Boundary Causality vs Hyperbolicity for Small Black Holes in Gauss-Bonnet

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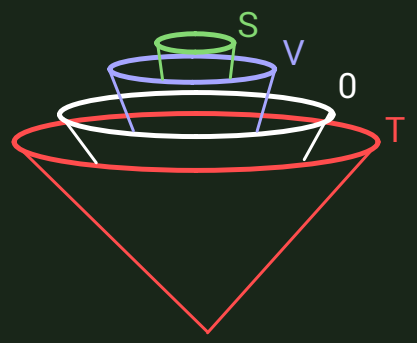
(1609.xxxxx with Elena Carceres and Tomas Andrade)

Propagation Speeds in Gauss-Bonnet

Different graviton modes
(e.g. scalar, tensor)
propagate along different characteristic
surfaces in Gauss-Bonnet.
These modes have different causal cones.



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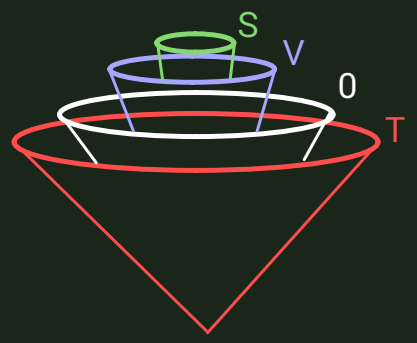

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1. Boundary causality violation
2. Slice-hyperbolicity failure: the initial value problem (IVP) is not well-defined on some slice.
   E.g. a particular slice may be spacelike for light waves, but not for tensor modes.
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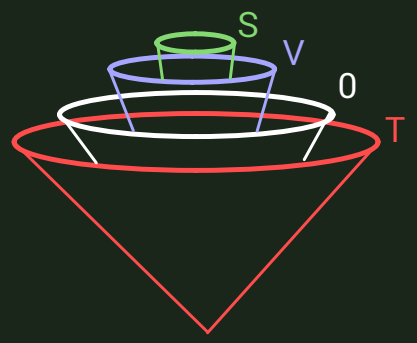

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Note these hyperbolicity failures are local not global as we are used to in AdS.

Black Holes in AdS$_5$ Gauss-Bonnet

Effective metrics

In sufficiently symmetric spacetimes, the different propagation speeds can be accounted for via effective metrics. For AdS$_5$ small black holes:

$$ds^2 = -\frac{f(r)}{f_\infty}dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{c_i(r)}d\Omega_3^2,$$

where λ is the Gauss-Bonnet coupling, μ depends on the black hole radius r_h, f_∞ is a constant, and

$$f(r) = r^2\left[\frac{L^2}{r^2} + \frac{1}{2\lambda}\left(1 - \sqrt{1 - 4\lambda + 4\lambda\frac{\mu}{r^4}}\right)\right],$$

$$c_i(r) = 1 - \sigma_i\left(\frac{(2\lambda\mu}{2\lambda\mu + (1-2\lambda)r^4}\right)$$

and the σ_i tells us which modes propagate along the effective metric's light cones. For the background metric, $\sigma = 0$, while it is set to $\sigma = (2, 1, -2)$ for (scalars, vectors, tensors) respectively.

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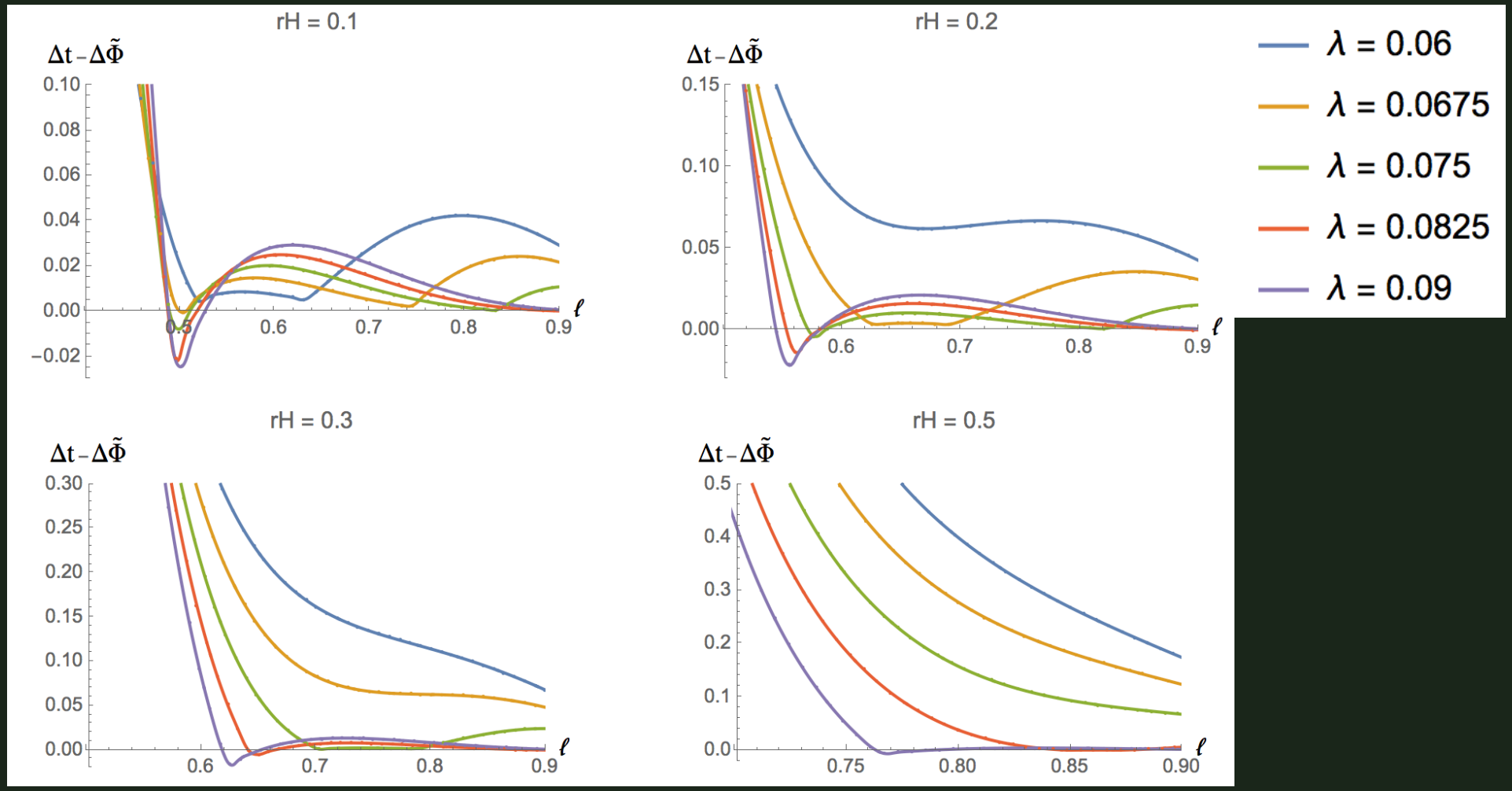
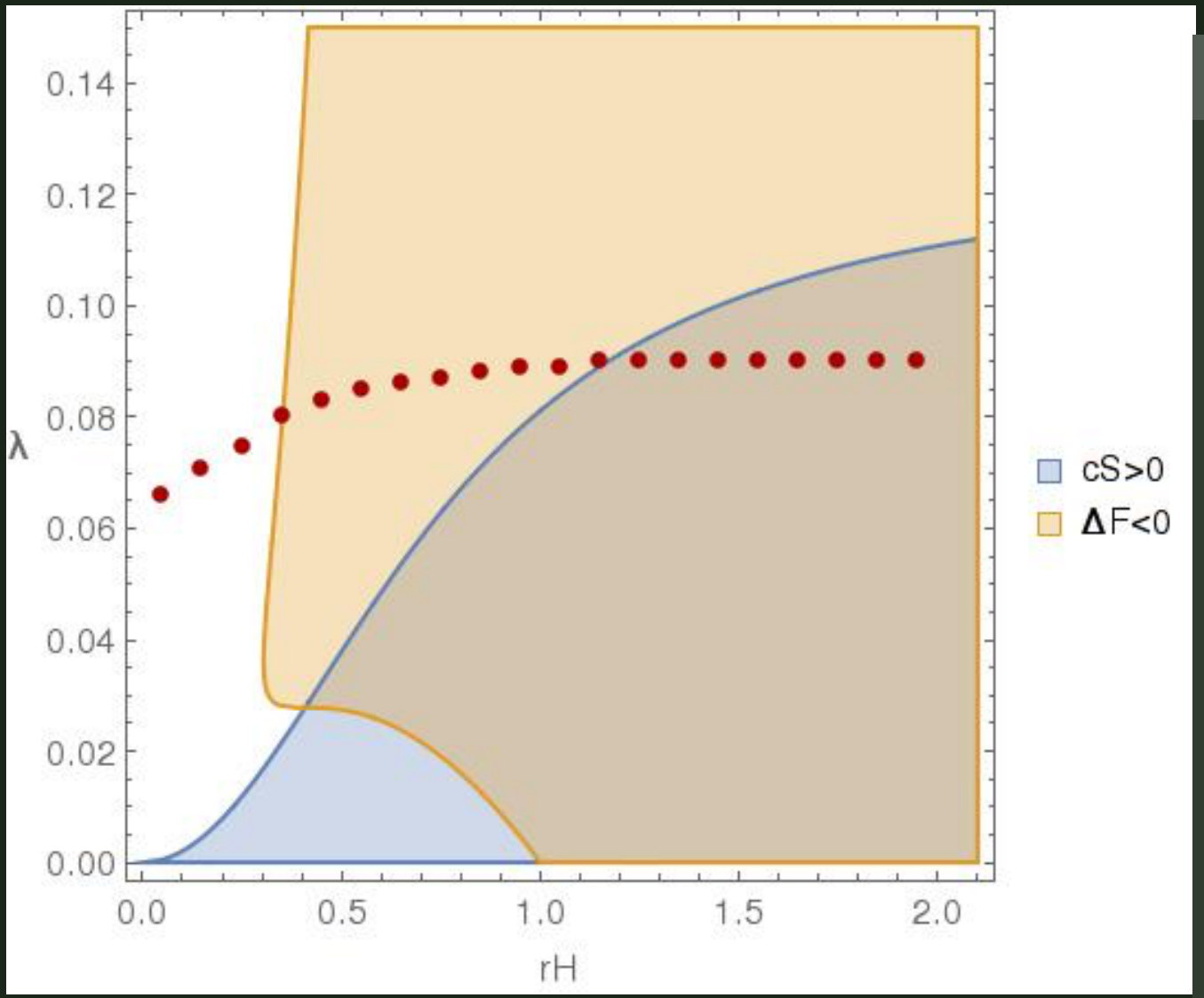
We examine black holes with small r_h, to find what values of λ are allowed if we insist on boundary causality.

Finding Causally Safe λ at Fixed r_h

1. Fix r_h and scan λ's
2. Compute $\Delta\phi$ and Δt for all geos through bulk.
3. Find first λ for which $\exists$ a geodesic with $\Delta\phi > \Delta t$.



Results



Comments

- Choice of theory = Choice of λ
- If a theory has a bad solution, do we throw out the theory or the solution?
- Need a reason to toss a solution.

Problem 2: Hyperbolicity violations

Big question: Is initial value problem well-defined?

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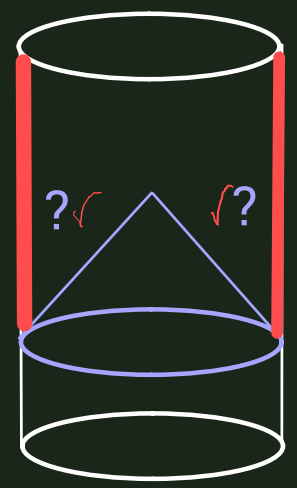

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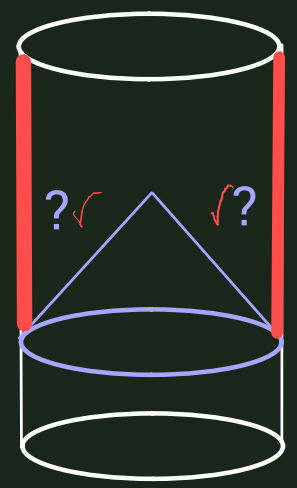

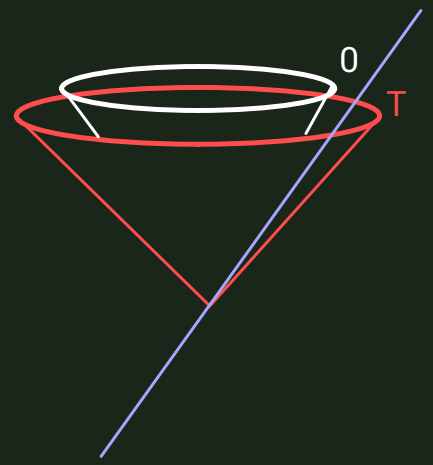

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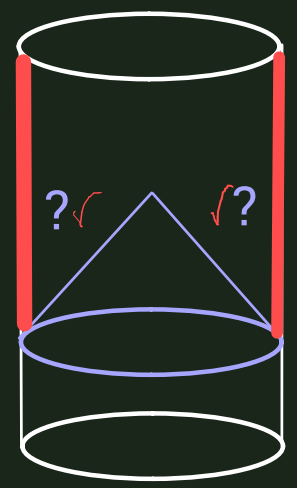
3. NO slice. This is our situation if $c_i < 0$ somewhere:



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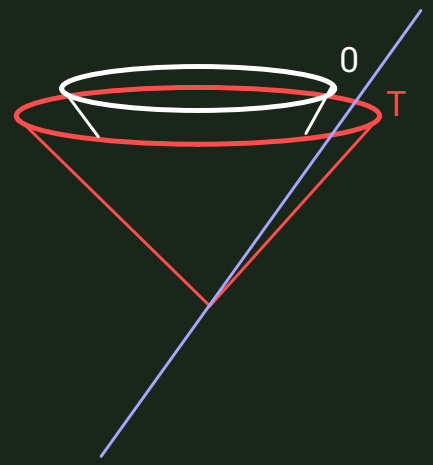
H. Reall, T. Norihiro, B. Way



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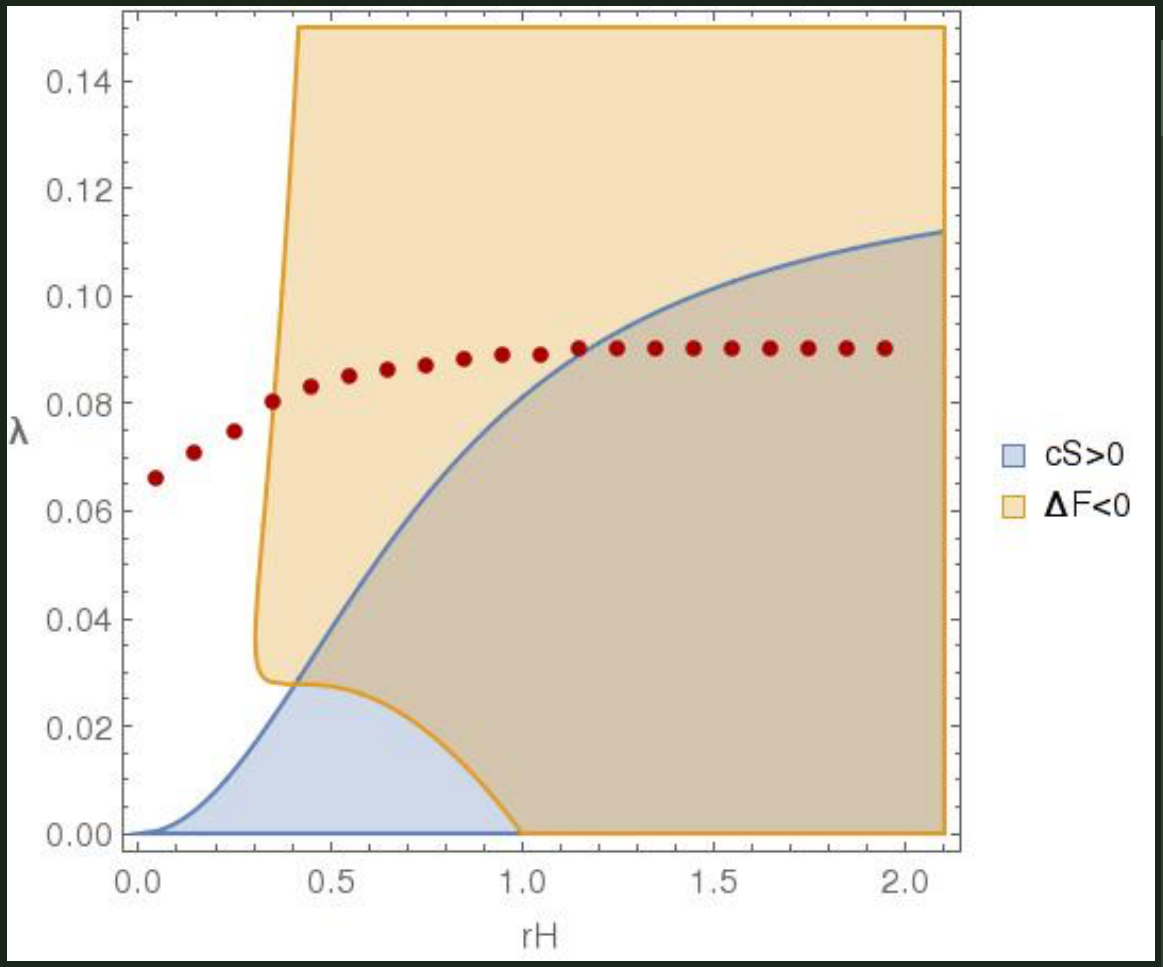

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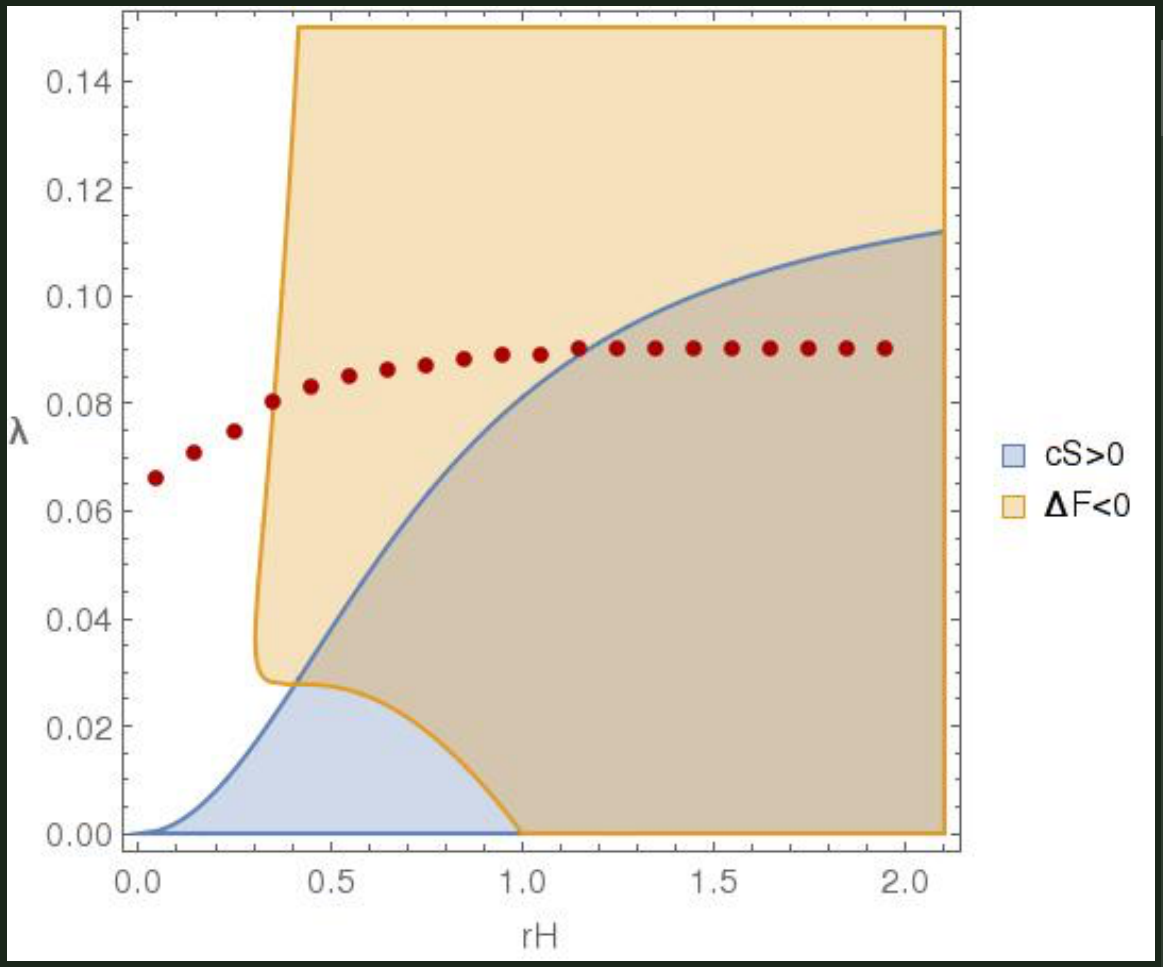

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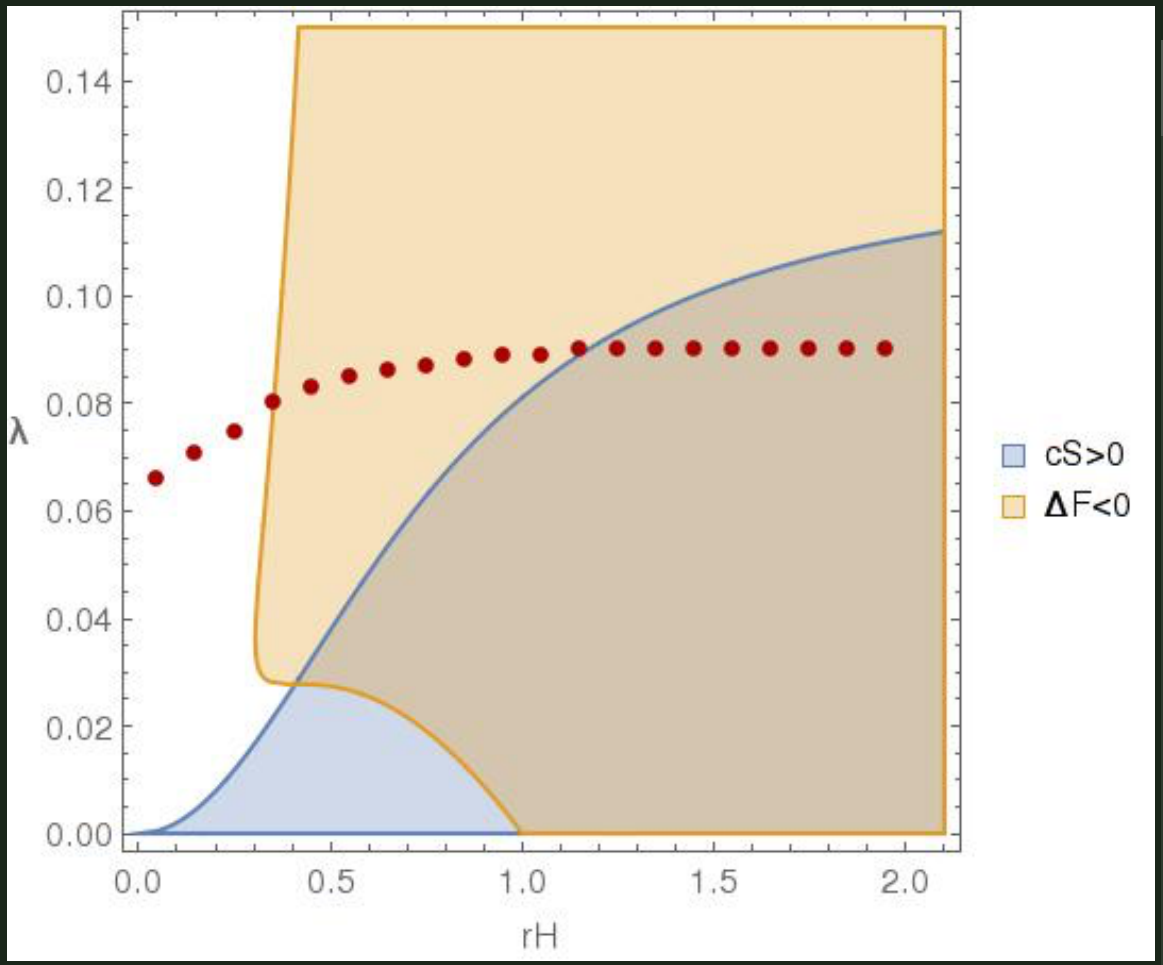

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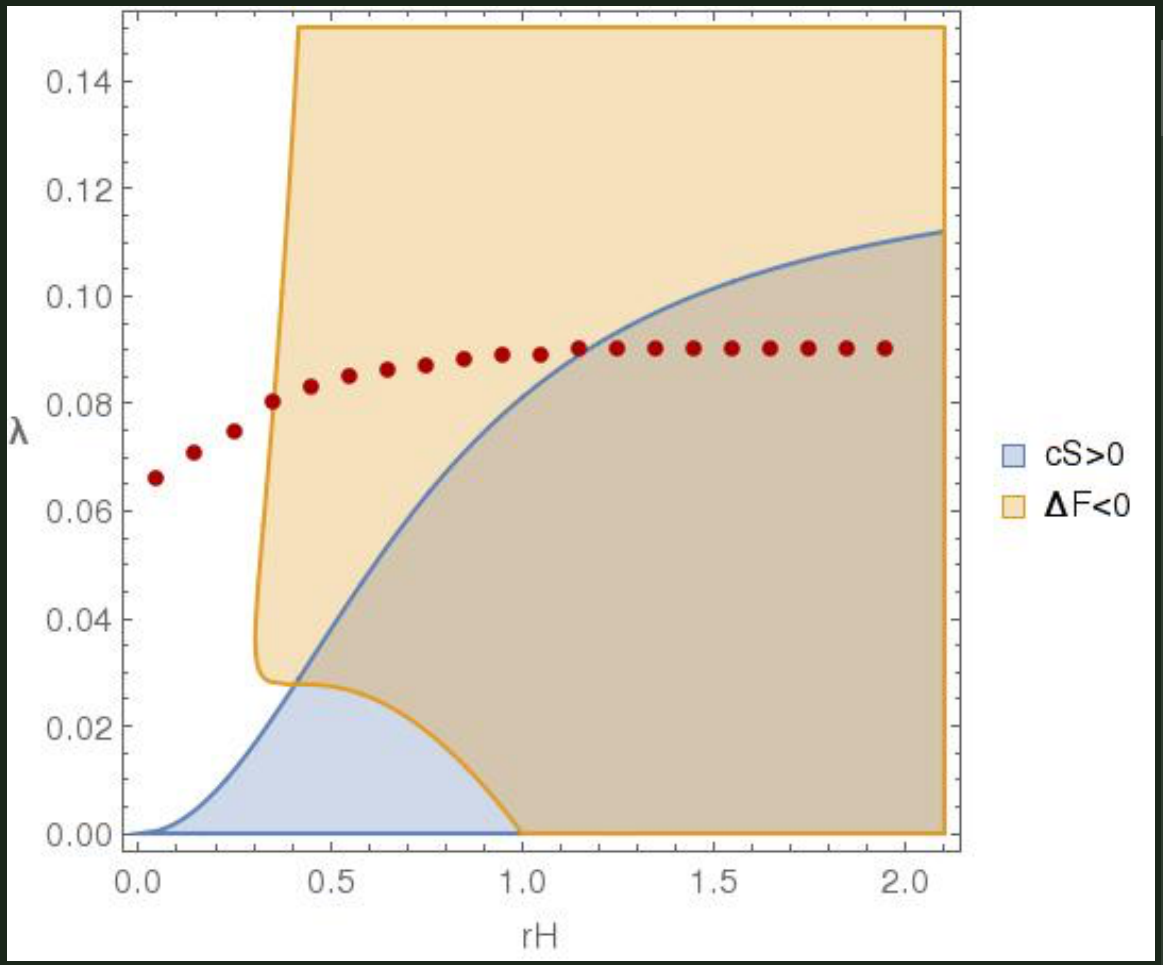

N.B. tensor modes cause bndy. causality violation but scalars cause 'no-slice' hyperbolicity violation.

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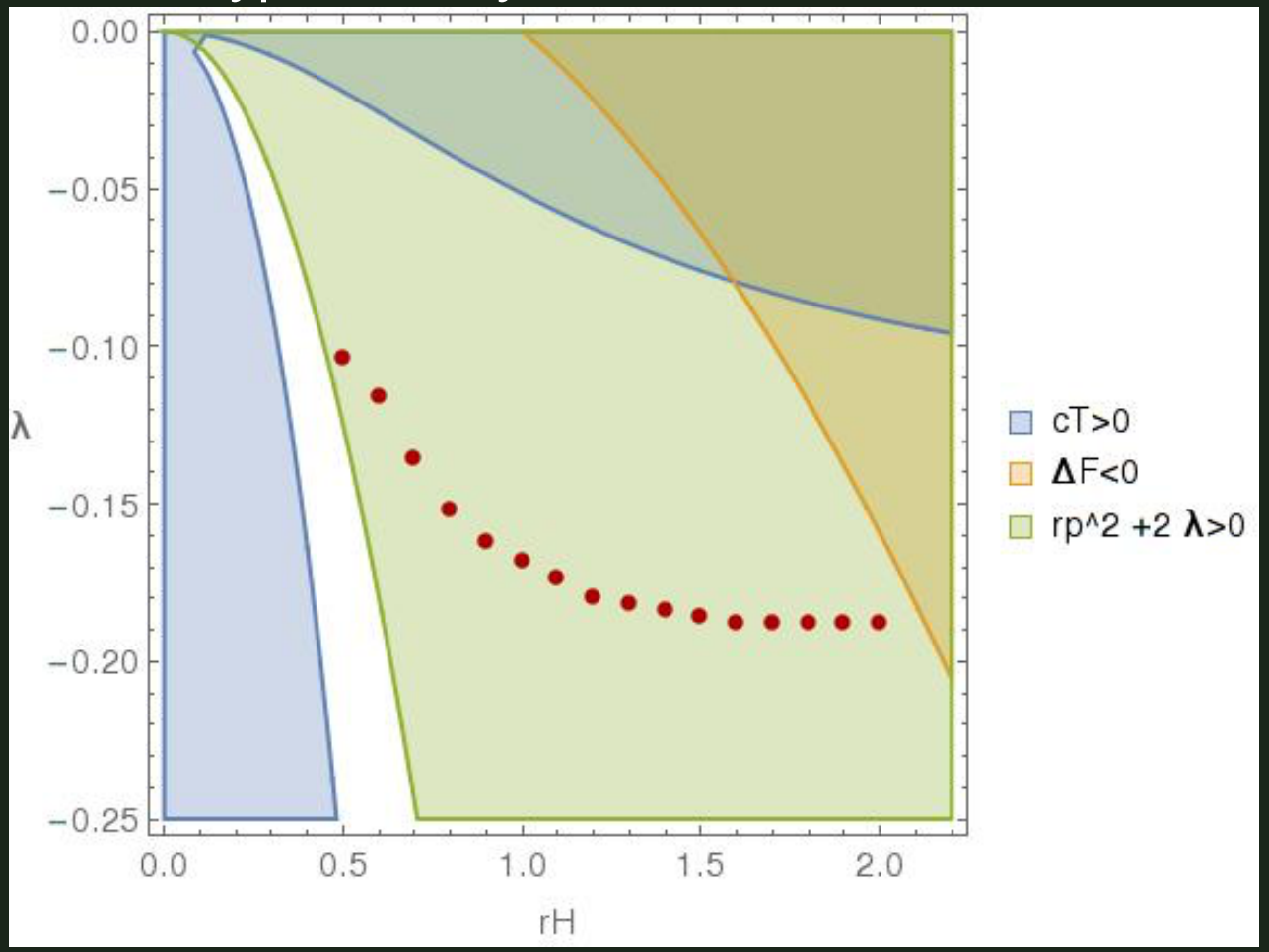
It is clear that hyperbolicity matters and we should decide what to do about it.

Other considerations of differing causal structure

- What happens in CFTs when hyperbolicity failure arises? Is it the duality that fails, or does something sick happen to the CFT?
- How do we build ‘light’ sheets?
  Needed for entropy bounds, entanglement wedges

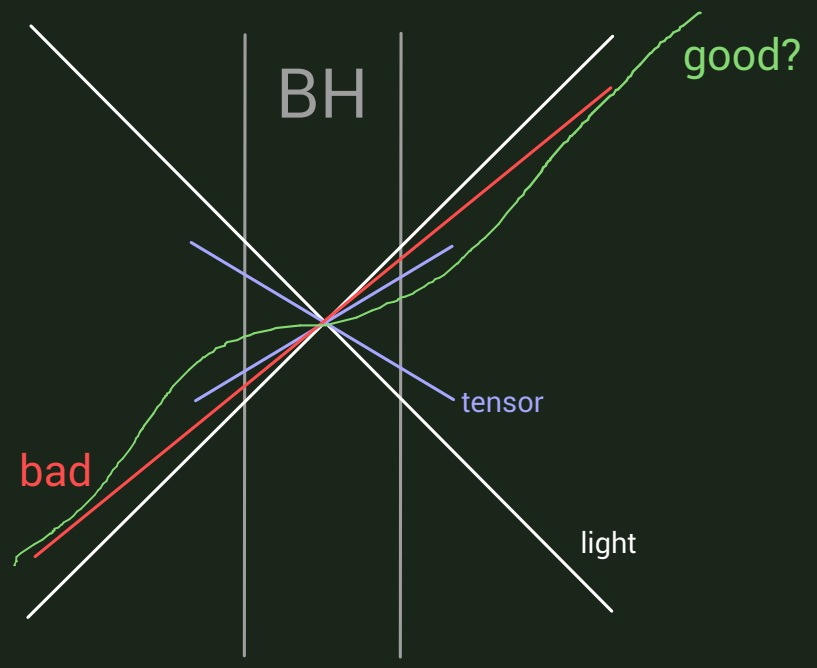
Negative λ

In this case, all black holes with boundary causality violation also violate hyperbolicity.



Aside about CEMZ via Papallo, Reall

- How fast can we boost a (small) black hole? Papallo and Reall give a limit (red slice is bad).



- What about green slice?
- How does CFT show slice violation?