Interacting higher spin *CFTs* in 3d

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Talk based on:

- arXiv: 1602.01682 (with Hampus Linander) and work in progress
- arXiv: 1312.5883, 1506.03328 and work in progress

Also relevant : topologically *gauged* $SO(N)$ $\mathcal{N} = 8$ CFT_3

- arXiv:1204.2521 (with Ulf Gran, Jesper Greitz and Paul Howe)
- arXiv:0809.4478 (with Ulf Gran), arXiv:0906.1655 (with Chu)
- *"Critical solutions in ...."*, arXiv: 1304.2270

Introduction and Content

(2+1)-dimensional higher spins ($s \geq 2$) + scalar CFT:

We will consider a *sourced unfolded equation for a scalar field* Φ

$$\mathcal{D}\Phi|0\rangle_q = S(\Phi)|0\rangle_q, \quad \textit{with} \quad \mathcal{D} = d + A \tag{1}$$

and suggest some non-linear source terms $S(\Phi)$.
This unfolded equation is related to the *hs field strength equation*

$$F(A) = T(\Phi, \mathcal{D}\Phi) \tag{2}$$

which however will be discussed in some detail only for $T = 0$.
The aim of this talk is to describe

- how to derive the spin 2 covariant spin 3 Cotton eq from $F = 0$
- and how to obtain the correct non-linear spin 2 - spin 0 equations from the sourced unfolded equation above

We work in a multi-commutator expansion of the star product $\star$ corresponding to Weyl ordering ($\star$ not written explicitly).

Motivation

MOTIVATION:

Interacting 3d higher spin (hs) CFTs coupled to scalars

- have not yet been constructed (compare to Vasiliev hs in AdS)
- seems to be part of the AdS_4/CFT_3 with N/mixed bc for spin ≥ 2

Related work: Horne-Witten (1988), Fradkin-Linetsky (1989),
Pope-Townsend (1989), Vasiliev (1992), Segal (2002),
Shaynkman-Vasiliev (2001,...), Vasiliev (2009, 2012,...)
Damour-Deser (1987), Bergshoeff et al (2009),
Henneaux et al (2015)

3d conformal higher spin basics

Basics

- We introduce two hermitian $so(2,1)$ spinorial operators q^α, p_α satisfying $[q^\alpha, p_\beta] = i\delta^\alpha_\beta$, $(\alpha, \beta = 1, 2)$. Then
  - $M^a = -\frac{1}{2}(\gamma^a)_\alpha{}^\beta q^\alpha p_\beta$ => $so(2,1)$, the 3d Lorentz algebra
  - all bilinears in q^α, p_α => the set P^a, M^a, D, K^a => $so(3,2)$, the 3d conformal algebra
  - the Weyl ordered form of all even polynomials => the generators of the hs algebra $hs(SO(3,2))$
- this corresponds to the simplest realization of this hs algebra –> compare to c_i, c^i in e.g. Günaydin-Warner (1986):
  - canonical pairs c_i, c^i, $a_i(r), a^i(r)$ and $b_i(r), b^i(r)$ with $i = 1, 2$ and $r = 1, 2, ..p$

The higher spin set-up

The higher spin one-form gauge potential A

- $A = \Sigma_{s=2}^{\infty}(-i)^{s-1}A_s$ where
  - $A_s = e^{a_1....a_{s-1}}(x)P_{a_1....a_{s-1}} + ... + f^{a_1....a_{s-1}}(x)K_{a_1....a_{s-1}}$
  - similar expansions valid for the field strength $F = dA + A^2$ and the gauge parameter Λ
- the action is "just" the Chern-Simons one for $hs(SO(3,2))$ written out explicitly by Fradkin-Linetsky in 1987 including all auxiliary fields
  - auxiliary fields = here Stückelberg and dependent fields
- we will instead study the theory *AFTER* eliminating all the auxiliary fields! ("solving" the Chern-Simons eq $F = 0$)

The higher spin set-up: spin 2

The standard spin 2 case: $A_2(x;q,p)$

$A_2 = e^a P_a + \omega^a M_a + bD + f^a K_a$

spin 2 generators with $n_q + n_p = 2$ where $(n_q, n_p) = (q,p)$-content

$$P^a(2,0) = -\frac{1}{2}(\sigma^a)_{\alpha\beta} q^\alpha q^\beta, \;\; M^a(1,1) = -\frac{1}{2}(\sigma^a)_\alpha{}^\beta q^\alpha p_\beta$$

$$D(1,1) = -\frac{1}{4}(q^\alpha p_\alpha + p_\alpha q^\alpha), \;\; K^a(0,2) = -\frac{1}{2}(\sigma^a)^{\alpha\beta} p_\alpha p_\beta$$

The higher spin set-up: spin 3

The spin 3 part: $A_3(x; q, p)$ with $n_q + n_p = 4$

$$A_3 = e^{ab}P_{ab} + \tilde{e}^{ab}\tilde{P}_{ab} + \tilde{e}^{a}\tilde{P}_{a} + \tilde{\omega}^{ab}\tilde{M}_{ab} + \tilde{\omega}^{b}\tilde{M}_{a} + \tilde{b}\tilde{D} + \tilde{f}^{a}\tilde{K}_{a} + \tilde{f}^{ab}\tilde{K}_{ab} + f^{ab}K_{ab}$$

- a tilde indicates that the flat vector indices $ab\ldots$ = irrep
- $P^{ab}(4,0)$ and $K^{ab}(0,4)$ are automatically irreducible (Fierz)

Spin 3 generators:

$$P^{ab}(4,0) = \frac{1}{4}(\sigma^a)_{\alpha\beta}(\sigma^b)_{\gamma\delta}q^\alpha q^\beta q^\gamma q^\delta,$$

$$\tilde{P}^{ab}(3,1) = \frac{1}{4}(\sigma^{(a})_{\alpha\beta}(\sigma^{b)})_{\gamma}{}^{\delta}q^\alpha q^\beta q^\gamma p_\delta,$$

$$\tilde{P}^{a}(3,1) = \frac{1}{16}(\sigma^a)_{\alpha\beta}(q^\alpha q^\beta q^\gamma p_\gamma + q^{(\alpha}q^\beta p_\gamma q^{\gamma)} + q^{(\alpha}p_\gamma q^\beta q^{\gamma)} + p_\gamma q^\alpha q^\beta q^\gamma)$$

etc to

$$K^{ab}(0,4) = \frac{1}{4}(\sigma^a)^{\alpha\beta}(\sigma^b)^{\gamma\delta}p_\alpha p_\beta p_\gamma p_\delta,$$

The Scalar field

The scalar field $\Phi(x;p)$ defined as acting on $|0\rangle_q$

Singleton: $(E_0, s) = (\frac{1}{2}, 0)$ repr of $SO(3,2)$

$\Phi(x;p) = \Sigma_{n=0}^{\infty}(-i)^n \phi^{a_1 \dots a_n}(x) K_{a_1 \dots a_n}(p)$,

that is

$\Phi(x;p) = \phi(x) - i\phi^a(x)K_a(p) - \phi^{ab}(x)K_{ab}(p) + ..,$

where

$$K_{a_1 \dots a_n}(p) = K_{a_1 \dots a_n}(0, 2n) = K_{a_1}K_{a_2} \dots . K_{a_n}$$

- We will now start "solving" $F = 0$ assuming that the dreibein is invertible and return to the scalar eq later!

The spin 2 equations

The pure spin 2 equations: all spin 3 and higher fields are set to zero

- $F_2 = 0$ in the gauge $b_\mu = 0$ (using $\Lambda^a(0,2)$):

$$\begin{array}{rl} P_a(2,0): & F^a(2,0) = T^a = 0,\ \textit{the torsion constraint} => \omega = \omega(e) \\ M_a(1,1): & F^a(1,1) = R^a - 2\epsilon^a{}_{bc}e^b \wedge f^c = 0, \textit{solved for } f_\mu{}^a\ \textit{(below)} \\ D(1,1): & F(1,1) = e^a \wedge f_a = 0,\ \textit{constraint on } f_\mu{}^a\ \ (f_{[\mu\nu]} = 0) \\ K_a(0,2): & F^a(0,2) = Df^a = 0,\ \textit{Cotton equation} \end{array}$$

Schouten tensor $S_{\mu\nu}$

$$f_{\mu\nu} = \frac{1}{2}(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R) = \frac{1}{2}S_{\mu\nu},$$

Field equation (3rd order in derivatives): Cotton equation

$$C_{\mu\nu} := \epsilon_\mu{}^{\rho\sigma}D_\rho S_{\sigma\nu} = 0$$

which is symmetric and traceless, i.e., in irrep **5** of $SO(2,1)$

The spin 3 equations

Analysis of the spin 3 equations: *(BN (2013)), (Linander, BN (2016))*
(here all fields with spin ≥ 4 *are set to zero)*

- recall the spin 3 generators: (n_q, n_p)
  $P_{ab}(4,0), \tilde{P}_{ab}(3,1), \tilde{P}_a(3,1), ..., K_{ab}(0,4)$
- the corresponding $F_3(n_q, n_p) = 0$ eqs divide into 3 groups:
  - equations which make it possible to solve for all fields in A_3 in terms of the frame field $e_\mu{}^{ab}$ (includes the "cascade" below)
  - after implementing a set of gauge conditions (eliminating all Stückelberg fields) some of the equations become *constraints*
  - the last component eq is the spin 3 Cotton equation:

$$F_3(0,4) = 0$$

- the spin 4 sector behaves the same way
  - linearly all spins behave like this (Pope-Townsend (1989))

The spin 3 cascade equations

The subset of equations in irrep **5** gives :

- the *cascade* solution $f_\mu{}^{ab} \to \tilde{f}_\mu{}^{ab} \to \tilde{\omega}_\mu{}^{ab} \to \tilde{e}_\mu{}^{ab} \to e_\mu{}^{ab}$
- and the 5'th order spin 3 Cotton equation (last equation below)
  - $F^{ab}(4,0) =$
    $De^{ab} + e^c \wedge \tilde{e}^{d(a}\epsilon_{cd}{}^{b)} - (e^{(a} \wedge \tilde{e}^{b)} - trace) = 0,$
  - $F^{ab}(3,1) =$
    $D\tilde{e}^{ab} - 2e^c \wedge \tilde{\omega}^{d(a}\epsilon_{cd}{}^{b)} - (e^{(a} \wedge \tilde{\omega}^{b)} - trace) - 4f^c \wedge e^{d(a}\epsilon_{cd}{}^{b)} = 0,$
  - $F^{ab}(2,2) =$
    $D\tilde{\omega}^{ab} + 3e^c \wedge \tilde{f}^{d(a}\epsilon_{cd}{}^{b)} - (e^{(a} \wedge \tilde{f}^{b)} + f^{(a} \wedge \tilde{e}^{b)} - trace)$
    $+3f^c \wedge \tilde{e}^{d(a}\epsilon_{cd}{}^{b)} = 0,$
  - $F^{ab}(1,3) =$
    $D\tilde{f}^{ab} - 4e^c \wedge f^{d(a}\epsilon_{cd}{}^{b)} + (f^{(a} \wedge \tilde{\omega}^{b)} - trace) - 2f^c \wedge \tilde{\omega}^{d(a}\epsilon_{cd}{}^{b)} = 0,$
  - $F^{ab}(0,4) =$
    $Df^{ab} + (f^{(a} \wedge \tilde{f}^{b)} - trace) + f^c \wedge \tilde{f}^{d(a}\epsilon_{cd}{}^{b)} = 0.$ (spin 3 Cotton eq)

- the terms in brackets simplify after choosing gauges (not unique)
  - remaining parts play a role later!

Spin 3 gauges

Spin 3 gauge choices

The spin 3 fields in irrep **3**: Using the gauge symmetry we can set

- $\tilde{e}_\mu{}^a(3,1) = 0$, $\tilde{\omega}_\mu{}^a(2,2) = e_\mu{}^a \hat{\omega}$, etc
- remaining spin 3 symmetry parameters:

$$\Lambda^{ab}(4,0), \quad \tilde{\Lambda}^{ab}(3,1), \quad \tilde{\Lambda}^{a}(3,1) \tag{3}$$

  - i.e. the spin 3 "translations", "Lorentz" and "dilatations"
- the spin 3 dilatation gauge field $\tilde{b}_\mu(2,2)$ and the above (hatted) fields are also determined by the zero field strength equations!

=> All fields are then given in terms of the spin 3 frame field $e_\mu{}^{ab}$ =>

- The spin 2 covariant spin 3 Cotton eq is quite complicated even when neglecting all spin ≥ 4 fields: (see the arXiv)
  - when expressed in terms of $e_\mu{}^{ab}$ it has more than 1300 terms!
  - about 900 terms in the metric version!

Spin 3 Cotton eq

Some spin 3 linearized equations

Define the operator $\mathcal{O}$ by $\tilde{e}_\mu{}^{ab}|_{lin}(e_\mu{}^{ab}) = (\mathcal{O}e)_\mu{}^{ab}$
Then

$$\tilde{\omega}_\mu{}^{ab}|_{lin}(e_\mu{}^{ab}) = -\tfrac{1}{2}(\mathcal{O}^2 e)_\mu{}^{ab}$$
$$= -\tfrac{1}{2}(-\Box e_\mu{}^{ab} + \partial_\mu \partial^\nu e_\nu{}^{ab} - \tfrac{4}{3} e_\mu{}^{(a}\partial_\nu\partial_\rho e^{|\nu\rho|b)} + \tfrac{4}{3} e_\mu{}^{(a}\Box e_\nu{}^{b)\nu}$$
$$+ \, another \;\; 12 \;\; terms).$$

Spin 3 Cotton equation

$$C_\mu{}^{ab}(e)|_{lin} := (\not{\epsilon}f(e)|_{lin})_\mu{}^{ab} = \tfrac{1}{4!}\not{\epsilon}(\mathcal{O}^4 e + \mathcal{O}\hat{\mathcal{O}}\mathcal{O}^2 e)_\mu{}^{ab} = 0.$$

- here $\not{\epsilon}$, $\mathcal{O}$ are constructed from an epsilon tensor and a derivative
  - $\hat{\mathcal{O}}$ is another operator similar to $\mathcal{O}$
- by turning ∂_μ into D_μ we get the first term in the full Cotton eq:

$$C_{\mu\nu\rho} = C_{\mu\nu\rho}(D^5) + C_{\mu\nu\rho}(R, D^3) + C_{\mu\nu\rho}(R^2, D)$$

Spin 3 Cotton eq: checks on the non-linear equations

Checks on the spin 2 covariant spin 3 Cotton equation (> 1300 terms)

- The Cotton eq is invariant under spin 3 translation, Lorentz and scale transformations related to $\Lambda^{ab}(4,0)$, $\tilde{\Lambda}^{ab}(3,1)$, $\tilde{\Lambda}^{a}(3,1)$! (this is true provided the other parameters are solved for and inserted into the transformations rules for translations etc)
- the linearized Cotton eq in the metric formulation coincides with the one of *Damour-Deser (1987)*
  - a nice way to write it is (Bergshoeff et al (2010), Henneaux et al (2015))
    $C_{\mu\nu\rho}|_{lin}(h) := (\not\partial)_{\mu}{}^{\sigma}(\not\partial)_{\nu}{}^{\tau}S_{\sigma\tau\rho}|_{lin} = 0$
    where the linearized Schouten tensor
    $S_{\mu\nu\rho}|_{lin}(h) := G_{\mu\nu\rho}|_{lin} - \frac{3}{4}\eta_{(\mu\nu}G_{\rho)}|_{lin}$ (note the second term)
    and the spin 3 “Einstein” tensor
    $G_{\mu\nu\rho}|_{lin}(h) := -\frac{1}{6}(\not\partial)_{\mu}{}^{a}(\not\partial)_{\nu}{}^{b}(\not\partial)_{\rho}{}^{c}h_{abc}$
- the non-linear field $\tilde{\omega}_{\mu}{}^{ab}(e)$ is Lorentz invariant when linearized but not otherwise! (i.e. has no metric formulation)
  - but $\tilde{\omega}_{\mu}{}^{ab}|_{lin}(h)$ coincides with the *Damour-Deser Ricci tensor*

Coupling to a scalar field: unfolding

Unfolding (BN:1312.5883)

A minimal requirement: the unfolded equation must reproduce the conformal scalar equation in an arbitrary curved background:

$$\Box\phi - \tfrac{1}{8}R\phi = 0$$

This is easy to demonstrate: recall $A(q,p)$ while $\Phi(p)$

$$\mathcal{D}\Phi|0\rangle_q = (d+A)\Phi|0\rangle_q = (d - iA_2 - A_3 + ...)\Phi|0\rangle_q = 0$$

- $|0\rangle_q$ component eq => $\phi_\mu = -\partial_\mu\phi$
  - dropping higher spin terms $s \geq 3$ here (${e_\mu}^a$ used)
- $K_a|0\rangle_q$ component eq => $\frac{1}{8}R\phi$ from $f^a(0,2)K_a(0,2)$ term in A_2
  - including the spin 3 sector gives
    $\Box\phi - \frac{1}{8}R\phi + \tilde{f}\phi = 0$
    where $\tilde{f} := {e_a}^\mu {\tilde{f}_\mu}{}^a(1,3)$ from solving $F_3 = 0$

Further requirements

Back reaction (BN:1506.03328)

The term $-\frac{1}{8}R\phi$ leads to back reaction in the spin 2 Cotton equation.

- => must add source terms to the unfolded equation!

This follows from from looking at the component eqs –>

In detail: expand $\mathcal{D}\Phi|0\rangle_q = 0$ in $K_{a_1\ldots a_n}|0\rangle_q$:

$$\begin{aligned}
n = 0: &\quad (\partial_\mu\phi + \phi_\mu + \mathcal{O}(e_\mu{}^{ab}\phi_{ab}(s=3)+\ldots))|0\rangle_q = 0,\\
n = 1: &\quad (D_\mu\phi^a + f_\mu{}^a\phi + 6\phi_\mu{}^a + \mathcal{O}(s\geq 3))K_a|0\rangle_q = 0,\\
n = 2: &\quad (D_\mu\phi^{ab} + f_\mu{}^{(a}\phi^{b)} + 15\phi_\mu{}^{ab} + \mathcal{O}(s\geq 3))K_{ab}|0\rangle_q = 0,
\end{aligned}$$

where $D = d + \omega_{s=2}$.
For $n \geq 1$ we split these into irreps (in μ and $a, b, ..$) with spin

$$n^- = n-1,\;\; n^0 = n,\;\; n^+ = n+1 \tag{4}$$

Background discussion: the classical case

The $n = 0$ eq => $\phi_\mu = -\partial_\mu \phi$ (as above).
Inserting this into the other equations gives

$$\begin{aligned} n = 1^- : &\qquad -\Box\phi + f_\mu{}^\mu \phi + \mathcal{O}(s \geq 3) = 0, \\ n = 1^0 : &\qquad \epsilon^{\mu\nu a} f_{\mu\nu} + \mathcal{O}(s \geq 3) = 0, \\ n = 1^+ : &\qquad -D_{(\mu}\partial_{\nu)}\phi + f_{(\mu\nu)} + 6\phi_{\mu\nu} + \mathcal{O}(s \geq 3) = 0. \end{aligned}$$

$$\begin{aligned} n = 2^- : &\qquad D_\mu \phi^{\mu a} + \frac{1}{2} f_\mu{}^\mu \phi^a + \frac{1}{6} f^{ab}\phi_b + \mathcal{O}(s \geq 3) = 0, \\ n = 2^0 : &\qquad \epsilon^{\mu\nu(a} D_\mu \phi_\nu{}^{b)} + \frac{1}{2}\epsilon^{\mu\nu(a} f_\mu{}^{b)}\phi_\nu + \mathcal{O}(s \geq 3) = 0, \\ n = 2^+ : &\qquad D_{(\mu}\phi_{\nu\rho)} + f_{(\mu\nu}\phi_{\rho)} + 15\phi_{\mu\nu\rho} + \mathcal{O}(s \geq 3) = 0. \end{aligned}$$

- note that the n^+ equations will just determine $\phi(x)_{\mu_1 \ldots \mu_{n+1}}$

Unfolding

From the equations on the previous slide we find that

- the 1^- eq => the linear Klein-Gordon equation (as above)
- the 2^- eq =>

$$(R_{\mu\nu} - 2f_{\mu\nu} - 2f_\rho{}^\rho g_{\mu\nu})\partial^\nu\phi - (D_\nu f_\mu{}^\nu - D_\mu f_\nu{}^\nu)\phi = 0.$$

  but this eq becomes an identity if we set, as found from $F_2 = 0$,

$$f_{\mu\nu} = \frac{1}{2}S_{\mu\nu} = \frac{1}{2}(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R),$$

  where $S_{\mu\nu}$ is the Schouten tensor.

- the 2^0 part eq =>

$$-\frac{1}{2}C_{\mu\nu}\phi = 0$$

  i.e. the Cotton equation without a source term!

What kind of coupled spin 2 / scalar theory are we aiming for? —–>

Topologically gauged BLG theory

Prototype field theory: gauged CFT_3 with $\mathcal{N} = 8$ susy

(Gran, BN (2008)), (Gran, Greitz, Howe, BN(2012))

- BLG scalars $X^i{}_a$: $i, j, ..$ $SO_R(8)$ and $a, b, ..$ gauge group indices,
- The bosonic part of L is

$$L = \frac{1}{g}L^{sugra}_{conf} + L^{BLG}_{cov} - \frac{1}{16}X^2R - V^{(tt)}_{new}$$

the total scalar potential has

- a contribution from L^{BLG}_{cov}

$$V^{(st)}_{BLG} = \frac{\lambda^2}{12}(X^i{}_a X^j{}_b X^k{}_c f^{abcd})(X^i{}_e X^j{}_f X^k{}_g f^{efg}{}_d)$$

- and a new term from the topological gauging

$$V^{(tt)}_{new} \quad = \quad \frac{eg^2}{2{\cdot}32{\cdot}32}\Big((X^2)^3 - 8(X^2)X^j_b X^j_c X^k_c X^k_b + 16X^i_c X^i_a X^j_a X^j_b X^k_b X^k_c\Big)$$

- setting $\lambda = 0$ => $SO(4)$ can be extended to $SO(N)$ for any N!

The Cotton equation in gauged CFTs

- The bosonic field equations relevant for this talk are obtained by replacing ${X^i}_a(x)$ by one scalar $\phi(x)$:

$$\Box\phi - \frac{1}{8}R\phi - \frac{27g^3}{32\cdot 32}\phi^5 = 0$$

and

$$\begin{aligned}
&C_{\mu\nu} - \frac{g}{16}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\phi^2 - \frac{g}{2}(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial^\rho\phi\partial_\rho\phi) \\
&-\frac{g}{8}(\phi\Box\phi + \partial^\rho\phi\partial_\rho\phi)g_{\mu\nu} + \frac{g}{8}(\phi D_\mu\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi) \\
&+\frac{9g^3}{2\cdot 32\cdot 32}g_{\mu\nu}\phi^6 = 0
\end{aligned}$$

The Cotton equation in gauged CFTs and unfolding

In the hs context we split all equations into irreps.
The irreducible spin 2 Cotton eq is in the irrep **5** and reads

- $C_{\mu\nu} - \frac{g}{16}(\phi^2 R_{\mu\nu} - 2\phi D_\mu \partial_\nu \phi + 6\partial_\mu \phi \partial_\nu \phi)|_{\mathbf{5}} = 0$

This is the equation we need to obtain from the unfolded equation!

- now, using the 2^- and 1^+ parts of the unfolded eq to eliminate $D_\mu \partial_\nu \phi$ in the above eq our prototype Cotton eq becomes:

$$C_{\mu\nu} = \frac{3g}{8}(\phi_\mu \phi_\nu - 2\phi\phi_{\mu\nu})|_{\mathbf{5}}$$

- But recall the 2^0 part of the unfolded eq $\mathcal{D}\Phi|0\rangle_q = 0$:

$$-\frac{1}{2}C_{\mu\nu}\phi = 0$$

i.e. the Cotton equation without a source term!

Source terms

Unfolded eq with sources

Thus we must add source terms to the unfolded equation

- the source term (1-form, $dim = L^{-\frac{1}{2}}$) that seems to do the job is

$$S_M = -i\lambda_1 M(\Phi^*\Phi)\Phi \;\; with \; M = dx^\mu e_\mu{}^a M_a$$

- in fact, expanding in powers of K_a gives

$$S_M|0\rangle_q = -i\lambda_1 M((\phi^a\phi^b - 2\phi\phi^{ab})\phi K_{ab} + ...)|0\rangle_q$$

- with this source term the 2^0 eq gives the Cotton equation above

$$C_{\mu\nu} = 36\lambda_1(\phi_\mu\phi_\nu - 2\phi\phi_{\mu\nu})|_{\mathbf{5}}$$

which indicates that we are on the right track!

Sixth order potential

The sixth order potential

From dimensional arguments alone it is possible to add also

$$S_K = -i\lambda_2 K(\Phi^*\Phi)^2\Phi \;\; \textit{with} \; K = dx^\mu e_\mu{}^a K_a$$

which gives the contributions to the unfolded equation

$$\begin{aligned} P^a 6\lambda K_a \Phi^5|0>_q \qquad &= 0, \quad n = 0, \\ &= -3\lambda_2\phi^5|0>_q, \quad n = 1^-, \\ &= -10\lambda_2\phi^b\phi^4 K_b|0>_q, \quad n = 2^-. \end{aligned}$$

- thus these new terms give the correct non-linear spin 2 Cotton and Klein-Gordon equations
- The 2^- equation (def of Schouten) remains the same due to new terms on both sides which cancel

Comments

- *Conclusions:*
  - we derived the spin 2 covariant spin 3 Cotton (complicated!)
  - we proposed source terms for the unfolded eq which seems to give the correct spin 2 Cotton/scalar theory (for $F = T$ see BN:1506.03328)
- *but many aspects are still unclear:*
  - what are the full set of source terms in the two higher spin eqs?
  - what is the Lagrangian in terms of $A(x; q, p)$ *and* $\Phi(x; p)$??
  - is there a hs invariant metric definition? (Campoleoni, Fredenhagen, ...)
    - does $\hat{g}_{\mu\nu} = Tr(e_{(\mu}f_{\nu)})$ (has dim=L^{-2}) work?
  - is 3d conformal hs-theory on AdS_3 a new theory? $(sl(\lambda) \times sl(\lambda) \leftarrow so(1,2) \times so(1,2) \rightarrow hs(so(3,2)))$
    - can we learn anything about derivative dressing in AdS? (Bekaert, Skvortsov, Taronna, , Erdmenger,...)
  - what is its exact role in AdS_4/CFT_3? (N/mixed bc) (Giombi, Yin, Klebanov, ...)