Precursors and Bulk Locality in AdS/CFT

Laurens Kabir

University of Amsterdam

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Based on "Precursors, Gauge Invariance and Quantum Error Correction in AdS/CFT"
by B. Freivogel, R. Jefferson, L.K.
hep-th/1602.04811

and ongoing work with A. Belin, B. Freivogel, R. Jefferson, L.K.

Delta Institute for Theoretical Physics

Outline

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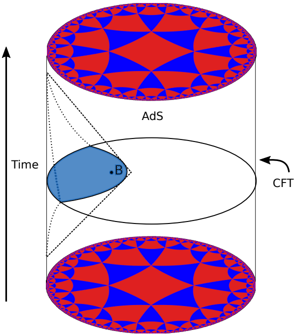
   - Bulk locality
2. Localizing the Precursor in a Holographic Toy Model
   - Using gauge freedom
   - Using entanglement
3. Bulk Locality Breaking Loose

Introduction: Puzzles in AdS/CFT

Non-uniqueness of bulk information

A local bulk operator can be reconstructed in the CFT using the HKLL prescription.



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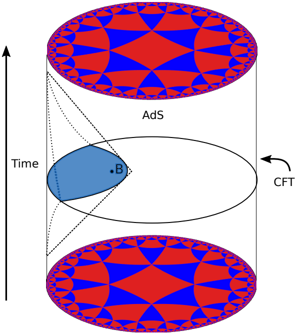
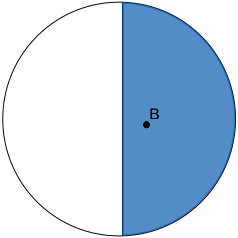


A bulk field $\Phi(B)$ seems to correspond with many different non-local boundary operators.



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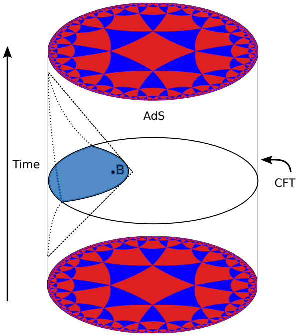

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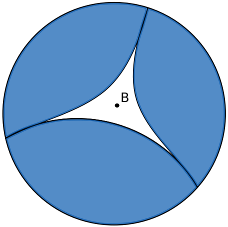
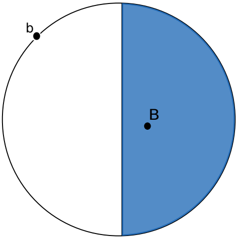


$$\Phi(B) = \int db\, K(B|b) O(b) + O(\frac{1}{N})$$

Bulk locality

As pointed out by ADH, given a local boundary operator $O(b)$, it is always possible to find a wedge reconstruction of $\Phi(B)$ that does not include $O(b)$ and hence $[\Phi(B), O(b)] = 0$.



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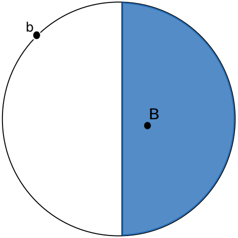

Consistency with the time slice axiom implies that different CFT representations of $\Phi(B)$ can not be equal as operators.

Localizing the Precursor in a Holographic Toy Model

A Holographic toy model

MPR suggests that the different CFT precursors are all equivalent when acting on gauge invariant states.
Building on their model, we took a massless scalar Φ in AdS_3 dual to a $\Delta = 2$ operator $\partial_+\phi^i\partial_-\phi^i$.

$$\Phi(B) = \int dx^+ dx^- \; K(B|x^+, x^-)\, \partial_+\phi^i\partial_-\phi^i$$

The global O(N) gauge invariance includes the freedom to add to any operator a linear combination of operators of the form $\alpha^i_{\nu_+}\alpha^i_{\nu_-}$ with $\nu_+\nu_- < 0$.

Can we understand the different CFT representations, and use it to localize the information of a bulk field $\Phi(B)$?

Localizing the precursor using gauge freedom

$K(B|b)$ is a distribution and not necessarily unique, i.e. we can add to it a function δK that satisfies

$$\int dx^+ dx^- \; \delta K(B|x^+, x^-) \partial_+ \phi^i \partial_- \phi^i = 0$$

This due to the absence of modes with $\nu^2 < k^2$ when solving $\Box \Phi = 0$ in global AdS.

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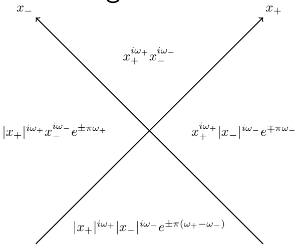


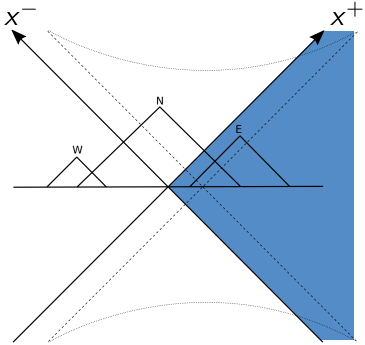
All gauge freedom can be fixed by demanding $\delta K = -K$ in three quadrants. The resulting $K + \delta K$ is non-zero in the right Rindler wedge and yields

$$\Phi(B) \propto \int d\omega_+ d\omega_- \; K_{\text{Rindler}} \; \hat{\beta}^E_{\omega_+}\hat{\beta}^E_{\omega_-}$$

Localizing the precursor using entanglement

Time evolve the boundary operator to $t = 0$ and use the entanglement in the Minkowski vacuum to map everything into the Eastern Rindler wedge

$$|0\rangle = \bigotimes_{\omega} \sum_{n} e^{-\pi\omega n} |n\rangle_W \otimes |n\rangle_E \;\Rightarrow\; \hat{\beta}^W_{\omega_\pm} |0\rangle = e^{-\pi\omega_\pm} \hat{\beta}^E_{-\omega_\pm} |0\rangle$$



Mapping the Rindler creation/annihilation operators from the West into the East again yields

$$\Phi(B) \propto \int d\omega_+ d\omega_- \; K_{\text{Rindler}} \; \hat{\beta}^E_{\omega_+} \hat{\beta}^E_{\omega_-}$$

Summary: gauge invariance VS quantum error correction

Our holographic toy model explicitly demonstrates the redundancy of bulk information as a consequence of gauge invariance or entanglement.

We showed that

1. MPR's boundary gauge invariance corresponds to an ambiguity of the smearing function that can be used to localize the precursor.
2. The same localization of the precursor can be obtained using the entanglement of the state, which is the crucial ingredient in ADH's QEC.

Bulk Locality Breaking Loose

Bulk locality breaking loose

Furthermore, a detailed analysis of the CFT shows some features that our model has, and hasn't:

✓	X
$\rho(E) \sim e^{E^{\frac{2}{3}}}$ for $1 \ll E \ll N$	Bulk locality

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$\rho(E) \sim e^{E^{\frac{2}{3}}}$ for $1 \ll E \ll N$	Bulk locality
$\rho(E) \sim e^{\sqrt{NE}}$ for $1 \ll N \ll E$	Modular invariance
	Extended Cardy regime

Conjecture: a CFT criterion for having a local bulk dual?

1. $\rho(E) \leq e^{E^{\frac{d-1}{d}}}$ for $1 \ll E \ll N$ and $d < \infty$
2. Modular invariance