Causality in CFT

and

constraints on graviton couplings

Tom Hartman

Cornell University

INWARD BEYOND: BLACK HOLES AND EMERGENT SPACETIME

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1509.00014 and 1601.07904 with

Sachin Jain

Sandipan Kundu

and work in progress

(+ Nima Afkhami-Jeddi, Venkatesa Chandrasekharan, and Amir Tajdini)

MOTIVATION

Which classical theories of gravity can (possibly) be UV-completed?

Naively,

$$S = \frac{1}{G_N} \int \sqrt{-g} \left(R - 2\Lambda + \frac{\lambda_2}{(M_{new})^2} R^2 + \frac{\lambda_3}{(M_{new})^4} R^3 + \cdots \right)$$

But this is not correct.

Assuming Lorentz invariance in the UV, the dimensionless couplings λ_i must obey various constraints.

Two basic types of causality constraints:

1. Sign constraints. For example [Gruzinov, Kleban '06]:

$$\lambda_4 > 0$$

2. Fine-tuning constraints [Camanho, Edelstein, Maldacena, Zhiboedov '14]:

$$\lambda_{2,3} \lesssim O(1)$$

ie, no large dimensionless ratio between the scale of coefficients and the scale of new physics. Certain higher curvature terms *must* be accompanied by new massive particles (or strings) at the same scale.

This is far below the perturbative unitarity constraint, due to the overall G_N — it is a constraint on the classical theory.

“Ultraviolet constraints on infrared couplings”

Derivations:

1. S-matrix + analyticity + optical theorem

2. Causality violation in nontrivial backgrounds

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi ’06]

[Bellazini, Cheung, Remmen ’15]

Both types of constraint also occur in scalar EFT:

$$\mathcal{L} = (\partial\phi)^2 + \frac{\lambda_4}{M^4}(\partial\phi)^4 + \frac{\lambda_8}{M^8}(\partial^2\phi)^4 + \cdots$$

must have

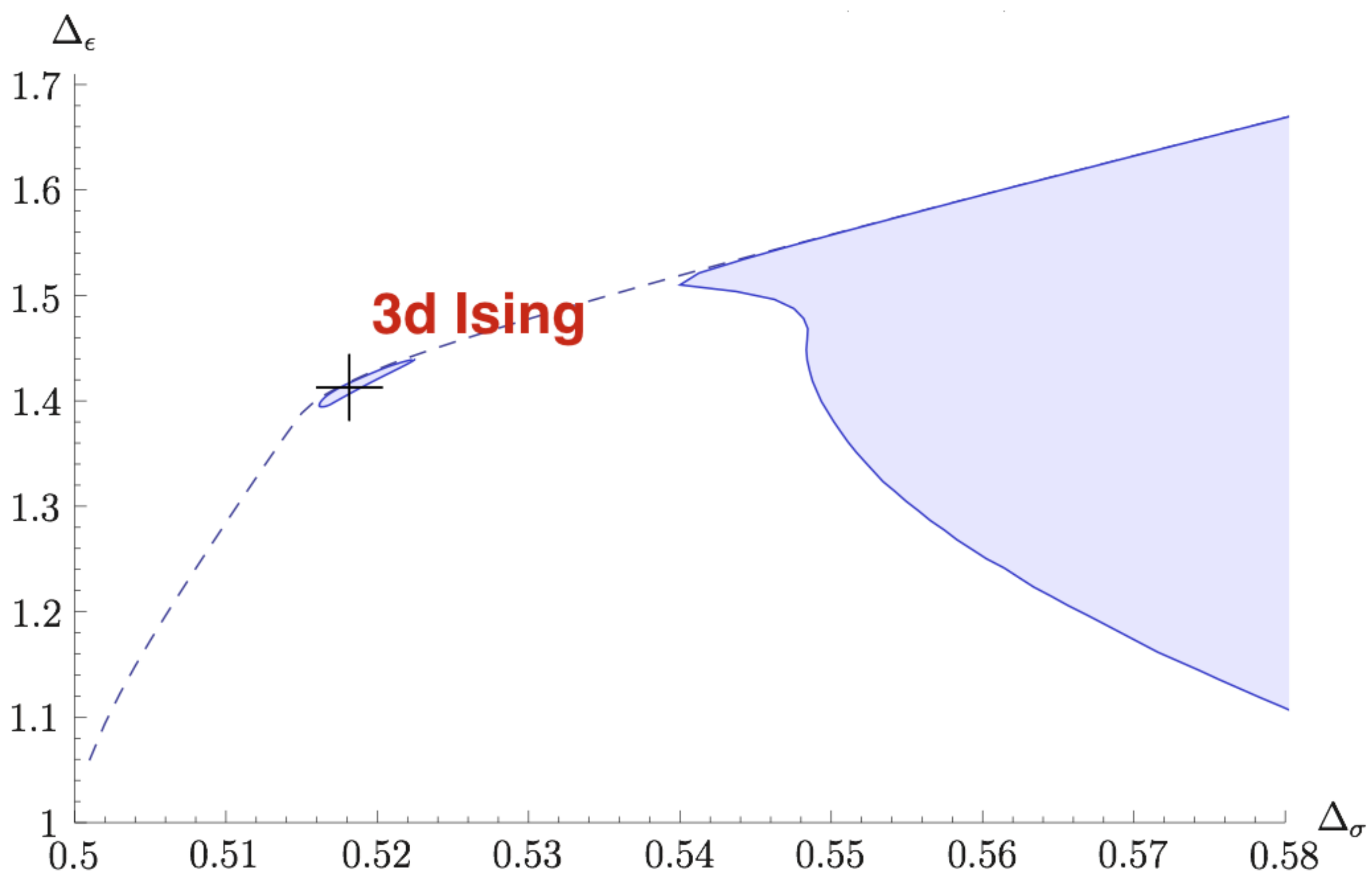
$$\lambda_4 > 0\ , \quad \lambda_8 \lesssim O(1), \quad \ldots$$

Comments:

- The $(\partial\phi)^4$ constraint plays a key role in the proof of the a-theorem [Komargodski, Schwimmer '11]
- The gravity constraints are even stronger: for example, the Gauss-Bonnet term in 5d *cannot appear* without new massive particles!
- This uniquely fixes the 3-graviton coupling in the IR; perhaps other couplings are also fixed?

These causality constraints are in weakly coupled theories, and the argument relies on the perturbative expansion.

Meanwhile, the **conformal bootstrap** constrains strongly interacting theories:



[Rattazzi, Rychkov, Tonni, Vichi '08; El-Showk et al; Kos, Poland, Simmons-Duffin; etc]

Goal of this talk is to merge these ideas in several examples:

Conformal bootstrap ⇒
Causality constraints at strong coupling

Constraints on infrared CFT data — spectrum and OPE coefficients — imposed by UV consistency.

By AdS/CFT, the causality constraints on R^2 gravity and $(\partial\phi)^4$ should map to constraints on CFTs.

The derivation is purely a QFT result, without assuming large-N or holography.

But at large N, it gives a CFT derivation for some of these causality constraints on EFT in AdS.

CAUSALITY REVIEW

$$\langle\Psi|[O(x),O(y)]|\Psi\rangle = 0 \qquad (x-y)^2 > 0$$

This is a Lorentzian statement.

But bootstrap is usually formulated in terms of Euclidean correlators.

So, first:

How is causality encoded in Euclidean correlators?

This was answered at least in principle long ago.

[ex: Streater and Wightman]

Euclidean correlators,

$$G(x_1, x_2, \dots) = \langle O(x_1)O(x_2)\cdots\rangle$$

are:

1. Permutation invariant

2. With singularities only at coincident points

3. and have no branch cuts (ie, single-valued)

Ex. conformal scalar:

$$\langle \phi(0)\phi(\tau, y)\rangle = (\tau^2 + y^2)^{-\Delta}$$

But in Lorentzian signature,

$$\langle \phi(0)\phi(t, y)\rangle = (-t^2 + y^2)^{-\Delta}$$

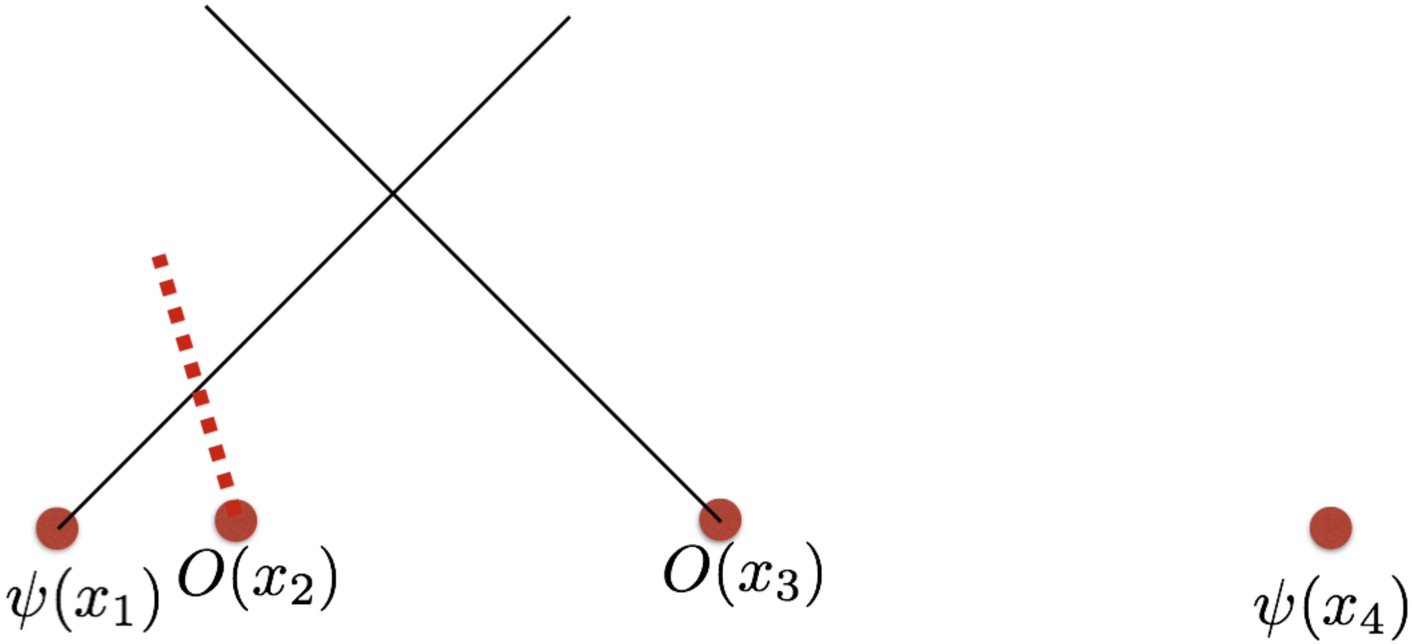
Lightcones $\Rightarrow$ *Branch cuts*

This leads to an ambiguity in analytic continuation. Different choices correspond to different operator orderings.

Commutator = discontinuity across the cut. *Ex:*

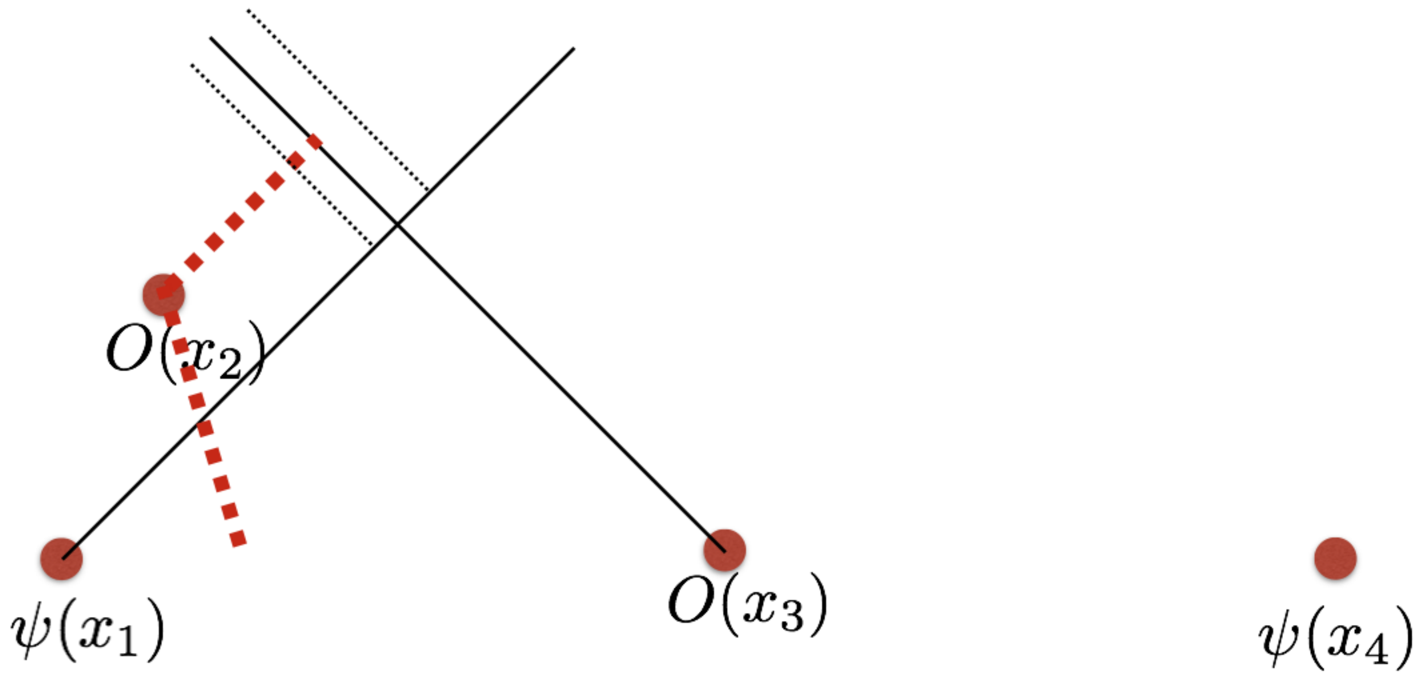
$$\langle\, [\phi(0), \phi(t,y)]\,\rangle = |t^2 - y^2|(e^{i\pi\Delta} - e^{-i\pi\Delta})$$

More generally, there is a branch cut whenever an operator crosses the lightcone of another operator.



The *first* branch point is always in the correct spot — on the Minkowski lightcone — by $SO(d)$ invariance of the Euclidean correlator.

But further singularities are *not fixed by symmetry*.



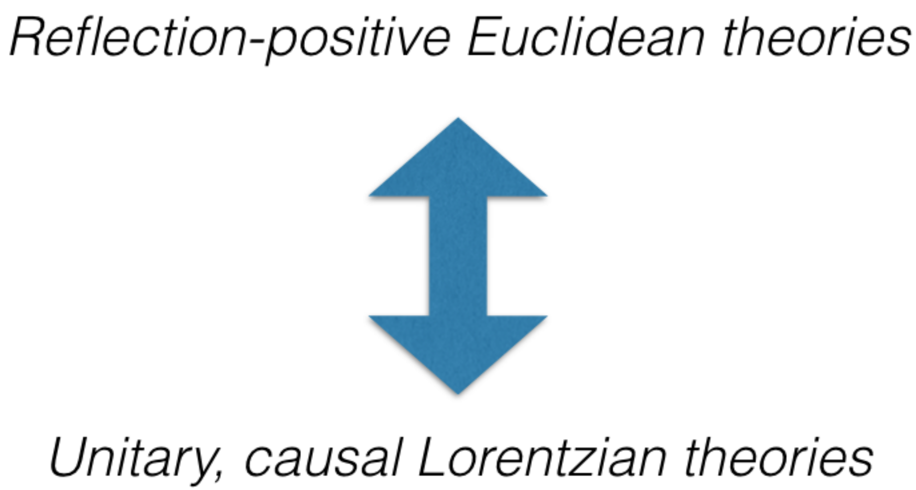
Shift in the branch cut $\Rightarrow$ time delay or time advance.

The upshot:

Causality requires the Euclidean correlator, upon analytic continuation, to be analytic on a certain domain of complexified spacetime.

This is not guaranteed by symmetry.

The extra ingredient in Euclidean QFT to ensure this is **reflection positivity**.



[Schwinger, Wightman, Osterwalder and Schrader, etc]

CFT

This was all in a general QFT.

In CFT, can be phrased in terms of conformal cross-ratios [Luscher, Mack '74]

$$z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad (1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

and causality is a question of where $G(z, \bar{z})$ is analytic in $C \times C$.

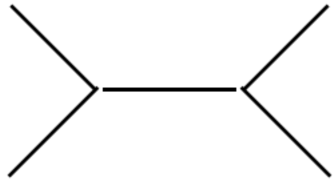
[Example later.]

Causality constraints in CFT [TH, Jain, Kundu '15-16]

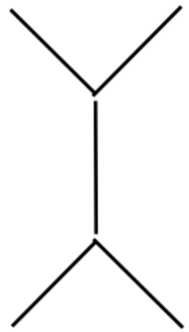
To derive constraints on CFT data, we will combine this with techniques from the conformal bootstrap.

Bootstrap:

$$G(z, \bar{z}) = \quad \sum$$



$$= \quad \sum$$



(Previous bootstrap results are mostly Euclidean — now we clearly need timelike separated operators.)

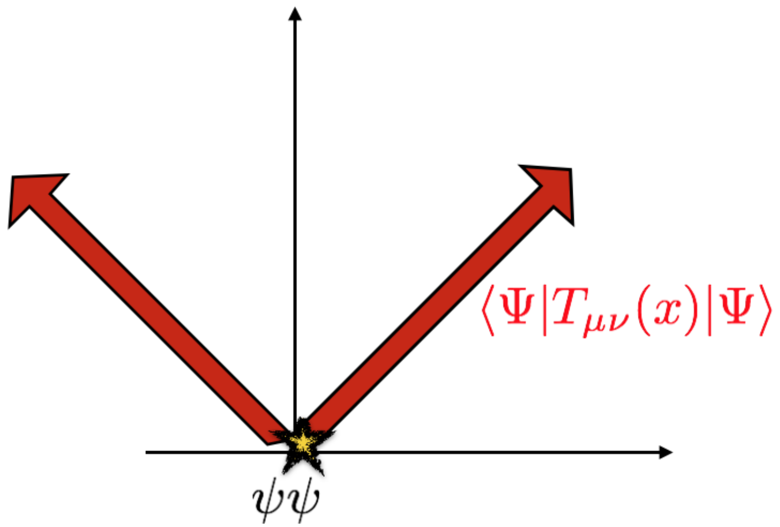
Shockwaves

Define the "shockwave state"

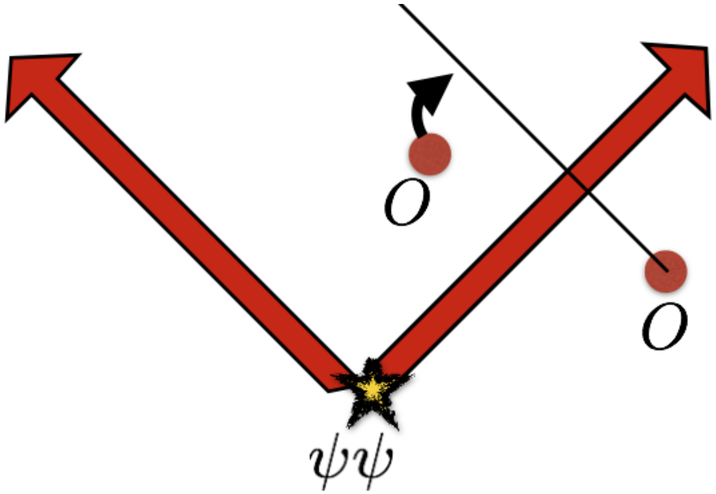
$$|\Psi\rangle = \psi(t = i\delta, \vec{x} = 0)|0\rangle$$

where ψ is a scalar primary.

For small δ, this state has a stress tensor supported on an expanding null shell:



Probe the shockwave with an operator O:



Causality:

$$\langle\Psi|[O,O]|\Psi\rangle = \text{disc.}\langle\psi(i\delta)O(x)O(y)\psi(-i\delta)\rangle$$

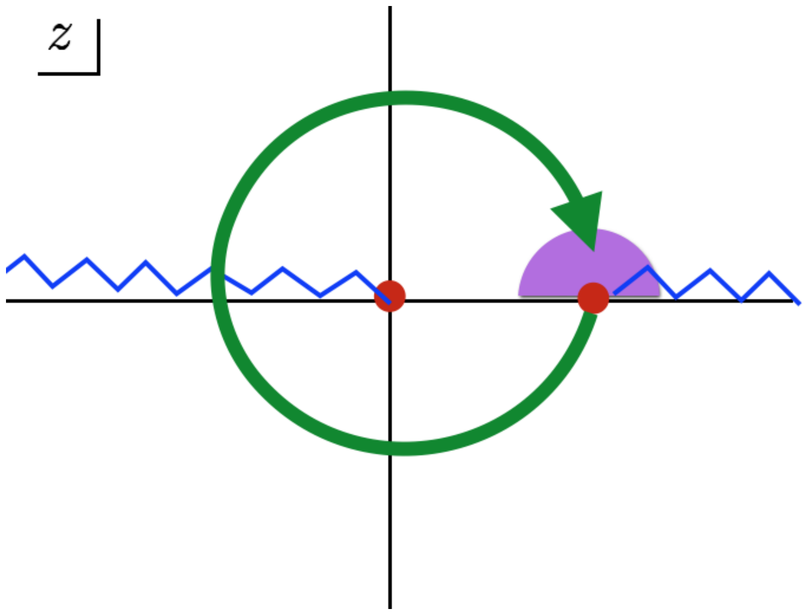
Conclude:

This 4pt function must be analytic on a region of complexified spacetime just before the lightcone.

Phrased in terms of the cross-ratios, the lightcone limit is

$$\bar{z} \to 1$$

Going through the first lightcone sends z around 0:

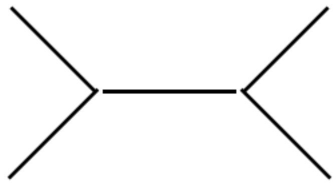
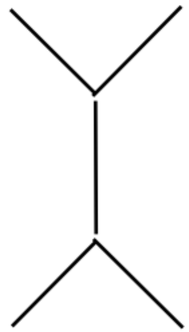


and causality is the statement that $G(z, \bar{z})$ is analytic near $z \sim 1$ on the '2nd sheet'.

cf: Chaos bound of Maldacena, Shenker, Stanford '15

Derivation of constraints

In general, the 4pt correlator can be decomposed as a sum of operator exchanges:

$$G(z, \bar{z}) = \sum$$  $$= \sum$$ 

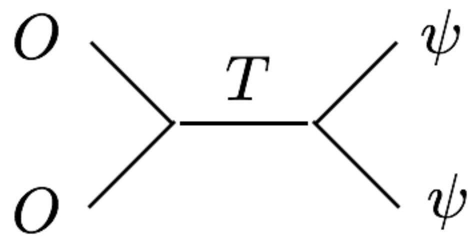
In some parts of the analytic (purple) region, this sum is dominated by low-dimension operators ("IR data")

In other parts, by high-dimension operators ("UV data")

A contour integral relates the two,

$$\oint dz G(z) = 0 \qquad \Rightarrow \qquad -\int_{IR} G = \int_{UV} G$$

IR piece: calculate via conformal block methods. Dominated by the exchange of low-dimension spinning operators, typically the stress tensor $T_{\mu\nu}$:



UV piece: cannot be calculated explicitly, but sign is fixed by expanding in the dual channel and using reflection positivity.

$\oint G = 0$ becomes a sum rule relating IR couplings to UV:

$$\lambda_{IR} = \int_{UV} (\text{positive})$$

A sum rules exists for the lowest-dimension operator of each spin ≥ 2.

For example, the spin-2 sum rule says

$$\lambda_T \equiv \langle OOT_{--} \rangle > 0$$

Applications

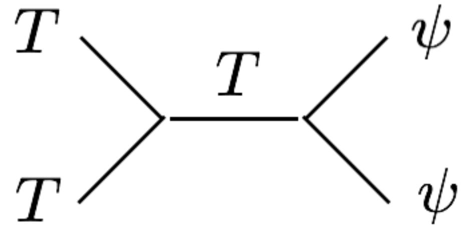
- Scalar probes: trivial, as conformal Ward implies $\lambda_T = \Delta_O$
- In a large-N theory, the constraint fixes the signs of anomalous

dimensions of composite operators,

$$\gamma(O\partial_\mu\partial_\nu O) < 0$$

This is the holographic dual of the sign constraint on $(\partial\phi)^4$ in the bulk Lagrangian.

- Probes with spin, such as T itself:



are related to constraints on graviton couplings via AdS/CFT.

Constraints on $\langle TTT \rangle$

Why bother?

Any theory of quantum gravity looks like Einstein gravity in the IR.

This is *suggested* by EFT, but (for 3pt functions) *required* by causality [Camanho et al '14].

In CFT, this translates into the conjecture that at large N,

$$\langle TTT \cdots \rangle_{CFT} = \langle TTT \cdots \rangle_{Einstein} + \cdots$$

where the dots are suppressed by the dimension of 'new physics' operators [see for example: Heemskerk, Penedones, Polchinski, Sully '09].

Where does this universality come from in CFT?

(ie, can we derive Einstein gravity from CFT? Bootstrap?)

This is not kinematics; the physics is all in the size of the corrections!

This question is already very interesting and nontrivial for 3pt functions $\langle TTT \rangle$.

$\langle TTT \rangle$ is fixed by conformal invariance up to 3 numbers:

$$
\begin{aligned}
\langle TTT \rangle \; = \; & a \;\times (\text{structure } \#1\ ) \\
& + \; c \;\times (\text{structure } \#2) \\
& + \; \lambda_3 \;\times (\text{structure } \#3)
\end{aligned}
$$

($a, c =$ Weyl anomaly coefficients in 4d).

The fine-tuning constraints mentioned at the beginning of the talk constrain the 3-graviton vertex, and therefore $\langle TTT \rangle$ in the dual CFT.

In CFT language, the AdS causality constraints are:

- At large impact parameter [Brigante et al; Hofman '08]:

$$\frac{1}{3} \leq \frac{a}{c} \leq \frac{31}{18}$$

- At small impact parameter [Camanho et al '14]:

$$\frac{a}{c} \approx 1 \, .$$

Our initial motivation to study causality in CFT was to try to prove $a \approx c$ in some class of CFTs.

(But haven't done this.)

The looser, large-impact parameter constraints were derived from CFT by Hofman and Maldacena assuming the average null energy condition:

$$\langle O \int T_{--} O \rangle > 0$$

But to push this to $a \approx c$ seems to require (at least) 2 new ingredients:

1. 4-point functions

2. causality

RESULTS

So can we derive $a = c$ from the bootstrap?

Not yet.

The CFT sum rule described above leads immediately to [TH, Jain, Kundu 1601]

$$\frac{13}{54} \leq \frac{a}{c} \leq \frac{31}{18}$$

Improvements to the sum rules by [Hofman et al 1603] squeeze this down to

$$\frac{1}{3} \leq \frac{a}{c} \leq \frac{31}{18} \, .$$

This agrees precisely with the Hofman-Maldacena bounds.

To find $a \approx c$, need to input 'holographic universality class':

1. large N

2. sparse spectrum of low-dimension operators

If these assumptions can be combined with the bootstrap sum rules, maybe will lead to $a \approx c$.

(*for comparison:* Cardy formula in 2d CFT)

This would be a derivation of *Einstein* gravity from CFT (for 3pt functions).

Thank you.