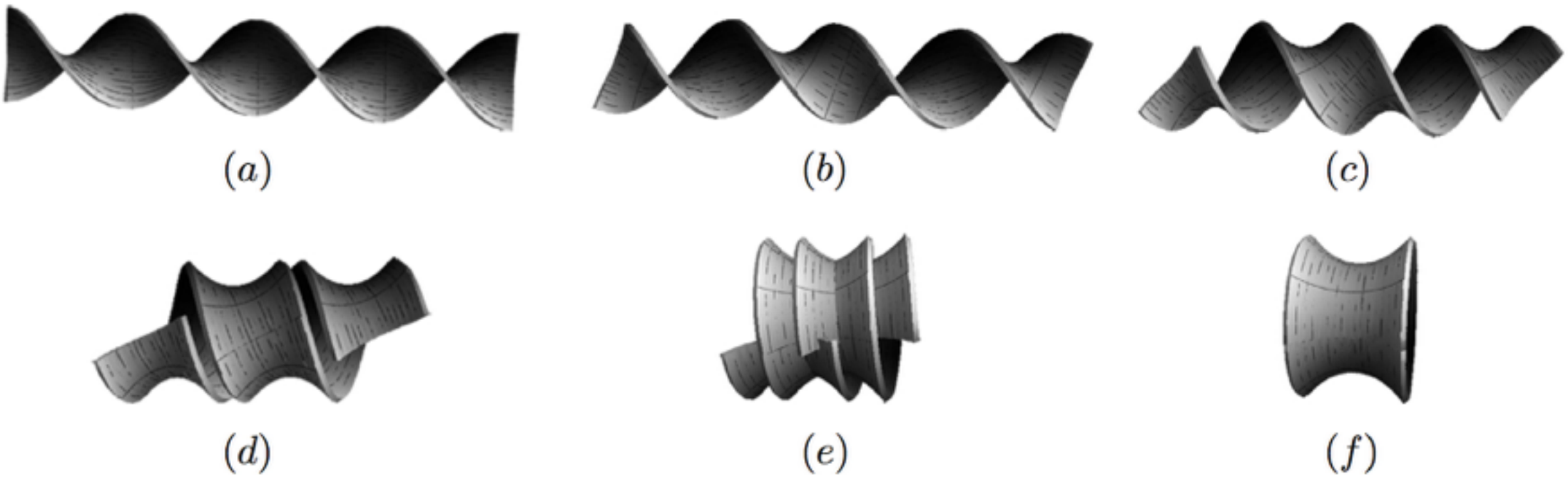


arXiv:1504.01393 by JA & M. Blau

The effective theory

Black Scherk Surfaces/Catenoids in Plane Waves:

$$ds^2 = -(1 + A(x^q))dt^2 + (1 - A(x^q))dy^2 - 2A(x^q)dtdy + d\mathbb{E}^2_{(D-2)}(x^q)$$



arXiv:1503.08834 by JA & M. Blau

Supergravity

$$G_{\mu\nu} \qquad C_p$$

$$\phi$$

Supergravity

SUPERGRAVITY BACKGROUND

$g_{\mu\nu}(L)$

(1) Same asymptotics $\frac{r_0}{R} \ll 1$

(2) Different asymptotics $\frac{r_0}{L} \ll 1$

Supergravity

Consider the democratic formulation:

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}_{10}}\left[\star R - \frac{1}{2}d\phi\wedge\star d\phi - \frac{1}{2}e^{-\phi}H_{(3)}\wedge\star H_{(3)} - \frac{1}{4}\sum_q e^{a_q\phi}\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}\right]$$

$$G_{\mu\nu} = 8\pi G T_{M\mu\nu}$$

$$\tilde{F}_{q+2} = (-1)^{[q/2]+1}e^{-a_q\phi}\star\tilde{F}_{D-q-2}$$

$$16\pi G\mathbf{T}^{\mu\nu}_{(F)} = \frac{e^{a_q\phi}}{(q+1)!}\left(F_{q+2}^{\mu\mu_1\ldots\mu_{q+1}}F_{q+2}{}^{\nu}{}_{\mu_1\ldots\mu_{q+1}} - \frac{1}{2(q+2)}g^{\mu\nu}F^2_{q+2}\right)$$

$$16\pi G\mathbf{T}^{\mu\nu}_{(\phi)} = \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\lambda\phi\partial^\lambda\phi \quad .$$

Supergravity

SUPERGRAVITY BACKGROUND

$$r \gg r_0$$

$$(G_{\mu\nu} - 8\pi G \mathbf{T}_{\mathbf{M}\mu\nu}) \,|_{\mathbf{linear}} = 8\pi G \mathbf{T}_{\mu\nu}$$

Supergravity

Coupling to the democratic formulation:

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}_{10}}\left[\star R - \frac{1}{2}d\phi\wedge\star d\phi - \frac{1}{2}e^{-\phi}H_{(3)}\wedge\star H_{(3)} - \frac{1}{4}\sum_q e^{a_q\phi}\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}\right]$$

$$G_{\mu\nu} = 8\pi G\left(\mathbf{T}_{\mathbf{M}\mu\nu} + \mathbf{T}_{\mu\nu}\right) \longrightarrow \nabla_\mu \mathbf{T}^{\mu\nu} = -\nabla_\mu \mathbf{T}^{\mu\nu}_{\mathbf{M}}$$

$$d\left(\star(e^{-\phi}H_{(3)}) - \sum_q \frac{1}{2!}e^{a_q\phi}\left[\star\tilde{F}_{(q)}\wedge C_{(q-3)}\right]\right) = -16\pi G\star j_{(2)} \quad ,$$

$$d\star\left(e^{a_q}\tilde{F}_{(q)}\right) + (-1)^q e^{a_{q+2}\phi}\left[\star\tilde{F}_{(q+2)}\wedge H_{(3)}\right] = (-1)^q 16\pi G\star J_{(q-1)} \quad ,$$

$$\Box\phi + \frac{1}{2}e^{-\phi}\star\left(H_{(3)}\wedge\star H_{(3)}\right) - \sum_q \frac{a_q}{4}e^{a_q\phi}\star\left(\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}\right) = -16\pi G j_\phi$$

Supergravity

Effective equations:

$$G_{\mu\nu} = 8\pi G\left(\mathbf{T}_{\mathbf{M}\mu\nu} + \mathbf{T}_{\mu\nu}\right) \longrightarrow \nabla_\mu \mathbf{T}^{\mu\nu} = -\nabla_\mu \mathbf{T}^{\mu\nu}_{\mathbf{M}}$$

$$d\left(\star(e^{-\phi}H_{(3)}) - \sum_q \frac{1}{2!}e^{a_q\phi}\left[\star\tilde{F}_{(q)}\wedge C_{(q-3)}\right]\right) = -16\pi G\star j_{(2)} \quad ,$$

$$d\star(e^{a_q}\tilde{F}_{(q)}) + (-1)^q e^{a_{q+2}\phi}\left[\star\tilde{F}_{(q+2)}\wedge H_{(3)}\right] = (-1)^q 16\pi G\star J_{(q-1)} \quad ,$$

$$\Box\phi + \frac{1}{2}e^{-\phi}\star(H_{(3)}\wedge\star H_{(3)}) - \sum_q \frac{a_q}{4}e^{a_q\phi}\star(\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}) = -16\pi G j_\phi$$

$$dH_3 = 16\pi G\star\mathbf{j}_6 \quad ,$$

$$d\tilde{F}_{q+2} - H_3\wedge\tilde{F}_q = 16\pi G\left(\star\boldsymbol{\mathcal{J}}_{D-q-3} - (\star\mathbf{j}_6\wedge C_{q-1})\right)$$

Supergravity

Effective equations:

$$\nabla_\mu \mathbf{T}^{\mu\nu} = \frac{1}{2!} H_3^{\nu\mu_1\mu_2} \mathbf{j}_{2\mu_1\mu_2} + \frac{e^{-\phi}}{6!} H_7^{\nu\mu_1\ldots\mu_6} \mathbf{j}_{6\mu_1\ldots\mu_6} + \mathbf{j}_\phi \partial^\nu \phi$$

$$+ \sum_q \frac{1}{(q+1)!} \left(\tilde{F}_{q+2}^{\nu\mu_1\ldots\mu_{q+1}} + (-1)^{q+1} \frac{q(q+1)}{2!} H_3^{\nu\mu_1\mu_2} C_{q-1}^{\mu_3\ldots\mu_{q+1}} \right) \mathbf{J}_{q+1\mu_1\ldots\mu_{q+1}}$$

$$+ \sum_q \frac{e^{a_q\phi}}{(\tilde{q}+1)!} \left(\tilde{F}_{\tilde{q}+2}^{\nu\mu_1\ldots\mu_{\tilde{q}+1}} + (-1)^{\tilde{q}+1} \frac{\tilde{q}(\tilde{q}-1)}{2!} H_3^{\nu\mu_1\mu_2} C_{\tilde{q}-1}^{\mu_3\ldots\mu_{q+1}} \right) \boldsymbol{\mathcal{J}}_{\tilde{q}+1\mu_1\ldots\mu_{\tilde{q}+1}}$$

$$+ \frac{1}{4!} \left(\tilde{F}_5^{\nu\mu_1\ldots\mu_4} + 3 H_3^{\nu\mu_1\mu_2} C_2^{\mu_3\ldots\mu_4} \right) \mathbf{J}_{4\mu_1\ldots\mu_4}$$

$$- \sum_q \frac{e^{a_q\phi}}{(q+2)!} \tilde{F}_{q+2}^{\mu_1\ldots\mu_{q+2}} [\star \mathbf{j}_6 \wedge C_{q-1}]^\nu{}_{\mu_1\ldots\mu_{q+2}} \quad ,$$

Supergravity

Current equations:

$$d\star\mathbf{J}_{q+1}+(-1)^{q+1}\star\mathbf{J}_{q+3}\wedge H_3+(-1)^{q+1}e^{a_{q+2}\phi}\star\tilde{F}_{q+4}\wedge\star\mathbf{j}_6=0\quad,\quad q=0,...,3$$

$$d\star\boldsymbol{\mathcal{J}}_{D-q-3}=0\quad,\quad q=-1,0\quad,\quad d\star\boldsymbol{\mathcal{J}}_{D-q-3}=H_3\wedge\star\boldsymbol{\mathcal{J}}_{D-q-1}\quad,\quad q=1,2\quad,$$

$$d\star\mathbf{j}_2=0\quad,\quad d\star\mathbf{j}_6=0\quad.$$

Supergravity

The Dp-F1 black brane:

$$\mathrm{d}s^2 = D^{\frac{1-p}{8}} H^{\frac{p-7}{8}} \left(-f u_a u_b + v_a v_b\right) \mathrm{d}\sigma^a \mathrm{d}\sigma^b + D^{\frac{9-p}{8}} H^{\frac{p-7}{8}} \left(\Delta_{ab} - v_a v_b\right) \mathrm{d}\sigma^a \mathrm{d}\sigma^b + D^{\frac{1-p}{8}} H^{\frac{p+1}{8}} \left(f^{-1}\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{8-p}\right) \ ,$$

$$B = e^{-a_{\mathrm{F1}}\varphi_0/2} \sin\xi \left(H^{-1} - 1\right) \coth\alpha \ u \wedge v \ ,$$

$$A_{p-1} = e^{-a_{p-2}\varphi_0/2} \tan\xi \left(DH^{-1} - 1\right) \ \star_{(p+1)} \left(u \wedge v\right) \ ,$$

$$A_{p+1} = e^{-a_p\varphi_0/2} \cos\xi \left(H^{-1} - 1\right) \coth\alpha \ \star_{(p+1)} \mathbf{1} \ ,$$

Supergravity

The Dp-F1 black brane:

$$\varepsilon = \frac{\Omega_{n+1}}{16\pi G} r_0^n (n+1+n\sinh^2\alpha) \ ,$$

$$P = -\frac{\Omega_{n+1}}{16\pi G}(1+n\sinh^2\alpha) \ , \quad P_\perp = -\frac{\Omega_{n+1}}{16\pi G}(1+n\sinh^2\alpha\cos^2\xi)$$

$$T_{ab} = \varepsilon u_a u_b + P v_a v_b + P_\perp \perp_{ab}$$

$$j_2 = \mathcal{Q}_{\mathrm{F1}} u \wedge v \ , \quad J_{p+1} = \mathcal{Q}_p *_{p+1} \mathbf{1}$$

$$\mathcal{Q}_{p-2} = \Phi_p \mathcal{Q}_{\mathrm{F1}} = \Phi_{\mathrm{F1}} \mathcal{Q}_p \ , \quad J_{p-1} = \mathcal{Q}_{p-2} *_{p+1} (u \wedge v)$$

Supergravity

Effective equations:

$$\nabla_a T^{ab} = \frac{1}{(p-1)!}\left[\mathcal{F}_p^{ba_1\ldots a_{p-1}} J_{p-1 a_1\ldots a_{p-1}} + (-1)^{p+1}\frac{1}{2}\mathcal{H}_3^{ba_1a_2}\mathcal{C}_{p-1}^{a_3\ldots a_{p+1}} J_{p+1 a_1\ldots a_{p+1}}\right.$$
$$\left. + \frac{1}{2}\mathcal{H}_3^{ba_1a_2} j_{2 a_1a_2} + j_\phi \partial^b \varphi\,,\right.$$

$$\nabla_a j_2^{ab} = 0\,, \quad \nabla_a J_{p-1}^{ab_1\ldots b_{p-2}} = \frac{1}{3!}\mathcal{H}_{3abc} J_{p+1}^{abcb_1\ldots b_{p-2}}\,, \quad \nabla_a J_{p+1}^{ab_1\ldots b_p} = 0\,,$$

$$j_\phi = \frac{1}{2}\left(a_{\mathrm{F}1}\mathcal{Q}_{\mathrm{F}1}\Phi_{\mathrm{F}1} + a_p Q_p \Phi_p\right)$$

Supergravity

Effective equations:

$$\mathcal{F}^{\mu}_{H} = \frac{1}{2}\mathcal{H}_3^{\mu\mu_1\mu_2}\left(j_{2\mu_1\mu_2} + (-1)^{p+1}\frac{1}{(p-1)!}\mathcal{C}_{p-1}^{\mu_3\dots\mu_{p+1}}J_{p+1\mu_1\dots\mu_{p+1}}\right)$$

$$\mathcal{F}^{\mu}_{F_p} = \frac{1}{(p-1)!}\mathcal{F}_p^{\mu\mu_1\dots\mu_{p-1}}J_{p-1\mu_1\dots\mu_{p-1}}\,,$$

$$\mathcal{F}^{\mu}_{\tilde{F}_{p+2}} = \frac{1}{(p+1)!}\tilde{\mathcal{F}}_{p+2}^{\mu\mu_1\dots\mu_{p+1}}J_{p+1\mu_1\dots\mu_{p+1}}\,, \quad \mathcal{F}^{\mu}_{\phi} = j_{\phi}\partial^{\mu}\varphi\ .$$

$$T^{ab}K_{ab}{}^{i} = n^{i}{}_{\mu}\left(\mathcal{F}^{\mu}_{H} + \mathcal{F}^{\mu}_{F_p} + \mathcal{F}^{\mu}_{F_{p+2}} + \mathcal{F}^{\mu}_{\phi}\right)$$

Supergravity

Using the partition function:

$$ln\mathcal{Z} = \beta I_{\mathrm{E}}[g_{\mu\nu}, A_{(p+1)}, \phi]$$

For a D3-brane, for example:

$$\ln \mathcal{Z} = -\beta \int_{\mathcal{B}_p} \left(\star_{(p)} P + \boldsymbol{Q}_p \mathbb{P}[\mathcal{C}_{q+1}] \right)$$

Supergravity

Taking the extremal limit:

$$P \to -e^{-a_p \varphi/2} \boldsymbol{Q}_p$$

$$S = -\frac{1}{\beta}\int dt \ln \mathcal{Z} = -T_p \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \; e^{-a_p\varphi/2}\sqrt{-\gamma} + T_p \int_{\mathcal{W}_{p+1}} \mathbb{P}[\mathcal{C}_{q+1}]$$

Summary and future work

- The method allows to scan for black hole horizons and topologies in higher dimensions
- The method can be applied to supergravity in flux backgrounds
- It would be interesting to find solutions to these equations in several setups
- Higher-order corrections can lead to a derivation of the DBI from gravity and to interesting transport coefficients

THANK

YOU