“Inward Bound: Black holes and emergent spacetime”
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Nonextremal Black Holes, Subtracted Geometry and Holography

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Outline:

I. General asymptotically flat black holes in four (&five) dimensions – prototype STU black holes
thermodynamics, suggestive of conformal symmetry

II. Subtracted Geometry: non-extremal black holes in asymptotically conical box
manifest conformal symmetry

III. Variational Principle & Subtracted Geometry
conserved charges and thermodynamics

IV. Holography via 2D Einstein-Maxwell-Dilaton gravity
full holographic dictionary

V. Outlook

Background:

Initial work on subtracted geometry

M.C., Finn Larsen 1106.3341, 1112.4846, 1406.4536

M.C., Gary Gibbons 1201.0601

M.C., Monica Guica, Zain Saleem 1301.7032

…

Recent: variational principle, conserved charges and thermodynamics of subtracted geometry

Ok Song An, M.C., Ioannis Papadimitriou, 1602.0150

Most recent: subtracted geometry and AdS_2 holography

M.C., Ioannis Papadimtiriou,1608….. (this week)

c.f., Ioannis Papadimitriou’s talk at the workshop

I. 4D non-extremal black holes in string theory w/ $\Lambda=0$

M-mass, Q_i- multi-charges, J- angular momentum

w/ $M > \Sigma_i |Q_i|$

Prototype solutions of a sector of maximally supersymmetric D=4 Supergravity

[effective action of toroidally compactified string theory] →

so-called STU model

STU Model Lagrangian

[A sector of toroidally compactified effective string theory]

$$2\kappa_4^2\mathcal{L}_4 = R\star 1 - \frac{1}{2}\star d\eta_a\wedge d\eta_a - \frac{1}{2}e^{2\eta_a}\star d\chi^a\wedge d\chi^a$$
$$-\frac{1}{2}e^{-\eta_0}\star F^0\wedge F^0 - \frac{1}{2}e^{2\eta_a-\eta_0}\star(F^a+\chi^a F^0)\wedge(F^a+\chi^a F^0)$$
$$+\frac{1}{2}C_{abc}\chi^a F^b\wedge F^c + \frac{1}{2}C_{abc}\chi^a\chi^b F^0\wedge F^c + \frac{1}{6}C_{abc}\chi^a\chi^b\chi^c F^0\wedge F^0$$

{a,b,c}={1,2,3}

w/ A^0 & three gauge fields A^a, the three dilatons η^a and the three axions χ^a.

Black holes: explicit solutions of equations of motion for the above Lagrangian w/ metric, four gauge potentials and three axio-dilatons

Prototype, four-charge rotating black hole, originally obtained via solution generating techniques

M.C., Youm 9603147
Chong, M.C., Lü, Pope 0411045

Four- SO(1,1) transfs.
time-reduced Kerr BH

$$H = \begin{pmatrix}\cosh\delta_i & \sinh\delta_i\\ \sinh\delta_i & \cosh\delta_i\end{pmatrix}$$

Full four-electric and four-magnetic charge solution only recently obtained

Chow, Compère 1310.1295;1404.2602

Metric of rotating four-charge black holes in STU model

M.C., Youm 9603147
Chong, M.C., Lü, Pope 0411045

$$ds_4^2 = -\Delta_0^{-1/2} G(dt + \mathcal{A})^2 + \Delta_0^{1/2}\left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G}\sin^2\theta d\phi^2\right)$$

$X = r^2 - 2mr + a^2 \quad = 0$ outer & inner horizon

$$G = r^2 - 2mr + a^2\cos^2\theta\ ,$$

$$\mathcal{A} = \frac{2ma\sin^2\theta}{G}\left[(\Pi_c - \Pi_s)r + 2m\Pi_s\right]d\phi\ ,$$

$$\Pi_c \equiv \prod_{I=0}^{3}\cosh\delta_I\ , \quad \Pi_s \equiv \prod_{I=0}^{3}\sinh\delta_I\ .$$

$$\Delta_0 = \prod_{I=0}^{3}(r + 2m\sinh^2\delta_I) + 2a^2\cos^2\theta[r^2 + mr\sum_{I=0}^{3}\sinh^2\delta_I + 4m^2(\Pi_c - \Pi_s)\Pi_s - 2m^2\sum_{I<J<K}\sinh^2\delta_I\sinh^2\delta_J\sinh^2\delta_K] + a^4\cos^4\theta\ .$$

$$G_4 M = \frac{1}{4}m\sum_{I=0}^{3}\cosh 2\delta_I$$ Mass

$$G_4 Q_I = \frac{1}{4}m\sinh 2\delta_I \quad (I = 0,1,2,3)$$ Four charges

$$G_4 J = ma(\Pi_c - \Pi_s)$$ Angular momentum

Special cases:

$\delta_I = \delta$ Kerr-Newman

& $a = 0$ Reisner-Nordström;

$\delta_I = 0$ Kerr

& $a = 0$ Schwarzschild;

$\delta_I \to \infty$ $m \to 0$ w/m $\exp(2\delta_I)$-finite
extremal (BPS) black hole

Or equivalently : m, a, δ_I (I=0,1,2,3)

Thermodynamics - suggestive of weakly interacting 2-dim. CFT w/ ``left-'' & ``right-moving'' excitations

M.C., Youm '96
M.C., Larsen '97

Area of outer horizon $S_+ = S_L + S_R$

[Area of inner horizon $S_- = S_L - S_R$]

$$S_L = \pi\, m^2\, (\Pi_c + \Pi_s)$$

$$S_R = \pi\, m \sqrt{m^2 - a^2}\, (\Pi_c - \Pi_s)$$

Surface gravity (inverse temperature) of

outer horizon $\beta_H = ½ (\beta_L + \beta_R)$

[inner horizon $\beta_- = ½ (\beta_L - \beta_R)$]

$$\beta_L = 2\pi\, m\, (\Pi_c - \Pi_s)$$

$$\beta_R = \frac{2\pi\, m^2}{\sqrt{m^2 - a^2}} (\Pi_c + \Pi_s)$$

Similar structure for angular velocities Ω_+, Ω_- and momenta J_+, J_-.

Depend only on four parameters: m, a, $\Pi_c \equiv \prod_{I=0}^{3} \cosh \delta_I$, $\Pi_s \equiv \prod_{I=0}^{3} \sinh \delta_I$

Shown more recently, all independent of the warp factor Δ_o !

M.C., Larsen '11

II. Subtracted Geometry - Motivation

Quantify this ``conventional wisdom'' M.C., Larsen '97-'99
that also non-extremal black holes might have microscopic explanation in terms of dual 2D CFT

Focus on the black hole "by itself" →
enclose the black hole in a box (à la Gibbons Hawking) →
an equilibrium system w/ conformal symmetry manifest

The box leads to a ``mildly'' modified geometry
changing only the warp factor $\Delta_0 \rightarrow \Delta$
(same horizon thermodynamic quantities)

⬇

Subtracted Geometry M.C., Larsen '11

Determination of new warp factor $\Delta_0 \rightarrow \Delta$

via massless scalar field wave eq.: wave eq. separable &

the radial part is solved by hypergeometric functions w/ $SL(2,R)^2$

(manifest conformal symmetry)

The general Laplacian (with warp factor Δ_0 – implicit):

$$\Delta_0^{-\frac{1}{2}}[\partial_r X \partial_r - \frac{1}{X}(\mathcal{A}_{\rm red}\partial_t - \partial_\phi)^2 + \frac{\mathcal{A}_{\rm red}^2 - \Delta_0}{G}\partial_t^2 + \frac{1}{\sin\theta}\partial_\theta \sin\theta \partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi^2]$$

w/ $\mathcal{A}_{\rm red} = \frac{G}{a\sin^2\theta}\mathcal{A} = 2m[(\Pi_c - \Pi_s)r + 2m\Pi_s]$

$$G = r^2 - 2mr + a^2\cos^2\theta$$

$\Delta_0 \rightarrow \Delta$ such that wave eq. is separable: f(r)+g(θ) (true for Δ_0 and Δ)

& the radial part is solved by hypergeometric functions:

f(r)+g(θ)-const. → uniquely fixes Δ

M.C., Larsen 1112.4856

Subtracted geometry for rotating four-charge black holes

$$ds_4^2 = -\Delta_0^{-1/2} G (dt + \mathcal{A})^2 + \Delta_0^{1/2} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G} \sin^2\theta d\phi^2 \right)$$

$$X = r^2 - 2mr + a^2 \ ,$$

$$G = r^2 - 2mr + a^2 \cos^2\theta \ ,$$

$$\mathcal{A} = \frac{2ma \sin^2\theta}{G} \left[(\Pi_c - \Pi_s) r + 2m\Pi_s \right] d\phi \ ,$$

$$\Delta_0 = \prod_{I=0}^{3} (r + 2m \sinh^2\delta_I) + 2a^2 \cos^2\theta [r^2 + mr \sum_{I=0}^{3} \sinh^2\delta_I + 4m^2 (\Pi_c - \Pi_s)\Pi_s$$
$$- 2m^2 \sum_{I<J<K} \sinh^2\delta_I \sinh^2\delta_J \sinh^2\delta_K] + a^4 \cos^4\theta \ .$$

$$\boxed{\Delta_0 \to \Delta = (2m)^3 r (\Pi_c^2 - \Pi_s^2) + (2m)^4 \Pi_s^2 - (2m)^2 (\Pi_c - \Pi_s)^2 a^2 \cos^2\theta}$$

Comments: while $\Delta_0 \sim r^4$, $\Delta \sim r$ (not asymptotically flat!)

subtracted geometry depends only on four parameters:

m, a, $\Pi_c \equiv \prod_{I=0}^{3} \cosh\delta_I \ , \quad \Pi_s \equiv \prod_{I=0}^{3} \sinh\delta_I$

Matter fields (gauge potentials and scalars)

M.C., Gibbons 1201.0601

Scalars: $\eta_1 = \eta_2 = \eta_3 \equiv \eta, \; \chi_1 = \chi_2 = \chi_3 \equiv \chi,$

Running dilaton: $e^\eta = \frac{(2m)^2}{\sqrt{\Delta}}, \qquad \chi = \frac{a\left(\Pi_c - \Pi_s\right)}{2m}\cos\theta$

Gauge potentials: $A^1 = A^2 = A^3 \equiv A.$

$$A^0 = \frac{(2m)^4 a\left(\Pi_c - \Pi_s\right)}{\Delta}\sin^2\theta d\phi + \frac{(2ma)^2\cos^2\theta\left(\Pi_c - \Pi_s\right)^2 + (2m)^4\Pi_c\Pi_s}{\left(\Pi_c^2 - \Pi_s^2\right)\Delta}dt,$$

$$A = \frac{2m\cos\theta}{\Delta}\left(\left[\Delta - (2ma)^2(\Pi_c - \Pi_s)^2\sin^2\theta\right]d\phi - 2ma\left(2m\Pi_s + r(\Pi_c - \Pi_s)\right)dt\right),$$

Magnetic frame

Non-extremal black hole immersed in constant magnetic field

w/ $\Delta = (2m)^3(\Pi_c^2 - \Pi_s^2)r + (2m)^4\Pi_s^2 - (2ma)^2(\Pi_c - \Pi_s)^2\cos^2\theta$

Remarks:

Asymptotic geometry of subtracted geometry is of Lifshitz-type w/ a deficit angle:

$$ds^2 = -(\frac{R}{R_0})^{2p} dt^2 + B^2 dR^2 + R^2(d\theta^2 + \sin^2\theta^2 d\phi^2)$$

p=3, B=4

→ black hole in an ``asymptotically conical box''

M.C., Gibbons 1201.0601

→ the box conformal to AdS_2 x S^2

→ confining, but ``softer'' than AdS

Origin of subtracted geometry

i. Subtracted geometry – as a scaling limit of near-horizon black hole w/ three-large charges Q, (mapped on m, a, Π_c, Π_s)

$$\epsilon \to 0 \qquad \tilde{r} = r\epsilon, \quad \tilde{t} = t\epsilon^{-1}, \quad \tilde{m} = m\epsilon\,, \qquad \tilde{a} = a\epsilon$$

M.C., Gibbons 1201.0601

$$2\tilde{m}\sinh^2\tilde{\delta} \equiv Q = 2m\epsilon^{-1/3}(\Pi_c^2 - \Pi_s^2)^{1/3}, \qquad \sinh^2\tilde{\delta}_0 = \frac{\Pi_s^2}{\Pi_c^2 - \Pi_s^2}$$

ii. Subtracted geometry - as an infinite boost Harrison transformations on the original BH

M.C., Gibbons 1201.0601
Virmani 1203.5088
Sahay, Virmani 1305.2800
M.C., Guica, Saleem 1302.7032..

SO(1,1): $H \sim \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \qquad \beta \to 1$

iii. Subtracted geometry – as turning off certain integration constants in harmonic functions of asymptotically flat black holes

Baggio, de Boer, Jottar, Mayerson 1210.7695
An, M.C., Papardimitriou 1602.0150

→ non-extremal black hole microscopic properties associated with its horizon are captured by a dual field theory of subtracted geometry

Lift of subtracted geometry

M.C., Larsen 1112.4856

on a circle S^1 to five-dimension turns out to locally factorize $AdS_3 \times S^2$

($[SL(2,R)^2 \times SO(3)]/Z_2$ symmetry)

[globally S^2 fibered over Bañados-Teitelboim-Zanelli (BTZ) black hole w/ mass M_3, angular momentum J_3 & 3d cosmol. const. $\Lambda=\ell^{-3}$]

$$ds_5^2 = (ds_{S^2}^2 + ds_{BTZ}^2)$$

$$ds_{S^2}^2 = \tfrac{1}{4}\ell^2 \left(d\theta^2 + \sin^2\theta d\bar{\phi}^2\right)$$

$$\bar{\phi} = \phi + \frac{16ma(\Pi_c - \Pi_s)}{\ell^3}(z+t)$$

$$ds_{BTZ}^2 = -\frac{(r_3^2 - r_{3+}^2)(r_3^2 - r_{3-}^2)}{\ell^2\, r_3^2}dt_3^2 + \frac{\ell^2 r_3^2}{(r_3^2 - r_{3+}^2)(r_3^2 - r_{3-}^2)}dr_3^2 + r_3^2(d\phi_3 + \frac{r_{3+}r_{3-}}{\ell r_3^2}dt_3)^2$$

$$\phi_3 = \frac{z}{R},$$

$$t_3 = \frac{\ell}{R}t,$$

$$r_3^2 = \frac{16(2mR)^2}{\ell^4}\left[2m(\Pi_c^2 - \Pi_s^2)r + (2m)^2\Pi_s^2 - a^2(\Pi_c - \Pi_s)^2\right]$$

Conformal symmetry of AdS_3 can be promoted to Virasoro algebra of dual two-dimensional CFT

à la Brown-Hennaux

Standard statistical entropy (via AdS_3/CFT_2)

à la Cardy

→ Reproduces entropy of 4D black holes

M.C., Larsen 1406.4536

Subtracted geometry [$\Delta_0 \rightarrow \Delta = A\, r + B \cos^2\theta + C$; A,B,C-horrendous]
also works for most general black holes of the STU Model
(specified by mass, four electric and four magnetics charges and angular momentum)

Chow, Compère 1310.1295;1404.2602

M.C., Larsen 1106.3341

All also works in parallel for subtracted geometry of most general five-dimensional black holes
(specified by mass, three charges and two angular momenta)

M.C., Youm 9603100

Further developments

Quantum aspects of subtracted geometries:

i) Quasi-normal modes - exact results for scalar fields
two damped branches → no black hole bomb

M.C., Gibbons 1312.2250, M.C., Gibbons, Saleem 1401.0544

ii) Entanglement entropy –minimally coupled scalar
M.C., Satz, Saleem 1407.0310

iii) Vacuum polarization $\langle\varphi^2\rangle$ analytic expressions
at the horizon: static M.C., Gibbons, Saleem, Satz 1411.4658
rotating M.C., Satz, Saleem 1506.07189
outside & inside horizon: rotating M.C., Satz 1609….

iv) Thermodynamics of subtracted geometry
via Komar integral: M.C., Gibbons, Saleem 1412.5996 (PRL)

III. Thermodynamics via variational principle

An, M.C., Papadimitriou 1602.0150

Following lessons from AdS geometries

Heningson, Skenderis '98; Balasubramanian ,Kraus '99; deBoer ,Verlinde[2] '99,…

achieved through an algorithmic procedure for subtracted geometry:

- Integration constants, parameterizing solutions of the eqs. of motion, separated into `normalizable' - free to vary & 'non-normalizable' modes – fixed
- Non-normalizable modes – fixed only up to transformations induced by local symmetries of the bulk theory (radial diffeomorphisms & gauge transf.)
- Covariant boundary term, S_{ct}, to the bulk action - determined by solving asymptotically the radial Hamilton-Jacobi eqn. →

  Skenderis,Papadimitriou '04, Papadimitriou '05

  Total action $S+S_{ct}$ independent of the radial coordinate
- First class constraints of Hamiltonian formalism lead to conserved charges associated with Killing vectors.
- Conserved charges satisfy the first law of thermodynamics

- Identify normalizable and non-rormalizable modes:

Introduce new coordinates:

Rescaled radial coord.: $\ell^4 r \leftarrow (2m)^3(\Pi_c^2 - \Pi_s^2) r + (2m)^4 \Pi_s^2 - (2ma)^2 (\Pi_c - \Pi_s)^2,$

Rescaled time: $\frac{k}{\ell^3} t \leftarrow \frac{1}{(2m)^3(\Pi_c^2 - \Pi_s^2)} t,$

Trade four parameters m, a, Π_c, Π_s for:

$$\ell^4 r_\pm = (2m)^3 m(\Pi_c^2 + \Pi_s^2) - (2ma)^2(\Pi_c - \Pi_s)^2 \pm \sqrt{m^2 - a^2}(2m)^3(\Pi_c^2 - \Pi_s^2)$$

$$\ell^3 \omega = 2ma(\Pi_c - \Pi_s), \qquad B = 2m,$$

r_+, r_-, ω - normalizable modes

B - non-renormalizable mode

(fixed up to bulk diffeomorphisms & global gauge symmetries)

``Vacuum” solution

obtained by turning off r_+, r_-, ω – three normalizable modes:

Asymptotically conical box – conformal to $AdS_2 \times S^2$

$$ds^2 = \sqrt{r}\left(\ell^2 \frac{dr^2}{r^2} - r k^2 dt^2 + \ell^2 d\theta^2 + \ell^2 \sin^2\theta d\phi^2\right)$$

$$e^\eta = \frac{B^2/\ell^2}{\sqrt{r}}, \qquad \chi = 0, \qquad A^0 = 0, \qquad A = B\cos\theta d\phi$$

Non-normalizable (fourth) mode B, along with ℓ and k, fixed up to radial diffeomorphism:

$$r \to \lambda^{-4} r \qquad k \to \lambda^3 k, \quad \ell \to \lambda \ell, \quad B \to B$$

and global U(1) symmetry:

$$e^\eta \to \mu^2 e^\eta, \quad \chi \to \mu^{-2}\chi, \quad A^0 \to \mu^3 A^0, \quad A \to \mu A, \quad ds^2 \to ds^2$$

which keep kB^3/ℓ^3 - fixed

- # Radial Hamiltonian formalism

to determine S_{ct}, to the bulk action S

Suitable radial coordinate u, such that constant-u slices Σ_u

$$\Sigma_u \to \partial\mathcal{M} \quad \text{as } u \to \infty.$$

Decomposition of the metric and gauge fields:

$$ds^2 = (N^2 + N_i N^i)du^2 + 2N_i du dx^i + \gamma_{ij} dx^i dx^j$$

$$A^L = a^\Lambda du + A_i^\Lambda dx^i,$$

Decomposition leads to the radial Lagrangian L w/ canonical momenta:

$$\pi^{ij} = \frac{\delta L}{\delta \dot{\gamma}_{ij}}$$

$$\pi_I = \frac{\delta L}{\delta \dot{\varphi}^I}$$

$$\pi^i_\Lambda = \frac{\delta L}{\delta \dot{A}^\Lambda_i}$$

w/ momenta conjugate to N, N_i, and a_Λ vanish.

Hamiltonian:

$$H = \int \mathrm{d}^3\mathbf{x} \left(\pi^{ij}\dot{\gamma}_{ij} + \pi_I \dot{\varphi}^I + \pi^i_\Lambda \dot{A}^\Lambda_i\right) - L = \int \mathrm{d}^3\mathbf{x} \left(N\mathcal{H} + N_i\mathcal{H}^i + a^\Lambda \mathcal{F}_\Lambda\right)$$

First class constraints $\mathcal{H} = \mathcal{H}^i = \mathcal{F}_\Lambda = 0,$ - Hamilton Jacobi eqs.:

& Momenta as gradients of Hamilton's principal function $\mathcal{S}(\gamma, A^\Lambda, \varphi^I)$:

$$\pi^{ij} = \frac{\delta \mathcal{S}}{\delta \gamma_{ij}}, \quad \pi^i_\Lambda = \frac{\delta \mathcal{S}}{\delta A^\Lambda_i}, \quad \pi_I = \frac{\delta \mathcal{S}}{\delta \varphi^I}.$$

w/ original.

$$\pi^{ij} = \frac{\delta L}{\delta \dot{\gamma}_{ij}}$$

$$\pi_I = \frac{\delta L}{\delta \dot{\varphi}^I}$$

$$\pi^i_\Lambda = \frac{\delta L}{\delta \dot{A}^\Lambda_i}$$

deBoer, Verlinde[2] '99,…Skenderis, Papadimitriou '04,…

Solve asymptotically (for \`vacuum' asymptotic solutions) for

$\mathcal{S}(\gamma, A^\Lambda, \varphi^I)$ = - S_{ct} !

$\mathcal{S}(\gamma, A^\Lambda, \varphi^I)$ coincides with the on-shell action, up to terms that remain finite as $\Sigma_u \rightarrow \partial\mathcal{M}$. In particular, divergent part of $\mathcal{S}[\gamma, A^\Lambda, \varphi^I]$ coincides with that of the on-shell action.

• Hamiltonian Formalism with ``Renormalized'' Action

$$S_{\rm reg} = S_4 + S_{\rm ct} \qquad S_{\rm ren} = \lim_{r\to\infty} S_{\rm reg}$$ Finite-independent of r

Covariant S_{ct} calculated for vacuum asymptotic sol.
(for non-flat, conformal to AdS_2 x S^2 geometry)

$$S_{\rm ct} = -\frac{1}{\kappa_4^2}\int d^3\mathbf{x}\sqrt{-\gamma}\,\frac{B}{4}e^{\eta/2}\left(\frac{4-\alpha}{B^2} + (\alpha-1)e^{-\eta}R[\gamma] - \frac{\alpha}{2}e^{-2\eta}F_{ij}F^{ij} + \frac{1}{4}e^{-4\eta}F^0_{ij}F^{0ij}\right)$$

Renormalized canonical momenta:

$$\Pi^{ij} = \pi^{ij} + \frac{\delta S_{\rm ct}}{\delta\gamma_{ij}}, \quad \Pi^i_\Lambda = \pi^i_\Lambda + \frac{\delta S_{\rm ct}}{\delta A^\Lambda_i}, \quad \Pi_I = \pi_I + \frac{\delta S_{\rm ct}}{\delta\varphi^I}$$

- # Conserved Charges:

Conserved currents, a consequence of the first class constraints

$F_\Lambda = 0$ Conserved currents for gauge potentials: $D_i\Pi^i = 0, \quad D_i\Pi^{0i} = 0.$

Conserved charges: $Q_4^{(m)} = -\int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\,\Pi^t, \qquad Q_4^{0(e)} = -\int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\,\Pi^{0t}$

$$= \frac{3B}{4G_4} \qquad\qquad = \frac{\ell^4}{4G_4B^3}\left(\sqrt{r_+r_-} + \omega^2\ell^2\right)$$

$\mathcal{H}_i = 0$ Conserved currents: $-2D_j\Pi^j_i + \Pi_\eta\partial_i\eta + \Pi_\chi\partial_i\chi + F^0_{ij}\Pi^{0j} + F_{ij}\Pi^j \approx 0$

Conserved ``charges'': $\mathcal{Q}[\zeta] = \int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\,\left(2\Pi^t_j + \Pi^{0t}A^0_j + \Pi^tA_j\right)\zeta^j$

Asymptotic Killing vector ζ_i

Mass: $M_4 = -\int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\,\left(2\Pi^t{}_t + \Pi^t_0A^0_t + \Pi^tA_t\right) = \frac{\ell k}{8G_4}(r_+ + r_-)$

Angular Momentum: $J_4 = \int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\,(2\Pi^t{}_\phi + \Pi^t_0A^0_\phi + \Pi^tA_\phi) = -\frac{\omega\ell^3}{2G_4}$

- # Thermodynamic relations and the first law

Free Energy: $I_4 = S^{\mathrm{E}}_{\mathrm{ren}} = -S_{\mathrm{ren}} = \beta_4 \mathcal{G}_4 = \frac{\beta_4 \ell k}{8G_4}\left((r_- - r_+) + 2\omega^2\ell^2\sqrt{\frac{r_-}{r_+}}\right)$

Quantum statistical relation: $\mathcal{G}_4 = M_4 - T_4 S_4 - \Omega_4 J_4 - \Phi^{0(e)} Q^{0(e)}$

First law: $\mathrm{d}M_4 - T_4\mathrm{d}S_4 - \Omega_4\mathrm{d}J_4 - \Phi_4^{0(e)}\mathrm{d}Q_4^{0(e)} - \Phi_4^{(m)}\mathrm{d}Q_4^{(m)} = 0.$

Smarr's Formula: $M_4 = 2S_4T_4 + 2\Omega_4 J_4 + Q_4^{0(e)}\Phi_4^{0(e)} + Q_4^{(m)}\Phi_4^{(m)}$

Varying parameters: r_+, r_-, ω, and B, k, ℓ subject to kB^3/ℓ^3 –fixed

⟺ original parameters m, a, Π_c, Π_s & a scaling parameter

IV. Holography via 2D Einstein-Maxwell-Dilaton

For more details, c.f., Ioannis Papadimitriou's talk
M.C., Papadimitriou 1608…..

4D STU fields can be consistently Kaluza-Klein reduced on S^2 by one-parameter family of Ansätze:

$$e^{-2\eta} = e^{-2\psi} + \lambda^2 B^2 \sin^2\theta, \qquad \chi = \lambda B \cos\theta$$

$$e^{-2\eta} A^0 = e^{-2\psi} A^{(2)} + \lambda B^2 \sin^2\theta d\phi, \quad A + \chi A^0 = B\cos\theta d\phi$$

$$e^{\eta} ds_4^2 = ds_2^2 + B^2 \left(d\theta^2 + \frac{\sin^2\theta}{1 + \lambda^2 B^2 e^{2\psi} \sin^2\theta} (d\phi - \lambda A^{(2)})^2 \right)$$

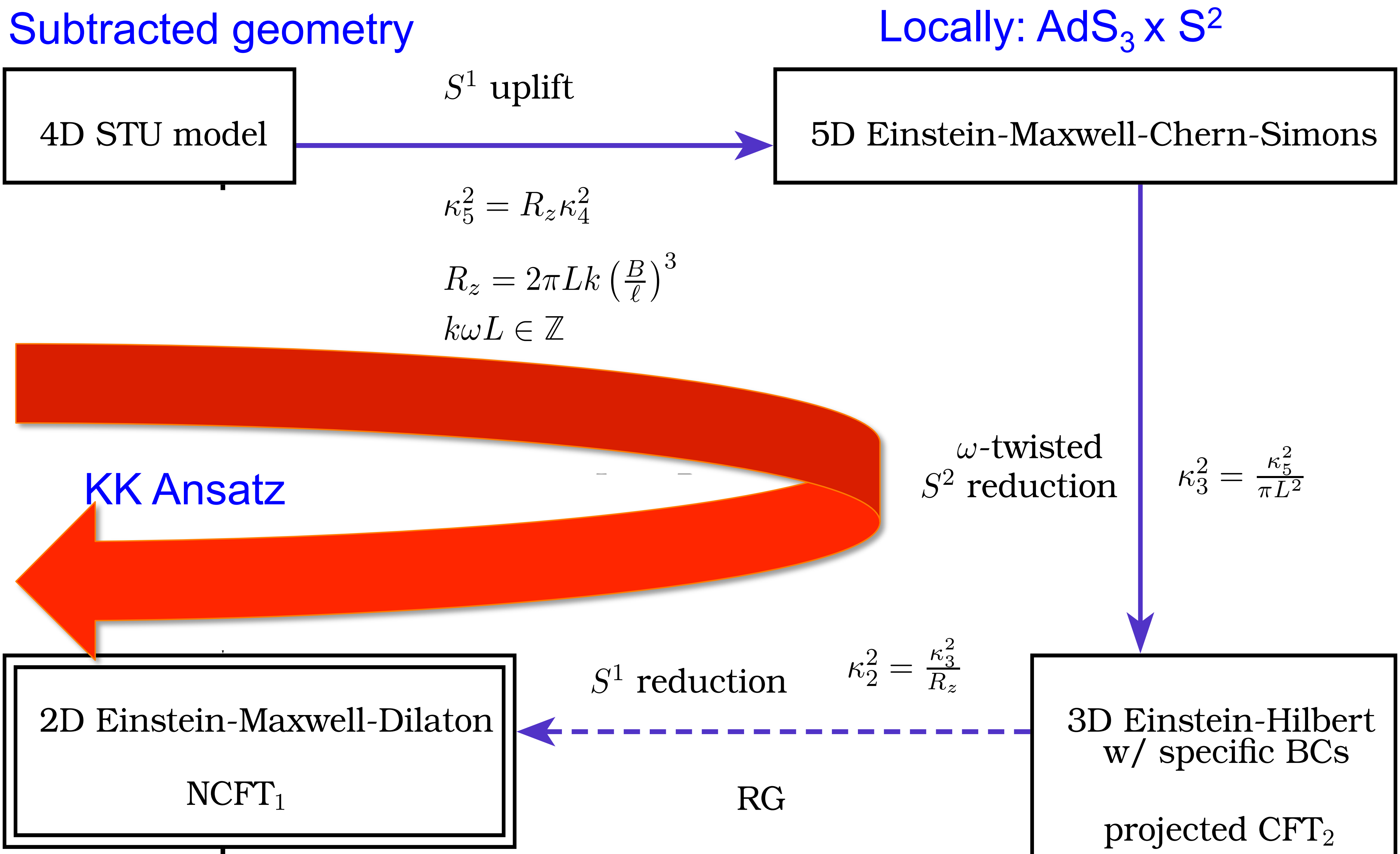
ds_2^2, Ψ, $A^{(2)}$ -fields of 2D Einstein-Maxwell-Dilaton Gravity:

$$S_{\rm 2D} = \frac{1}{2\kappa_2^2} \left(\int \mathrm{d}^2\mathbf{x}\sqrt{-g}\, e^{-\psi} \left(R[g] + \frac{2}{L^2} - \frac{1}{4} e^{-2\psi} F_{ab}F^{ab} \right) + \int \mathrm{d}t \sqrt{-\gamma}\, e^{-\psi} 2K \right)$$

B = 2L; λ-independent

$\lambda = \omega \ell^3 / B^3$ rotational parameter of subtracted geometry

Web of Theories



General solution of 2D EMD Gravity – running dilaton

Feffeman-Graham gauge: $ds^2 = du^2 + \gamma_{tt}(u,t)dt^2, \quad A_u = 0$

Anaytic general solution:

$$e^{-\psi} = \beta(t)e^{u/L}\sqrt{\left(1 + \frac{m - \beta'^2(t)/\alpha^2(t)}{4\beta^2(t)}L^2 e^{-2u/L}\right)^2 - \frac{Q^2L^2}{4\beta^4(t)}e^{-4u/L}}$$

$$\sqrt{-\gamma} = \frac{\alpha(t)}{\beta'(t)}\partial_t e^{-\psi}$$

$$A_t = \mu(t) + \frac{\alpha(t)}{2\beta'(t)}\partial_t \log\left(\frac{4L^{-2}e^{2u/L}\beta^2(t) + m - \beta'^2(t)/\alpha^2(t) - 2Q/L}{4L^{-2}e^{2u/L}\beta^2(t) + m - \beta'^2(t)/\alpha^2(t) + 2Q/L}\right)$$

Leading asymptotic behavior:

$$\gamma_{tt} = -\alpha^2(t)e^{2u/L} + \mathcal{O}(1), \quad e^{-\psi} \sim \beta(t)e^{u/L} + \mathcal{O}(e^{-u/L}), \quad A_t = \mu(t) + \mathcal{O}(e^{-2u/L})$$

running dilaton

- Arbitrary functions *α(t), β(t)* and *μ(t)* identified with the sources of the corresponding dual operators
- 4D uplift results in asymptotically conformally $AdS_2 \times S^2$ subtracted geometries, generalized to include arbitrary time-dependent sources

Repeat Radial Hamiltonian Formalism in 2D

Radial ADM decomposition: $ds^2 = (N^2 + N_t N^t)du^2 + 2N_t du dt + \gamma_{tt} dt^2$

Countertern Action: $S_{\mathrm{ct}} = -\frac{1}{\kappa_2^2} \int \mathrm{d}t \sqrt{-\gamma}\, L^{-1} \left(1 - u_o L \Box_t\right) e^{-\psi}$

Renormalized one-point functions: $\mathcal{T} = 2\widehat{\pi}^t_t, \quad \mathcal{O}_\psi = -\widehat{\pi}_\psi, \quad \mathcal{J}^t = -\widehat{\pi}^t$

$$\widehat{\pi}^t_t = \frac{1}{2\kappa_2^2} \lim_{u\to\infty} e^{u/L} \left(\partial_u e^{-\psi} - e^{-\psi} L^{-1}\right)$$

$$\widehat{\pi}^t = \lim_{u\to\infty} \frac{e^{u/L}}{\sqrt{-\gamma}} \pi^t$$

$$\widehat{\pi}_\psi = -\frac{1}{\kappa_2^2} \lim_{u\to\infty} e^{u/L} e^{-\psi} \left(K - L^{-1}\right)$$

Explicit one-point functions:

$$\mathcal{T} = -\frac{L}{2\kappa_2^2}\left(\frac{m}{\beta} - \frac{\beta'^2}{\beta\alpha^2}\right), \quad \mathcal{J}^t = \frac{1}{\kappa_2^2}\frac{Q}{\alpha}, \quad \mathcal{O}_\psi = \frac{L}{2\kappa_2^2}\left(\frac{m}{\beta} - \frac{\beta'^2}{\beta\alpha^2} - 2\frac{\beta'\alpha'}{\alpha^3} + 2\frac{\beta''}{\alpha^2}\right)$$

Ward Identities: $\partial_t \mathcal{T} - \mathcal{O}_\psi \partial_t \log\beta = 0, \qquad \mathcal{D}_t \mathcal{J}^t = 0$

Conformal anomaly: $\mathcal{T} + \mathcal{O}_\psi = \frac{L}{\kappa_2^2}\left(\frac{\beta''}{\alpha^2} - \frac{\beta'\alpha'}{\alpha^3}\right) = \frac{L}{\kappa_2^2\alpha}\partial_t\left(\frac{\beta'}{\alpha}\right) \equiv \mathcal{A}$

Exact generating function ($\mathcal{T} = \frac{\delta S_{\text{ren}}}{\delta\alpha}, \quad \mathcal{O}_\psi = \frac{\beta}{\alpha}\frac{\delta S_{\text{ren}}}{\delta\beta}, \quad \mathcal{J}^t = -\frac{1}{\alpha}\frac{\delta S_{\text{ren}}}{\delta\mu}$):

$$S_{\text{ren}}[\alpha,\beta,\mu] = -\frac{L}{2\kappa_2^2}\int \mathrm{d}t\left(\frac{m\alpha}{\beta} + \frac{\beta'^2}{\beta\alpha} + \frac{2\mu Q}{L}\right)$$

Legandre transformed generating function (w/ $\alpha(t) = \beta(t)$):

$$\Gamma_{\text{eff}} = S_{\text{ren}} + \int \mathrm{d}t\,\alpha\,(\mathcal{T} + \mathcal{O}_\psi) = \frac{L}{\kappa_2^2}\int \mathrm{d}t\,(\{\tau,t\} - \mu Q/L - m)$$

$$\{\tau,t\} = \frac{\tau'''}{\tau'} - \frac{3}{2}\frac{\tau''^2}{\tau'^2}$$

Schwarzian derivative

``dynamic time''

$$-\alpha^2(t)dt^2 = -(d\tau(t))^2$$

c.f., Sadchev-Ye-Kitaev'93,… Almeheiri-Polochinski'14;Maldacena, Stanford, Yang'16, Engelsoy,Merens,Verlinde'16,…

Asymptotic symmetries and conserved charges

Asymptotic symmetries: subset of Penrose-Brown-Henneaux (PBH) transformations (diffeomorphisms and gauge transformations preserving the Fefferman-Graham gauge) that preserve boundary conditions:

$$\delta_{\mathrm{PBH}}\alpha = \partial_t(\varepsilon\alpha) + \alpha\sigma/L, \qquad \delta_{\mathrm{PBH}}\beta = \varepsilon\beta' + \beta\sigma/L, \qquad \delta_{\mathrm{PBH}}\mu = \partial_t(\varepsilon\mu + \varphi)$$

$\delta_{\mathrm{PBH}}\,(\text{sources}) = 0$ → constrain functions ε(t), σ(t) and φ(t) in term of two constants $\xi_{1,2}$

Conserved Charges: boundary terms obtained by varying the action with respect to the asymptotic symmetries (and Ward identities) →

U(1)xU(1): $$\mathcal{Q}_1 = -\left(\beta\mathcal{T} - \frac{L}{2\kappa_2^2}\frac{\beta'^2}{\alpha^2}\right) = \frac{mL}{2\kappa_2^2}, \qquad \mathcal{Q}_2 = \alpha\mathcal{J}^t = \frac{Q}{\kappa_2^2}.$$

3D perspective: two copies of the Virasoro algebra with the Brown-Henneaux central charge. Only $L^{\pm}_0$ are realized non-trivially in 2D.

Constant dilaton solutions and AdS_2 holography

c.f., Strominger'98, …Castro, Grumiller, Larsen, McNees '08,…
Compère, Song, Strominger '13,…Castro, Song'14,…

Holography depends on the structure of non-extremal constant dilaton solutions and choice of boundary conditions →

Provided systematic holographic dictionary for each choice

c.f., Papadimitriou's talk &1608….
No Time

Note: non-extremal running dilaton solution $\xrightarrow{Q=mL/2}$

extremal running-dilaton solution
with RG flow to IR fixed point   VEV of irrelevant scalar op.
extremal constant dilaton solution →

non-extremal constant dilaton branch (`Coulomb phase')
(does not lift into subtracted geometry)

Summary/Outlook with focus on AdS_2 Holography

- Provided a family of consistent KK Ansätze that allow us to uplift any solution of 2D EMD gravity to 4D STU solutions, which are non-extremal 4D black holes, asymptotically (conformally) $AdS_2 \times S^2$ – subtracted geometry. [Should work also for 5D solutions asymptotically (conformally) AdS_2 x S^3.]

- Constructed complete holographic dictionary of 2D EMD gravity theory obtained by an S^2 reduction of 4D STU subtracted geometry & constant dilaton solutions.

- 2D EMD gravity has a well defined UV fixed point, described by a sector of 2D CFT.

- Many aspects of the holographic description are generic and should apply to more generic 2D dilaton gravity theories.