Towards Higher Spin bulk reconstruction from thermal O(N) correlators

Irene Amado

Stockholm U.

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W/ B. Sundborg, L. Thorlacius & N. Wintergerst, *coming soon...*

Motivation

Reconstruction of bulk physics in AdS/CFT

- Black holes $\longleftrightarrow$ Large T phases of large N gauge theories

  - Hawking-Page & confinement-deconf. in strongly coupled N=4 SYM

    [Witten '98]

  - Hagedorn-deconfinement in weakly coupled large N gauge th.

    [Sundborg '99]
    [Aharony et al. '03]

- Use weakly coupled bdry th. to probe bulk.

- Singlet sector of O(N) vector model $\longleftrightarrow$ Higher Spin theory in AdS

  [Klebanov,Polyakov '02]

Motivation

- Singlet sector of O(N) vector model: phase transition at finite T

[Shenker,Yin '11]

low T phase

higher spin gas in AdS

[Shenker,Yin '11]

high T phase

hints of Sch-BH

(evanescent modes)

[Jevicki et al. '15]

- Bulk reconstruction from correlators: learn about HS high T phase, geometrical interpretation: HS BH ??

Outline

- O(N) model & phase transition
- Thermal correlators
- Further interpretation

Free O(N) model

- O(N) gauge theory on $S^2 x S^1$ and N_f fundamental scalars

$$S = \int \frac{k}{2\pi}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) + |D\phi|^2 - \frac{1}{4}|\phi|^2$$

- Free limit $k \to \infty$ ⟶ singlet constraint

$$S_{\rm eff}^0 = -\sum_{i<j} 2\ln|\sin\left(\frac{\lambda_i-\lambda_j}{2}\right)| - 2N_f\sum_{k=1}^{\infty}\frac{1}{k}z_S(x^k)\sum_i \cos(k\lambda_i)$$

eigenvalues of the gauge holonomy

$$z_S(x) = x^{\frac{1}{2}}\frac{1+x}{(1-x)^2}, \quad x = e^{-\beta}$$

$$\boxed{\begin{aligned} Z[\beta] = \frac{1}{N!}\int \mathcal{D}\lambda \exp\Bigg[&N^2\int d\lambda\, d\lambda'\rho(\lambda)\rho(\lambda')\ln\left|\sin\left(\frac{\lambda-\lambda'}{2}\right)\right| \\ &+ 2NN_f\int d\lambda\rho(\lambda)\sum_{k=1}^{\infty}\frac{1}{k}z_S(x^k)\cos(k\lambda)\Bigg] \end{aligned}}$$

Gross-Witten phase transition

- Saddle point: $\rho(\lambda) = \frac{1}{2\pi} + \frac{N_f}{N}\tilde{\rho}(\lambda)$; $\tilde{\rho}(\lambda) = \sum_{k=1}^{\infty} z_S(x^k)\frac{1}{\pi}\cos(k\lambda)$

- At large T: $z_S(x^k) \sim 2\frac{T^2}{k^2}$ $\longrightarrow$ $\tilde{\rho}(\lambda) = T^2\left(-\frac{\pi}{6} + \frac{(|\lambda|-\pi)^2}{2\pi}\right)$

- Phase transition: $\rho(\pm\pi) = 0$ $\longrightarrow$ $T = T_c \equiv \frac{\sqrt{3}}{\pi}\sqrt{\frac{N}{N_f}}$

T< Tc → $\rho(\lambda) \neq 0 \quad \forall\lambda$

T>Tc → $\rho(\lambda > \lambda_m) = 0 \qquad \rho(\lambda_m) = 0\,; \quad \lambda_m \to \frac{\pi}{\sqrt{3}}\frac{\beta}{\beta_c} \quad \text{for } \beta \to 0$

Free energy

$$F = \log Z(\beta, 0) \approx \begin{cases} \frac{4N_f^2 \zeta(5)}{\beta^4} & \text{for } T \lesssim T_c \\ \frac{4N_f N \zeta(3)}{\beta^2} & \text{for } T \gg T_c \end{cases}$$

3rd order PT: [Shenker,Yin '11]

$$F_{HT} = F_{LT} + O(\delta\beta^3)$$

Low T phase = Higher spin gas in AdS

High T phase: dual description? $\longrightarrow$ BH background or other geometry?

Thermal correlators

- Source in generating function, scalar action: $\delta S_\phi = \int J\phi + J^\dagger \phi^\dagger$

$$S_{\text{eff}} = S^0_{\text{eff}} + \beta^{-1} \sum_{i,n,l,m} \frac{\left| j^\dagger_{lmn} \cdot \Psi^i \right|^2}{\beta^{-2}\left(2\pi n + \lambda_i\right)^2 + \frac{1}{4}(2l+1)^2}$$

$$\begin{aligned} Z[\beta, J] = \frac{1}{N!} \int \mathcal{D}\lambda \exp \Bigg[& N^2 \int d\lambda d\lambda' \rho(\lambda)\rho(\lambda') \ln \left| \sin\left(\frac{\lambda - \lambda'}{2}\right)\right| \\ & + 2N N_f \int d\lambda \rho(\lambda) \sum_{k=1}^{\infty} \frac{1}{k} z_S(x^k) \cos(k\lambda) \\ & + \beta^{-1} N \int d\lambda \rho(\lambda) \sum_{n,l,m} \frac{\left| j^\dagger_{lmn} \cdot \Psi(\lambda)\right|^2}{\beta^{-2}\left(2\pi n + \lambda\right)^2 + \frac{1}{4}(2l+1)^2} \Bigg] \end{aligned}$$

1-point function of bi-local operators

$$\left\langle \mathrm{Tr}\,\phi(0)^\dagger \phi(y) \right\rangle = \frac{1}{2\sqrt{\pi}} Z[\beta,0]^{-1} \sum_{lmn} \sum_{\bar{l}\bar{n}} \mathrm{Tr}\, \frac{\delta}{\delta j_{\bar{l}0\bar{n}}} \frac{\delta}{\delta j^\dagger_{lmn}} Z[\beta, J] \Big|_{j=0} Y_{\bar{l},0}(0)\, Y_{l,m}(y)$$

- On the saddle point:

$$\boxed{\left\langle \mathrm{Tr}\,\phi(0)^\dagger \phi(y) \right\rangle = \frac{N}{4\pi} \frac{1}{\sqrt{2(1-\cos\theta)}} + \frac{N}{2\pi} \sum_{n=1}^{\infty} \frac{\rho_n}{\sqrt{2}\sqrt{\cosh n\beta - \cos\theta}}}$$

- Momenta of eigenvalue distribution: $\rho_n \equiv \int d\lambda\, \rho(\lambda)\, \cos(n\lambda)$

$$\rho_n = \begin{cases} \frac{N_f}{N} z_S(x^n) \sim \frac{N_f}{N} \frac{2}{n^2\beta^2} & \text{for } T \leq T_c \\ \frac{N_f}{N} \frac{2}{\pi\beta^2} \frac{1}{n^3} \left(n\pi - n(\pi - \lambda_m) \cos n\lambda_m - \sin n\lambda_m \right) & \text{for } T > T_c \end{cases}$$

$$\rho(\lambda_m) = 0\,; \quad \lambda_m \to \frac{\pi}{\sqrt{3}} \frac{\beta}{\beta_c} \quad \text{for } \beta \to 0$$

Low energy phase: 1<<T<Tc

$$\langle \mathrm{Tr}\,\phi^\dagger(0)\phi(y)\rangle_T = \frac{N_f}{\pi\beta^2}\sum_{n=1}^{\infty}\frac{1}{n^2\sqrt{2(\cosh n\beta - \cos\theta)}}$$

- Short distance $\theta \ll \beta$:

$$\langle \mathrm{Tr}\,\phi(0)^\dagger\phi(y)\rangle = \begin{cases} \boxed{\frac{N}{4\pi\theta}} + \frac{N_f e^{-\beta}}{2\pi} & \text{for } \beta \gg 1 \\ \boxed{\frac{N}{4\pi\theta}} + \frac{N_f}{\pi\beta^3}\zeta(3) & \text{for } \beta \ll 1 \end{cases}$$

- Long distance $\theta \gg \beta$:

$$\langle \mathrm{Tr}\,\phi(0)^\dagger\phi(y)\rangle = \frac{N}{4\pi}\left(1 + 2\frac{\beta_c^2}{\beta^2}\right)\boxed{\frac{1}{\sqrt{2(1-\cos\theta)}}}$$

High Temperature phase: T>Tc

$$\left\langle \mathrm{Tr}\,\phi^\dagger(0)\phi(y)\right\rangle_T = \frac{N_f}{\pi^2\beta^2}\sum_{n=1}^{\infty}\frac{\pi n - n(\pi-\lambda_m)\cos n\lambda_m - \sin n\lambda_m}{n^3\sqrt{2(\cosh n\beta - \cos\theta)}}$$

- Short distance $\theta \ll \beta$:

$$\left\langle \mathrm{Tr}\,\phi(0)^\dagger\phi(y)\right\rangle = \begin{cases} \frac{N}{4\pi\theta} + \frac{N_f}{\pi\beta^3}\zeta(3) & \text{for } \beta \lesssim \beta_c \\ \frac{N}{4\pi\theta} - \frac{N}{2\pi\beta}\log\beta & \text{for } \beta \ll \beta_c \end{cases}$$

- Long distance $\theta \gg \beta$: for $n\lambda_m \ll 1 \quad \rightarrow \quad \rho_n \sim \lambda_m^2$

$\frac{1}{\sqrt{N}} \gg \theta \gg \beta$: different suppression for different modes

$\theta \gg \frac{1}{\sqrt{N}} > \beta$: large n modes highly suppressed as for T<Tc

High Temperature phase: T>Tc

$$\langle \mathrm{Tr}\, \phi^\dagger(0)\phi(y)\rangle_T = \frac{N_f}{\pi^2\beta^2}\sum_{n=1}^{\infty}\frac{\pi n - n(\pi-\lambda_m)\cos n\lambda_m - \sin n\lambda_m}{n^3\sqrt{2(\cosh n\beta - \cos\theta)}}$$

- Short distance $\theta \ll \beta$:

$$\langle \mathrm{Tr}\, \phi(0)^\dagger\phi(y)\rangle = \begin{cases} \boxed{\frac{N}{4\pi\theta}} + \frac{N_f}{\pi\beta^3}\zeta(3) & \text{for } \beta \lesssim \beta_c \\ \boxed{\frac{N}{4\pi\theta}} - \frac{N}{2\pi\beta}\log\beta & \text{for } \beta \ll \beta_c \end{cases}$$

- Long distance $\theta \gg \beta$:

$$\boxed{\frac{1}{\sqrt{N}} \gg \theta \gg \beta \ : \ \langle \mathrm{Tr}\, \phi(0)^\dagger\phi(y)\rangle = -\frac{N}{2\pi\beta}\log\theta}$$

$$\theta \gg \frac{1}{\sqrt{N}} > \beta \ : \ \langle \mathrm{Tr}\, \phi(0)^\dagger\phi(y)\rangle = \boxed{\frac{N}{4\pi\sqrt{2(1-\cos\theta)}}}\left(2\sqrt{3}\frac{\beta_c}{\beta} - 1\right)$$

Unconstrained theory:

No singlet constraint imposed: $\rho_n = 1$

$$\langle \mathrm{Tr}\, \phi^\dagger(0)\phi(y)\rangle_T = \frac{N}{2\pi}\sum_{n=1}^{\infty}\frac{1}{\sqrt{2(\cosh n\beta - \cos\theta)}}$$

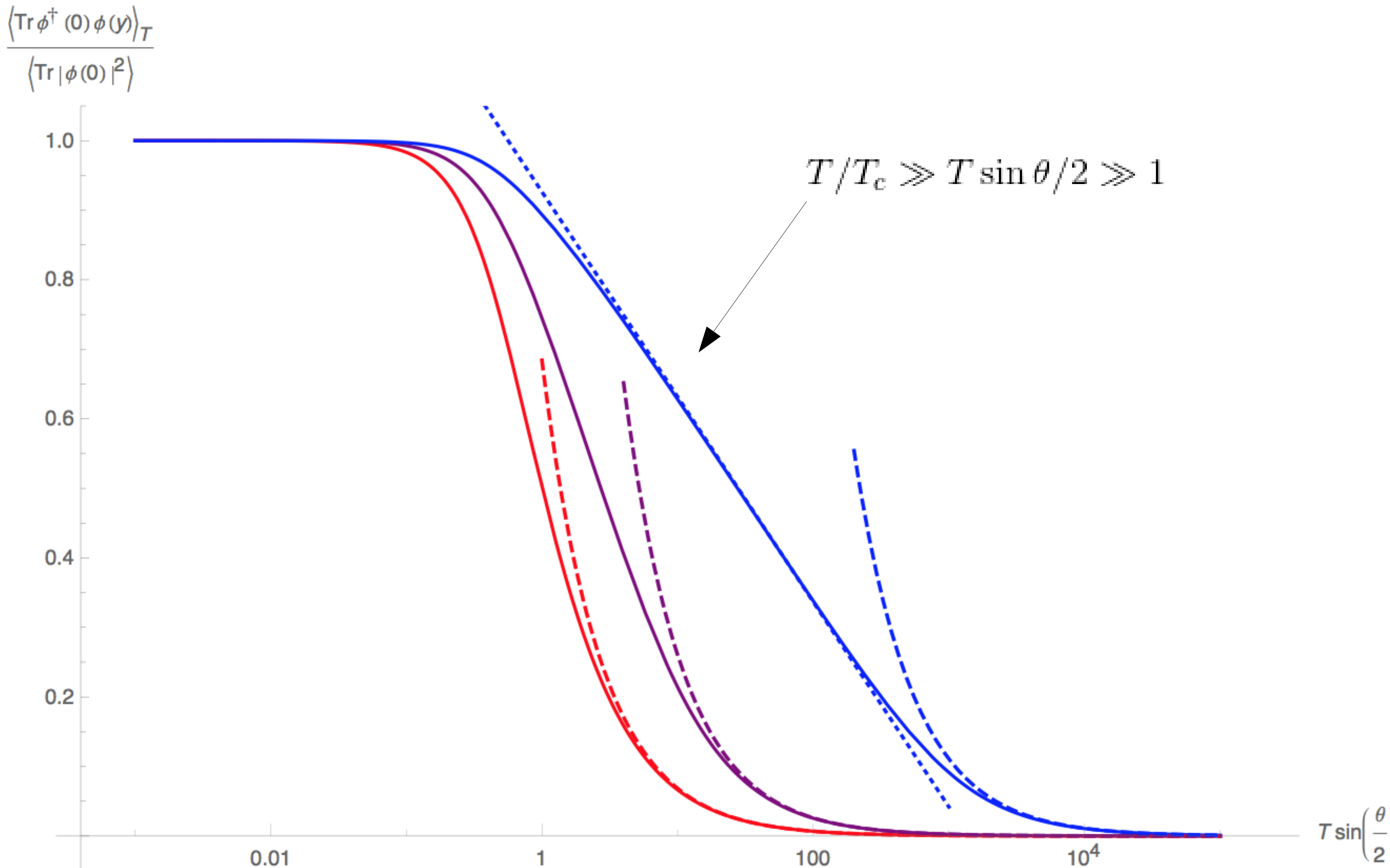
- Short distance $\theta \ll \beta$:

$$\langle \mathrm{Tr}\, \phi(0)^\dagger \phi(y)\rangle = \begin{cases} \frac{N}{4\pi\theta} + \frac{N_f e^{-\beta/2}}{2\pi} & \text{for } \beta \gg 1 \\ \frac{N}{4\pi\theta} - \frac{N}{2\pi\beta}\log\beta & \text{for } \beta \ll 1 \end{cases}$$

- Long distance $\theta \gg \beta$:

$$\boxed{\langle \mathrm{Tr}\, \phi(0)^\dagger \phi(y)\rangle = -\frac{N}{2\pi\beta}\log\theta}$$

1-point function of bi-local ops



2-point function of local operators

$$\left\langle \mathrm{Tr}\,|\phi(0)|^2\,\mathrm{Tr}\,|\phi(y)|^2 \right\rangle = \frac{1}{4\pi}\frac{1}{\sqrt{2(1-\cos\theta)}}\left\langle \mathrm{Tr}\,\phi(0)^\dagger\phi(y)\right\rangle + \frac{N}{16\pi^2}\sum_{n_1 n_2}\frac{\rho_{n_1-n_2}+\rho_{n_1+n_2}}{\sqrt{(\cosh n_1\beta-\cos\theta)(\cosh n_2\beta-\cos\theta)}}$$

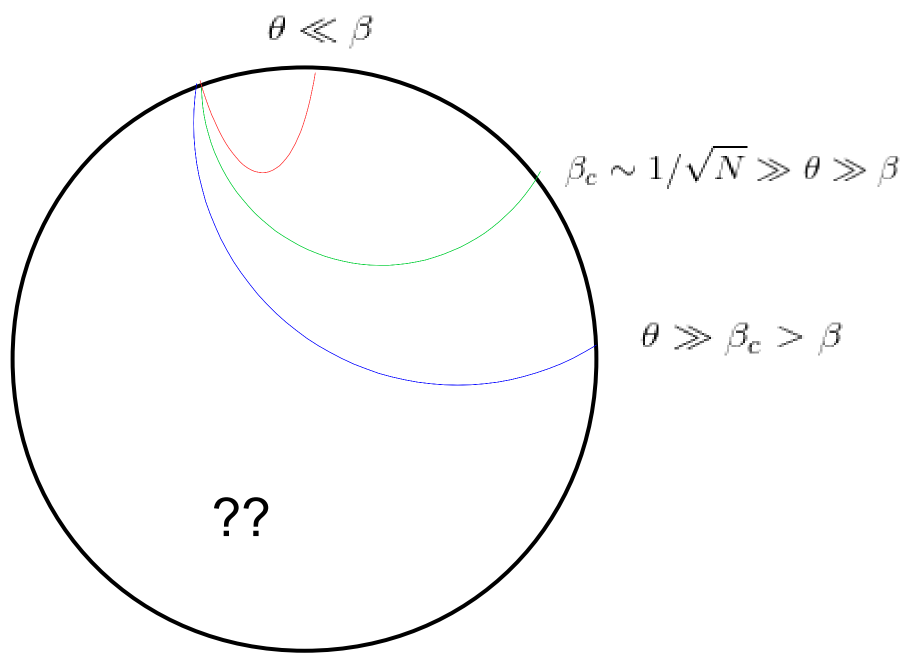
with $\rho_{n_1-n_2}=1$ for $n_1=n_2$

2-point function of local operators

$\langle \mathrm{Tr}\lvert\phi(0)\rvert^2 \mathrm{Tr}\lvert\phi(y)\rvert^2 \rangle$	*Free*	$T \lesssim T_c$	$T \gg T_c$
$\theta \ll \beta$	$\frac{1}{\theta^2} - \frac{2}{\theta\beta}\log\beta$	$\frac{1}{\theta^2} + \frac{12\beta_c^2}{\pi^2\beta^3}\frac{\zeta(3)}{\theta}$	$\frac{1}{\theta^2} - \frac{2}{\theta\beta}\log\beta$
$\frac{1}{\sqrt{N}} \gg \theta \gg \beta$	$\boxed{\frac{1}{\beta^2}(\log\theta)^2}$	$\frac{\pi-\theta}{\beta\sin\theta}\left(1 + 2\frac{\beta_c^2}{\beta^2}\right)$	$\boxed{\frac{1}{\beta^2}(\log\theta)^2}$
$\theta \gg \frac{1}{\sqrt{N}} > \beta$			$\frac{\pi-\theta}{\beta\sin\theta}\left(2\sqrt{3}\frac{\beta_c}{\beta} - 1\right)$

Bulk reconstruction

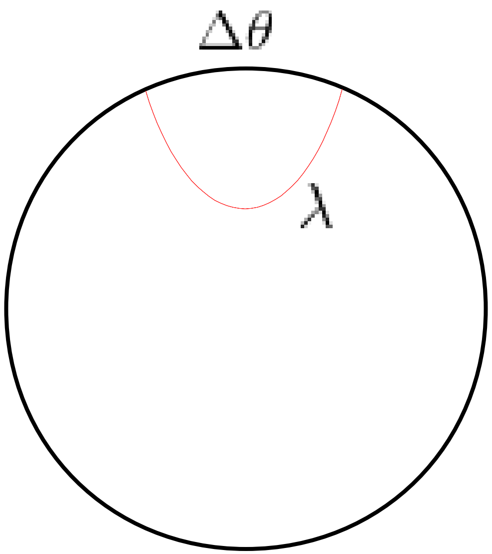
- Construct geometric model s.t. reproduces the bdry thermal correlators



- In particular: characteristic logarithmic behavior for $\beta_c \sim 1/\sqrt{N} \gg \theta \gg \beta$

Bulk reconstruction

- Consider fixed time geodesics in vacuum AdS



$$ds^2 = -(u+1)dt^2 + \frac{du^2}{4u(u+1)} + u\,d\theta^2 + u\,\sin^2\theta d\Omega_{d-2}^2$$

$$\begin{cases} \dot{\theta} = \frac{P_\theta}{u} \\ 1 = \frac{\dot{u}^2}{4u(u+1)} + \frac{P_\theta^2}{u} \end{cases} \rightarrow \qquad d\lambda = \frac{du}{2\sqrt{u+1}\sqrt{u-P_\theta^2}}$$

$$\frac{d\theta}{du} = \frac{\dot{\theta}}{\dot{u}} \qquad \rightarrow \qquad \frac{\Delta\theta}{2} = \lim_{u\to\infty}\theta(u) = \arctan\frac{1}{P_\theta}$$

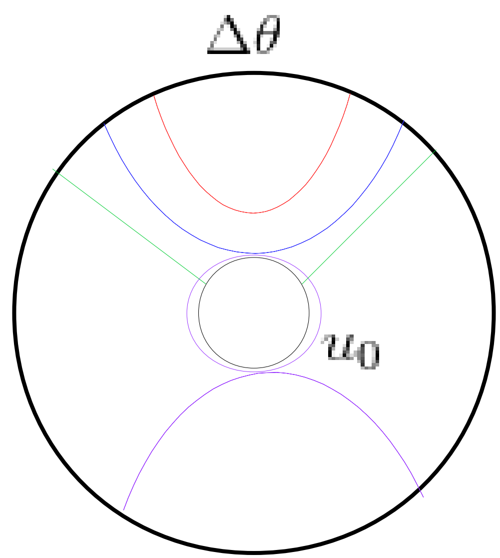
- Regulated length: $L = 2\lim_{u\to\infty}\left(\lambda(u,P_\theta) - \lambda(u,0)\right) = -\log(1+P_\theta^2)$

$$G \sim e^{-L} = 1 + P_\theta^2 = \frac{1}{\sin^2(\Delta\theta/2)}$$

~ low T correlators

Bulk reconstruction

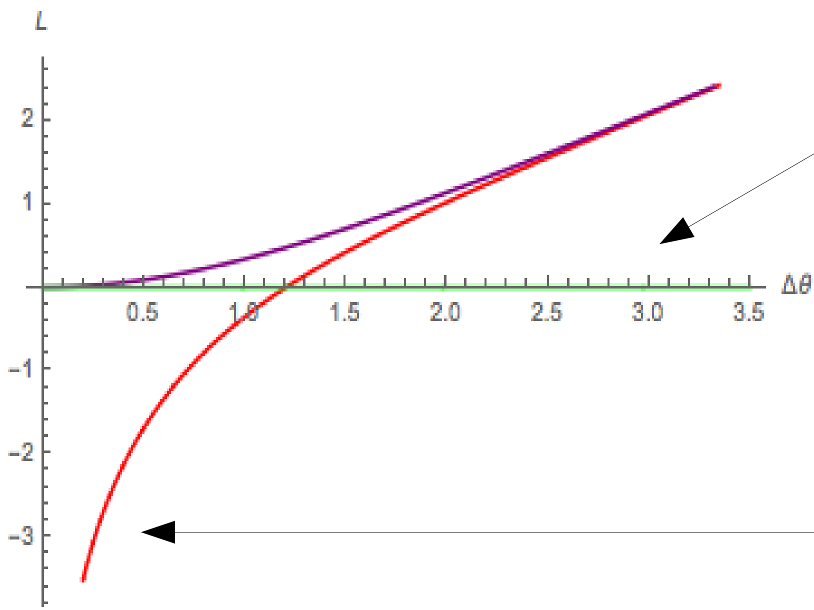
- Consider fixed time geodesics in AdS w/ blackening factor



$$ds^2 = -(u-u_0)f(u)dt^2 + \frac{du^2}{4u(u-u_0)f(u)} + ud\theta^2 + \dots$$

$$\begin{cases} \dot{\theta} = \frac{P_\theta}{u} \\ 1 = \frac{\dot{u}^2}{4u(u-u_0)f(u)} + \frac{P_\theta^2}{u} \end{cases} \rightarrow \quad d\lambda = \frac{du}{2\sqrt{f(u)}\sqrt{(u-u_0)(u-P_\theta^2)}}$$

$$\frac{d\theta}{du} = \frac{P_\theta}{2u\sqrt{f(u)(u-u_0)(u-P_\theta^2)}}$$



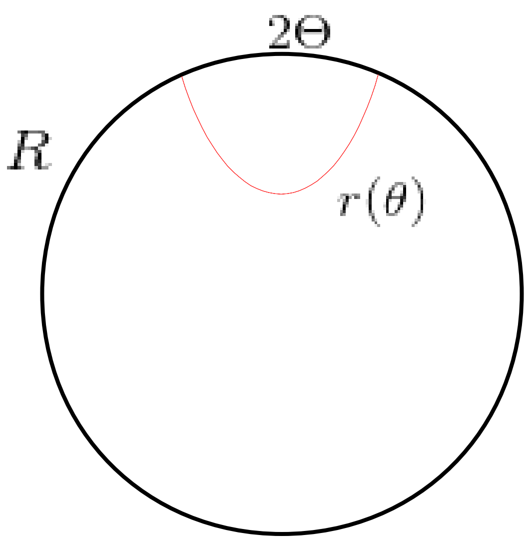
For large separation: no angle dep.

No obvious logarithmic regime...

For short separation ~ vacuum

Bulk reconstruction

- Use HJ to infer metric: geodesics in the disk



$$ds^2 = h(r)^2 dr^2 + r^2 d\theta^2$$

$$g(\Theta)^{-2} = S(\Theta, R) = \int_0^{\Theta} d\theta \sqrt{h(r)^2 \dot{r}^2 + r^2}$$

Hamilton-Jacobi

$$0 = H + \frac{\partial S}{\partial \Theta} \rightarrow 0 = r^2 - \frac{r^2}{h(r)} \left(\frac{\partial S}{\partial r}\right)^2 - \left(\frac{\partial S}{\partial \Theta}\right)^2$$

Method of characteristics

$$\int_{p_\Theta}^{R} dr\, h(r) = S(p_\Theta, 0) = \int_{p_\Theta}^{R} \frac{dr}{r} \left(\sqrt{r^2 - p_\Theta^2} + \frac{p_\Theta^2}{\sqrt{r^2 - p_\Theta^2}} \right) h(r) + g(\Theta)^{-2}$$

- Look for *h(r)* s.t. correlators behave as for the high T phase

Conclusions

- We know the behavior of bdry correlators as a function of T and separation
- T>Tc: new scale, same behavior as the free theory
- At larger distance: back to the low T behavior
- Next step: extract effective geometry
- Large object in the bulk localized?