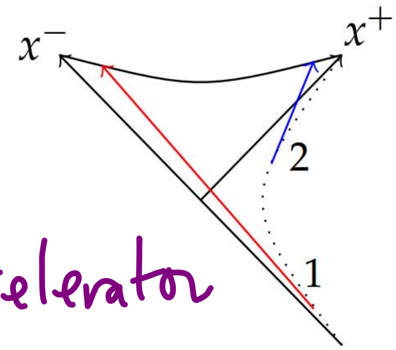
Six point string scattering: simulation of horizon infallers

with M. Dodelson

arXiv:1504.05536, arXiv:1504.05537, paper 3 in progress

cf '14 E.S. arXiv:1402.1486 ; Puhm Rojas Ugajin (open string production

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \left(1-\frac{r_s}{r}\right)^{-1}dr^2$$
$$= -\frac{2r_s}{r}e^{1-r/r_s}dx^+\,dx^-.$$



Black Hole as accelerator

I. Intro

string spreading

Susskind '94 + Lowe Polchinski Thorlacius Uglum '95...Brower Polchinski Strassler Tan'06

II. S-matrix analysis * 6 points

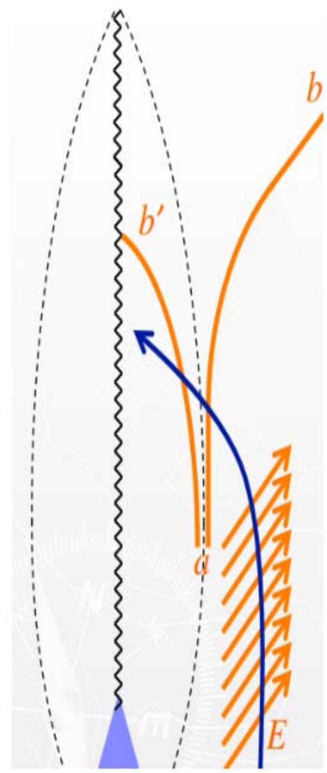
What is the leading breakdown of effective field theory (including GR) at a horizon, in string theory?

Naive estimate: EFT valid for small curvature in units of the string tension, $\alpha' R$ (and small tidal forces).

Despite weak curvature, over long relative infall times a large energy can develop. This, combined with previously proposed string spreading dynamics, violates the above estimate. The string-theoretic modification of GR this indicates is potentially important for black hole physics.

Hawking evaporation calculation gave thermal (information-losing) result => Since AdS/CFT says otherwise, something has to give.

**Hawking radiation thermal (no info); comes from vacuum near horizon: if unitary, then not vacuum near horizon
--monogamy of entanglement
*`Complementarity' was an idea that this problem cannot be seen by a single observer. ...AMPS '12...:
--There are observers who can see too much for `complementarity'
--Thought exp't constraints so far rule out all existing theories, something must give
1)Unitarity: AdS/CFT (Hawking agrees!)
2) EFT outside horizon, smooth infall
How get around above? so far all attempts at smooth horizon violate QM
*Horizon drama for infaller (e.g. `firewall', or variant)? **Need dynamics**



Various interesting attempts at two extremes:

EFT after all? (soft hair, or attempts at nonlocal deformation of QFT)

Non-perturbative theory (AdS/CFT, quantum info, fuzzballs...) Ironically some QI based proposals violate quantum mechanics at some level with state-dependent operators.

---Intermediate approach within tree-level string theory (and perhaps large-Nc gauge theory) Regardless of BH information, the leading breakdown of EFT in QG (e.g. string theory) is a crucial and very basic question.

Subtle problem cf Lowe Polchinski Thorlacius Susskind Uglum, et al.

The fact that it is hard to immediately kill or establish such a long range nonlocal interaction is remarkable in itself.

Effective Field Theory and dangerous irrelevance:

Standard method parameterizing our ignorance of high(er) energy physics

GR breaks down for $\lambda_G \to 1$ (or before)

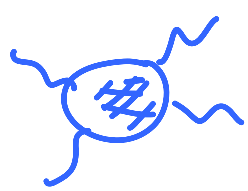
classical & Quantum corrections

$$S = \int \left(\frac{R}{G_N} - V(\phi)\right)\left(1 + R\left(\frac{c_1}{M_*^2} + \tilde{c}_1 G_N\right) + \cdots\right)$$

$$+ \int (\partial\phi)^2 + k_1 \frac{(\partial\phi)^4}{M_*^2} + \cdots$$

$\leftarrow$ scale of "new physics"

with corrections sensitive to short-distance physics



e.g. scalar quantum fields

$$S = \int d^4x \left\{ \dot{\phi}^2 - (\vec{\partial}\phi)^2 - \frac{1}{2}\underbrace{\lambda_2 M_*^2}_{m^2}\phi^2 \right.$$

$$+ \frac{\lambda_4}{4!}\phi^4 + \frac{\lambda_6}{6!\,M_*^2}\phi^6 + \cdots$$

$$\left. - \lambda_{4,2}\frac{(\partial\phi)^2\phi^2}{M_*^2} + \cdots \frac{\lambda\, \mathcal{O}_\Delta}{M_*^{\Delta-4}} \right\}$$

ϕ has dimensions of Energy

$\mathcal{O}_\Delta$ corrections $\sim \left(\frac{\text{Energy}}{M_*}\right)^{\Delta-4}$

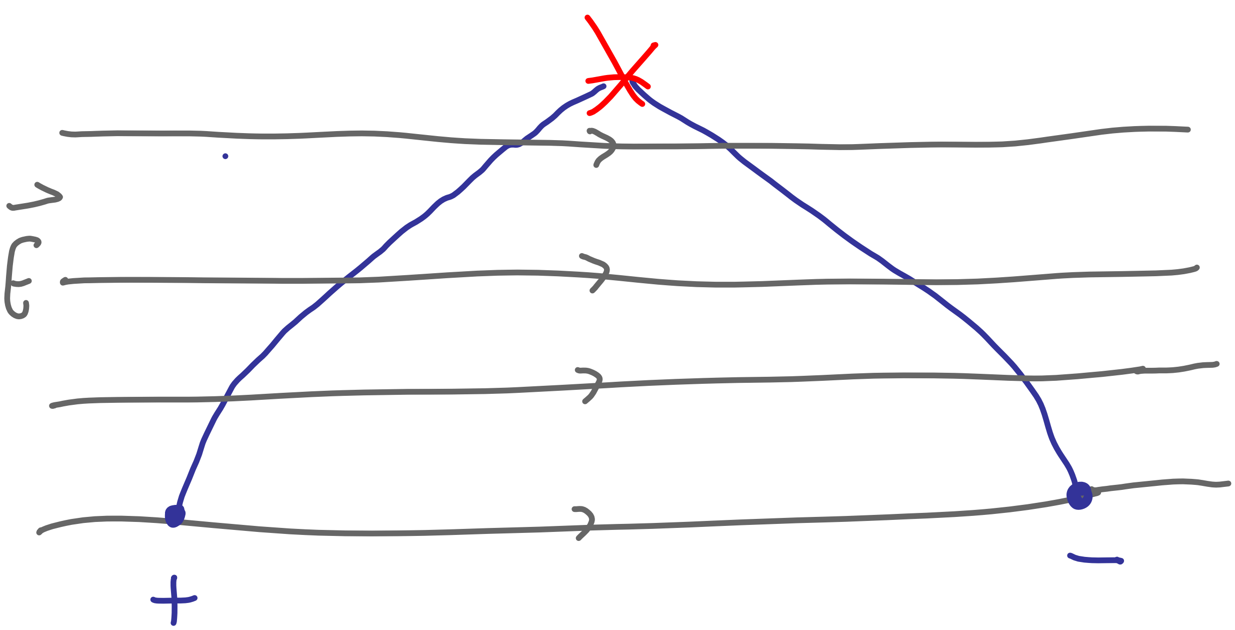
$$\mathcal{O}_\Delta \text{ corrections} \sim \left(\frac{\text{Energy}}{M_*}\right)^{\Delta-4}$$

There is an infinite sequence of `irrelevant' perturbations, those with $\Delta > 4$. Since these die out at low Energy, we can often make reliable physical predictions despite our ignorance of this infinite sequence.

However, physics *can* become sensitive to `UV completion' even in systems with low input energies.

This subtlety arises in the presence of long time evolution and/or large field excursions.

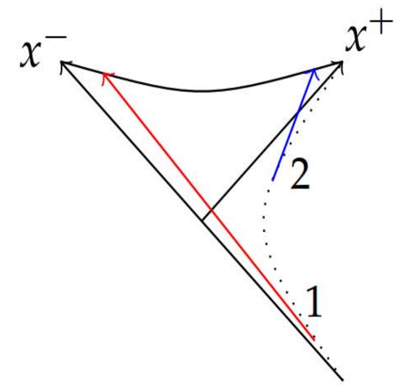
An electromagnetic example:
Consider a weak electric field permeating space, with two charges initially sitting at rest.



The weak field accelerates charges over a long time, producing a large invariant energy

A similar effect occurs in weakly curved geometries with a horizon: evolution of trajectories of (say) two probes sent in with modest energy leads to a large *nonlocal* invariant energy in the near horizon region.

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \left(1-\frac{r_s}{r}\right)^{-1}dr^2$$
$$= -\frac{2r_s}{r}\underbrace{e^{1-r/r_s}dx^+\,dx^-}_{r\to r_s}.$$

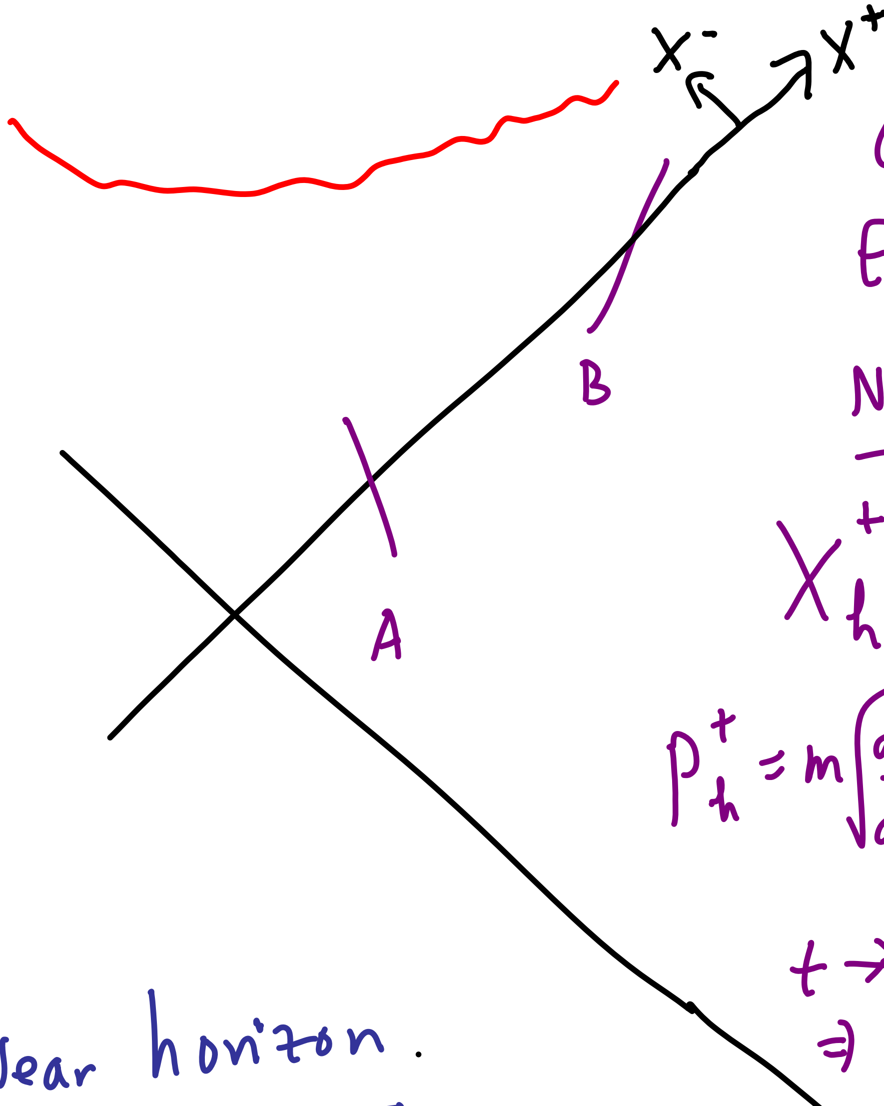


get patch of flat (Minkowski/Rindler) spacetime $ds^2 = -d\hat{t}^2 + d\hat{x}^2$

$$= -2\,dx^+dx^-$$

$x^\pm = \hat{t} \pm \hat{x}$ light cone coordinates

$$ds^2 = -\frac{2r_s}{r} e^{1-\frac{r}{r_s}} dX^+ dX^- + r^2 d\Omega^2$$



Outside :

E, m fixed

Near Horizon :

$$X^+_h = 2r_s\sqrt{e}\,\frac{E}{m}e^{\eta}$$

$$p^+_h = m\sqrt{\frac{dX^+}{dX^-}}\bigg|_h = m e^{\eta}$$

$$t \to t + \Delta t$$

$$\Rightarrow \eta \to \eta + \frac{\Delta t}{2r_s}$$

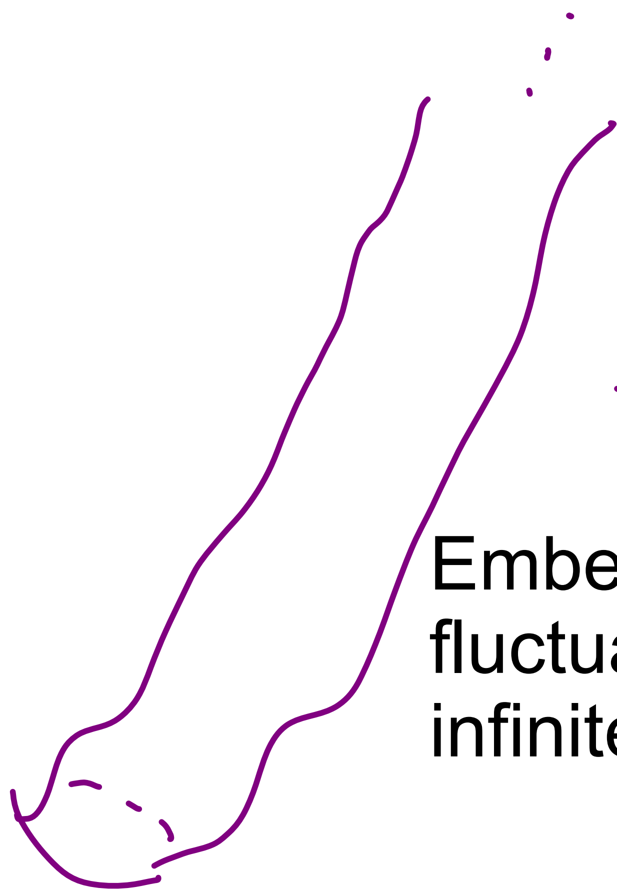
Near horizon.

- $E^2_{n.h.} \sim 2p^+_{B,h}\,p^-_{A,h} \sim e^{\frac{\Delta t}{2r_s}} m^2$
- $X^+_B - X^+_A \propto p^+_B \propto e^{\frac{\Delta t}{2r_s}}$

Exponentially large, non-local energy in A-B system

Near horizon: large energy, but separated in X+: non-local large energy invariant.

String Theory



$X^\mu(\sigma, \tau)$

Embedding in spacetime fluctuates: formally infinite mean square size

$$< XX >$$

because of high-frequency modes. Need high energy probe to

String Spreading

Susskind '94 + Lowe Polchinski Thorlacius Uglum '95...Brower Polchinski Strassler Tan'06

Light Cone gauge $X^- \sim p^- \tau$,

Constraint determines X^+ in terms of $X^\perp$

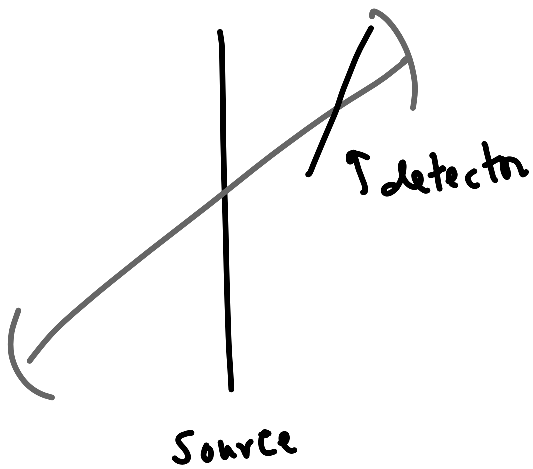
$$\langle \psi | (X_\perp - x_\perp)^2 | \psi \rangle = \sum_n^{n_{max}} \frac{1}{n} = \log \frac{n_{max}}{n_0} + \mathcal{O}\left(\frac{1}{n_{max}}\right)$$

Direction of relative motion:

$$\langle \psi | (X^+ - x^+)^2 | \psi \rangle \simeq \frac{1}{(p^-)^2} \sum^{n_{max}} n \simeq \frac{n_{max}^2}{(p^-)^2}$$

$n_{max} \leftrightarrow$ light cone time resolution

Light cone time resolution (refined prediction)



Note that this effect is causal, just non-local (the string is spread out, so can potentially interact before its center reaches the detector).

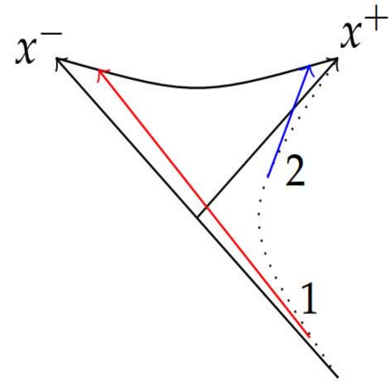
Measurement degrades as $\uparrow p_\perp$: conservatively

$$\Delta x^- \sim \frac{p_\perp^2 + m^2}{p^+_{det}}, \quad n_{max} \sim \frac{p_s^- \, p_d^+}{p_\perp^2 + m^2}$$

$$\boxed{\Rightarrow \underset{\text{RMS}}{\Delta X^+_{spr}} \sim \frac{E_{det}}{p_\perp^2}}$$

Agrees precisely with BPST '06 2->2 calculation, and with Gross-Mende

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \left(1-\frac{r_s}{r}\right)^{-1}dr^2$$
$$= -\frac{2r_s}{r}e^{1-r/r_s}dx^+\,dx^-.$$



Trajectories: system 2 (its secondary offshoots) can have the required light cone time resolution to detect 1.

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*Precisely the same prediction arises from the Gross-Mende saddle point, if interpreted as indicative of the dominant contribution at real embedding coordinate X.

*A final piece of intuition, which we will explicitly see in our results: string amplitudes are UV-soft. This implies that the probability must spread out in position space. (The question is the extent and direction(s) of this spread.)

**On the other hand, the pole structure is somewhat similar in ST and QFT)

This effect, if not an artifact of a particular gauge choice (i.e. choice of coordinates), leads to an interaction between early and late infallers that goes beyond the predictions of naive effective field theory (i.e. not controlled by the weak curvature). The acceleration of the trajectories exponentially amplifies the energy. However, to get a substantial effect in the most conservative calculations, the late infalling system needs somewhat high local energy (e.g. photons grazing very near the horizon).

A similar setup to the black hole appears at six points in tree level flat space string scattering

$AB \to 1'3$

$C1' \to 21$

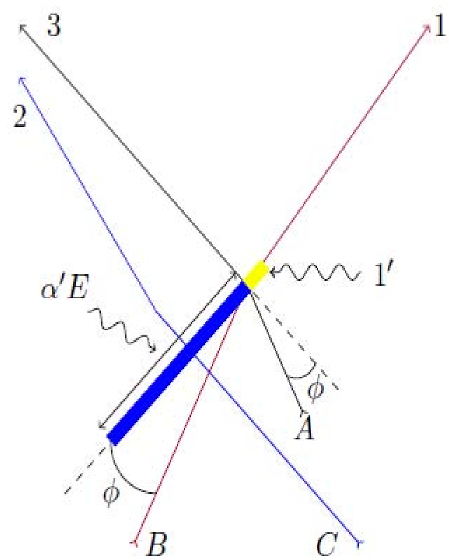
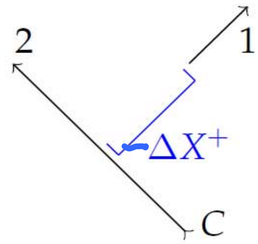


Figure 1: The setup for one version of our process, with peak trajectories shown. The central trajectories of strings A and B collide at $T = X = Y = 0$, producing outgoing strings 1 and 3. We introduce the third incoming string C, with kinematics such that string 1 (and the closely related Pomeron $1'$) has optimal light cone time resolution to detect the longitudinal spreading of strings C and 2 (shown in blue), for any value of the angle ϕ as in (2.15). The amplitude for the process decomposes into the $AB \to 1'3$ and $C1' \to 21$ four-point amplitudes times the $1'$ Pomeron propagator. It has a phase which implies peak support for the $C \to 2$ trajectories propagating through the interaction region a time T_{C*} of order $\alpha' E$ before the center of mass collision of A and B, independently of the value of ϕ, consistently with the longitudinal spreading predictions of [1, 2]. We will refer to this process in the kinematic regime well described by $1'$ exchange as 'case $1'$'.

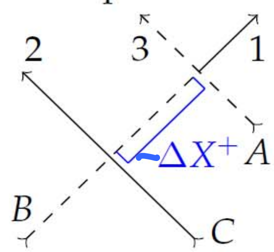
Light cone/Gross-Mende predict C and 1' interact at long range.

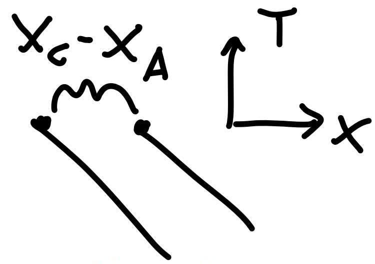
SIMULATING HORIZON PHYSICS IN FLAT SPACE

- We're interested in extracting the large longitudinal scale $\Delta X^+ \sim \alpha' E$ from some gauge-invariant quantity. The strategy is to set up a situation like in the black hole and compute $A(\Delta X^+)$,



- This is not an S-matrix element. But we can add a few auxiliary particles (dashed lines) to set up the above picture in an on-shell way:





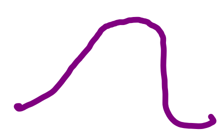
- This is a six-point function. Longitudinal spreading predicts a long range over which $A(\Delta X^+)$ is supported, $\Delta X^+ \sim \alpha' E_B$. Of course we need to make sure that there is no early collision between the dashed and solid lines.

<u>Calculable</u>

$$\langle 123 | e^{-iHT} | ABC \rangle$$

$$= \int DX \, e^{iS} \prod_{I=1}^{6} \psi_I [X(z_I)]$$

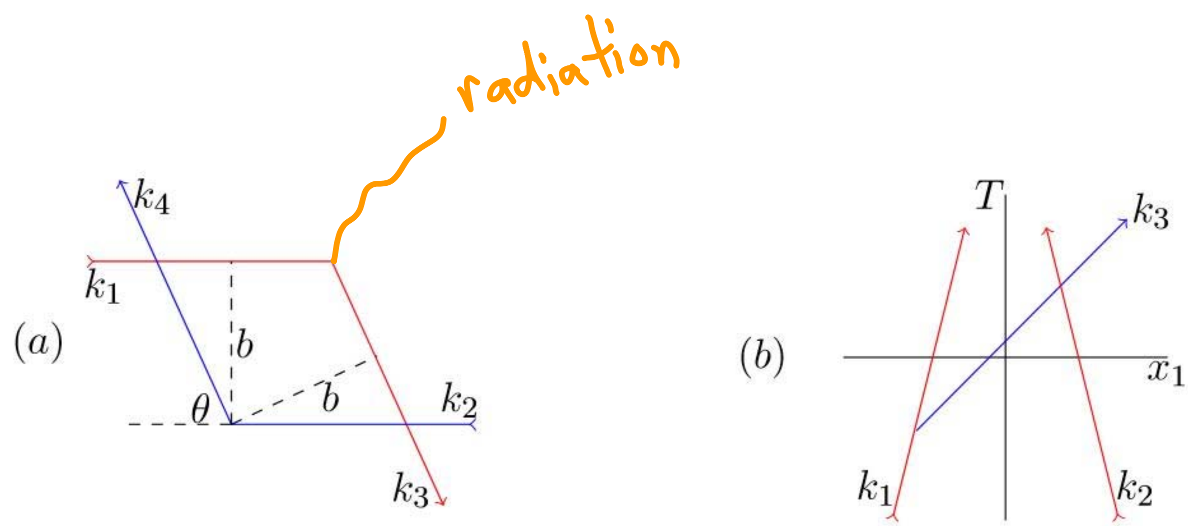
localized wave-packets



e.g. for $\tilde{X}_c - \tilde{X}_A \leftrightarrow \delta \tilde{p}_c - \delta \tilde{p}_A$

Previous S-matrix analysis at 4 & 5 points:

*Tracing back Gaussian wavepackets' peak trajectories (from phases), one finds that they meet, and assuming they do directly bend into each other (fits with peak trajectory of corresponding Bremsstrahlung and intermediate string solutions), and that scattering in fact includes contributions from these peaks, there is early interaction. Did not reveal the large ~Eα' scale, and depended on these assumptions.



(now also determining full profile)

At six points, we will find the long scale $E\alpha'$ appearing directly, in string theory but not tree-level QFT.

There are two main technical issues we will address explicitly in our setup:

1) Quantum Noise: The prediction ($X_{spr} \sim E\alpha'$) is in momentum space, while the geometry is in position space.
Can control with appropriate wavepackets (linear combos of on-shell vertex ops).
Also want to understand contributions from tails of wavefunctions (still in progress, but see below).

2) Systematics of the scattering experiment: which strings are actually interacting? Evade ambiguity using open string gauge invariance to exclude unwanted interactions.

Kinematics

$$k_a = (\omega_a, p_a, q_a), \qquad a = A, B, C$$
$$k_j = (-\omega_j, -p_j, -q_j), \qquad j = 1, 2, 3$$

$$\omega_I = +\sqrt{p_I^2 + q_I^2}, \qquad I = A - C, 1 - 3$$

Mandelstam Invariants

$$K_{IJ} = 2\alpha' k_I \cdot k_J$$

$E \sim \omega \simeq |p| \gg |q|$

$$k_{1'}^2 \simeq -4\omega_1(\underbrace{p_c}_{} - p_2) + O(q^2)$$ in a certain regime

☆ Strong dependence on detector $\omega_1 \sim E$

- $\delta \tilde{p}_c \sim -\delta \tilde{p}_A \longleftrightarrow \tilde{X} \equiv \tilde{X}_c - \tilde{X}_A$
- $A(\{K_{IJ}\}) = A(k_{1'}^2, \dots)$

The six point amplitude is not known in closed form, but it is tractable (a) near one pole and/or (b) in Regge limits where saddle points dominate. Open string ordering A3C21B (avoids direct BC joining interaction):

$$\mathcal{A}_{ST} = \frac{1}{K_{A3}} B(k_1'^2, K_{B1}) B(K_{C2}, K_{12})\, {}_3F_2(-K_{B2}, k_1'^2, K_{C2}; k_1'^2 + K_{B1}, K_{12} + K_{C2}; 1)$$

$$\frac{1}{K_{A3}} \sum_n \frac{(-K_{B2})_n}{n!} B(k_1'^2 + n, K_{B1}) B(K_{C2} + n, K_{12})$$

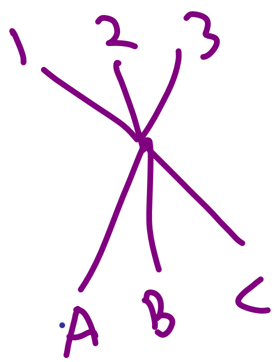
$K_{12}^{-K_{C2}}$

✓ Regge 4pt

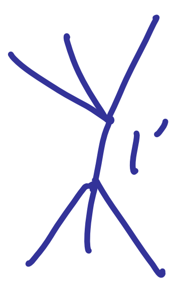
$$\frac{1}{K_{A3}} \frac{1}{K_{12}} \underbrace{B(k_1'^2, K_{B1} - 1)}_{\int} \quad \text{for} \quad K_{C2} = 1, \quad K_{12} + K_{C2} = -K_{B2}$$

$$\begin{aligned}
&\sim \quad \frac{1}{k_{1'}^2} (B1 - 1)^{-k_{1'}^2} \quad \text{for} \quad k_{1'}^2 \ll 1 \ll B1 - 1 \\
&\sim \quad (\frac{k_{1'}^2}{B1 - 1})^{k_{1'}^2} \quad \text{for} \quad 1 \ll k_{1'}^2 \ll B1 - 1 \\
&\sim \quad (\frac{B1 - 1}{k_{1'}^2})^{B1-1} \quad \text{for} \quad 1 \ll B1 - 1 \ll k_{1'}^2
\end{aligned}$$

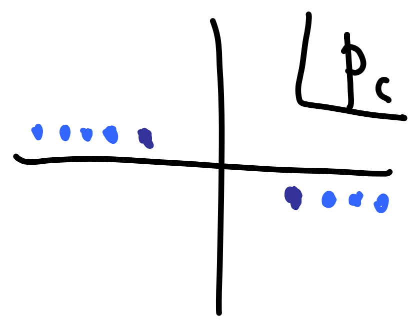
Will compare this to several tree-level QFT models to understand relevant distinctions:



$$A \sim i\lambda_6$$

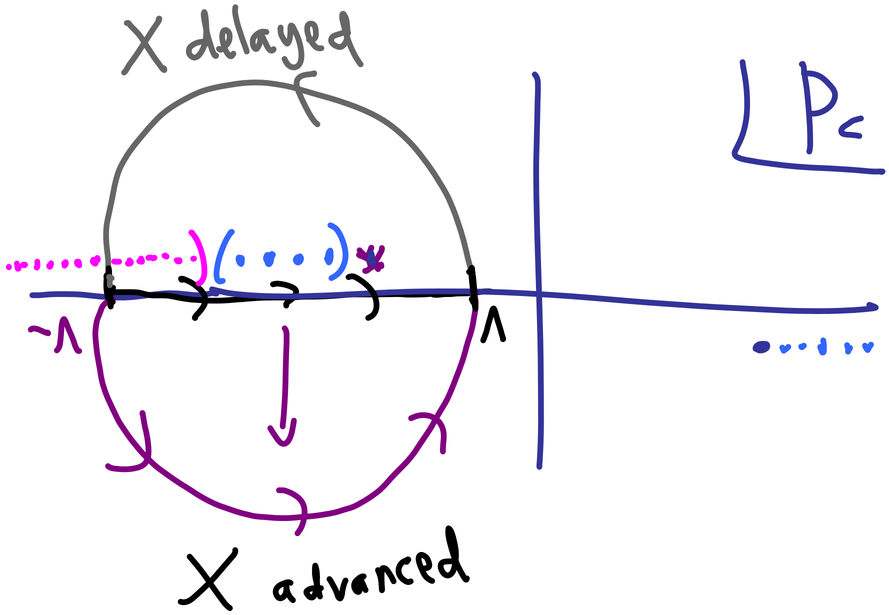


$$A \sim \frac{i}{k_{1'}^2 - i\varepsilon}$$



$$A \sim \sum_{m_j} \frac{i}{k_{1'}^2 + m_j - i\varepsilon}$$

Finite number of poles (same iε), not soft. ST contains EFT processes, but may have more. (Also true that string production *not* well approximated by summing up particle production for massive tower. cf Bachas, McAllister/Mitra, ES/Senatore/Zaldarriaga+Polchinski, Kleban et al, Puhm Rojas Ugajin...)



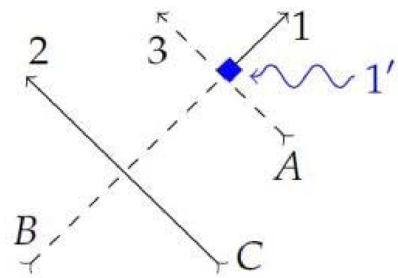
(Full Fourier transform has $x \gtrsim 0$ from both directions of motion $\Rightarrow$ no sharp step ftn.)

$$\int_{-\Lambda}^{\Lambda} d\delta p_c \, \frac{e^{-i\delta p_c X}}{\delta p_c - i\varepsilon} \, \theta(-X)$$

$$+ \, \theta(+X)\left(\text{poles} + \frown\right)$$

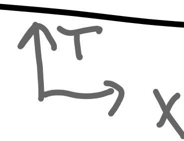
In tree-level point particle scattering, the early contribution must be from a tail of the wavefunction (QFT has commutators of local ops). The range of pC integration is dictated partly by wavefunction and partly by amplitude (via stress-energy conservation). [The late contribution from the string of poles is also calculably spread out in ST]

- For the black hole application, we are particularly interested in the case where $1'$ is produced by the AB collision,

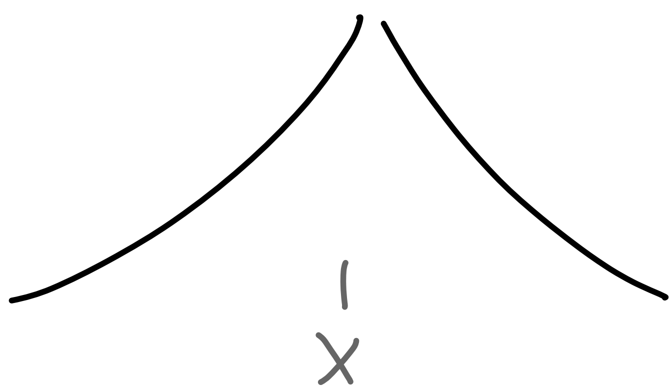


Direct BC interaction is impossible in this open string ordering A3C21B (also in comparison QFTs above).

Wavefunction e.g. $\dot{c}$ $\dot{A}$



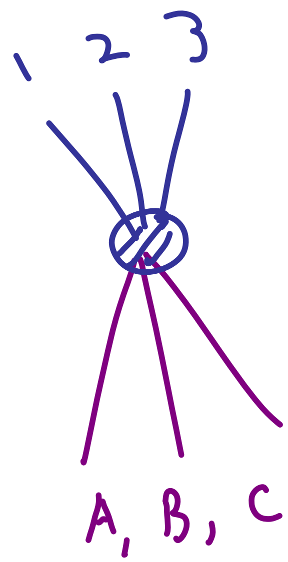
$$\psi_{X \equiv X_c - X_A}(\check{X})$$



e.g. $$\psi_X(\check{X}) = \frac{i}{\check{X} - X + T} - \frac{i}{\check{X} - X - T} + \delta(\check{X} - X + T) + \delta(\check{X} - X - T)$$

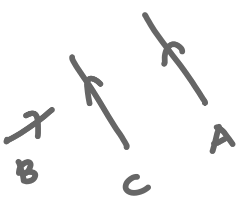
$$\psi_X(\check{p}_c) = e^{-iX\check{p}_c}$$

Energy-momentum conservation



$k_{1,2,3}$ fixed $\Rightarrow \omega_{Tot},\ \underline{P},\ Q \equiv 0$

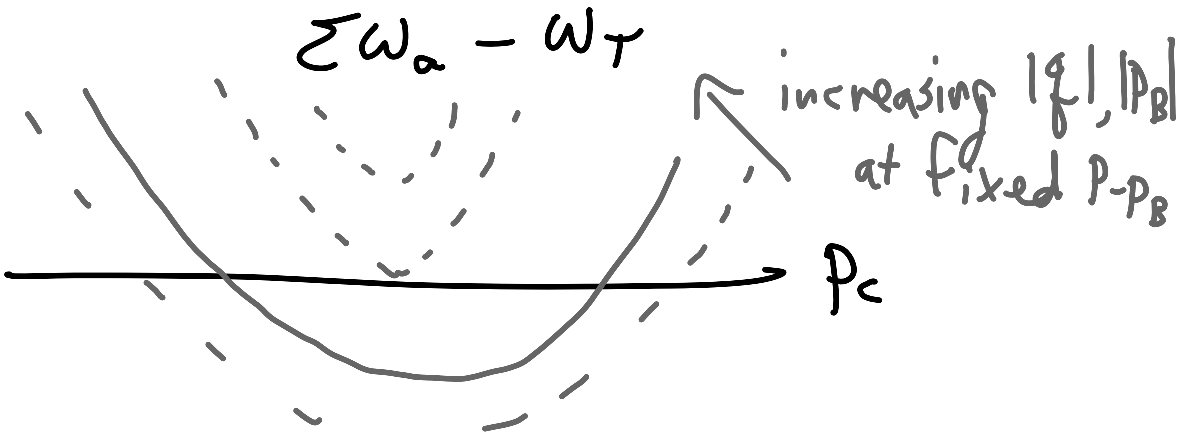
$$\left.\begin{aligned} q_B = 0, \quad \tilde{q}_A &= -\tilde{q}_C \\ \tilde{p}_A &= \underline{P} - \tilde{p}_B - \tilde{p}_C \end{aligned}\right\} \text{mom cons.}$$



$$\delta\left(\tilde{\omega}_A + \tilde{\omega}_B + \tilde{\omega}_C - \omega_T\right)$$

$$\|$$

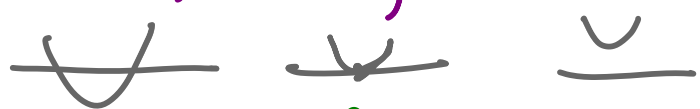
$$\sqrt{(P - \tilde{p}_B - \tilde{p}_C)^2 + \tilde{q}^2} + |\tilde{p}_B| + \sqrt{p_c^2 + \tilde{q}^2}$$



for range of p_B & q supported in ψ

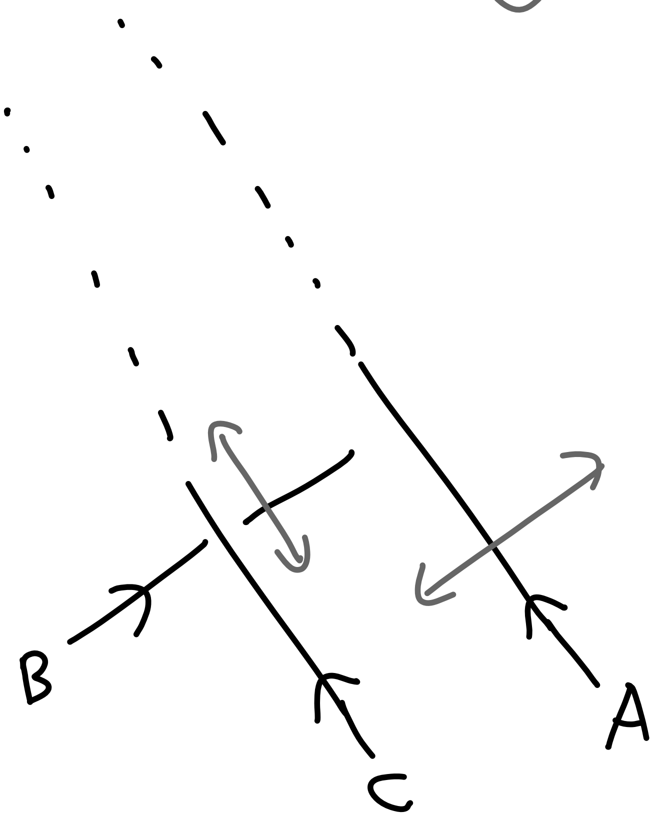
$$A(x) = \int \prod d\tilde{p}\, \psi_x(\tilde{p}) \underbrace{A(\tilde{p})}_{\delta\left(\sum \tilde{\omega} - \omega_{Tot}\right)\hat{A}}$$

First consider momentum eigenstates for q, p_B $\Rightarrow$ 2, 1, or 0 sol'ns for p_C

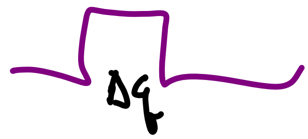


ABC intersection goes to ∞:

$p_A \to p_C = \dfrac{p - p_B}{2}$

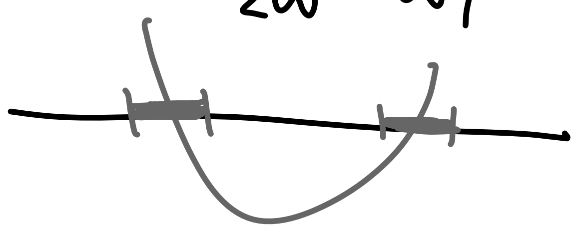


Next consider $\psi(\tilde{q})$

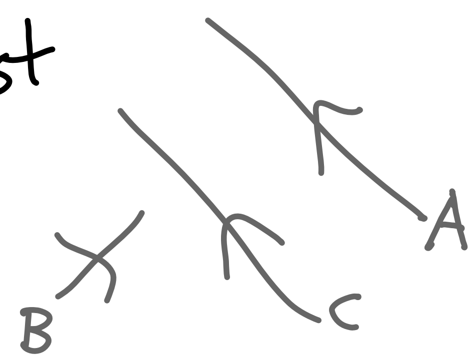


$$\psi(\tilde{p}_B) = \delta(\tilde{p}_B - p_B)$$

and again $\psi_x(\tilde{p}_c) = e^{-ix\tilde{p}_c}$



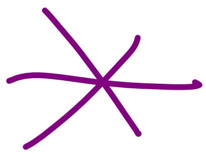
Wavefunction supported for all $\tilde{p}_c$,
but $A \propto \delta(\Sigma\tilde{\omega} - \omega_T)$ supports
small interval $\Delta p_c << |p_c|$
within kinematics of interest
(and simple form of ST
amplitude)



$$A(X) = \frac{\int d\tilde{p}_c \, e^{-i\delta\tilde{p}_c X}}{\left\{ \left| \frac{\partial \sum \tilde{\omega}}{\partial q} \right|_{q_*(\tilde{p}_c, p_B)} \right|} \cdot \mathcal{A}$$

(varies weakly)

$$\mathcal{A} \uparrow \text{QFT vs ST}$$

QFT_0  $\mathcal{A} \sim \lambda_6$

⋮

ST $\quad \mathcal{A} \sim \frac{1}{k_{A3}} k_{12}^{-k_{c2}} \, B(\underbrace{k_{1'}^2, k_{B1} - k_{c2}}_{\sim e^{-k_{1'}^2 \log k_{B1}}})$

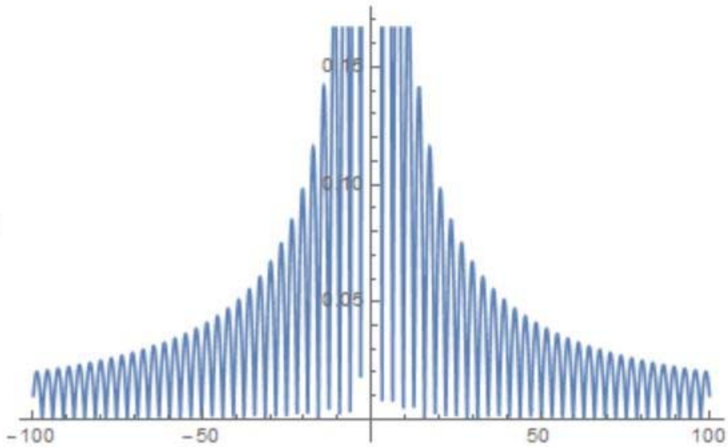
$$\left\{ \begin{array}{l} \int_{-\Delta p_c}^{\Delta p_c} d\delta\tilde{p}_c \, e^{-i\delta\tilde{p}_c X} \\ \int_{-\Delta p_c}^{\Delta p_c} d\delta\tilde{p}_c \, e^{-i\delta\tilde{p}_c (X + i X_{max})} \end{array} \right.$$

$$X_{max} \sim \epsilon \log k_{B1}$$

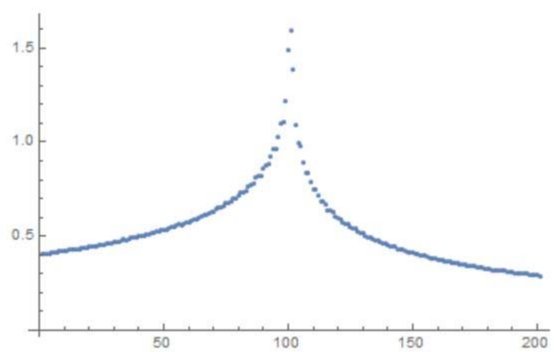
Resulting |A(X)| for 3 QFT cases

Relatively sharply peaked, rapidly varying near origin, well within the Eα' scale.

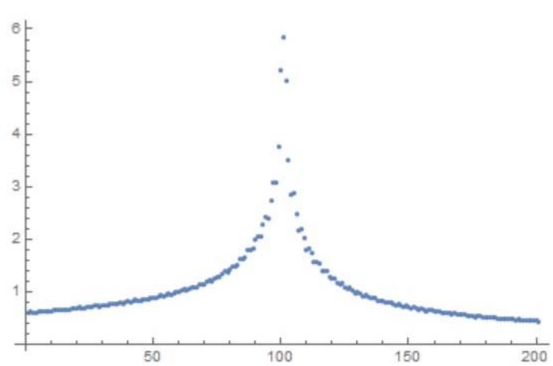
```
Plot[Abs[2 Sin[Xn] / Xn], {Xn, -100, 100}]
```




```
Table[
  Abs[NIntegrate[Exp[-I Xn deltPn] (1 / ((40 (deltPn + (1))) + (1 / 10) - (I / 10)))
     Sqrt[(10 - deltPn) (10 + deltPn)], {deltPn, -1, 1}, WorkingPrecision → 20]], {Xn, -100, 100}];
ListPlot[NewintQFT2, PlotRange → All]
```

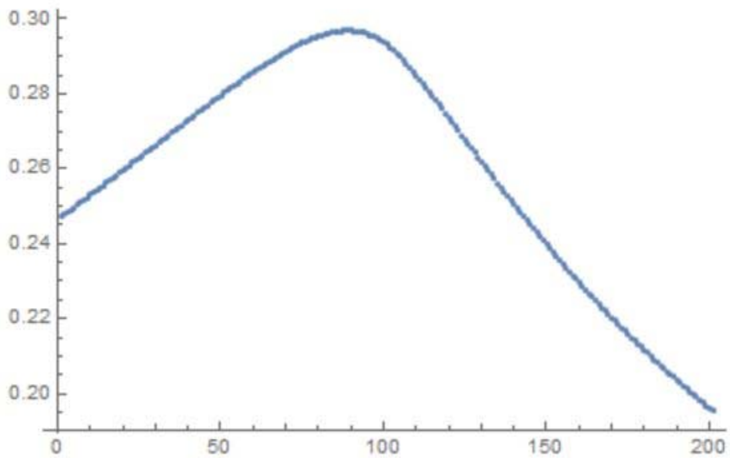



```
fmassive2[x_] := Sum[1 / ((40 (x + 1)) + (1 / 10) - (I / 10) + n), {n, 0, 5}]
NewintQFTmassive =
  Table[Abs[NIntegrate[Exp[-I Xn deltPn] fmassive2[deltPn] Sqrt[(10 - deltPn) (10 + deltPn)],
     {deltPn, -1, 1}, WorkingPrecision → 20]], {Xn, -100, 100}];
ListPlot[NewintQFTmassive, PlotRange → All]
```



Resulting |A(X)| for string theory

```
NewintST2 =
  Table[
   Abs[NIntegrate[Exp[-I Xn deltPn] Beta[(40 (deltPn + (1))) + (1/10) - (I/10), 5]
      Sqrt[(10 - deltPn) (10 + deltPn)], {deltPn, -1, 1}]], {Xn, -100, 100}];
ListPlot[NewintST2]
```



This is nearly constant over the predicted longitudinal spreading scale ~Eα', in sharp contrast to the tree-level QFT models.

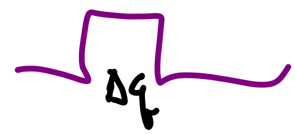
Note this X is peak of wavefunction Ψ, not necessarily the scattering position. Regardless, at this level it shows clear contrast with QFT in longitudinal direction. Accords with basic intuition: UV softness-> spreading of probability in X.

These results can be understood very simply from

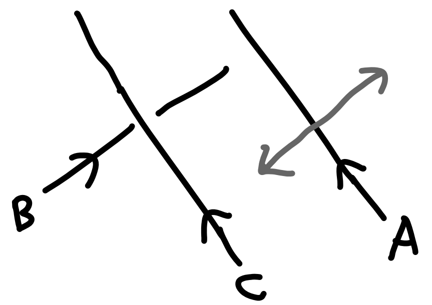
$$A(X) \begin{cases} e^{-ip_{c0}X} \displaystyle\int_{-\Delta p_c}^{\Delta p_c} d\delta\tilde{p}_c \, e^{-i\delta\tilde{p}_c X} & \text{QFT} \\[2em] e^{-ip_{c0}X} \displaystyle\int_{-\Delta p_c}^{\Delta p_c} d\delta\tilde{p}_c \, e^{-i\delta\tilde{p}_c (X + i \underset{\uparrow \atop \sim \epsilon \log K_{B1}}{X_{max}})} & \text{ST} \end{cases}$$

$$\sim \begin{cases} \dfrac{\sin \Delta p_c X}{X} \\[2em] \dfrac{\sin \Delta p_c (X + i X_{max})}{X + i X_{max}} \sim \dfrac{e^{\Delta p_c X_{max}} \, e^{i\Delta p_c X}}{X + i X_{max}} \end{cases}$$

Finally consider $\psi(\tilde{q})$



$\psi(\tilde{p}_B) =$



↳ suppresses ABC intersection, hence QFT amplitude

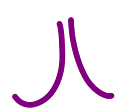
and again $\psi_X(\tilde{p}_c) = e^{-iX\tilde{p}_c}$

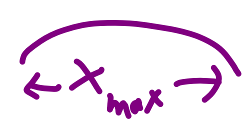
$$\int \underline{d\tilde{p}_B} \; A_{\left\{\substack{QFT \\ ST}\right.}\left(X \,;\, p_{c0}(\tilde{p}_B), \delta\tilde{p}_c(\tilde{p}_B)\right)$$

$$e^{\underline{\delta\tilde{p}_B}\, c(iX)} \quad \text{vs} \quad e^{\underline{\delta\tilde{p}_B}\, c(iX + X_{max})}$$

↑ oscillates away for $X \neq 0$

↖ oscillates away only for $|X| \gg X_{max}$





(To be finished ...)

Summary

Applied to black hole, longitudinal spreading implies that a late infalling system can sense early infaller (e.g. matter that formed black hole) in a way that goes beyond effective QFT and GR. Beyond-GR physics from intrinsic string-theoretic non-locality (open question: what about QCD...?) could exploit large energy built up in system of early and late infallers near horizon of black hole. Ongoing work to analyze and apply this to to to thought (and real??) experiments.