Constraining the Bulk with Boundary Causality

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Based on:

N.E., S. Fischetti arXiv:1604.03944

Bulk from Boundary

- Plenty of evidence for spacetime being emergent, at least in string theory
- But what do we know about the spacetime that emerges?
- More precisely, what *must* be true about a (holographic) spacetime that emerges from quantum d.o.f.?
- Fundamentals of QFT $\Leftrightarrow$ fundamentals of the bulk

What does an emergent spacetime look like?

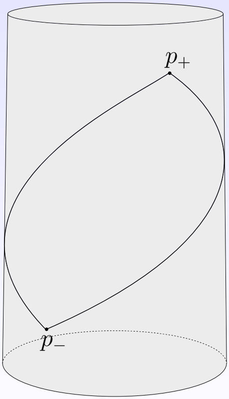
In gauge/gravity, we can use properties of the CFT that are always true to constrain emergent geometry.

Constraining the Bulk from the Boundary

- Unitarity: QFT on conformal boundary evolves unitarily, so e.g. black hole evaporation should be unitary
- Entanglement: inequalities imply spatially-averaged energy condition in certain bulk spacetimes
  - Lashkari, McDermott, Van Raamsdonk; Faulkner, Guica, Hartman, Van Raamsdonk; Lashkari, Rabideau, Sabelle-Garnier, Van Raamsdonk; Swingle, Van Raamsdonk
- Causality: Signals cannot travel faster than light. Just as fundamental as unitarity or entanglement inequalities.

Violations of Boundary Causality

- How could the bulk violate boundary causality?



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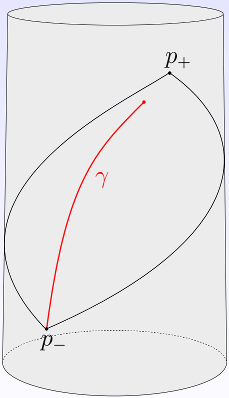
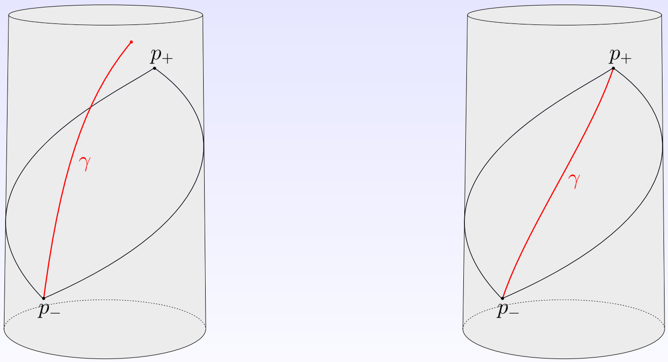


The Boundary Causality Condition

Bulk signals must behave in one of two ways:



Boundary Causality

Signals cannot travel faster in the bulk than on the boundary, or else boundary causality is violated.

The Boundary Causality Condition (BCC)

Boundary causality is preserved by the bulk if and only if boundary points that are acausal on the boundary are acausal in the bulk.

Subject of this talk: find the property of the bulk that is equivalent to this.

Causality in AdS/CFT

Partial Results

- Necessary (but not sufficient) conditions [Page, Surya, Woolgar; Kleban, McGreevy, Thomas]
- Sufficient condition from Gao-Wald: boundary causality is preserved by any asymptotically AdS spacetime obeying the Averaged Null Curvature Condition (+other technical assumptions):

$$\int_{\gamma} R_{ab} k^a k^b \geq 0$$

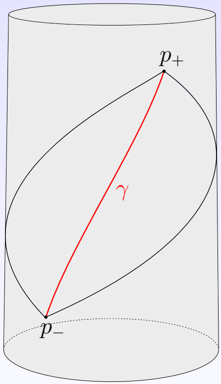
where γ is any null geodesic with generator k, and the integral is over the entire geodesic.

Finding a Necessary and Sufficient Condition

- Strict large N, large λ limit of string theory in the bulk obeys the ANCC, so boundary causality is satisfied classically
- **Away from this limit, the ANCC can be violated.** Gao-Wald can no longer guarantee that boundary causality is preserved.
- Working in perturbative quantum (or stringy) gravity, what bulk condition guarantees boundary causality?

Saturating the BCC

- Generically, bulk signals in the large N, large λ limit experience time delay
- Pure AdS: bulk and boundary signals travel equally fast
- In pure AdS, perturbations run into the danger of **violating boundary causality**.



Pure AdS

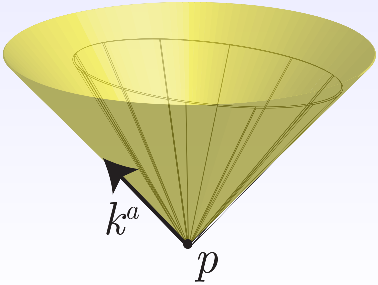
- Boundary points are null-separated on bdy iff they are null-separated through the bulk
- Only spacetime known to have this property of saturating the BCC (should other such spacetimes exist, similar results should follow straightforwardly)
- For deriving a necessary and sufficient bulk condition for bdy causality under perturbative corrections, we need only worry about perturbations of pure AdS.

The Gravity Dual of Causality

The Dual of Boundary Causality

- Expectation: perturbed bulk light cones should not open up when compared with the lightcones of pure AdS
- If the perturbation is $\bar{g}_{ab} - g_{ab} = \delta g_{ab}$, might expect that at every point:

$$\bar{g}_{ab}k^a k^b\Big|_p = \delta g_{ab}k^a k^b\Big|_p \geq 0$$



The Dual of Boundary Causality

Pointwise condition is not gauge-invariant and too strong.

BCC in the Bulk

Let δg_{ab} be a regular perturbation of AdS.
The bulk preserves boundary causality **if and only if**:

$$I = \int_{\gamma} \delta g_{ab} k^a k^b \geq 0.$$

(and perturbations of AdS are the only ones that matter!) For proof, see 1604.03944.

Relation to the Averaged Null Curvature Condition

Relation to the ANCC

- Recall the ANCC:
$$\int_{\gamma} R_{ab}k^a k^b \geq 0$$

- Gao-Wald Theorem says this is sufficient for boundary causality

- Is it also necessary? i.e. is it equivalent to or stronger than our condition?

Relation to the ANCC

The BCC: $$I = \int_\gamma \delta g_{ab} k^a k^b \geq 0$$

Averaged Null Curvature Condition $\neq$ Boundary Causality

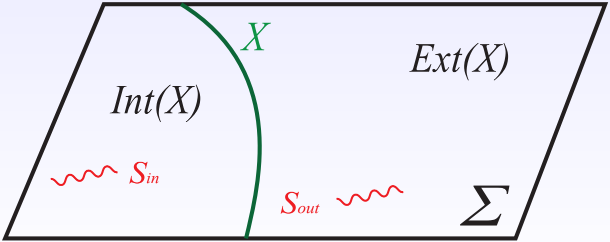
The ANCC implies boundary causality, but boundary causality doesn't imply the ANCC.
$I \geq 0$ is a *bona fide* new condition on the bulk.

- To see the ANCC implies $I \geq 0$: this is just the Gao-Wald Theorem.
- To see that $I \geq 0$ does not imply the ANCC, get a counterexample.

Relation to Other Conditions

- ANCC is not the only "reasonable" condition in perturbative quantum gravity
- Other conditions involve the generalized entropy:

$$S_{\text{gen}}(X) = \frac{\text{Area}(X)}{4G\hbar} + S_{\text{out}} + \text{c.t.}$$



The Generalized Entropy as "Quantum Area"

- The generalized entropy is often thought of as a quantum-corrected area
- Quantum Focussing Conjecture: "quantum" focussing theorem (second derivative of S_{gen} in null direction is negative) [Bousso, Fisher, Leichenauer, Wall]
- Generalized Second Law: S_{gen} increases along causal horizons

How are these related to the boundary causality condition?

Summary

- We've found the (perturbative) bulk dual to boundary causality:

$$\int_{\gamma} \delta g_{ab} k^a k^b \geq 0$$

- Weaker condition than the averaged null curvature condition
- Relation with S_{gen} conditions unclear: locality in space vs. time
- Work in progress: background-independent formulation
- Generalization of condition to situations with perturbatively "fuzzy" causal structure?