Covariant Entanglement Constructs

Veronika Hubeny

UC DAVIS
UNIVERSITY OF CALIFORNIA

Physics Department & center for Quantum Mathematics and Physics

INWARD BOUND
conference 2016

NORDITA, Aug.20, 2016

based on earlier works w/ {M. Headrick, A. Lawrence, H. Maxfield, M. Rangamani, T. Takayanagi, E. Tonni}
& on work in progress w/ M. Headrick

Motivation

- Elucidate holography
  - Fundamental nature of spacetime & its relation to entanglement
  - Structure/characterization of CFTs (& states) w/ gravity dual
- Start w/ situations with large amount of symmetry (e.g. pure AdS)
  - Explicit calculations possible, can obtain analytical expressions
  - Use these to guess duality relations → entry in gauge/gravity dictionary
- But this has limitations
  - How to generalize? (e.g. time dependence)
  - Often symmetry brings degeneracy between logically distinct concepts
- Need to "covariantize"
  - Define a quantity which is purely geometrical (e.g. independent of any choice of coordinate systems) and fully general

Utility of covariant constructs

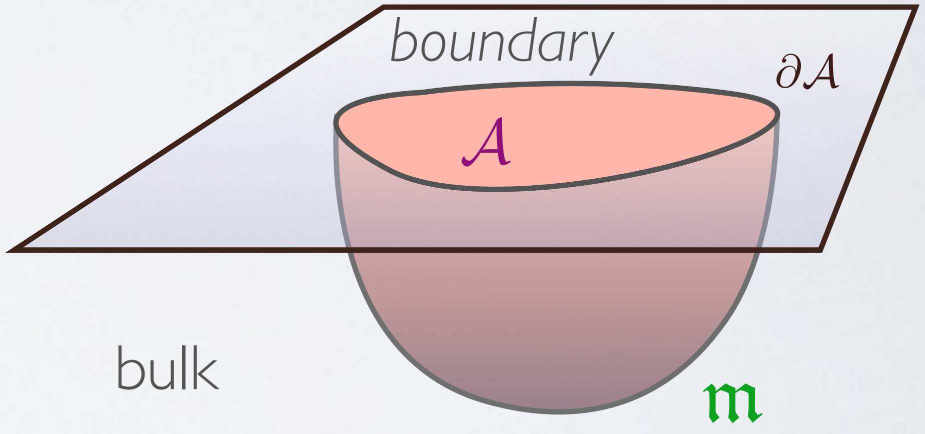
- Gives a general prescription
  - Definition of a quantity is equally robust on both sides of duality
  - Once beyond analytically tractable cases, might as well go for full generality (within the class of systems we want to consider)
- Time dependence interesting in its own right
  - Novel phenomena in out-of-equilibrium systems
  - New insight into the structure of the theory
- Breaks degeneracy between distinct constructs
  - Allows us to identify the true dual → underlying nature of the map
- Natural covariant constructs motivate new relations
  - Even if a given construct is not the sought dual, it eventually finds its use

Example: Holographic EE

Proposal [RT=Ryu & Takayanagi, '06] for *static* configurations:

In the bulk, entanglement entropy $S_{\mathcal{A}}$ for a boundary region $\mathcal{A}$ is captured by the area of a minimal co-dimension-2 bulk surface $\mathfrak{m}$ *at constant t* anchored on entangling surface $\partial\mathcal{A}$ & homologous to $\mathcal{A}$

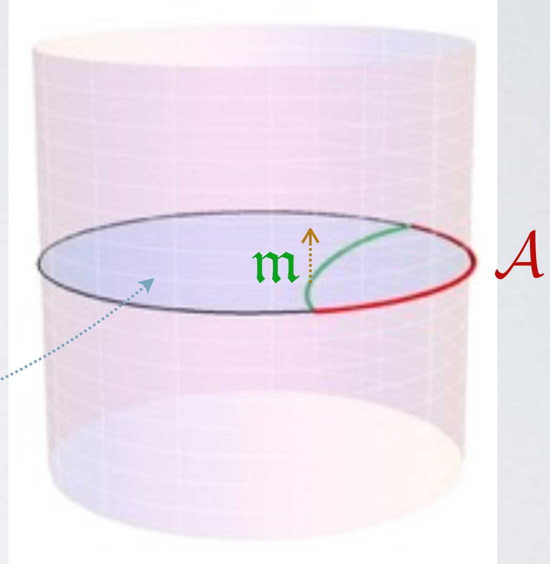
$$S_{\mathcal{A}} = \min_{\partial\mathfrak{m}=\partial\mathcal{A}} \frac{\text{Area}(\mathfrak{m})}{4\,G_N}$$



Covariant Holographic EE

But the RT prescription is not well-defined outside the context of static configurations:

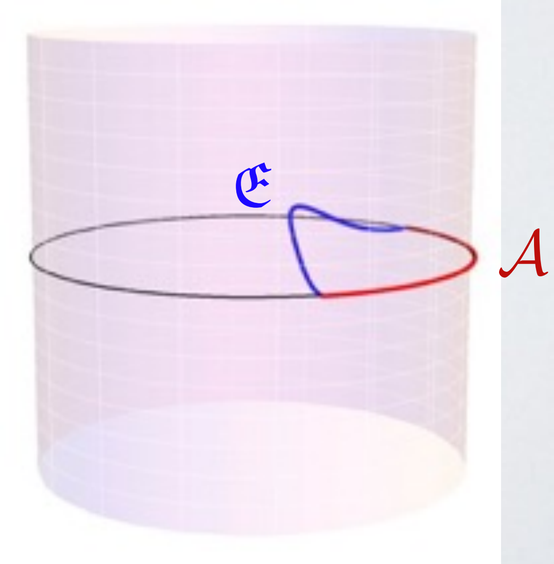
- In Lorentzian geometry, we can decrease the area arbitrarily by timelike deformations
- In time-dependent context, no natural notion of "const. t" slice…



In *time-dependent* situations, RT prescription must be covariantized:

Simplest candidate: [HRT = VH, Rangamani, Takayanagi '07]

minimal surface $\mathfrak{m}$ at constant time → extremal surface $\mathfrak{E}$ in the full bulk



This gives a well-defined quantity in any (arbitrarily time-dependent asymptotically AdS) spacetime

⇒ equally robust as in CFT

Covariant Holographic EE

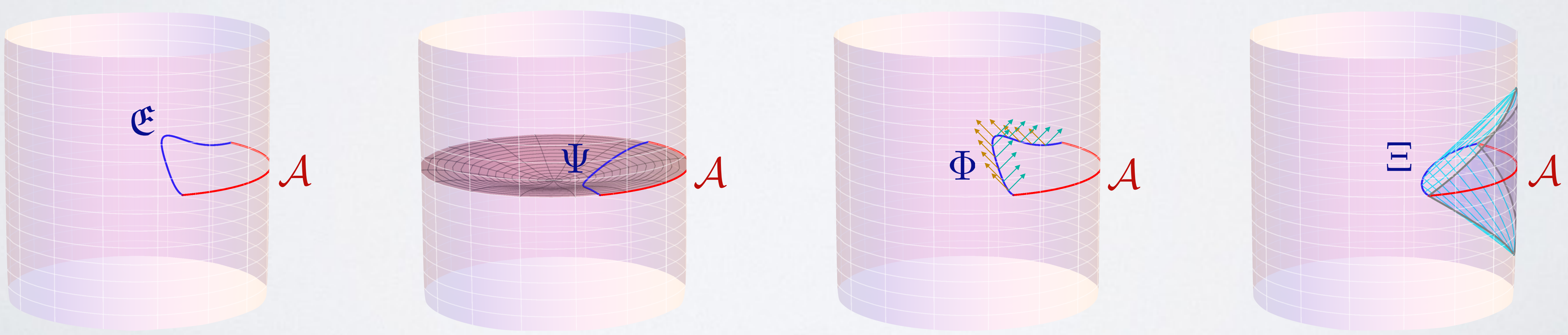
In fact, [Hubeny, Rangamani, Takayanagi '07] identified 4 natural candidates:

(all co-dim.2 surfaces ending on $\partial\mathcal{A}$, and coincident for ball regions $\mathcal{A}$ in pure AdS)

- $\mathfrak{E}$ = Extremal surface
- Ψ = Minimal-area surface on maximal-volume slice
- Φ = Surface with zero null expansions
- Ξ = Causal wedge rim

$\mathfrak{E} = \Phi$ is correct
= 'HRT prescription'

Later known as Causal Information Surface;
w/ area = causal holographic information χ [Hubeny, Rangamani '12]



Power of covariant constructs

- 'Natural' geometrical constructs (defined for general bulk spacetimes, independent of coordinates) provide useful candidates for dual of 'natural' quantities in CFT
- e.g. dual of $\rho_{\mathcal{A}}$? [Bousso, Leichenauer, Rosenhaus; Czech, Karczmarek, Nogueira, Van Raamsdonk;…]
- In generic Lorentzian spacetime, null congruences which define a causal set provide useful characterization of 'natural' bulk regions.

2 options:

…starting from bdy:

$D[\mathcal{A}] \rightsquigarrow$ Causal Wedge:

= future and past causally-separated from bdy region determined by $\rho_{\mathcal{A}}$

[VH & Rangamani]

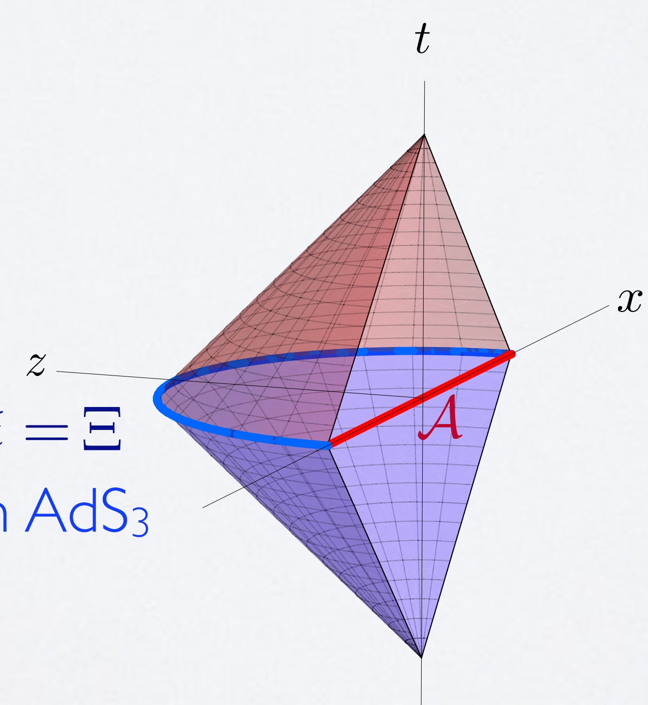
…starting from bulk:

$\mathfrak{E} \rightsquigarrow$ Entanglement Wedge:

= spacelike-separated (toward $\mathcal{A}$) from $\mathfrak{E}$

[Headrick, VH, Lawrence, Rangamani]

$\mathfrak{E} = \Xi$ in AdS_3



NB: in pure AdS, & for spherical $\mathcal{A}$, these coincide, but not in general.

Causal wedge vs. Entanglement wedge

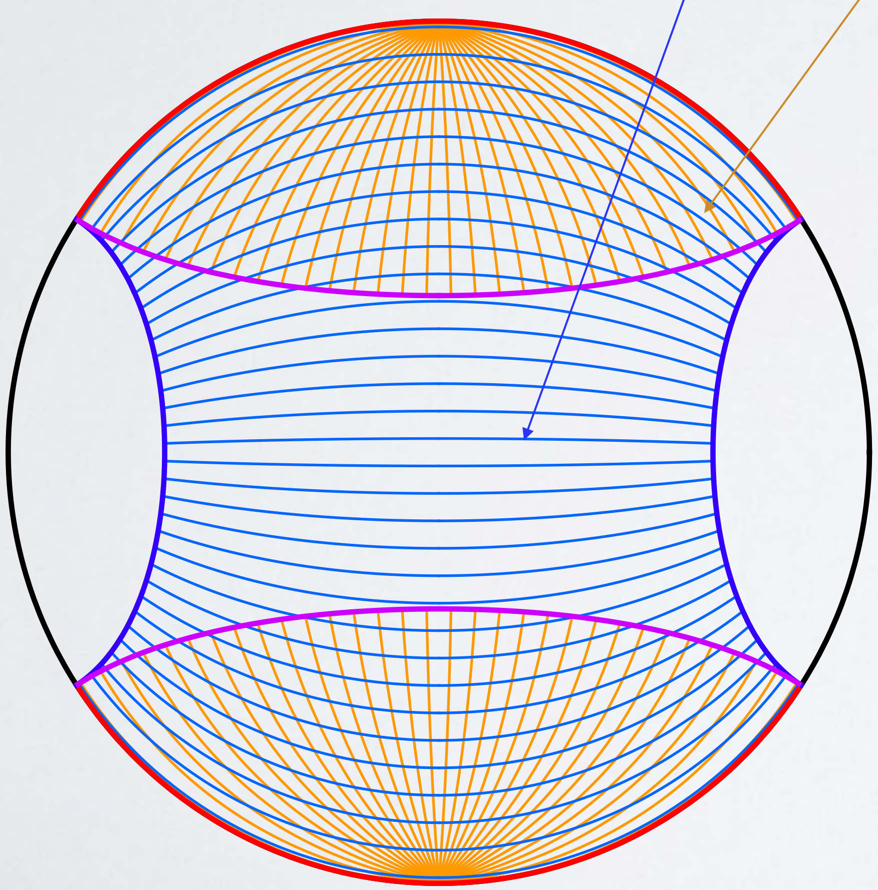
- Even in pure AdS3, these can differ for composite regions $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$

Entanglement Wedge

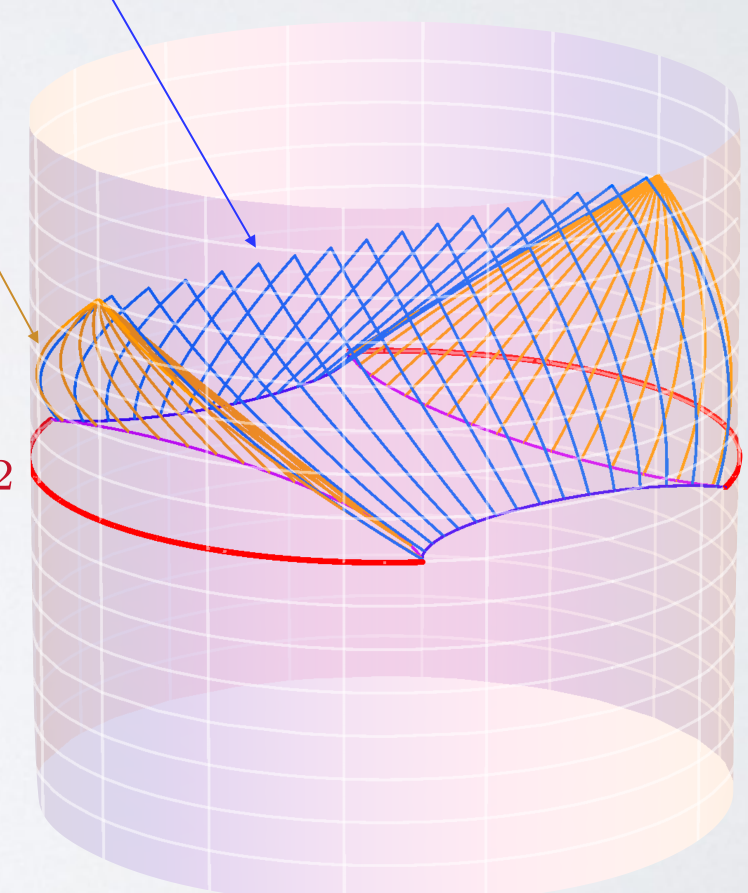
contains

Causal Wedge

$\mathcal{A}_1$



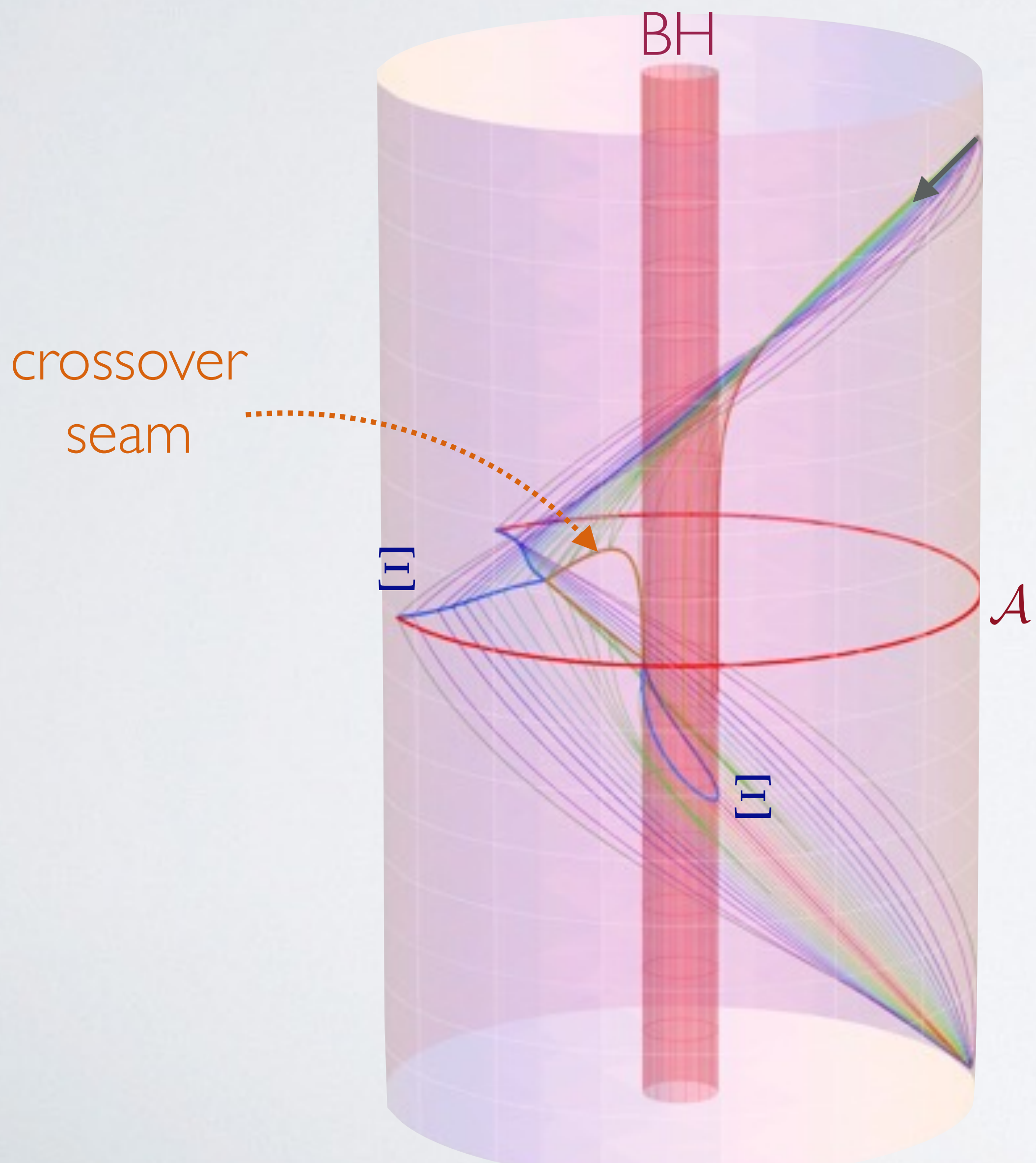
$\mathcal{A}_2$



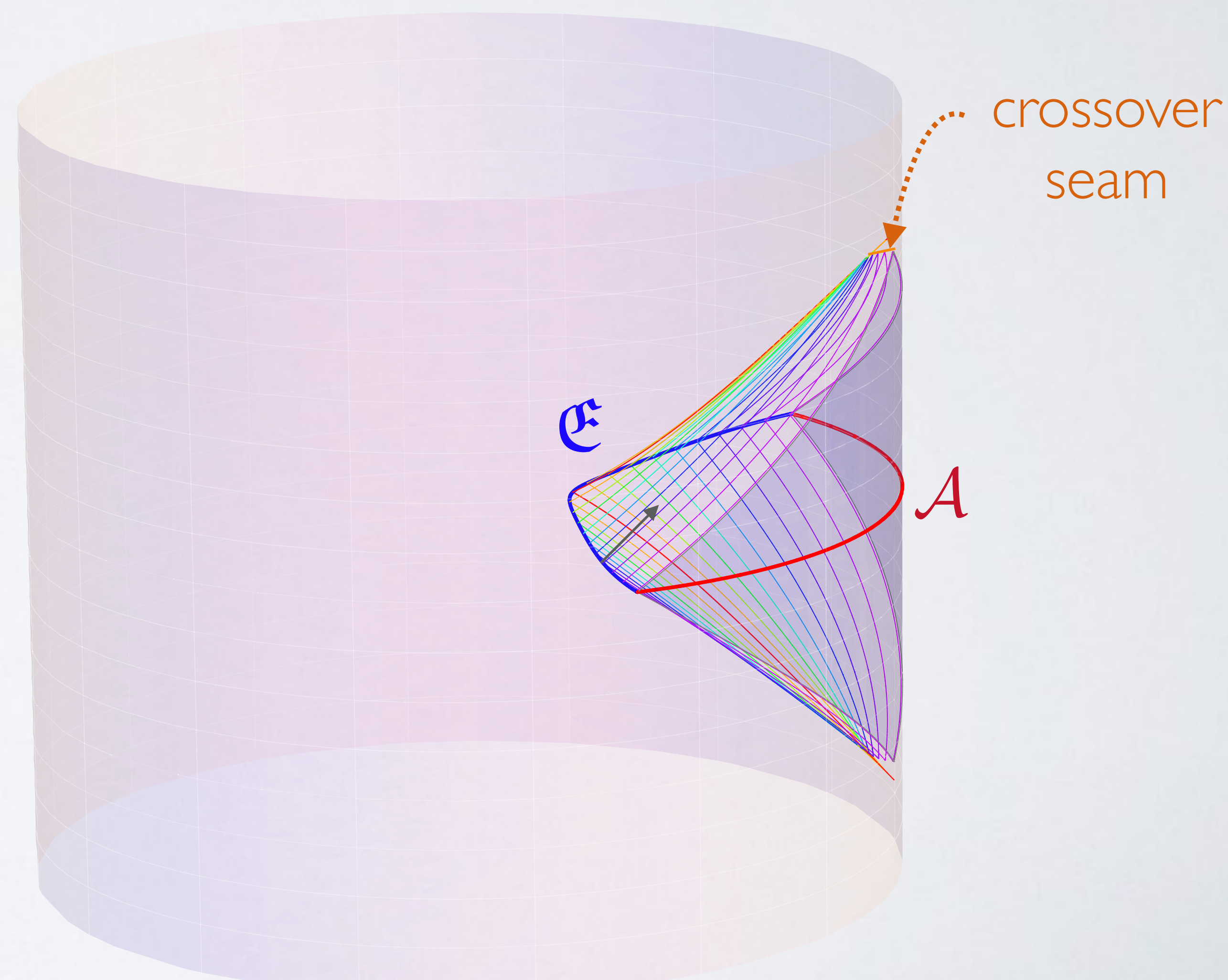
Causal wedge vs. Entanglement wedge

- crucial difference: in which direction can null generators cross…

$D[\mathcal{A}] \rightsquigarrow$ Causal Wedge:



$\mathfrak{E} \rightsquigarrow$ Entanglement Wedge:



Power of covariant constructs

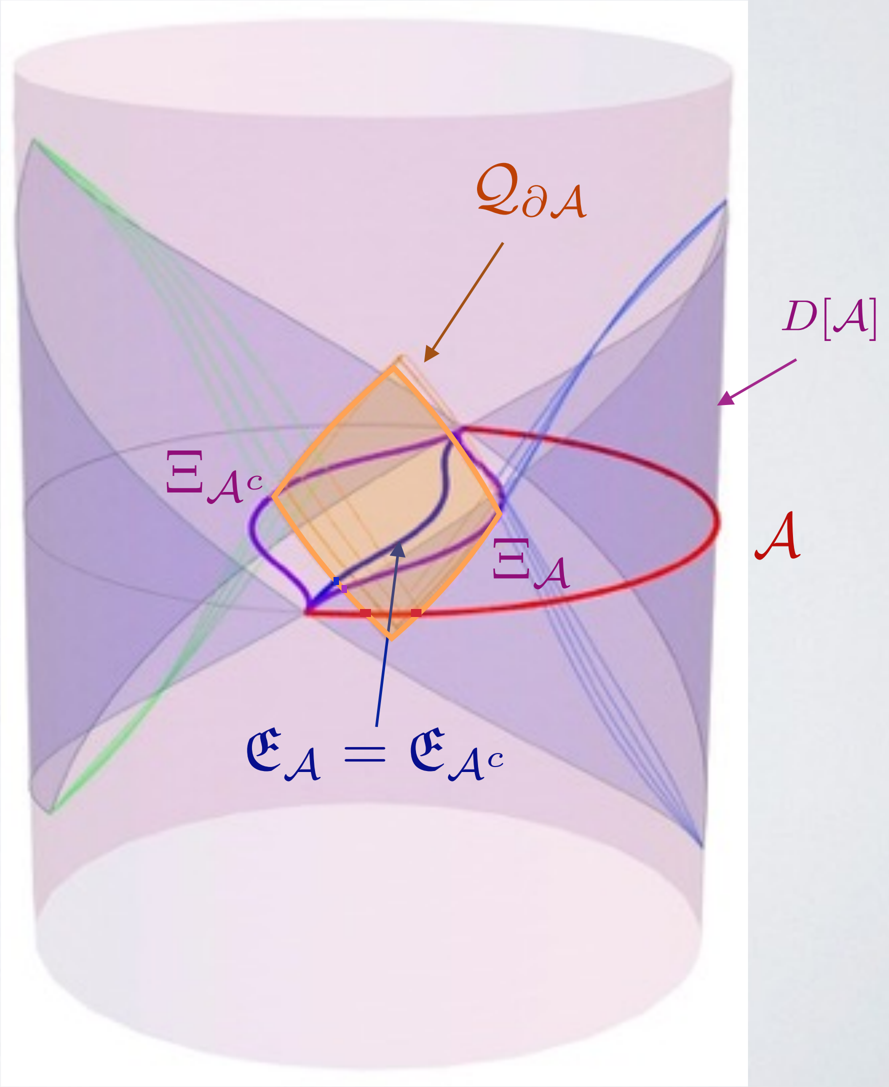
$D[\mathcal{A}] \rightsquigarrow$ Causal Wedge: $\mathfrak{E} \rightsquigarrow$ Entanglement Wedge:

…continued past Ξ: $\rightsquigarrow$ Causal Shadow $\mathcal{Q}_{\partial\mathcal{A}}$

- We can prove the inclusion property [Headrick, VH, Lawrence, Rangamani; Wall]

  CW $\subset$ EW

  or equivalently, $\mathfrak{E} \subset \mathcal{Q}_{\partial\mathcal{A}}$



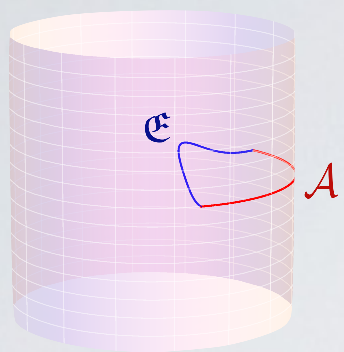
- Consequences:
  - HRT is consistent with CFT causality (= non-trivial check of HRT)
  - Entanglement plateaux
  - Entanglement wedge can reach deep inside a black hole!

Covariant re-formulations

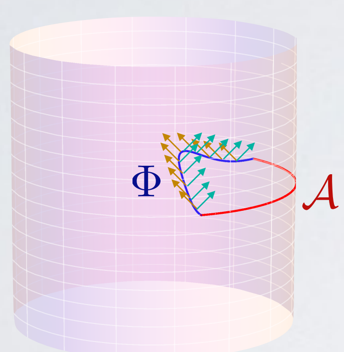
- Covariance is pre-requisite to construct being physically meaningful, but it need not be unique
  - Distinct geometrical formulations can turn out equivalent (cf. $\mathfrak{E} = \Phi$)

- This redundancy is useful
  - Each formulation can have its own advantages
  - e.g. different properties may be manifest in different formulations (cf. gauge / coordinate choice)
  - Re-formulation can reveal deeper relations (cf. ER=EPR [Maldacena, Susskind])

Covariant re-formulations of HEE



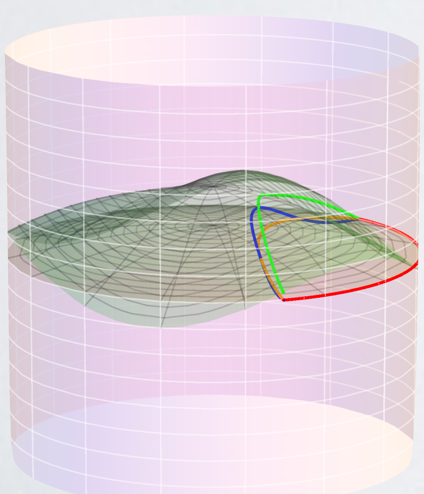
- $\mathfrak{E}$ = Extremal surface
    - (relatively) easy to find
    - minimal set of ingredients required in specification
    - need to include homology constraint as extra requirement



- Φ = Surface with zero null expansions
    - (cf. light sheet construction & covariant entropy bound [Bousso, '99]:

      Bulk entropy through light sheet of surface $\sigma \leq$ Area(σ)/4

      Φ = surface admitting a light sheet closest to bdy



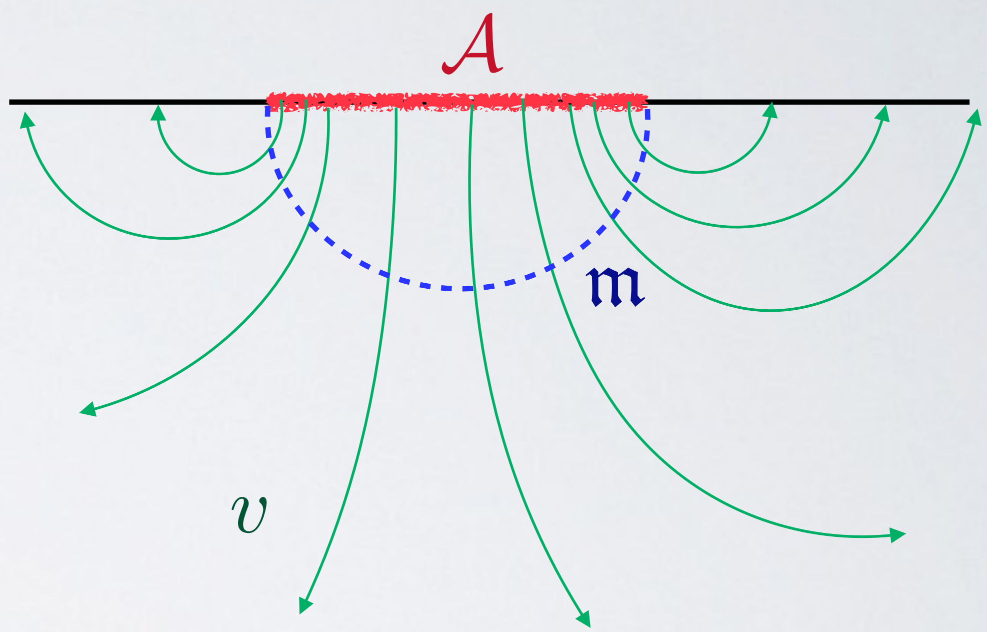
- Maximin surface [Wall, '12]
    - maximize over minimal-area surface on a spacelike slice
    - requires the entire collection of slices & surfaces
    - implements homology constraint automatically
    - useful for proofs (e.g. SSA)

- But none of these elucidate the relation to quantum information

Bit thread picture of (static) EE

- Reformulate EE in terms of flux of flow lines [Freedman & Headrick, '16]
  - let v be a vector field satisfying $\nabla \cdot v = 0$ and $|v| \leq 1$. Then EE is given by

$$S_{\mathcal{A}} = \max_{v} \int_{\mathcal{A}} v$$



  - By Max Flow - Min Cut theorem, equivalent to RT: (bottleneck for flow = minimal surface)

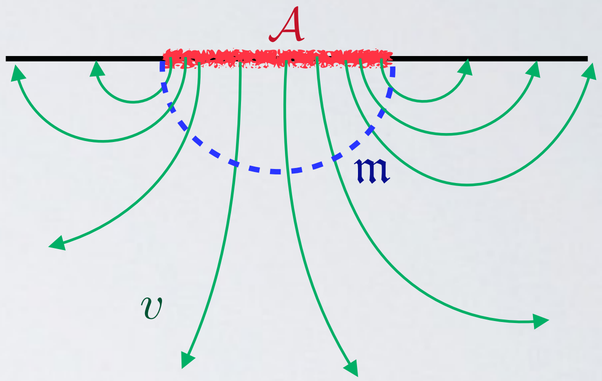
- Useful reformulation of holographic EE
  - flow continuous under varying region (cf. minimal surfaces can jump discontinuously)
  - implements QI meaning of EE and its inequalities more naturally
  - provides more intuition: think of each bit thread as connecting an EPR pair

- How does this extend to time-dependent settings?

Covariantizing bit threads

1. Identify the correct geometrical quantities of interest

   Analogous to flow lines (vector field v) in



2. Identify the constraints they must satisfy

   Analogous to $\nabla \cdot v = 0$ and $|v| \leq 1$

3. Identify the expression for EE obtained from these

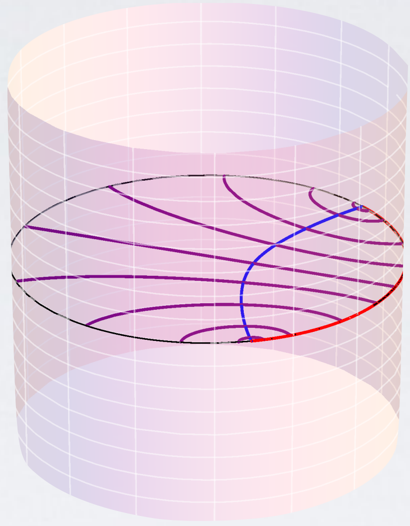
   Analogous to $S_{\mathcal{A}} = \max_v \int_{\mathcal{A}} v$

4. Test that it fulfills all requisite requirements

5. Extract lessons / implications

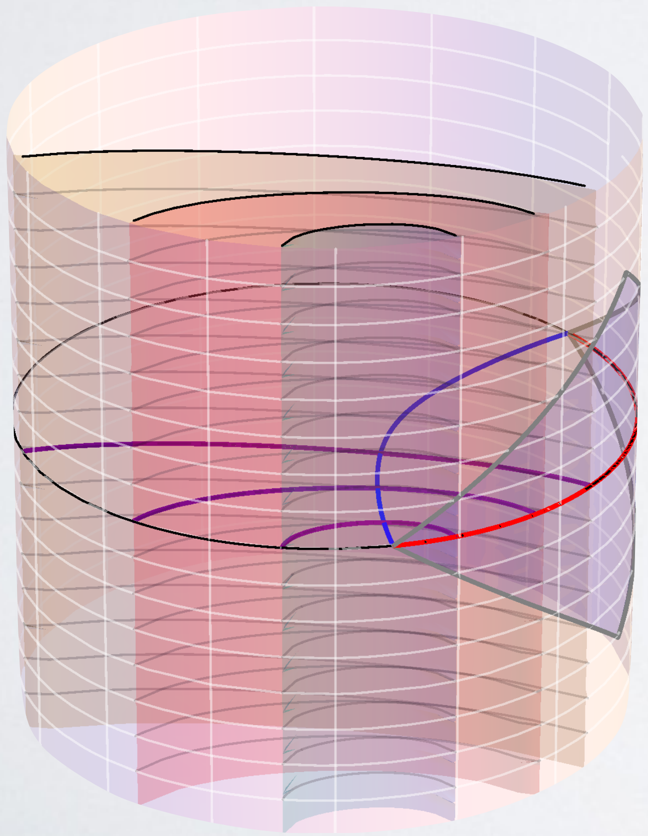
Two natural possibilities

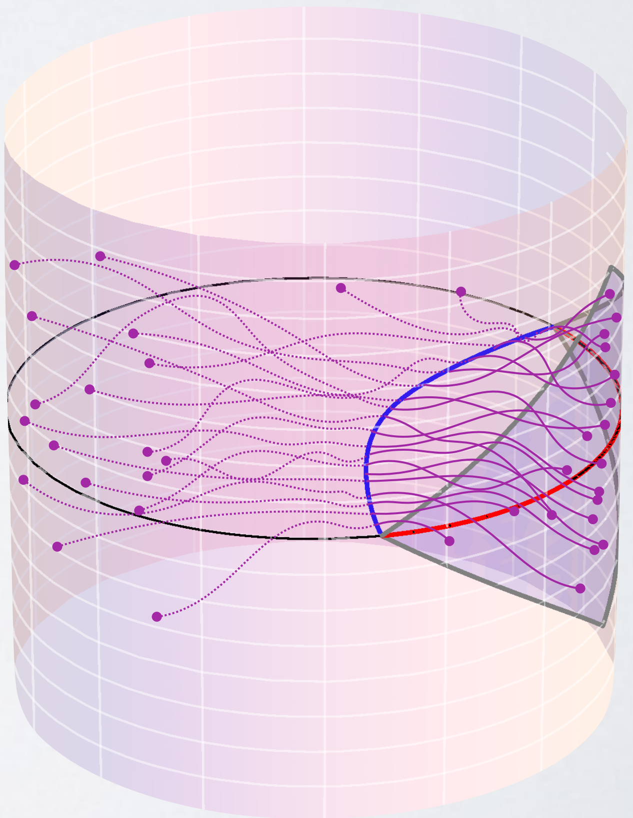
Step 1:



extend threads in time
flow sheets

keep 1-d threads
flow lines

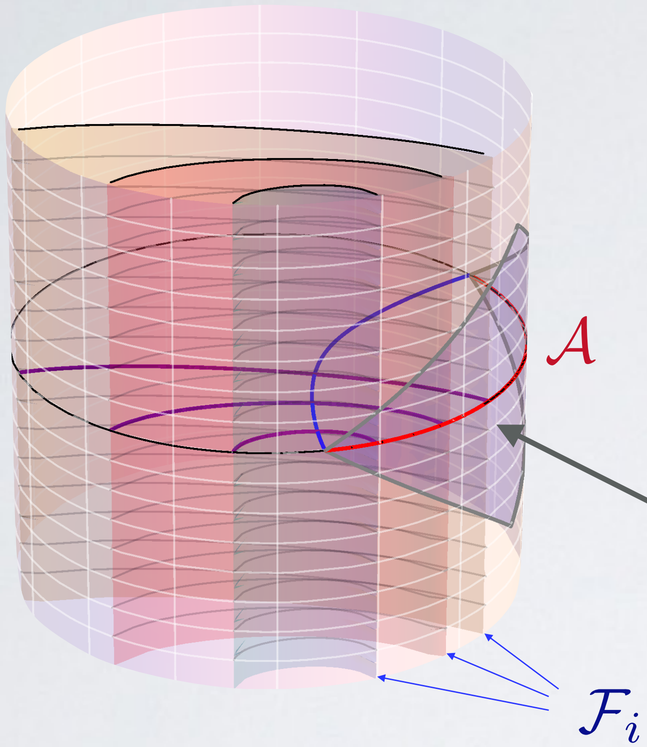




Requirements on constructs

- Imperative:
  - Reduces to bit threads in static case
  - Equivalent to HRT (when null energy condition (NEC) is obeyed)
  - Depends only on $D[\mathcal{A}]$ (i.e. $\partial\mathcal{A}$ + orientation), not on $\mathcal{A}$ itself
- Useful:
  - Manifests CFT causality (directly rather than via equivalence to HRT)
  - Manifests area law, positivity, subadditivity, SSA, etc.
  - Elucidates role of NEC
  - Elucidates role of homology constraint w/ time-dependence

Flow sheets



Require:

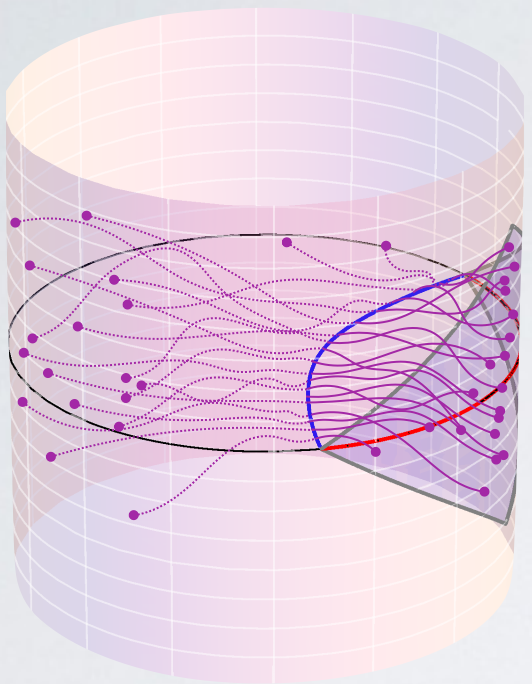
- flow sheets are timelike everywhere
- flow sheets cannot end in the bulk
- density is bounded

$$\rho = \frac{\sum_i \mathrm{Area}[\mathcal{F}_i \cap \mathcal{R}]}{\mathrm{Vol}[\mathcal{R}]} \leq 1$$

EE = sheets through $D[\mathcal{A}]$

- Most "obvious" generalization of bit threads
  - entanglement lasts in time & cannot be changed a-causally ✔
- Danger:
  - Potentially too global (e.g. future singularity may prevent sheets in past)
  - Too many sheets through $D[\mathcal{A}]$ by local boost

Flow lines



Require:

- flow lines are spacelike everywhere
- flow lines don't end: i.e. keep v s.t. $\nabla \cdot v = 0$
- but use integrated norm bound:

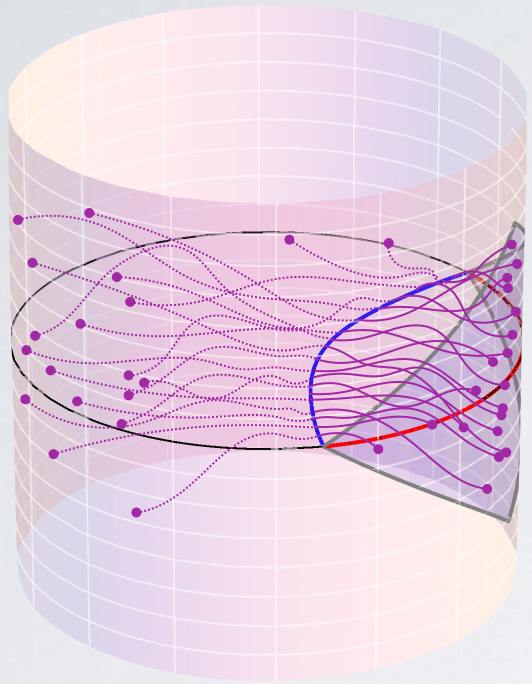
For a unit normal vector w on any worldline γ , $\int_\gamma w \cdot v \leq 1$

$\therefore$ Over full lifetime, any observer sees at most 1 thread / 4 Planck areas

EE counts bit threads in $D[\mathcal{A}]$: $S_{\mathcal{A}} = \max_v \int_{D[\mathcal{A}]} v$

- Covariant construct which works…
  - reduces to bit threads at const time in static case ✔
  - threads must all pass through extremal surface (for max flow)
  - endpoints are floppy and can lie anywhere within $D[\mathcal{A}]$
  - Bonus: naturally picks out the entanglement wedge
  - does not depend on spacetime in the far future ✔

Flow lines



But what is the QI interpretation ?

- Entanglement entropy counted by events ?
  - e.g. # of indep. measurements that can be performed within $D[\mathcal{A}]$
  - novel interpretation…

- Why are 1-d structures natural?
  - why is a specific measurement connected to another instantaneous event somewhere in $\mathcal{A}^c$?


Thank you