HOLOGRAPHIC BLACK HOLE CHEMISTRY

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Nordita - Inward Bound
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INTENT

Holographic origin of generalized Smarr formula

$$\boxed{(d-3)U = (d-2)TS - 2p_b V_b + (d-3)\mu Q}$$

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$$M = \frac{(d-2)}{(d-3)} TS \boxed{- \frac{2}{(d-3)} P_b V_b} + \mu_i Q_i$$

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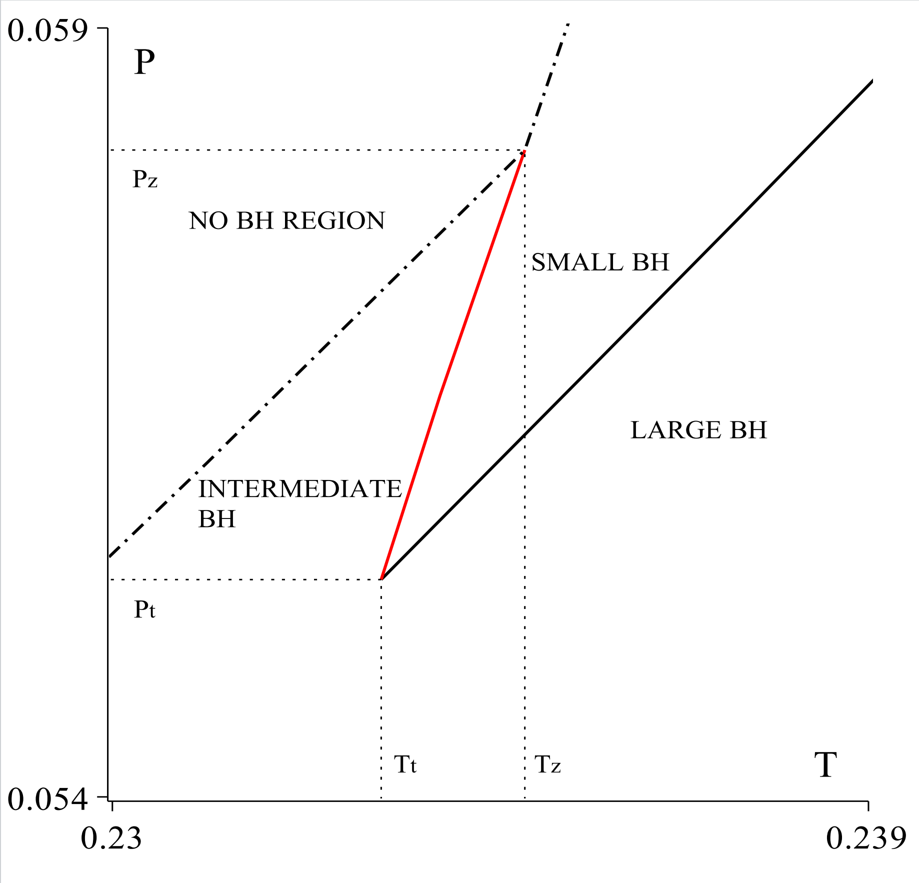

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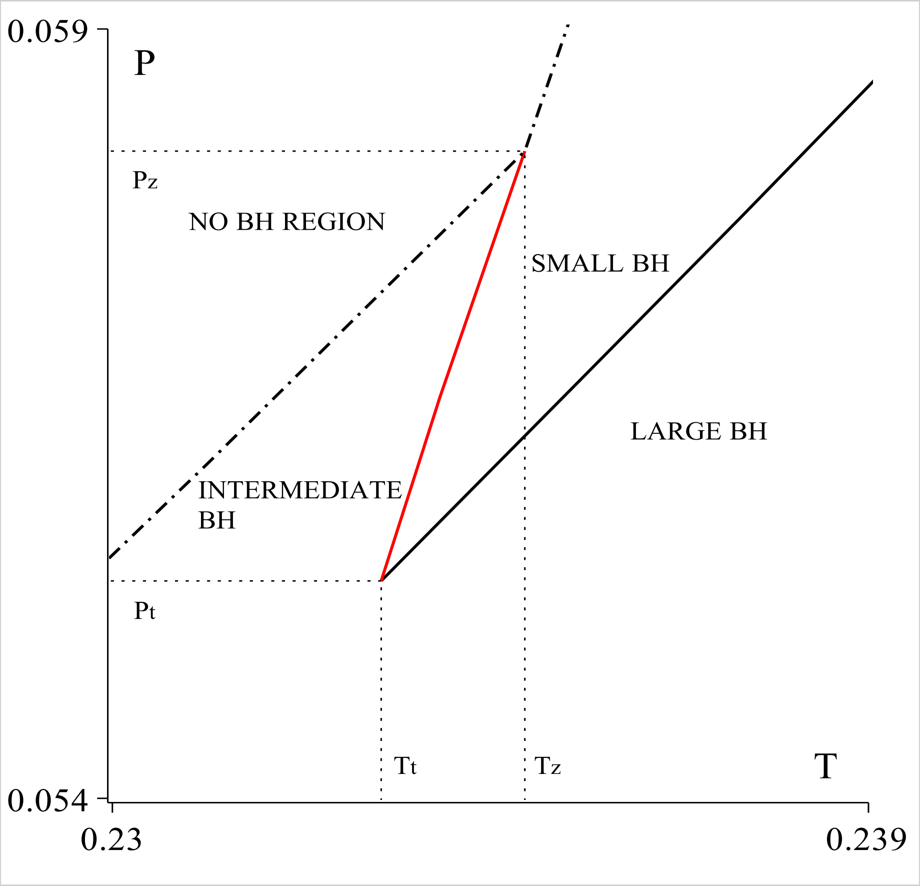

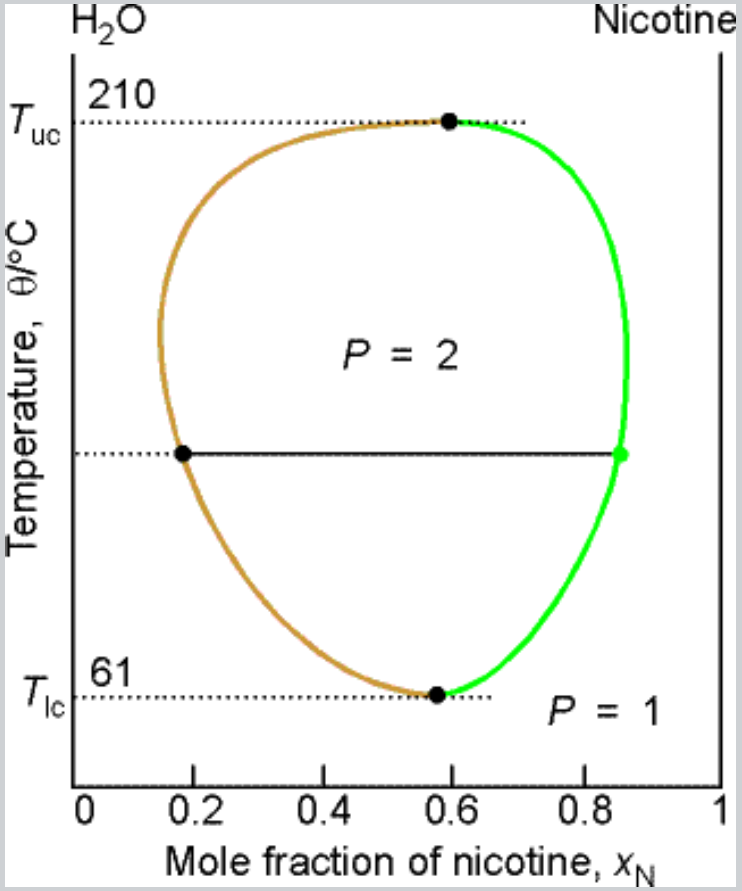

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$$\left.\begin{aligned} dU &= U d\lambda \\ dV &= -(d-2)V d\lambda \end{aligned}\right\} \quad \boxed{U = (d-2)pV}$$

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ALL FROM LARGE N SCALING IN FIELD THEORY!

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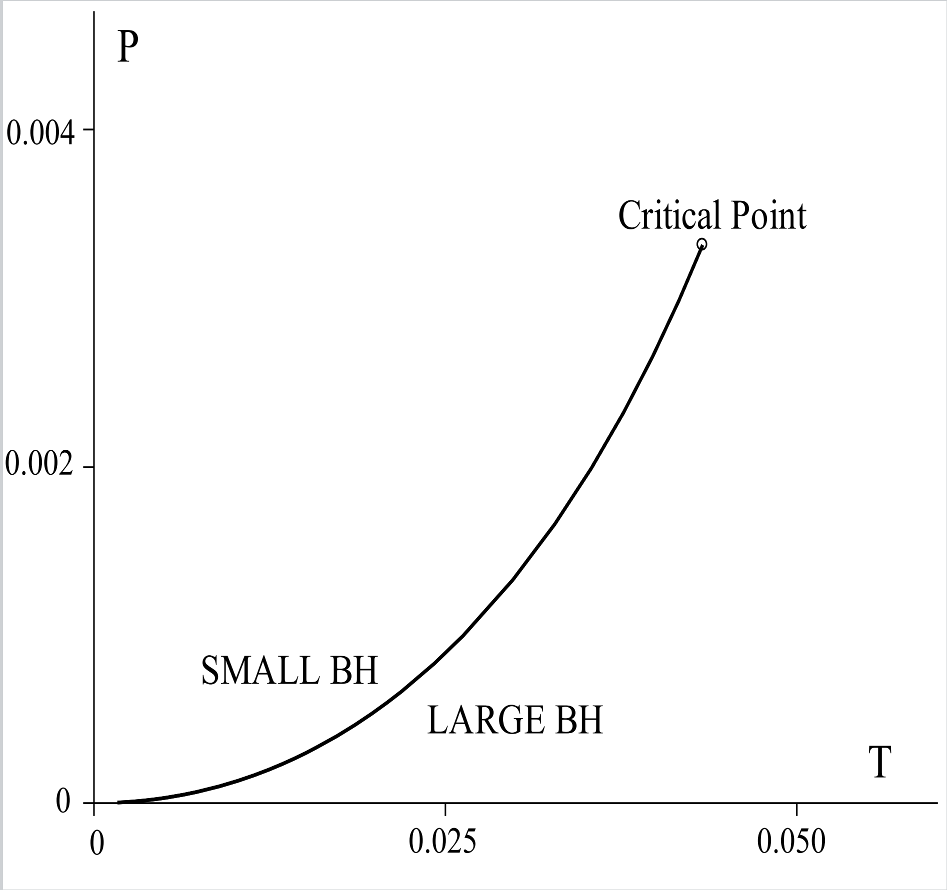

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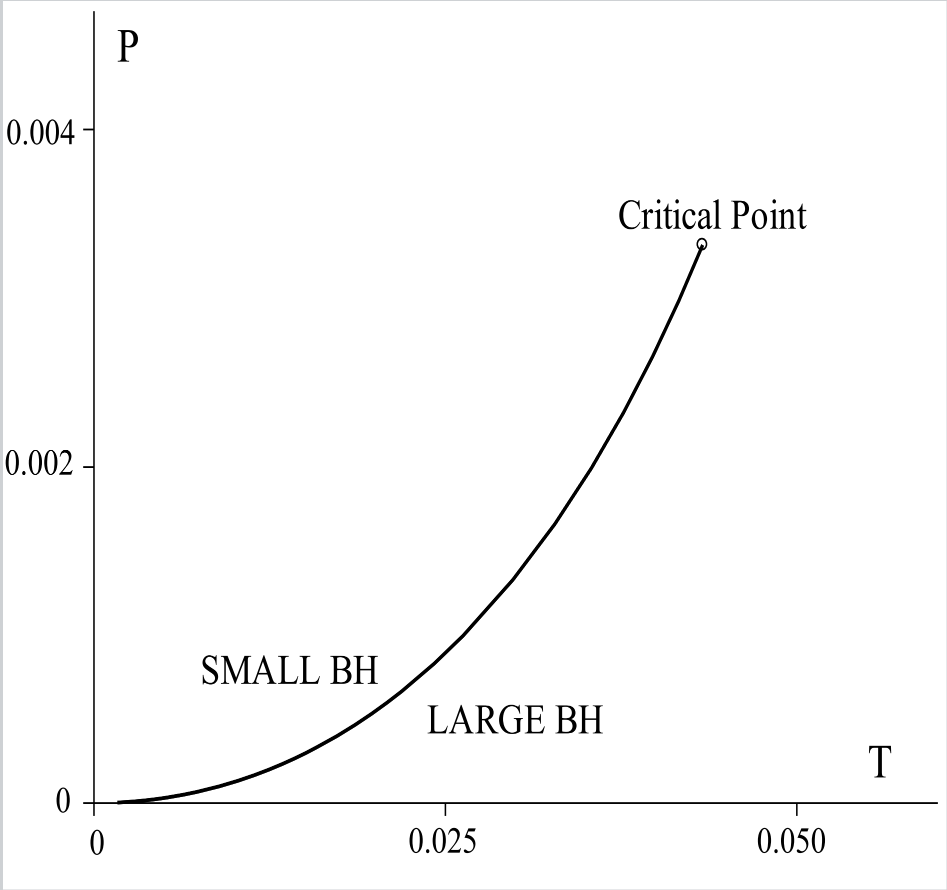

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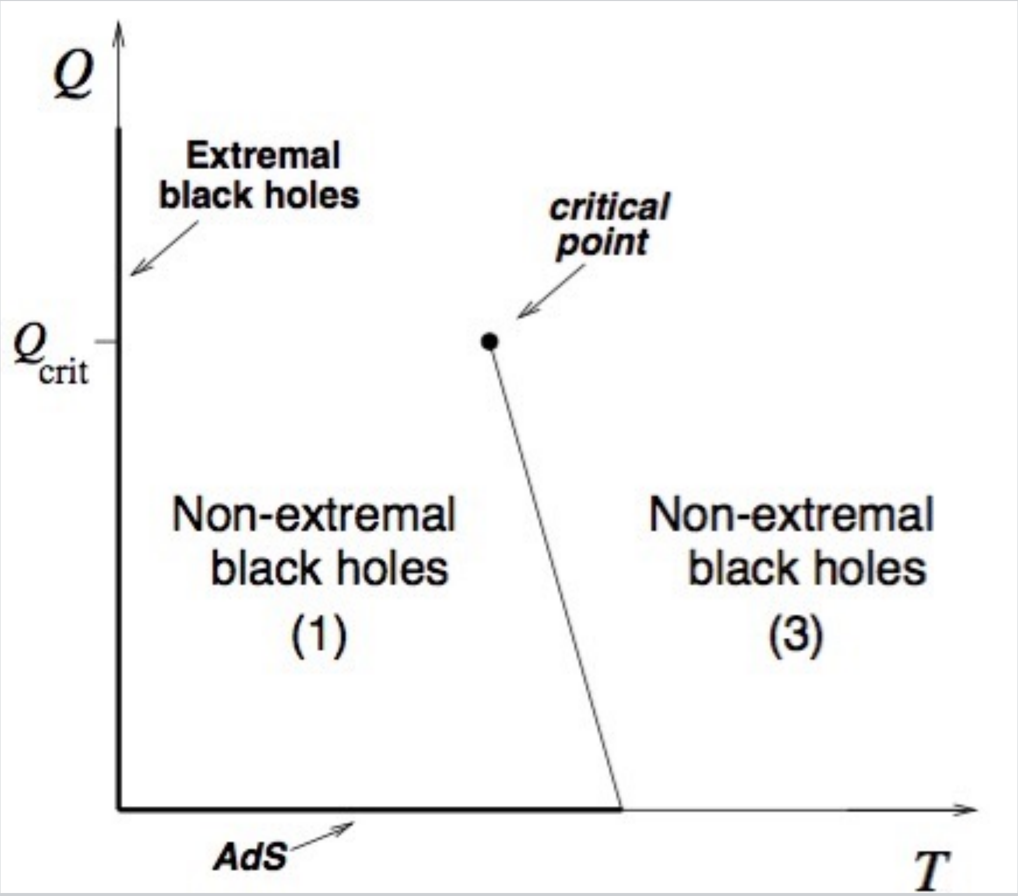

[Myers, Emperan, Chamblin, Johnson]

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QUESTIONS