Extended black hole and entanglement thermodynamics in holography

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Based on:

ELENA CÁCERES, PHUC NGUYEN & JP [ARXIV:1605.00595]
"HOLOGRAPHIC ENTANGLEMENT CHEMISTRY"
⟶ SEE ALSO [ARXIV:1507.06069]

Outline

Motivation

- The notion of entanglement has played a crucial role in our understanding of quantum gravity and the emergence of spacetime
- Key ingredients: BH thermodynamics, holography

$$dM = \frac{\kappa}{8\pi G} dA \qquad \longleftrightarrow \qquad dE = TdS$$

- For BHs with charge and angular momentum:

$$dE = TdS + \Omega dJ + \Phi dQ$$

What about a PV term? Einstein's equation suggests:

$$P = -\frac{\Lambda}{8\pi G}$$

- Considering Λ as an additional thermodynamical variable is known as extended BH thermodynamics or BH chemistry [Kastor, Ray & Traschen], [Cvetic, Gibbons, Kubiznak & Pope], [Dolan], [Kubiznak & Mann],...

Motivation: extended BH thermodynamics

- The ADM mass M is the enthalpy H:

$$M = H = U + PV$$

and the volume is defined as the thermodynamical conjugate to P:

$$V := \left(\frac{\partial H}{\partial P}\right)_S$$

- In simple cases V coincides with a naive integration over the BH interior $V = \frac{4}{3}\pi r_+^3$ (in $d = 3$). Notice it depends on the foliation!
- The extended first law is:

$$dM = TdS + \Omega dJ + \Phi dQ + VdP$$

- However, the holographic interpretation is radically different...

Motivation: holographic interpretation

- First, note that P (or Λ) is a parameter of the gravity action, while M, Q and J are just parameters of the solutions
- In string/M theory constructions, $\Lambda = -d(d-1)/2L^2$ is set by the value of the Planck length, and the number of branes N. In a Dp system one generally finds:

$$\frac{L^{d-1}}{G} \sim N^2$$

- Varying P changes N and hence changes the dual theory. Naively, V can be interpreted as a chemical potential for color [Dolan]
- However, varying P also changes the radius of curvature R of the CFT metric (which also changes the theory) [Karch & Robinson]

$$\partial_{N^2}|_R = \partial_{G^{-1}}|_L\,, \qquad \partial_R|_{N^2} = \partial_L|_{L^3/G}\,.$$

Motivation: the extended PV space

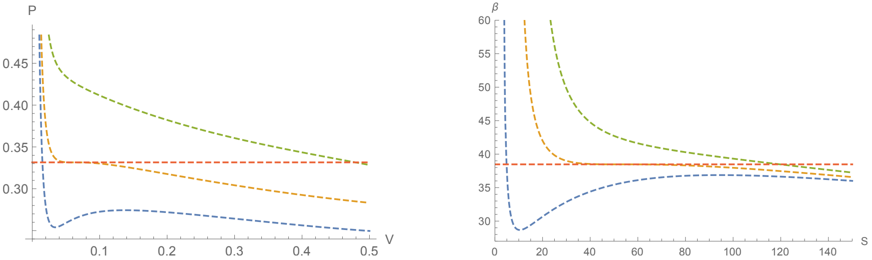


Figure: Typical PV and TS diagrams of AdS-RN

- Critical point can be specified by (P_c, V_c, β_c) or (β_c, S_c, Q_c)
- In the canonical ensemble:

$$P_c = \frac{1}{384\pi Q^2}\,, \qquad V_c = 64\sqrt{6}\pi Q^3\,, \qquad \beta_c = 6\sqrt{6}\pi Q$$

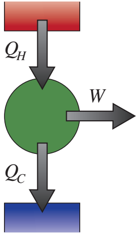
- The EOS leaves us with one dimensionless parameter, e.g. $\zeta = \beta^2 P$

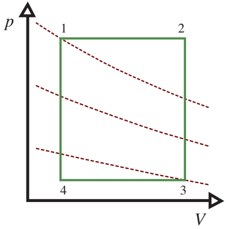
Motivation: the extended PV space

- At fixed (Q, V) we can vary either β or P to reach the CP
  - Field theory interpretation is completely different! [Caceres, Nguyen & JP]
  - Significance of the VdW transition also differs:

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P \geq 0\,, \qquad \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \geq 0$$

  - Also explains why is it possible to derive the generalized Smarr relation from standard CFT thermodynamics [Karch & Robinson]
- Varying P has also motivated the “holographic heat engines” [Johnson]





Motivation: extended laws of entanglement?

- Entanglement entropy for a subregion A also obeys a first law:

$$\delta S_A = \delta\langle H_A\rangle \,.$$

where $S_A = -\mathrm{tr}[\rho_A \log \rho_A]$ and $\rho_A \simeq e^{-H_A}$

- H_A not know in general. For a spherical region in the vacuum:

$$H_A = 2\pi \int_A d^{d-1}x \frac{R^2 - |\vec{x} - \vec{x}_0|^2}{2R} T_{00}$$

- In holography, S_A is strikingly similar to S [Ryu & Takayanagi]

$$S_A = \frac{\mathcal{A}}{4G}$$

and the origin of the first law is well understood

- Important: first law + RT = linearized EOM in the bulk! [Lashkari, McDermott & Van Raamsdonk], [Faulkner, Guica, Hartman, Myers & Van Raamsdonk]

Motivation: extended laws of entanglement?

Two natural questions we may ask:

Q1: Can we derive an extended version for the first law?
Q2: If so, what new information codifies?

In order to answer these we made use of the Iyer-Wald formalism...

Review of Iyer-Wald

- The Iyer-Wald formalism is an application of Noether's Theorem to diffeomorphism invariance
- Consider a diffeomorphism-invariant theory of gravity, with Lagrangian $\mathcal{L}$, e.g. in Einstein gravity $\mathcal{L} = (R - 2\Lambda)\varepsilon$
- The algorithm is to compute 4 differential forms (in this order): Θ (symplectic potential current), J (Noether current), Q (Noether charge), and χ
- Θ is the boundary term obtained by varying the Lagrangian under some metric variation δg:

$$\delta\mathcal{L} = (\mathrm{e.o.m.})\delta g + d\Theta(\delta g)$$

- For Einstein gravity:

$$\Theta(\delta g) = \frac{1}{16\pi G} g^{ac} g^{bd} (\nabla_b \delta g_{cd} - \nabla_c \delta g_{bd})\varepsilon_a$$

The Noether current J and Noether charge Q

- Once we have Θ for an arbitrary perturbation, we specialize to the case when δg is induced by a diffeomorphism:

$$\delta g = \mathcal{L}_X g$$

where X is the vector field generating the diffeomorphism

- The Noether current is then:

$$J = \Theta(\mathcal{L}_X g) - X \cdot \mathcal{L}$$

- After some manipulation, J can be brought to the form:

$$J = dQ + \text{e.o.m.}$$

for some Noether charge form Q.

- For Einstein gravity,

$$Q = -\frac{1}{16\pi G} \nabla^a X^b \varepsilon_{ab}$$

The form χ and the 1st law

- Next, consider a different kind of δg: an arbitrary on-shell perturbation (i.e. it satisfies the linearized e.o.m.), and define:

$$\chi := \delta Q - X \cdot \Theta(\delta g)$$

where Θ is evaluated for this new perturbation, and δQ is the variation of Q under this new perturbation.

- If X is a Killing vector field ξ then χ is closed:

$$d\chi = 0$$

- For BH thermodynamics $\xi = \partial_t$. Stokes theorem yields:

$$\int_\infty \chi - \int_{\mathcal{H}} \chi = \delta E - \delta S = 0$$

How to do Iyer-Wald with $\delta\Lambda$ and δG?

- Note that $\delta\Lambda$ and δG are qualitatively different from δM, δQ, δJ: Λ appears both as a coupling in the action and a parameter in the solution; G only appears in the action.
- Solution: [Urano, Tomimatsu & Saida]
  - Extend the definition of $\delta\mathcal{L}$ at the beginning to account for $\delta\Lambda$:

$$\delta\mathcal{L} = (\mathrm{e.o.m})\delta g + \frac{\partial\mathcal{L}}{\partial\Lambda}\delta\Lambda + \frac{\partial\mathcal{L}}{\partial G}\delta G$$

  - Compute the contribution to χ due to a shift of $\delta\Lambda$ in the metric.
- In the end, we find the modified statement:

$$\delta\Lambda\int_\Sigma \frac{\partial\mathcal{L}}{\partial\Lambda}\xi\cdot\epsilon + \delta G\int_\Sigma \frac{\partial\mathcal{L}}{\partial G}\xi\cdot\epsilon + \int_\infty \chi - \int_{\mathcal{H}}\chi = 0$$

Application: deriving the black hole volume

- Consider the 4D AdS-Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{r^2}{L^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{r^2}{L^2}\right)} + r^2 d\Omega^2$$

- In this case:

$$\delta\Lambda \int_\Sigma \frac{\partial \mathcal{L}}{\partial \Lambda} \xi \cdot \epsilon = -\frac{\delta\Lambda}{8\pi G} \int_{r_+}^{\infty} \sqrt{-g}\, dr d\Omega = -\frac{\delta\Lambda}{6G}(r_c^3 - r_+^3)$$

where $r_c >> r_+$ is a radial cutoff $\rightarrow$ divergent!

- This divergence cancels with another one:

$$\int_\infty \chi = \frac{r_c^3}{6G}\delta\Lambda$$

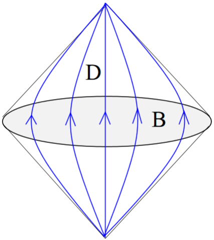
- We find the term $\frac{r_+^3}{6G}\delta\Lambda$ or $V\delta P$ with $V = \frac{4}{3}\pi r_+^3$
  $\rightarrow$ we recovered the thermodynamical volume!

Entanglement thermodynamics in Einstein gravity

- Recall that $\delta S_A = \delta\langle H_A\rangle$, with $S_A = -\mathrm{tr}[\rho_A \log \rho_A]$ and $\rho_A \simeq e^{-H_A}$

- For a spherical boundary region, H_A is flow along a conformal Killing vector field:

$$\zeta = -\frac{2\pi}{R} t x^i \partial_i + \frac{\pi}{R}(R^2 - \vec{x}^2 - t^2)\partial_t$$



- The flow is confined to the causal development $\mathcal{D}$ of the ball.

- For pure AdS, the CKVF ζ extends into a bifurcate KVF in the bulk:

$$\xi = -\frac{2\pi}{R} t(z\partial_z + x^i \partial_i) + \frac{\pi}{R}(R^2 - \vec{x}^2 - t^2 - z^2)\partial_t$$

such that the bifurcation surface is the RT surface

- Iyer-Wald formalism can be applied to derive 1st law!

Entanglement chemistry in Einstein gravity

- At the end we get (see paper for details):

$$\delta S_A = \delta\langle H_A\rangle + S_A\left((d-1)\frac{\delta L}{L} - \frac{\delta G}{G}\right)$$

  - Extra terms are due to change of the parameters of the theory!
  - Variations in δL, δG can be easily written in terms of δN, δR
  - Can be conveniently written in terms of central charges:

$$(d=2) \qquad c = \frac{3L}{2G}$$

$$(d=4) \qquad c = a = \frac{45\pi L^3}{G}$$

$$\boxed{\delta S_A = \delta\langle H_A\rangle + \frac{S_A}{c}\delta c}$$

  which follows from scaling of S_A with c. Compare to [Karch & Robinson]

- Does it buy us anything from bulk perspective? Yes: L and G appear explicitly so we can retrieve the gravity couplings

Entanglement chemistry in higher derivative gravity

- Q: Why is higher derivative gravity interesting in this context?
  $\rightarrow$ New and exotic phase transitions, e.g. multiple reentrant phase transitions and isolated critical points [Frassino, Kubiznak, Mann & Simovic], [Hennigar, Brenna & Mann], [Dolan, Kostouki, Kubiznak & Mann]
- Prototypical example: Gauss-Bonnet theory:

$$\mathcal{L} = \frac{R - 2\Lambda}{16\pi G} + \alpha \left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 \right)$$

- GB serves as a toy model for a theory with different central charges

$$c = \frac{45\pi L^3}{G}\left(1 - \frac{64\pi G\alpha}{L^2}\right)$$

$$a = \frac{45\pi L^3}{G}\left(1 - \frac{192\pi G\alpha}{L^2}\right)$$

- Scaling of thermodynamical quantities with a and c is not universal.

Entanglement chemistry in higher derivative gravity

- In higher derivative gravity the EE functional is modified [Dong], [Camps]
- In Gauss-Bonnet gravity:

$$S = \frac{1}{4G} \int d^{d-1}x\sqrt{h}[1 + 32\pi G\alpha\mathcal{R}]$$

- For a spherical boundary region, the surface is same as in Einstein gravity i.e. a hemisphere $\rightarrow$ same Killing vector field can be used.
- Since the surface is a Killing horizon, its extrinsic curvature vanishes $\rightarrow$ EE coincides with the Wald entropy $\rightarrow$ Iyer-Wald can be used!
- To study entanglement chemistry consider $\delta\Lambda$, δG and $\delta\alpha$.
- Can be easily generalized to Lovelock theories (see the paper for further details)

Entanglement chemistry in higher derivative gravity

- For Gauss-Bonnet in arbitrary dimensions the result is:

$$\delta S_A = \delta\langle H_A\rangle + S_A\left(\Psi_L \delta L - \Psi_G \delta G - \Psi_\alpha \delta\alpha\right)$$

where

$$\Psi_L = \frac{(d-1)}{L}\left(\frac{L^2 - 32\pi G\alpha(d-2)(d-3)}{L^2 - 32\pi G\alpha(d-1)(d-2)}\right)$$
$$\Psi_G = \frac{1}{G}\left(\frac{L^2}{L^2 - (d-1)(d-2)32\pi G\alpha}\right)$$
$$\Psi_\alpha = -\frac{32\pi G(d-1)(d-2)}{L^2 - (d-1)(d-2)32\pi G\alpha}$$

- Again, extended first law includes information about gravity couplings
- In $d = 3$ GB is topological $\rightarrow$ EOM the same as in Einstein gravity. Varying the coupling α gives a non-trivial effect
- Results agree with [Kastor, Ray & Traschen] based on the Hamiltonian formulation of GR

Outlook

Still a lot of work to do:

- Shape dependence [Faulkner, Leigh, Parrikar & Wang]
- Extended first law for excited states [Bhattacharya, Nozaki, Takayanagi & Ugajin], [Allahbakhshi, Alishahiha & Naseh], [Wong, Klich, Pando-Zayas & Vaman], explain VdW transition [Caceres, Nguyen, JP]
- General higher derivative theories
- Examples of the extended first law in field theory
- Interplay with quantum corrections [Faulkner, Lewkowycz & Maldacena], [Engelhardt & Wall]
- String/M theory realizations [Dolan], [Belhaj, Chabab, Moumni, Masmar & Sedra], [Chabab, Moumni & Masmar]
- Relation with complexity? [Alishahiha], [Brown, Roberts, Susskind, Swingle & Zhao], [Momeni, Myrzakulov & Myrzakulov]