A CFT Perspective on the Black Hole Horizon

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Based on:
arXiv:1505.05069
arXiv:1608.xxyyzz w/ Aitor Lewkowycz and Gustavo Turiaci

Motivation:

Reconstruction of bulk physics in AdS/CFT is an important problem that will likely teach us new things about the nature of black hole horizons and space-time.

Bulk locality should be an approximate, emergent, dynamical property of the CFT.

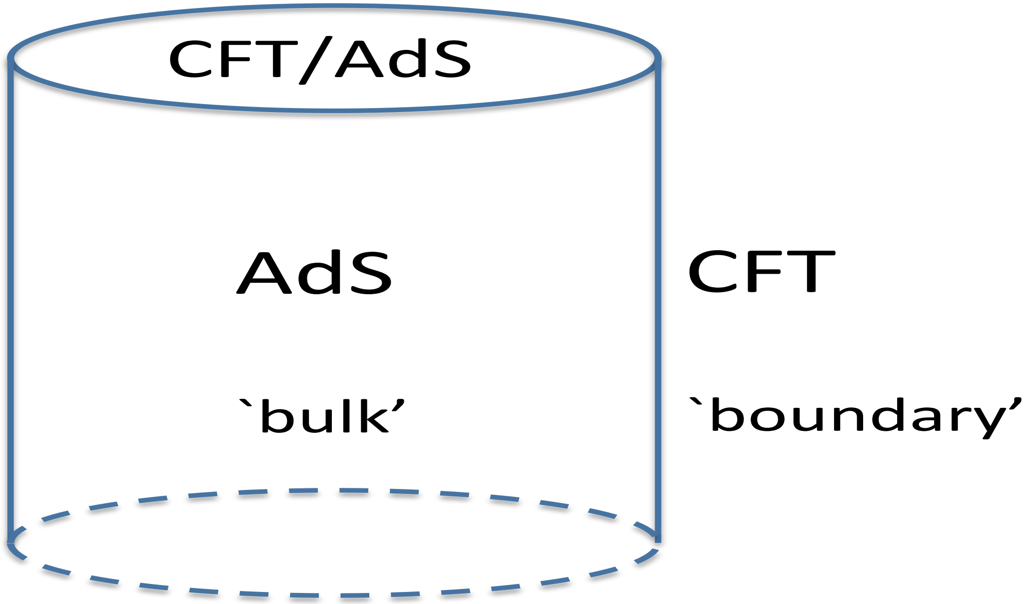
Do there exist natural CFT observables that appear to behave like gravitationally dressed `local' operators in the bulk?

Can we use CFT bootstrap techniques as a helpful tool in bulk reconstruction?

Plan: study the AdS bulk by means of a `local' bulk operators*

Principle: The CFT is smarter than us => use CFT as our guide.

* satisfying the properties (i) and (ii)



(i) $$\Box_{\text{bulk}} \Phi_h = m_h^2 \, \Phi_h$$

+ 1/N corrections

(ii) $$\lim_{y \to 0} y^{-2h} \Phi_h(y, x) = \mathcal{O}_h(x).$$

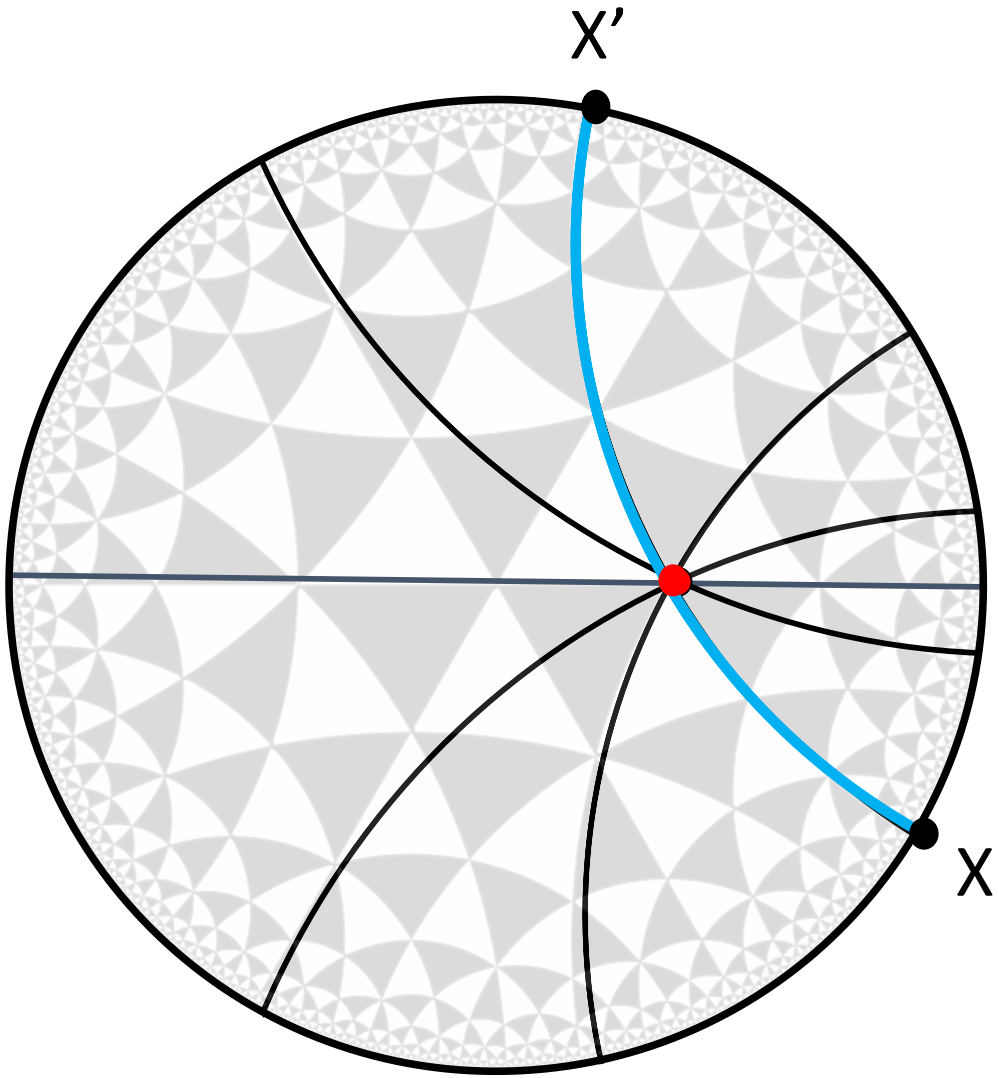
The `standard' construction of bulk operators is to use Green's theorem to solve eqns (i) and (ii). This gives the HKLL prescription:

$$\Phi_h^{\mathrm{HKLL}}(X) = \int d^2x\, K(X;x)\, \mathcal{O}_h(x) \quad + \quad 1/N \quad \text{corrections}$$

K(X;x) = some suitable `smearing function'.

This prescription is somewhat unsatisfactory because it presumes the existence of a bulk space-time, and makes explicit use of the bulk geometry and interactions. → It looks `state-dependent'.

The family of geodesics through a bulk point specifies an antipodal Z_2 pairing $X \leftrightarrow X'$ between boundary points.



The map $X \leftrightarrow X'$ defines an orientation reversing involution on the AdS boundary.

A geometric reformulation of HKLL defines bulk operators as solutions to the cross-cap boundary state conditions

$$M_{ab}\big|\Phi(0)\big\rangle = (P_a + K_a)\big|\Phi(0)\big\rangle = 0$$

Miyaji, Numasawa, Takayanagi, Watanabe

Nakayama, Ooguri

These conditions select a unique bulk point, that is left invariant by the corresponding global AdS isometries. This reproduces HKLL.

Specializing to AdS_3/CFT_2, these conditions take the form:

$$(L_0 - \bar{L}_0)\big|\Phi(0)\big\rangle = (L_1 + \bar{L}_{-1})\big|\Phi(0)\big\rangle = (L_{-1} + \bar{L}_1)\big|\Phi(0)\big\rangle = 0$$

At finite N, we need to include the coupling to gravity. This `gravitational dressing' should ensure that bulk fields solve the wave equation in general backgrounds.

A natural proposal: promote the bulk operator to a Virasoro cross cap state
= the unique linear sum of descendants of the primary state |h, h >, such that

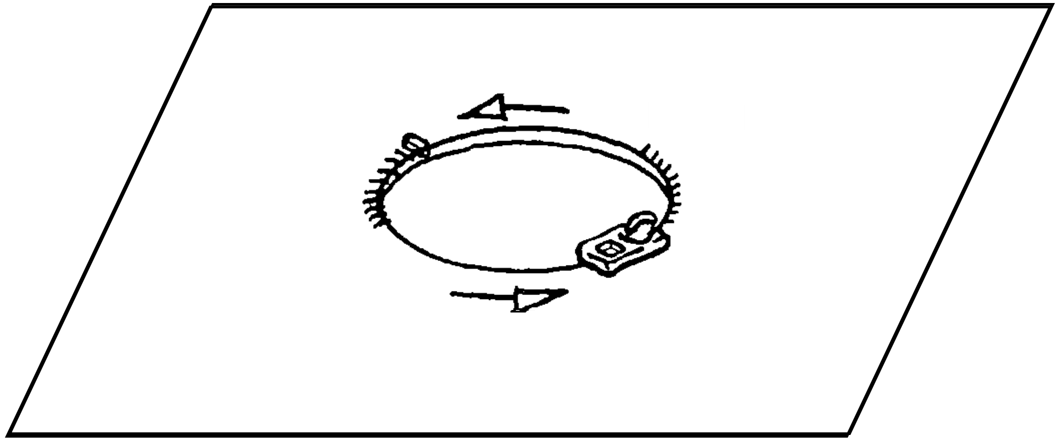
$$\boxed{\left(L_{-n} - (-1)^n \bar{L}_n\right) \| h \rangle\!\rangle_{\otimes} = 0.}$$

Ishibashi

Using the operator state correspondence/radial quantization, we define the local bulk operator (located at the origin $(z, \bar{z}) = (0,0)$ and radial location y) via

$$\boxed{\Phi_h(0, y)\big|0\rangle = y^{L_0 + \bar{L}_0} \, \| h \rangle\!\rangle_{\otimes}.}$$

Geometrically, $\Phi(y, z_0, \bar{z}_0)$ cuts a hole of size y centered around $(z_0, \bar{z}_0)$ and glues the opposite points on the edge together, via the antipodal identification



$$\boxed{\bar{z}' - \bar{z}_0 = -\frac{y^2}{z - z_0}}$$

Adding a cross cap changes the Euler character by 1, and adds 3 moduli to the surface on which the CFT lives. The three moduli $(y, z, \bar{z})$ comprise an SL(2,R) element g via

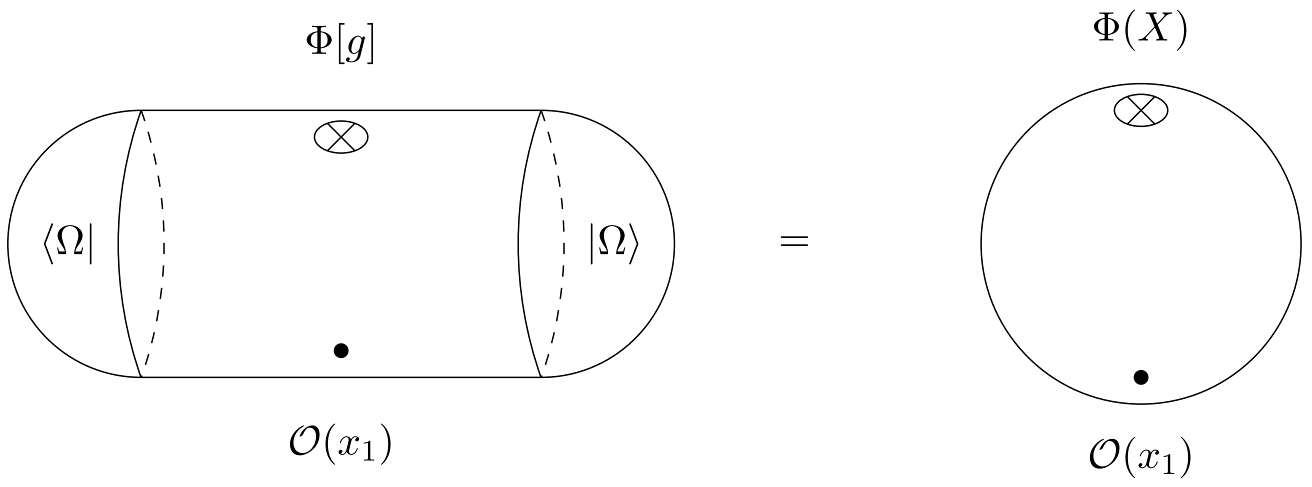
$$\bar{z}' = -\frac{az+b}{cz+d} \qquad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{\bar{z}_0}{y} & \frac{z_0\bar{z}_0}{y} - y \\ -\frac{1}{y} & \frac{z_0}{y} \end{pmatrix} \in SL(2, \mathbb{R})$$

Background independence

$$\boxed{(\Box_\Omega + m^2)\Phi(X) \;=\; 0.}$$

$$ds^2 \;=\; \frac{1}{y^2}\left(dy^2 + dz d\bar{z}\right) + \Omega dz^2 + \overline{\Omega} d\bar{z}^2 + y^2 \Omega \bar{\Omega} dy^2$$

$$\langle \Omega | T(z) | \Omega \rangle \;=\; \frac{c}{6}\Omega(z) \;=\; \frac{c}{12}\{Z, z\}$$



follows from the uniformization theorem

$$\boxed{\big\langle \Omega \big| \Phi[g]\, \mathcal{O}(x) \big| \Omega \big\rangle \;=\; \big\langle 0 \big| \Phi(X)\, \mathcal{O}(x) \big| 0 \big\rangle.}$$

$$ds^2 \;=\; \frac{1}{Y^2}\left(dY^2 + dZ d\bar{Z}\right).$$

Φ includes descendants created by L_{-n}'s. These terms are subleading at large N.

E.g.

$$\left\| L_n |0\rangle \right\|^2 = \frac{c}{12}(n^3 - n)$$

Fitzpatrick et al

This argument works in pure AdS, for the vacuum state with < T(z) > = 0. However, for semi-classical states and at large c, one can always find a local coordinate system (*Z*,*Z*) such that

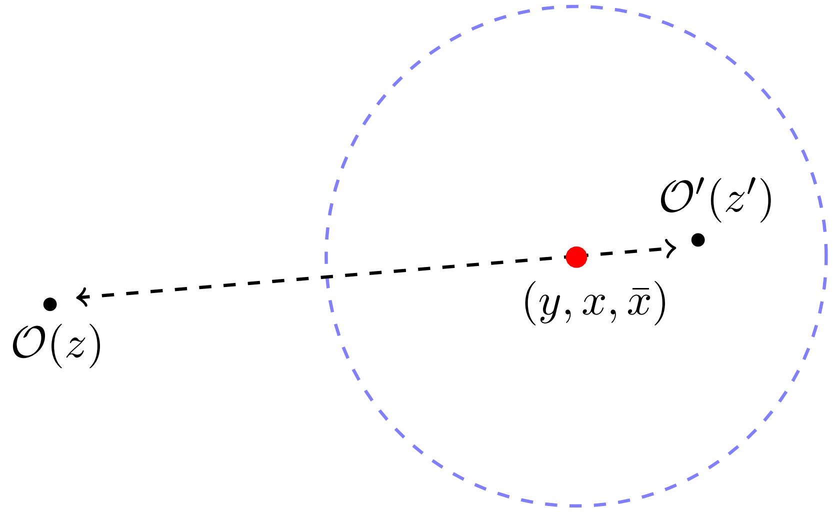
$$\langle T(Z) \rangle = \langle \bar{T}(\bar{Z}) \rangle = 0.$$

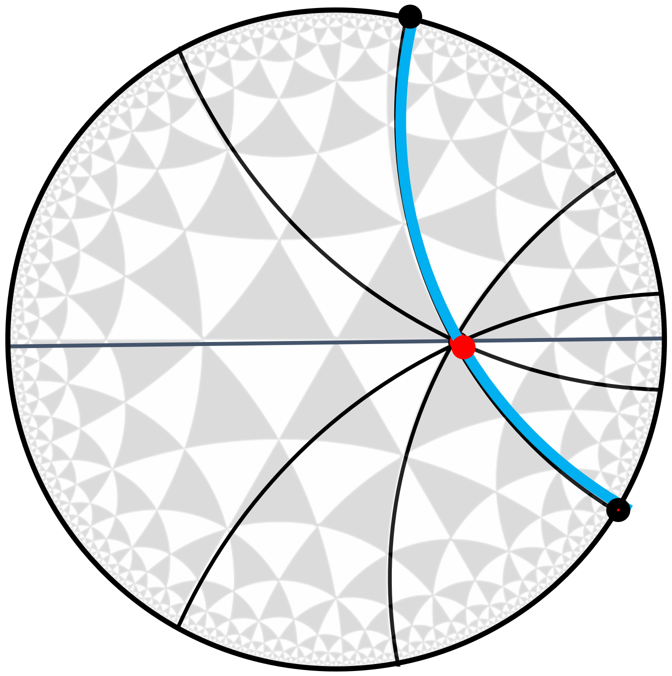
E.g:

$$\langle T(z) \rangle = \Delta / z^2 \quad \rightarrow \quad Z(z) = z^{ir_+}$$

$$r_+^2 = \frac{24\Delta}{c} - 1.$$

Correlation functions are computed via the method of images:





$$\langle \mathcal{O}(x_1, \bar{x}_1)\Phi_h(y, x_2, \bar{x}_2)\rangle = \big\langle \mathcal{O}(x_1, \bar{x}_1)\,\mathcal{O}(\tilde{x}_1, \bar{\tilde{x}}_1)\big\rangle = \frac{y^{2h}}{(y^2 + x_{12}\bar{x}_{12})^{2h}}$$

$$
\begin{aligned}
\langle T(z)\mathcal{O}(x_1,\bar{x}_1)\Phi(y,x_2,\bar{x}_2)\rangle &= \left\langle T(z)\,\mathcal{O}(x_1,\bar{x}_1)\,\mathcal{O}(\tilde{x}_1,\bar{\tilde{x}}_1)\right\rangle \\
&= \frac{h\ y^{2h}}{(y^2+x_{12}\bar{x}_{12})^{2h-2}(z-x_1)^2(y^2+(z-x_2)\bar{x}_{12})^2},
\end{aligned}
$$

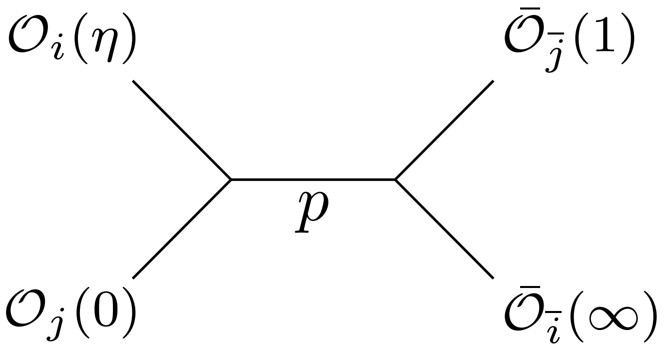
$$
\langle h|T(z)\bar{T}(\bar{w})|\Phi^{(0)}\rangle = \frac{h^2}{(-z\bar{w})^2}\ {}_2F_1\left(2,2,2h;-\frac{1}{\bar{w}z}\right)
$$

$$
\langle h|T(z)\bar{T}(\bar{w})|\Phi\rangle = \frac{h^2}{(-z\bar{w})^2}+\frac{2h}{(-z\bar{w})(1+z\bar{w})^2}+\frac{c/2}{(1+z\bar{w})^4}
$$

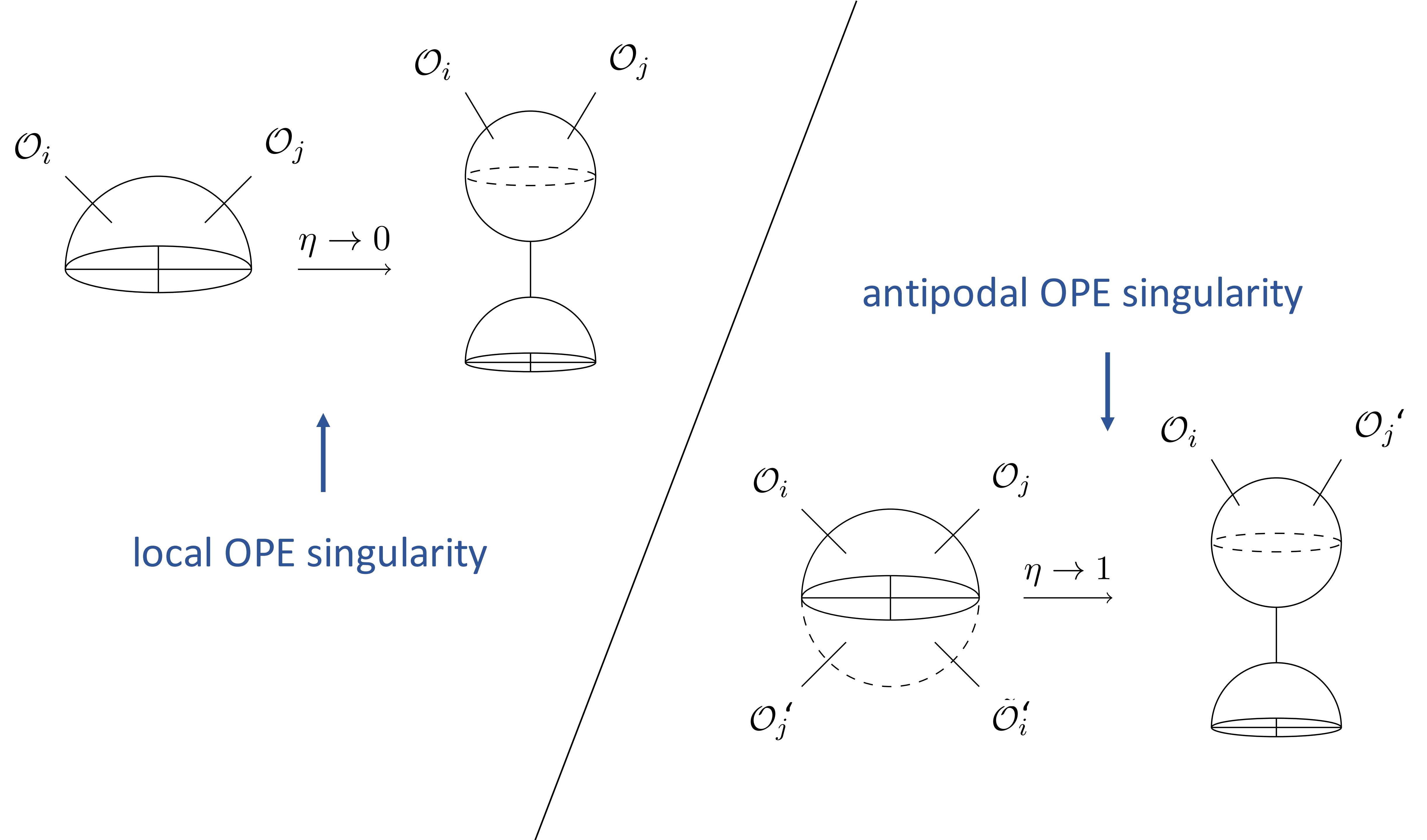
Three point functions are equal to chiral conformal blocks!

$$\big\langle \mathcal{O}_i \big| \, \mathcal{O}_j(x) \, \big| \, p \big\rangle\!\big\rangle \; = \; \eta^{h_i - h_j} G_{ijp}(\eta), \qquad \eta \equiv \frac{1}{1+x^2}$$

$$G_{ijp}(\eta) \; = \; \big\langle \mathcal{O}_i(0)\mathcal{O}_j(\eta)\, P_p\, \bar{\mathcal{O}}_{\mathrm{i}}(1)\bar{\mathcal{O}}_{\mathrm{j}}(\infty)\big\rangle \; =$$



$$G_{ijp}(\eta) \underset{c\to\infty}{=} C_{ijp}\, \eta^{h_p} \; {}_2F_1\big(h_p + h_{ij}, h_p - h_{ij}, 2h_p; \eta\big)$$



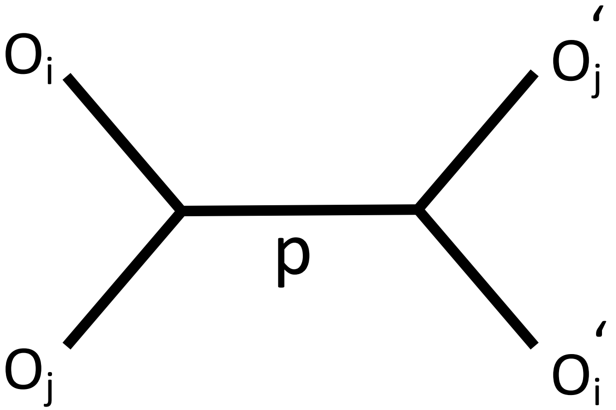
$\mathcal{O}_i$
$\mathcal{O}_j$
$\eta \to 0$
local OPE singularity
antipodal OPE singularity
$\mathcal{O}_i$
$\mathcal{O}_j$
$\eta \to 1$
$\mathcal{O}_j'$
$\tilde{\mathcal{O}}_i'$

Restoring bulk locality at next leading order in 1/N, via HKLL expansion:

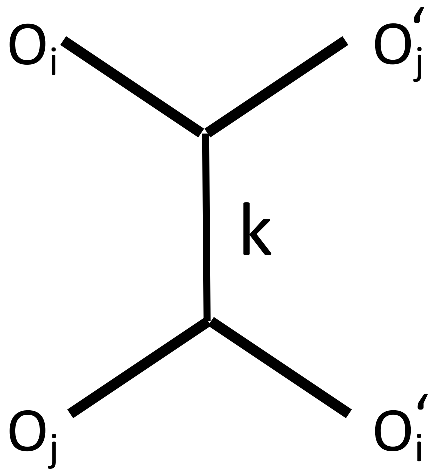
$$\left|\Phi\right\rangle = \sum_{h_p \geq h} \Phi_p \left| p \right\rangle\!\rangle$$

Coefficients of double trace terms are fixed by the bootstrap condition:

$$\sum_p \Phi_p$$

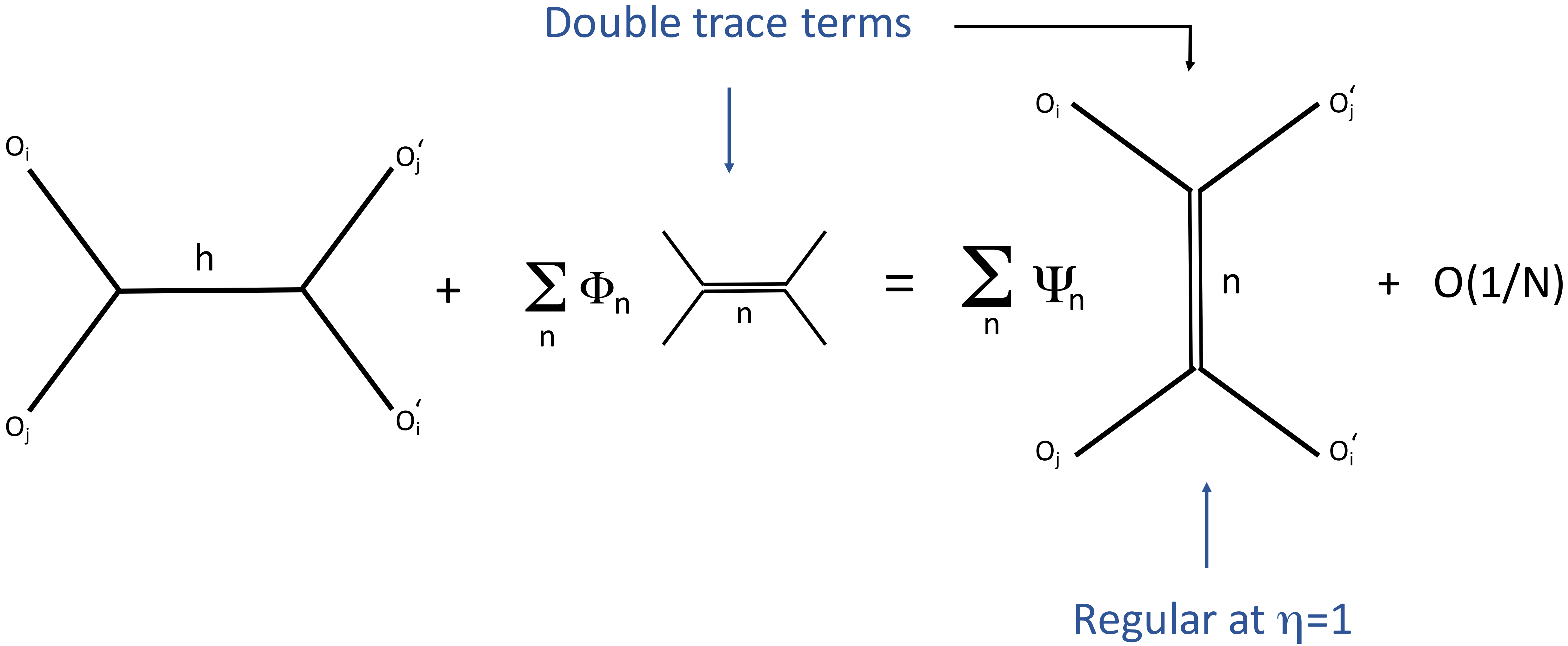


$$= \sum_k \Psi_k$$



$$\sum_p \Phi_p G_{ijp}(1-\eta) = \sum_k \Psi_k G_{ijk}(\eta),$$

Restoring bulk locality at next leading order in 1/N, via HKLL expansion:



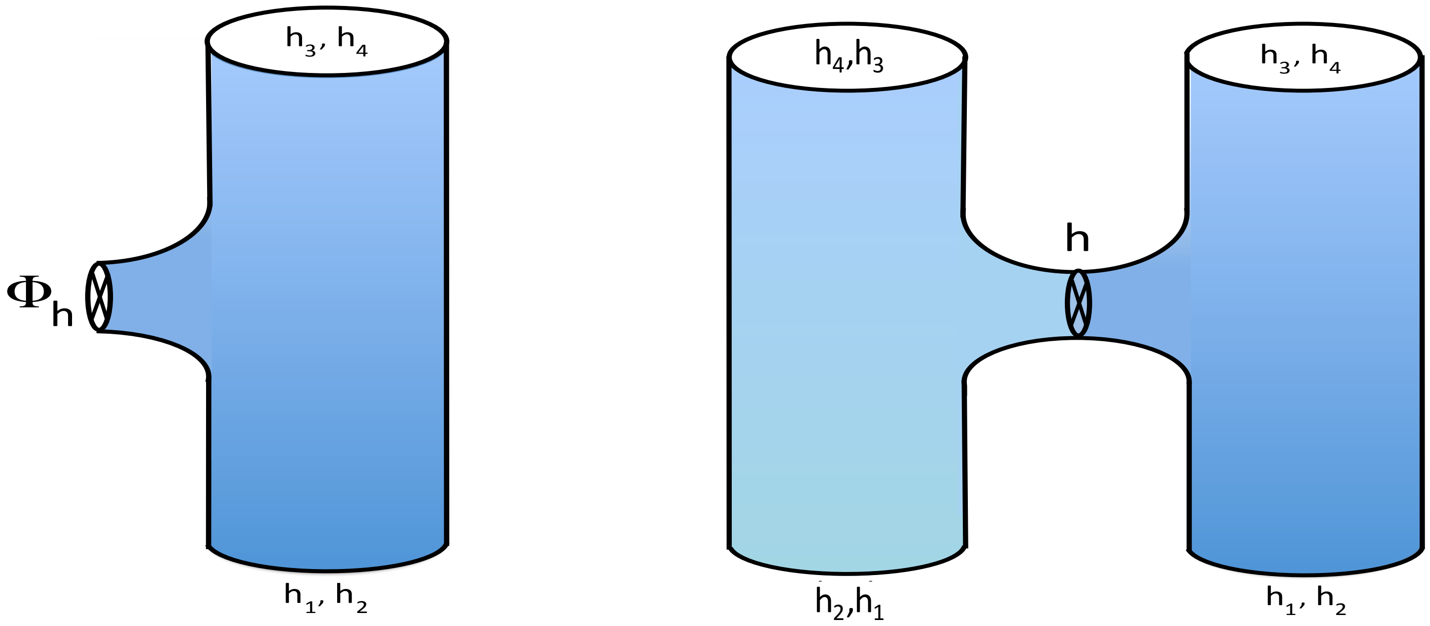
The matrix element of Φ(g) between two highly excited primary states

$$\boxed{\phi_h\left[\begin{smallmatrix}1 & 2\\ 3 & 4\end{smallmatrix}\right](g) = \left\langle h_3, h_4 \middle| \Phi_h(g) \middle| h_1, h_2 \right\rangle}$$

$h_1 = h_2 = \frac{1}{2}\Delta$
$h_3 = \frac{1}{2}(\Delta + \omega + \ell)$
$h_4 = \frac{1}{2}(\Delta + \omega - \ell)$

should coincide with the mode-function $f_{\omega\ell}$(g) of a scalar field of mass m_h in the BTZ black hole geometry dual to the excited state $|h_1, h_2\rangle$.

Again we consider the CFT on the `Schottky double':

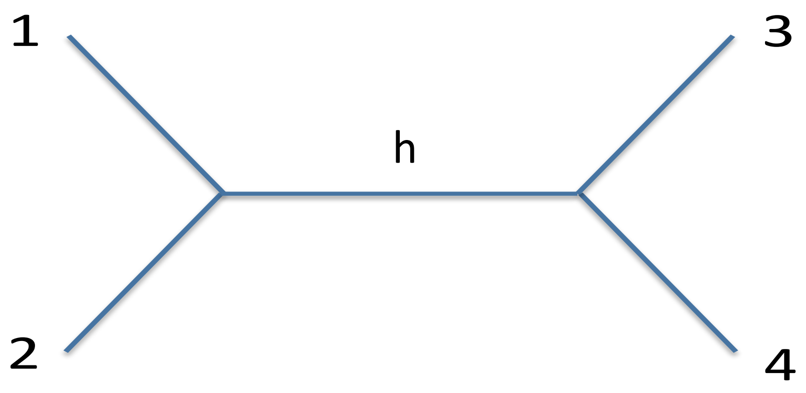


$$\langle h_3, h_4 | \Phi_h(g) | h_1, h_2 \rangle \simeq \langle \mathcal{O}_4(\infty)\mathcal{O}_2(1) | \mathcal{P}_h | \mathcal{O}_3(Z)\mathcal{O}_1(0) \rangle_{\text{chiral}}$$

identifies the matrix element of $\Phi_h(g)$ with a chiral conformal block.

We need to know the explicit form of a 2D conformal block:

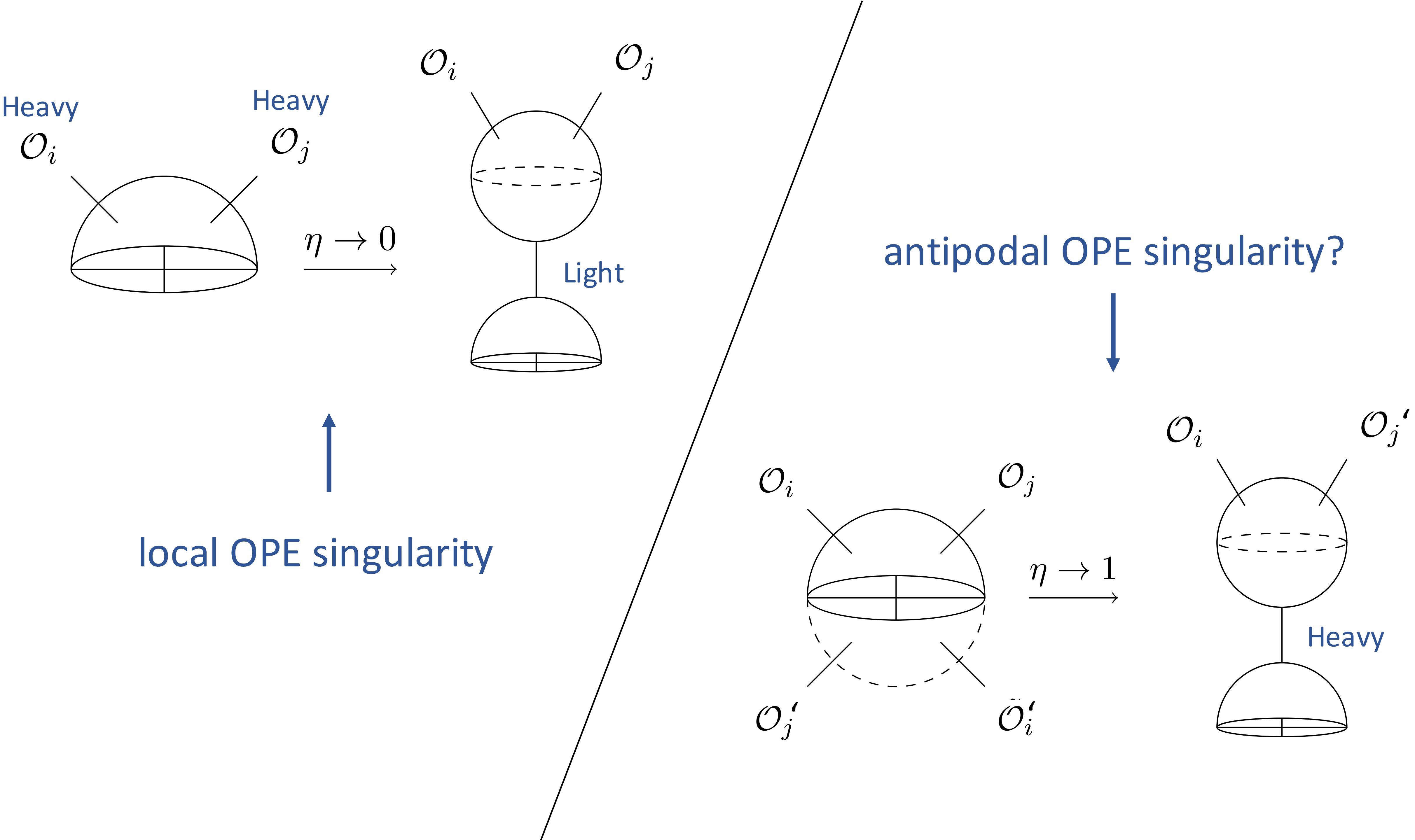
$$\Psi_h\left[\begin{smallmatrix}1&2\\3&4\end{smallmatrix}\right](Z) =$$



Our proposal predicts that the semi-classical Virasoro block is given by:

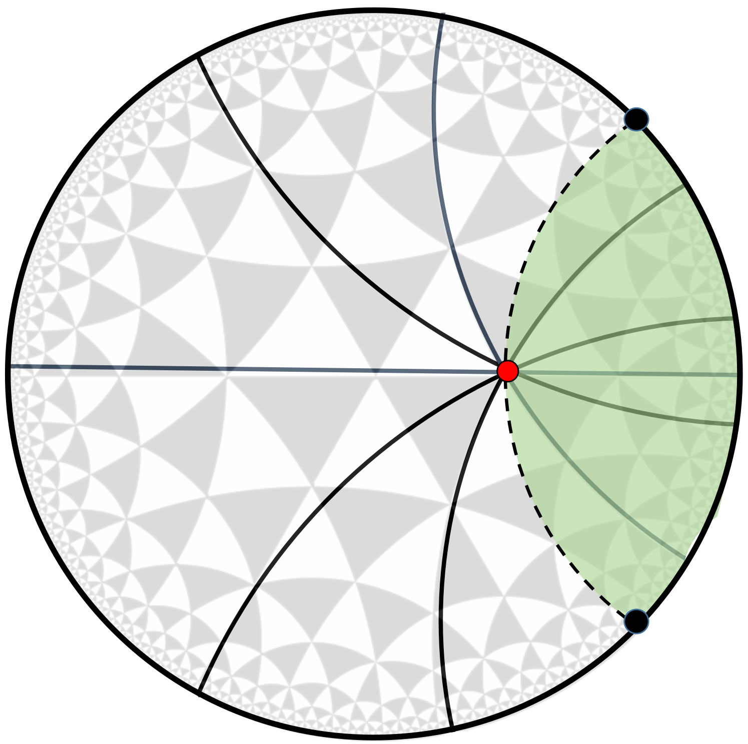
$$\boxed{\Psi_h\left[\begin{matrix}1&2\\3&4\end{matrix}\right](Z) = Z^h\,{}_2F_1\left(h+\frac{i}{r_+}h_{13},\, h+\frac{i}{r_+}h_{24},\, 2h;\, Z\right)}$$

$$Z\frac{\partial}{\partial Z} = \frac{i}{r_+}z\frac{\partial}{\partial z}$$

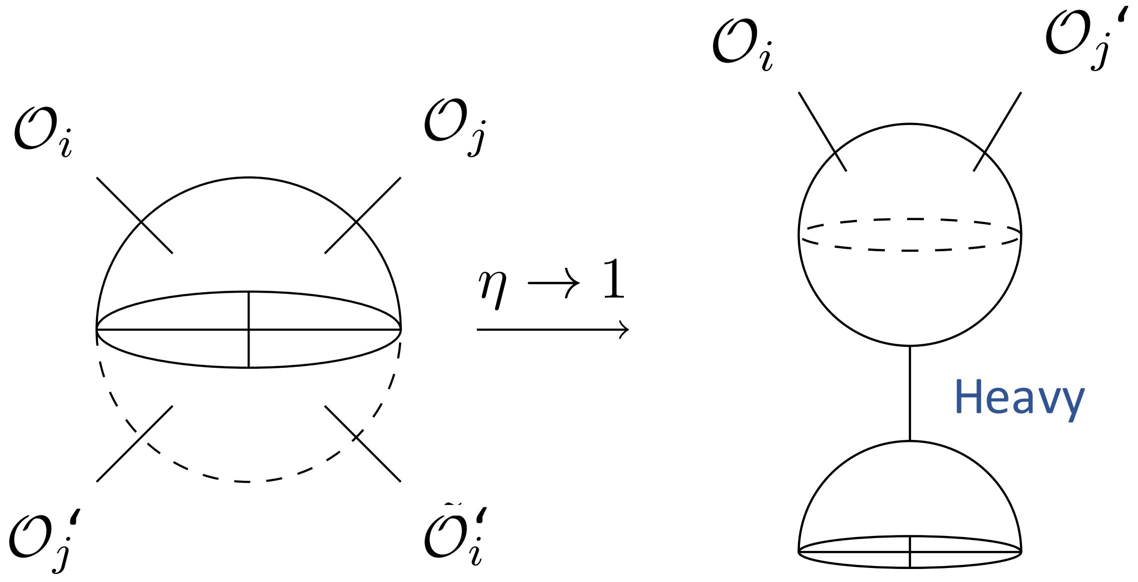


Heavy
$\mathcal{O}_i$
Heavy
$\mathcal{O}_j$
$\xrightarrow{\eta \to 0}$
$\mathcal{O}_i$
$\mathcal{O}_j$
Light
local OPE singularity
antipodal OPE singularity?
$\mathcal{O}_i$
$\mathcal{O}_j$
$\mathcal{O}_j$'
$\tilde{\mathcal{O}}_i$'
$\xrightarrow{\eta \to 1}$
$\mathcal{O}_i$
$\mathcal{O}_j$'
Heavy

Heavy



antipodal OPE singularity?



Conclusion:

Reconstruction of bulk physics in AdS/CFT is an important problem that will likely teach us new things about the nature of black hole horizons and space-time.

Bulk locality should be an approximate, emergent, dynamical property of the CFT.

Cross cap operators are natural CFT observables that appear to behave like gravitationally dressed `local’ operators in the bulk.

CFT bootstrap techniques become helpful tools in bulk reconstruction.