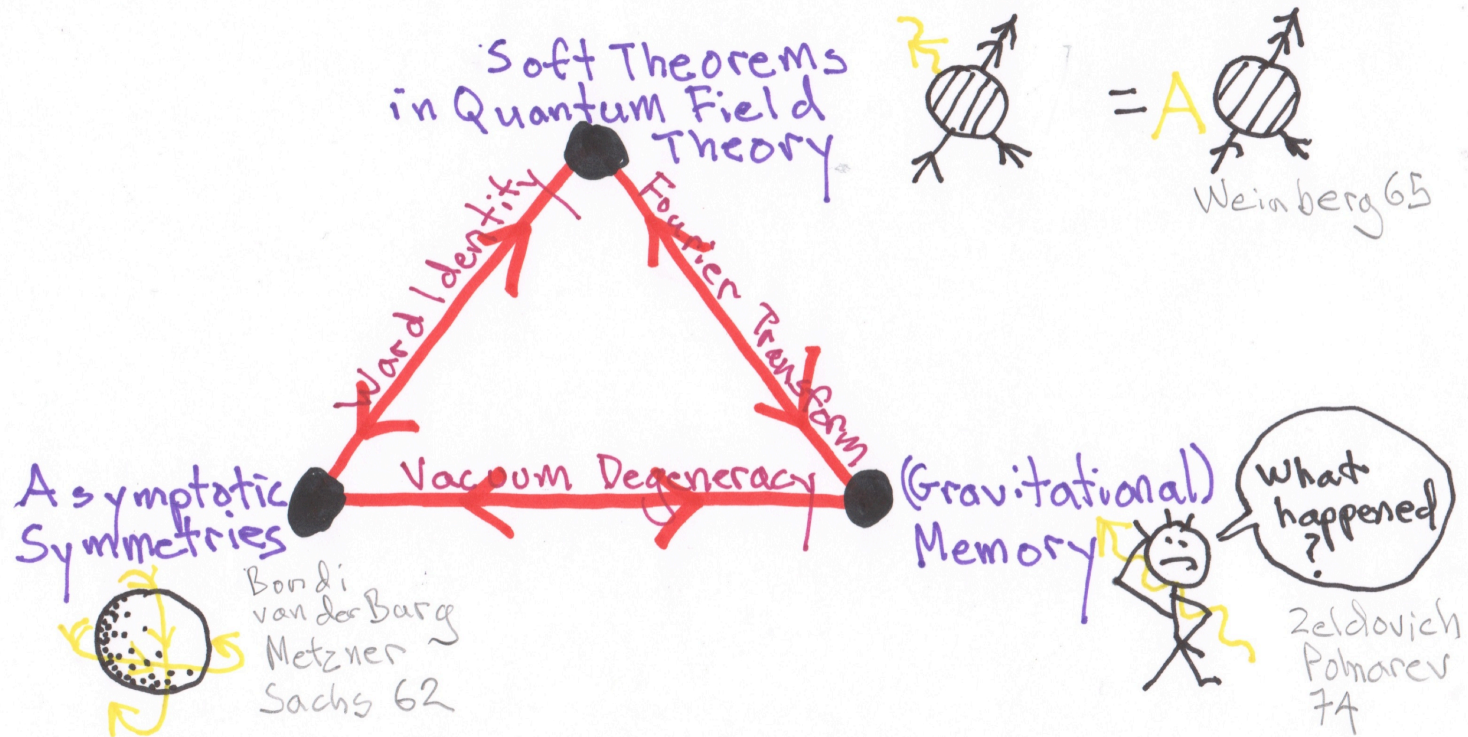


Soft Hair on Black Holes

NORDITA, August 2016

Andy Strominger

Over the last 3 years, an exact mathematical equivalence has been discovered of 3 previously disparate phenomena, each studied for half a century:



This has led to surprising new insights into the low-energy structure of gravitational & electromagnetic theories. It also has profound implications for black hole information, the focus of this lecture.

OUTLINE

I. Describe some basics of the \triangle , and the consequent \propto of conservation laws in all theories with gravity or electromagnetism.

He, Dumitrescu, Kapec, Iysor, Mitra, Pasterski, Pate, Porfyriadis, Zhiboedov, AS 2013-2015

II. Review the black hole information paradox
Hawking 75

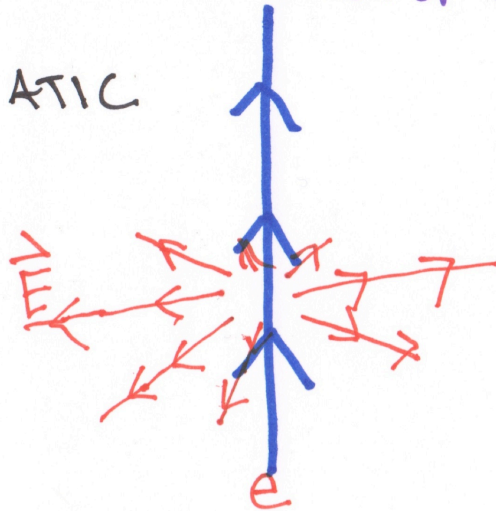
III. Describe implications of \triangle for paradox
Pasterski, Zhiboedov, AS 2014, and the existence
of 'soft quantum hair' on black holes
Hawking, Perry, AS 2016

IV. Conclude

∞ of conserved charges in E&M

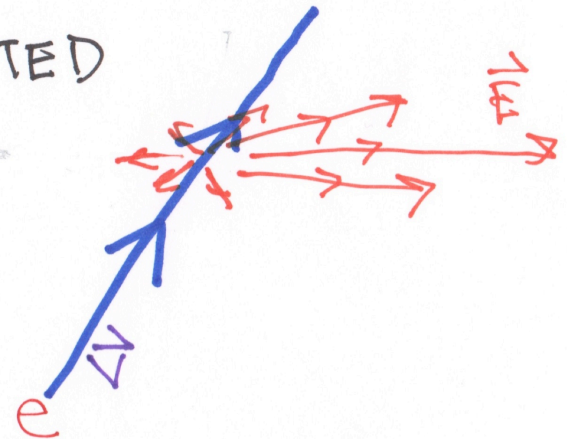
Lienard-Wiechert radial electric field

STATIC



$$F_{rt} = \frac{e}{4\pi r^2}$$

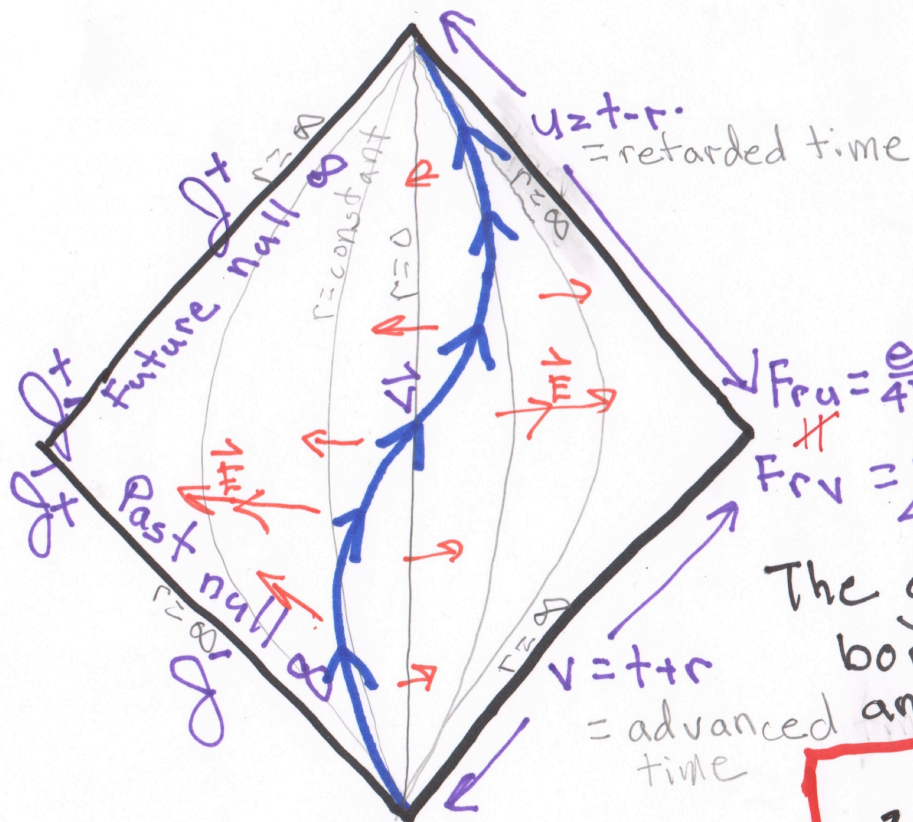
BOOSTED



$$F_{rt} = \frac{e\sqrt{1-v^2}(r^2 - t\vec{x}\cdot\vec{v})}{4\pi r[(1-v^2)(t-\vec{x}\cdot\vec{v})^2 - t^2 + r^2]^{3/2}}$$

The BOOSTED field is not single-valued near spatial ∞ . We need to understand this carefully.

Penrose diagram for a moving charge in Minkowski space



$$F_{ru} = \frac{e}{4\pi} \frac{(1-v^2)}{(r-v \cdot \hat{x})^2}$$

$$F_{rv} = \frac{e}{4\pi} \frac{(1-v^2)}{(r+v \cdot \hat{x})^2}$$

different sign

The general Lorentz-invariant boundary condition is antipodal:

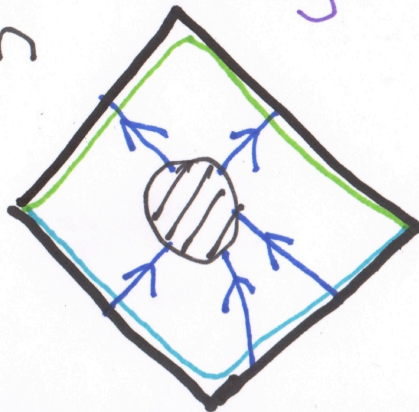
$$r^2 F_{ru}(\theta^A) \Big|_{\mathcal{H}^+} = r^2 F_{rv}(\tilde{\theta}^A) \Big|_{\mathcal{H}^-}$$

\uparrow angle on sphere \mathcal{H}^+ antipodal \uparrow angle on sphere \mathcal{H}^-

This implies an ∞ of 'antipodal' conservation laws

$$\boxed{Q_{\epsilon}^{+} \equiv \int_{\mathcal{I}^{+}} d^2\Omega r^2 F_{ru} \epsilon(\theta^A)} \\ = Q_{\epsilon}^{-} \equiv \int_{\mathcal{I}^{-}} d^2\Omega r^2 F_{rv} \epsilon(\theta^A)}$$

where $\epsilon(\theta^A)|_{\mathcal{I}^{+}} = \epsilon(\theta^A)|_{\mathcal{I}^{-}}$ is any function on the sphere. For the special case $\epsilon=1$ using Gauss's law $\nabla^{\mu} F_{\mu\nu} = j_{\nu}^{\mu}$ this is global charge conservation



$$\sum_k e_k^{\text{in}} \\ = \sum_k e_k^{\text{out}}$$

AS hep-th/1308.0589
He. Mitra Porfyriadis AS
hep-th/1407.3789
Campiglia & Laddha
hep-th/1505.05346
Kapec, Pate AS
hep-th/1506.02906

7

But what are the conservation laws
when $\partial_A \epsilon(\theta^A) \neq 0$???

Integrating by parts and using Gauss's law,
the conservation laws are

$$\underbrace{\sum_k \epsilon(\theta_k^A) e_k^{\text{in}}}_{\text{incoming charges weighted by angle}} + \underbrace{\text{in}}_{\text{strange duck}} = \sum_k \epsilon(\theta_k^A) e_k^{\text{out}} + \text{out}$$

$$\text{in} = \int d^3\Omega \int_{-\infty}^{\infty} dv \partial_A \epsilon F^A{}_v = \text{soft photon w/ polarization } \partial_A \epsilon$$

soft = zero-energy

At the classical level, this cons. law equates the sum of a zero mode of the incoming EM field and a moment of the incoming charge distribution to its antipodal outgoing counterpart.

Quantum Conservation Laws


In QM $|out\rangle = S |in\rangle$
 \nwarrow s-matrix

the ∞ of conservation laws are

He Mitra Portyriadis
 Kapec Pate Campiglia
 Laddha AS

$$\langle out | Q_\epsilon^+ S - S Q_\epsilon^- | in \rangle = 0$$

for any pair $|in\rangle, |out\rangle$ and any ϵ .

 $= \lim_{\omega \rightarrow 0} \int d^3x \int_{-\infty}^{\infty} dt e^{i\omega t} \partial_\mu \epsilon F^\mu_\nu$

creates/annihilates
 soft photons.

Bizzare conservation law

$$\lim_{\omega \rightarrow 0}$$



$$= A(\epsilon, \dots)$$



but it was discovered long ago Low (58) Weinberg (65)
 and = **SOFT PHOTON THEOREM** all EM theories e.g. QED
 Can reverse logic: soft photon theorem $\Rightarrow Q_\epsilon^+ = Q_\epsilon^-$

Conservation Laws \rightarrow Symmetries

$$[Q_\epsilon^\dagger, A_B]_{gt} = i \partial_B \epsilon$$

are "large gauge transformations" that go to angle-dependent constants at null infinity. They act non-trivially on physical states & can be measured via the electromagnetic memory effect: EM analog of well-known gravitational memory effect.

Bieri, Garfinkle gr-qc 1307.5098

Pasterski hep-th 1505.00716

Susskind hep-th 1507.02584

∞ Vacuum Degeneracy

$$H|0\rangle = 0$$

$$H Q_{\epsilon}^{\dagger} |0\rangle = 0$$

$$\langle 0 | Q_{\epsilon}^{\dagger} |0\rangle = 0$$

$$\Rightarrow Q_{\epsilon}^{\dagger} |0\rangle \neq |0\rangle$$

= an additional soft photon on $|0\rangle$

$\exists \infty$ many degenerate vacua w/ different angular momenta: quantum vacuum has 'soft hair'

Large gauge symmetries are spontaneously broken.

SOFT PHOTON = NAMBU-GOLDSTONE BOSON

Ditto for gravity!!!

I. 'Newtonian potential' in GR obeys $g_{00}(\theta^A)|_{\mathcal{I}^+} = g_{00}(\tilde{\theta}^A)|_{\mathcal{I}^-}$

II. ∞ of conserved 'supertranslation charges' generalizing the total mass & creating 'soft gravitons' $Q_A^+ = Q_A^-$ $f = f(\theta^A)$

III. Quantum conservation law = Weinberg's 1965 soft graviton theorem Hc Mitra Lysov AS hep-th 1401.7026

IV. Symmetry = Bondi, van der Burg, Metzner, Sachs 1962
BMS supertranslations

V. ∞ -degenerate vacua measured via Zeldovich-Polnarev 1974 gravitational memory effect Zhukovskiy AS hep-th 1411.5745

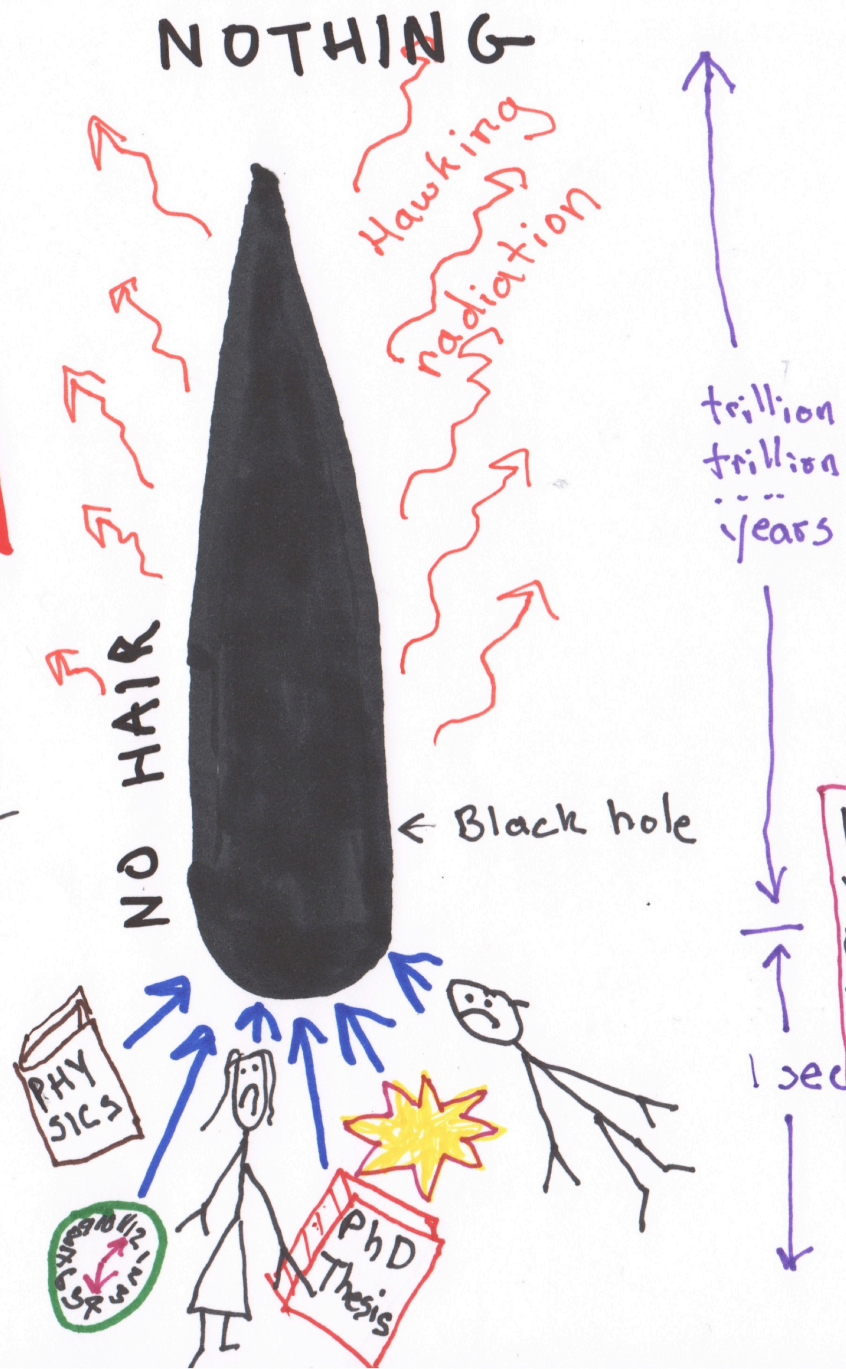
Oddly, though technically more complex, gravity was understood earlier than Maxwell theory!

THE BLACK HOLE INFORMATION PARADOX

Hawking 1975

In the far future, there is no record whatsoever of the PhD thesis.

~~DETERMINISM~~



In the last 40 years, no a priori reason to doubt the assumptions has surfaced.

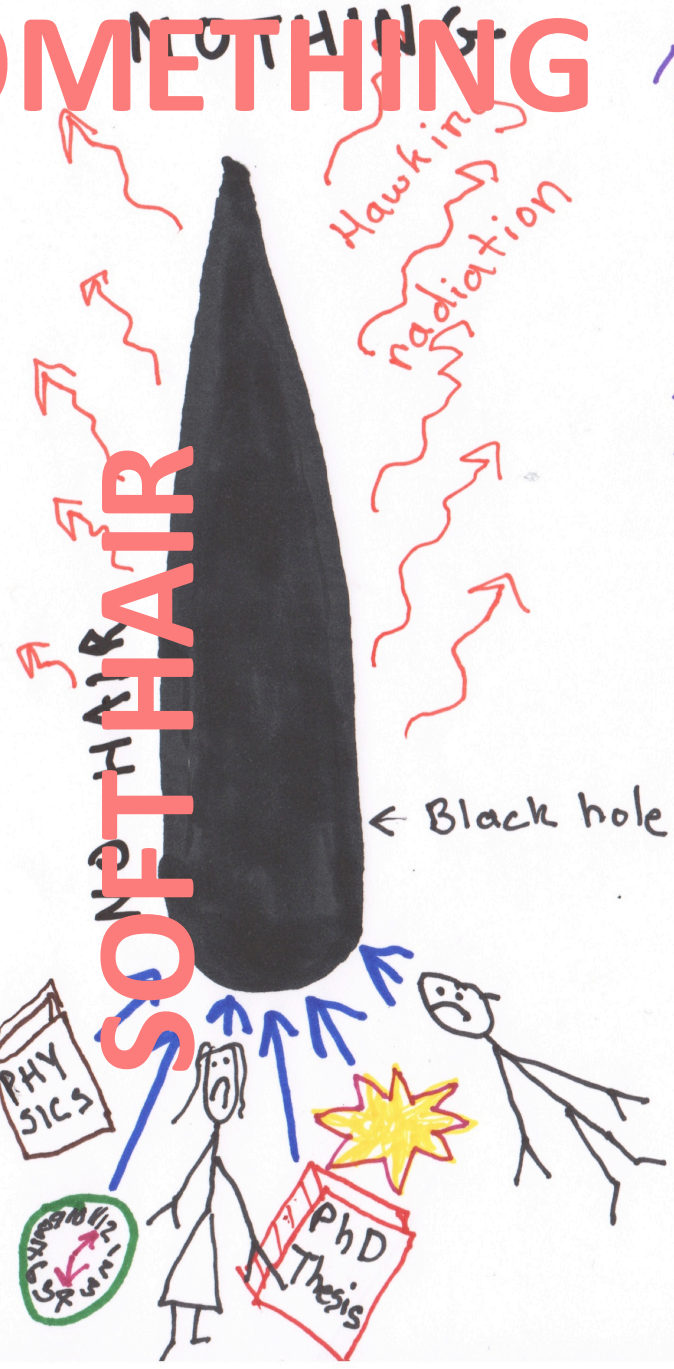
SOMETHING

THE BLACK HOLE INFORMATION PARADOX

Hawking 1975

In the far future, there is no record whatsoever of the PhD thesis.

~~DETERMINISM~~



trillion
trillion
years

NOW
THERE
IS

In the last 40 years, no a priori reason to doubt the assumptions has surfaced.

1 sec

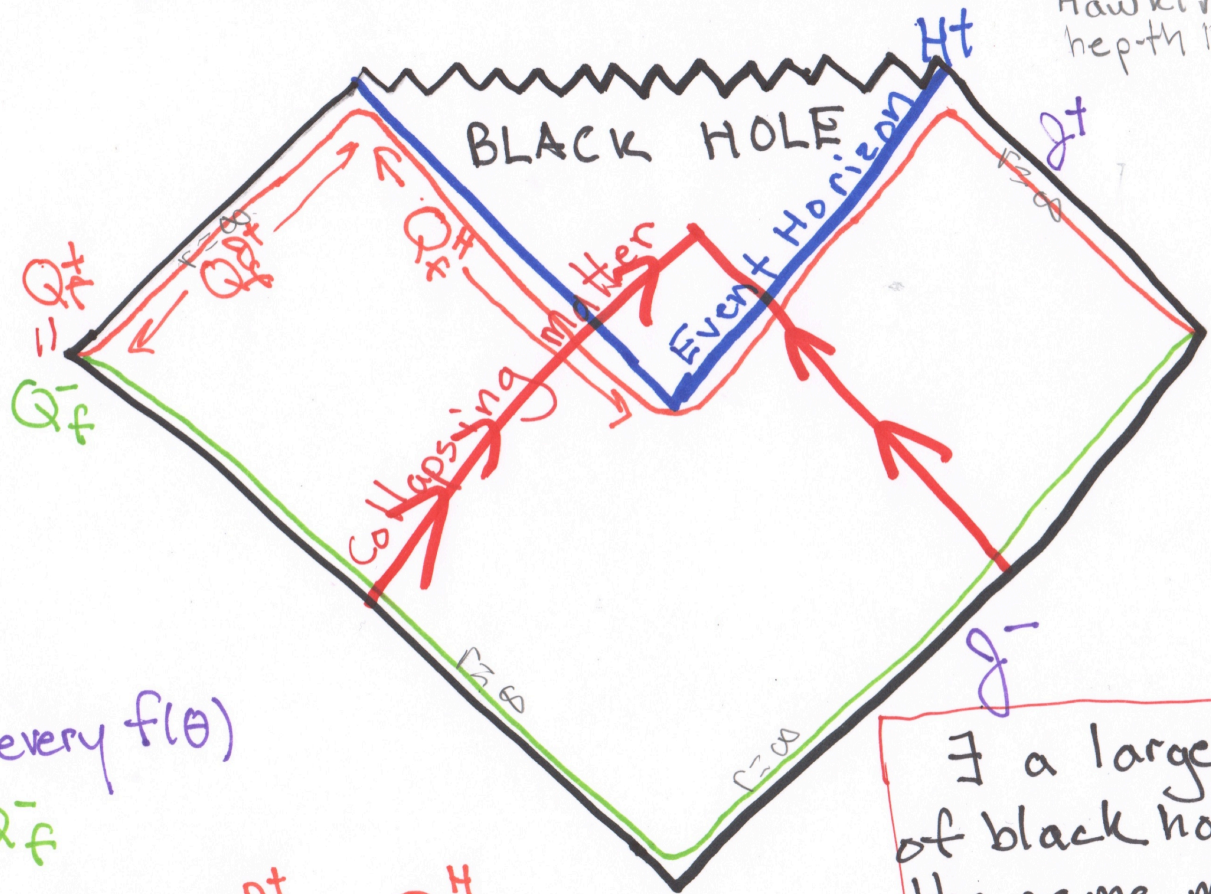
SUMMARY SO FAR

1. We have found a flaw in the assumptions underlying the information loss argument.
2. We have not resolved the information paradox.
3. The nature of the flaw suggests concrete new avenues of investigation.

onward....

SOFT HAIR ON BLACK HOLES

Hawking, Perry AS
hep-th/1407015



for every $f(\theta)$

Q_f^-

$$Q_f^- = Q_{fH}^+ + Q_{fH}^-$$

creates soft graviton on g^+

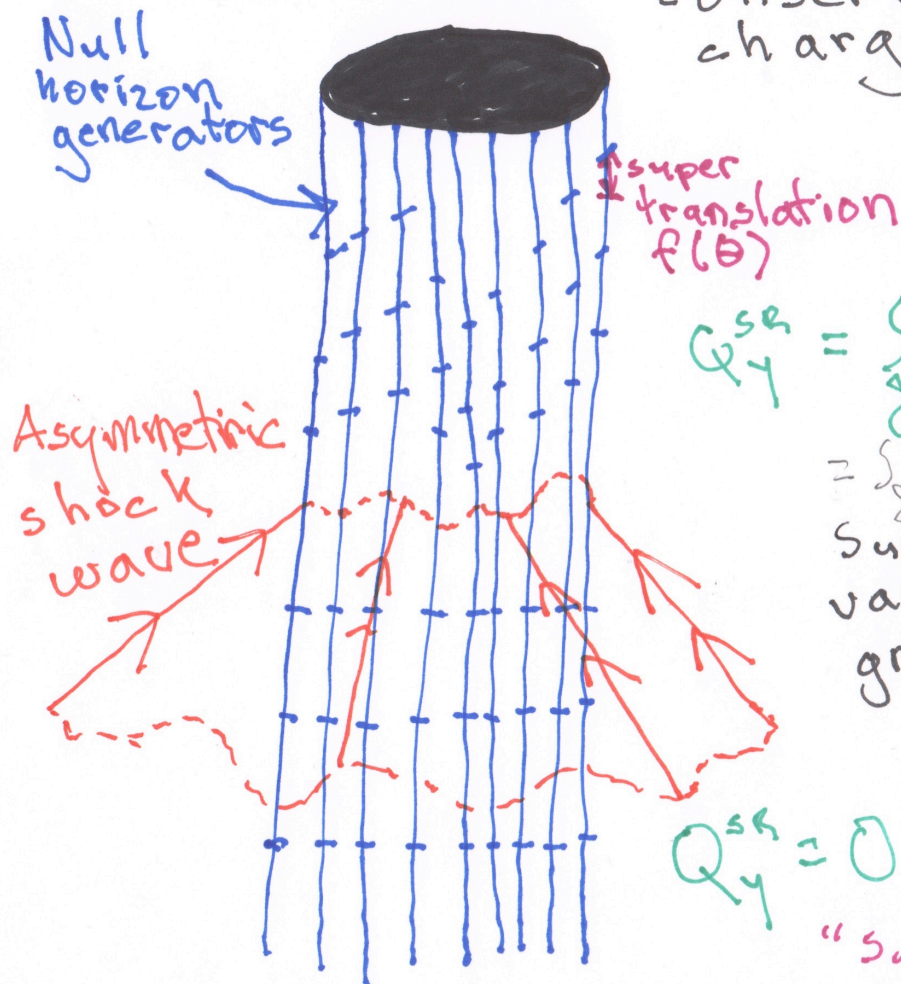
creates soft graviton on the horizon,

= pixel in the hologram at H^+

\exists a large degeneracy of black holes with the same mass but different numbers of soft gravitons = soft hair

Supertranslated black holes are classically distinguishable via conserved superrotation charges.

~~No-hair theorem~~



$$Q_Y^{sr} = \int_{\mathcal{H}^+} d^2\Omega m_B Y^A \partial_A f \neq 0$$

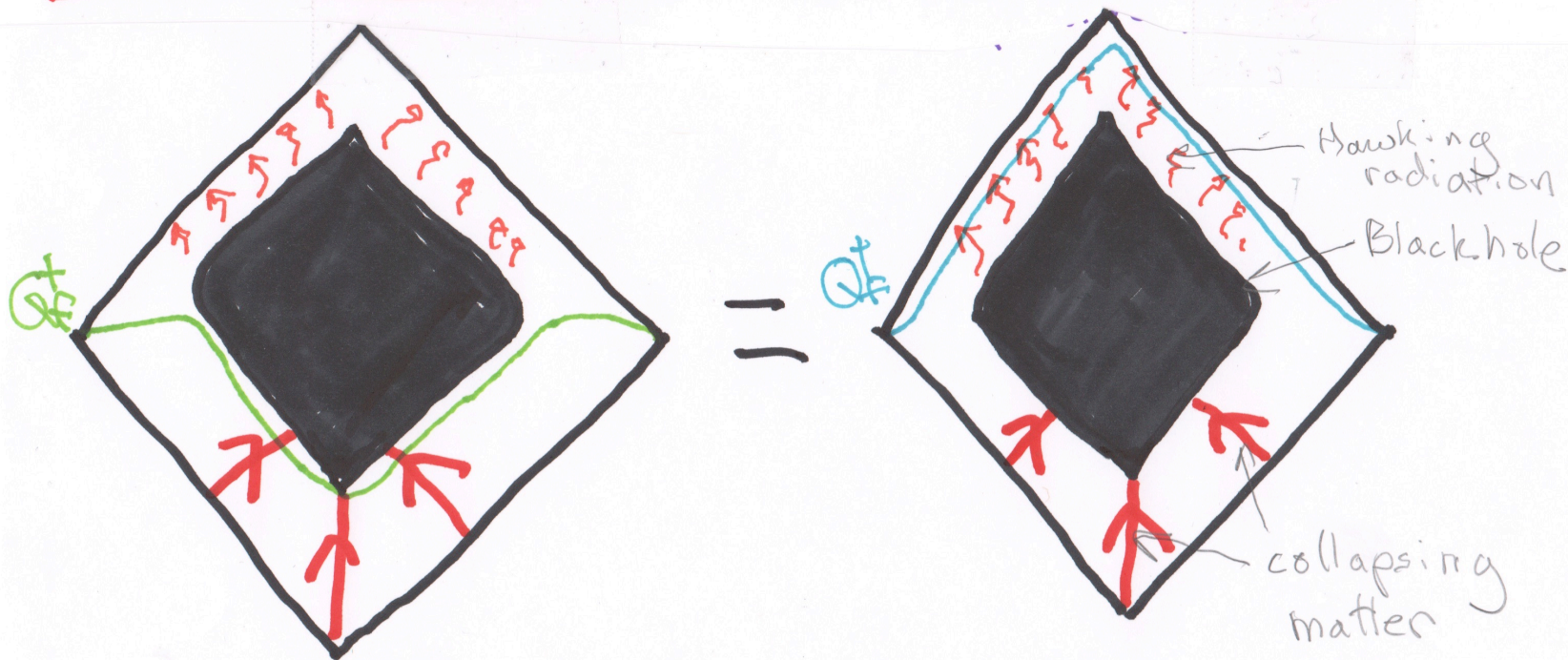
$$= \int_{\mathcal{H}^+} d^2\Omega Y^A N_A \leftarrow \begin{matrix} \text{angular momentum} \\ \text{aspect} \end{matrix}$$

Supertranslation charges vanish because the group is abelian.

$$Q_Y^{sr} = 0$$

Hawking, Perry AS
"Supertranslation Hair on Black Holes" to appear

Does this affect quantum black hole formation/evaporation?



$$|f \quad |4_{in}\rangle \rightarrow |4_{out}\rangle$$

$$\text{then } Q_f^\dagger |4_{in}\rangle \rightarrow Q_f^\dagger |4_{out}\rangle,$$

thereby distinguishing deterministically the out-states from BHs with or without soft hair in other words...

Yes!

Concluding Comment

Black holes carry quantum hair in the form of soft gravitons (or photons) arising from supertranslations. These comprise some of the pixels in the hologram at the boundary H^+ of the horizon. We think it unlikely they are all the pixels or store all the information. However more symmetries (e.g. superrotations) \Rightarrow more pixels. Whether or not a complete picture of black hole information can be obtained in this manner is a concrete & interesting challenge for future research.

